Intermediate Algebra I

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- Extracted the three chapters from *Introductory Algebra* (BC Campus) by Izabela Mazur to create a separate book.
- Adjusted formatting and textboxes so the content displays well in printed format.
- Fixed some errors

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Contents

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PART I

CHAPTER 3 RATIO, PROPORTION, AND PERCENT

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When you apply for a mortgage, the loan officer will compare your total debt to your total income to decide if you qualify for the loan. This comparison is called the debt-to-income ratio. A ratio compares two quantities that are measured with the same unit. If we compare a and b, the ratio is written as $a \text{ to } b,\ \frac{a}{b},\ \text{or a:b}.$

3.1 Ratios and Rate

Learning Objectives

By the end of this section, you will be able to:

- Write a ratio as a fraction
- Find unit rates
- Find unit price
- Translate phrases to expressions with fractions

Write a Ratio as a Fraction

Ratios

A ratio compares two numbers or two quantities that are measured with the same unit. The ratio of a to b is written $a \text{ to } b,\ \frac{a}{b},\ \text{or a:b}.$

In this section, we will use the fraction notation. When a ratio is written in fraction form, the fraction should be simplified. If it is an improper fraction, we do not change it to a mixed number. Because a ratio compares two quantities, we would leave a ratio as $\frac{4}{1}$ instead of simplifying it to 4 so that we can see the two parts of the ratio.

EXAMPLE 1

Write each ratio as a fraction: a)$15 \text{ to } 27$ b)$45 \text{ to } 18.$

Solution

a)

	$15 \text{ to } 27$
Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{15}{27}$
Simplify the fraction.	$\frac{5}{9}$

We leave the ratio in b) as an improper fraction.

b)

	$45 \text{ to } 18$
Write as a fraction with the first number in the numerator and the second in the denominator.	$\frac{45}{18}$
Simplify.	$\frac{5}{2}$

TRY IT 1.1

Write each ratio as a fraction: a) $21 \text{ to } 56$ b) $48 \text{ to } 32$.

Answer

a. $\frac{3}{8}$
b. $\frac{3}{2}$

TRY IT 1.2

Write each ratio as a fraction: a)$27 \text{ to } 72$ b) $51 \text{ to } 34$.

Answer

a. $\frac{3}{8}$
b. $\frac{3}{2}$

Ratios Involving Decimals

We will often work with ratios of decimals, especially when we have ratios involving money. In these cases, we can eliminate the decimals by using the Equivalent Fractions Property to convert the ratio to a fraction with whole numbers in the numerator and denominator.

For example, consider the ratio $0.8 \text{ to } 0.05$. We can write it as a fraction with decimals and then multiply the numerator and denominator by 100 to eliminate the decimals.

$$\frac{0.8}{0.05}$$

$$\frac{(0.8)100}{(0.05)100}$$

$$\frac{80}{5}$$

Do you see a shortcut to find the equivalent fraction? Notice that $0.8 = \frac{8}{10}$ and $0.05 = \frac{5}{100}$. The least common denominator of $\frac{8}{10}$ and $\frac{5}{100}$ is 100. By multiplying the numerator and denominator of $\frac{0.8}{0.05}$ by 100, we 'moved' the decimal two places to the right to get the equivalent fraction with no decimals. Now that we understand the math behind the process, we can find the fraction with no decimals like this:

"Move" the decimal 2 places.	$\frac{0.80}{0.05} = \frac{80}{5}$
Simplify.	$\frac{16}{1}$

You do not have to write out every step when you multiply the numerator and denominator by powers of ten. As long as you move both decimal places the same number of places, the ratio will remain the same.

EXAMPLE 2

Write each ratio as a fraction of whole numbers:

a) 4.8 to 11.2

b) 2.7 to 0.54

Solution

a) $4.8 \text{ to } 11.2$	
Write as a fraction.	$\frac{4.8}{11.2}$
Rewrite as an equivalent fraction without decimals, by moving both decimal points 1 place to the right.	$\frac{48}{112}$
Simplify.	$\frac{3}{7}$

So 4.8 to 11.2 is equivalent to $\frac{3}{7}$.

b) The numerator has one decimal place and the denominator has 2. To clear both decimals we need to move the decimal 2 places to the right. $2.7 \text{ to } 0.54$	
Write as a fraction.	$\frac{2.7}{0.54}$
Move both decimals right two places.	$\frac{270}{54}$
Simplify.	$\frac{5}{1}$

So $2.7 \text{ to } 0.54$ is equivalent to $\frac{5}{1}$.

TRY IT 2.1

Write each ratio as a fraction: a) $4.6 \text{ to } 11.5$ b) $2.3 \text{ to } 0.69$.

Answer

a. $\frac{2}{5}$
b. $\frac{10}{3}$

TRY IT 2.2

Write each ratio as a fraction: a) $3.4 \text{ to } 15.3$ b) $3.4 \text{ to } 0.68$.

Answer

a. $\frac{2}{9}$
b. $\frac{5}{1}$

Some ratios compare two mixed numbers. Remember that to divide mixed numbers, you first rewrite them as improper fractions.

EXAMPLE 3

Write the ratio of $1\frac{1}{4} \text{ to } 2\frac{3}{8}$ as a fraction.

Solution

Write " $1\frac{1}{4}$ to $2\frac{3}{8}$ " as a fraction.	$\frac{1\frac{1}{4}}{2\frac{3}{8}}$
Convert the numerator and denominator to improper fractions.	$\frac{\frac{5}{4}}{\frac{19}{8}}$
Rewrite as a division of fractions.	$\frac{5}{4} \div \frac{19}{8}$
Invert the divisor and multiply.	$\frac{5}{4} \cdot \frac{8}{19}$
Simplify.	$\frac{10}{19}$

TRY IT 3.1

Write each ratio as a fraction: $1\frac{3}{4}$ to $2\frac{5}{8}$.

Answer
$\frac{2}{3}$

TRY IT 3.2

Write each ratio as a fraction: $1\frac{1}{8}$ to $2\frac{3}{4}$.

Answer
$\frac{9}{22}$

Applications of Ratios

One real-world application of ratios that affects many people involves measuring cholesterol in blood. The ratio of total cholesterol to HDL cholesterol is one way doctors assess a person's overall health. A ratio of less than 5 to 1 is considered good.

EXAMPLE 4

Hector's total cholesterol is 249 mg/dl and his HDL cholesterol is 39 mg/dl. a) Find the ratio of his total cholesterol to his HDL cholesterol. b) Assuming that a ratio less than 5 to 1 is considered good, what would you suggest to Hector?

Solution

a) First, write the words that express the ratio. We want to know the ratio of Hector's total cholesterol to his HDL cholesterol.

Write as a fraction.	$\frac{\text{total cholesterol}}{\text{HDL cholesterol}}$
Substitute the values.	$\frac{249}{39}$
Simplify.	$\frac{83}{13}$

b) Is Hector's cholesterol ratio ok? If we divide 83 by 13 we obtain approximately 6.4, so $\frac{83}{13} \approx \frac{6.4}{1}$. Hector's cholesterol ratio is high! Hector should either lower his total cholesterol or raise his HDL cholesterol.

TRY IT 4.1

Find the patient's ratio of total cholesterol to HDL cholesterol using the given information.

Total cholesterol is 185 mg/dL and HDL cholesterol is 40 mg/dL.

Answer
$\frac{37}{8}$

TRY IT 4.2

Find the patient's ratio of total cholesterol to HDL cholesterol using the given information.

Total cholesterol is 204 mg/dL and HDL cholesterol is 38 mg/dL.

Answer
$\frac{102}{19}$

Ratios of Two Measurements in Different Units

To find the ratio of two measurements, we must make sure the quantities have been measured with the same unit. If the measurements are not in the same units, we must first convert them to the same units.

We know that to simplify a fraction, we divide out common factors. Similarly in a ratio of measurements, we divide out the common unit.

EXAMPLE 5

The Canadian National Building Code (CNBC) Guidelines for wheel chair ramps require a maximum vertical rise of 1 inch for every 1 foot of horizontal run. What is the ratio of the rise to the run?

Solution
In a ratio, the measurements must be in the same units. We can change feet to inches, or inches to feet. It is usually easier to convert to the smaller unit, since this avoids introducing more fractions into the problem.

Write the words that express the ratio.

	Ratio of the rise to the run
Write the ratio as a fraction.	$\frac{\text{rise}}{\text{run}}$
Substitute in the given values.	$\frac{1\text{ inch}}{1\text{ foot}}$
Convert 1 foot to inches.	$\frac{1\text{ inch}}{12\text{ inches}}$
Simplify, dividing out common factors and units.	$\frac{1}{12}$

So the ratio of rise to run is 1 to 12. This means that the ramp should rise 1 inch for every 12 inches of horizontal run to comply with the guidelines.

TRY IT 5.1

Find the ratio of the first length to the second length: 32 inches to 1 foot.

Answer
$\frac{8}{3}$

TRY IT 5.2

Find the ratio of the first length to the second length: 1 foot to 54 inches.

Answer
$\frac{2}{9}$

Write a Rate as a Fraction

Frequently we want to compare two different types of measurements, such as miles to gallons. To make this comparison, we use a rate. Examples of rates are 120 miles in 2 hours, 160 words in 4 minutes, and $\$5$ dollars per 64 ounces.

Rate

A rate compares two quantities of different units. A rate is usually written as a fraction.

When writing a fraction as a rate, we put the first given amount with its units in the numerator and the second amount with its units in the denominator. When rates are simplified, the units remain in the numerator and denominator.

EXAMPLE 6

Bob drove his car 525 miles in 9 hours. Write this rate as a fraction.

Solution

	$525 \text{ miles in } 9 \text{ hours}$
Write as a fraction, with 525 miles in the numerator and 9 hours in the denominator.	$\frac{525 \text{ miles}}{9 \text{ hours}}$
	$\frac{175 \text{ miles}}{3 \text{ hours}}$

So 525 miles in 9 hours is equivalent to $\frac{175 \text{ miles}}{3 \text{ hours}}$.

TRY IT 6.1

Write the rate as a fraction: 492 miles in 8 hours.

Answer
$\frac{123 \text{ miles}}{2 \text{ hours}}$

TRY IT 6.2

Write the rate as a fraction: 242 miles in 6 hours.

Answer
$\frac{121 \text{ miles}}{3 \text{ hours}}$

Find Unit Rates

In the last example, we calculated that Bob was driving at a rate of $\frac{175 \text{ miles}}{3 \text{ hours}}$. This tells us that every three hours, Bob will travel 175 miles. This is correct, but not very useful. We usually want the rate to reflect the number of miles in one hour. A rate that has a denominator of 1 unit is referred to as a unit rate.

Unit Rate

A unit rate is a rate with denominator of 1 unit.

Unit rates are very common in our lives. For example, when we say that we are driving at a speed of 68 miles per hour we mean that we travel 68 miles in 1 hour. We would write this rate as 68 miles/hour (read 68 miles per hour). The common abbreviation for this is 68 mph. Note that when no number is written before a unit, it is assumed to be 1.

So 68 miles/hour really means $68 \text{ miles}/1 \text{ hour}.$

Two rates we often use when driving can be written in different forms, as shown:

Example	Rate	Write	Abbreviate	Read
68 miles in 1 hour	$\frac{68 \text{ miles}}{1 \text{ hour}}$	68 miles/hour	68 mph	$68 \text{ miles per hour}$
36 miles to 1 gallon	$\frac{36 \text{ miles}}{1 \text{ gallon}}$	36 miles/gallon	36 mpg	$36 \text{ miles per gallon}$

Another example of unit rate that you may already know about is hourly pay rate. It is usually expressed as the amount of money earned for one hour of work. For example, if you are paid $\$12.50$ for each hour you work, you could write that your hourly (unit) pay rate is $\$12.50/\text{hour}$ (read $\$12.50$ per hour.)

To convert a rate to a unit rate, we divide the numerator by the denominator. This gives us a denominator of 1.

EXAMPLE 7

Anita was paid $\$384$ last week for working 32 hours. What is Anita's hourly pay rate?

Solution

Start with a rate of dollars to hours. Then divide.	$\$384 \text{ last week for } 32 \text{ hours}$
Write as a rate.	$\frac{\$384}{32 \text{ hours}}$
Divide the numerator by the denominator.	$\frac{\$12}{1 \text{ hour}}$
Rewrite as a rate.	$\$12/\text{hour}$

Anita's hourly pay rate is $\$12$ per hour.

TRY IT 7.1

Find the unit rate: $\$630$ for 35 hours.

Answer
\$18.00/hour

TRY IT 7.2

Find the unit rate: $\$684$ for 36 hours.

Answer
\$19.00/hour

EXAMPLE 8

Sven drives his car 455 miles, using 14 gallons of gasoline. How many miles per gallon does his car get?

Solution
Start with a rate of miles to gallons. Then divide.

	$455 \text{ miles to } 14 \text{ gallons of gas}$
Write as a rate.	$\frac{455 \text{ miles}}{14 \text{ gallons}}$
Divide 455 by 14 to get the unit rate.	$\frac{32.5 \text{ miles}}{1 \text{ gallon}}$

Sven's car gets 32.5 miles/gallon, or 32.5 mpg.

TRY IT 8.1

Find the unit rate: 423 miles to 18 gallons of gas.

Answer
23.5 mpg

TRY IT 8.2

Find the unit rate: 406 miles to 14.5 gallons of gas.

Answer
28 mpg

Find Unit Price

Sometimes we buy common household items 'in bulk', where several items are packaged together and sold for one price. To compare the prices of different sized packages, we need to find the unit price. To find the unit price, divide the total price by the number of items. A unit price is a unit rate for one item.

Unit price

A unit price is a unit rate that gives the price of one item.

EXAMPLE 9

The grocery store charges $\$3.99$ for a case of 24 bottles of water. What is the unit price?

Solution
What are we asked to find? We are asked to find the unit price, which is the price per bottle.

Write as a rate.	$\frac{\$3.99}{24\text{ bottles}}$
Divide to find the unit price.	$\frac{\$0.16625}{1\text{ bottle}}$
Round the result to the nearest penny.	$\frac{\$0.17}{1\text{ bottle}}$

The unit price is approximately $\$0.17$ per bottle. Each bottle costs about $\$0.17$.

TRY IT 9.1

Find the unit price. Round your answer to the nearest cent if necessary.

24-pack of juice boxes for $\$6.99$

Answer
$0.29/box

TRY IT 9.2

Find the unit price. Round your answer to the nearest cent if necessary.

24-pack of bottles of ice tea for $\$12.72$

Answer
$0.53/bottle

Unit prices are very useful if you comparison shop. The *better buy* is the item with the lower unit price. Most grocery stores list the unit price of each item on the shelves.

EXAMPLE 10

Paul is shopping for laundry detergent. At the grocery store, the liquid detergent is priced at $\$14.99$ for 64 loads of laundry and the same brand of powder detergent is priced at $\$15.99$ for 80 loads.

Which is the better buy, the liquid or the powder detergent?

Solution
To compare the prices, we first find the unit price for each type of detergent.

	Liquid	Powder
Write as a rate.	$\frac{\$14.99}{64 \text{ loads}}$	$\frac{\$15.99}{80 \text{ loads}}$
Find the unit price.	$\frac{\$0.234\ldots}{1 \text{ load}}$	$\frac{\$0.199\ldots}{1 \text{ load}}$
Round to the nearest cent.	$\$0.23/\text{load}$ $(23 \text{ cents per load.})$	$\$0.20/\text{load}$ $(20 \text{ cents per load})$

Now we compare the unit prices. The unit price of the liquid detergent is about $\$0.23$ per load and the unit price of the powder detergent is about $\$0.20$ per load. The powder is the better buy.

TRY IT 10.1

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand A Storage Bags, $\$4.59$ for 40 count, or Brand B Storage Bags, $\$3.99$ for 30 count

Answer
Brand A costs $0.11 per bag. Brand B costs $0.13 per bag. Brand A is the better buy.

TRY IT 10.2

Find each unit price and then determine the better buy. Round to the nearest cent if necessary.

Brand C Chicken Noodle Soup, $\$1.89$ for 26 ounces, or Brand D Chicken Noodle Soup, $\$0.95$ for 10.75 ounces

Answer

Brand C costs $0.07 per ounce. Brand D costs $0.09 per ounce. Brand C is the better buy.

Notice in the above example that we rounded the unit price to the nearest cent. Sometimes we may need to carry the division to one more place to see the difference between the unit prices.

Translate Phrases to Expressions with Fractions

Have you noticed that the examples in this section used the comparison words *ratio of*, *to*, *per*, *in*, *for*, *on*, and *from*? When you translate phrases that include these words, you should think either ratio or rate. If the units measure the same quantity (length, time, etc.), you have a ratio. If the units are different, you have a rate. In both cases, you write a fraction.

EXAMPLE 11

Translate the word phrase into an algebraic expression:

a) 427 miles per h hours

b) x students to 3 teachers

c) y dollars for 18 hours

Solution

a)	$427 \text{ miles per } h \text{ hours}$
Write as a rate.	$\frac{427 \text{ miles}}{h \text{ hours}}$

b)	$x \text{ students to } 3 \text{ teachers}$
Write as a rate.	$\frac{x \text{ students}}{3 \text{ teachers}}$

c)	$y \text{ dollars for } 18 \text{ hours}$
Write as a rate.	$\frac{\$y}{18 \text{ hours}}$

TRY IT 11.1

Translate the word phrase into an algebraic expression.

a) 689 miles per h hours b) y parents to 22 students c) d dollars for 9 minutes

Answer

a. 689 mi/h hours
b. y parents/22 students
c. \$$d$/9 min

TRY IT 11.2

Translate the word phrase into an algebraic expression.

a)m miles per 9 hours b) x students to 8 buses c) y dollars for 40 hours

Answer

a. m mi/9 h
b. x students/8 buses
c. \$$y$/40 h

Access to Additional Online R

- Ratios
- Write Ratios as a Simplified Fractions Involving Decimals and Fractions
- Write a Ratio as a Simplified Fraction
- Rates and Unit Rates
- Unit Rate for Cell Phone Plan

Glossary

ratio
A ratio compares two numbers or two quantities that are measured with the same unit. The ratio of a to b is written a to b, $\frac{a}{b}$, or $a : b$.

rate
A rate compares two quantities of different units. A rate is usually written as a fraction.

unit rate
A unit rate is a rate with denominator of 1 unit.
unit price
A **unit price** is a unit rate that gives the price of one item.

Practice Makes Perfect

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction.

1. 20 to 36	2. 20 to 32
3. 42 to 48	4. 45 to 54
5. 49 to 21	6. 56 to 16
7. 84 to 36	8. 6.4 to 0.8
9. 0.56 to 2.8	10. 1.26 to 4.2
11. $1\frac{2}{3}$ to $2\frac{5}{6}$	12. $1\frac{3}{4}$ to $2\frac{5}{8}$
13. $4\frac{1}{6}$ to $3\frac{1}{3}$	14. $5\frac{3}{5}$ to $3\frac{3}{5}$
15. $\$18$ to $\$63$	16. $\$16$ to $\$72$
17. $\$1.21$ to $\$0.44$	18. $\$1.38$ to $\$0.69$
19. 28 ounces to 84 ounces	20. 32 ounces to 128 ounces
21. 12 feet to 46 feet	22. 15 feet to 57 feet
23. 246 milligrams to 45 milligrams	24. 304 milligrams to 48 milligrams
25. total cholesterol of 175 to HDL cholesterol of 45	26. total cholesterol of 215 to HDL cholesterol of 55
27. 27 inches to 1 foot	28. 28 inches to 1 foot

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction.

29. 140 calories per 12 ounces	30. 180 calories per 16 ounces
31. 8.2 pounds per 3 square inches	32. 9.5 pounds per 4 square inches
33. 488 miles in 7 hours	34. 527 miles in 9 hours
35. $\$595$ for 40 hours	36. $\$798$ for 40 hours

Find Unit Rates

In the following exercises, find the unit rate. Round to two decimal places, if necessary.

37. 140 calories per 12 ounces	38. 180 calories per 16 ounces
39. 8.2 pounds per 3 square inches	40. 9.5 pounds per 4 square inches
41. 488 miles in 7 hours	42. 527 miles in 9 hours
43. $\$595$ for 40 hours	44. $\$798$ for 40 hours
45. 576 miles on 18 gallons of gas	46. 435 miles on 15 gallons of gas
47. 43 pounds in 16 weeks	48. 57 pounds in 24 weeks
49. 46 beats in 0.5 minute	50. 54 beats in 0.5 minute
51. The bindery at a printing plant assembles $96,000$ magazines in 12 hours. How many magazines are assembled in one hour?	52. The pressroom at a printing plant prints $540,000$ sections in 12 hours. How many sections are printed per hour?

Find Unit Price

In the following exercises, find the unit price. Round to the nearest cent.

53. Soap bars at 8 for $\$8.69$	54. Soap bars at 4 for $\$3.39$
55. Women's sports socks at 6 pairs for $\$7.99$	56. Men's dress socks at 3 pairs for $\$8.49$
57. Snack packs of cookies at 12 for $\$5.79$	58. Granola bars at 5 for $\$3.69$
59. CD-RW discs at 25 for $\$14.99$	60. CDs at 50 for $\$4.49$
61. The grocery store has a special on macaroni and cheese. The price is $\$3.87$ for 3 boxes. How much does each box cost?	62. The pet store has a special on cat food. The price is $\$4.32$ for 12 cans. How much does each can cost?

In the following exercises, find each unit price and then identify the better buy. Round to three decimal places.

63. Mouthwash, 50.7-ounce size for $\$6.99$ or 33.8-ounce size for $\$4.79$	64. Toothpaste, 6 ounce size for $\$3.19$ or $7.8-ounce$ size for $\$5.19$
65. Breakfast cereal, 18 ounces for $\$3.99$ or 14 ounces for $\$3.29$	66. Breakfast Cereal, 10.7 ounces for $\$2.69$ or 14.8 ounces for $\$3.69$
67. Ketchup, 40-ounce regular bottle for $\$2.99$ or 64-ounce squeeze bottle for $\$4.39$	68. Mayonnaise 15-ounce regular bottle for $\$3.49$ or 22-ounce squeeze bottle for $\$4.99$
69. Cheese $\$6.49$ for 1 lb. block or $\$3.39$ for $\frac{1}{2}$ lb. block	70. Candy $\$10.99$ for a 1 lb. bag or $\$2.89$ for $\frac{1}{4}$ lb. of loose candy

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.

71. 793 miles per p hours	72. 78 feet per r seconds
73. $\$3$ for 0.5 lbs.	74. j beats in 0.5 minutes
75. 105 calories in x ounces	76. 400 minutes for m dollars
77. the ratio of y and $5x$	78. the ratio of $12x$ and y

Everyday Math

79. One elementary school in Saskatchewan has 684 students and 45 teachers. Write the student-to-teacher ratio as a unit rate.	80. The average Canadian produces about 350 pounds of paper trash per year (365 days). How many pounds of paper trash does the average Canadian produce each day? (Round to the nearest tenth of a pound.)
81. A popular fast food burger weighs 7.5 ounces and contains 540 calories, 29 grams of fat, 43 grams of carbohydrates, and 25 grams of protein. Find the unit rate of a) calories per ounce b) grams of fat per ounce c) grams of carbohydrates per ounce d) grams of protein per ounce. Round to two decimal places.	82. A $16-ounce$ chocolate mocha coffee with whipped cream contains 470 calories, 18 grams of fat, 63 grams of carbohydrates, and 15 grams of protein. Find the unit rate of a) calories per ounce b) grams of fat per ounce c) grams of carbohydrates per ounce d) grams of protein per ounce.

Writing Exercises

83. Would you prefer the ratio of your income to your friend's income to be $3/1$ or $1/3?$ Explain your reasoning.	84. The parking lot at the airport charges $\$0.75$ for every 15 minutes. a) How much does it cost to park for 1 hour? b) Explain how you got your answer to part a). Was your reasoning based on the unit cost or did you use another method?
85. Kathryn ate a $4-ounce$ cup of frozen yogurt and then went for a swim. The frozen yogurt had 115 calories. Swimming burns 422 calories per hour. For how many minutes should Kathryn swim to burn off the calories in the frozen yogurt? Explain your reasoning.	86. Arjun had a $16-ounce$ cappuccino at his neighbourhood coffee shop. The cappuccino had 110 calories. If Arjun walks for one hour, he burns 246 calories. For how many minutes must Arjun walk to burn off the calories in the cappuccino? Explain your reasoning.

Answers

1. $\frac{5}{9}$	3. $\frac{7}{8}$	5. $\frac{7}{3}$
7. $\frac{7}{3}$	9. $\frac{1}{5}$	11. $\frac{10}{17}$
13. $\frac{5}{4}$	15. $\frac{2}{7}$	17. $\frac{11}{4}$
19. $\frac{1}{3}$	21. $\frac{6}{23}$	23. $\frac{82}{15}$
25. $\frac{35}{9}$	27. $\frac{9}{4}$	29. $\frac{35 \text{ calories}}{3 \text{ ounces}}$
31. $\frac{41 \text{ lbs}}{15 \text{ sq. in.}}$	33. $\frac{488 \text{ miles}}{7 \text{ hours}}$	35. $\frac{\$119}{8 \text{ hours}}$
37. 11.67 calories/ounce	39. 2.73 lbs./sq. in.	41. 69.71 mph
43. $14.88/hour	45. 32 mpg	47. 2.69 lbs./week
49. 92 beats/minute	51. 8,000	53. $1.09/bar
55. $1.33/pair	57. $0.48/pack	59. $0.60/disc
61. $1.29/box	63. The 50.7-ounce size costs $0.138 per ounce. The 33.8-ounce size costs $0.142 per ounce. The 50.7-ounce size is the better buy.	65. The 18-ounce size costs $0.222 per ounce. The 14-ounce size costs $0.235 per ounce. The 18-ounce size is a better buy.
67. The regular bottle costs $0.075 per ounce. The squeeze bottle costs $0.069 per ounce. The squeeze bottle is a better buy.	69. The half-pound block costs $6.78/lb, so the 1-lb. block is a better buy.	71. $\frac{793 \text{ miles}}{p \text{ hours}}$
73. $\frac{?3}{0.5 \text{ lbs.}}$	75. $\frac{105 \text{ calories}}{x \text{ ounces}}$	77. $\frac{y}{5x}$
79. 15.2 students per teacher	81. a) 72 calories/ounce b) 3.87 grams of fat/ounce c) 5.73 grams carbs/once d) 3.33 grams protein/ounce	83. Answers will vary.
85. Answers will vary.		

Attributions

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3.2 Understand Percent

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of percent
- Convert percents to fractions and decimals
- Convert decimals and fractions to percents

Use the Definition of Percent

How many cents are in one dollar? There are 100 cents in a dollar. How many years are in a century? There are 100 years in a century. Does this give you a clue about what the word "percent" means? It is really two words, "per cent," and means per one hundred. A percent is a ratio whose denominator is 100. We use the percent symbol $\%$, to show percent.

Percent

A percent is a ratio whose denominator is 100.

According to data from the Statistics Canada, 57% of Canadian Internet users reported a cyber security incident, including being redirected to fraudulent websites that asked for personal information or getting a virus or other computer infection. This means 57 out of every 100 Canadian internet users reported cyber security incidents as (Figure 1) shows. Out of the 100 squares on the grid, 57 are shaded, which we write as the ratio $\frac{57}{100}$.

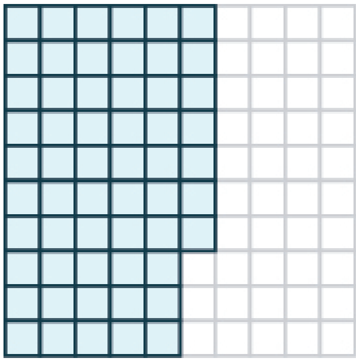


Figure 1

Similarly, 25% means a ratio of $\frac{25}{100}$, 3% means a ratio of $\frac{3}{100}$ and 100% means a ratio of $\frac{100}{100}$. In words, “one hundred percent” means the total 100% is $\frac{100}{100}$, and since $\frac{100}{100} = 1$, we see that 100% means 1 whole.

EXAMPLE 1

According to a survey done by Universities Canada (2017), 71% of Canada’s Universities are working to include Indigenous representation within their governance or leadership structures.Write this percent as a ratio.

Solution

The amount we want to convert is 71%.	71%
Write the percent as a ratio. Remember that *percent* means per 100.	$\frac{71}{100}$

TRY IT 1.1

Write the percent as a ratio.

According to a survey, 89% of college students have a smartphone.

Answer
$\frac{89}{100}$

TRY IT 1.2

Write the percent as a ratio.

A study found that 72% of Canadian teens send text messages regularly.

Answer
$\frac{72}{100}$

EXAMPLE 2

In 2018, according to a Universities Canada survey, 56 out of every 100 of today’s undergraduates benefit from experiential learning such as co-ops, internships and service learning. Write this as a ratio and then as a percent.

Solution

The amount we want to convert is 56 out of 100.	56 out of 100
Write as a ratio.	$\frac{56}{100}$
Convert the 56 per 100 to percent.	56%

TRY IT 2.1

Write as a ratio and then as a percent: According to Statistics Canada, only 10 out of 100 young Canadians cross a provincial border to complete their university degree.

Answer
$\frac{10}{100}$, 10%

TRY IT 2.2

Write as a ratio and then as a percent: According to an international comparison done by the British Council, 55 out of 100 current professional leaders across 30 countries and in all sectors, are liberal arts grads with bachelor's degrees in the social sciences or humanities.

Answer
$\frac{55}{100}$, 55%

Convert Percents to Fractions and Decimals

Since percents are ratios, they can easily be expressed as fractions. Remember that percent means per 100, so the denominator of the fraction is 100.

Convert a Percent to a Fraction.

1. Write the percent as a ratio with the denominator 100.
2. Simplify the fraction if possible.

EXAMPLE 3

Convert each percent to a fraction:

a. 36%
b. 125%

Solution

a)	36%	b)	125%
Write as a ratio with denominator 100.	$\frac{36}{100}$	Write as a ratio with denominator 100.	$\frac{125}{100}$
Simplify.	$\frac{9}{25}$	Simplify.	$\frac{5}{4}$

TRY IT 3.1

Convert each percent to a fraction:

a. 48%
b. 110%

Answer

a. $\frac{12}{25}$
b. $\frac{11}{10}$

TRY IT 3.2

Convert each percent to a fraction:

a. 64%
b. 150%

Answer

a. $\frac{16}{25}$
b. $\frac{3}{2}$

The previous example shows that a percent can be greater than 1. We saw that 125% means $\frac{125}{100}$, or $\frac{5}{4}$. These are improper fractions, and their values are greater than one.

EXAMPLE 4

Convert each percent to a fraction:

a. 24.5%
b. $33\frac{1}{3}\%$

Solution

a)	24.5%
Write as a ratio with denominator 100.	$\frac{24.5}{100}$
Clear the decimal by multiplying numerator and denominator by 10.	$\frac{24.5(10)}{100(10)}$
Multiply.	$\frac{245}{1000}$
Rewrite showing common factors.	$\frac{5 \cdot 49}{5 \cdot 200}$
Simplify.	$\frac{49}{200}$

b)	$33\frac{1}{3}\%$
Write as a ratio with denominator 100.	$\frac{33\frac{1}{3}}{100}$
Write the numerator as an improper fraction.	$\frac{\frac{100}{3}}{100}$
Rewrite as fraction division, replacing 100 with $\frac{100}{1}$.	$\frac{100}{3} \div \frac{100}{1}$
Multiply by the reciprocal.	$\frac{100}{3} \cdot \frac{1}{100}$
Simplify.	$\frac{1}{3}$

TRY IT 4.1

Convert each percent to a fraction:

a. 64.4%
b. $66\frac{2}{3}\%$

Answer

a. $\frac{161}{250}$
b. $\frac{2}{3}$

TRY IT 4.2

Convert each percent to a fraction:

a. 42.5%
b. $8\frac{3}{4}\%$

Answer

a. $\frac{17}{40}$
b. $\frac{7}{80}$

To convert a percent to a decimal, we first convert it to a fraction and then change the fraction to a decimal.

HOW TO: *Convert a Percent to a Decimal*

1. Write the percent as a ratio with the denominator 100.
2. Convert the fraction to a decimal by dividing the numerator by the denominator.

EXAMPLE 5

Convert each percent to a decimal:

a. 6%
b. 78%

Solution
Because we want to change to a decimal, we will leave the fractions with denominator 100 instead of removing common factors.

a)	6%
Write as a ratio with denominator 100.	$\frac{6}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.06

b)	78%
Write as a ratio with denominator 100.	$\frac{78}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.78

TRY IT 5.1

Convert each percent to a decimal:

a. 9%
b. 87%

Answer

a. 0.09
b. 0.87

TRY IT 5.2

Convert each percent to a decimal:

a. 3%
b. 91%

Answer

a. 0.03
b. 0.91

EXAMPLE 6

Convert each percent to a decimal:

a. 135%
b. 12.5%

Solution

a)	135%
Write as a ratio with denominator 100.	$\frac{135}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	1.35

b)	12.5%
Write as a ratio with denominator 100.	$\frac{12.5}{100}$
Change the fraction to a decimal by dividing the numerator by the denominator.	0.125

TRY IT 6.1

Convert each percent to a decimal:

a. 115%
b. 23.5%

Answer

a. 1.15
b. 0.235

TRY IT 6.2

Convert each percent to a decimal:

a. 123%
b. 16.8%

Answer

a. 1.23
b. 0.168

Let's summarize the results from the previous examples in the table below, and look for a pattern we could use to quickly convert a percent number to a decimal number.

Percent	**Decimal**
6%	0.06
78%	0.78
135%	1.35
12.5%	0.125

Do you see the pattern?

To convert a percent number to a decimal number, we move the decimal point two places to the left and remove the $\%$ sign. (Sometimes the decimal point does not appear in the percent number, but just like we can think of the integer 6 as 6.0, we can think of 6% as 6.0%.) Notice that we may need to add zeros in front of the number when moving the decimal to the left.

(Figure 2) uses the percents in the table above and shows visually how to convert them to decimals by moving the decimal point two places to the left.

Percent	Decimal
006.%	0.06
078.%	0.78
135.%	1.35
012.5%	0.125

Figure 2

EXAMPLE 7

Among a group of business leaders, 77% believe that poor math and science education in Canada will lead to higher unemployment rates.

Convert the percent to: a) a fraction b) a decimal

Solution

a) Write 77% as a ratio with denominator 100.	$\frac{77}{100}$

b) Change the fraction $\frac{77}{100}$ to a decimal by dividing the numerator by the denominator.	0.77

TRY IT 7.1

Convert the percent to: a) a fraction and b) a decimal

Twitter's share of web traffic jumped 24% when one celebrity tweeted live on air.

Answer

a. $\frac{6}{25}$
b. 0.24

TRY IT 7.2

Convert the percent to: a) a fraction and b) a decimal

Statistics Canada shows that in 2016, 29% of adults aged 25 to 64 had a bachelor degree.

Answer

a. $\frac{29}{100}$
b. 0.29

EXAMPLE 8

There are four suits of cards in a deck of cards—hearts, diamonds, clubs, and spades. The probability of randomly choosing a heart from a shuffled deck of cards is 25%. Convert the percent to:

a. a fraction
b. a decimal

(credit: Riles32807, Wikimedia Commons)

Solution

a) Write 25% as a ratio with denominator 100.	$\frac{25}{100}$
Simplify.	$\frac{1}{4}$

b) Change the fraction $\frac{1}{4}$ to a decimal by dividing the numerator by the denominator.	0.25

TRY IT 8.1

Convert the percent to: a) a fraction, and b) a decimal

The probability that it will rain Monday is 30%.

Answer

a. $\frac{3}{10}$
b. 0.3

TRY IT 8.2

Convert the percent to: a) a fraction, and b) a decimal

The probability of getting heads three times when tossing a coin three times is 12.5%.

Answer

a. $\frac{12.5}{100}$
b. 0.125

Convert Decimals and Fractions to Percents

To convert a decimal to a percent, remember that percent means per hundred. If we change the decimal to a fraction whose denominator is 100, it is easy to change that fraction to a percent.

HOW TO: *Convert a Decimal to a Percent*

1. Write the decimal as a fraction.
2. If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.
3. Write this ratio as a percent.

EXAMPLE 9

Convert each decimal to a percent: a) 0.05 b) 0.83

Solution

a) Write 0.05 as a fraction. The denominator is 100.	$\frac{5}{100}$
Write this ratio as a percent.	5%

b) Write 0.83 as a fraction. The denominator is 100.	$\frac{83}{100}$
Write this ratio as a percent.	83%

TRY IT 9.1

Convert each decimal to a percent: a)0.01 b)0.17.

Answer

a. 1%
b. 17%

TRY IT 9.2

Convert each decimal to a percent: a)0.04 b)0.41

Answer

a. 4%
b. 41%

To convert a mixed number to a percent, we first write it as an improper fraction.

EXAMPLE 10

Convert each decimal to a percent: a) 1.05 b) 0.075

Solution

a) Write 1.05 as a fraction.	$1\frac{5}{100}$
Write as an improper fraction. The denominator is 100.	$\frac{105}{100}$
Write this ratio as a percent.	105%

Notice that since $1.05 > 1$, the result is more than 100%.

b) Write 0.075 as a fraction. The denominator in this case is 1,000.	$\frac{75}{1{,}000}$
Divide the numerator and denominator by 10, so that the denominator is 100.	$\frac{7.5}{100}$
Write this ratio as a percent.	7.5%

TRY IT 10.1

Convert each decimal to a percent: a)1.75 b)0.0825

Answer

a. 175%
b. 8.25%

TRY IT 10.2

Convert each decimal to a percent: a)2.25 b)0.0925

Answer

a. 225%
b. 9.25%

Let's summarize the results from the previous examples in the table below so we can look for a pattern.

Decimal	**Percent**
0.05	5%
0.83	83%
1.05	105%
0.075	7.5%

Do you see the pattern? To convert a decimal to a percent, we move the decimal point two places to the right and then add the percent sign.

(Figure.3) uses the decimal numbers in the table above and shows visually to convert them to percent by moving the decimal point two places to the right and then writing the $\%$ sign.

Decimal	Percent
0.05	5%
0.83	83%
1.05	105%
0.075	7.5%

Figure. 3

Now we also know how to change decimals to percents. So to convert a fraction to a percent, we first change it to a decimal and then convert that decimal to a percent.

HOW TO: *Convert a Fraction to a Percent*

1. Convert the fraction to a decimal.
2. Convert the decimal to a percent.

EXAMPLE 11

Convert each fraction or mixed number to a percent: a) $\frac{3}{4}$ b) $\frac{11}{8}$ c) $2\frac{1}{5}$

Solution

To convert a fraction to a decimal, divide the numerator by the denominator.

a) Change $\frac{3}{4}$ to a decimal.	$3 \div 4 = 0.75$
Write as a percent by moving the decimal two places to the right.	$0.75 = 75\%$

b) Change $\frac{11}{8}$ to a decimal.	$11 \div 8$
Write as a percent by moving the decimal two places to the right.	$1.375 = 137.5\%$

c) Write $2\frac{1}{5}$ as an improper fraction.	$\frac{11}{5}$
Change $\frac{11}{5}$ to a decimal.	$11 \div 5$
Write as a percent.	$2.20 = 220\%$

Notice that we needed to add zeros at the end of the number when moving the decimal two places to the right.

TRY IT 11.1

Convert each fraction or mixed number to a percent: a) $\frac{5}{8}$ b) $\frac{11}{4}$ c) $3\frac{2}{5}$

Answer

a. 62.5%
b. 275%
c. 340%

TRY IT 11.2

Convert each fraction or mixed number to a percent: a)$\frac{7}{8}$ b)$\frac{9}{4}$ c)$1\frac{3}{5}$

Answer

a. 87.5%
b. 225%
c. 160%

Sometimes when changing a fraction to a decimal, the division continues for many decimal places and we will round off the quotient. The number of decimal places we round to will depend on the situation. If the decimal involves money, we round to the hundredths place. For most other cases in this book we will round the number to the nearest thousandth, so the percent will be rounded to the nearest tenth.

EXAMPLE 12

Convert $\frac{5}{7}$ to a percent.

Solution
To change a fraction to a decimal, we divide the numerator by the denominator.

To convert $\frac{5}{7}$ to a decimal	$5 \div 7$
Rounding to the nearest thousandth.	0.714
Write as a percent.	71.4%

TRY IT 12.1

Convert the fraction to a percent: $\frac{3}{7}$

Answer
42.9%

TRY IT 12.2

Convert the fraction to a percent: $\frac{4}{7}$

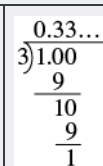
Answer
57.1%

When we first looked at fractions and decimals, we saw that fractions converted to a repeating decimal. When we converted the fraction $\frac{4}{3}$ to a decimal, we wrote the answer as $1.\overline{3}$. We will use this same notation, as well as fraction notation, when we convert fractions to percents in the next example.

EXAMPLE 13

Statistics Canada reported in 2018 that approximately $\frac{1}{3}$ of Canadian adults are obese. Convert the fraction $\frac{1}{3}$ to a percent.

Solution

Change $\frac{1}{3}$ to a decimal by dividing 1 by 3.	
Write as a repeating decimal.	$0.333\ldots$
Write as a percent.	$33\frac{1}{3}\%$

We could also write the percent as $33.\overline{3}\%$.

TRY IT 13.1

Convert the fraction to a percent:

According to the Canadian Census 2016, about $\frac{33}{50}$ people within the population of Canada are between the ages of 15 and 64.

Answer
66%

TRY IT 13.2

Convert the fraction to a percent:

According to the Canadian Census 2015, about $\frac{1}{6}$ of Canadian residents under age 18 are low income.

Answer

$16.\overline{6}\,\%$, or $16\frac{2}{3}\%$

Key Concepts

- **Convert a percent to a fraction.**
  1. Write the percent as a ratio with the denominator 100.
  2. Simplify the fraction if possible.
- **Convert a percent to a decimal.**
  1. Write the percent as a ratio with the denominator 100.
  2. Convert the fraction to a decimal by dividing the numerator by the denominator.
- **Convert a decimal to a percent.**
  1. Write the decimal as a fraction.
  2. If the denominator of the fraction is not 100, rewrite it as an equivalent fraction with denominator 100.
  3. Write this ratio as a percent.
- **Convert a fraction to a percent.**
  1. Convert the fraction to a decimal.
  2. Convert the decimal to a percent.

Glossary

percent
: A percent is a ratio whose denominator is 100.

Practice Makes Perfect

Use the Definition of Percents

In the following exercises, write each percent as a ratio.

1. In 2014, the unemployment rate for those with only a high school degree was 6.0%.	2. In 2015, among the unemployed, 29% were long-term unemployed.
3. The unemployment rate for those with Bachelor's degrees was 3.2% in 2014.	4. The unemployment rate in Canada in 2019 was 13.7%. In the following exercises, write as a) a ratio and b) a percent
5. 57 out of 100 nursing candidates received their degree at a community college.	6. 80 out of 100 firefighters and law enforcement officers were educated at a community college.
7. 42 out of 100 first-time freshmen students attend a community college.	8. 71 out of 100 full-time community college faculty have a master's degree.

Convert Percents to Fractions and Decimals

In the following exercises, convert each percent to a fraction and simplify all fractions.

9. 4%	10. 8%
11. 17%	12. 19%
13. 52%	14. 78%
15. 125%	16. 135%
17. 37.5%	18. 42.5%
19. 18.4%	20. 46.4%
21. $9\frac{1}{2}\%$	22. $8\frac{1}{2}\%$
23. $5\frac{1}{3}\%$	24. $6\frac{2}{3}\%$

In the following exercises, convert each percent to a decimal.

25. 5%	26. 9%
27. 1%	28. 2%
29. 63%	30. 71%
31. 40%	32. 50%
33. 115%	34. 125%
35. 150%	36. 250%
37. 21.4%	38. 39.3%
39. 7.8%	40. 6.4%

In the following exercises, convert each percent to

a) a simplified fraction and

b) a decimal

41. In $2010, 1.5\%$ of home sales had owner financing. (Source: Bloomberg Businessweek, 5/23–29/2011)	42. In $2016, 22.3\%$ of the Canadian population was a visible minority. (Source: www12.statcan.gc.ca)
43. According to government data, in 2013 the number of cell phones in India was 70.23% of the population.	44. According to the Survey of Earned Doctorates, among Canadians age 25 or older who had doctorate degrees in $2006, 44\%$ are women.
45. A couple plans to have two children. The probability they will have two girls is 25%.	46. Javier will choose one digit at random from 0 through 9. The probability he will choose 3 is 10%.
47. According to the local weather report, the probability of thunderstorms in New York City on July 15 is 60%.	48. A club sells 50 tickets to a raffle. Osbaldo bought one ticket. The probability he will win the raffle is 2%.

Convert Decimals and Fractions to Percents

In the following exercises, convert each decimal to a percent.

49. 0.01	50. 0.03
51. 0.18	52. 0.15
53. 1.35	54. 1.56
55. 3	56. 4
57. 0.009	58. 0.008
59. 0.0875	60. 0.0625
61. 1.5	62. 2.2
63. 2.254	64. 2.317

In the following exercises, convert each fraction to a percent.

65. $\frac{1}{4}$	66. $\frac{1}{5}$
67. $\frac{3}{8}$	68. $\frac{5}{8}$
69. $\frac{7}{4}$	70. $\frac{9}{8}$
71. $6\frac{4}{5}$	72. $5\frac{1}{4}$
73. $\frac{5}{12}$	74. $\frac{11}{12}$
75. $2\frac{2}{3}$	76. $1\frac{2}{3}$
77. $\frac{3}{7}$	78. $\frac{6}{7}$
79. $\frac{5}{9}$	80. $\frac{4}{9}$

In the following exercises, convert each fraction to a percent.

81. $\frac{1}{4}$ of washing machines needed repair.	82. $\frac{1}{5}$ of dishwashers needed repair.

In the following exercises, convert each fraction to a percent.

83. According to the Government of Canada, in 2017, $\frac{16}{25}$ of Canadian adults were overweight or obese.	84. Statistics Canada showed that in 2016, 15.4% of Canadian workers are using more than one language at work.

In the following exercises, complete the table.

85.

Fraction	Decimal	Percent
$\frac{1}{2}$		
	0.45	
		18%
$\frac{1}{3}$		
	0.0008	
2		

86.

Fraction	Decimal	Percent
$\frac{1}{4}$		
	0.65	
		22%
$\frac{2}{3}$		
	0.0004	
3		

Writing Exercises

87. Convert $25\%, 50\%, 75\%, \text{and } 100\%$ to fractions. Do you notice a pattern? Explain what the pattern is.	88. Convert $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, and $\frac{9}{10}$ to percents. Do you notice a pattern? Explain what the pattern is.
89. When the Szetos sold their home, the selling price was 500% of what they had paid for the house 30 years ago. Explain what 500% means in this context.	90. According to cnn.com, cell phone use in 2008 was 600% of what it had been in 2001. Explain what 600% means in this context.

Answers

1. $\frac{6}{100}$	3. $\frac{32}{1000}$	5. a) $\frac{57}{100}$ b) 57%
7. a) $\frac{42}{100}$ b) 42%	9. $\frac{1}{25}$	11. $\frac{17}{100}$
13. $\frac{13}{25}$	15. $\frac{5}{4}$	17. $\frac{3}{8}$
19. $\frac{23}{125}$	21. $\frac{19}{200}$	23. $\frac{4}{75}$
25. 0.05	27. 0.01	29. 0.63
31. 0.4	33. 1.15	35. 1.5
37. 0.214	39. 0.078	41. a) $\frac{3}{200}$ b) 0.015
43. a) $\frac{7023}{10,000}$ b) 0.7023	45. a) $\frac{1}{4}$ b) 0.25	47. a) $\frac{3}{5}$ b) 0.6
49. 1%	51. 18%	53. 135%
55. 300%	57. 0.9%	59. 8.75%
61. 150%	63. 225.4%	65. 25%
67. 37.5%	69. 175%	71. 680%
73. 41.7%	75. $266.\overline{6}\%$	77. 42.9%
79. 55.6%	81. 25%	83. 64%
87. $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1.$	89. The Szetos sold their home for five times what they paid 30 years ago.	

Attributions

3.3 Solve Proportions and their Applications

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of proportion
- Solve proportions
- Solve applications using proportions
- Write percent equations as proportions
- Translate and solve percent proportions

Use the Definition of Proportion

When two ratios or rates are equal, the equation relating them is called a proportion.

Proportion

A proportion is an equation of the form $\frac{a}{b}=\frac{c}{d}$, where $b\neq 0, d\neq 0$.

The proportion states two ratios or rates are equal. The proportion is read "a is to b, as c is to d".

The equation $\frac{1}{2}=\frac{4}{8}$ is a proportion because the two fractions are equal. The proportion $\frac{1}{2}=\frac{4}{8}$ is read "1 is to 2 as 4 is to 8".

If we compare quantities with units, we have to be sure we are comparing them in the right order. For example, in the proportion $\frac{20\text{ students}}{1\text{ teacher}}=\frac{60\text{ students}}{3\text{ teachers}}$ we compare the number of students to the number of teachers. We put students in the numerators and teachers in the denominators.

EXAMPLE 1

Write each sentence as a proportion:

a. 3 is to 7 as 15 is to 35.
b. 5 hits in 8 at bats is the same as 30 hits in 48 at-bats.
c. $\$1.50$ for 6 ounces is equivalent to $\$2.25$ for 9 ounces.

Solution

a)	3 is to 7 as 15 is to 35.
Write as a proportion.	$\frac{3}{7} = \frac{15}{35}$

b)	5 hits in 8 at-bats is the same as 30 hits in 48 at-bats.
Write each fraction to compare hits to at-bats.	$\frac{\text{hits}}{\text{at-bats}} = \frac{\text{hits}}{\text{at-bats}}$
Write as a proportion.	$\frac{5}{8} = \frac{30}{48}$

c)	$1.50 for 6 ounces is equivalent to $2.25 for 9 ounces.
Write each fraction to compare dollars to ounces.	$\frac{?}{\text{ounces}} = \frac{?}{\text{ounces}}$
Write as a proportion.	$\frac{1.50}{6} = \frac{2.25}{9}$

TRY IT 1.1

Write each sentence as a proportion:

a. 5 is to 9 as 20 is to 36.
b. 7 hits in 11 at-bats is the same as 28 hits in 44 at-bats.
c. $\$2.50$ for 8 ounces is equivalent to $\$3.75$ for 12 ounces.

Answer

a. $\frac{5}{9} = \frac{20}{36}$
b. $\frac{7}{11} = \frac{28}{44}$
c. $\frac{2.50}{8} = \frac{3.75}{12}$

TRY IT 1.2

Write each sentence as a proportion:

a. 6 is to 7 as 36 is to 42.
b. 8 adults for 36 children is the same as 12 adults for 54 children.

a. $\$3.75$ for 6 ounces is equivalent to $\$2.50$ for 4 ounces.

Answer

a. $\frac{6}{7} = \frac{36}{42}$
b. $\frac{8}{36} = \frac{12}{54}$
c. $\frac{3.75}{6} = \frac{2.50}{4}$

Look at the proportions $\frac{1}{2} = \frac{4}{8}$ and $\frac{2}{3} = \frac{6}{9}$. From our work with equivalent fractions we know these equations are true. But how do we know if an equation is a proportion with equivalent fractions if it contains fractions with larger numbers?

To determine if a proportion is true, we find the **cross products** of each proportion. To find the cross products, we multiply each denominator with the opposite numerator (diagonally across the equal sign). The results are called a cross products because of the cross formed. The cross products of a proportion are equal.

$$8 \cdot 1 = 8 \qquad 2 \cdot 4 = 8 \qquad\qquad 9 \cdot 2 = 18 \qquad 3 \cdot 6 = 18$$
$$\frac{1}{2} = \frac{4}{8} \qquad\qquad\qquad \frac{2}{3} = \frac{6}{9}$$

Cross Products of a Proportion

For any proportion of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$, its cross products are equal.

$$a \cdot d = b \cdot c$$
$$\frac{a}{b} = \frac{c}{d}$$

Cross products can be used to test whether a proportion is true. To test whether an equation makes a proportion, we find the cross products. If they are the equal, we have a proportion.

EXAMPLE 2

Determine whether each equation is a proportion:

a. $\frac{4}{9} = \frac{12}{28}$

b. $\frac{17.5}{37.5} = \frac{7}{15}$

Solution

To determine if the equation is a proportion, we find the cross products. If they are equal, the equation is a proportion.

a)	$\frac{4}{9} = \frac{12}{28}$
Find the cross products.	$\frac{4}{9} = \frac{12}{28}$ $28 \cdot 4 = 112 \qquad 9 \cdot 12 = 108$

Since the cross products are not equal, $28 \cdot 4 \neq 9 \cdot 12$, the equation is not a proportion.

b)	$\frac{17.5}{37.5} = \frac{7}{15}$
Find the cross products.	$\frac{17.5}{37.5} = \frac{7}{15}$ $15 \cdot 17.5 = 262.5 \qquad 37.5 \cdot 7 = 262.5$

Since the cross products are equal, $15 \cdot 17.5 = 37.5 \cdot 7$, the equation is a proportion.

TRY IT 2.1

Determine whether each equation is a proportion:

a. $\frac{7}{9} = \frac{54}{72}$
b. $\frac{24.5}{45.5} = \frac{7}{13}$

Answer

a. no
b. yes

TRY IT 2.2

Determine whether each equation is a proportion:

a. $\frac{8}{9} = \frac{56}{73}$
b. $\frac{28.5}{52.5} = \frac{8}{15}$

Answer

a. no
b. no

Solve Proportions

To solve a proportion containing a variable, we remember that the proportion is an equation. All of the techniques we have used so far to solve equations still apply. In the next example, we will solve a proportion by multiplying by the Least Common Denominator (LCD) using the Multiplication Property of Equality.

EXAMPLE 3

Solve: $\frac{x}{63} = \frac{4}{7}$.

Solution

Given fraction:	$\frac{x}{63} = \frac{4}{7}$
To isolate x, multiply both sides by the LCD, 63.	${\color{red}63}\left(\frac{x}{63}\right) = {\color{red}63}\left(\frac{4}{7}\right)$
Simplify.	$x = \frac{9\cdot\cancel{7}\cdot 4}{\cancel{7}}$
Divide the common factors.	$x = 36$
Check: To check our answer, we substitute into the original proportion.	$\frac{x}{63} = \frac{4}{7}$
Substitute $x = {\color{red}36}$	$\frac{{\color{red}36}}{63} \stackrel{?}{=} \frac{4}{7}$
Show common factors.	$\frac{4\cdot 9}{7\cdot 9} \stackrel{?}{=} \frac{4}{7}$
Simplify.	$\frac{4}{7} = \frac{4}{7}\checkmark$

TRY IT 3.1

Solve the proportion: $\frac{n}{84} = \frac{11}{12}$.

Answer

77

TRY IT 3.2

Solve the proportion: $\frac{y}{96} = \frac{13}{12}$.

Answer

104

When the variable is in a denominator, we'll use the fact that the cross products of a proportion are equal to solve the proportions.

We can find the cross products of the proportion and then set them equal. Then we solve the resulting equation using our familiar techniques.

EXAMPLE 4

Solve: $\frac{144}{a} = \frac{9}{4}$.

Solution

Notice that the variable is in the denominator, so we will solve by finding the cross products and setting them equal.

Given fraction	$\frac{144}{a} = \frac{9}{4}$
Find the cross products and set them equal.	$4 \cdot 144 = a \cdot 9$
Simplify.	$576 = 9a$
Divide both sides by 9.	$\frac{576}{9} = \frac{9a}{9}$
Simplify.	$64 = a$
Check your answer:	$\frac{144}{a} = \frac{9}{4}$
Substitute $a = \color{red}{64}$	$\frac{144}{\color{red}{64}} \stackrel{?}{=} \frac{9}{4}$
Show common factors.	$\frac{9 \cdot 16}{4 \cdot 16} \stackrel{?}{=} \frac{9}{4}$
Simplify.	$\frac{9}{4} = \frac{9}{4} \checkmark$

Another method to solve this would be to multiply both sides by the LCD, $4a$. Try it and verify that you get the same solution.

TRY IT 4.1

Solve the proportion: $\frac{91}{b} = \frac{7}{5}$.

Answer

65

TRY IT 4.2

Solve the proportion: $\frac{39}{c} = \frac{13}{8}$.

Answer

24

EXAMPLE 5

Solve: $\frac{52}{91} = \frac{-4}{y}$.

Solution

Find the cross products and set them equal.	$\frac{52}{91} = \frac{-4}{y}$ $y \cdot 52 = 91 \cdot (-4)$
Simplify.	$52y = -364$
Divide both sides by 52.	$\frac{52y}{52} = \frac{-364}{52}$
Simplify.	$y = -7$
Check:	$\frac{52}{91} \stackrel{?}{=} \frac{-4}{y}$
Substitute $y = 7$	$\frac{52}{91} \stackrel{?}{=} \frac{-4}{-7}$
Show common factors.	$\frac{13 \cdot 4}{13 \cdot 7} \stackrel{?}{=} \frac{-4}{-7}$
Simplify.	$\frac{4}{7} = \frac{4}{7} \checkmark$

TRY IT 5.1

Solve the proportion: $\frac{84}{98} = \frac{-6}{x}$.

Answer

−7

TRY IT 5.2

Solve the proportion: $\frac{-7}{y} = \frac{105}{135}$.

Answer

−9

Solve Applications Using Proportions

The strategy for solving applications that we have used earlier in this chapter, also works for proportions, since proportions are equations. When we set up the proportion, we must make sure the units are correct—the units in the numerators match and the units in the denominators match.

EXAMPLE 6

When pediatricians prescribe acetaminophen to children, they prescribe 5 millilitres (ml) of acetaminophen for every 25 pounds of the child's weight. If Zoe weighs 80 pounds, how many millilitres of acetaminophen will her doctor prescribe?

Solution

Identify what you are asked to find.	How many ml of acetaminophen the doctor will prescribe
Choose a variable to represent it.	Let $a =$ ml of acetaminophen.
Write a sentence that gives the information to find it.	If 5 ml is prescribed for every 25 pounds, how much will be prescribed for 80 pounds?
Translate into a proportion.	$\frac{\text{ml}}{\text{pounds}} = \frac{\text{ml}}{\text{pounds}}$
Substitute given values—be careful of the units.	$\frac{5}{25} = \frac{a}{80}$
Multiply both sides by 80.	$80 \cdot \frac{5}{25} = 80 \cdot \frac{a}{80}$
Multiply and show common factors.	$\frac{16 \cdot 5 \cdot 5}{5 \cdot 5} = \frac{80a}{80}$
Simplify.	$16 = a$
Check if the answer is reasonable.	Yes. Since 80 is about 3 times 25, the medicine should be about 3 times 5.
Write a complete sentence.	The pediatrician would prescribe 16 ml of acetaminophen to Zoe.

You could also solve this proportion by setting the cross products equal.

TRY IT 6.1

Pediatricians prescribe 5 millilitre s (ml) of acetaminophen for every 25 pounds of a child's weight. How many millilitre s of acetaminophen will the doctor prescribe for Emilia, who weighs 60 pounds?

Answer

12 ml

TRY IT 6.2

For every 1 kilogram (kg) of a child's weight, pediatricians prescribe 15 milligrams (mg) of a fever reducer. If Isabella weighs 12 kg, how many milligrams of the fever reducer will the pediatrician prescribe?

Answer

180 mg

EXAMPLE 7

One brand of microwave popcorn has 120 calories per serving. A whole bag of this popcorn has 3.5 servings. How many calories are in a whole bag of this microwave popcorn?

Solution

Identify what you are asked to find.	How many calories are in a whole bag of microwave popcorn?
Choose a variable to represent it.	Let c = number of calories.
Write a sentence that gives the information to find it.	If there are 120 calories per serving, how many calories are in a whole bag with 3.5 servings?
Translate into a proportion.	$\frac{\text{calories}}{\text{serving}} = \frac{\text{calories}}{\text{serving}}$
Substitute given values.	$\frac{120}{1} = \frac{c}{3.5}$
Multiply both sides by 3.5.	$(3.5)\frac{120}{1} = (3.5)\frac{c}{3.5}$
Simplify.	$420 = c$
Check if the answer is reasonable.	Yes. Since 3.5 is between 3 and 4, the total calories should be between 360 (3·120) and 480 (4·120).
Write a complete sentence.	The whole bag of microwave popcorn has 420 calories.

TRY IT 7.1

Marissa loves the Caramel Macchiato at the coffee shop. The 16 oz. medium size has 240 calories. How many calories will she get if she drinks the large 20 oz. size?

Answer

300

TRY IT 7.2

Yaneli loves Starburst candies, but wants to keep her snacks to 100 calories. If the candies have 160 calories for 8 pieces, how many pieces can she have in her snack?

Answer

5

EXAMPLE 8

Josiah went to Mexico for spring break and changed $\$325$ dollars into Mexican pesos. At that time, the exchange rate had $\$1$ U.S. is equal to 12.54 Mexican pesos. How many Mexican pesos did he get for his trip?

Solution

Identify what you are asked to find.	How many Mexican pesos did Josiah get?
Choose a variable to represent it.	Let p = number of pesos.
Write a sentence that gives the information to find it.	If $1 U.S. is equal to 12.54 Mexican pesos, then $325 is how many pesos?
Translate into a proportion.	$\dfrac{\$}{\text{pesos}} = \dfrac{\$}{\text{pesos}}$
Substitute given values.	$\dfrac{1}{12.54} = \dfrac{325}{p}$
The variable is in the denominator, so find the cross products and set them equal.	$p \cdot 1 = 12.54(325)$
Simplify.	$c = 4,075.50$
Check if the answer is reasonable.	Yes, $100 would be $1,254 pesos. $325 is a little more than 3 times this amount.
Write a complete sentence.	Josiah has 4075.50 pesos for his spring break trip.

TRY IT 8.1

Yurianna is going to Europe and wants to change $\$800$ dollars into Euros. At the current exchange rate, $\$1$ Canadian dollar is equal to 0.65 Euro. How many Euros will she have for her trip?

Answer

520 Euros

TRY IT 8.2

Corey and Nicole are traveling to Japan and need to exchange CAD $\$600$ into Japanese yen. If each dollar is 90.27 yen, how many yen will they get?

Answer

54,162 yen

Write Percent Equations As Proportions

Previously, we solved percent equations by applying the properties of equality we have used to solve equations throughout this text. Some people prefer to solve percent equations by using the proportion method. The proportion method for solving percent problems involves a percent proportion. A **percent proportion** is an equation where a percent is equal to an equivalent ratio.

For example, $60\% = \frac{60}{100}$ and we can simplify $\frac{60}{100} = \frac{3}{5}$. Since the equation $\frac{60}{100} = \frac{3}{5}$ shows a percent equal to an equivalent ratio, we call it a percent proportion. Using the vocabulary we used earlier:

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$
$$\frac{3}{5} = \frac{60}{100}$$

Percent Proportion

The amount is to the base as the percent is to 100.

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

If we restate the problem in the words of a proportion, it may be easier to set up the proportion:

$$\text{The amount is to the base as the percent is to one hundred.}$$

We could also say:

$$\text{The amount out of the base is the same as the percent out of one hundred.}$$

First we will practice translating into a percent proportion. Later, we'll solve the proportion.

EXAMPLE 9

Translate to a proportion. What number is 75% of 90?

Solution

If you look for the word “of”, it may help you identify the base.

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}}$ is $\underbrace{75\%}_{\text{percent}}$ of $\underbrace{90?}_{\text{base}}$
Restate as a proportion.	What number out of 90 is the same as 75 out of 100?
Set up the proportion. Let $n = \text{number.}$	$\frac{n}{90} = \frac{75}{100}$

TRY IT 9.1

Translate to a proportion: What number is 60% of $105?$

Answer

$\frac{n}{105} = \frac{60}{100}$

TRY IT 9.2

Translate to a proportion: What number is 40% of $85?$

Answer

$\frac{n}{85} = \frac{40}{100}$

EXAMPLE 10

Translate to a proportion. 19 is 25% of what number?

Solution

Identify the parts of the percent proportion.	$\underbrace{19}_{\text{amount}}$ is $\underbrace{25\%}_{\text{percent}}$ of $\underbrace{\text{what number?}}_{\text{base}}$
Restate as a proportion.	19 out of what number is the same as 25 out of 100?
Set up the proportion. Let $n = \text{number.}$	$\frac{19}{n} = \frac{25}{100}$

TRY IT 10.1

Translate to a proportion: 36 is 25% of what number?

Answer
$\frac{36}{n} = \frac{25}{100}$

TRY IT 10.2

Translate to a proportion: 27 is 36% of what number?

Answer
$\frac{27}{n} = \frac{36}{100}$

EXAMPLE 11

Translate to a proportion. What percent of 27 is 9?

Solution

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{percent}$ of $\underbrace{27}_{\text{base}}$ is $\underbrace{9?}_{\text{amount}}$
Restate as a proportion.	9 out of 27 is the same as what number out of 100?
Set up the proportion. Let $p = \text{percent}$.	$\frac{9}{27} = \frac{p}{100}$

TRY IT 11.1

Translate to a proportion: What percent of 52 is 39?

Answer
$\frac{n}{100} = \frac{39}{52}$

TRY IT 11.2

Translate to a proportion: What percent of 92 is 23?

Answer

$\frac{n}{100} = \frac{23}{92}$

Translate and Solve Percent Proportions

Now that we have written percent equations as proportions, we are ready to solve the equations.

EXAMPLE 12

Translate and solve using proportions: What number is 45% of $80?$

Solution

Identify the parts of the percent proportion.	$\underbrace{\text{What number}}_{\text{amount}} \text{ is } \underbrace{45\%}_{\text{percent}} \text{ of } \underbrace{80?}_{\text{base}}$
Restate as a proportion.	What number out of 80 is the same as 45 out of 100?
Set up the proportion. Let $n =$ number.	$\frac{n}{80} = \frac{45}{100}$
Find the cross products and set them equal.	$100 \cdot n = 80 \cdot 45$
Simplify.	$100n = 3,600$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,600}{100}$
Simplify.	$n = 36$
Check if the answer is reasonable.	Yes. 45 is a little less than half of 100 and 36 is a little less than half 80.
Write a complete sentence that answers the question.	36 is 45% of 80.

TRY IT 12.1

Translate and solve using proportions: What number is 65% of $40?$

Answer

26

TRY IT 12.2

Translate and solve using proportions: What number is 85% of 40?

Answer
34

In the next example, the percent is more than 100, which is more than one whole. So the unknown number will be more than the base.

EXAMPLE 13

Translate and solve using proportions: 125% of 25 is what number?

Solution

Identify the parts of the percent proportion.	$\underbrace{125\%}_{\text{percent}}$ is $\underbrace{25}_{\text{base}}$ of $\underbrace{\text{what number}}_{\text{amount}}$?
Restate as a proportion.	What number out of 25 is the same as 125 out of 100?
Set up the proportion. Let n = number.	$\frac{n}{25} = \frac{125}{100}$
Find the cross products and set them equal.	$100 \cdot n = 25 \cdot 125$
Simplify.	$100n = 3,125$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,125}{100}$
Simplify.	$n = 31.25$
Check if the answer is reasonable.	Yes. 125 is more than 100 and 31.25 is more than 25.
Write a complete sentence that answers the question.	125% of 25 is 31.25.

TRY IT 13.1

Translate and solve using proportions: 125% of 64 is what number?

Answer
80

TRY IT 13.2

Translate and solve using proportions: 175% of 84 is what number?

Answer

147

Percents with decimals and money are also used in proportions.

EXAMPLE 14

Translate and solve: 6.5% of what number is $\$1.56?$

Solution

Identify the parts of the percent proportion.	$\underbrace{6.5\%}_{\text{percent}}$ of $\underbrace{\text{what number}}_{\text{base}}$ is $\underbrace{\$1.56?}_{\text{amount}}$
Restate as a proportion.	$\$1.56$ out of what number is the same as 6.5 out of $100?$
Set up the proportion. Let n = number.	$\frac{1.56}{n}=\frac{6.5}{100}$
Find the cross products and set them equal.	$100(1.56)=n\cdot 6.5$
Simplify.	$156=6.5n$
Divide both sides by 6.5 to isolate the variable.	$\frac{156}{6.5}=\frac{6.5n}{6.5}$
Simplify.	$24=n$
Check if the answer is reasonable.	Yes. 6.5% is a small amount and \$1.56 is much less than \$24.
Write a complete sentence that answers the question.	6.5% of \$24 is \$1.56.

TRY IT 14.1

Translate and solve using proportions: 8.5% of what number is $\$3.23?$

Answer

38

TRY IT 14.2

Translate and solve using proportions: 7.25% of what number is $\$4.64$?

Answer

64

EXAMPLE 15

Translate and solve using proportions: What percent of 72 is 9?

Solution

Identify the parts of the percent proportion.	$\underbrace{\text{What percent}}_{\text{percent}}\ \text{of}\ \underbrace{72}_{\text{base}}\ \text{is}\ \underbrace{9?}_{\text{amount}}$
Restate as a proportion.	9 out of 72 is the same as what number out of 100?
Set up the proportion. Let n $=$ number.	$\frac{9}{72}=\frac{n}{100}$
Find the cross products and set them equal.	$72\cdot n=100\cdot 9$
Simplify.	$72n=900$
Divide both sides by 72.	$\frac{72n}{72}=\frac{900}{72}$
Simplify.	$n=12.5$
Check if the answer is reasonable.	Yes. 9 is $\frac{1}{8}$ of 72 and $\frac{1}{8}$ is 12.5%.
Write a complete sentence that answers the question.	12.5% of 72 is 9.

TRY IT 15.1

Translate and solve using proportions: What percent of 72 is 27?

Answer

37.5%

TRY IT 15.2

Translate and solve using proportions: What percent of 92 is 23?

Answer
25%

Key Concepts

- **Proportion**
  - A proportion is an equation of the form $\frac{a}{b}=\frac{c}{d}$, where $b\neq 0, d\neq 0$.The proportion states two ratios or rates are equal. The proportion is read "a is to b, as c is to d".
- **Cross Products of a Proportion**
  - For any proportion of the form $\frac{a}{b}=\frac{c}{d}$, where $b\neq 0$, its cross products are equal: $a\cdot d=b\cdot c$.
- **Percent Proportion**
  - The amount is to the base as the percent is to 100. $\frac{\text{amount}}{\text{base}}=\frac{\text{percent}}{100}$

Glossary

proportion
A proportion is an equation of the form $\frac{a}{b}=\frac{c}{d}$, where $b\neq 0, d\neq 0$.The proportion states two ratios or rates are equal. The proportion is read "a is to b, as c is to d".

Practice Makes Perfect

Use the Definition of Proportion

In the following exercises, write each sentence as a proportion.

1. 4 is to 15 as 36 is to 135.	2. 7 is to 9 as 35 is to 45.
3. 12 is to 5 as 96 is to 40.	4. 15 is to 8 as 75 is to 40.
5. 5 wins in 7 games is the same as 115 wins in 161 games.	6. 4 wins in 9 games is the same as 36 wins in 81 games.
7. 8 campers to 1 counsellor is the same as 48 campers to 6 counsellors.	8. 6 campers to 1 counselor is the same as 48 campers to 8 counselors.
9. $\$9.36$ for 18 ounces is the same as $\$2.60$ for 5 ounces.	10. $\$3.92$ for 8 ounces is the same as $\$1.47$ for 3 ounces.
11. $\$18.04$ for 11 pounds is the same as $\$4.92$ for 3 pounds.	12. $\$12.42$ for 27 pounds is the same as $\$5.52$ for 12 pounds.

In the following exercises, determine whether each equation is a proportion.

13. $\frac{7}{15} = \frac{56}{120}$	14. $\frac{5}{12} = \frac{45}{108}$
15. $\frac{11}{6} = \frac{21}{16}$	16. $\frac{9}{4} = \frac{39}{34}$
17. $\frac{12}{18} = \frac{4.99}{7.56}$	18. $\frac{9}{16} = \frac{2.16}{3.89}$
19. $\frac{13.5}{8.5} = \frac{31.05}{19.55}$	20. $\frac{10.1}{8.4} = \frac{3.03}{2.52}$

Solve Proportions

In the following exercises, solve each proportion.

21. $\frac{x}{56} = \frac{7}{8}$	22. $\frac{n}{91} = \frac{8}{13}$
23. $\frac{49}{63} = \frac{z}{9}$	24. $\frac{56}{72} = \frac{y}{9}$
25. $\frac{5}{a} = \frac{65}{117}$	26. $\frac{4}{b} = \frac{64}{144}$
27. $\frac{98}{154} = \frac{-7}{p}$	28. $\frac{72}{156} = \frac{-6}{q}$
29. $\frac{a}{-8} = \frac{-42}{48}$	30. $\frac{b}{-7} = \frac{-30}{42}$
31. $\frac{2.6}{3.9} = \frac{c}{3}$	32. $\frac{2.7}{3.6} = \frac{d}{4}$
33. $\frac{2.7}{j} = \frac{0.9}{0.2}$	34. $\frac{2.8}{k} = \frac{2.1}{1.5}$
35. $\frac{\frac{1}{2}}{1} = \frac{m}{8}$	36. $\frac{\frac{1}{3}}{3} = \frac{9}{n}$

Solve Applications Using Proportions

In the following exercises, solve the proportion problem.

37. Pediatricians prescribe 5 millilitre s (ml) of acetaminophen for every 25 pounds of a child's weight. How many millilitres of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?	38. Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child's weight. How many milligrams will the doctor prescribe?
39. At the gym, Carol takes her pulse for 10 sec and counts 19 beats. How many beats per minute is this? Has Carol met her target heart rate of 140 beats per minute?	40. Kevin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds. How many beats per minute is this? Has Kevin met his target heart rate?
41. A new energy drink advertises 106 calories for 8 ounces. How many calories are in 12 ounces of the drink?	42. One 12 ounce can of soda has 150 calories. If Josiah drinks the big 32 ounce size from the local mini-mart, how many calories does he get?
43. Karen eats $\frac{1}{2}$ cup of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?	44. An oatmeal cookie recipe calls for $\frac{1}{2}$ cup of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?
45. Janice is traveling to the US and will change $\$250$ Canadian dollars into US dollars. At the current exchange rate, $\$1$ Canadian is equal to $\$0.70$ US. How many Canadian dollars will she get for her trip?	46. Todd is traveling to Mexico and needs to exchange $\$450$ into Mexican pesos. If each dollar is worth 17.20 pesos, how many pesos will he get for his trip?
47. Steve changed $\$782$ into 507.08 Euros. How many Euros did he receive per Canadian dollar?	48. Martha changed $\$350$ Canadian into 392.28 Australian dollars. How many Australian dollars did she receive per US dollar?
49. At the laundromat, Lucy changed $\$12.00$ into quarters. How many quarters did she get?	50. When she arrived at a casino, Gerty changed $\$20$ into nickels. How many nickels did she get?
51. Jesse's car gets 30 miles per gallon of gas. If Toronto is 285 miles away, how many gallons of gas are needed to get there and then home? If gas is $\$3.09$ per gallon, what is the total cost of the gas for the trip?	52. Danny wants to drive to Banff to see his grandfather. Banff is 370 miles from Danny's home and his car gets 18.5 miles per gallon. How many gallons of gas will Danny need to get to and from Banff? If gas is $\$3.19$ per gallon, what is the total cost for the gas to drive to see his grandfather?
53. Hugh leaves early one morning to drive from his home in White Rock to go to Edmonton, 702 miles away. After 3 hours, he has gone 190 miles. At that rate, how long will the whole drive take?	54. Kelly leaves her home in Seattle to drive to Spokane, a distance of 280 miles. After 2 hours, she has gone 152 miles. At that rate, how long will the whole drive take?
55. Phil wants to fertilize his lawn. Each bag of fertilizer covers about $4,000$ square feet of lawn. Phil's lawn is approximately $13,500$ square feet. How many bags of fertilizer will he have to buy?	56. April wants to paint the exterior of her house. One gallon of paint covers about 350 square feet, and the exterior of the house measures approximately 2000 square feet. How many gallons of paint will she have to buy?

Write Percent Equations as Proportions

In the following exercises, translate to a proportion.

57. What number is 35% of 250?	58. What number is 75% of 920?
59. What number is 110% of 47?	60. What number is 150% of 64?
61. 45 is 30% of what number?	62. 25 is 80% of what number?
63. 90 is 150% of what number?	64. 77 is 110% of what number?
64. 77 is 110% of what number?	65. What percent of 85 is 17?
66. What percent of 92 is 46?	67. What percent of 260 is 340?
68. What percent of 180 is 220?	

Translate and Solve Percent Proportions

In the following exercises, translate and solve using proportions.

69. What number is 65% of $180?$	70. What number is 55% of $300?$
71. 18% of 92 is what number?	72. 22% of 74 is what number?
73. 175% of 26 is what number?	74. 250% of 61 is what number?
75. What is 300% of $488?$	76. What is 500% of $315?$
77. 17% of what number is $\$7.65?$	78. 19% of what number is $\$6.46?$
79. $\$13.53$ is 8.25% of what number?	80. $\$18.12$ is 7.55% of what number?
81. What percent of 56 is $14?$	82. What percent of 80 is $28?$
83. What percent of 96 is $12?$	84. What percent of 120 is $27?$

Everyday Math

85. **Mixing a concentrate** Sam bought a large bottle of concentrated cleaning solution at the warehouse store. He must mix the concentrate with water to make a solution for washing his windows. The directions tell him to mix 3 ounces of concentrate with 5 ounces of water. If he puts 12 ounces of concentrate in a bucket, how many ounces of water should he add? How many ounces of the solution will he have altogether?	86. **Mixing a concentrate** Travis is going to wash his car. The directions on the bottle of car wash concentrate say to mix 2 ounces of concentrate with 15 ounces of water. If Travis puts 6 ounces of concentrate in a bucket, how much water must he mix with the concentrate?

Writing Exercises

87. To solve "what number is 45% of 350" do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.	88. To solve "what percent of 125 is 25" do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.

Answers

1. $\frac{4}{15} = \frac{36}{135}$	3. $\frac{12}{5} = \frac{96}{40}$	5. $\frac{5}{7} = \frac{115}{161}$
7. $\frac{8}{1} = \frac{48}{6}$	9. $\frac{9.36}{18} = \frac{2.60}{5}$	11. $\frac{18.04}{11} = \frac{4.92}{3}$
13. yes	15. no	17. no
19. yes	21. 49	23. 47
25. 9	27. -11	29. 7
31. 2	33. 0.6	35. 4
37. 9 ml	39. 114, no	41. 159 cal
43. $\frac{3}{4}$ cup	45. $175.00	47. 0.65
49. 48 quarters	51. 19, $58.71	53. 11.1 hours
55. 4 bags	57. $\frac{n}{250} = \frac{35}{100}$	59. $\frac{n}{47} = \frac{110}{100}$
61. $\frac{45}{n} = \frac{30}{100}$	63. $\frac{90}{n} = \frac{150}{100}$	65. $\frac{17}{85} = \frac{p}{100}$
67. $\frac{340}{260} = \frac{p}{100}$	69. 117	70. 165
71. 16.56	73. 45.5	75. 1464
77. $45	79. $164	81. 25%
83. 12.5%	85. 20, 32	87. Answers will vary.

Attributions

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3.4 Solve General Applications of Percent

Learning Objectives

By the end of this section, you will be able to:

- Translate and solve basic percent equations
- Solve applications of percent
- Find percent increase and percent decrease

Translate and Solve Basic Percent Equations

In the last section, we solved percent problems by setting them up as proportions. That is the best method available when you did not have the tools of algebra. Now, in this section we will translate word sentences into algebraic equations, and then solve the percent equations.

We'll look at a common application of percent—tips to a server at a restaurant—to see how to set up a basic percent application.

When Kim and her friends went on a road trip to Vancouver, they ate lunch at Marta's Cafe Tower. The bill came to $\$80$. They wanted to leave a 20% tip. What amount would the tip be?

To solve this, we want to find what *amount* is 20% of $\$80$. The $\$80$ is called the *base*. The amount of the tip would be $0.20\,(80)$, or $\$16$ See (Figure 1). To find the amount of the tip, we multiplied the percent by the base.

A 20% tip for an $\$80$ restaurant bill comes out to $\$16$.

Figure 1.(credit: Marta Oraniewicz)

In the next examples, we will find the amount. We must be sure to change the given percent to a decimal when we translate the words into an equation.

EXAMPLE 1

What number is 35% of $90?$

Solution

Translate into algebra. Let n $=$the number.	$\underbrace{\text{What number}}_{\text{n}}\ \underbrace{\text{is}}_{=}\ \underbrace{35\%}_{0.35}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{90?}_{90}$ $n=0.35\cdot 90$
Multiply.	$n=31.5$
	31.5 is 35% of 90

TRY IT 1.1

What number is 45% of $80?$

Answer
36

TRY IT 1.2

What number is 55% of $60?$

Answer
33

EXAMPLE 2

125% of 28 is what number?

Solution

Translate into algebra. Let a $=$the number.	$\underbrace{125\%}_{1.25}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{28}_{28}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what number?}}_{a}$ $1.25 \cdot 28 = a$
Multiply.	$35 = a$
	125% of 28 is 35.

Remember that a percent over 100 is a number greater than 1. We found that 125% of 28 is 35, which is greater than 28.

TRY IT 2.1

150% of 78 is what number?

Answer

117

TRY IT 2.2

175% of 72 is what number?

Answer

126

In the next examples, we are asked to find the base.

EXAMPLE 3

Translate and solve: 36 is 75% of what number?

Solution

Translate. Let b = the number.	$\underbrace{36}_{36}\ \underbrace{\text{is}}_{=}\ \underbrace{75\%}_{0.75}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{what number?}}_{b}$ $36 = 0.75 \cdot b$
Divide both sides by 0.75.	$\frac{36}{0.75} = \frac{0.75b}{0.75}$
Simplify.	$48 = b$
	36 is 75% of 48

TRY IT 3.1

17 is 25% of what number?

Answer

68

TRY IT 3.2

40 is 62.5% of what number?

Answer

64

EXAMPLE 4

6.5% of what number is $\$1.17$?

Solution

Translate. Let b = the number.	$\underbrace{6.5\%}_{0.065}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{what number}}_{\text{b}}\ \underbrace{\text{is}}_{=}\ \underbrace{\$1.17?}_{1.17}$ $0.065 \cdot b = 1.17$
Divide both sides by 0.065.	$\frac{0.065n}{0.065} = \frac{1.17}{0.065}$
Simplify.	$n = 18$
	6.5% of $\$18$ is $\$1.17$

TRY IT 4.1

7.5% of what number is $\$1.95$?

Answer
$26

TRY IT 4.1

8.5% of what number is $\$3.06$?

Answer
$36

In the next examples, we will solve for the percent.

EXAMPLE 5

What percent of 36 is 9?

Solution

Translate into algebra. Let $p =$ the percent.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{36}_{36}\ \underbrace{\text{is}}_{=}\ \underbrace{9?}_{9}$ $p \cdot 36 = 9$
Divide by 36.	$\frac{36p}{36} = \frac{9}{36}$
Simplify.	$p = \frac{1}{4}$
Convert to decimal form.	$p = 0.25$
Convert to percent.	$p = 25\%$
	25% of 36 is 9

TRY IT 5.1

What percent of 76 is 57?

Answer

75%

TRY IT 5.2

What percent of 120 is $96?$

Answer
80%

EXAMPLE 6

144 is what percent of $96?$

Solution

Translate into algebra. Let p $=$ the percent.	$\underbrace{144}_{144}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{96?}_{96}$ $144 = 9 \cdot 96$
Divide by 96.	$\frac{144}{\color{red}{96}} = \frac{96p}{\color{red}{96}}$
Simplify.	$1.5 = p$
Convert to percent.	$150\% = p$
	144 is 150% of 96.

TRY IT 6.1

110 is what percent of $88?$

Answer
125%

TRY IT 6.2

126 is what percent of $72?$

Answer

175%

Solve Applications of Percent

Many applications of percent occur in our daily lives, such as tips, sales tax, discount, and interest. To solve these applications we'll translate to a basic percent equation, just like those we solved in the previous examples in this section. Once you translate the sentence into a percent equation, you know how to solve it.

We will update the strategy we used in our earlier applications to include equations now. Notice that we will translate a sentence into an equation.

HOW TO: *Solve an Application*

1. Identify what you are asked to find and choose a variable to represent it.
2. Write a sentence that gives the information to find it.
3. Translate the sentence into an equation.
4. Solve the equation using good algebra techniques.
5. Check the answer in the problem and make sure it makes sense.
6. Write a complete sentence that answers the question.

Now that we have the strategy to refer to, and have practiced solving basic percent equations, we are ready to solve percent applications. Be sure to ask yourself if your final answer makes sense—since many of the applications we'll solve involve everyday situations, you can rely on your own experience.

EXAMPLE 7

Dezohn and his girlfriend enjoyed a dinner at a restaurant, and the bill was $\$68.50$. They want to leave an 18% tip. If the tip will be 18% of the total bill, how much should the tip be?

Solution

What are you asked to find?	The amount of the tip
Choose a variable to represent it.	Let t $=$ amount of tip.
Write a sentence that give the information to find it.	The tip is 18% of the total bill.
Translate the sentence into an equation.	$\underbrace{\text{The tip}}_{t}\ \underbrace{\text{is}}_{=}\ \underbrace{18\%}_{0.018}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\$68.50?}_{68.50}$ $t = 0.18 \cdot 68.50$
Multiply.	$t = 12.33$
Check. Is this answer reasonable?	If we approximate the bill to \$70 and the percent to 20%, we would have a tip of \$14. So a tip of \$12.33 seems reasonable.
Write a complete sentence that answers the question.	The couple should leave a tip of \$12.33.

TRY IT 7.1

Cierra and her sister enjoyed a special dinner in a restaurant, and the bill was $\$81.50$. If she wants to leave 18% of the total bill as her tip, how much should she leave?

Answer

\$14.67

TRY IT 7.2

Kimngoc had lunch at her favorite restaurant. She wants to leave 15% of the total bill as her tip. If her bill was $\$14.40$, how much will she leave for the tip?

Answer

\$2.16

EXAMPLE 8

The label on Masao's breakfast cereal said that one serving of cereal provides 85 milligrams (mg) of potassium, which is 2% of the recommended daily amount. What is the total recommended daily amount of potassium?

Nutrition Facts

Serving Size: 1 cup (47g)

Servings Per Container: About 7

Amount Per Serving	Cereal	With Milk
Calories	180	230
Calories from Fat	10	20
	% Daily Value*	
Total Fat 1g	**2%**	**2%**
Saturated Fat 0g	**0%**	**0%**
Trans Fat 0g		
Polyunsaturated Fat 0.5g		
Monounsaturated Fat 0.5g		
Cholesterol 0mg	**0%**	**0%**
Sodium 190mg	**8%**	**11%**
Potassium 85mg	**2%**	**8%**
Total Carbohydrate 40g	**13%**	**15%**
Dietary Fiber 1g	**4%**	**4%**
Sugars 8g		
Protein 3g		

Solution

What are you asked to find?	the total amount of potassium recommended
Choose a variable to represent it.	Let $a =$ total amount of potassium.
Write a sentence that gives the information to find it.	85 mg is 2% of the total amount.
Translate the sentence into an equation.	$\underbrace{85\text{ mg}}_{85}\ \underbrace{\text{is}}_{=}\ \underbrace{2\%}_{0.02}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{a?}_{a}$ $85 = 0.02 \cdot a$
Divide both sides by 0.02.	$\frac{85}{\color{red}{0.02}} = \frac{0.02a}{\color{red}{0.02}}$
Simplify.	$4,250 = a$
Check: Is this answer reasonable?	Yes. 2% is a small percent and 85 is a small part of 4,250.
Write a complete sentence that answers the question.	The amount of potassium that is recommended is 4250 mg.

TRY IT 8.1

One serving of wheat square cereal has 7 grams of fiber, which is 29% of the recommended daily amount. What is the total recommended daily amount of fiber?

Answer

24.1 grams

TRY IT 8.2

One serving of rice cereal has 190 mg of sodium, which is 8% of the recommended daily amount. What is the total recommended daily amount of sodium?

Answer

2,375 mg

EXAMPLE 9

Mitzi received some gourmet brownies as a gift. The wrapper said each brownie was 480 calories, and had 240 calories of fat. What percent of the total calories in each brownie comes from fat?

Solution

What are you asked to find?	the percent of the total calories from fat
Choose a variable to represent it.	Let p = percent from fat.
Write a sentence that gives the information to find it.	What percent of 480 is 240?
Translate the sentence into an equation.	$\underbrace{\text{What percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{480}_{480}\ \underbrace{\text{is}}_{=}\ \underbrace{240?}_{240}$ $p \cdot 480 = 240$
Divide both sides by 480.	$\frac{480p}{\color{red}{480}} = \frac{240}{\color{red}{480}}$
Simplify.	$p = 0.5$
Convert to percent form.	$p = 50\%$
Check. Is this answer reasonable?	Yes. 240 is half of 480, so 50% makes sense.
Write a complete sentence that answers the question.	Of the total calories in each brownie, 50% is fat.

TRY IT 9.1

Veronica is planning to make muffins from a mix. The package says each muffin will be 230 calories and 60 calories will be from fat. What percent of the total calories is from fat? (Round to the nearest whole percent.)

Answer

26%

Exercises

The brownie mix Ricardo plans to use says that each brownie will be 190 calories, and 70 calories are from fat. What percent of the total calories are from fat?

Answer

37%

Find Percent Increase and Percent Decrease

People in the media often talk about how much an amount has increased or decreased over a certain period of time. They usually express this increase or decrease as a percent.

To find the percent increase, first we find the amount of increase, which is the difference between the new amount and the original amount. Then we find what percent the amount of increase is of the original amount.

HOW TO: *Find Percent Increase*

Step 1. Find the amount of increase.

- $\text{increase} = \text{new amount} - \text{original amount}$

Step 2. Find the percent increase as a percent of the original amount.

EXAMPLE 10

In 2017, university tuition fees in Canada for domestic students increased from $\$26$ per school year to $\$36$ per school year. Find the percent increase. (Round to the nearest tenth of a percent.)

Solution

What are you asked to find?	the percent increase
Choose a variable to represent it.	Let p = percent.
Find the amount of increase.	$\text{increase} = \text{new amount} - \text{original amount}$ $= \$36 - \$26 = \$10$
Find the percent increase.	The increase is what percent of the original amount?
Translate to an equation.	$\underbrace{10}_{10}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{26?}_{26}$ $10 = p \cdot 26$
Divide both sides by 26.	$\frac{10}{26} = \frac{26p}{26}$
Round to the nearest thousandth.	$0.384 = p$
Convert to percent form.	$38.4\% = p$
Write a complete sentence.	The new fees represent a 38.4% increase over the old fees.

TRY IT 10.1

In 2011, the IRS increased the deductible mileage cost to 55.5 cents from 51 cents. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

8.8%

TRY IT 10.2

In 1984, the standard bus fare in Vancouver was $\$1.25$. In 2008, the standard bus fare was $\$2.50$. Find the percent increase. (Round to the nearest tenth of a percent.)

Answer

100%

Finding the percent decrease is very similar to finding the percent increase, but now the amount of decrease is the difference between the original amount and the final amount. Then we find what percent the amount of decrease is of the original amount.

HOW TO: *Find Percent Decrease*

1. Find the amount of decrease.
   - $\text{decrease} = \text{original amount} - \text{new amount}$
2. Find the percent decrease as a percent of the original amount.

EXAMPLE 11

The average price of a gallon of gas in one city in June 2014 was $\$3.71$. The average price in that city in July was $\$3.64$. Find the percent decrease.

Solution

What are you asked to find?	the percent decrease
Choose a variable to represent it.	Let p = percent.
Find the amount of decrease.	$\text{decrease} = \text{original amount} - \text{new amount}$ $\$3.71 - \$3.64 = \$0.07$
Find the percent of decrease.	The decrease is what percent of the original amount?
Translate to an equation.	$\underbrace{0.07}_{0.07}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{what percent}}_{p}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{3.71?}_{3.71}$ $0.07 = p \cdot 3.71$
Divide both sides by 3.71.	$\dfrac{0.07}{\color{red}{3.71}} = \dfrac{3.71p}{\color{red}{0.07}}$
Round to the nearest thousandth.	$0.019 = p$
Convert to percent form.	$1.9\% = p$
Write a complete sentence.	The price of gas decreased 1.9%.

TRY IT 11.1

The population of one city was about $672,000$ in 2010. The population of the city is projected to be about $630,000$ in 2020. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

6.3%

TRY IT 11.2

Last year Sheila's salary was $\$42,000$. Because of furlough days, this year her salary was $\$37,800$. Find the percent decrease. (Round to the nearest tenth of a percent.)

Answer

10%

Access Additional Online Resources

- Percent Increase and Percent Decrease Visualization

Key Concepts

- **Solve an application.**
  1. Identify what you are asked to find and choose a variable to represent it.
  2. Write a sentence that gives the information to find it.
  3. Translate the sentence into an equation.
  4. Solve the equation using good algebra techniques.
  5. Write a complete sentence that answers the question.
  6. Check the answer in the problem and make sure it makes sense.
- **Find percent increase.**
  1. Find the amount of increase:
     $\text{increase} = \text{new amount} - \text{original amount}$
  2. Find the percent increase as a percent of the original amount.
- **Find percent decrease.**
  1. Find the amount of decrease.
     $\text{decrease} = \text{original amount} - \text{new amount}$
  2. Find the percent decrease as a percent of the original amount.

Glossary

percent increase
The percent increase is the percent the amount of increase is of the original amount.
percent decrease
The percent decrease is the percent the amount of decrease is of the original amount.

Practice Makes Perfect

Translate and Solve Basic Percent Equations

In the following exercises, translate and solve.

1. What number is 45% of $120?$	2. What number is 65% of $100?$
3. What number is 24% of $112?$	4. What number is 36% of $124?$
5. 250% of 65 is what number?	6. 150% of 90 is what number?
7. 800% of $2,250$ is what number?	8. 600% of $1,740$ is what number?
9. 28 is 25% of what number?	10. 36 is 25% of what number?
11. 81 is 75% of what number?	12. 93 is 75% of what number?
13. 8.2% of what number is $\$2.87?$	14. 6.4% of what number is $\$2.88?$
15. 11.5% of what number is $\$108.10?$	16. 12.3% of what number is $\$92.25?$
17. What percent of 260 is $78?$	18. What percent of 215 is $86?$
19. What percent of $1,500$ is $540?$	20. What percent of $1,800$ is $846?$
21. 30 is what percent of $20?$	22. 50 is what percent of $40?$
23. 840 is what percent of $480?$	24. 790 is what percent of $395?$

Solve Applications of Percents

In the following exercises, solve the applications of percents.

25. Geneva treated her parents to dinner at their favorite restaurant. The bill was $\$74.25$. She wants to leave 16% of the total bill as a tip. How much should the tip be?	26. When Hiro and his co-workers had lunch at a restaurant the bill was $\$90.50$. They want to leave 18% of the total bill as a tip. How much should the tip be?
27. Trong has 12% of each paycheck automatically deposited to his savings account. His last paycheck was $\$2,165$. How much money was deposited to Trong's savings account?	28. Cherise deposits 8% of each paycheck into her retirement account. Her last paycheck was $\$1,485$. How much did Cherise deposit into her retirement account?
29. One serving of oatmeal has 8 grams of fiber, which is 33% of the recommended daily amount. What is the total recommended daily amount of fiber?	30. One serving of trail mix has 67 grams of carbohydrates, which is 22% of the recommended daily amount. What is the total recommended daily amount of carbohydrates?
31. A bacon cheeseburger at a popular fast food restaurant contains $2,070$ milligrams (mg) of sodium, which is 86% of the recommended daily amount. What is the total recommended daily amount of sodium?	32. A grilled chicken salad at a popular fast food restaurant contains 650 milligrams (mg) of sodium, which is 27% of the recommended daily amount. What is the total recommended daily amount of sodium?
33. The nutrition fact sheet at a fast food restaurant says the fish sandwich has 380 calories, and 171 calories are from fat. What percent of the total calories is from fat?	34. The nutrition fact sheet at a fast food restaurant says a small portion of chicken nuggets has 190 calories, and 114 calories are from fat. What percent of the total calories is from fat?
35. Emma gets paid $\$3,000$ per month. She pays $\$750$ a month for rent. What percent of her monthly pay goes to rent?	36. Dimple gets paid $\$3,200$ per month. She pays $\$960$ a month for rent. What percent of her monthly pay goes to rent?

Find Percent Increase and Percent Decrease

In the following exercises, find the percent increase or percent decrease.

37. Tamanika got a raise in her hourly pay, from $\$15.50$ to $\$17.55$. Find the percent increase.	38. Ayodele got a raise in her hourly pay, from $\$24.50$ to $\$25.48$. Find the percent increase.
39. According to Statistics Canada, annual international graduate student fees in Canada rose from about $\$13,000$ in 2015 to about $\$15,000$ in 2019. Find the percent increase.	40. The price of a share of one stock rose from $\$12.50$ to $\$50$. Find the percent increase.
41. According to Time magazine $(7/19/2011)$ annual global seafood consumption rose from 22 pounds per person in 1960 to 38 pounds per person today. Find the percent increase. (Round to the nearest tenth of a percent.)	42. In one month, the median home price in the Northeast rose from $\$225,400$ to $\$241,500$. Find the percent increase. (Round to the nearest tenth of a percent.)
43. A grocery store reduced the price of a loaf of bread from $\$2.80$ to $\$2.73$. Find the percent decrease.	44. The price of a share of one stock fell from $\$8.75$ to $\$8.54$. Find the percent decrease.
45. Hernando's salary was $\$49,500$ last year. This year his salary was cut to $\$44,055$. Find the percent decrease.	46. From 2000 to 2010, the population of Detroit fell from about $951,000$ to about $714,000$. Find the percent decrease. (Round to the nearest tenth of a percent.)
47. In one month, the median home price in the West fell from $\$203,400$ to $\$192,300$. Find the percent decrease. (Round to the nearest tenth of a percent.)	48. Sales of video games and consoles fell from $\$1,150$ million to $\$1,030$ million in one year. Find the percent decrease. (Round to the nearest tenth of a percent.)

Everyday Math

49. **Tipping** At the campus coffee cart, a medium coffee costs $\$1.65$. MaryAnne brings $\$2.00$ with her when she buys a cup of coffee and leaves the change as a tip. What percent tip does she leave?	50. **Late Fees** Alison was late paying her credit card bill of $\$249$. She was charged a 5% late fee. What was the amount of the late fee?

Writing Exercises

51. Without solving the problem "44 is 80% of what number", think about what the solution might be. Should it be a number that is greater than 44 or less than 44? Explain your reasoning.	52. Without solving the problem "What is 20% of 300?" think about what the solution might be. Should it be a number that is greater than 300 or less than 300? Explain your reasoning.
53. After returning from vacation, Alex said he should have packed 50% fewer shorts and 200% more shirts. Explain what Alex meant.	54. Because of road construction in one city, commuters were advised to plan their Monday morning commute to take 150% of their usual commuting time. Explain what this means.

Answers

1. 54	3. 26.88	5. 162.5
7. 18,000	9. 112	11. 108
13. $35	15. $940	17. 30%
19. 36%	21. 150%	23. 175%
25. $11.88	27. $259.80	29. 24.2 grams
31. 2,407 grams	33. 45%	35. 25%
37. 13.2%	39. 15%	41. 72.7%
43. 2.5%	45. 11%	47. 5.5%
49. 21.2%	51. The original number should be greater than 44.80% is less than 100%, so when 80% is converted to a decimal and multiplied to the base in the percent equation, the resulting amount of 44 is less. 44 is only the larger number in cases where the percent is greater than 100%.	53. Alex should have packed half as many shorts and twice as many shirts.

Attributions

This chapter has been adapted from "Solve General Applications of Percent" in *Prealgebra* (OpenStax) by Lynn Marecek,

3.5 Chapter Review

Review Exercises

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction. Simplify the answer if possible.

1. 56 to 32	2. 28 to 40
3. 1.2 to 1.8	4. 3.5 to 0.5
5. $2\frac{1}{3}$ to $5\frac{1}{4}$	6. $1\frac{3}{4}$ to $1\frac{5}{8}$
7. 28 inches to 3 feet	8. 64 ounces to 30 ounces

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction. Simplify the answer if possible.

9. 90 pounds per 7.5 square inches	10. 180 calories per 8 ounces
11.$\$612.50$ for 35 hours	12. 126 miles in 4 hours

Find Unit Rates

In the following exercises, find the unit rate.

13. 90 pounds per 7.5 square inches	14. 180 calories per 8 ounces
15. $\$612.50$ for 35 hours	16. 126 miles in 4 hours

Find Unit Price

In the following exercises, find the unit price.

17.Highlighters: 6 for $\$2.52$	18. T-shirts: 3 for $\$8.97$
19. Anna bought a pack of 8 kitchen towels for $\$13.20$. How much did each towel cost? Round to the nearest cent if necessary.	20. An office supply store sells a box of pens for $\$11$. The box contains 12 pens. How much does each pen cost?

In the following exercises, find each unit price and then determine the better buy.

21.Vitamins: 60 tablets for $\$6.49$ or 100 for $\$11.99?$	22. Shampoo: 12 ounces for $\$4.29$ or 22 ounces for $\$7.29?$

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.

23. a adults to 45 children	24. 535 miles per h hours
25. the ratio of 19 and the sum of 3 and n	26. the ratio of $4y$ and the difference of x and 10

In the following exercises, write each percent as a ratio.

27. 32% admission rate for the university	28. 53.3% rate of college students with student loans

In the following exercises, write as a ratio and then as a percent.

29. 13 out of 100 architects are women.	30. 9 out of every 100 nurses are men.

In the following exercises, convert each percent to a fraction.

31. 48%	32. 175%
33. 64.1%	34. $8\frac{1}{4}\%$

In the following exercises, convert each percent to a decimal.

35. 6%	36. 23%
37. 128%	38. 4.9%

In the following exercises, convert each percent to a) a simplified fraction and b) a decimal.

39. In $2016, 17\%$ of the Canadian population was age 65 or over. (Source: www12.statcan.gc.ca)	40. In $2016, 16.6\%$ of the Canadian population was under 15 years old. (Source: www12.statcan.gc.ca)
41. When a die is tossed, the probability it will land with an even number of dots on the top side is 50%.	42. A couple plans to have three children. The probability they will all be girls is 12.5%.

In the following exercises, convert each decimal to a percent.

43. 0.04	44. 0.15
45. 2.82	46. 3
47. 0.003	48. 1.395

In the following exercises, convert each fraction to a percent.

49. $\frac{3}{4}$	50. $\frac{11}{5}$
51. $3\frac{5}{8}$	52. $\frac{2}{9}$
53. According to the Centers for Disease Control, $\frac{2}{5}$ of adults do not take a vitamin or supplement.	54. According to the Centers for Disease Control, among adults who do take a vitamin or supplement, $\frac{3}{4}$ take a multivitamin.

In the following exercises, translate and solve.

55. What number is 46% of $350?$	56. 120% of 55 is what number?
57. 84 is 35% of what number?	58. 15 is 8% of what number?
59. 200% of what number is $50?$	60. 7.9% of what number is $\$4.74?$
61. What percent of 120 is $81.6?$	62. What percent of 340 is $595?$

Solve General Applications of Percents

In the following exercises, solve.

63. When Aurelio and his family ate dinner at a restaurant, the bill was $\$83.50$. Aurelio wants to leave 20% of the total bill as a tip. How much should the tip be?	64. One granola bar has 2 grams of fiber, which is 8% of the recommended daily amount. What is the total recommended daily amount of fiber?
65. The nutrition label on a package of granola bars says that each granola bar has 190 calories, and 54 calories are from fat. What percent of the total calories is from fat?	66. Elsa gets paid $\$4,600$ per month. Her car payment is $\$253$. What percent of her monthly pay goes to her car payment?

67. Marta got a gift of $\$1900$ from her uncle. She spent 35% of that money for her trip to Victoria. How much money she has left?.	68. Last year Bernard bought a new car for $\$30,000$. If the value of the car depreciated 20% every year , find the value of the car this year.

Solve Proportions and their Applications

In the following exercises, write each sentence as a proportion.

69. 3 is to 8 as 12 is to 32.	70. 95 miles to 3 gallons is the same as 475 miles to 15 gallons.
71. 1 teacher to 18 students is the same as 23 teachers to 414 students.	72. $\$7.35$ for 15 ounces is the same as $\$2.94$ for 6 ounces.

In the following exercises, determine whether each equation is a proportion.

73. $\frac{5}{13} = \frac{30}{78}$	74. $\frac{16}{7} = \frac{48}{23}$
75. $\frac{12}{18} = \frac{6.99}{10.99}$	76. $\frac{11.6}{9.2} = \frac{37.12}{29.44}$

In the following exercises, solve each proportion.

77. $\frac{x}{36} = \frac{5}{9}$	78. $\frac{7}{a} = \frac{-6}{84}$
79. $\frac{1.2}{1.8} = \frac{d}{6}$	80. $\frac{\frac{1}{2}}{2} = \frac{m}{20}$

In the following exercises, solve the proportion problem.

81. The children's dosage of acetaminophen is 5 millilitre s (ml) for every 25 pounds of a child's weight. How many millilitre s of acetaminophen will be prescribed for a 60 pound child?	82. After a workout, Dennis takes his pulse for 10 sec and counts 21 beats. How many beats per minute is this?
83. An 8 ounce serving of ice cream has 272 calories. If Lavonne eats 10 ounces of ice cream, how many calories does she get?	84. Alma is going to Europe and wants to exchange $\$1,200$ into Euros. If each dollar is 0.65 Euros, how many Euros will Alma get?
85. Zack wants to drive from Abbotsford to Banff, a distance of 494 miles. If his car gets 38 miles to the gallon, how many gallons of gas will Zack need to get to Banff?	86. Teresa is planning a party for 100 people. Each gallon of punch will serve 18 people. How many gallons of punch will she need?

In the following exercises, translate to a proportion.

87. What number is 62% of $395?$	88. 42 is 70% of what number?
89. What percent of $1,000$ is $15?$	90. What percent of 140 is $210?$

In the following exercises, translate and solve using proportions.

91. What number is 85% of $900?$	92. 6% of what number is $\$24?$
93. $\$3.51$ is 4.5% of what number?	94. What percent of $3,100$ is $930?$

In the following exercises, convert each percent to a) a decimal b) a simplified fraction.

95. 24%	96. 5%
97. 350%	

In the following exercises, convert each fraction to a percent. (Round to 3 decimal places if needed.)

98. $\frac{7}{8}$	99. $\frac{1}{3}$
100. $\frac{11}{12}$	

In the following exercises, solve the percent problem.

101. 65 is what percent of $260?$	102. What number is 27% of $3,000?$
103. 150% of what number is $60?$	104. Write as a proportion: 4 gallons to 144 miles is the same as 10 gallons to 360 miles.
105. Vin read 10 pages of a book in 12 minutes. At that rate, how long will it take him to read 35 pages?	

Review Answers

1. $\frac{7}{4}$	3. $\frac{2}{3}$	5. $\frac{4}{9}$
7. $\frac{7}{9}$	9. $\frac{\text{90 pounds}}{\text{7.5 square inches}}$	11. $\frac{\$612.50}{\text{35 hours}}$
13. 12 pounds/sq.in.	15. $17.50/hour	17. $0.42
19. $1.65	21. $0.11, $0.12; 60 tablets for $6.49	23. $\frac{a\text{ adults}}{\text{45 children}}$
25. $\frac{19}{3+n}$	27. $\frac{32}{100}$	29. $\frac{13}{100}, 13\%$
31. $\frac{12}{25}$	33. $\frac{641}{1000}$	35. 0.06
37. 1.28	39. a) $\frac{17}{100}$ b) 0.17	41. a) $\frac{1}{2}$ b) 0.5
43. 4%	45. 282%	47. 0.3%
49. 75%	51. 362.5%	53. 40%
55. 161	57. 240	59. 25
61. 68%	63. $16.70	65. 28.4%
67. 1235	69. $\frac{3}{8} = \frac{12}{32}$	71. $\frac{1}{18} = \frac{23}{414}$
73. yes	75. no	77. 20
79. 4	81. 12	83. 340
87. $\frac{x}{395} = \frac{62}{100}$	89. $\frac{x}{100} = \frac{15}{1000}$	91.765
93. $78	95.$0.24, \frac{6}{25}$	97. $3.5, 3\frac{1}{2}$
99. 33.333%	101.25%	103. 40
105. 42		

Chapter Test

1. Write a ratio as a fraction. Simplify the answer if possible. 42 to 28	2. Write a rate as a fraction. Simplify the answer if possible. 80 pounds per 6.5 square inches
3. Find the unit rate. $\$868.80$ for 24 hours	4. Marta bought a pack of 6 paint brushes for $\$32.20$. How much did each brush cost? Round to the nearest cent if necessary
5. Find each unit price and then the better buy. Laundry detergent: 64 ounces for $\$10.99$ or 48 ounces for $\$8.49$	6. Convert a percent to a fraction: 245%
7. Convert a decimal to a percent: 0.07	8. Convert a fraction to a percent. (Round to 3 decimal places if needed.) $\frac{11}{8}$
9. What number is 36% of 450?	10. 8% of what number is $\$34$?
11. 57.6 is what percent of 360?	12. One granola bar has 3 grams of fiber, which is 12% of the recommended daily amount. What is the total recommended daily amount of fiber?
13. Klaudia is going to Poland and wants to exchange $\$1{,}400$ into Polish zlotych. If each dollar is 2.91 zlotych, how many zlotych will Klaudia get?	14. Solve a proportion: $\frac{24}{x} = \frac{3}{7}$
15. Solve a proportion: $\frac{x}{6} = \frac{9}{24}$	16.Solve a proportion: $\frac{2.4}{1.6} = \frac{t}{6.2}$

Test Answers

1. $\frac{3}{2}$	2. $\frac{160}{13}$	3. \$36.20
4. \$5.37	5. 64 ounces for \$10.99 is the better buy	6. $\frac{49}{20}$
7. 7%	8. 137.5%	9. 162
10. 425	11. 16%	12. 25 grams
13. 4074 zlotych	14. 56	15. 2.25
16. 9.3		

PART II

CHAPTER 4 MEASUREMENT, PERIMETER, AREA, AND VOLUME

Note the many individual shapes in this building.

We are surrounded by all sorts of geometry. Architects use geometry to design buildings. Artists create vivid images out of colorful geometric shapes. Street signs, automobiles, and product packaging all take advantage of geometric properties. In this chapter, we will begin with learning about two measurement systems used in Canada and then we will explore geometry and solve problems related to everyday situations.

Attributions

4.1 Systems of Measurement

Learning Objectives

By the end of this section, you will be able to:

- Make unit conversions in the imperial system
- Use mixed units of measurement in the imperial system
- Make unit conversions in the metric system
- Use mixed units of measurement in the metric system
- Convert between the imperial and the metric systems of measurement
- Convert between Fahrenheit and Celsius temperatures

Make Unit Conversions in the Imperial System

There are two systems of measurement commonly used around the world. Most countries use the metric system. Canada uses the metric system, and the United States use the imperial system of measurement. However, people in Canada often use imperial measurements as well. We will look at the imperial system first.

The imperial system of measurement uses units of inch, foot, yard, and mile to measure length and pound and ton to measure weight. For capacity, the units used are cup, pint, quart, and gallons. Both the imperial system and the metric system measure time in seconds, minutes, and hours.

The equivalencies of measurements are shown in the table below. The table also shows, in parentheses, the common abbreviations for each measurement.

Imperial System of Measurement

Length	$\begin{array}{rcl} 1\text{ foot (ft.)} & = & 12\text{ inches (in.)} \\ 1\text{ yard (yd.)} & = & 3\text{ feet (ft.)} \\ 1\text{ mile (mi.)} & = & 5,280\text{ feet (ft.)} \end{array}$	Volume	$\begin{array}{rcl} 3\text{ teaspoons (t)} & = & 1\text{ tablespoon (T)} \\ 16\text{ tablespoons (T)} & = & 1\text{ cup (C)} \\ 1\text{ cup (C)} & = & 8\text{ fluid ounces (fl. oz.)} \\ 1\text{ pint (pt.)} & = & 2\text{ cups (C)} \\ 1\text{ quart (qt.)} & = & 2\text{ pints (pt.)} \\ 1\text{ gallon (gal)} & = & 4\text{ quarts (qt.)} \end{array}$
Weight	$\begin{array}{rcl} 1\text{ pound (lb.)} & = & 16\text{ ounces (oz.)} \\ 1\text{ ton} & = & 2000\text{ pounds (lb.)} \end{array}$	Time	$\begin{array}{rcl} 1\text{ minute (min)} & = & 60\text{ seconds (sec)} \\ 1\text{ hour (hr)} & = & 60\text{ minutes (min)} \\ 1\text{ day} & = & 24\text{ hours (hr)} \\ 1\text{ week (wk)} & = & 7\text{ days} \\ 1\text{ year (yr)} & = & 365\text{ days} \end{array}$

In many real-life applications, we need to convert between units of measurement, such as feet and yards, minutes and seconds, quarts and gallons, etc. We will use the identity property of multiplication to do these conversions. We'll restate the identity property of multiplication here for easy reference.

Identity Property of Multiplication

For any real number a:

$a\cdot 1=a$	$1\cdot a=a$

1 is the **multiplicative identity**.

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to change inches to feet. We know that 1 foot is equal to 12 inches, so we will write 1 as the fraction $\frac{1\text{ foot}}{12\text{ inches}}$. When we multiply by this fraction we do not change the value, but just change the units.

But $\frac{12\text{ inches}}{1\text{ foot}}$ also equals 1. How do we decide whether to multiply by $\frac{1\text{ foot}}{12\text{ inches}}$ or $\frac{12\text{ inches}}{1\text{ foot}}$? We choose the fraction that will make the units we want to convert *from* divide out. Treat the unit words like factors and "divide out" common units like we do common factors. If we want to convert 66 inches to feet, which multiplication will eliminate the inches?

$$66\text{ inches}\cdot\frac{1\text{ foot}}{12\text{ inches}}\quad\text{or}\quad\cancel{66\text{ inches}\cdot\frac{12\text{ inches}}{1\text{ foot}}}$$

$$\text{The first form works since } 66\ \cancel{\text{inches}}\cdot\frac{1\text{ foot}}{12\ \cancel{\text{inches}}}.$$

The inches divide out and leave only feet. The second form does not have any units that will divide out and so will not help us.

EXAMPLE 1

MaryAnne is 66 inches tall. Convert her height into feet.

Solution

Step 1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.	Multiply 66 inches by 1, writing 1 as a fraction relating inches and feet. We need inches in the denominator so that the inches will divide out!	$66 \text{ inches} \cdot 1$ $66 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}}$
Step 2. Multiply.	Think of 66 inches as $\frac{66 \text{ inches}}{1}$	$\frac{66 \text{ inches} \cdot 1 \text{ foot}}{12 \text{ inches}}$
Step 3. Simplify the fraction.	Notice: inches divide out.	$\frac{66 \cancel{\text{inches}} \cdot 1 \text{ foot}}{12 \cancel{\text{inches}}}$ $= \frac{66 \text{ feet}}{12}$
Step 4. Simplify.	Divide 66 by 12.	5.5 feet

TRY IT 1.1

Lexie is 30 inches tall. Convert her height to feet.

Answer
2.5 feet

TRY IT 1.2

Rene bought a hose that is 18 yards long. Convert the length to feet.

Answer
54 feet

HOW TO: *Make unit conversions*

1. Multiply the measurement to be converted by 1; write 1 as a fraction relating the units given and the units needed.
2. Multiply.
3. Simplify the fraction.
4. Simplify.

When we use the identity property of multiplication to convert units, we need to make sure the units we want to change from will divide out. Usually this means we want the conversion fraction to have those units in the denominator.

EXAMPLE 2

A female orca in the Salish Sea weighs almost 3.2 tons. Convert her weight to pounds.

Solution

We will convert 3.2 tons into pounds. We will use the identity property of multiplication, writing 1 as the fraction $\frac{2000 \text{ pounds}}{1 \text{ ton}}$.

	3.2 tons
Multiply the measurement to be converted, by 1.	$3.2 \text{ tons} \cdot 1$
Write 1 as a fraction relating tons and pounds.	$3.2 \text{ tons} \cdot \frac{2{,}000 \text{ pounds}}{1 \text{ ton}}$
Simplify.	$\frac{3.2 \cancel{\text{tons}} \cdot 2,000 \text{ pounds}}{1 \cancel{\text{ton}}}$
Multiply.	$6{,}400 \text{ pounds}$
	The female orca weighs almost 6,400 pounds.

TRY IT 2.1

Arnold's SUV weighs about 4.3 tons. Convert the weight to pounds.

Answer

8,600 pounds

TRY IT 2.2

The Carnival *Destiny* cruise ship weighs 51,000 tons. Convert the weight to pounds.

Answer

102,000,000 pounds

Sometimes, to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.

EXAMPLE 3

Juliet is going with her family to their summer home. She will be away from her boyfriend for 9 weeks. Convert the time to minutes.

Solution
To convert weeks into minutes we will convert weeks into days, days into hours, and then hours into minutes. To do this we will multiply by conversion factors of 1.

	9 weeks
Write 1 as $\frac{7\text{ days}}{1\text{ week}}$, $\frac{24\text{ hours}}{1\text{ day}}$, and $\frac{60\text{ minutes}}{1\text{ hour}}$.	$\frac{9\text{ wk}}{1}\cdot\frac{7\text{ days}}{1\text{ wk}}\cdot\frac{24\text{ hr}}{1\text{ day}}\cdot\frac{60\text{ min}}{1\text{ hr}}$
Divide out the common units.	$\frac{9\cancel{\text{ wk}}}{1}\cdot\frac{7\cancel{\text{ days}}}{1\cancel{\text{ wk}}}\cdot\frac{24\cancel{\text{ hr}}}{1\cancel{\text{ day}}}\cdot\frac{60\text{ min}}{1\cancel{\text{ hr}}}$
Multiply.	$\frac{9\cdot 7\cdot 24\cdot 60\text{ min}}{1\cdot 1\cdot 1\cdot 1}$
Multiply.	$90{,}720\text{ min}$

Juliet and her boyfriend will be apart for 90,720 minutes (although it may seem like an eternity!).

TRY IT 3.1

The distance between the earth and the moon is about 250,000 miles. Convert this length to yards.

Answer
440,000,000 yards

TRY IT 3.2

The astronauts of Expedition 28 on the International Space Station spend 15 weeks in space. Convert the time to minutes.

Answer
151,200 minutes

EXAMPLE 4

How many ounces are in 1 gallon?

Solution
We will convert gallons to ounces by multiplying by several conversion factors. Refer to the table on Imperial Systems of Measurement.

	1 gallon
Multiply the measurement to be converted by 1.	$\frac{1\text{ gallon}}{1}\cdot\frac{4\text{ quarts}}{1\text{ gallon}}\cdot\frac{2\text{ pints}}{1\text{ quart}}\cdot\frac{2\text{ cups}}{1\text{ pint}}\cdot\frac{8\text{ ounces}}{1\text{ cup}}$
Use conversion factors to get to the right unit. Simplify.	$\frac{1\cancel{\text{ gallon}}}{1}\cdot\frac{4\cancel{\text{ quarts}}}{1\cancel{\text{ gallon}}}\cdot\frac{2\cancel{\text{ pints}}}{1\cancel{\text{ quart}}}\cdot\frac{2\cancel{\text{ cups}}}{1\cancel{\text{ pint}}}\cdot\frac{8\text{ ounces}}{1\cancel{\text{ cup}}}$
Multiply.	$\frac{1\cdot4\cdot2\cdot2\cdot8\text{ ounces}}{1\cdot1\cdot1\cdot1\cdot1}$
Simplify.	128 ounces There are 128 ounces in a gallon.

TRY IT 4.1

How many cups are in 1 gallon?

Answer
16 cups

TRY IT 4.2

How many teaspoons are in 1 cup?

Answer
48 teaspoons

Use Mixed Units of Measurement in the Imperial System

We often use mixed units of measurement in everyday situations. Suppose Joe is 5 feet 10 inches tall, stays at work for 7 hours and 45 minutes, and then eats a 1 pound 2 ounce steak for dinner—all these measurements have mixed units.

Performing arithmetic operations on measurements with mixed units of measures requires care. Be sure to add or subtract like units!

EXAMPLE 5

Seymour bought three steaks for a barbecue. Their weights were 14 ounces; 1 pound, 2 ounces; and 1 pound, 6 ounces. How many total pounds of steak did he buy?

Solution
We will add the weights of the steaks to find the total weight of the steaks.

Add the ounces. Then add the pounds.	$\begin{array}{rr} & 14\text{ ounces} \\ 1\text{ pound} & 2\text{ ounces} \\ +1\text{ pound} & 6\text{ ounces} \\ \hline 2\text{ pounds} & 22\text{ ounces} \end{array}$
Convert 22 ounces to pounds and ounces.	1 pound, 6 ounces
Add the pounds and ounces.	2 pounds + 1 pound + 6 ounces
Answer	Seymour bought 3 pounds 6 ounces of steak.

TRY IT 5.1

Laura gave birth to triplets weighing 3 pounds 6 ounces, 3 pounds 5 ounces, and 2 pounds 13 ounces. What was the total birth weight of the three babies?

Answer
9 lbs. 8 oz

TRY IT 5.2

Stan cut two pieces of crown molding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the molding?

Answer
21 ft. 6 in.

EXAMPLE 6

Anthony bought four planks of wood that were each 6 feet 4 inches long. What is the total length of the wood he purchased?

Solution
We will multiply the length of one plank to find the total length.

Multiply the inches and then the feet.	$\begin{array}{lr} 6\text{ feet} & 4\text{ inches} \\ \times & 4 \\ \hline 24\text{ feet} & 16\text{ inches} \end{array}$
Convert the 16 inches to feet. Add the feet.	$24\text{ feet} + 1\text{ foot}4\text{ inches} = 25\text{ feet}4\text{ inches}$
Anthony bought 25 feet and 4 inches of wood.	

TRY IT 6.1

Henri wants to triple his vegan spaghetti sauce recipe that uses 1 pound 8 ounces of black beans. How many pounds of black beans will he need?

Answer
4 lbs. 8 oz.

TRY IT 6.2

Joellen wants to double a solution of 5 gallons 3 quarts. How many gallons of solution will she have in all?

Answer
11 gallons 2 qt.

Make Unit Conversions in the Metric System

In the metric system, units are related by powers of 10. The roots words of their names reflect this relation. For example, the basic unit for measuring length is a metre. One kilometre is 1,000 metres; the prefix *kilo* means *thousand*. One centimetre is $\frac{1}{100}$ of a metre, just like one cent is $\frac{1}{100}$ of one dollar.

The equivalencies of measurements in the metric system are shown in the table below. The common abbreviations for each measurement are given in parentheses.

Metric System of Measurement

Length	Mass	Capacity
1 kilometre (km) = 1,000 m 1 hectometre (hm) = 100 m 1 dekametre (dam) = 10 m 1 metre (m) = 1 m 1 decimetre (dm) = 0.1 m 1 centimetre (cm) = 0.01 m 1 millimetre (mm) = 0.001 m	1 kilogram (kg) = 1,000 g 1 hectogram (hg) = 100 g 1 dekagram (dag) = 10 g 1 gram (g) = 1 g 1 decigram (dg) = 0.1 g 1 centigram (cg) = 0.01 g 1 milligram (mg) = 0.001 g	1 kilolitre (kL) = 1,000 L 1 hectolitre (hL) = 100 L 1 dekalitre (daL) = 10 L 1 litre (L) = 1 L 1 decilitre (dL) = 0.1 L 1 centilitre (cL) = 0.01 L 1 millilitre (mL) = 0.001 L
1 metre = 100 centimetres 1 metre = 1,000 millimetres	1 gram = 100 centigrams 1 gram = 1,000 milligrams	1 litre = 100 centilitres 1 litre = 1,000 millilitres

To make conversions in the metric system, we will use the same technique we did in the Imperial system. Using the identity property of multiplication, we will multiply by a conversion factor of one to get to the correct units.

Have you ever run a 5K or 10K race? The length of those races are measured in kilometres. The metric system is commonly used in Canada when talking about the length of a race.

EXAMPLE 7

Nick ran a 10K race. How many metres did he run?

Solution
We will convert kilometres to metres using the identity property of multiplication.

Nick ran	10 kilometres
Multiply the measurement to be converted by 1.	10 kilometres × 1
Write 1 as a fraction relating kilometres and metres.	$10 \text{ kilometres} \times \frac{1,000 \text{ metres}}{1 \text{ kilometres}}$
Simplify.	$10 \cancel{\text{kilometres}} \times \frac{1,000 \text{ metres}}{1 \cancel{\text{kilometres}}}$
Multiply.	10,000 metres
Nick ran 10,000 metres.	

TRY IT 7.1

Sandy completed her first 5K race! How many metres did she run?

Answer
5,000 metres

TRY IT 7.2

Herman bought a rug 2.5 metres in length. How many centimetres is the length?

Answer
250 centimetres

EXAMPLE 8

Eleanor's newborn baby weighed 3,200 grams. How many kilograms did the baby weigh?

Solution
We will convert grams into kilograms.

Eleanor's baby weighs	$3200\ \text{grams}$
Multiply the measurement to be converted by 1.	$3200\ \text{grams} \cdot 1$
Write 1 as a function relating kilograms and grams.	$3,200\ \text{grams} \cdot \frac{1\text{kg}}{1,000\ \text{grams}}$
Simplify.	$3,200\ \cancel{\text{grams}} \cdot \frac{1\text{kg}}{1,000\ \cancel{\text{grams}}}$
Multiply.	$\frac{3{,}200\ \text{kilograms}}{1,000}$
Divide.	3.2 kilograms
The baby weighed 3.2 kilograms.	

TRY IT 8.1

Kari's newborn baby weighed 2,800 grams. How many kilograms did the baby weigh?

Answer
2.8 kilograms

TRY IT 8.2

Anderson received a package that was marked 4,500 grams. How many kilograms did this package weigh?

Answer
4.5 kilograms

As you become familiar with the metric system you may see a pattern. Since the system is based on multiples of ten, the calculations involve multiplying by multiples of ten. We have learned how to simplify these calculations by just moving the decimal.

To multiply by 10, 100, or 1,000, we move the decimal to the right one, two, or three places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left one, two, or three places, respectively.

We can apply this pattern when we make measurement conversions in the metric system. In Example 8, we changed 3,200 grams to kilograms by multiplying by $\frac{1}{1000}$ (or 0.001). This is the same as moving the decimal three places to the left.

Figure.1

EXAMPLE 9

Convert a) 350 L to kilolitres b) 4.1 L to millilitres.

Solution

a. We will convert litres to kilolitres. In the Metric System of Measurement table, we see that $1 \text{ kilolitre} = 1{,}000 \text{ litres.}$

Given amount	350 L
Multiply by 1, writing 1 as a fraction relating litres to kilolitres.	$350 \text{ L} \cdot \frac{1 \text{ kL}}{1{,}000 \text{ L}}$
Simplify.	$350\cancel{\text{L}} \cdot \frac{1 \text{ kL}}{1,000\cancel{\text{L}}}$
Move the decimal 3 units to the left.	0.35 kL

b. We will convert litres to millilitres. From Metric System of Measurement table we see that $1 \text{ litre} = 1{,}000 \text{ millilitres.}$

Given amount	4.1L
Multiply by 1, writing 1 as a fraction relating litres to millilitres.	$4.1\text{L} \cdot \dfrac{1,000\text{mL}}{1\text{L}}$
Simplify.	$4.1\cancel{\text{L}} \cdot \dfrac{1,000\text{mL}}{1\cancel{\text{L}}}$
Move the decimal 3 units to the right.	4.100 mL = $4,100\text{mL}$

TRY IT 9.1

Convert: a) 725 L to kilolitres b) 6.3 L to millilitres

Answer
a) 0.725 kilolitres b) 6,300 millilitres

TRY IT 9.2

Convert: a) 350 hL to litres b) 4.1 L to centilitres

Answer
a) 35,000 litres b) 410 centilitres

Use Mixed Units of Measurement in the Imperial System

Performing arithmetic operations on measurements with mixed units of measures in the imperial system requires the same care we used in the Canadian system. Make sure to add or subtract like units.

EXAMPLE 10

Ryland is 1.6 metres tall. His younger brother is 85 centimetres tall. How much taller is Ryland than his younger brother?

Solution
We can convert both measurements to either centimetres or metres. Since metres is the larger unit, we will subtract the lengths in metres. We convert 85 centimetres to metres by moving the decimal 2 places to the left.

Write the 85 centimetres as metres.	$\begin{array}{r} 1.60\text{m} \\ -0.85\text{m} \\ \hline 0.75\text{m} \end{array}$

Ryland is 0.75 m taller than his brother.

TRY IT 10.1

Mariella is 1.58 metres tall. Her daughter is 75 centimetres tall. How much taller is Mariella than her daughter? Write the answer in centimetres.

Answer
83 centimetres

TRY IT 10.2

The fence around Hank's yard is 2 metres high. Hank is 96 centimetres tall. How much shorter than the fence is Hank? Write the answer in metres.

Answer
1.04 metres

EXAMPLE 11

Dena's recipe for lentil soup calls for 150 millilitres of olive oil. Dena wants to triple the recipe. How many litres of olive oil will she need?

Solution
We will find the amount of olive oil in millileters then convert to litres.

What do we need to do?	Triple 150 mL
Translate to algebra.	$3 \cdot 150 \text{ mL}$
Multiply.	450 mL
Convert to litres.	$450 \cdot \dfrac{0.001 \text{ L}}{1 \text{ mL}}$
Simplify.	0.45 L
Dena needs 0.45 litres of olive oil.	

TRY IT 11.1

A recipe for Alfredo sauce calls for 250 millilitre s of milk. Renata is making pasta with Alfredo sauce for a big party and needs to multiply the recipe amounts by 8. How many litres of milk will she need?

Answer

2 litres

TRY IT 11.2

To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Answer

2.4 kilograms

Convert Between the Imperial and the Metric Systems of Measurement

Many measurements in Canada are made in metric units. Our soda may come in 2-litre bottles, our calcium may come in 500-mg capsules, and we may run a 5K race. To work easily in both systems, we need to be able to convert between the two systems.

The table below shows some of the most common conversions.

Conversion Factors Between Imperial and Metric Systems

Length	Mass	Capacity
$1 \text{ in.} = 2.54 \text{ cm}$ $1 \text{ ft.} = 0.305 \text{ m}$ $1 \text{ yd.} = 0.914 \text{ m}$ $1 \text{ mi.} = 1.609 \text{ km}$ $1 \text{ m} = 3.28 \text{ ft.}$	$1 \text{ lb.} = 0.45 \text{ kg}$ $1 \text{ oz.} = 28 \text{ g}$ $1 \text{ kg} = 2.2 \text{ lb.}$	$1 \text{ qt.} = 0.95 \text{ L}$ $1 \text{ fl. oz.} = 30 \text{ mL}$ $1 \text{ L} = 1.06 \text{ qt.}$

(Figure.2) shows how inches and centimetres are related on a ruler.

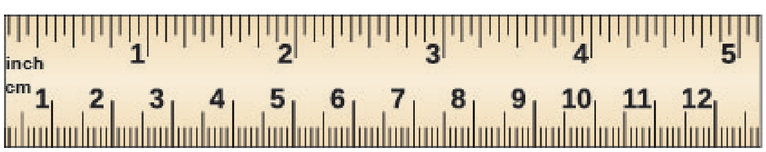


Figure.2

(Figure.3) shows the ounce and millilitre markings on a measuring cup.

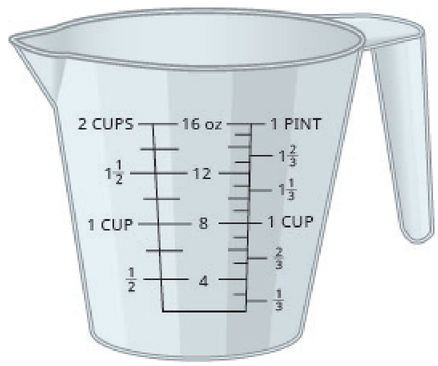


Figure.3

(Figure.4) shows how pounds and kilograms marked on a bathroom scale.

Figure.4

We make conversions between the systems just as we do within the systems—by multiplying by unit conversion factors.

EXAMPLE 12

Lee's water bottle holds 500 mL of water. How many ounces are in the bottle? Round to the nearest tenth of an ounce.

Solution

Given amount	500 mL
Multiply by a unit conversion factor relating mL and ounces.	$500\cancel{\text{ millilitres}} \cdot \dfrac{1\text{ ounce}}{30\cancel{\text{ millilitres}}}$
Simplify.	$\dfrac{500\text{ ounce}}{30}$
Divide.	16.7 ounces.
The water bottle has 16.7 ounces.	

TRY IT 12.1

How many quarts of soda are in a 2-L bottle? Round your answer to nearest tenth.

Answer
2.1 quarts

TRY IT 12.2

How many litres are in 4 quarts of milk?

Answer
3.8 litres

EXAMPLE 13

Soleil was on a road trip and saw a sign that said the next rest stop was in 100 kilometres. How many miles until the next rest stop? Round your answer to nearest mile.

Solution

Distance to the next stop	100 kilometres
Multiply by a unit conversion factor relating km and mi.	$100 \text{ kilometres} \cdot \dfrac{1 \text{ mile}}{1.609 \text{ kilometre}}$
Simplify.	$\dfrac{100 \text{ miles}}{1.609}$
Divide.	62 miles.
Soleil will travel 62 miles.	

TRY IT 13.1

The height of Mount Kilimanjaro is 5,895 metres. Convert the height to feet. Round your answer to nearest feet.

Answer
19,336 feet or 19,341 feet

TRY IT 13.2

The flight distance from Toronto to Vancouver is 3,364 kilometres. Convert the distance to miles. Round your answer to nearest mile.

Answer
2,090 miles or 2,091 miles

Convert between Fahrenheit and Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for 71°F what does that mean?

The Canadian and imperial systems use different scales to measure temperature. The Canadian system uses degrees Celsius, written °C. The imperial system uses degrees Fahrenheit, written°F. (Figure.5) shows the relationship between the two systems.
The diagram shows normal body temperature, along with the freezing and boiling temperatures of water in degrees Fahrenheit and degrees Celsius.

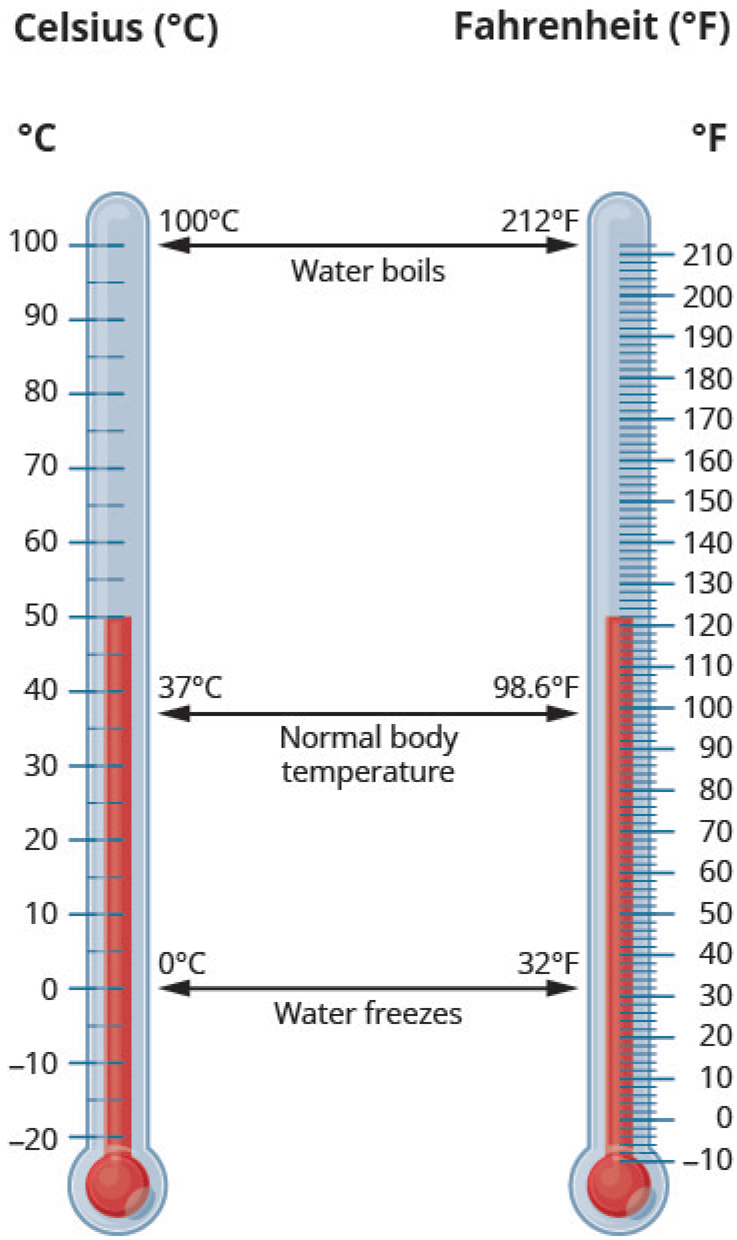


Figure.5

Temperature Conversion

To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula

$$C = \frac{5}{9}(F - 32).$$

To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula

$$F = \frac{9}{5}C + 32.$$

EXAMPLE 14

Convert 50° Fahrenheit into degrees Celsius.

Solution

We will substitute 50°F into the formula to find C.

	$C = \frac{5}{9}(F - 32)$
Substitute 50 for F.	$C = \frac{5}{9}(50 - 32)$
Simplify in parentheses.	$C = \frac{5}{9}(18)$
Multiply.	$C = 10$
So we found that 50°F is equivalent to 10°C.	

TRY IT 14.1

Convert the Fahrenheit temperature to degrees Celsius: 59° Fahrenheit.

Answer
15°C

TRY IT 14.2

Convert the Fahrenheit temperature to degrees Celsius: 41° Fahrenheit.

Answer
5°C

EXAMPLE 15

While visiting Paris, Woody saw the temperature was 20° Celsius. Convert the temperature into degrees Fahrenheit.

Solution

We will substitute 20°C into the formula to find F.

	$F = \frac{9}{5}C + 32$
Substitute 20 for C.	$F = \frac{9}{5}(20) + 32$
Multiply.	$F = 36 + 32$
Add.	$F = 68$
So we found that 20°C is equivalent to 68°F.	

TRY IT 15.1

Convert the Celsius temperature to degrees Fahrenheit: the temperature in Helsinki, Finland, was 15° Celsius.

Answer
59°F

TRY IT 15.2

Convert the Celsius temperature to degrees Fahrenheit: the temperature in Sydney, Australia, was 10° Celsius.

Answer
50° F

Key Concepts

- **Metric System of Measurement**
  - **Length**

$$\begin{array}{lcl} 1 \text{ kilometre (km)} & = & 1{,}000 \text{ m} \\ 1 \text{ hectometre (hm)} & = & 100 \text{ m} \\ 1 \text{ dekametre (dam)} & = & 10 \text{ m} \\ 1 \text{ metre (m)} & = & 1 \text{ m} \\ 1 \text{ decimetre (dm)} & = & 0.1 \text{ m} \\ 1 \text{ centimetre (cm)} & = & 0.01 \text{ m} \\ 1 \text{ millimetre (mm)} & = & 0.001 \text{ m} \\ 1 \text{ metre} & = & 100 \text{ centimetres} \\ 1 \text{ metre} & = & 1{,}000 \text{ millimetres} \end{array}$$

  - **Mass**

$$\begin{array}{lcl} 1 \text{ kilogram (kg)} & = & 1{,}000 \text{ g} \\ 1 \text{ hectogram (hg)} & = & 100 \text{ g} \\ 1 \text{ dekagram (dag)} & = & 10 \text{ g} \\ 1 \text{ gram (g)} & = & 1 \text{ g} \\ 1 \text{ decigram (dg)} & = & 0.1 \text{ g} \\ 1 \text{ centigram (cg)} & = & 0.01 \text{ g} \\ 1 \text{ milligram (mg)} & = & 0.001 \text{ g} \\ 1 \text{ gram} & = & 100 \text{ centigrams} \\ 1 \text{ gram} & = & 1{,}000 \text{ milligrams} \end{array}$$

- **Capacity**

$$\begin{array}{lcl} 1 \text{ kilolitre (kL)} & = & 1{,}000 \text{ L} \\ 1 \text{ hectolitre (hL)} & = & 100 \text{ L} \\ 1 \text{ dekalitre (daL)} & = & 10 \text{ L} \\ 1 \text{ litre (L)} & = & 1 \text{ L} \\ 1 \text{ decilitre (dL)} & = & 0.1 \text{ L} \\ 1 \text{ centilitre (cL)} & = & 0.01 \text{ L} \\ 1 \text{ millilitre (mL)} & = & 0.001 \text{ L} \\ 1 \text{ litre} & = & 100 \text{ centilitres} \\ 1 \text{ litre} & = & 1{,}000 \text{ millilitres} \end{array}$$

- **Temperature Conversion**
  - To convert from Fahrenheit temperature, F, to Celsius temperature, C, use the formula $\mathrm{C} = \frac{5}{9}(\mathrm{F} - 32)$
  - To convert from Celsius temperature, C, to Fahrenheit temperature, F, use the formula $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$

Practice Makes Perfect

Make Unit Conversions in the Imperial System

In the following exercises, convert the units.

1. A park bench is 6 feet long. Convert the length to inches.	2. A floor tile is 2 feet wide. Convert the width to inches.
3. A ribbon is 18 inches long. Convert the length to feet.	4. Carson is 45 inches tall. Convert his height to feet.
5. A football field is 160 feet wide. Convert the width to yards.	6. On a baseball diamond, the distance from home plate to first base is 30 yards. Convert the distance to feet.
7. Ulises lives 1.5 miles from school. Convert the distance to feet.	8. Denver, Colorado, is 5,183 feet above sea level. Convert the height to miles.
9. A killer whale weighs 4.6 tons. Convert the weight to pounds.	10. Blue whales can weigh as much as 150 tons. Convert the weight to pounds.
11. An empty bus weighs 35,000 pounds. Convert the weight to tons.	12. At take-off, an airplane weighs 220,000 pounds. Convert the weight to tons.
13. Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.	14. Misty's surgery lasted $2\frac{1}{4}$ hours. Convert the time to seconds.
15. How many teaspoons are in a pint?	16. How many tablespoons are in a gallon?
17. JJ's cat, Posy, weighs 14 pounds. Convert her weight to ounces.	18. April's dog, Beans, weighs 8 pounds. Convert his weight to ounces.
19. Crista will serve 20 cups of juice at her son's party. Convert the volume to gallons.	20. Lance needs 50 cups of water for the runners in a race. Convert the volume to gallons.
21. Jon is 6 feet 4 inches tall. Convert his height to inches.	22. Faye is 4 feet 10 inches tall. Convert her height to inches.
23. The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days.	24. Lynn's cruise lasted 6 days and 18 hours. Convert the time to hours.
25. Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.	26. Baby Audrey weighted 6 pounds 15 ounces at birth. Convert her weight to ounces.

Use Mixed Units of Measurement in the Imperial System

In the following exercises, solve.

27. Eli caught three fish. The weights of the fish were 2 pounds 4 ounces, 1 pound 11 ounces, and 4 pounds 14 ounces. What was the total weight of the three fish?	28. Judy bought 1 pound 6 ounces of almonds, 2 pounds 3 ounces of walnuts, and 8 ounces of cashews. How many pounds of nuts did Judy buy?
29. One day Anya kept track of the number of minutes she spent driving. She recorded 45, 10, 8, 65, 20, and 35. How many hours did Anya spend driving?	30. Last year Eric went on 6 business trips. The number of days of each was 5, 2, 8, 12, 6, and 3. How many weeks did Eric spend on business trips last year?
31. Renee attached a 6 feet 6 inch extension cord to her computer's 3 feet 8 inch power cord. What was the total length of the cords?	32. Fawzi's SUV is 6 feet 4 inches tall. If he puts a 2 feet 10 inch box on top of his SUV, what is the total height of the SUV and the box?
33. Leilani wants to make 8 placemats. For each placemat she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?	34. Mireille needs to cut 24 inches of ribbon for each of the 12 girls in her dance class. How many yards of ribbon will she need altogether?

Make Unit Conversions in the Metric System

In the following exercises, convert the units.

35. Ghalib ran 5 kilometres. Convert the length to metres.	36. Kitaka hiked 8 kilometres. Convert the length to metres.
37. Estrella is 1.55 metres tall. Convert her height to centimetres.	38. The width of the wading pool is 2.45 metres. Convert the width to centimetres.
39. Mount Whitney is 3,072 metres tall. Convert the height to kilometres.	40. The depth of the Mariana Trench is 10,911 metres. Convert the depth to kilometres.
41. June's multivitamin contains 1,500 milligrams of calcium. Convert this to grams.	42. A typical ruby-throated hummingbird weights 3 grams. Convert this to milligrams.
43. One stick of butter contains 91.6 grams of fat. Convert this to milligrams.	44. One serving of gourmet ice cream has 25 grams of fat. Convert this to milligrams.
45. The maximum mass of an airmail letter is 2 kilograms. Convert this to grams.	46. Dimitri's daughter weighed 3.8 kilograms at birth. Convert this to grams.
47. A bottle of wine contained 750 millilitre s. Convert this to litres.	48. A bottle of medicine contained 300 millilitre s. Convert this to litres.

Use Mixed Units of Measurement in the Metric System

In the following exercises, solve.

49. Matthias is 1.8 metres tall. His son is 89 centimetres tall. How much taller is Matthias than his son?	50. Stavros is 1.6 metres tall. His sister is 95 centimetres tall. How much taller is Stavros than his sister?
51. A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, of the weights of a duck and a dove?	52. Concetta had a 2-kilogram bag of flour. She used 180 grams of flour to make biscotti. How many kilograms of flour are left in the bag?
53. Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?	54. One glass of orange juice provides 560 milligrams of potassium. Linda drinks one glass of orange juice every morning. How many grams of potassium does Linda get from her orange juice in 30 days?
55. Jonas drinks 200 millilitre s of water 8 times a day. How many litres of water does Jonas drink in a day?	56. One serving of whole grain sandwich bread provides 6 grams of protein. How many milligrams of protein are provided by 7 servings of whole grain sandwich bread?

Convert Between the Imperial and the Metric Systems of Measurement

In the following exercises, make the unit conversions. Round to the nearest tenth.

57. Bill is 75 inches tall. Convert his height to centimetres.	58. Frankie is 42 inches tall. Convert his height to centimetres.
59. Marcus passed a football 24 yards. Convert the pass length to metres	60. Connie bought 9 yards of fabric to make drapes. Convert the fabric length to metres.
61. According to research conducted by the CRC, Canadians regrettably produce more garbage per capita than any other country on earth, at 2,172.6 pounds per person annually. Convert the waste to kilograms.	62. An average Canadian will throw away 163,000 pounds of trash over his or her lifetime. Convert this weight to kilograms.
63. A 5K run is 5 kilometres long. Convert this length to miles.	64. Kathryn is 1.6 metres tall. Convert her height to feet.
65. Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.	66. Jackson's backpack weighed 15 kilograms. Convert the weight to pounds.
67. Ozzie put 14 gallons of gas in his truck. Convert the volume to litres.	68. Bernard bought 8 gallons of paint. Convert the volume to litres.

Convert between Fahrenheit and Celsius Temperatures

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

69. 86° Fahrenheit	70. 77° Fahrenheit
71. 104° Fahrenheit	72. 14° Fahrenheit
73. 72° Fahrenheit	74. 4° Fahrenheit
75. 0° Fahrenheit	76. 120° Fahrenheit

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

77. 5° Celsius	78. 25° Celsius
79. -10° Celsius	80. -15° Celsius
81. 22° Celsius	82. 8° Celsius
83. 43° Celsius	84. 16° Celsius

Everyday Math

85. **Nutrition** Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?	86. **Reflectors** The reflectors in each lane-marking stripe on a highway are spaced 16 yards apart. How many reflectors are needed for a one mile long lane-marking stripe?

Writing Exercises

87. Some people think that 65° to 75° Fahrenheit is the ideal temperature range. a) What is your ideal temperature range? Why do you think so? b) Convert your ideal temperatures from Fahrenheit to Celsius.	88. a) Did you grow up using the Canadian. or the Imperial system of measurement? b) Describe two examples in your life when you had to convert between the two systems of measurement.

Answers

1. 72 inches	3. 1.5 feet	5. $53\frac{1}{3}$ yards
7. 7,920 feet	9. 9,200 pounds	11. $17\frac{1}{2}$ tons
13. 5,400 s	15. 96 teaspoons	17. 224 ounces
19. $1\frac{1}{4}$ gallons	21. 76 in.	23. 65 days
25. 115 ounces	27. 8 lbs. 13 oz.	29. 3.05 hours
31. 10 ft. 2 in.	33. 4 yards	35. 5,000 metres
37. 155 centimetres	39. 3.072 kilometres	41. 1.5 grams
43. 91,600 milligrams	45. 2,000 grams	47. 0.75 litres
49. 91 centimetres	51. Typically, a duck weighs $855g$ more than a dove.	
53. 2.1 kilograms	55. 1.6 litres	57. 190.5 centimetres
59. 21.9 metres	61. 985.5 kilograms	63. 3.1 miles
65. 44 pounds	67. 53.2 litres	69. 30°C
71. 40°C	73. 22.2°C	75. -17.8°C
77. 41°F	79. 14°F	81. 71.6°F
83. 109.4°F	85. 14.6 kilograms	87. Answers may vary.

Attributions

4.2 Use Properties of Rectangles, Triangles, and Trapezoids

Learning Objectives

By the end of this section, you will be able to:

- Understand linear, square, and cubic measure
- Use properties of rectangles
- Use properties of triangles
- Use properties of trapezoids

Understand Linear, Square, and Cubic Measure

When you measure your height or the length of a garden hose, you use a ruler or tape measure (Figure.1). A tape measure might remind you of a line—you use it for linear measure, which measures length. Inch, foot, yard, mile, centimetre and metre are units of linear measure.
This tape measure measures inches along the top and centimetres along the bottom.

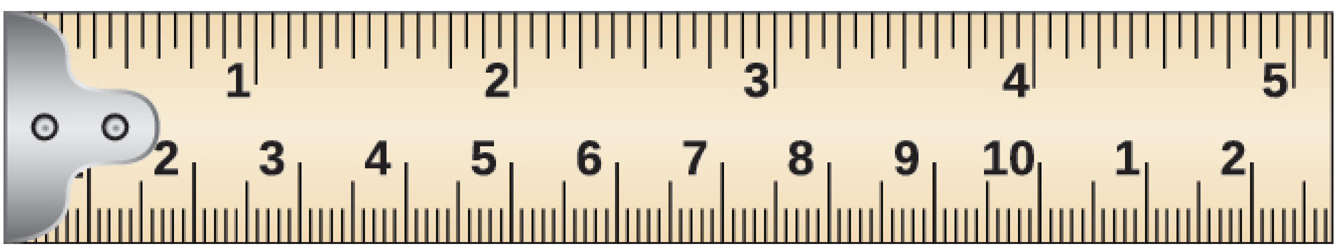


Figure.1

When you want to know how much tile is needed to cover a floor, or the size of a wall to be painted, you need to know the area, a measure of the region needed to cover a surface. Area is measured is square units. We often use square inches, square feet, square centimetres, or square miles to measure area. A square centimetre is a square that is one centimetre (cm) on each side. A square inch is a square that is one inch on each side (Figure.2).

Square measures have sides that are each 1 unit in length.

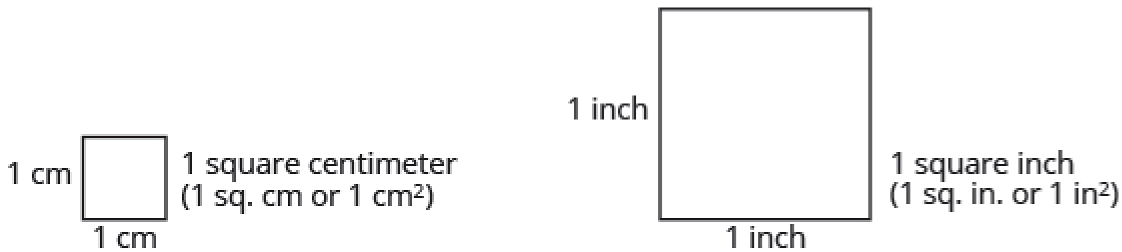


Figure.2

(Figure.3) shows a rectangular rug that is 2 feet long by 3 feet wide. Each square is 1 foot wide by 1 foot long, or 1 square foot. The rug is made of 6 squares. The area of the rug is 6 square feet.

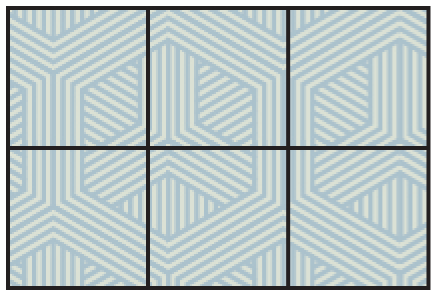


Figure 3 The rug contains six squares of 1 square foot each, so the total area of the rug is 6 square feet.

When you measure how much it takes to fill a container, such as the amount of gasoline that can fit in a tank, or the amount of medicine in a syringe, you are measuring volume. Volume is measured in cubic units such as cubic inches or cubic centimetres. When measuring the volume of a rectangular solid, you measure how many cubes fill the container. We often use cubic centimetres, cubic inches, and cubic feet. A cubic centimetre is a cube that measures one centimetre on each side, while a cubic inch is a cube that measures one inch on each side (Figure.4).

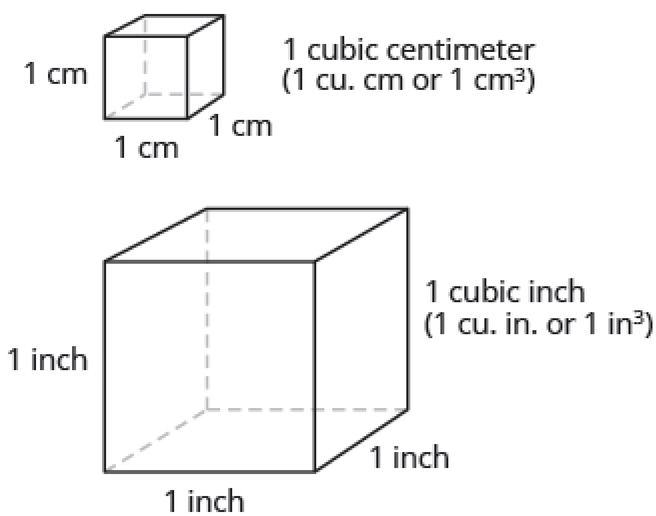


Figure 4 Cubic measures have sides that are 1 unit in length.

Suppose the cube in (Figure.5) measures 3 inches on each side and is cut on the lines shown. How many little cubes does it contain? If we were to take the big cube apart, we would find 27 little cubes, with each one measuring one inch on all sides. So each little cube has a volume of 1 cubic inch, and the volume of the big cube is 27 cubic inches.

A cube that measures 3 inches on each side is made up of 27 one-inch cubes, or 27 cubic inches.

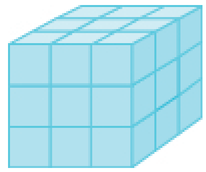


Figure.5

EXAMPLE 1

For each item, state whether you would use linear, square, or cubic measure:

a) amount of carpeting needed in a room

b) extension cord length

c) amount of sand in a sandbox

d) length of a curtain rod

e) amount of flour in a canister

f) size of the roof of a doghouse.

Solution

a) You are measuring how much surface the carpet covers, which is the area.	square measure
b) You are measuring how long the extension cord is, which is the length.	linear measure
c) You are measuring the volume of the sand.	cubic measure
d) You are measuring the length of the curtain rod.	linear measure
e) You are measuring the volume of the flour.	cubic measure
f) You are measuring the area of the roof.	square measure

TRY IT 1.1

Determine whether you would use linear, square, or cubic measure for each item.

a) amount of paint in a can b) height of a tree c) floor of your bedroom d) diametre of bike wheel e) size of a piece of sod f) amount of water in a swimming pool

Answer

a. cubic
b. linear
c. square
d. linear
e. square
f. cubic

TRY IT 1.2

Determine whether you would use linear, square, or cubic measure for each item.

a) volume of a packing box b) size of patio c) amount of medicine in a syringe d) length of a piece of yarn e) size of housing lot f) height of a flagpole

Answer

a. cubic
b. square
c. cubic
d. linear
e. square
f. linear

Many geometry applications will involve finding the perimeter or the area of a figure. There are also many applications of perimeter and area in everyday life, so it is important to make sure you understand what they each mean.

Picture a room that needs new floor tiles. The tiles come in squares that are a foot on each side—one square foot. How many of those squares are needed to cover the floor? This is the area of the floor.

Next, think about putting new baseboard around the room, once the tiles have been laid. To figure out how many strips are needed, you must know the distance around the room. You would use a tape measure to measure the number of feet around the room. This distance is the perimeter.

Perimeter and Area

The perimeter is a measure of the distance around a figure.

The area is a measure of the surface covered by a figure.

(Figure. 6) shows a square tile that is 1 inch on each side. If an ant walked around the edge of the tile, it would walk 4 inches. This distance is the perimeter of the tile.

Since the tile is a square that is 1 inch on each side, its area is one square inch. The area of a shape is measured by determining how many square units cover the shape.

$$\text{Perimeter} = 4 \text{ inches}$$
$$\text{Area} = 1 \text{ square inch}$$

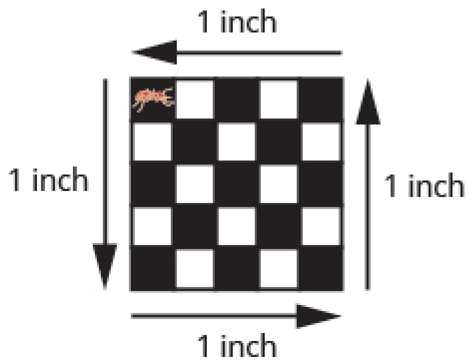


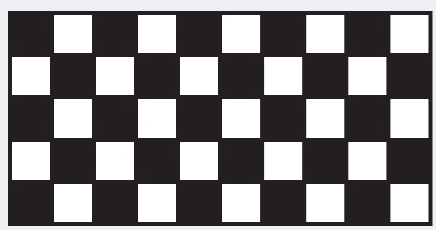
Figure 6 When the ant walks completely around the tile on its edge, it is tracing the perimeter of the tile. The area of the tile is 1 square inch.

EXAMPLE 2

Each of two square tiles is 1 square inch. Two tiles are shown together.

a) What is the perimeter of the figure?

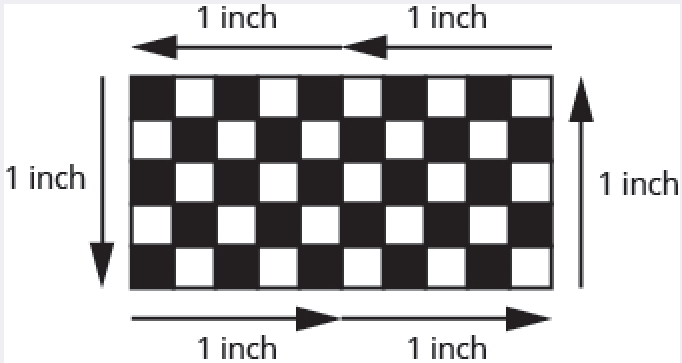
b) What is the area?



Solution

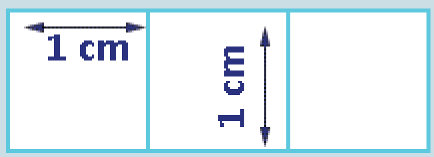
a) The perimeter is the distance around the figure. The perimeter is 6 inches.

b) The area is the surface covered by the figure. There are 2 square inch tiles so the area is 2 square inches.



TRY IT 2.1

Find the a) perimeter and b) area of the figure:

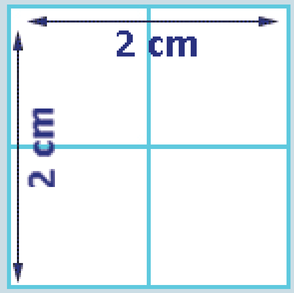


Answer

a. 8 cm
b. 3 sq. cm

TRY IT 2.2

Find the a) perimeter and b) area of the figure:



Answer

a. 8 centimetres
b. 4 sq. centimetres

Use the Properties of Rectangles

A rectangle has four sides and four right angles. The opposite sides of a rectangle are the same length. We refer to one side of the rectangle as the length, L, and the adjacent side as the width, W. See (Figure.7).

A rectangle has four sides, and four right angles. The sides are labeled L for length and W for width.

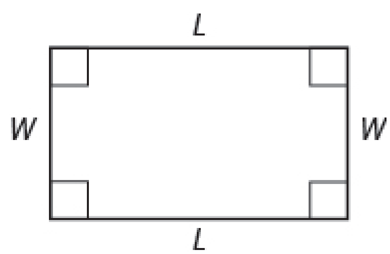


Figure.7

The perimeter, P, of the rectangle is the distance around the rectangle. If you started at one corner and walked around the rectangle, you would walk $L + W + L + W$ units, or two lengths and two widths. The perimeter then is

$$P = L + W + L + W$$

or

$$P = 2L + 2W$$

What about the area of a rectangle? Remember the rectangular rug from the beginning of this section. It was 2 feet long by 3 feet wide, and its area was 6 square feet. See (Figure.8). Since $A = 2 \cdot 3$, we see that the area, A, is the length, L, times the width, W, so the area of a rectangle is $A = L \cdot W$.

The area of this rectangular rug is 6 square feet, its length times its width.

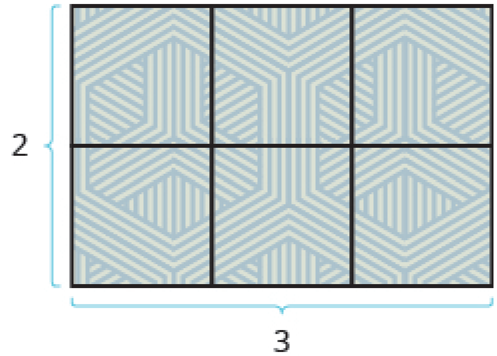


Figure.8

Properties of Rectangles

- Rectangles have four sides and four right $(90)°$ angles.
- The lengths of opposite sides are equal.
- The perimeter, P, of a rectangle is the sum of twice the length and twice the width. See (Figure 7).
  $P = 2L + 2W$
- The area, A, of a rectangle is the length times the width. $A = L \cdot W$

For easy reference as we work the examples in this section, we will state the Problem Solving Strategy for Geometry Applications here.

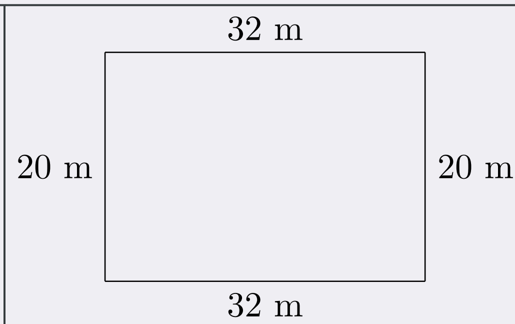
HOW TO: *Use a Problem Solving Strategy for Geometry Applications*

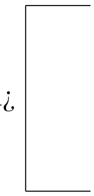
1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

EXAMPLE 3

The length of a rectangle is 32 metres and the width is 20 metres. Find a) the perimeter, and b) the area.

Solution

a) Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the perimeter of a rectangle
Step 3. **Name.** Choose a variable to represent it.	Let P = the perimeter
Step 4. **Translate.** Write the appropriate formula. Substitute.	$\underbrace{P}_{P} \underbrace{=}_{=} 2\underbrace{L}_{2(32)} \underbrace{+}_{+} \underbrace{W}_{2(20)}$
Step 5. **Solve** the equation.	$P = 64 + 40$ $P = 104$
Step 6. **Check:**	$P \stackrel{?}{=} 104$ $20 + 32 + 20 + 32 \stackrel{?}{=} 104$ $104 = 104 \checkmark$
Step 7. **Answer** the question.	The perimeter of the rectangle is 104 metres.

b) Step 1. **Read** the problem. Draw the figure and label it with the given information.	$32 < spanid = ”eip - id1168468395600”$ $20 < spanid = ”eip - id1168468395600” data - type = ”media” data - alt = ”.” >$ m¡/span¿  $32 < spanid = ”eip - id1168468395600”$
Step 2. **Identify** what you are looking for.	the area of a rectangle
Step 3. **Name.** Choose a variable to represent it.	Let A = the area
Step 4. **Translate.** Write the appropriate formula. Substitute.	$\underbrace{A}_{A} \underbrace{=}_{=} \underbrace{L}_{32\text{ m}} \underbrace{\cdot}_{\cdot} \underbrace{W}_{20\text{ m}}$
Step 5. **Solve** the equation.	$A = 640 \text{ m}^2$
Step 6. **Check:**	$A \stackrel{?}{=} 640$ $32 \cdot 20 \stackrel{?}{=} 640$ $640 = 640\checkmark$
Step 7. **Answer** the question.	The area of the rectangle is 60 square metres.

TRY IT 3.1

The length of a rectangle is 120 yards and the width is 50 yards. Find a) the perimeter and b) the area.

Answer

a. 340 yd
b. 6000 sq. yd

TRY IT 3.2

The length of a rectangle is 62 feet and the width is 48 feet. Find a) the perimeter and b) the area.

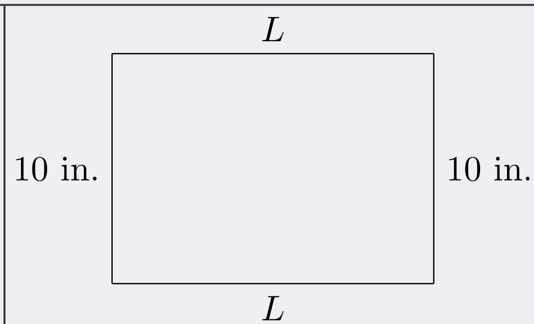
Answer

a. 220 ft
b. 2976 sq. ft

EXAMPLE 4

Find the length of a rectangle with perimeter 50 inches and width 10 inches.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the length of the rectangle
Step 3. **Name.** Choose a variable to represent it.	Let L = the length
Step 4. **Translate.** Write the appropriate formula. Substitute.	$\underbrace{P}_{50} \underbrace{=}_{=} 2\underbrace{L}_{2(L)} \underbrace{+}_{+} \underbrace{W}_{2(10)}$
Step 5. **Solve** the equation.	$50 - 20 = 2L + 20 - 20$ $30 = 2L$ $\frac{30}{2} = \frac{2L}{2}$ $15 = L$
Step 6. **Check:**	$P = 50$ $15 + 10 + 15 + 10 \stackrel{?}{=} 50$ $50 = 50\checkmark$
Step 7. **Answer** the question.	The length is 15 inches.

TRY IT 4.1

Find the length of a rectangle with a perimeter of 80 inches and width of 25 inches.

Answer
15 in.

TRY IT 4.2

Find the length of a rectangle with a perimeter of 30 yards and width of 6 yards.

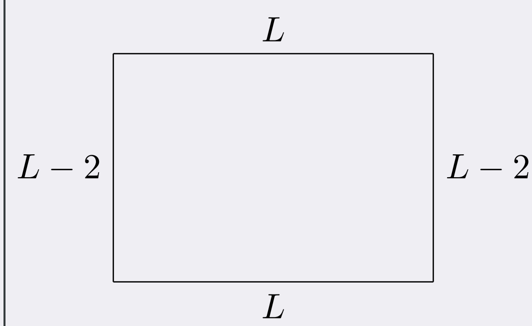
Answer
9 yd

In the next example, the width is defined in terms of the length. We'll wait to draw the figure until we write an expression for the width so that we can label one side with that expression.

EXAMPLE 5

The width of a rectangle is two inches less than the length. The perimeter is 52 inches. Find the length and width.

Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what you are looking for.	the length and width of the rectangle
Step 3. **Name.** Choose a variable to represent it. Now we can draw a figure using these expressions for the length and width.	Since the width is defined in terms of the length, we let L = length. The width is two feet less that the length, so we let L − 2 = width 
Step 4.**Translate.** Write the appropriate formula. The formula for the perimeter of a rectangle relates all the information. Substitute in the given information.	$\underbrace{P}_{52} \underbrace{=}_{=} 2\underbrace{L}_{2(L)} \underbrace{+}_{+} \underbrace{W}_{2(L-2)}$
Step 5. **Solve** the equation.	$52 = 2L + 2L - 4$
Combine like terms.	$52 = 4L - 4$
Add 4 to each side.	$56 = 4L$
Divide by 4.	$\frac{56}{4} = \frac{4L}{4}$
	$14 = L$
	The length is 14 inches.
Now we need to find the width.	
The width is L − 2.	$L - 2$ $= 14 - 2$ $= 12$ The width is 12 inches.
Step 6. **Check:**	Since $14 + 12 + 14 + 12 = 52\checkmark$, this works!
Step 7. **Answer** the question.	The length is 14 feet and the width is 12 feet.

TRY IT 5.1

The width of a rectangle is seven metres less than the length. The perimeter is 58 metres. Find the length and width.

Answer

18 m, 11 m

TRY IT 5.2

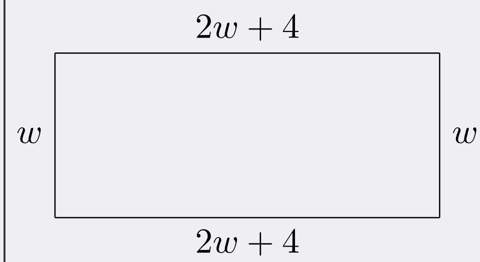
The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

Answer
11 ft , 19 ft

EXAMPLE 6

The length of a rectangle is four centimetres more than twice the width. The perimeter is 32 centimetres. Find the length and width.

Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what you are looking for.	the length and width
Step 3. **Name.** Choose a variable to represent it.	let w = width The length is four more than twice the width. $2w + 4 = \text{length}$ 
Step 4.**Translate.** Write the appropriate formula and substitute in the given information.	$\underbrace{P}_{32} \underbrace{=}_{=} 2 \underbrace{L}_{2(2w+4)} \underbrace{+}_{+} \underbrace{W}_{2w}$
Step 5. **Solve** the equation.	$32 = 4w + 8 + 2w$ $32 = 6w + 8$ $24 = 6w$ $\frac{24}{6} = \frac{6w}{6}$ $4 = w$ $\text{length} = 2w + 4$ $= 2(2) + 4$ $= 12cm$ The length is 12 cm.
Step 6. **Check:**	$P = 2L + 2W$ $32 \stackrel{?}{=} 2 \cdot 12 + 2 \cdot 4$ $32 = 32\checkmark$
Step 7. **Answer** the question.	The length is 12 cm and the width is 4 cm.

TRY IT 6.1

The length of a rectangle is eight more than twice the width. The perimeter is 64 feet. Find the length and width.

Answer
8 ft, 24 ft

TRY IT 6.2

The width of a rectangle is six less than twice the length. The perimeter is 18 centimetres. Find the length and width.

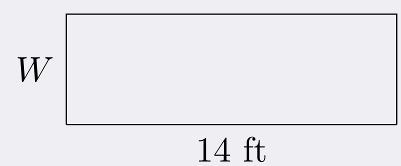
Answer

5 cm, 4 cm

EXAMPLE 7

The area of a rectangular room is 168 square feet. The length is 14 feet. What is the width?

Solution

Step 1. **Read** the problem.	 $\text{Area} = 168\ \text{ft}^2$
Step 2. **Identify** what you are looking for.	the width of a rectangular room
Step 3. **Name.** Choose a variable to represent it.	Let W = width
Step 4.**Translate.** Write the appropriate formula and substitute in the given information.	$A = LW$ $168 = 14W$
Step 5. **Solve** the equation.	$\frac{168}{\color{red}{14}} = \frac{14W}{\color{red}{14}}$ $12 = W$
Step 6. **Check:**	$A = LW$ $168 \stackrel{?}{=} 14 \cdot 12$ $168 = 168\checkmark$
Step 7. **Answer** the question.	The width of the room is 12 feet.

TRY IT 7.1

The area of a rectangle is 598 square feet. The length is 23 feet. What is the width?

Answer

26 ft

TRY IT 7.2

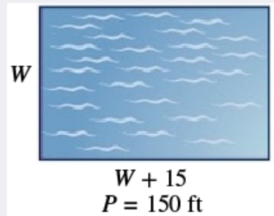
The width of a rectangle is 21 metres. The area is 609 square metres. What is the length?

Answer
29 m

EXAMPLE 8

The perimeter of a rectangular swimming pool is 150 feet. The length is 15 feet more than the width. Find the length and width.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the length and width of the pool
Step 3. **Name.** Choose a variable to represent it. The length is 15 feet more than the width.	Let $W = \text{width}$ $W + 15 = \text{length}$
Step 4.**Translate.** Write the appropriate formula and substitute.	$\underbrace{P}_{150}\ \underbrace{=}_{=}\ 2\ \underbrace{L}_{2(2w+15)}\ \underbrace{+}_{+}\ \underbrace{W}_{2w}$
Step 5. **Solve** the equation.	$150 = 2w + 30 + 2w$ $150 = 4w + 30$ $120 = 4w$ $\frac{120}{4} = \frac{4w}{4}$ $30 = w$ The width of the pool = 30 ft $\text{The length of the pool} = w + 15$ $= (30) + 15$ $= 45cm$
Step 6. **Check:**	$P = 2L + 2W$ $150 \stackrel{?}{=} 2(45) + 2(30)$ $150 = 150\checkmark$
Step 7. **Answer** the question.	The length of the pool is 45 feet and the width is 30 feet.

TRY IT 8.1

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Answer

30 ft, 70 ft

TRY IT 8.2

The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

Answer
60 yd, 90 yd

Use the Properties of Triangles

We now know how to find the area of a rectangle. We can use this fact to help us visualize the formula for the area of a triangle. In the rectangle in (Figure.9), we've labeled the length b and the width h, so it's area is bh.

The area of a rectangle is the base, b, times the height, h.

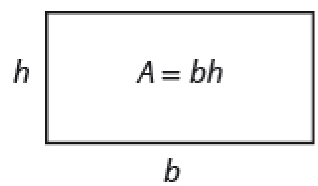


Figure.9

We can divide this rectangle into two congruent triangles (Figure.10). Triangles that are congruent have identical side lengths and angles, and so their areas are equal. The area of each triangle is one-half the area of the rectangle, or $\frac{1}{2}bh$. This example helps us see why the formula for the area of a triangle is $A = \frac{1}{2}bh$.

A rectangle can be divided into two triangles of equal area. The area of each triangle is one-half the area of the rectangle.

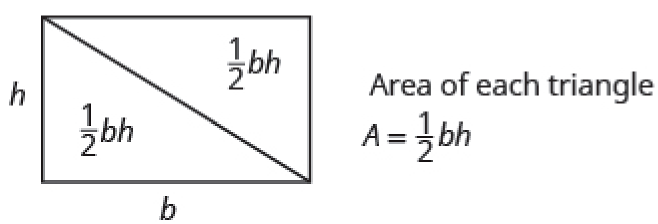


Figure.10

The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base and h is the height.

To find the area of the triangle, you need to know its base and height. The base is the length of one side of the triangle, usually the side at the bottom. The height is the length of the line that connects the base to the opposite vertex, and makes a $90°$ angle with the base. (Figure.11) shows three triangles with the base and height of each marked.

The height h of a triangle is the length of a line segment that connects the the base to the opposite vertex and makes a $90°$ angle with the base.

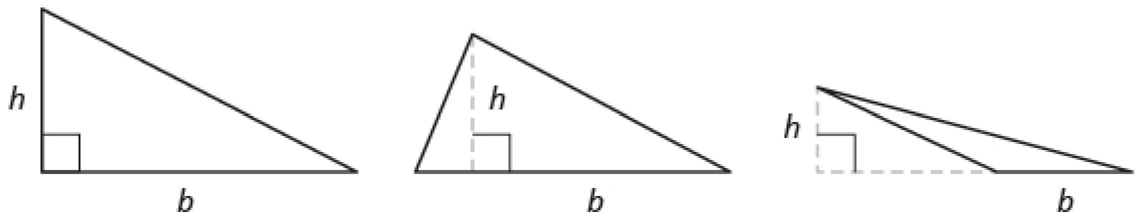


Figure.11

Triangle Properties

For any triangle ΔABC, the sum of the measures of the angles is $180°$.

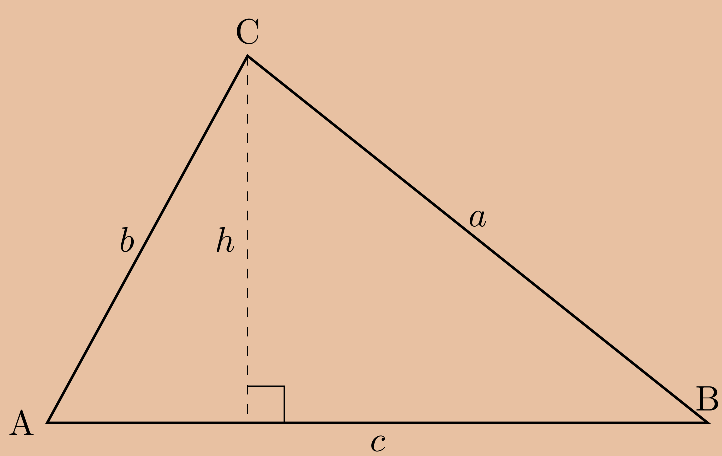
$$m\angle A + m\angle B + m\angle C = 180°$$

The perimeter of a triangle is the sum of the lengths of the sides.

$$P = a + b + c$$

The area of a triangle is one-half the base, b, times the height, h.

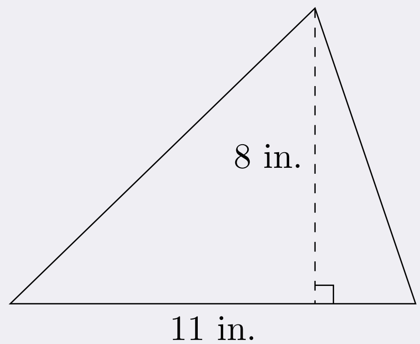
$$A = \frac{1}{2}bh$$



EXAMPLE 9

Find the area of a triangle whose base is 11 inches and whose height is 8 inches.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the area of the triangle
Step 3. **Name.** Choose a variable to represent it.	let A = area of the triangle
Step 4.**Translate.** Write the appropriate formula. Substitute.	$\underbrace{A}_{A} \quad \underbrace{=\frac{1}{2}}_{=\frac{1}{2}} \underbrace{\cdot}_{\cdot} \underbrace{b}_{11} \underbrace{h}_{8}$
Step 5. **Solve** the equation.	$A = 44\text{inches}^2$
Step 6. **Check:**	$A = \frac{1}{2}bh$ $44 \stackrel{?}{=} \frac{1}{2}(11)8$ $44 = 44\checkmark$
Step 7. **Answer** the question.	The area is 44 square inches.

TRY IT 9.1

Find the area of a triangle with base 13 inches and height 2 inches.

Answer

13 sq. in.

TRY IT 9.2

Find the area of a triangle with base 14 inches and height 7 inches.

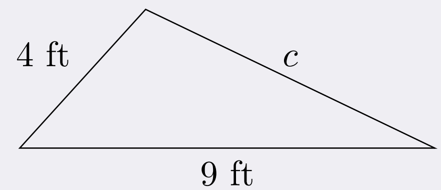
Answer

49 sq. in.

EXAMPLE 10

The perimeter of a triangular garden is 24 feet. The lengths of two sides are 4 feet and 9 feet. How long is the third side?

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	 P=24 ft
Step 2. **Identify** what you are looking for.	length of the third side of a triangle
Step 3. **Name.** Choose a variable to represent it.	Let c = the third side
Step 4.**Translate.** Write the appropriate formula. Substitute in the given information.	$\underbrace{P}_{24}\ \underbrace{=}_{=}\ \underbrace{a}_{4}\ \underbrace{+}_{+}\ \underbrace{b}_{9}\ \underbrace{+}_{+}\ \underbrace{c}_{c}$
Step 5. **Solve** the equation.	$24=13+c$ $24-13=13+c-13$ $11=c$
Step 6. **Check:**	$P=a+b+c$ $24\stackrel{?}{=}4+9+11$ $24=24\checkmark$
Step 7. **Answer** the question.	The third side is 11 feet long.

TRY IT 10.1

The perimeter of a triangular garden is 48 feet. The lengths of two sides are 18 feet and 22 feet. How long is the third side?

Answer

8 ft

TRY IT 10.2

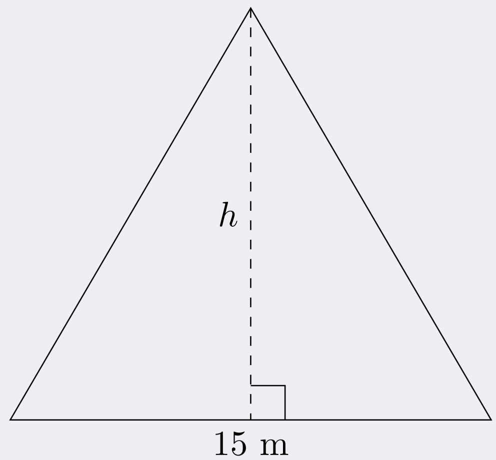
The lengths of two sides of a triangular window are 7 feet and 5 feet. The perimeter is 18 feet. How long is the third side?

Answer
6 ft

EXAMPLE 11

The area of a triangular church window is 90 square metres. The base of the window is 15 metres. What is the window's height?

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	height of a triangle
Step 3. **Name.** Choose a variable to represent it.	Let h = the height
Step 4.**Translate.** Write the appropriate formula. Substitute in the given information.	$\underbrace{A}_{90} \quad \underbrace{=\tfrac{1}{2}}_{=\tfrac{1}{2}} \underbrace{\cdot}_{\cdot} \underbrace{b}_{15} \underbrace{h}_{h}$
Step 5. **Solve** the equation.	$90 = \frac{15}{2}h$ $90 \times \frac{2}{15} = \frac{15}{2} \times \frac{2}{15}h$ $12 = h$
Step 6. **Check:**	$A = \frac{1}{2}bh$ $90 \stackrel{?}{=} \frac{1}{2} \cdot 15 \cdot 12$ $90 = 90\checkmark$
Step 7. **Answer** the question.	The height of the triangle is 12 metres.

TRY IT 11.1

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Answer

14 in.

TRY IT 11.2

A triangular tent door has an area of 15 square feet. The height is 5 feet. What is the base?

Answer
6 ft

Isosceles and Equilateral Triangles

Besides the right triangle, some other triangles have special names. A triangle with two sides of equal length is called an isosceles triangle. A triangle that has three sides of equal length is called an equilateral triangle. (Figure.12) shows both types of triangles.

In an isosceles triangle, two sides have the same length, and the third side is the base. In an equilateral triangle, all three sides have the same length.

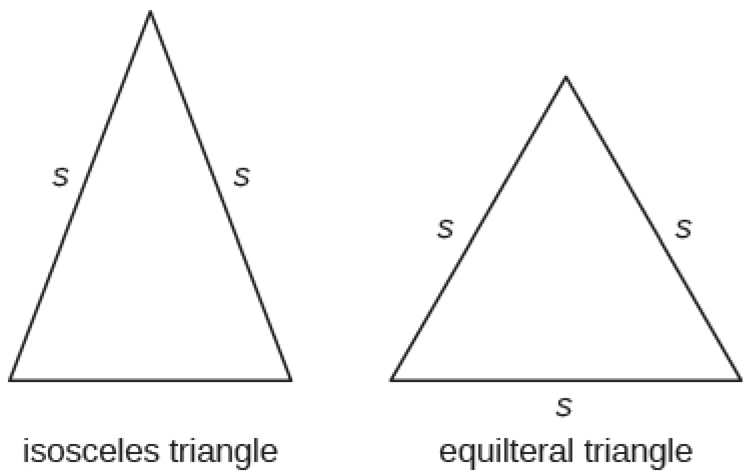


Figure.12

Isosceles and Equilateral Triangles

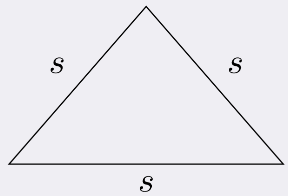
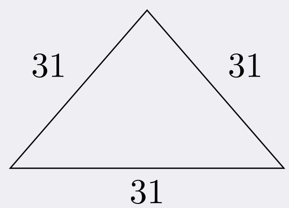
An **isosceles** triangle has two sides the same length.

An **equilateral** triangle has three sides of equal length.

EXAMPLE 12

The perimeter of an equilateral triangle is 93 inches. Find the length of each side.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	 Perimeter = 93 in.
Step 2. **Identify** what you are looking for.	length of the sides of an equilateral triangle
Step 3. **Name.** Choose a variable to represent it.	Let s = length of each side
Step 4.**Translate.** Write the appropriate formula. Substitute.	$\underbrace{P}_{93}\ \underbrace{=}_{=}\ \underbrace{a}_{s}\ \underbrace{+}_{+}\ \underbrace{b}_{s}\ \underbrace{+}_{+}\ \underbrace{c}_{s}$
Step 5. **Solve** the equation.	$93=3s$ $\frac{93}{3}=\frac{3}{3}s$ $31=s$
Step 6. **Check:**	 $93\stackrel{?}{=}31+31+31$ $93=93\checkmark$
Step 7. **Answer** the question.	Each side is 31 inches

TRY IT 12.1

Find the length of each side of an equilateral triangle with perimeter 39 inches.

Answer

13 in.

TRY IT 12.2

Find the length of each side of an equilateral triangle with perimeter 51 centimetres.

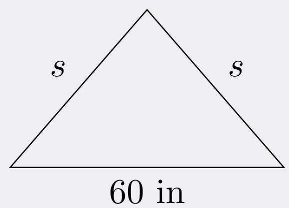
Answer

17 cm

EXAMPLE 13

Arianna has 156 inches of beading to use as trim around a scarf. The scarf will be an isosceles triangle with a base of 60 inches. How long can she make the two equal sides?

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	 P = 156 in.
Step 2. **Identify** what you are looking for.	the lengths of the two equal sides
Step 3. **Name.** Choose a variable to represent it.	Let s = the length of each side
Step 4.**Translate.** Write the appropriate formula. Substitute in the given information.	$\underbrace{P}_{156}\ \underbrace{=}_{=}\ \underbrace{a}_{s}\ \underbrace{+}_{+}\ \underbrace{b}_{60}\ \underbrace{+}_{+}\ \underbrace{c}_{s}$
Step 5. **Solve** the equation.	$156 = 2s + 60$ $156 - 60 = 2s + 60 - 60$ $96 = 2s$ $\frac{96}{2} = \frac{2s}{2}$ $48 = s$
Step 6. **Check:**	$p = a + b + c$ $156 \stackrel{?}{=} 48 + 60 + 48$ $156 = 156\checkmark$
Step 7. **Answer** the question.	Arianna can make each of the two equal sides 48 inches

TRY IT 13.1

A backyard deck is in the shape of an isosceles triangle with a base of 20 feet. The perimeter of the deck is 48 feet. How long is each of the equal sides of the deck?

Answer
14 ft

TRY IT 13.2

A boat's sail is an isosceles triangle with base of 8 metres. The perimeter is 22 metres. How long is each of the equal sides of the sail?

Answer
7 m

Use the Properties of Trapezoids

A trapezoid is four-sided figure, a *quadrilateral*, with two sides that are parallel and two sides that are not. The parallel sides are called the bases. We call the length of the smaller base b, and the length of the bigger base B. The height, h, of a trapezoid is the distance between the two bases as shown in (Figure.13).

A trapezoid has a larger base, B, and a smaller base, b. The height h is the distance between the bases.

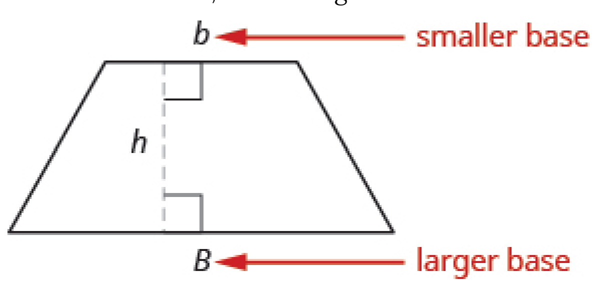


Figure.13

Formula for the Area of a Trapezoid

$$\text{Area}_{\text{trapezoid}} = \frac{1}{2}h\left(b + B\right)$$

Splitting the trapezoid into two triangles may help us understand the formula. The area of the trapezoid is the sum of the areas of the two triangles. See (Figure.14).

Splitting a trapezoid into two triangles may help you understand the formula for its area.

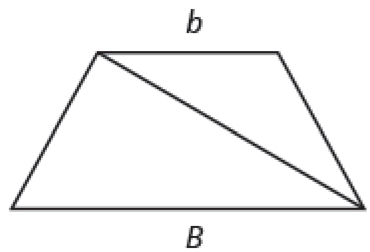


Figure.14

The height of the trapezoid is also the height of each of the two triangles. See (Figure.15).

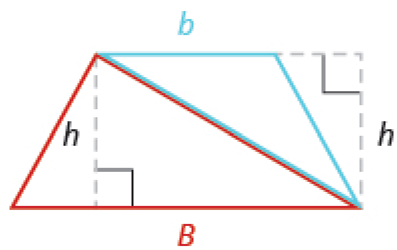


Figure.15

The formula for the area of a trapezoid is

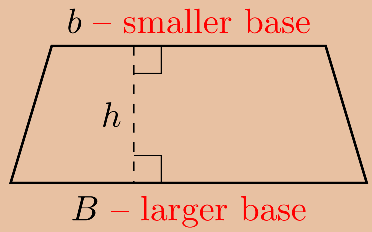
$$\text{Area}_{\text{trapezoid}} = \frac{1}{2}h(b + B)$$

If we distribute, we get,

$$\text{Area}_{trapezoid} = \frac{1}{2}bh + \frac{1}{2}Bh$$

$$\text{Area}_{trapezoid} = A_{blue\Delta} + A_{red\Delta}$$

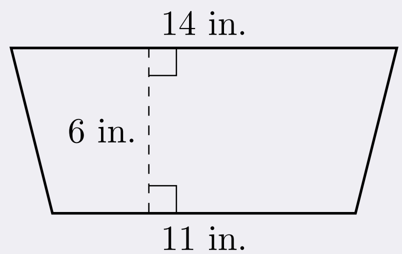
Properties of Trapezoids

• A trapezoid has four sides. • Two of its sides are parallel and two sides are not. • The area, A, of a trapezoid is $\mathrm{A} = \frac{1}{2}h\left(b+B\right)$.	

EXAMPLE 14

Find the area of a trapezoid whose height is 6 inches and whose bases are 14 and 11 inches.

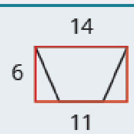
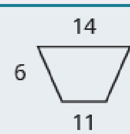
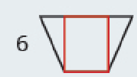
Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the area of the trapezoid
Step 3. **Name.** Choose a variable to represent it.	Let $A =$ the area
Step 4. **Translate.** Write the appropriate formula. Substitute.	$\underbrace{A}_{A} \quad \underbrace{=\tfrac{1}{2}}_{=\tfrac{1}{2}} \quad \underbrace{\cdot}_{\cdot} \quad \underbrace{h}_{6} \quad \underbrace{\cdot}_{\cdot} \quad \underbrace{(b+B)}_{(11+14)}$
Step 5. **Solve** the equation.	$A = \frac{1}{2} \cdot 6(25)$ $A = 3(25)$ $A = 75 \text{ square inches}$

Step 6. **Check:** Is this answer reasonable?

If we draw a rectangle around the trapezoid that has the same big base B and a height h, its area should be greater than that of the trapezoid.

If we draw a rectangle inside the trapezoid that has the same little base b and a height h, its area should be smaller than that of the trapezoid.

		
$A_{\text{rectangle}} = bh$	$A_{\text{trapezoid}} = \frac{1}{2}h(b+B)$	$A_{\text{rectangle}} = bh$
$A_{\text{rectangle}} = 14 \cdot 6$	$A_{\text{trapezoid}} = \frac{1}{2} \cdot 6(11+14)$	$A_{\text{rectangle}} = 11 \cdot 6$
$A_{\text{rectangle}} = 84$ sq. in.	$A_{\text{trapezoid}} = 75$ sq. in.	$A_{\text{rectangle}} = 66$ sq. in.

The area of the larger rectangle is 84 square inches and the area of the smaller rectangle is 66 square inches. So it makes sense that the area of the trapezoid is between 84 and 66 square inches

Step 7. **Answer** the question.	The area of the trapezoid is 75 square inches.

TRY IT 14.1

The height of a trapezoid is 14 yards and the bases are 7 and 16 yards. What is the area?

Answer
161 sq. yd

TRY IT 14.2

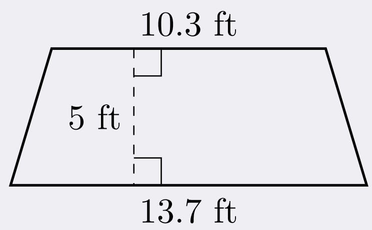
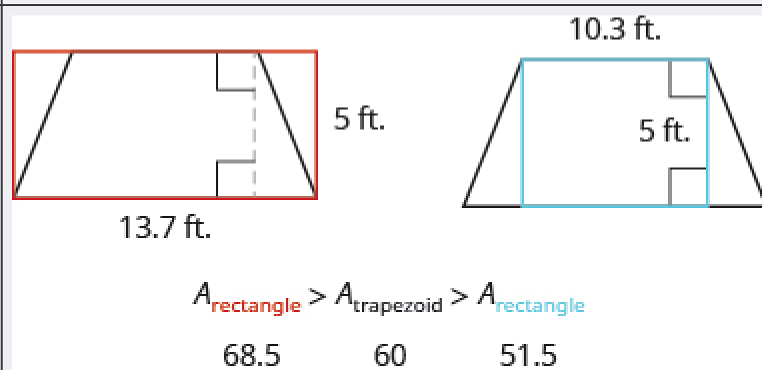
The height of a trapezoid is 18 centimetres and the bases are 17 and 8 centimetres. What is the area?

Answer
225 sq. cm

EXAMPLE 15

Find the area of a trapezoid whose height is 5 feet and whose bases are 10.3 and 13.7 feet.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the area of the trapezoid
Step 3. **Name.** Choose a variable to represent it.	Let A = the area
Step 4.**Translate.** Write the appropriate formula. Substitute.	$\underbrace{A}_{A} \underbrace{=\frac{1}{2}}_{=\frac{1}{2}} \underbrace{\cdot}_{\cdot} \underbrace{h}_{5} \underbrace{\cdot}_{\cdot} \underbrace{(b+B)}_{(10.3+13.7)}$
Step 5. **Solve** the equation.	$A=\frac{1}{2}\cdot 5(24)$ $A=12\cdot 5$ $A=60\text{square feet}$
Step 6. **Check:** Is this answer reasonable? The area of the trapezoid should be less than the area of a rectangle with base 13.7 and height 5, but more than the area of a rectangle with base 10.3 and height 5.	
Step 7. **Answer** the question.	The area of the trapezoid is 60 square feet.

TRY IT 15.1

The height of a trapezoid is 7 centimetres and the bases are 4.6 and 7.4 centimetres. What is the area?

Answer
42 sq. cm

TRY IT 15.2

The height of a trapezoid is 9 metres and the bases are 6.2 and 7.8 metres. What is the area?

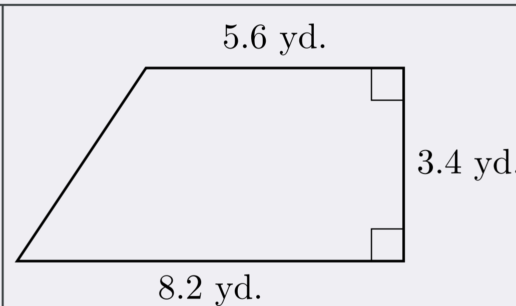
Answer

63 sq. m

EXAMPLE 16

Vinny has a garden that is shaped like a trapezoid. The trapezoid has a height of 3.4 yards and the bases are 8.2 and 5.6 yards. How many square yards will be available to plant?

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the area of a trapezoid
Step 3. **Name.** Choose a variable to represent it.	Let A = the area
Step 4. **Translate.** Write the appropriate formula. Substitute.	$\underbrace{A}_{A} \underbrace{=\frac{1}{2}}_{=\frac{1}{2}} \underbrace{\cdot}_{\cdot} \underbrace{h}_{3.4} \underbrace{\cdot}_{\cdot} \underbrace{(b+B)}_{(5.6+8.2)}$
Step 5. **Solve** the equation.	$A = \frac{1}{2}(3.4)(13.8)$ $A = 23.46 \text{ square yards}$
Step 6. **Check:** Is this answer reasonable? Yes. The area of the trapezoid is less than the area of a rectangle with a base of 8.2 yd and height 3.4 yd, but more than the area of a rectangle with base 5.6 yd. and height 3.4 yd.	
Step 7. **Answer** the question.	Vinny has 23.46 square yards in which he can plan

$A_{rectangle} = Bh$ $= (8.2)(3.4)$ $= 27.88 \text{ yd}^2$	$A_{trapezoid} = \frac{1}{2}(3.4 \text{ yd})(5.6 = 8.2)$ $= 23.46 \text{ yd}^2$	$A_{rectangle} = bh$ $= (5.6)(3.4)$ $= 19.04 \text{ yd}^2$

$$A_{rectangle} > A_{trapezoid} > A_{rectangle}$$
$$27.88 \quad 23.46 \quad 19.04$$

TRY IT 16.1

Lin wants to sod his lawn, which is shaped like a trapezoid. The bases are 10.8 yards and 6.7 yards, and the height is 4.6 yards. How many square yards of sod does he need?

Answer

40.25 sq. yd

TRY IT 16.2

Kira wants cover his patio with concrete pavers. If the patio is shaped like a trapezoid whose bases are 18 feet and 14 feet and whose height is 15 feet, how many square feet of pavers will he need?

Answer

240 sq. ft.

Access Additional Online Resources

- Perimeter of a Rectangle
- Area of a Rectangle
- Perimeter and Area Formulas
- Area of a Triangle
- Area of a Triangle with Fractions
- Area of a Trapezoid

Key Concepts

- **Properties of Rectangles**
  - Rectangles have four sides and four right (90°) angles.
  - The lengths of opposite sides are equal.
  - The perimeter, P, of a rectangle is the sum of twice the length and twice the width.
    - $P = 2L + 2W$
  - The area, A, of a rectangle is the length times the width.
    - $A = L \cdot W$
- **Triangle Properties**
  - For any triangle ΔABC, the sum of the measures of the angles is 180°.
    - $m\angle A + m\angle B + m\angle C = 180°$
  - The perimeter of a triangle is the sum of the lengths of the sides.

- $P = a + b + c$
- The area of a triangle is one-half the base, b, times the height, h.
  - $A = \frac{1}{2}bh$

Glossary

area
The area is a measure of the surface covered by a figure.

equilateral triangle
A triangle with all three sides of equal length is called an equilateral triangle.

isosceles triangle
A triangle with two sides of equal length is called an isosceles triangle.

perimeter
The perimeter is a measure of the distance around a figure.

rectangle
A rectangle is a geometric figure that has four sides and four right angles.

trapezoid
A trapezoid is four-sided figure, a quadrilateral, with two sides that are parallel and two sides that are not.

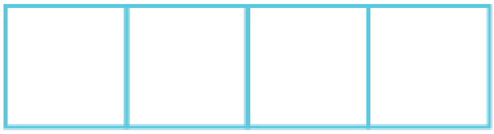
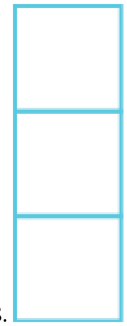
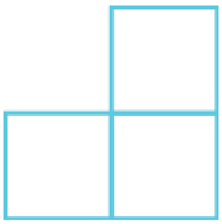
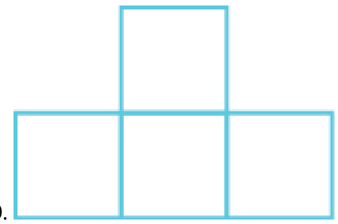
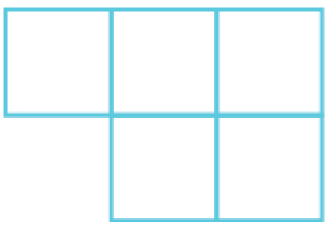
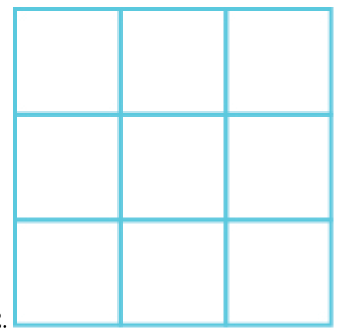
Practice Makes Perfect

Understand Linear, Square, and Cubic Measure

In the following exercises, determine whether you would measure each item using linear, square, or cubic units.

1. amount of water in a fish tank	2. length of dental floss
3. living area of an apartment	4. floor space of a bathroom tile
5. height of a doorway	6. capacity of a truck trailer

In the following exercises, find the a) perimeter and b) area of each figure. Assume each side of the square is 1 cm.

7. 	8. 
9. 	10. 
11. 	12. 

Use the Properties of Rectangles

In the following exercises, find the a) perimeter and b) area of each rectangle.

13. The length of a rectangle is 85 feet and the width is 45 feet.	14. The length of a rectangle is 26 inches and the width is 58 inches.
15. A rectangular room is 15 feet wide by 14 feet long.	16. A driveway is in the shape of a rectangle 20 feet wide by 35 feet long.

In the following exercises, solve.

17. Find the length of a rectangle with perimeter 124 inches and width 38 inches.	18. Find the length of a rectangle with perimeter 20.2 yards and width of 7.8 yards.
19. Find the width of a rectangle with perimeter 92 metres and length 19 metres.	20. Find the width of a rectangle with perimeter 16.2 metres and length 3.2 metres.
21. The area of a rectangle is 414 square metres. The length is 18 metres. What is the width?	22. The area of a rectangle is 782 square centimetres. The width is 17 centimetres. What is the length?
23. The length of a rectangle is 9 inches more than the width. The perimeter is 46 inches. Find the length and the width.	24. The width of a rectangle is 8 inches more than the length. The perimeter is 52 inches. Find the length and the width.
25. The perimeter of a rectangle is 58 metres. The width of the rectangle is 5 metres less than the length. Find the length and the width of the rectangle.	26. The perimeter of a rectangle is 62 feet. The width is 7 feet less than the length. Find the length and the width.
27. The width of the rectangle is 0.7 metres less than the length. The perimeter of a rectangle is 52.6 metres. Find the dimensions of the rectangle.	28. The length of the rectangle is 1.1 metres less than the width. The perimeter of a rectangle is 49.4 metres. Find the dimensions of the rectangle.
29. The perimeter of a rectangle of 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.	30. The length of a rectangle is three times the width. The perimeter is 72 feet. Find the length and width of the rectangle.
31. The length of a rectangle is 3 metres less than twice the width. The perimeter is 36 metres. Find the length and width.	32. The length of a rectangle is 5 inches more than twice the width. The perimeter is 34 inches. Find the length and width.
33. The width of a rectangular window is 24 inches. The area is 624 square inches. What is the length?	34. The length of a rectangular poster is 28 inches. The area is 1316 square inches. What is the width?
35. The area of a rectangular roof is 2310 square metres. The length is 42 metres. What is the width?	36. The area of a rectangular tarp is 132 square feet. The width is 12 feet. What is the length?
37. The perimeter of a rectangular courtyard is 160 feet. The length is 10 feet more than the width. Find the length and the width.	38. The perimeter of a rectangular painting is 306 centimetres. The length is 17 centimetres more than the width. Find the length and the width.
39. The width of a rectangular window is 40 inches less than the height. The perimeter of the doorway is 224 inches. Find the length and the width.	40. The width of a rectangular playground is 7 metres less than the length. The perimeter of the playground is 46 metres. Find the length and the width.

Use the Properties of Triangles

In the following exercises, solve using the properties of triangles.

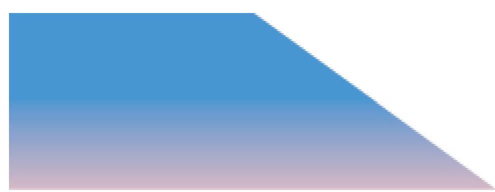
41. Find the area of a triangle with base 12 inches and height 5 inches.	42. Find the area of a triangle with base 45 centimetres and height 30 centimetres.
43. Find the area of a triangle with base 8.3 metres and height 6.1 metres.	44. Find the area of a triangle with base 24.2 feet and height 20.5 feet.
45. A triangular flag has base of 1 foot and height of 1.5 feet. What is its area?	46. A triangular window has base of 8 feet and height of 6 feet. What is its area?
47. If a triangle has sides of 6 feet and 9 feet and the perimeter is 23 feet, how long is the third side?	48. If a triangle has sides of 14 centimetres and 18 centimetres and the perimeter is 49 centimetres, how long is the third side?
49. What is the base of a triangle with an area of 207 square inches and height of 18 inches?	50. What is the height of a triangle with an area of 893 square inches and base of 38 inches?
51. The perimeter of a triangular reflecting pool is 36 yards. The lengths of two sides are 10 yards and 15 yards. How long is the third side?	52. A triangular courtyard has perimeter of 120 metres. The lengths of two sides are 30 metres and 50 metres. How long is the third side?
53. An isosceles triangle has a base of 20 centimetres. If the perimeter is 76 centimetres, find the length of each of the other sides.	54. An isosceles triangle has a base of 25 inches. If the perimeter is 95 inches, find the length of each of the other sides.
55. Find the length of each side of an equilateral triangle with a perimeter of 51 yards.	56. Find the length of each side of an equilateral triangle with a perimeter of 54 metres.
57. The perimeter of an equilateral triangle is 18 metres. Find the length of each side.	58. The perimeter of an equilateral triangle is 42 miles. Find the length of each side.
59. The perimeter of an isosceles triangle is 42 feet. The length of the shortest side is 12 feet. Find the length of the other two sides.	60. The perimeter of an isosceles triangle is 83 inches. The length of the shortest side is 24 inches. Find the length of the other two sides.
61. A dish is in the shape of an equilateral triangle. Each side is 8 inches long. Find the perimeter.	62. A floor tile is in the shape of an equilateral triangle. Each side is 1.5 feet long. Find the perimeter.
63. A road sign in the shape of an isosceles triangle has a base of 36 inches. If the perimeter is 91 inches, find the length of each of the other sides.	64. A scarf in the shape of an isosceles triangle has a base of 0.75 metres. If the perimeter is 2 metres, find the length of each of the other sides.
65. The perimeter of a triangle is 39 feet. One side of the triangle is 1 foot longer than the second side. The third side is 2 feet longer than the second side. Find the length of each side.	66. The perimeter of a triangle is 35 feet. One side of the triangle is 5 feet longer than the second side. The third side is 3 feet longer than the second side. Find the length of each side.
67. One side of a triangle is twice the smallest side. The third side is 5 feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.	68. One side of a triangle is three times the smallest side. The third side is 3 feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

Use the Properties of Trapezoids

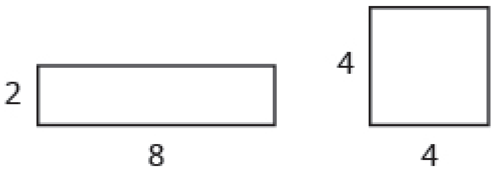
In the following exercises, solve using the properties of trapezoids.

69. The height of a trapezoid is 12 feet and the bases are 9 and 15 feet. What is the area?	70. The height of a trapezoid is 24 yards and the bases are 18 and 30 yards. What is the area?
71. Find the area of a trapezoid with a height of 51 metres and bases of 43 and 67 metres.	72. Find the area of a trapezoid with a height of 62 inches and bases of 58 and 75 inches.
73. The height of a trapezoid is 15 centimetres and the bases are 12.5 and 18.3 centimetres. What is the area?	74. The height of a trapezoid is 48 feet and the bases are 38.6 and 60.2 feet. What is the area?
75. Find the area of a trapezoid with a height of 4.2 metres and bases of 8.1 and 5.5 metres.	76. Find the area of a trapezoid with a height of 32.5 centimetres and bases of 54.6 and 41.4 centimetres.
77. Laurel is making a banner shaped like a trapezoid. The height of the banner is 3 feet and the bases are 4 and 5 feet. What is the area of the banner?	78. Niko wants to tile the floor of his bathroom. The floor is shaped like a trapezoid with width 5 feet and lengths 5 feet and 8 feet. What is the area of the floor?
79. Theresa needs a new top for her kitchen counter. The counter is shaped like a trapezoid with width 18.5 inches and lengths 62 and 50 inches. What is the area of the counter?	80. Elena is knitting a scarf. The scarf will be shaped like a trapezoid with width 8 inches and lengths 48.2 inches and 56.2 inches. What is the area of the scarf?

Everyday Math

81. **Fence** Jose just removed the children's playset from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep out the dog. He has a 50 foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other side if he wants to use the entire roll of fence?	82. **Gardening** Lupita wants to fence in her tomato garden. The garden is rectangular and the length is twice the width. It will take 48 feet of fencing to enclose the garden. Find the length and width of her garden.
83. **Fence** Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are 6 feet, 8 feet, and 10 feet. The fence costs $\$10$ per foot. How much will it cost for Christa to fence in her flowerbed?	84. **Painting** Caleb wants to paint one wall of his attic. The wall is shaped like a trapezoid with height 8 feet and bases 20 feet and 12 feet. The cost of the painting one square foot of wall is about $?0.05$. About how much will it cost for Caleb to paint the attic wall? 

Writing Exercises

	86. If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.
87. Look at the two figures.  a) Which figure looks like it has the larger area? Which looks like it has the larger perimeter? b) Now calculate the area and perimeter of each figure. Which has the larger area? Which has the larger perimeter?	88. The length of a rectangle is 5 feet more than the width. The area is 50 square feet. Find the length and the width. a) Write the equation you would use to solve the problem. b) Why can't you solve this equation with the methods you learned in the previous chapter?

Answers

1. cubic	3. square	5. linear
7. a) 10 cm, b) 4 sq. cm	9. a) 8 cm, b) 3 sq. cm	11. a) 10 cm, b) 5 sq. cm
13. a) 260 ft, b) 3825 sq. ft	15. a) 58 ft, b) 210 sq. ft	17. 24 inches
19. 27 metres	21. 23 m	23. 7 in., 16 in.
25. 17 m, 12 m	27. 13.5 m, 12.8 m	29. 25 ft, 50 ft
31. 7 m, 11 m	33. 26 in.	35. 55 m
37. 35 ft, 45 ft	39. 76 in., 36 in.	41. 60 sq. in.
43. 25.315 sq. m	45. 0.75 sq. ft	47. 8 ft
49. 23 in.	51. 11 ft	53. 28 cm
55. 17 ft	57. 6 m	59. 15 ft
61. 24 in.	63. 27.5 in.	65. 12 ft, 13 ft, 14 ft
67. 3 ft, 6 ft, 8 ft	69. 144 sq. ft	71. 2805 sq. m
73. 231 sq. cm	75. 28.56 sq. m	77. 13.5 sq. ft
79. 1036 sq. in.	81. 15 ft	83. $24
85. Answers will vary.	87. Answers will vary.	

Attributions

4.3 Solve Geometry Applications: Volume and Surface Area

Learning Objectives

By the end of this section, you will be able to:

- Find volume and surface area of rectangular solids
- Find volume and surface area of spheres
- Find volume and surface area of cylinders
- Find volume of cone

In this section, we will find the volume and surface area of some three-dimensional figures. Since we will be solving applications, we will once again show our Problem-Solving Strategy for Geometry Applications.

Problem Solving Strategy for Geometry Applications

1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Find Volume and Surface Area of Rectangular Solids

A cheer leading coach is having the squad paint wooden crates with the school colors to stand on at the games. (See Figure.1). The amount of paint needed to cover the outside of each box is the surface area, a square measure of the total area of all the sides. The amount of space inside the crate is the volume, a cubic measure.
This wooden crate is in the shape of a rectangular solid.

Figure.1

Each crate is in the shape of a rectangular solid. Its dimensions are the length, width, and height. The rectangular solid shown in Figure.2 has length 4 units, width 2 units, and height 3 units. Can you tell how many cubic units there are altogether? Let's look layer by layer.

Breaking a rectangular solid into layers makes it easier to visualize the number of cubic units it contains. This 4 by 2 by 3 rectangular solid has 24 cubic units.

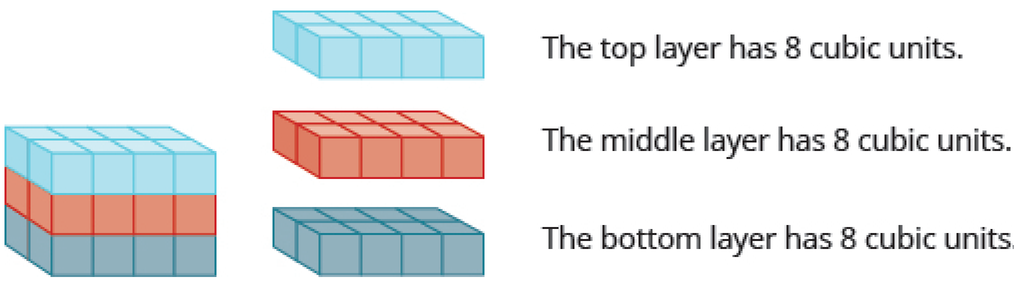


Figure.2

Altogether there are 24 cubic units. Notice that 24 is the $\text{length} \quad \times \quad \text{width} \quad \times \quad \text{height.}$

$$\underbrace{V}_{24} \underbrace{=}_{=} \underbrace{L}_{4} \underbrace{\cdot}_{\cdot} \underbrace{W}_{2} \underbrace{\cdot}_{\cdot} \underbrace{H}_{3}$$

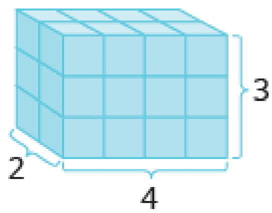
The volume, V, of any rectangular solid is the product of the length, width, and height.

$$V = LWH$$

We could also write the formula for volume of a rectangular solid in terms of the area of the base. The area of the base, B, is equal to $\text{length} \times \text{width.}$
$\mathrm{B} = \mathrm{L} \cdot \mathrm{W}$
We can substitute B for $\mathrm{L} \cdot \mathrm{W}$ in the volume formula to get another form of the volume formula.

$$V = L \cdot W \cdot H$$
$$V = (L \cdot W) \cdot H$$
$$V = Bh$$

We now have another version of the volume formula for rectangular solids. Let's see how this works with the $4 \times 2 \times 3$ rectangular solid we started with. See Figure.3.



$$\begin{aligned}
V &= Bh\\
V &= \text{Base} \times \text{height}\\
V &= (4 \cdot 2) \times \text{height}\\
V &= (4 \cdot 2) \times 3\\
V &= 8 \times 3\\
V &= 24 \text{ cubic units}
\end{aligned}$$

Figure.3

To find the *surface area* of a rectangular solid, think about finding the area of each of its faces. How many faces does the rectangular solid above have? You can see three of them.

$$\begin{array}{lll}
A_{\text{front}} = L \times W & A_{\text{side}} = L \times W & A_{\text{top}} = L \times W \\
A_{\text{front}} = 4 \cdot 3 & A_{\text{side}} = 2 \cdot 3 & A_{\text{top}} = 4 \cdot 2 \\
A_{\text{front}} = 12 & A_{\text{side}} = 6 & A_{\text{top}} = 8
\end{array}$$

Notice for each of the three faces you see, there is an identical opposite face that does not show.

$$\begin{aligned}
S &= (\text{front} + \text{back}) + (\text{left side} + \text{right side}) + (\text{top} + \text{bottom})\\
S &= (2 \cdot \text{front}) + (2 \cdot \text{left side}) + (2 \cdot \text{top})\\
S &= 2 \cdot 12 + 2 \cdot 6 + 2 \cdot 8\\
S &= 24 + 12 + 16\\
S &= 52 \text{ sq. units}
\end{aligned}$$

The surface area S of the rectangular solid shown in (Figure.3) is 52 square units.

In general, to find the surface area of a rectangular solid, remember that each face is a rectangle, so its area is the product of its length and its width (see Figure.4). Find the area of each face that you see and then multiply each area by two to account for the face on the opposite side.

$$S = 2LH + 2LW + 2WH$$

For each face of the rectangular solid facing you, there is another face on the opposite side. There are 6 faces in all.

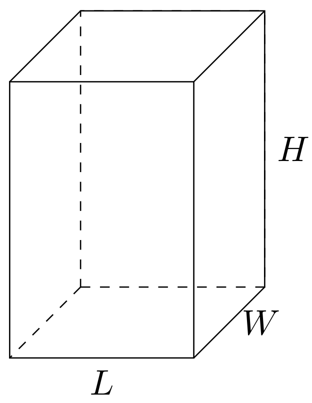
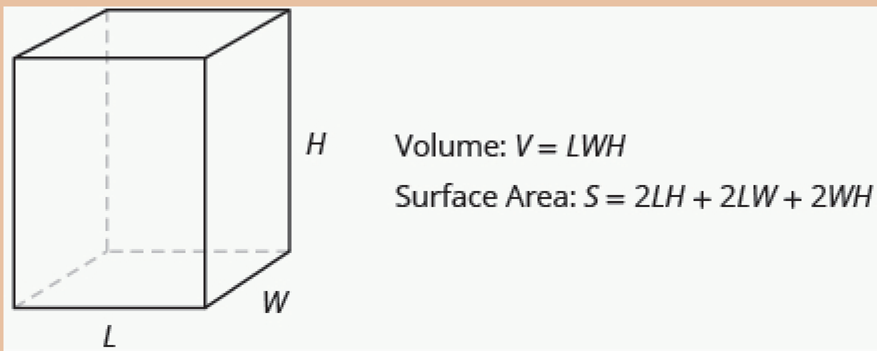


Figure 4

Volume and Surface Area of a Rectangular Solid

For a rectangular solid with length L, width W, and height H :

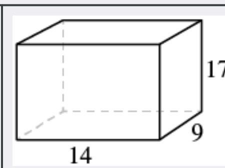


EXAMPLE 1

For a rectangular solid with length 14 cm, height 17 cm, and width 9 cm, find the a) volume and b) surface area.

Solution

Step 1 is the same for both a) and b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the rectangular solid
Step 3. **Name.** Choose a variable to represent it.	Let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute.	$V = LWH$ $V = 14 \cdot 9 \cdot 17$
Step 5. **Solve** the equation.	$V = 2,142$
Step 6. **Check** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume is $2,142$ cubic centimetres.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the solid
Step 3. **Name.** Choose a variable to represent it.	Let S= surface area
Step 4. **Translate.** Write the appropriate formula. Substitute.	$S = 2LH + 2LW + 2WH$ $S = 2\left(14 \cdot 17\right) + 2\left(14 \cdot 9\right) + 2\left(9 \cdot 17\right)$
Step 5. **Solve the equation.**	$S = 1,034$
Step 6. **Check:** Double-check with a calculator.	
Step 7. **Answer** the question.	The surface area is 1,034 square centimetres.

TRY IT 1.1

Find the a) volume and b) surface area of rectangular solid with the: length 8 feet, width 9 feet, and height 11 feet.

Answer

a. 792 cu. ft
b. 518 sq. ft

TRY IT 1.2

Find the a) volume and b) surface area of rectangular solid with the: length 15 feet, width 12 feet, and height 8 feet.

Answer

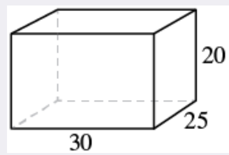
a. 1,440 cu. ft

b. 792 sq. ft

EXAMPLE 2

A rectangular crate has a length of 30 inches, width of 25 inches, and height of 20 inches. Find its a) volume and b) surface area.

Solution
Step 1 is the same for both a) and b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the crate
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute.	$V = LWH$ $V = 30 \cdot 25 \cdot 20$
Step 5. **Solve** the equation.	$V = 15,000$
Step 6. **Check:** Double check your math.	
Step 7. **Answer** the question.	The volume is 15,000 cubic inches.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the crate
Step 3. **Name.** Choose a variable to represent it.	let S= surface area
Step 4. **Translate.** Write the appropriate formula. Substitute.	$S = 2LH + 2LW + 2WH$ $S = 2\left(30 \cdot 20\right) + 2\left(30 \cdot 25\right) + 2\left(25 \cdot 20\right)$
Step 5. **Solve** the equation.	$S = 3,700$
Step 6. **Check:** Check it yourself!	
Step 7. **Answer** the question.	The surface area is 3,700 square inches.

TRY IT 2.1

A rectangular box has length 9 feet, width 4 feet, and height 6 feet. Find its a) volume and b) surface area.

Answer

a. 216 cu. ft
b. 228 sq. ft

TRY IT 2.2

A rectangular suitcase has length 22 inches, width 14 inches, and height 9 inches. Find its a) volume and b) surface area.

Answer

a. 2,772 cu. in.
b. 1,264 sq. in.

Volume and Surface Area of a Cube

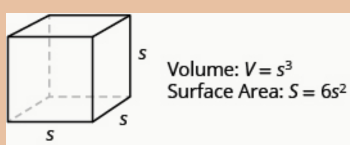
A cube is a rectangular solid whose length, width, and height are equal. See Volume and Surface Area of a Cube, below. Substituting, s for the length, width and height into the formulas for volume and surface area of a rectangular solid, we get:

$$\begin{array}{ll} V = LWH & S = 2LH + 2LW + 2WH \\ V = \mathrm{s}\cdot\mathrm{s}\cdot\mathrm{s} & S = 2\mathrm{s}\cdot\mathrm{s} + 2\mathrm{s}\cdot\mathrm{s} + 2\mathrm{s}\cdot\mathrm{s} \\ V = \mathrm{s}^3 & S = 2s^2 + 2s^2 + 2s^2 \\ & S = 6s^2 \end{array}$$

So for a cube, the formulas for volume and surface area are $V = s^3$ and $S = 6s^2$.

Volume and Surface Area of a Cube

For any cube with sides of length s,

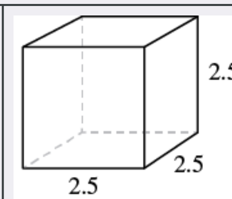


EXAMPLE 3

A cube is 2.5 inches on each side. Find its a) volume and b) surface area.

Solution

Step 1 is the same for both a) and b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the cube
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula.	$V = s^3$
Step 5. **Solve.** Substitute and solve.	$V = (2.5)^3$ $V = 15.625$
Step 6. **Check:** Check your work.	
Step 7. **Answer** the question.	The volume is 15.625 cubic inches.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the cube
Step 3. **Name.** Choose a variable to represent it.	let S = surface area
Step 4. **Translate.** Write the appropriate formula.	$S = 6s^2$
Step 5. **Solve.** Substitute and solve.	$S = 6 \cdot (2.5)^2$ $S = 37.5$
Step 6. **Check:** The check is left to you.	
Step 7. **Answer** the question.	The surface area is 37.5 square inches.

TRY IT 3.1

For a cube with side 4.5 metres, find the a) volume and b) surface area of the cube.

Answer

a. 91.125 cu. m
b. 121.5 sq. m

TRY IT 3.2

For a cube with side 7.3 yards, find the a) volume and b) surface area of the cube.

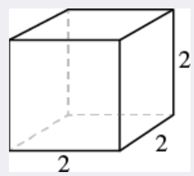
Answer

a. 389.017 cu. yd.
b. 319.74 sq. yd.

EXAMPLE 4

A notepad cube measures 2 inches on each side. Find its a) volume and b) surface area.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the cube
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula.	$V = s^3$
Step 5. **Solve** the equation.	$V = 2^3$ $V = 8$
Step 6. **Check:** Check that you did the calculations correctly.	
Step 7. **Answer** the question.	The volume is 8 cubic inches.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the cube
Step 3. **Name.** Choose a variable to represent it.	let S = surface area
Step 4. **Translate.** Write the appropriate formula.	$S = 6s^2$
Step 5. **Solve** the equation.	$S = 6 \cdot 2^2$ $S = 24$
Step 6. **Check:** The check is left to you.	
Step 7. **Answer** the question.	The surface area is 24 square inches.

TRY IT 4.1

A packing box is a cube measuring 4 feet on each side. Find its a) volume and b) surface area.

Answer

a. 64 cu. ft
b. 96 sq. ft

TRY IT 4.2

An unopened tissue box is a cube measuring 5 inch on each side. Find its a) volume and b) surface area.

Answer

a. 125 cu. in
b. 150 sq. in

Find the Volume and Surface Area of Spheres

A sphere is the shape of a basketball, like a three-dimensional circle. Just like a circle, the size of a sphere is determined by its radius, which is the distance from the centre of the sphere to any point on its surface. The formulas for the volume and surface area of a sphere are given below.

Showing where these formulas come from, like we did for a rectangular solid, is beyond the scope of this course. We will approximate π with 3.14. Remember, that we approximate π with 3.14 or $\frac{22}{7}$ depending on whether the radius of the circle is given as a decimal or a fraction. If you use the π key on your calculator to do the calculations in this section, your answers will be slightly different from the answers shown. That is because the π key uses more than two decimal places.

Volume and Surface Area of a Sphere

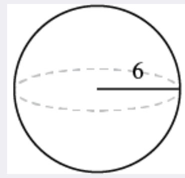
For a sphere with radius r:

EXAMPLE 5

A sphere has a radius 6 inches. Find its a) volume and b) surface area.

Solution

Step 1 is the same for both a) and b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the sphere
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula.	$V = \frac{4}{3}\pi r^3$
Step 5. **Solve.**	$V \approx \frac{4}{3}(3.14)\,6^3$ $V \approx 904.32\text{ cubic inches}$
Step 6. **Check:** Double-check your math on a calculator.	
Step 7. **Answer** the question.	The volume is approximately 904.32 cubic inches.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the cube
Step 3. **Name.** Choose a variable to represent it.	let S = surface area
Step 4. **Translate.** Write the appropriate formula.	$S = 4\pi r^2$
Step 5. **Solve.**	$S \approx 4(3.14)\,6^2$ $S \approx 452.16$
Step 6. **Check:** Double-check your math on a calculator	
Step 7. **Answer** the question.	The surface area is approximately 452.16 square inches.

TRY IT 5.1

Find the a) volume and b) surface area of a sphere with radius 3 centimetres.

Answer

a. 113.04 cu. cm
b. 113.04 sq. cm

TRY IT 5.2

Find the a) volume and b) surface area of each sphere with a radius of 1 foot

Answer

a. 4.19 cu. ft
b. 12.56 sq. ft

EXAMPLE 6

A globe of Earth is in the shape of a sphere with radius 14 centimetres. Find its a) volume and b) surface area. Round the answer to the nearest hundredth.

Solution

Step 1. **Read** the problem. Draw a figure with the given information and label it.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the sphere
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$V=\frac{4}{3}\pi r^3$ $V\approx\frac{4}{3}\left(3.14\right)14^3$
Step 5. **Solve.**	$V\approx 11,488.21$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume is approximately 11,488.21 cubic inches.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the sphere
Step 3. **Name.** Choose a variable to represent it.	let S = surface area
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$S=4\pi r^2$ $S\approx 4\left(3.14\right)14^2$
Step 5. **Solve.**	$S\approx 2461.76$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The surface area is approximately 2461.76 square inches.

TRY IT 6.1

A beach ball is in the shape of a sphere with radius of 9 inches. Find its a) volume and b) surface area.

Answer

a. 3052.08 cu. in.
b. 1017.36 sq. in.

TRY IT 6.2

A Roman statue depicts Atlas holding a globe with radius of 1.5 feet. Find the a) volume and b) surface area of the globe.

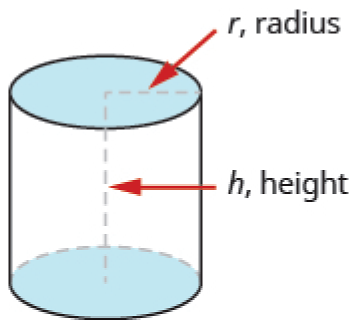
Answer

a. 14.13 cu. ft
b. 28.26 sq. ft

Find the Volume and Surface Area of a Cylinder

If you have ever seen a can of soda, you know what a cylinder looks like. A cylinder is a solid figure with two parallel circles of the same size at the top and bottom. The top and bottom of a cylinder are called the bases. The height h of a cylinder is the distance between the two bases. For all the cylinders we will work with here, the sides and the height, h , will be perpendicular to the bases.

A cylinder has two circular bases of equal size. The height is the distance between the bases.



Rectangular solids and cylinders are somewhat similar because they both have two bases and a height. The formula for the volume of a rectangular solid, $V = Bh$, can also be used to find the volume of a cylinder.

For the rectangular solid, the area of the base, B , is the area of the rectangular base, length × width. For a cylinder, the area of the base, B, is the area of its circular base, πr^2. (Figure.5) compares how the formula $V = Bh$ is used for rectangular solids and cylinders.

Seeing how a cylinder is similar to a rectangular solid may make it easier to understand the formula for the volume of a cylinder.

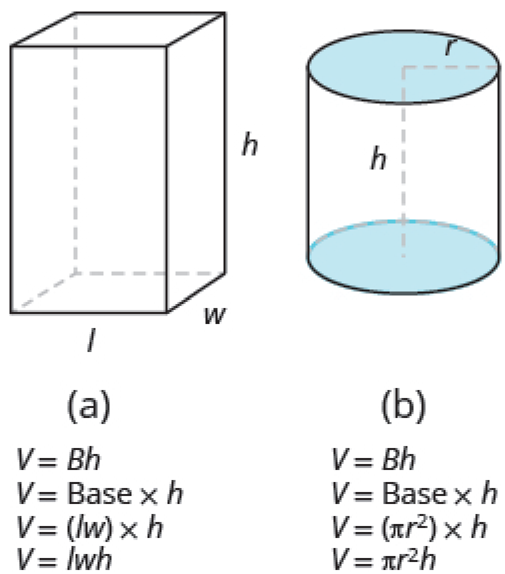


Figure.5

To understand the formula for the surface area of a cylinder, think of a can of vegetables. It has three surfaces: the top, the bottom, and the piece that forms the sides of the can. If you carefully cut the label off the side of the can and unroll it, you will see that it is a rectangle. See (Figure.6).
By cutting and unrolling the label of a can of vegetables, we can see that the surface of a cylinder is a rectangle. The length of the rectangle is the circumference of the cylinder's base, and the width is the height of the cylinder.

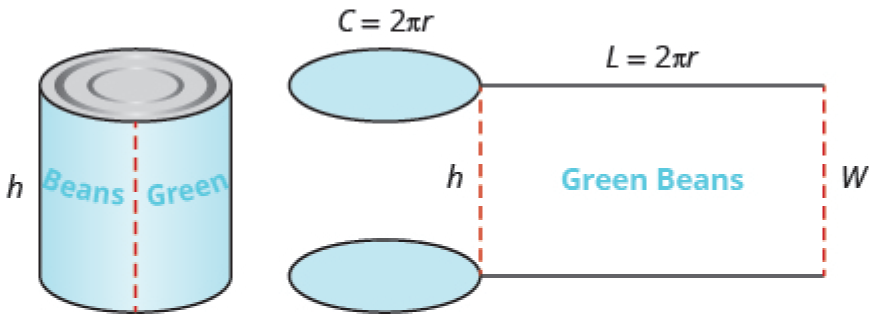


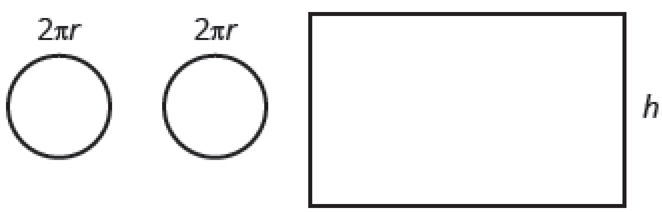
Figure.6

The distance around the edge of the can is the circumference of the cylinder's base it is also the length L of the rectangular label. The height of the cylinder is the width W of the rectangular label. So the area of the label can be represented as

$$A = L \cdot W$$

$$A = 2\pi r \cdot h$$

To find the total surface area of the cylinder, we add the areas of the two circles to the area of the rectangle.



$$S = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{rectangle}}$$

$$S = \underbrace{\pi r^2 + \pi r^2} + 2\pi r \cdot h$$

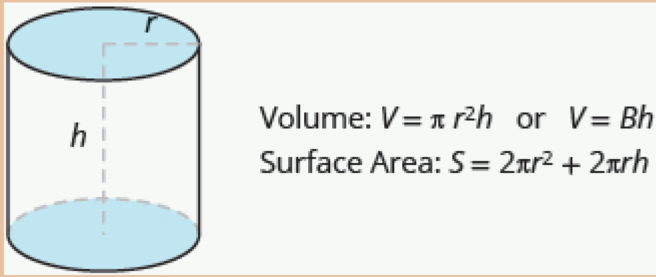
$$S = 2 \cdot \pi r^2 + 2\pi rh$$

$$S = 2\pi r^2 + 2\pi rh$$

The surface area of a cylinder with radius r and height h, is
$S = 2\pi r^2 + 2\pi rh$

Volume and Surface Area of a Cylinder

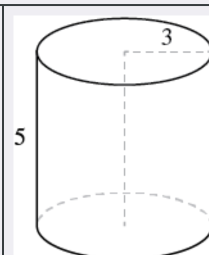
For a cylinder with radius r and height h :



EXAMPLE 7

A cylinder has height 5 centimetres and radius 3 centimetres. Find the a) volume and b) surface area.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the cylinder
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$V = \pi r^2 h$ $V \approx (3.14)\,3^2 \cdot 5$
Step 5. **Solve.**	$V \approx 141.3$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume is approximately 141.3 cubic inches.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the cylinder
Step 3. **Name.** Choose a variable to represent it.	let S = surface area
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$S = 2\pi r^2 + 2\pi rh$ $S \approx 2\left(3.14\right)3^2 + 2\left(3.14\right)\left(3\right)5$
Step 5. **Solve.**	$S \approx 150.72$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The surface area is approximately 150.72 square inches.

TRY IT 7.1

Find the a) volume and b) surface area of the cylinder with radius 4 cm and height 7cm.

Answer

a. 351.68 cu. cm
b. 276.32 sq. cm

TRY IT 7.2

Find the a) volume and b) surface area of the cylinder with given radius 2 ft and height 8 ft.

Answer

a. 100.48 cu. ft
b. 125.6 sq. ft

EXAMPLE 8

Find the a) volume and b) surface area of a can of soda. The radius of the base is 4 centimetres and the height is 13 centimetres. Assume the can is shaped exactly like a cylinder.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	

a)	
Step 2. **Identify** what you are looking for.	the volume of the cylinder
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$V = \pi r^2 h$ $V \approx (3.14)\, 4^2 \cdot 13$
Step 5. **Solve.**	$V \approx 653.12$
Step 6. **Check:** We leave it to you to check.	
Step 7. **Answer** the question.	The volume is approximately 653.12 cubic centimetres.

b)	
Step 2. **Identify** what you are looking for.	the surface area of the cylinder
Step 3. **Name.** Choose a variable to represent it.	let S = surface area
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$S = 2\pi r^2 + 2\pi r h$ $S \approx 2\,(3.14)\, 4^2 + 2\,(3.14)\,(4)\, 13$
Step 5. **Solve.**	$S \approx 427.04$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The surface area is approximately 427.04 square centimetres.

TRY IT 8.1

Find the a) volume and b) surface area of a can of paint with radius 8 centimetres and height 19 centimetres. Assume the can is shaped exactly like a cylinder.

Answer

a. 3,818.24 cu. cm
b. 1,356.48 sq. cm

TRY IT 8.2

Find the a) volume and b) surface area of a cylindrical drum with radius 2.7 feet and height 4 feet. Assume the drum is shaped exactly like a cylinder.

Answer

a. 91.5624 cu. ft
b. 113.6052 sq. ft

Find the Volume of Cones

The first image that many of us have when we hear the word 'cone' is an ice cream cone. There are many other applications of cones (but most are not as tasty as ice cream cones). In this section, we will see how to find the volume of a cone.

In geometry, a cone is a solid figure with one circular base and a vertex. The height of a cone is the distance between its base and the vertex.The cones that we will look at in this section will always have the height perpendicular to the base. See (Figure.6).
The height of a cone is the distance between its base and the vertex.

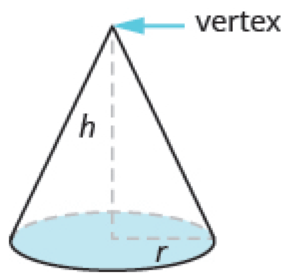


Figure.6

Earlier in this section, we saw that the volume of a cylinder is $V = \pi r^2 h$. We can think of a cone as part of a cylinder. Figure.7 shows a cone placed inside a cylinder with the same height and same base. If we compare the volume of the cone and the cylinder, we can see that the volume of the cone is less than that of the cylinder.
The volume of a cone is less than the volume of a cylinder with the same base and height.

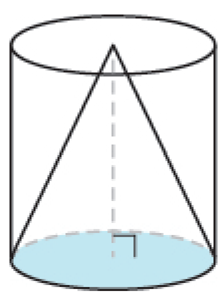


Figure.7

In fact, the volume of a cone is exactly one-third of the volume of a cylinder with the same base and height. The volume of a cone is

$$V = \frac{1}{3}Bh$$

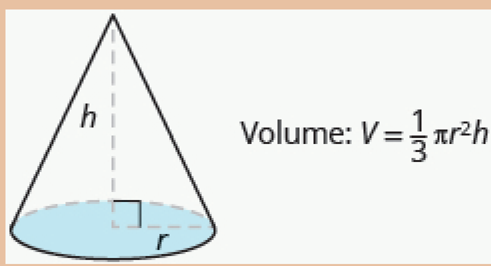
Since the base of a cone is a circle, we can substitute the formula of area of a circle, πr^2 , for B *to get the formula for volume of a cone.*

$$V = \frac{1}{3}\pi r^2 h$$

In this book, we will only find the volume of a cone, and not its surface area.

Volume of a Cone

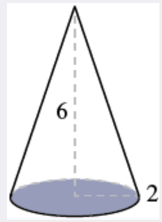
For a cone with radius r and height h.



EXAMPLE 9

Find the volume of a cone with height 6 inches and radius of its base 2 inches.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the volume of the cone
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π)	$V = \frac{1}{3} \quad \pi \quad r^2 \quad h$ $V \approx \frac{1}{3} \quad 3.14 \quad (2)^2 \quad (6)$
Step 5. **Solve.**	$V \approx 25.12$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume is approximately 25.12 cubic inches.

TRY IT 9.1

Find the volume of a cone with height 7 inches and radius 3 inches

Answer
65.94 cu. in.

TRY IT 9.2

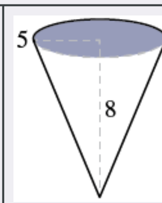
Find the volume of a cone with height 9 centimetres and radius 5 centimetres

Answer
235.5 cu. cm

EXAMPLE 10

Marty's favorite gastro pub serves french fries in a paper wrap shaped like a cone. What is the volume of a conic wrap that is 8 inches tall and 5 inches in diametre? Round the answer to the nearest hundredth.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information. Notice here that the base is the circle at the top of the cone.	
Step 2. **Identify** what you are looking for.	the volume of the cone
Step 3. **Name.** Choose a variable to represent it.	let V = volume
Step 4. **Translate.** Write the appropriate formula. Substitute. (Use 3.14 for π, and notice that we were given the distance across the circle, which is its diametre. The radius is 2.5 inches.)	$V = \frac{1}{3} \quad \pi \quad r^2 \quad h$ $V \approx \frac{1}{3} \quad 3.14 \quad (2.5)^2 \quad (8)$
Step 5. **Solve.**	$V \approx 52.33$
Step 6. **Check:** We leave it to you to check your calculations.	
Step 7. **Answer** the question.	The volume of the wrap is approximately 52.33 cubic inches.

TRY IT 10.1

How many cubic inches of candy will fit in a cone-shaped piñata that is 18 inches long and 12 inches across its base? Round the answer to the nearest hundredth.

Answer

678.24 cu. in.

TRY IT 10.2

What is the volume of a cone-shaped party hat that is 10 inches tall and 7 inches across at the base? Round the answer to the nearest hundredth.

Answer

128.2 cu. in.

ACCESS ADDITIONAL ONLINE RESOURCES

- [Volume of a Cone]

Key Concepts

- **Volume and Surface Area of a Rectangular Solid**
  - $V = LWH$
  - $S = 2LH + 2LW + 2WH$
- **Volume and Surface Area of a Cube**
  - $V = s^3$
  - $S = 6s^2$
- **Volume and Surface Area of a Sphere**
  - $V = \frac{4}{3}\pi r^3$
  - $S = 4\pi r^2$
- **Volume and Surface Area of a Cylinder**
  - $V = \pi r^2 h$
  - $S = 2\pi r^2 + 2\pi rh$
- **Volume of a Cone**
  - For a cone with radius r and height h:
    Volume: $V = \frac{1}{3}\pi r^2 h$

Glossary

cone
A cone is a solid figure with one circular base and a vertex.

cube
A cube is a rectangular solid whose length, width, and height are equal.

cylinder
A cylinder is a solid figure with two parallel circles of the same size at the top and bottom.

Practice Makes Perfect

Find Volume and Surface Area of Rectangular Solids

In the following exercises, find a) the volume and b) the surface area of the rectangular solid with the given dimensions.

1. length 2 metres, width 1.5 metres, height 3 metres	2. length 5 feet, width 8 feet, height 2.5 feet
3. length 3.5 yards, width 2.1 yards, height 2.4 yards	4. length 8.8 centimetres, width 6.5 centimetres, height 4.2 centimetres

In the following exercises, solve.

5. **Moving van** A rectangular moving van has length 16 feet, width 8 feet, and height 8 feet. Find its a) volume and b) surface area.	6. **Gift box** A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches. Find its a) volume and b) surface area.
7. **Carton** A rectangular carton has length 21.3 cm, width 24.2 cm, and height 6.5 cm. Find its a) volume and b) surface area.	8.**Shipping container** A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet. Find its a) volume and b) surface area.

In the following exercises, find a) the volume and b) the surface area of the cube with the given side length.

9. 5 centimetres	10. 6 inches
11. 10.4 feet	12. 12.5 metres

In the following exercises, solve.

13. **Science center** Each side of the cube at the Discovery Science Center in Santa Ana is 64 feet long. Find its a) volume and b) surface area.	14. **Museum** A cube-shaped museum has sides 45 metres long. Find its a) volume and b) surface area.
15. **Base of statue** The base of a statue is a cube with sides 2.8 metres long. Find its a) volume and b) surface area.	16. **Tissue box** A box of tissues is a cube with sides 4.5 inches long. Find its a) volume and b) surface area.

Find the Volume and Surface Area of Spheres

In the following exercises, find a) the volume and b) the surface area of the sphere with the given radius. Round answers to the nearest hundredth.

17. 3 centimetres	18. 9 inches
19. 7.5 feet	20. 2.1 yards

In the following exercises, solve. Round answers to the nearest hundredth.

21. **Exercise ball** An exercise ball has a radius of 15 inches. Find its a) volume and b) surface area.	22. **Balloon ride** The Great Park Balloon is a big orange sphere with a radius of 36 feet . Find its a) volume and b) surface area.
23. **Golf ball** A golf ball has a radius of 4.5 centimetres. Find its a) volume and b) surface area.	24. **Baseball** A baseball has a radius of 2.9 inches. Find its a) volume and b) surface area.

Find the Volume and Surface Area of a Cylinder

In the following exercises, find a) the volume and b) the surface area of the cylinder with the given radius and height. Round answers to the nearest hundredth.

25. radius 3 feet, height 9 feet	26. radius 5 centimetres, height 15 centimetres
27. radius 1.5 metres, height 4.2 metres	28. radius 1.3 yards, height 2.8 yards

In the following exercises, solve. Round answers to the nearest hundredth.

29. **Coffee can** A can of coffee has a radius of 5 cm and a height of 13 cm. Find its a) volume and b) surface area.	30. **Snack pack** A snack pack of cookies is shaped like a cylinder with radius 4 cm and height 3 cm. Find its a) volume and b) surface area.
31. **Barber shop pole** A cylindrical barber shop pole has a diametre of 6 inches and height of 24 inches. Find its a) volume and b) surface area.	32. **Architecture** A cylindrical column has a diametre of 8 feet and a height of 28 feet. Find its a) volume and b) surface area.

Find the Volume of Cones

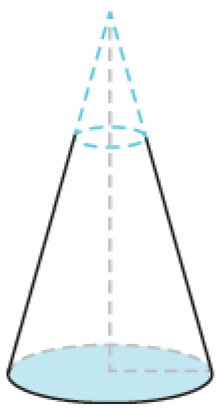
In the following exercises, find the volume of the cone with the given dimensions. Round answers to the nearest hundredth.

33. height 9 feet and radius 2 feet	34. height 8 inches and radius 6 inches
35. height 12.4 centimetres and radius 5 cm	36. height 15.2 metres and radius 4 metres

In the following exercises, solve. Round answers to the nearest hundredth.

37. **Teepee** What is the volume of a cone-shaped teepee tent that is 10 feet tall and 10 feet across at the base?	38. **Popcorn cup** What is the volume of a cone-shaped popcorn cup that is 8 inches tall and 6 inches across at the base?
39. **Silo** What is the volume of a cone-shaped silo that is 50 feet tall and 70 feet across at the base?	40. **Sand pile** What is the volume of a cone-shaped pile of sand that is 12 metres tall and 30 metres across at the base?

Everyday Math

41. **Street light post** The post of a street light is shaped like a truncated cone, as shown in the picture below. It is a large cone minus a smaller top cone. The large cone is 30 feet tall with base radius 1 foot. The smaller cone is 10 feet tall with base radius of 0.5 feet. To the nearest tenth, a) find the volume of the large cone. b) find the volume of the small cone. c) find the volume of the post by subtracting the volume of the small cone from the volume of the large cone. 	42. **Ice cream cones** A regular ice cream cone is 4 inches tall and has a diametre of 2.5 inches. A waffle cone is 7 inches tall and has a diametre of 3.25 inches. To the nearest hundredth, a) find the volume of the regular ice cream cone. b) find the volume of the waffle cone. c) how much more ice cream fits in the waffle cone compared to the regular cone?

Writing Exercises

43. The formulas for the volume of a cylinder and a cone are similar. Explain how you can remember which formula goes with which shape.	44. Which has a larger volume, a cube of sides of 8 feet or a sphere with a diametre of 8 feet? Explain your reasoning.

Answers

1. a) 9 cu. m b) 27 sq. m	3. a) 17.64 cu. yd. b) 41.58 sq. yd.	5. a) 1,024 cu. ft b) 640 sq. ft
7. a) 3,350.49 cu. cm b) 1,622.42 sq. cm	9. a) 125 cu. cm b) 150 sq. cm	11. a) 1124.864 cu. ft. b) 648.96 sq. ft
13. a) 262,144 cu. ft b) 24,576 sq. ft	15. a) 21.952 cu. m b) 47.04 sq. m	17. a) 113.04 cu. cm b) 113.04 sq. cm
19. a) 1,766.25 cu. ft b) 706.5 sq. ft	21. a) 14,130 cu. in. b) 2,826 sq. in.	23. a) 381.51 cu. cm b) 254.34 sq. cm
25. a) 254.34 cu. ft b) 226.08 sq. ft	27. a) 29.673 cu. m b) 53.694 sq. m	29. a) 1,020.5 cu. cm b) 565.2 sq. cm
31. a) 678.24 cu. in. b) 508.68 sq. in.	33. 37.68 cu. ft	35. 324.47 cu. cm
37. 261.67 cu. ft	39. 64,108.33 cu. ft	41. a) 31.4 cu. ft b) 2.6 cu. ft c) 28.8 cu. ft
43. Answers will vary.		

Attributions

- This chapter has been adapted from "Solve Geometry Applications: Volume and Surface Area" in [Prealgebra](#) (OpenStax) by Lynn Marecek, MaryAnne Anthony-Smith, and Andrea Honeycutt Mathis, which is under a [CC BY 4.0](#)

4.4 Solve Geometry Applications: Circles and Irregular Figures

Learning Objectives

By the end of this section, you will be able to:

- Use the properties of circles
- Find the area of irregular figures

In this section, we'll continue working with geometry applications. We will add several new formulas to our collection of formulas. To help you as you do the examples and exercises in this section, we will show the Problem Solving Strategy for Geometry Applications here.

Problem Solving Strategy for Geometry Applications

1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Use the Properties of Circles

We'll refer to the properties of circles as we use them to solve applications.

Properties of Circles

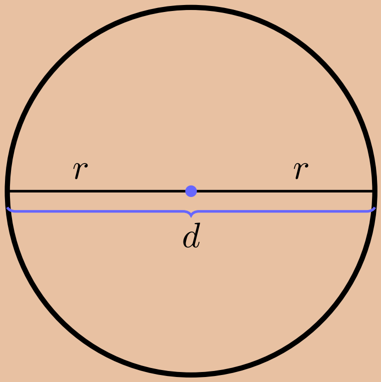
- r is the length of the radius
- d is the length of the diametre

$d = 2r$

- Circumference is the perimeter of a

circle. The formula for circumference is $C = 2\pi r$

- The formula for area of a circle is $A = \pi r^2$



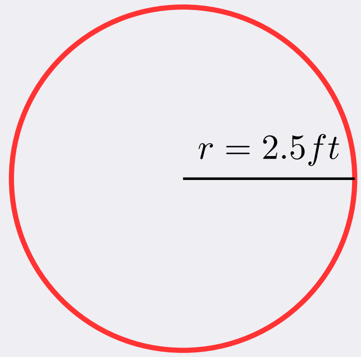
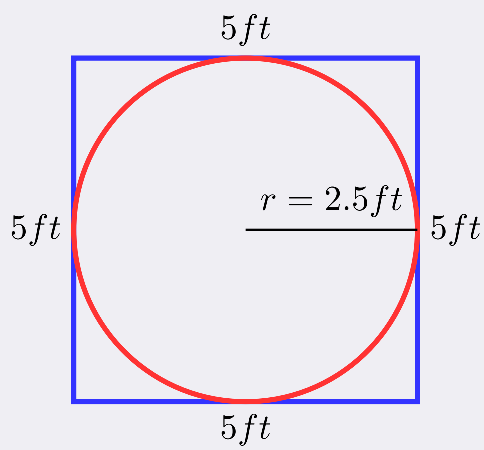
Remember, that we approximate π with 3.14 or $\frac{22}{7}$ depending on whether the radius of the circle is given as a decimal or a fraction. If you use the π key on your calculator to do the calculations in this section, your answers will be slightly different from the answers shown. That is because the π key uses more than two decimal places.

EXAMPLE 1

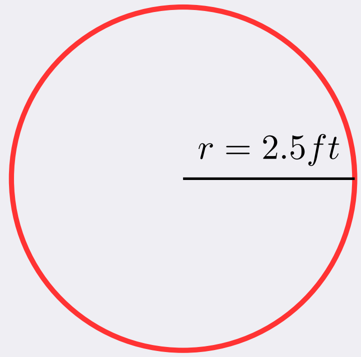
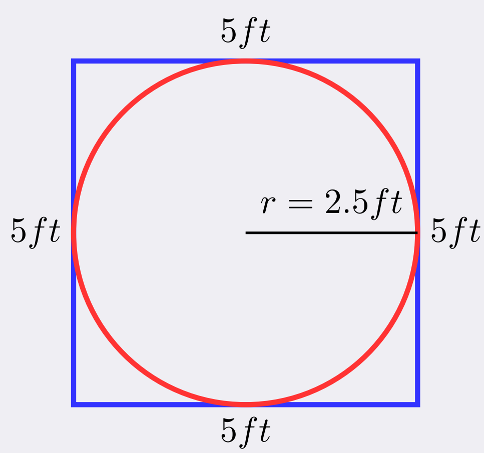
A circular sandbox has a radius of 2.5 feet. Find the a) circumference and b) area of the sandbox.

Solution

a)

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the circumference of the circle
Step 3. **Name.** Choose a variable to represent it.	Let c = circumference of the circle
Step 4. **Translate.** Write the appropriate formula Substitute	$C = 2\pi r$ $C = 2\pi\left(2.5\right)$
Step 5. **Solve** the equation.	$C \approx 2\left(3.14\right)\left(2.5\right)$ $C \approx 15.7\text{ft}$
Step 6. **Check.** Does this answer make sense? Yes. If we draw a square around the circle, its sides would be 5 ft (**twice the radius**), so its perimeter would be 20 ft. This is slightly more than the circle's circumference, 15.7 ft.	
Step 7. **Answer** the question.	The circumference of the sandbox is 15.7 feet.

b)

Step 1. **Read** the problem. Draw the figure and label it with the given information	
Step 2. **Identify** what you are looking for.	the area of the circle
Step 3. **Name.** Choose a variable to represent it.	Let A = the area of the circle
Step 4. **Translate.** Write the appropriate formula Substitute	$A = \pi r^2$ $A = \pi(2.5)^2$
Step 5. **Solve** the equation.	$A \approx (3.14)\,(2.5)^2$ $A \approx 19.625\,\text{sq. ft}$
Step 6. **Check.** Yes. If we draw a square around the circle, its sides would be 5 ft, as shown in part a). So the area of the square would be 25 sq. ft. This is slightly more than the circle's area, 19.625 sq. ft.	
Step 7. **Answer** the question.	The area of the circle is 19.625 square feet.

TRY IT 1.1

A circular mirror has radius of 5 inches. Find the a) circumference and b) area of the mirror.

Answer

a. 31.4 in.
b. 78.5 sq. in.

TRY IT 1.2

A circular spa has radius of 4.5 feet. Find the a) circumference and b) area of the spa.

Answer

a. 28.26 ft
b. 63.585 sq. ft

We usually see the formula for circumference in terms of the radius r of the circle:
$C = 2\pi r$

But since the diametre of a circle is two times the radius, we could write the formula for the circumference in terms of d.

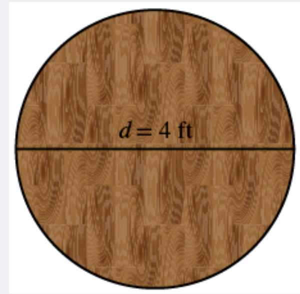
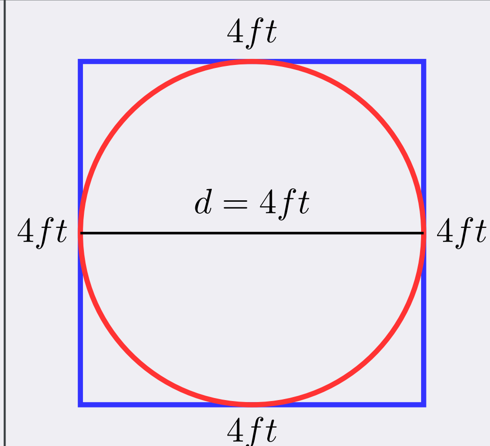
$$\begin{array}{rc} & C = 2\pi r \\ \text{Using the commutative property, we get} & C = \pi \cdot 2r \\ \text{Then substituting } d = 2r & C = \pi \cdot \text{d} \\ \text{So} & C = \pi d \end{array}$$

We will use this form of the circumference when we're given the length of the diametre instead of the radius.

EXAMPLE 2

A circular table has a diametre of four feet. What is the circumference of the table?

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the circumference of the table
Step 3. **Name.** Choose a variable to represent it.	Let c = the circumference of the table
Step 4. **Translate.** Write the appropriate formula for the situation. Substitute.	$C = \pi d$ $C = \pi\left(4\right)$
Step 5. **Solve** the equation, using 3.14 for π.	$C \approx \left(3.14\right)\left(4\right)$ $C \approx 12.56\ \text{sq. feet}$
Step 6. **Check:** If we put a square around the circle, its side would be 4. The perimeter would be 16. It makes sense that the circumference of the circle, 12.56, is a little less than 16.	
Step 7. **Answer** the question.	The diametre of the table is 12.56 square feet

TRY IT 2.1

Find the circumference of a circular fire pit whose diametre is 5.5 feet.

Answer

17.27 ft

TRY IT 2.2

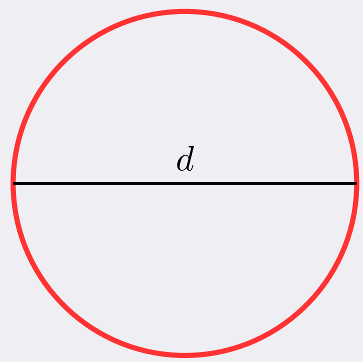
If the diametre of a circular trampoline is 12 feet, what is its circumference?

Answer
37.68 ft

EXAMPLE 3

Find the diametre of a circle with a circumference of 47.1 centimetres.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	 C=47.1 cm
Step 2. **Identify** what you are looking for.	the diametre of the circle
Step 3. **Name.** Choose a variable to represent it.	Let *d* = the diametre of the circle
Step 4. **Translate.**	
Write the formula. Substitute, using 3.14 to approximate π.	$C = \pi d$ $47.1 \approx 3.14d$
Step 5. **Solve.**	$\frac{47.1}{3.14} \approx \frac{3.14d}{3.14}$ $15 \approx d$
Step 6. **Check:**	$C = \pi d$ $47.1 \stackrel{?}{=} (3.14)(15)$ $47.1 = 47.1\checkmark$
Step 7. **Answer the question.**	The diametre of the circle is approximately 15 centimetres.

TRY IT 3.1

Find the diametre of a circle with circumference of 94.2 centimetres.

Answer
30 cm

TRY IT 3.2

Find the diametre of a circle with circumference of 345.4 feet.

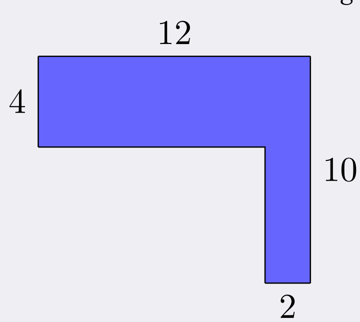
Answer
110 ft

Find the Area of Irregular Figures

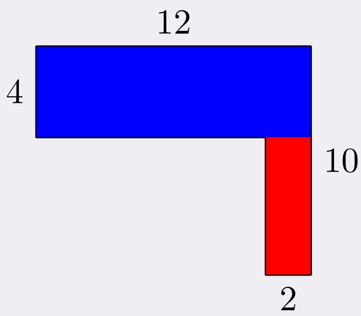
So far, we have found area for rectangles, triangles, trapezoids, and circles. An irregular figure is a figure that is not a standard geometric shape. Its area cannot be calculated using any of the standard area formulas. But some irregular figures are made up of two or more standard geometric shapes. To find the area of one of these irregular figures, we can split it into figures whose formulas we know and then add the areas of the figures.

EXAMPLE 4

Find the area of the shaded region.

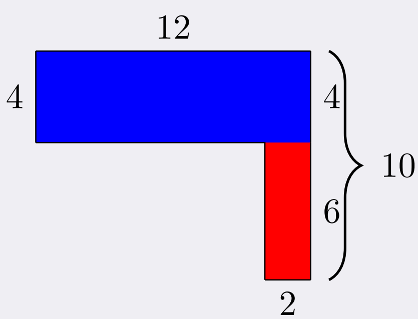


Solution
The given figure is irregular, but we can break it into two rectangles. The area of the shaded region will be the sum of the areas of both rectangles.



$$\mathrm{A}_{\text{figure}} = {\color{blue}A_{\text{rectangle}}} + {\color{red}A_{\text{rectangle}}}$$

The blue rectangle has a width of 12 and a length of 4. The red rectangle has a width of 2, but its length is not labeled. The right side of the figure is the length of the red rectangle plus the length of the blue rectangle. Since the right side of the blue rectangle is 4 units long, the length of the red rectangle must be 6 units.



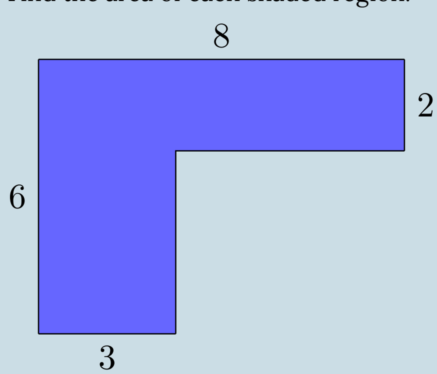
$$\begin{aligned}
A_{\text{figure}} &= {\color{blue}A_{\text{rectangle}}} + {\color{red}A_{\text{rectangle}}} \\
A_{\text{figure}} &= {\color{blue}bh} + {\color{red}bh} \\
A_{\text{figure}} &= 12 \cdot 4 + 2 \cdot 6 \\
A_{\text{figure}} &= 48 + 12 \\
A_{\text{figure}} &= 60
\end{aligned}$$

The area of the figure is 60 square units.

Is there another way to split this figure into two rectangles? Try it, and make sure you get the same area.

TRY IT 4.1

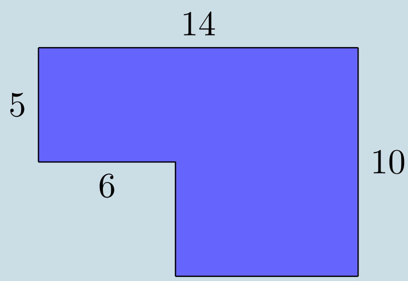
Find the area of each shaded region:



Answer
28 sq. units

TRY IT 4.2

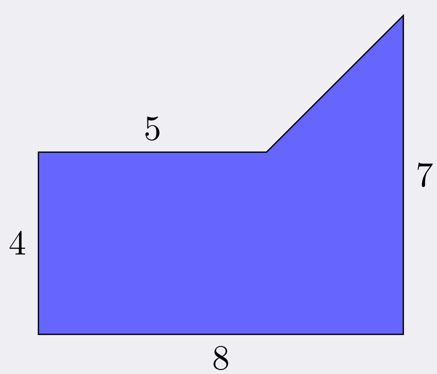
Find the area of each shaded region:



Answer
110 sq. units

EXAMPLE 5

Find the area of the shaded region.



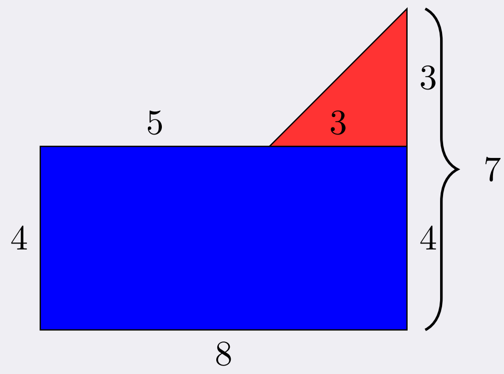
Solution
We can break this irregular figure into a triangle and rectangle. The area of the figure will be the sum of the areas of triangle and rectangle.

The rectangle has a length of 8 units and a width of 4 units.

We need to find the base and height of the triangle.

Since both sides of the rectangle are 4, the vertical side of the triangle is 3, which is $7-4$.

The length of the rectangle is 8, so the base of the triangle will be 3, which is $8-4$.



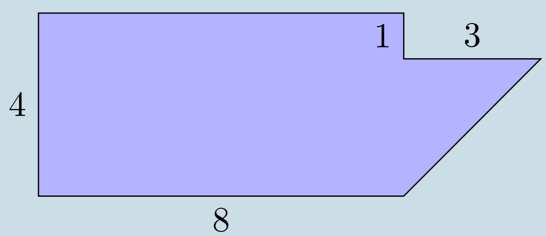
Now we can add the areas to find the area of the irregular figure.

$$\begin{aligned} A_{\text{figure}} &= A_{\text{rectangle}} + A_{\text{triangle}} \\ A_{\text{figure}} &= lw + \frac{1}{2}bh \\ A_{\text{figure}} &= 8 \cdot 4 + \frac{1}{2} \cdot 3 \cdot 3 \\ A_{\text{figure}} &= 32 + 4.5 \\ A_{\text{figure}} &= 36.5 \text{ sq. units} \end{aligned}$$

The area of the figure is 36.5 square units.

TRY IT 5.1

Find the area of each shaded region.

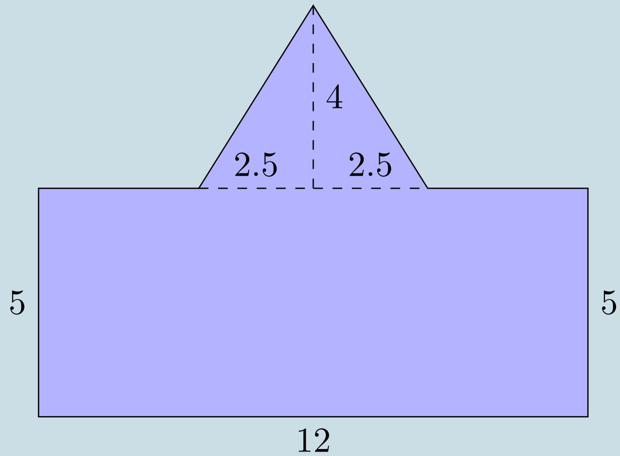


Answer

36.5 sq. units

TRY IT 5.2

Find the area of each shaded region.



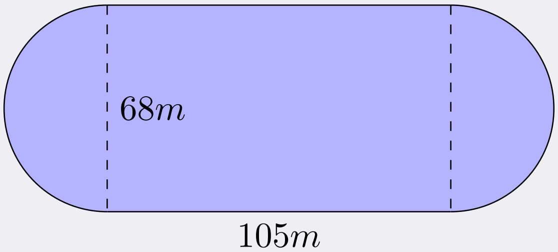
Answer
70 sq. units

EXAMPLE 6

A high school track is shaped like a rectangle with a semi-circle (half a circle) on each end. The rectangle has length 105 metres and width 68 metres. Find the area enclosed by the track. Round your answer to the nearest hundredth.

Solution

We will break the figure into a rectangle and two semi-circles. The area of the figure will be the sum of the areas of the rectangle and the semicircles.



The rectangle has a length of 105 m and a width of 68 m. The semi-circles have a diametre of 68 m, so each has a radius of 34 m.

$$A_{\text{figure}} = A_{\text{rectangle}} + A_{\text{semicircles}}$$

$$A_{\text{figure}} = bh + 2\left(\frac{1}{2}\pi \cdot r^2\right)$$

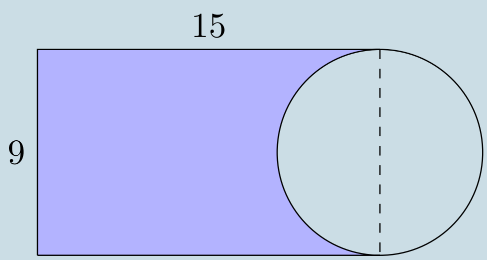
$$A_{\text{figure}} \approx 105 \cdot 68 + 2\left(\frac{1}{2} \cdot 3.14 \cdot 34^2\right)$$

$$A_{\text{figure}} \approx 7140 + 3629.84$$

$$A_{\text{figure}} \approx 10,769.84 \text{ square meters}$$

TRY IT 6.1

Find the area of the shaded shape:

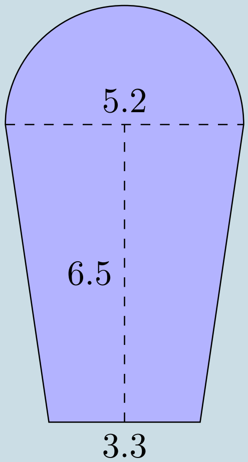


Answer

103.2 sq. units

TRY IT 6.2

Find the area:



Answer
38.24 sq. units

Access Additional Online Resources

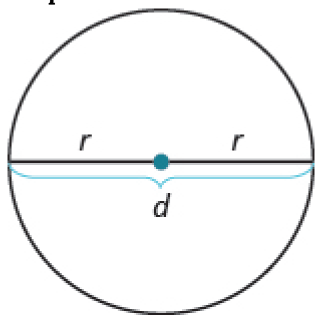
- Circumference of a Circle
- Area of a Circle
- Area of an L-shaped polygon
- Area of an L-shaped polygon with Decimals
- Perimeter Involving a Rectangle and Circle
- Area Involving a Rectangle and Circle

Key Concepts

- **Problem Solving Strategy for Geometry Applications**

1. Read the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. Identify what you are looking for.
3. Name what you are looking for. Choose a variable to represent that quantity.
4. Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. Solve the equation using good algebra techniques.
6. Check the answer in the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

- **Properties of Circles**



- $d=2r$
- **Circumference:** $C=2\pi r$ or $C=\pi d$
- **Area:** $A=\pi r^2$

Glossary

irregular figure
An irregular figure is a figure that is not a standard geometric shape. Its area cannot be calculated using any of the standard area formulas.

Practice Makes Perfect

Use the Properties of Circles

In the following exercises, solve using the properties of circles.

1. The lid of a paint bucket is a circle with radius 7 inches. Find the a) circumference and b) area of the lid.	2. An extra-large pizza is a circle with radius 8 inches. Find the a) circumference and b) area of the pizza.
3. A farm sprinkler spreads water in a circle with radius of 8.5 feet. Find the a) circumference and b) area of the watered circle.	4. A circular rug has radius of 3.5 feet. Find the a) circumference and b) area of the rug.
5. A reflecting pool is in the shape of a circle with diametre of 20 feet. What is the circumference of the pool?	6. A turntable is a circle with diametre of 10 inches. What is the circumference of the turntable?
7. A circular saw has a diametre of 12 inches. What is the circumference of the saw?	8. A round coin has a diametre of 3 centimetres. What is the circumference of the coin?
9. A barbecue grill is a circle with a diametre of 2.2 feet. What is the circumference of the grill?	10. The top of a pie tin is a circle with a diametre of 9.5 inches. What is the circumference of the top?
11. A circle has a circumference of 163.28 inches. Find the diametre.	12. A circle has a circumference of 59.66 feet. Find the diametre.
13. A circle has a circumference of 17.27 metres. Find the diametre.	14. A circle has a circumference of 80.07 centimetres. Find the diametre.

In the following exercises, find the radius of the circle with given circumference.

15. A circle has a circumference of 150.72 feet.	16. A circle has a circumference of 251.2 centimetres.
17. A circle has a circumference of 40.82 miles.	18. A circle has a circumference of 78.5 inches.

Find the Area of Irregular Figures

In the following exercises, find the area of the irregular figure. Round your answers to the nearest hundredth.

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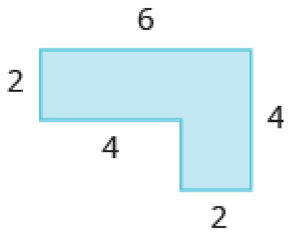

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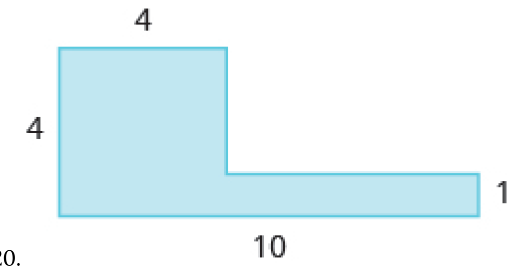

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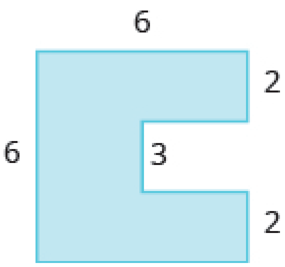

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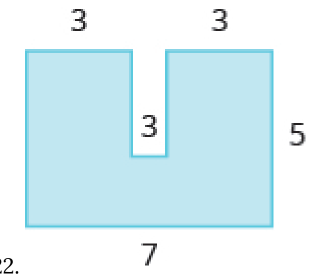

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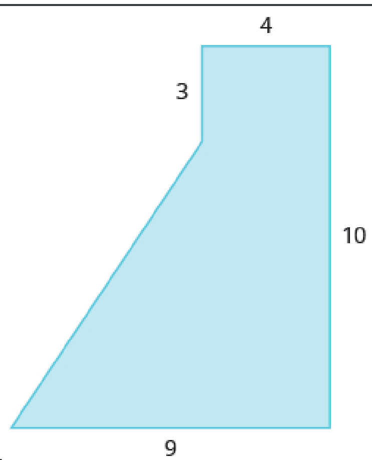

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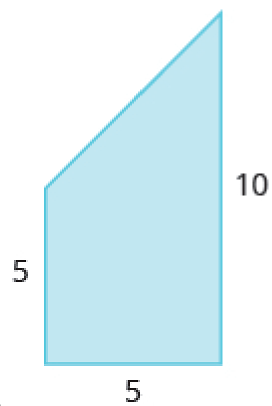

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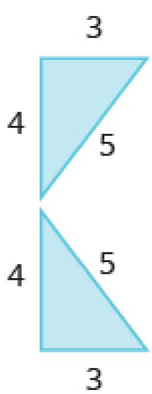

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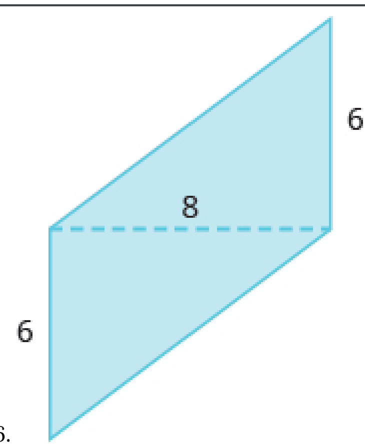

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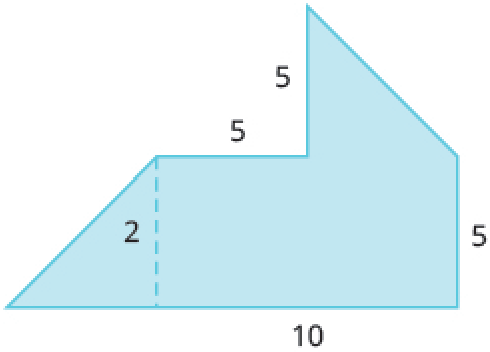

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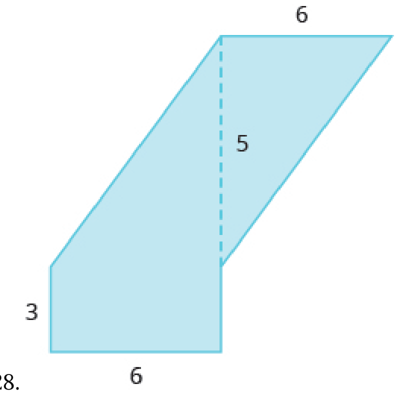

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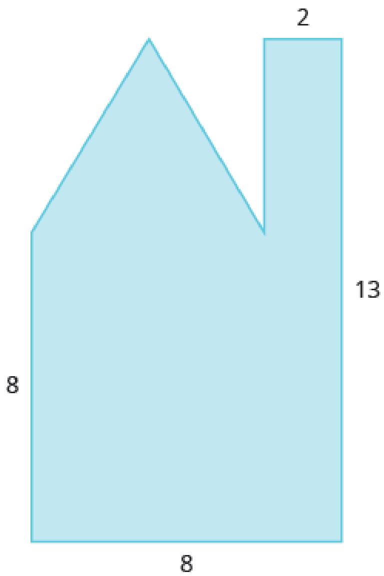

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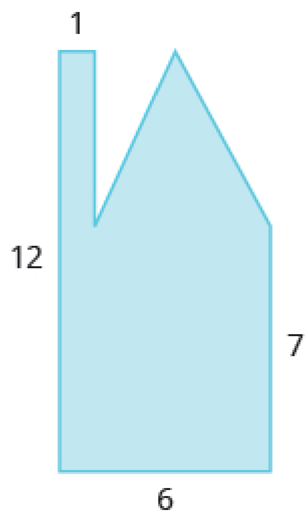

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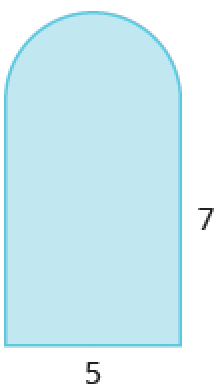

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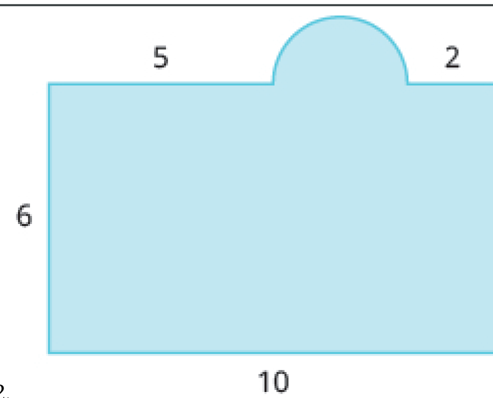

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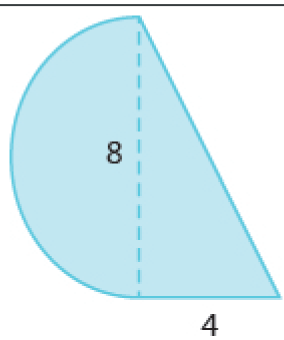

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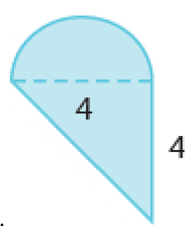

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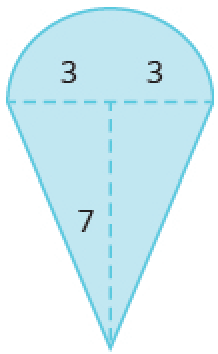

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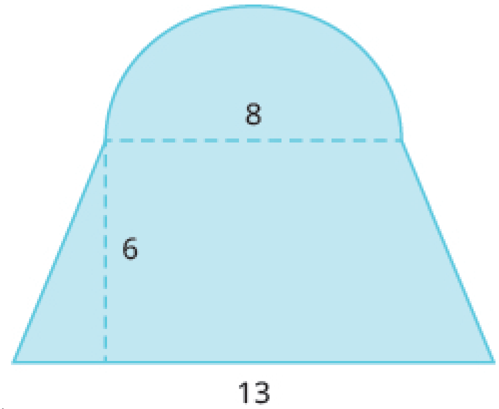

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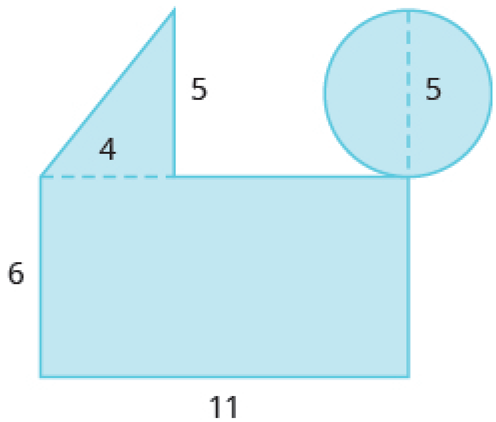

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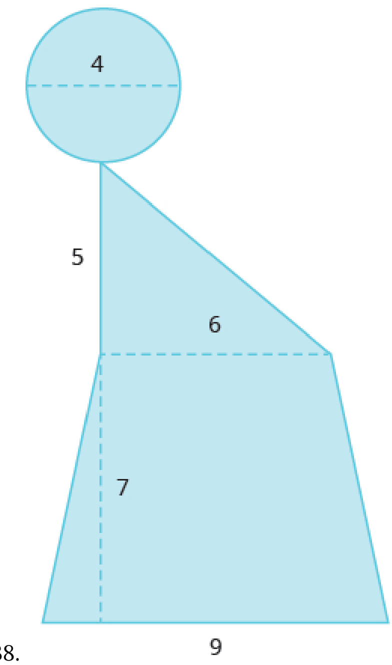
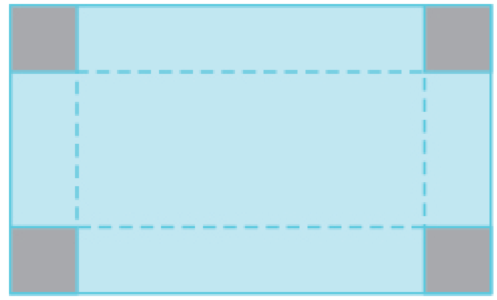


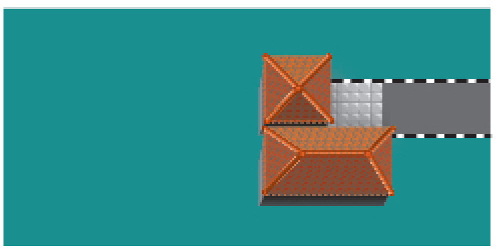
In the following exercises, solve.

39. A city park covers one block plus parts of four more blocks, as shown. The block is a square with sides 250 feet long, and the triangles are isosceles right triangles. Find the area of the park.

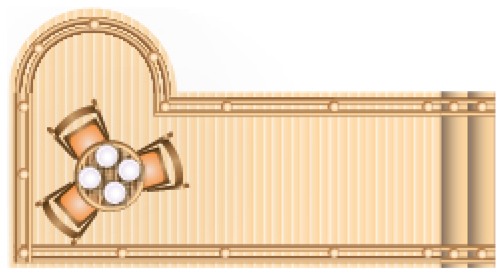
40. A gift box will be made from a rectangular piece of cardboard measuring 12 inches by 20 inches, with squares cut out of the corners of the sides, as shown. The sides of the squares are 3 inches. Find the area of the cardboard after the corners are cut out.



41. Perry needs to put in a new lawn. His lot is a rectangle with a length of 120 feet and a width of 100 feet. The house is rectangular and measures 50 feet by 40 feet. His driveway is rectangular and measures 20 feet by 30 feet, as shown. Find the area of Perry's lawn.



42. Denise is planning to put a deck in her back yard. The deck will be a 20-ft by 12-ft rectangle with a semicircle of diametre 6 feet, as shown below. Find the area of the deck.



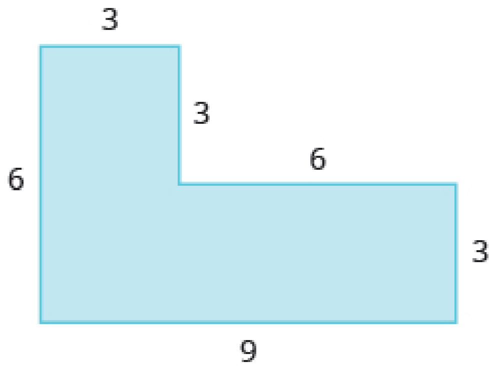
Everyday Math

43. **Area of a Tabletop** Yuki bought a drop-leaf kitchen table. The rectangular part of the table is a 1-ft by 3-ft rectangle with a semicircle at each end, as shown. a) Find the area of the table with one leaf up. b) Find the area of the table with both leaves up.

44. **Painting** Leora wants to paint the nursery in her house. The nursery is an 8-ft by 10-ft rectangle, and the ceiling is 8 feet tall. There is a 3-ft by 6.5-ft door on one wall, a 3-ft by 6.5-ft closet door on another wall, and one 4-ft by 3.5-ft window on the third wall. The fourth wall has no doors or windows. If she will only paint the four walls, and not the ceiling or doors, how many square feet will she need to paint?

Writing Exercises

45. Describe two different ways to find the area of this figure, and then show your work to make sure both ways give the same area.



46. A circle has a diametre of 14 feet. Find the area of the circle a) using 3.14 for π b) using $\frac{22}{7}$ for π. c) Which calculation to do prefer? Why?

Answers

1. a) 43.96 in. b) 153.86 sq. in.	3. a) 53.38 ft b) 226.865 sq. ft	5. 62.8 ft
7. 37.68 in.	9. 6.908 ft	11. 52 in.
13. 5.5 m	15. 24 ft	17. 6.5 mi
19. 16 sq. units	21. 30 sq. units	23. 57.5 sq. units
25. 12 sq. units	27. 67.5 sq. units	29. 89 sq. units
31. 44.81 sq. units	33. 41.12 sq. units	35. 35.13 sq. units
37. 95.625 sq. units	39. 187,500 sq. ft	41. 9400 sq. ft
43. a) 6.5325 sq. ft b) 10.065 sq. ft	45. Answers will vary.	

Attributions

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4.5 Chapter Review

Review Exercises

Systems of Measurement

In the following exercises, convert between Imperial units. Round to the nearest tenth.

1. A picture frame is 42 inches wide. Convert the width to feet.	2. A floral arbor is 7 feet tall. Convert the height to inches.
3. A playground is 45 feet wide. Convert the width to yards.	4. Kelly is 5 feet 4 inches tall. Convert her height to inches.
5. An orca whale in the Salish Sea weighs 4.5 tons. Convert the weight to pounds.	6. The height of Mount Shasta is $14,179$ feet. Convert the height to miles.
7. How many tablespoons are in a quart?	8. The play lasted $1\frac{3}{4}$ hours. Convert the time to minutes.
9. Trinh needs 30 cups of paint for her class art project. Convert the volume to gallons.	10. Naomi's baby weighed 5 pounds 14 ounces at birth. Convert the weight to ounces.

In the following exercises, solve, and state your answer in mixed units.

11. Every day last week, Pedro recorded the amount of time he spent reading. He read for $50, 25, 83, 45, 32, 60, \text{and } 135$ minutes. How much time, in hours and minutes, did Pedro spend reading?	12. John caught 4 lobsters. The weights of the lobsters were 1 pound 9 ounces, 1 pound 12 ounces, 4 pounds 2 ounces, and 2 pounds 15 ounces. What was the total weight of the lobsters?
13. Dalila wants to make pillow covers. Each cover takes 30 inches of fabric. How many yards and inches of fabric does she need for 4 pillow covers?	14. Fouad is 6 feet 2 inches tall. If he stands on a rung of a ladder 8 feet 10 inches high, how high off the ground is the top of Fouad's head?

In the following exercises, convert between metric units.

15. Mount Everest is $8,850$ metres tall. Convert the height to kilometres.	16. Donna is 1.7 metres tall. Convert her height to centimetres.
17. One cup of yogurt contains 13 grams of protein. Convert this to milligrams.	18. One cup of yogurt contains 488 milligrams of calcium. Convert this to grams.
19. A bottle of water contained 650 millilitre s. Convert this to litres.	20. Sergio weighed 2.9 kilograms at birth. Convert this to grams.

In the following exercises, solve.

21. Selma had a 1-liter bottle of water. If she drank 145 millilitres, how much water, in millilitres, was left in the bottle?	22. Minh is 2 metres tall. His daughter is 88 centimetres tall. How much taller, in metres, is Minh than his daughter?
23. One ounce of tofu provides 2 grams of protein. How many milligrams of protein are provided by 5 ounces of tofu?	24. One serving of cranberry juice contains 30 grams of sugar. How many kilograms of sugar are in 30 servings of cranberry juice?

In the following exercises, convert between Imperial and metric units. Round to the nearest tenth.

25. A college basketball court is 84 feet long. Convert this length to metres.	26. Majid is 69 inches tall. Convert his height to centimetres.
27. Lucas weighs 78 kilograms. Convert his weight to pounds.	28. Caroline walked 2.5 kilometres. Convert this length to miles.
29. A box of books weighs 25 pounds. Convert this weight to kilograms.	30. Steve's car holds 55 litres of gas. Convert this to gallons.

In the following exercises, convert the Fahrenheit temperatures to degrees Celsius. Round to the nearest tenth.

31. 23°F	32. 95°F
33. 64°F	34. 20°F

In the following exercises, convert the Celsius temperatures to degrees Fahrenheit. Round to the nearest tenth.

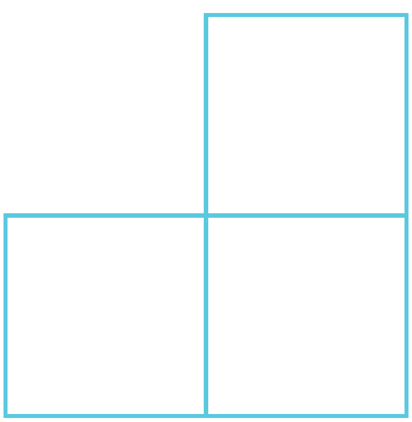
35. -5°C	36. 30°C
37. 24°C	38. -12°C

Understand Linear, Square, Cubic Measure

In the following exercises, would you measure each item using linear, square, or cubic measure?

39. amount of sand in a sandbag	40. height of a tree
41. size of a patio	42. length of a highway

In the following exercises, find a) the perimeter b) the area of each figure



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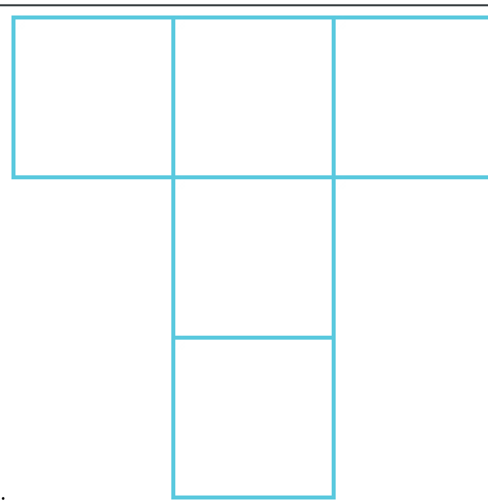

44.

Use Properties of Rectangles

In the following exercises, find the a) perimeter b) area of each rectangle

45. The length of a rectangle is 42 metres and the width is 28 metres.	46. The length of a rectangle is 36 feet and the width is 19 feet.
47. A sidewalk in front of Kathy's house is in the shape of a rectangle 4 feet wide by 45 feet long.	48. A rectangular room is 16 feet wide by 12 feet long.

In the following exercises, solve.

49. Find the length of a rectangle with perimeter of 220 centimetres and width of 85 centimetres.	50. Find the width of a rectangle with perimeter 39 and length 11.
51. The area of a rectangle is 2356 square metres. The length is 38 metres. What is the width?	52. The width of a rectangle is 45 centimetres. The area is 2700 square centimetres. What is the length?
53. The length of a rectangle is 12 centimetres more than the width. The perimeter is 74 centimetres. Find the length and the width.	54. The width of a rectangle is 3 more than twice the length. The perimeter is 96 inches. Find the length and the width.

Use Properties of Triangles

In the following exercises, solve using the properties of triangles.

55. Find the area of a triangle with base 18 inches and height 15 inches.	56. Find the area of a triangle with base 33 centimetres and height 21 centimetres.
57. A triangular road sign has base 30 inches and height 40 inches. What is its area?	58. If a triangular courtyard has sides 9 feet and 12 feet and the perimeter is 32 feet, how long is the third side?
59. A tile in the shape of an isosceles triangle has a base of 6 inches. If the perimeter is 20 inches, find the length of each of the other sides.	60. Find the length of each side of an equilateral triangle with perimeter of 81 yards.
61. The perimeter of a triangle is 59 feet. One side of the triangle is 3 feet longer than the shortest side. The third side is 5 feet longer than the shortest side. Find the length of each side.	62. One side of a triangle is three times the smallest side. The third side is 9 feet more than the shortest side. The perimeter is 39 feet. Find the lengths of all three sides.

Use Properties of Trapezoids

In the following exercises, solve using the properties of trapezoids.

63. The height of a trapezoid is 8 feet and the bases are 11 and 14 feet. What is the area?	64. The height of a trapezoid is 5 yards and the bases are 7 and 10 yards. What is the area?
65. Find the area of the trapezoid with height 25 metres and bases 32.5 and 21.5 metres.	66. A flag is shaped like a trapezoid with height 62 centimetres and the bases are 91.5 and 78.1 centimetres. What is the area of the flag?

Use Properties of Circles

In the following exercises, solve using the properties of circles. Round answers to the nearest hundredth.

67. A circular mosaic has radius 3 metres. Find the a) circumference b) area of the mosaic	68. A circular fountain has radius 8 feet. Find the a) circumference b) area of the fountain
69. Find the diametre of a circle with circumference 150.72 inches.	70. Find the radius of a circle with circumference 345.4 centimetres

Find the Area of Irregular Figures

In the following exercises, find the area of each shaded region.

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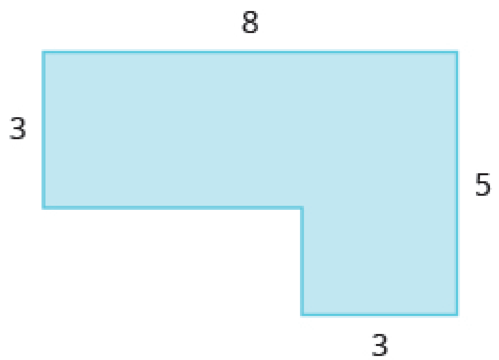

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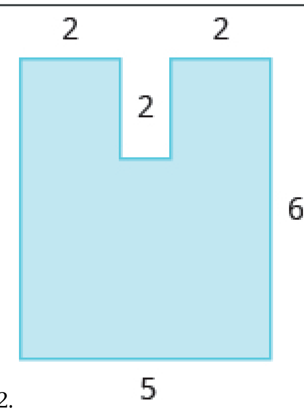

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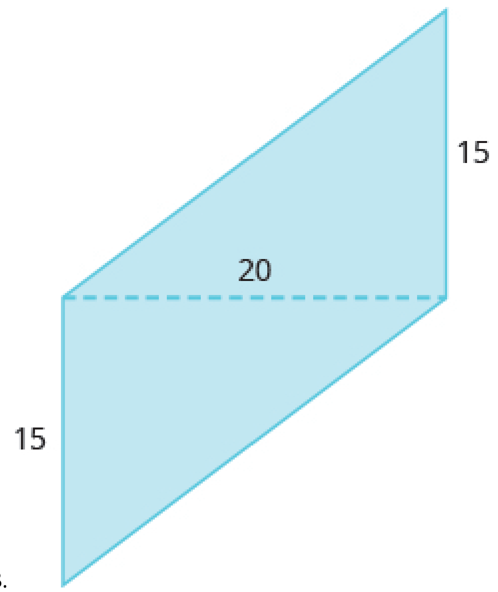

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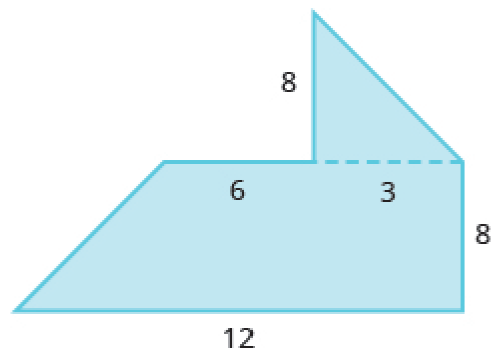

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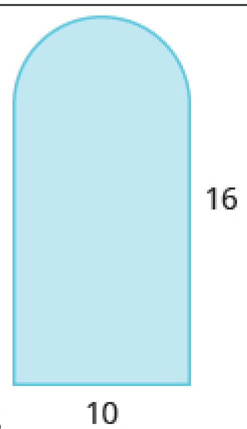

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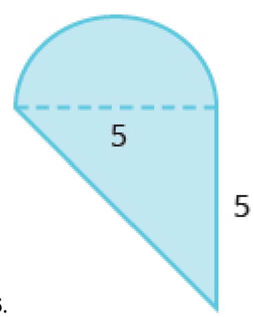


Find Volume and Surface Area of Rectangular Solids

In the following exercises, find the a) volume b) surface area of the rectangular solid

77. A rectangular solid with length 14 centimetres, width 4.5 centimetres, and height 10 centimetres	78. A cube with sides that are 3 feet long
79. A cube of tofu with sides 2.5 inches	80. A rectangular carton with length 32 inches, width 18 inches, and height 10 inches

Find Volume and Surface Area of Spheres

In the following exercises, find the a) volume b) surface area of the sphere.

81. a sphere with radius 4 yards	82. a sphere with radius 12 metres
83. a baseball with radius 1.45 inches	84. a soccer ball with radius 22 centimetres

Find Volume and Surface Area of Cylinders

In the following exercises, find the a) volume b) surface area of the cylinder

85. A cylinder with radius 2 yards and height 6 yards	86. A cylinder with diametre 18 inches and height 40 inches
87. A juice can with diametre 8 centimetres and height 15 centimetres	88. A cylindrical pylon with diametre 0.8 feet and height 2.5 feet

Find Volume of Cones

In the following exercises, find the volume of the cone.

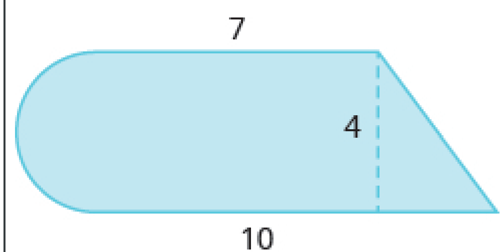
89. A cone with height 5 metres and radius 1 metre	90. A cone with height 24 feet and radius 8 feet
91. A cone-shaped water cup with diametre 2.6 inches and height 2.6 inches	92. A cone-shaped pile of gravel with diametre 6 yards and height 5 yards

Review Answers

1. 3.5 feet	3. 15 yards	5. 9000 pounds
7. 64 tablespoons	9. 1.9 gallons	11. 7 hours 10 minutes
13. 3 yards, 12 inches	15. 8.85 kilometres	17. 13,000 milligrams
19. 0.65 litres	21. 855 millilitre s	23. 10,000 milligrams
25. 25.6 metres	27. 171.6 pounds	29. 11.4 kilograms
31. -5°C	33. 17.8°C	35. 23°F
37. 75.2°F	39. cubic	41. square
43. a) 8 units b) 3 sq. units	45. a) 140 m b) 1176 sq. m	47. a) 98 ft. b) 180 sq. ft.
49. 25 cm	51. 62 m	53. 24.5 in., 12.5 in.
55. 135 sq. in.	57. 600 sq. in.	59. 7 in., 7 in.
61. 17 ft., 20 ft., 22 ft.	63. 100 sq. ft.	65. 675 sq. m
67. a) 18.84 m b) 28.26 sq. m	69. 48 in.	71. 30 sq. units
73. 300 sq. units	75. 199.25 sq. units	77. a) 630 cu. cm b) 496 sq. cm
79. a) 15.625 cu. in. b) 37.5 sq. in.	81. a) 267.95 cu. yd. b) 200.96 sq. yd.	83. a) 12.76 cu. in. b) 26.41 sq. in.
85. a) 75.36 cu. yd. b) 100.48 sq. yd.	87. a) 753.6 cu. cm b) 477.28 sq. cm	89. 5.233 cu. m
91. 4.599 cu. in.		

Practice Test

In the following exercises, solve using the appropriate unit conversions.

1. One cup of milk contains 276 milligrams of calcium. Convert this to grams. $(1 \text{ milligram} = 0.001 \text{ gram})$	2. Azize walked $4\frac{1}{2}$ miles. Convert this distance to feet. $(1 \text{ mile} = 5{,}280 \text{ feet})$.
3. Janice ran 15 kilometres. Convert this distance to miles. Round to the nearest hundredth of a mile. $(1 \text{ mile} = 1.61 \text{ kilometres})$	4. Larry had 5 phone customer phone calls yesterday. The calls lasted $28, 44, 9, 75, \text{and } 55$ minutes. How much time, in hours and minutes, did Larry spend on the phone? $(1 \text{ hour} = 60 \text{ minutes})$
5. Use the formula $F = \frac{9}{5}C + 32$ to convert 35°C to degrees F	6. Yolie is 63 inches tall. Convert her height to centimetres. Round to the nearest centimetre. $(1 \text{ inch} = 2.54 \text{ centimetres})$
7. A triangular poster has base 80 centimetres and height 55 centimetres. Find the area of the poster.	8. The length of a rectangle is 2 feet more than five times the width. The perimeter is 40 feet. Find the dimensions of the rectangle.
9. A circular pool has diametre 90 inches. What is its circumference? Round to the nearest *tenth.*	10. A trapezoid has height 14 inches and bases 20 inches and 23 inches. Find the area of the trapezoid.
11. Find the volume of a rectangular room with width 12 feet, length 15 feet, and height 8 feet.	12. Find the area of the shaded region. Round to the nearest tenth. 
13. A traffic cone has height 75 centimetres. The radius of the base is 20 centimetres. Find the volume of the cone. Round to the nearest tenth.	14. A coffee can is shaped like a cylinder with height 7 inches and radius 5 inches. Find (a) the surface area and (b) the volume of the can. Round to the nearest tenth.

Practice Test Answers

1. .276 grams	2. 23760 feet	3. 9.317 miles
4. 211 minutes, 3 hours and 31 minutes	5. 95°F	6. 160 centimetres
7. 2,200 square centimetres	8. 11 feet, 9 feet	9. 282.6 inches
10. 201 feet	11. 1,440 cubic feet	12. 10.3 square inches
13. 31,400 cubic inches	14. a) 534.1 square inches b) 1335 cubic inches	

PART III
CHAPTER 5 TRIGONOMETRY

Trigonometry is a part of geometry that takes its origin in the ancient study of the relationship of the sides and angles of a right triangle. “Trigon” from Greek means triangle and “metron” means measure.

Applications of trigonometry are essential to many disciplines like carpentry, engineering, surveying, and astronomy, just to name a few.

How tall is the Riverpole? Do we have to climb the pole to find out? Fortunately, with the knowledge of trigonometry, we can find out the measurements of tall objects without too much hassle.

In this chapter we will explore the basic properties of angles and triangles, and the applications of the Pythagorean Theorem and trigonometric ratios.

Riverpole by Vaughn Warren – Kamloops, BC.

5.1 Use Properties of Angles, Triangles, and the Pythagorean Theorem

Learning Objectives

By the end of this section, you will be able to:

- Use the properties of angles
- Use the properties of triangles
- Use the Pythagorean Theorem

Use the Properties of Angles

Are you familiar with the phrase 'do a 180' ? It means to make a full turn so that you face the opposite direction. It comes from the fact that the measure of an angle that makes a straight line is 180 degrees. See (Figure 1).

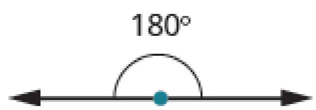


Figure 1

An angle is formed by two rays that share a common endpoint. Each ray is called a side of the angle and the common endpoint is called the vertex. An angle is named by its vertex. In (Figure 2), $\angle A$ is the angle with vertex at point A. The measure of $\angle A$ is written $m\angle A$.
$\angle A$ is the angle with vertex at $\text{point } A$.

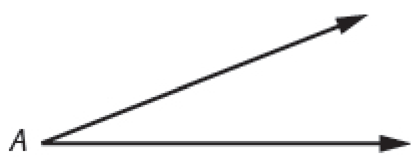


Figure 2

We measure angles in degrees, and use the symbol ° to represent degrees. We use the abbreviation m to for the *measure* of an angle. So if $\angle A$ is 27°, we would write $m\angle A = 27$.

If the sum of the measures of two angles is 180°, then they are called supplementary angles. In (Figure 3), each pair of angles is supplementary because their measures add to 180°. Each angle is the *supplement* of the other.
The sum of the measures of supplementary angles is 180°.

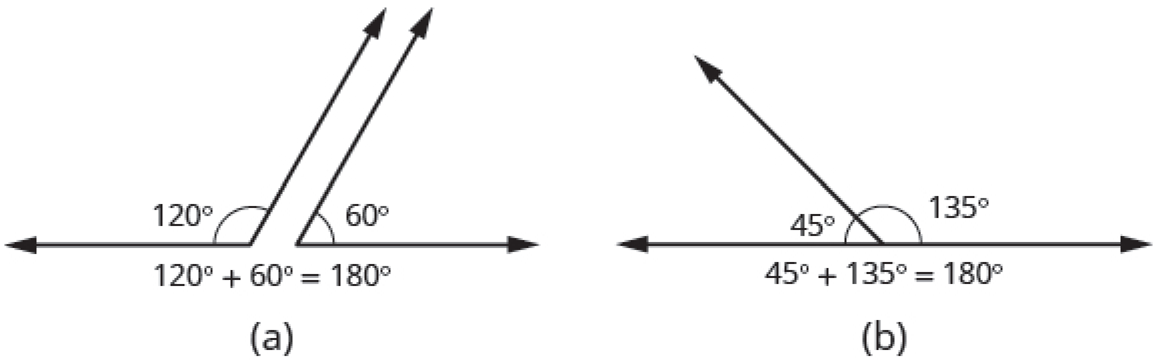


Figure 3

If the sum of the measures of two angles is $90°$, then the angles are complementary angles. In [(Figure 4)](#), each pair of angles is complementary, because their measures add to $90°$. Each angle is the *complement* of the other.
The sum of the measures of complementary angles is $90°$.

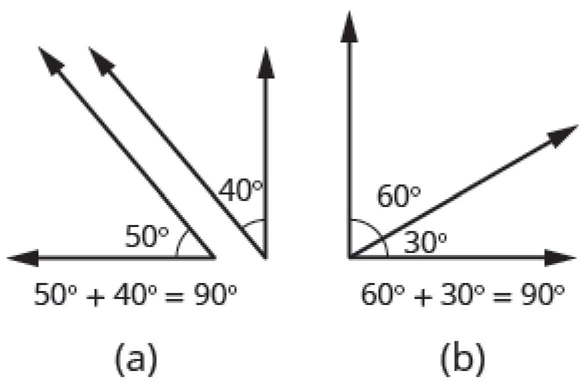


Figure 4

Supplementary and Complementary Angles

If the sum of the measures of two angles is $180°$, then the angles are supplementary.

If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180°$.

If the sum of the measures of two angles is $90°$, then the angles are complementary.

If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90°$.

In this section and the next, you will be introduced to some common geometry formulas. We will adapt our Problem Solving Strategy for Geometry Applications. The geometry formula will name the variables and give us the equation to solve.

In addition, since these applications will all involve geometric shapes, it will be helpful to draw a figure and then label it with the information from the problem. We will include this step in the Problem Solving Strategy for Geometry Applications.

HOW TO: *Use a Problem Solving Strategy for Geometry Applications*

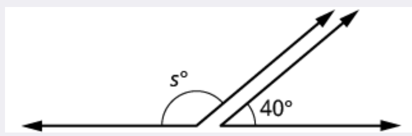
1. **Read** the problem and make sure you understand all the words and ideas. Draw a figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for and choose a variable to represent it.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

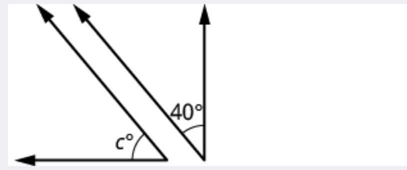
The next example will show how you can use the Problem Solving Strategy for Geometry Applications to answer questions about supplementary and complementary angles.

EXAMPLE 1

An angle measures $40°$. Find a) its supplement, and b) its complement.

Solution

a)	
Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the supplement of a $40°$ angle.
Step 3. **Name.** Choose a variable to represent it.	let s $=$ the measure of the supplement
Step 4. **Translate.** Write the appropriate formula for the situation and substitute in the given information.	$m\angle A + m\angle B = 180°$ $s + 40° = 180°$
Step 5. **Solve** the equation.	$s = 140$
Step 6. **Check:**	$140° + 40° \stackrel{?}{=} 180°$ $180° = 180° \checkmark$
Step 7. **Answer** the question.	The supplement of the $40°$ angle is $140°$

b)	
Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the complement of a $40°$ angle.
Step 3. **Name.** Choose a variable to represent it.	let $c =$ the measure of the complement
Step 4. **Translate.** Write the appropriate formula for the situation and substitute in the given information.	$m\angle A + m\angle B = 90°$ $c + 40° = 90°$
Step 5. **Solve** the equation.	$c = 50°$
Step 6. **Check:**	$50° + 40° \stackrel{?}{=} 90°$ $90° = 90° \checkmark$
Step 7. **Answer** the question.	The complement of the $40°$ angle is $50°$.

TRY IT 1.1

An angle measures $25°$. Find its: a) supplement b) complement.

Answer

a. 155°
b. 65°

TRY IT 1.2

An angle measures $77°$. Find its: a) supplement b) complement.

Answer

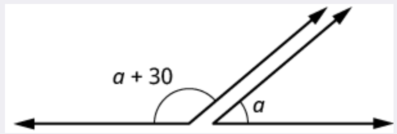
a. 103°
b. 13°

Did you notice that the words complementary and supplementary are in alphabetical order just like 90 and 180 are in numerical order?

EXAMPLE 2

Two angles are supplementary. The larger angle is $30°$ more than the smaller angle. Find the measure of both angles.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the measures of both angles
Step 3. **Name.** Choose a variable to represent it. The larger angle is 30° more than the smaller angle.	let $a =$ measure of smaller angle $a + 3 =$ measure of larger angle
Step 4. **Translate.** Write the appropriate formula and substitute.	$m\angle A + m\angle B = 180°$
Step 5. **Solve** the equation.	$(a+30)+a=180$ $2a+30=180$ $2a=150$ $\text{measure of smaller angle } a = 75$ $\text{measure of larger angle} = a + 30$ $75+30=105$
Step 6. **Check:**	$m\angle A + m\angle B = 180°$ $75° + 105° \stackrel{?}{=} 180°$ $180° = 180° \checkmark$
Step 7. **Answer** the question.	The measures of angles are $75°$ and $105°$.

TRY IT 2.1

Two angles are supplementary. The larger angle is $100°$ more than the smaller angle. Find the measures of both angles.

Answer

40°, 140°

TRY IT 2.2

Two angles are complementary. The larger angle is $40°$ more than the smaller angle. Find the measures of both angles.

Answer
25°, 65°

Use the Properties of Triangles

What do you already know about triangles? Triangle have three sides and three angles. Triangles are named by their vertices. The triangle in (Figure 5) is called ΔABC, read 'triangle ABC'. We label each side with a lower case letter to match the upper case letter of the opposite vertex.
ΔABC has vertices $A, B, \text{ and } C$ and sides $a, b, \text{and } c.$

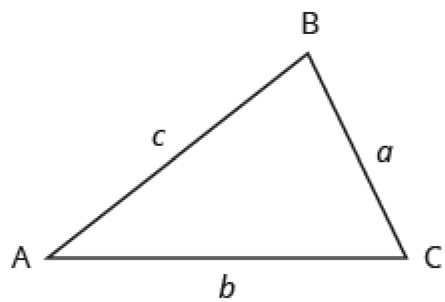


Figure 5

The three angles of a triangle are related in a special way. The sum of their measures is $180°$.
$m\angle A + m\angle B + m\angle C = 180°$

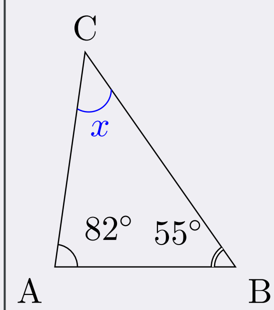
Sum of the Measures of the Angles of a Triangle

For any ΔABC, the sum of the measures of the angles is $180°$.
$m\angle A + m\angle B + m\angle C = 180°$

EXAMPLE 3

The measures of two angles of a triangle are $55°$ and $82°$. Find the measure of the third angle.

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the measure of the third angle in a triangle
Step 3. **Name.** Choose a variable to represent it.	let x = the measure of the angle
Step 4. **Translate.** Write the appropriate formula and substitute.	$m\angle A + m\angle B + m\angle C = 180°$
Step 5. **Solve** the equation.	$55 + 82 + x = 180$ $137 + x = 180$ $x = 43$
Step 6. **Check:**	$m\angle A + m\angle B + m\angle C = 180°$ $82° + 55° + 43° \stackrel{?}{=} 180°$ $180° = 180° \checkmark$
Step 7. **Answer** the question.	The measure of the third angle is $43°$.

TRY IT 3.1

The measures of two angles of a triangle are $31°$ and $128°$. Find the measure of the third angle.

Answer

21°

TRY IT 3.2

A triangle has angles of $49°$ and $75°$. Find the measure of the third angle.

Answer
56°

Right Triangles

Some triangles have special names. We will look first at the right triangle. A right triangle has one $90°$ angle, which is often marked with the symbol shown in (Figure 6).

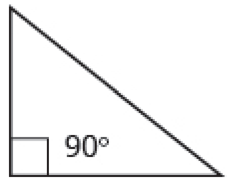


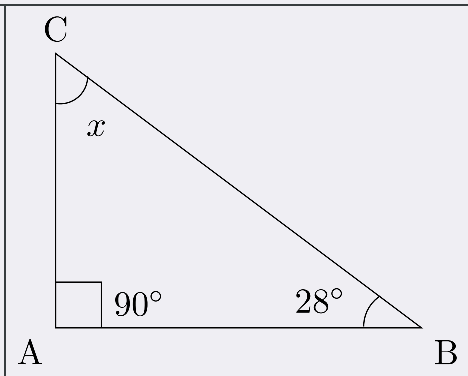
Figure 6

If we know that a triangle is a right triangle, we know that one angle measures $90°$ so we only need the measure of one of the other angles in order to determine the measure of the third angle.

EXAMPLE 4

One angle of a right triangle measures $28°$. What is the measure of the third angle?

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	
Step 2. **Identify** what you are looking for.	the measure of an angle
Step 3. **Name.** Choose a variable to represent it.	Let x $=$ the measure of the angle
Step 4. **Translate.** Write the appropriate formula and substitute.	$m\angle A + m\angle B + m\angle C = 180°$ $90 + 28 + x = 180°$
Step 5. **Solve** the equation.	$118 + x = 180$ $x = 62°$
Step 6. **Check:**	$m\angle A + m\angle B + m\angle C = 180°$ $90° + 28° + 62° \stackrel{?}{=} 180°$ $180° = 180° \checkmark$
Step 7. **Answer** the question.	The measure of the third angle is $62°$.

TRY IT 4.1

One angle of a right triangle measures $56°$. What is the measure of the other angle?

Answer

34°

TRY IT 4.2

One angle of a right triangle measures $45°$. What is the measure of the other angle?

Answer

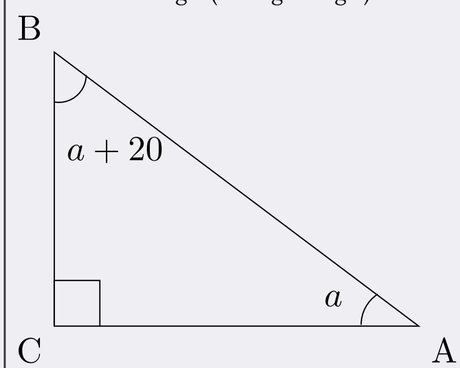
45°

In the examples so far, we could draw a figure and label it directly after reading the problem. In the next example, we will have to define one angle in terms of another. So we will wait to draw the figure until we write expressions for all the angles we are looking for.

EXAMPLE 5

The measure of one angle of a right triangle is $20°$ more than the measure of the smallest angle. Find the measures of all three angles.

Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what you are looking for.	the measures of all three angles
Step 3. **Name.** Choose a variable to represent it. Now draw the figure and label it with the given information.	Let $a = 1^{st}$ angle $a + 20 = 2^{nd}$ angle $90 = 3^{rd}$ angle (the right angle) 
Step 4. **Translate.** Write the appropriate formula and substitute into the formula.	$m\angle A + m\angle B + m\angle C = 180°$ $a + (a + 20) + 90 = 180$
Step 5. **Solve** the equation.	$2a + 110 = 180°$ $2a = 70$ $a = 35°\text{first angle}$ $\text{second angle} = a + 20$ $= 35 + 20$ $= 55°$ $\text{third angle} = 90°$
Step 6. **Check:**	$m\angle A + m\angle B + m\angle C = 180°$ $35° + 55° + 90° \stackrel{?}{=} 180°$ $180° = 180° \checkmark$
Step 7. **Answer** the question.	The three angles measure $35°, 55° \text{and} 90°$.

TRY IT 5.1

The measure of one angle of a right triangle is $50°$ more than the measure of the smallest angle. Find the measures of all three angles.

Answer
20°, 70°, 90°

TRY IT 5.2

The measure of one angle of a right triangle is $30°$ more than the measure of the smallest angle. Find the measures of all three angles.

Answer
30°, 60°, 90°

Similar Triangles

When we use a map to plan a trip, a sketch to build a bookcase, or a pattern to sew a dress, we are working with similar figures. In geometry, if two figures have exactly the same shape but different sizes, we say they are similar figures. One is a scale model of the other. The corresponding sides of the two figures have the same ratio, and all their corresponding angles are have the same measures.

The two triangles in (Figure 7) are similar. Each side of ΔABC is four times the length of the corresponding side of ΔXYZ and their corresponding angles have equal measures.

ΔABC and ΔXYZ are similar triangles. Their corresponding sides have the same ratio and the corresponding angles have the same measure.

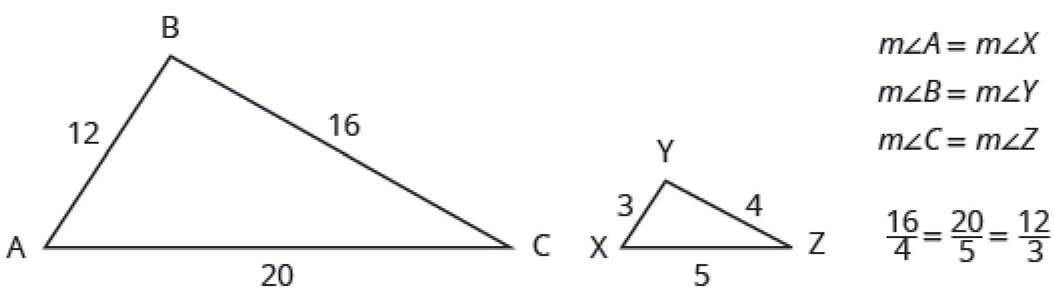
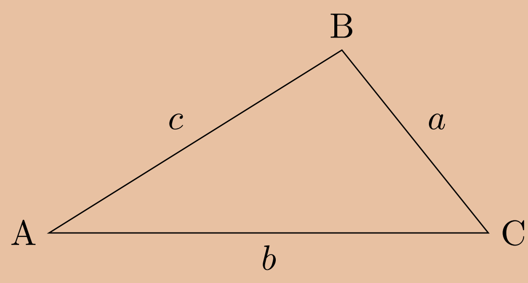
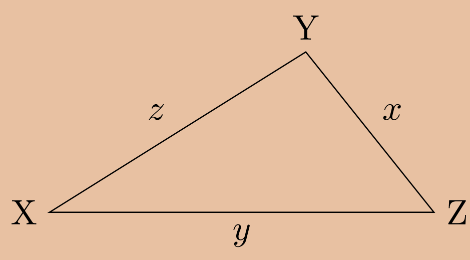


Figure 7

Properties of Similar Triangles

If two triangles are similar, then their corresponding angle measures are equal and their corresponding side lengths are in the same ratio.





$m\angle A = m\angle X$
$m\angle B = m\angle Y$
$m\angle C = m\angle Z$
$\dfrac{a}{X} = \dfrac{b}{y} = \dfrac{c}{Z}$

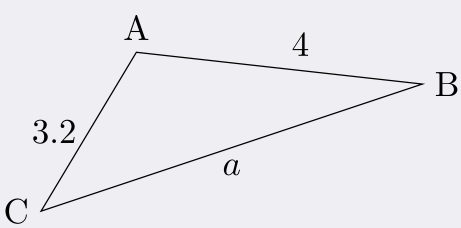
The length of a side of a triangle may be referred to by its endpoints, two vertices of the triangle. For example, in ΔABC:

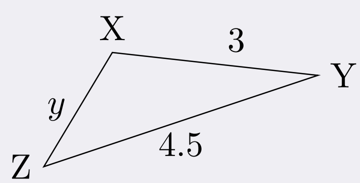
the length a can also be written BC
the length b can also be written AC
the length c can also be written AB

We will often use this notation when we solve similar triangles because it will help us match up the corresponding side lengths.

EXAMPLE 6

ΔABC and ΔXYZ are similar triangles. The lengths of two sides of each triangle are shown. Find the lengths of the third side of each triangle.



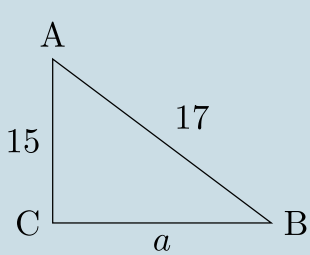


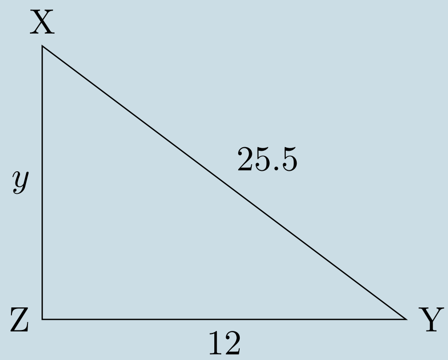
Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.	The figure is provided.
Step 2. **Identify** what you are looking for.	The length of the sides of similar triangles
Step 3. **Name.** Choose a variable to represent it.	Let *a* = length of the third side of ΔABC *y* = length of the third side ΔXYZ
Step 4. **Translate.**	The triangles are similar, so the corresponding sides are in the same ratio. So $\frac{AB}{XY}=\frac{BC}{YZ}=\frac{AC}{XZ}$ Since the side $AB=4$ corresponds to the side $XY=3$, we will use the ratio $\frac{\text{AB}}{\text{XY}}=\frac{4}{3}$ to find the other sides. Be careful to match up corresponding sides correctly. $\begin{array}{lcc} & \text{To find } a: & \text{To find } y: \\ \text{sides of large triangle} \longrightarrow & \frac{AB}{XY}=\frac{BC}{YZ} & \frac{AB}{XY}=\frac{AC}{XZ} \\ \text{sides of small triangle} \longrightarrow & \frac{4}{3}=\frac{a}{4.5} & \frac{4}{3}=\frac{3.2}{y} \end{array}$
Step 5. **Solve** the equation.	$\begin{aligned} 3a &= 4(4.5) & 4y &= 3(3.2) \\ 3a &= 18 & 4y &= 9.6 \\ a &= 6 & y &= 2.4 \end{aligned}$
Step 6. **Check.**	$\begin{aligned} \frac{4}{3} &\stackrel{?}{=} \frac{6}{4.5} & \frac{4}{3} &\stackrel{?}{=} \frac{3.2}{2.4} \\ 4(4.5) &\stackrel{?}{=} 6(3) & 4(2.4) &\stackrel{?}{=} 3.2(3) \\ 18 &= 18\checkmark & 9.6 &= 9.6\checkmark \end{aligned}$
Step 7. **Answer** the question.	The third side of ΔABC is 6 and the third side of ΔXYZ is 2.4.

TRY IT 6.1

ΔABC is similar to ΔXYZ. Find a.

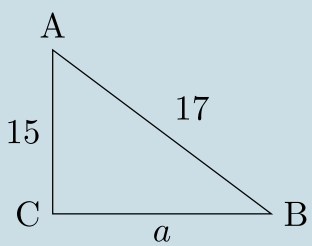


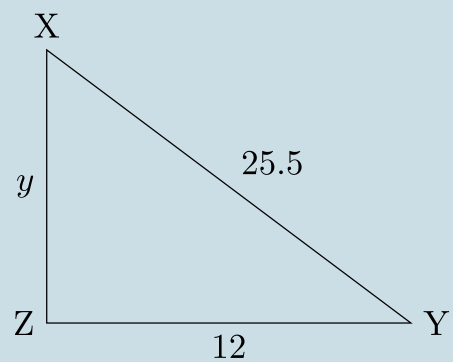


Answer
8

TRY IT 6.2

ΔABC is similar to ΔXYZ. Find y.





Answer
22.5

Use the Pythagorean Theorem

The Pythagorean Theorem is a special property of right triangles that has been used since ancient times. It is named after the Greek philosopher and mathematician Pythagoras who lived around 500 BCE.

Remember that a right triangle has a $90°$ angle, which we usually mark with a small square in the corner. The side of the triangle opposite the $90°$ angle is called the hypotenuse, and the other two sides are called the legs. See (Figure 8).

In a right triangle, the side opposite the $90°$ angle is called the hypotenuse and each of the other sides is called a leg.

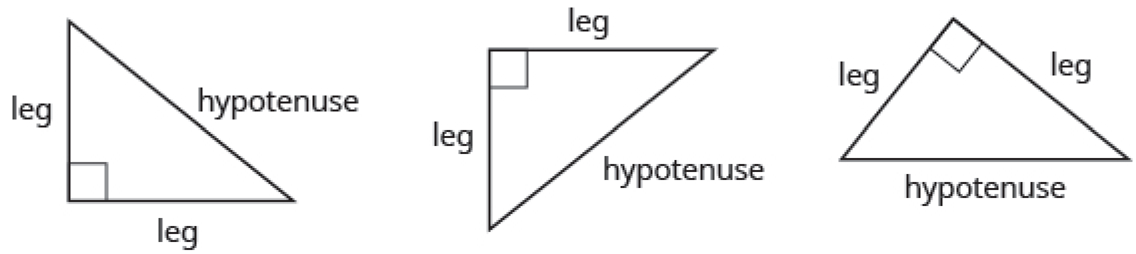


Figure 8

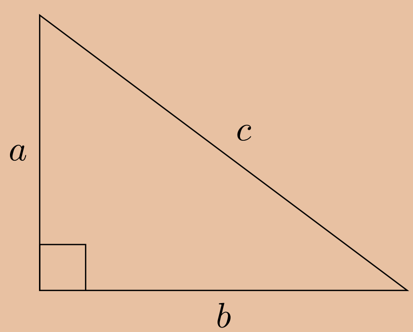
The Pythagorean Theorem tells how the lengths of the three sides of a right triangle relate to each other. It states that in any right triangle, the sum of the squares of the two legs equals the square of the hypotenuse.

The Pythagorean Theorem

In any right triangle ΔABC,

$a^2 + b^2 = c^2$

where c is the length of the hypotenuse a and b are the lengths of the legs.



To solve problems that use the Pythagorean Theorem, we will need to find square roots. We defined the notation $\sqrt{m}$ in this way:

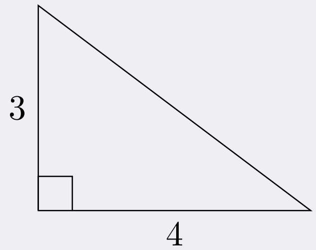
If $m = n^2, \text{ then } \sqrt{m} = n$ for $n \geq 0$

For example, we found that $\sqrt{25}$ is 5 because $5^2 = 25$.

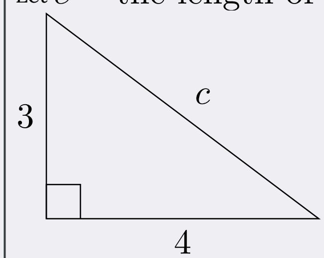
We will use this definition of square roots to solve for the length of a side in a right triangle.

EXAMPLE 7

Use the Pythagorean Theorem to find the length of the hypotenuse.

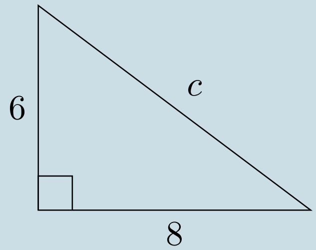


Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what you are looking for.	the length of the hypotenuse of the triangle
Step 3. **Name.** Choose a variable to represent it.	Let $c = \text{the length of the hypotenuse}$ 
Step 4. **Translate.** Write the appropriate formula. Substitute.	$a^2 + b^2 = c^2$ $3^2 + 4^2 = c^2$
Step 5. **Solve** the equation.	$9 + 16 = c^2$ $25 = c^2$ $\sqrt{25} = c^2$ $5 = c$
Step 6. **Check:**	$3^2 + 4^2 = 5^2$ $9 + 16 \stackrel{?}{=} 25$ $25 = 25\checkmark$
Step 7. **Answer** the question.	The length of the hypotenuse is 5.

TRY IT 7.1

Use the Pythagorean Theorem to find the length of the hypotenuse.

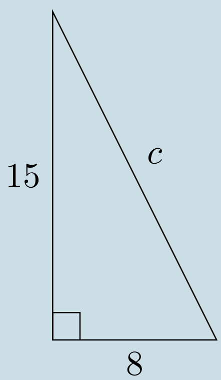


Answer

10

TRY IT 7.2

Use the Pythagorean Theorem to find the length of the hypotenuse.

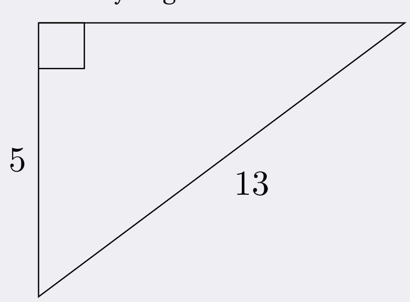


Answer

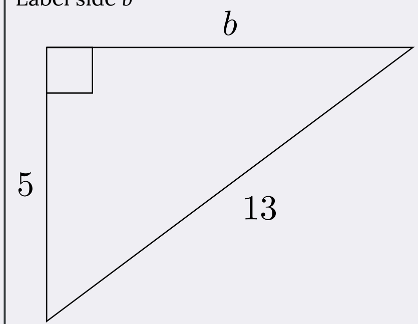
17

EXAMPLE 8

Use the Pythagorean Theorem to find the length of the longer leg.

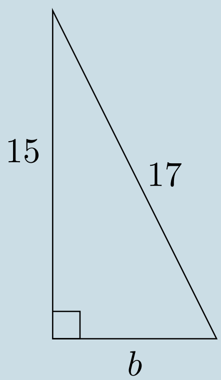


Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what you are looking for.	The length of the leg of the triangle
Step 3. **Name.** Choose a variable to represent it.	Let $b = \text{the leg of the triangle}$ Label side b 
Step 4. **Translate.** Write the appropriate formula. Substitute.	$a^2 + b^2 = c^2$ $5^2 + b^2 = 13^2$
Step 5. **Solve** the equation. Isolate the variable term. Use the definition of the square root. Simplify.	$25 + b^2 = 169$ $b^2 = 144$ $b^2 = \sqrt{144}$ $b = 12$
Step 6. **Check:**	$5^2 + 12^2 \stackrel{?}{=} 13^2$ $25 + 144 \stackrel{?}{=} 169$ $169 = 169$
Step 7. **Answer** the question.	The length of the leg is 12.

TRY IT 8.1

Use the Pythagorean Theorem to find the length of the leg.

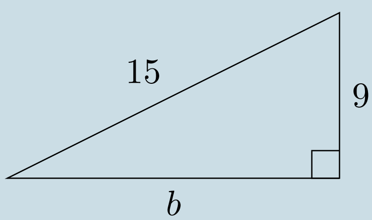


Answer

8

TRY IT 8.2

Use the Pythagorean Theorem to find the length of the leg.

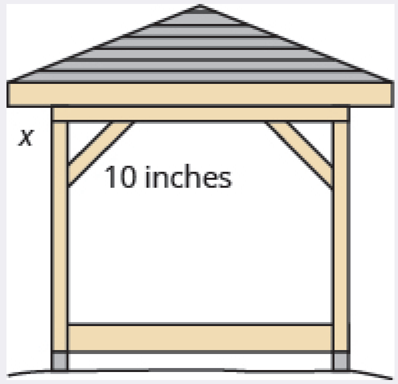


Answer

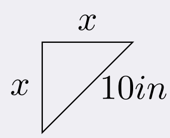
12

EXAMPLE 9

Kelvin is building a gazebo and wants to brace each corner by placing a 10-inch wooden bracket diagonally as shown. How far below the corner should he fasten the bracket if he wants the distances from the corner to each end of the bracket to be equal? Approximate to the nearest tenth of an inch.



Solution

Step 1. **Read** the problem.	
Step 2. **Identify** what you are looking for.	the distance from the corner that the bracket should be attached
Step 3. **Name.** Choose a variable to represent it.	Let *x* = the distance from the corner 
Step 4. **Translate.** Write the appropriate formula. Substitute.	$a^2 + b^2 = c^2$ $x^2 + x^2 = 10^2$
Step 5. **Solve** the equation. Isolate the variable. Use the definition of the square root. Simplify. Approximate to the nearest tenth.	$2x^2 = 100$ $x^2 = 50$ $x = \sqrt{50}$ $b \approx 7.1$
Step 6. **Check:**	$a^2 + b^2 = c^2$ $(7.1)^2 + (7.1)^2 \overset{?}{\approx} 10^2 \checkmark$
Step 7. **Answer** the question.	Kelvin should fasten each piece of wood approximately 7.1″ from the corner.

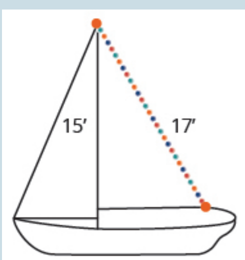
TRY IT 9.1

John puts the base of a 13-ft ladder 5 feet from the wall of his house. How far up the wall does the ladder reach?

Answer
12 feet

TRY IT 9.2

Randy wants to attach a 17-ft string of lights to the top of the 15-ft mast of his sailboat. How far from the base of the mast should he attach the end of the light string?



Answer
8 feet

Key Concepts

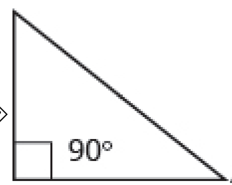
- **Supplementary and Complementary Angles**
  - If the sum of the measures of two angles is 180°, then the angles are supplementary.
  - If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180$.
  - If the sum of the measures of two angles is 90°, then the angles are complementary.

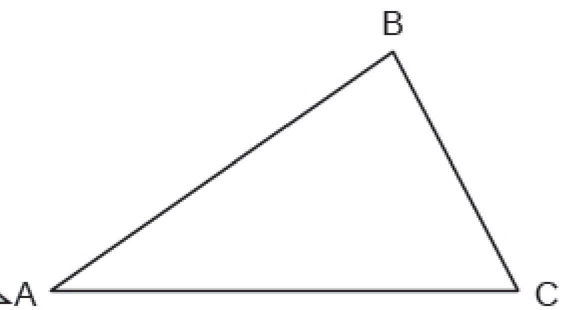
- If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90$.

- **Solve Geometry Applications**

1. Read the problem and make sure you understand all the words and ideas. Draw a figure and label it with the given information.
2. Identify what you are looking for.
3. Name what you are looking for and choose a variable to represent it.
4. Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. Solve the equation using good algebra techniques.
6. Check the answer in the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

- **Sum of the Measures of the Angles of a Triangle**
  - For any ΔABC, the sum of the measures is 180°
  - $m\angle A + m\angle B + m\angle C = 180$
- **Right Triangle**
  - A right triangle is a triangle that has one 90° angle, which is often marked with a ⟐ symbol.





- **Properties of Similar Triangles**
  - If two triangles are similar, then their corresponding angle measures are equal and their corresponding side lengths have the same ratio.

Glossary

angle
An angle is formed by two rays that share a common endpoint. Each ray is called a side of the angle.
complementary angles
If the sum of the measures of two angles is $90°$, then they are called complementary angles.
hypotenuse
The side of the triangle opposite the 90° angle is called the hypotenuse.
legs of a right triangle
The sides of a right triangle adjacent to the right angle are called the legs.
right triangle
A right triangle is a triangle that has one $90°$ angle.
similar figures
In geometry, if two figures have exactly the same shape but different sizes, we say they are similar figures.
supplementary angles
If the sum of the measures of two angles is $180°$, then they are called supplementary angles.
triangle
A triangle is a geometric figure with three sides and three angles.

vertex of an angle

When two rays meet to form an angle, the common endpoint is called the vertex of the angle.

Practice Makes Perfect

Use the Properties of Angles

In the following exercises, find a) the supplement and b) the complement of the given angle.

1. $53°$	2. $16°$
3. $29°$	4. $72°$

In the following exercises, use the properties of angles to solve.

5. Find the supplement of a $135°$ angle.	6. Find the complement of a $38°$ angle.
7. Find the complement of a $27.5°$ angle.	8. Find the supplement of a $109.5°$ angle.
9. Two angles are supplementary. The larger angle is $56°$ more than the smaller angle. Find the measures of both angles.	10. Two angles are supplementary. The smaller angle is $36°$ less than the larger angle. Find the measures of both angles.
11. Two angles are complementary. The smaller angle is $34°$ less than the larger angle. Find the measures of both angles.	12. Two angles are complementary. The larger angle is $52°$ more than the smaller angle. Find the measures of both angles.

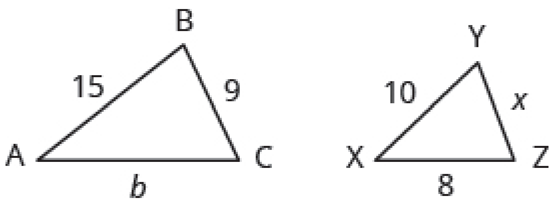
Use the Properties of Triangles

In the following exercises, solve using properties of triangles.

13. The measures of two angles of a triangle are $26°$ and $98°$. Find the measure of the third angle.	14. The measures of two angles of a triangle are $61°$ and $84°$. Find the measure of the third angle.
15. The measures of two angles of a triangle are $105°$ and $31°$. Find the measure of the third angle.	16. The measures of two angles of a triangle are $47°$ and $72°$. Find the measure of the third angle.
17. One angle of a right triangle measures $33°$. What is the measure of the other angle?	18. One angle of a right triangle measures $51°$. What is the measure of the other angle?
19. One angle of a right triangle measures $22.5°$. What is the measure of the other angle?	20. One angle of a right triangle measures $36.5°$. What is the measure of the other angle?
21. The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.	22. The measure of the smallest angle of a right triangle is $20°$ less than the measure of the other small angle. Find the measures of all three angles.
23. The angles in a triangle are such that the measure of one angle is twice the measure of the smallest angle, while the measure of the third angle is three times the measure of the smallest angle. Find the measures of all three angles.	24. The angles in a triangle are such that the measure of one angle is $20°$ more than the measure of the smallest angle, while the measure of the third angle is three times the measure of the smallest angle. Find the measures of all three angles.

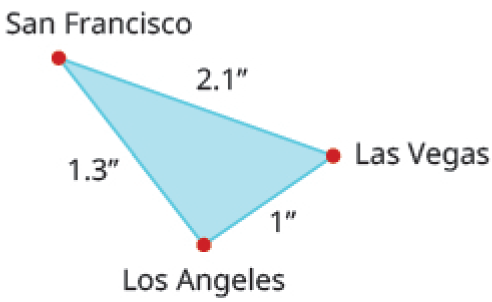
Find the Length of the Missing Side

In the following exercises, ΔABC is similar to ΔXYZ. Find the length of the indicated side.



25. side b	26. side x

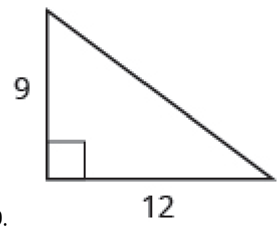
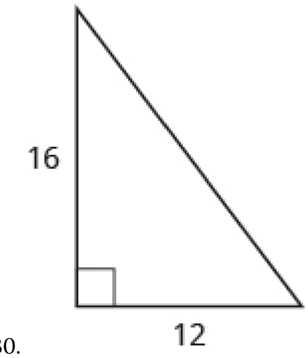
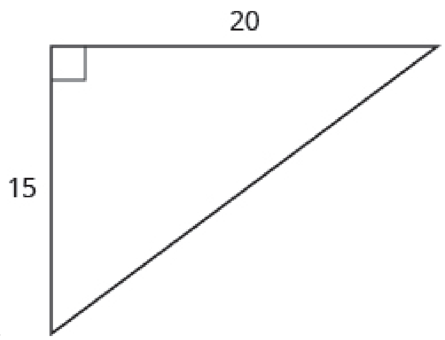
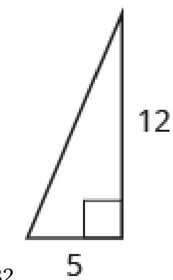
On a map, San Francisco, Las Vegas, and Los Angeles form a triangle whose sides are shown in the figure below. The actual distance from Los Angeles to Las Vegas is 270 miles.



27. Find the distance from Los Angeles to San Francisco.	28. Find the distance from San Francisco to Las Vegas.

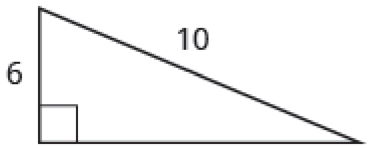
Use the Pythagorean Theorem

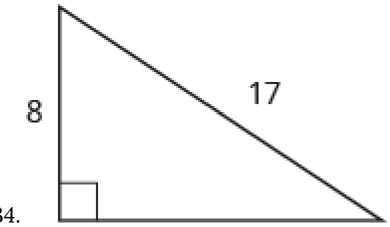
In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

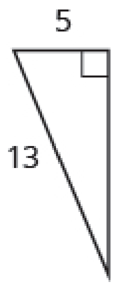
29. 	30. 
31. 	32. 

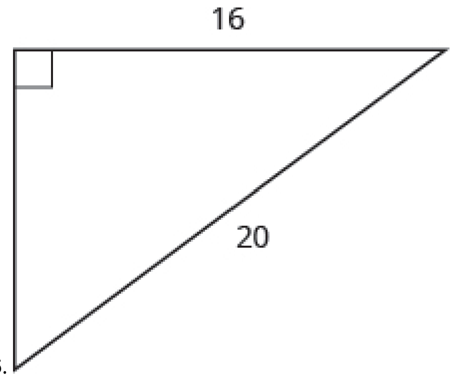
Find the Length of the Missing Side

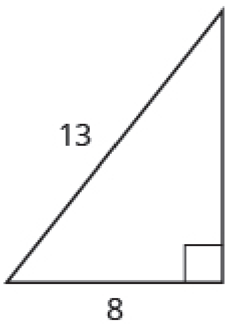
In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

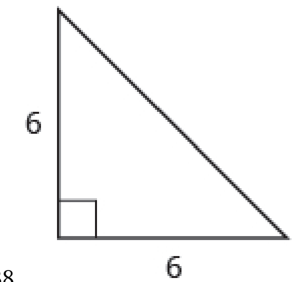
33. 

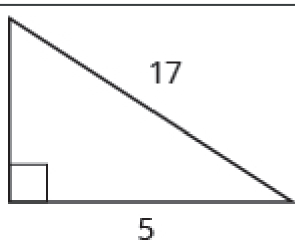
34. 

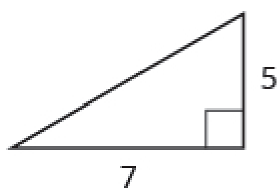
35. 

36. 

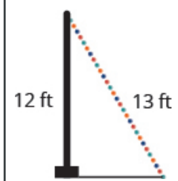
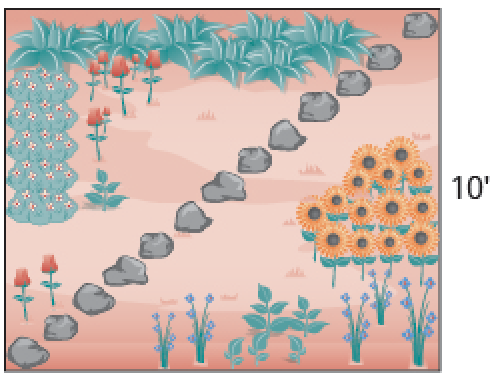
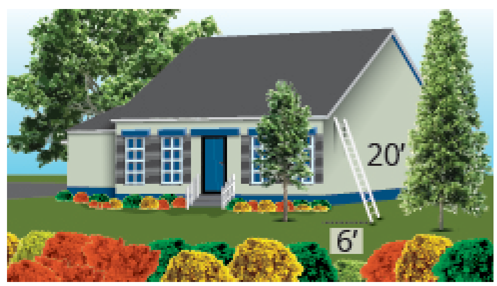
37. 

38. 

39. 

40. 

In the following exercises, solve. Approximate to the nearest tenth, if necessary.

41. A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display. How far from the base of the pole should the end of the string of lights be anchored? 	42. Pam wants to put a banner across her garage door to congratulate her son on his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door? 
43. Chi is planning to put a path of paving stones through her flower garden. The flower garden is a square with sides of 10 feet. What will the length of the path be? 	44. Brian borrowed a 20-foot extension ladder to paint his house. If he sets the base of the ladder 6 feet from the house, how far up will the top of the ladder reach? 

Answers

1. a) 127° b) 37°	3. a) 151° b) 61°	5. 45°
7. 62.5°	9. 62°, 118°	11. 62°, 28°
13. 56°	15. 44°	17. 57°
19. 67.5°	21. 45°, 45°, 90°	23. 30°, 60°, 90°
25. 12	27. 351 miles	29. 15
31. 25	33. 8	35. 12
37. 10.2	39. 8	41. 5 feet
43. 14.1 feet		

Attributions

This chapter has been adapted from "Use Properties of Angles, Triangles, and the Pythagorean Theorem" in *Prealgebra* (OpenStax) by Lynn Marecek, MaryAnne Anthony-Smith, and Andrea Honeycutt Mathis, which is under a CC BY 4.0 Licence. Adapted by Izabela Mazur. See the Copyright page for more information.

5.2 Solve Applications: Sine, Cosine and Tangent Ratios.

Learning Objectives

By the end of this section, you will be able to:

- Find missing side of a right triangle using sine, cosine, or tangent ratios
- Find missing angle of a right triangle using sine, cosine, or tangent ratios
- Solve applications using right angle trigonometry

Now, that we know the fundamentals of algebra and geometry associated with a right triangle, we can start exploring trigonometry. Many real life problems can be represented and solved using right angle trigonometry.

Sine, Cosine, and Tangent Ratios

We know that any right triangle has three sides and a right angle. The side opposite to the right angle is called the hypotenuse. The other two angles in a right triangle are acute angles (with a measure less than 90 degrees). One of those angles we call reference angle and we use θ (theta) to represent it.

The hypotenuse is always the longest side of a right triangle. The other two sides are called opposite side and adjacent side. The names of those sides depends on which of the two acute angles is being used as a reference angle.

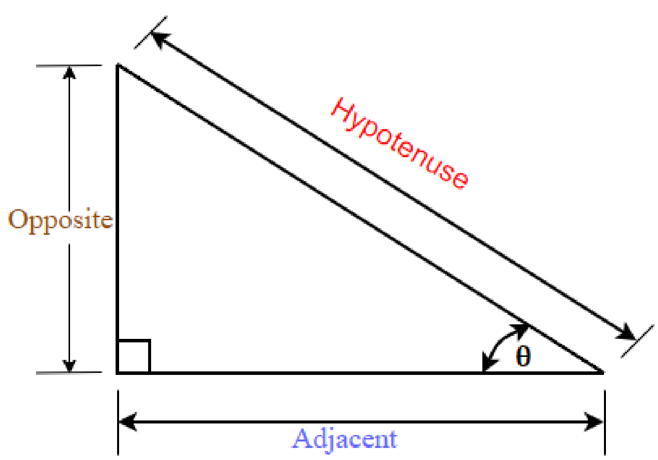
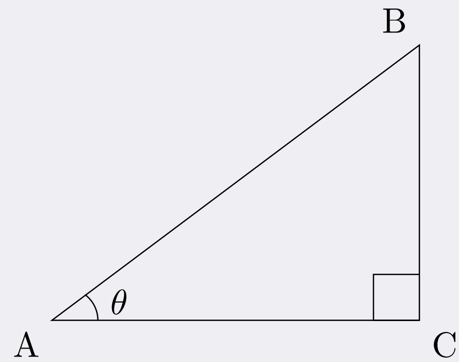


Figure 1.

In the right triangle each side is labeled with a lowercase letter to match the uppercase letter of the opposite vertex.

EXAMPLE 1

Label the sides of the triangle and find the hypotenuse, opposite, and adjacent.



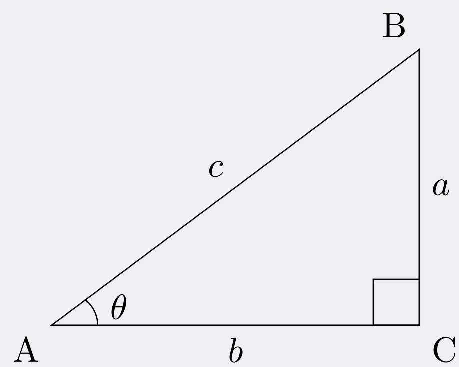
Solution

We labeled the sides with a lowercase letter to match the uppercase letter of the opposite vertex.

c is hypotenuse

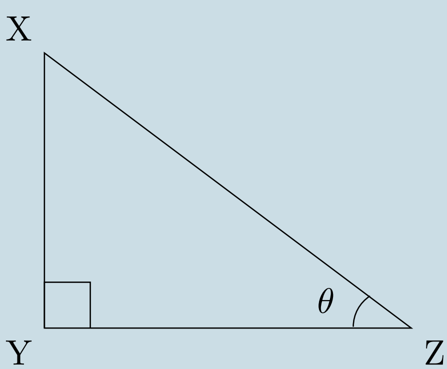
a is opposite

b is adjacent



TRY IT 1.1

Label the sides of the triangle and find the hypotenuse, opposite and adjacent.



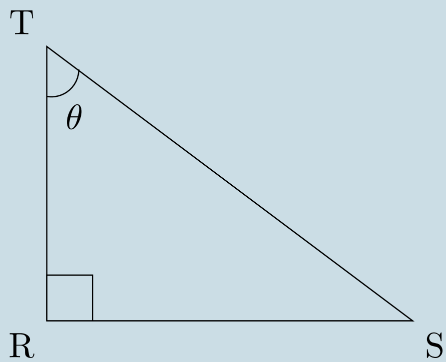
Answer
y is hypotenuse

z is opposite

x is adjacent

TRY IT 1.2

Label the sides of the triangle and find the hypotenuse, opposite and adjacent.



Answer
r is hypotenuse

t is opposite

s is adjacent

Trigonometric Ratios

Trigonometric ratios are the ratios of the sides in the right triangle. For any right triangle we can define three basic trigonometric ratios: sine, cosine, and tangent.

Let us refer to Figure 1 and define the three basic trigonometric ratios as:

Three Basic Trigonometric Ratios

- sine θ = $\frac{\text{the length of the opposite side}}{\text{the length of the hypotenuse side}}$
- cosine θ = $\frac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse side}}$
- tangent θ = $\frac{\text{the length of the opposite side}}{\text{the length of the adjacent side}}$

Where θ is the measure of a reference angle measured in degrees.

Very often we use the abbreviations for sine, cosine, and tangent ratios.

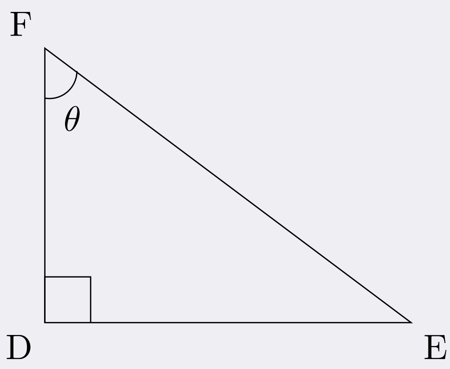
- $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
- $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
- $\tan\theta = \frac{\text{opp}}{\text{adj}}$

Some people remember the definition of the trigonometric ratios as SOH CAH TOA.

Let's use the ΔDEF from Example 1 to find the three ratios.

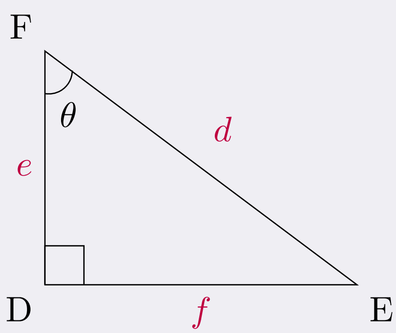
EXAMPLE 2

For the given triangle find the sine, cosine and tangent ratio.



Solution

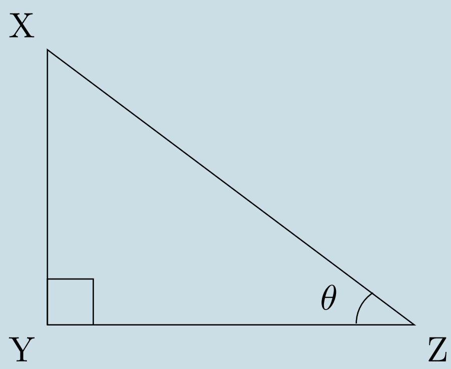
First let's label the sides of the triangle:



$\sin\theta = \frac{f}{d}$ $\cos\theta = \frac{e}{d}$ $\tan\theta = \frac{f}{e}$

TRY IT 2.1

For the given triangle find the sine cosine and tangent ratio.



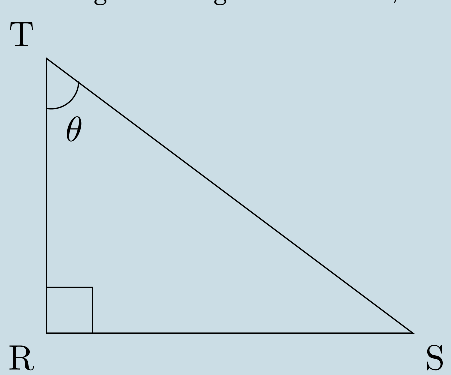
Answer
sin θ = $\frac{z}{y}$

cos θ = $\frac{x}{y}$

tan θ = $\frac{z}{x}$

TRY IT 2.2

For the given triangle find the sine, cosine and tangent ratio.



Answer
sin θ = $\frac{t}{r}$

cos θ = $\frac{s}{r}$

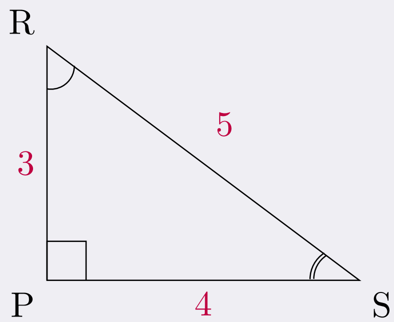
tan θ = $\frac{t}{s}$

In Example 2, our reference angles can be $\angle E$ or $\angle F$. Using the definition of trigonometric ratios, we can write sinE=$\frac{e}{d}$, cosE=$\frac{f}{d}$, and tanE= $\frac{e}{f}$.

When calculating we will usually round the ratios to four decimal places and at the end our final answer to one decimal place unless stated otherwise.

EXAMPLE 3

For the given triangle find the sine, cosine and tangent ratios. If necessary round to four decimal places.



Solution

We have two possible reference angles: R and S.

Using the definitions, the trigonometric ratios for angle R are:

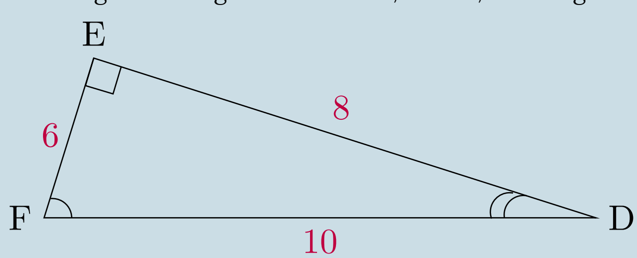
sin R= $\frac{4}{5}$ = 0.8	cos R= $\frac{3}{5}$ = 0.6	tan R=$\frac{4}{3}$ = 1.3333

Using the definitions, the trigonometric ratios for angle S are:

sin S = $\frac{3}{5}$ = 0.6	cos S = $\frac{4}{5}$ = 0.8	tan S = $\frac{3}{4}$ = 0.75

TRY IT 3.1

For the given triangle find the sine, cosine, and tangent ratios. If necessary round to four decimal places.

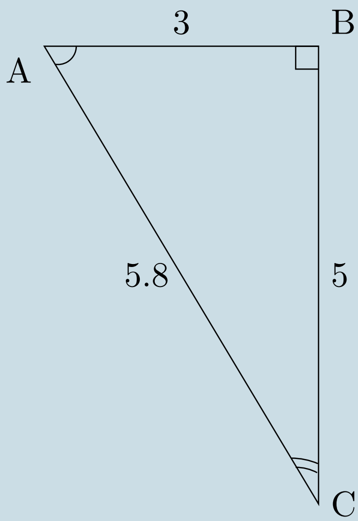


Answer

sin F = $\frac{8}{10}$ = 0.8	cos F = $\frac{6}{10}$ =0.6	tan F = $\frac{8}{6}$ = 1.3333
sin D =$\frac{6}{10}$ = 0.6	cos D =$\frac{8}{10}$ = 0.8	tan D =$\frac{6}{8}$ = 0.75

TRY IT 3.2

For given triangle find the sine, cosine and tangent ratios. If necessary round to four decimal places.



Answer

sin A = $\frac{5}{5.8}$ = 0.8621	cos A = $\frac{3}{5.8}$ =0.5172	tan A = $\frac{5}{3}$ = 1.6667
sin C = $\frac{3}{5.8}$ = 0.5172	cos C = $\frac{5}{5.8}$ = 0.8621	tan C = $\frac{3}{5}$ = 0.6

Now, let us use a scientific calculator to find the trigonometric ratios. Can you find the sin, cos, and tan buttons on your calculator? To find the trigonometric ratios make sure your calculator is in Degree Mode.

EXAMPLE 4

Using a calculator find the trigonometric ratios. If necessary, round to 4 decimal places.

a) sin 30°

b) cos 45°

c) tan 60°

Solution

Make sure your calculator is in Degree Mode. Using a calculator find:

a) sin 30° = 0.5

b) cos 45° = 0.7071 Rounded to 4 decimal places.

c) tan 60° = 1.7321 Rounded to 4 decimal places.

TRY IT 4.1

Find the trigonometric ratios. If necessary, round to 4 decimal places.

a) sin 60°

b) cos 30°

c) tan 45°

Answer
a) sin 60° = 0.8660

b) cos 30° = 0.8660

c) tan 45° = 1

TRY IT 4.2

Find the trigonometric ratios. If necessary, round to 4 decimal places.

a) sin 35°

b) cos 67°

c) tan 83°

Answer
a) sin 35° = 0.5736

b) cos 67 ° = 0.3907

c) tan 83° = 8.1443

Finding Missing Sides of a Right Triangle

In this section you will be using trigonometric ratios to solve right triangle problems. We will adapt our problem solving strategy for trigonometry applications. In addition, since those problems will involve the right triangle, it is helpful to draw it (if the drawing is not given) and label it with the given information.We will include this in the first step of the problem solving strategy for trigonometry applications.

HOW TO: *Solve Trigonometry Applications*

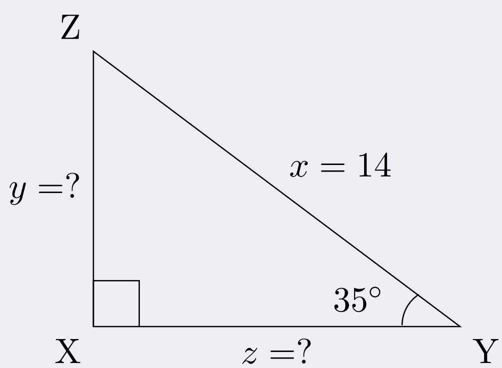
1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.
2. **Identify** what we are looking for.

3. **Label** what we are looking for by choosing a variable to represent it.
4. **Find** the required trigonometric ratio.
5. **Solve** the ratio using good algebra techniques.
6. **Check** the answer by substituting it back into the ratio in step 4 and by making sure it makes sense in the context of the problem.
7. **Answer** the question with a complete sentence.

In the next few examples, having given the measure of one acute angle and the length of one side of the right triangle, we will solve the right triangle for the missing sides.

EXAMPLE 5

Find the missing sides. Round your final answer to two decimal places

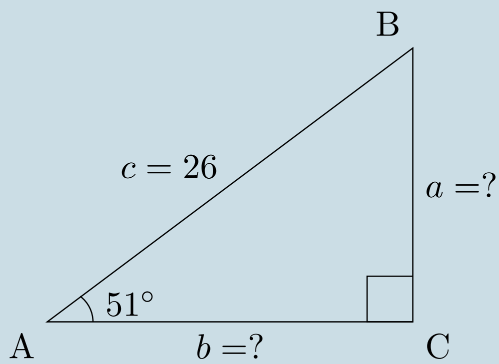


Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	A drawing is given. Angle Y is our reference angle, y is opposite side, z is adjacent side, and x=14 is the hypotenuse.	
2. **Identify** what we are looking for.	a) the opposite side	b) adjacent side
3.**Label** what we are looking for by choosing a variable to represent it.	y=?	z=?
4. **Find** the required trigonometric ratio.	$\sin 35° = \frac{y}{14}$	$\cos 35° = \frac{z}{14}$
5. **Solve** the ratio using good algebra techniques.	$14 \sin 35° = y$ $8.03 = y$	$14 \cos 35° = z$ $11.47 = z$
6. **Check** the answer in the problem and by making sure it makes sense.	$0.57 \stackrel{?}{=} 8.03 \div 14$ $0.57 = 0.57$ ✓	$0.82 \stackrel{?}{=} 11.47 \div 14$ $0.82 = 0.82$ ✓
7. **Answer** the question with a complete sentence.	The opposite side is 8.03	The adjacent side is 11.47

TRY IT 5.1

Find the missing sides. Round your final answer to one decimal place.



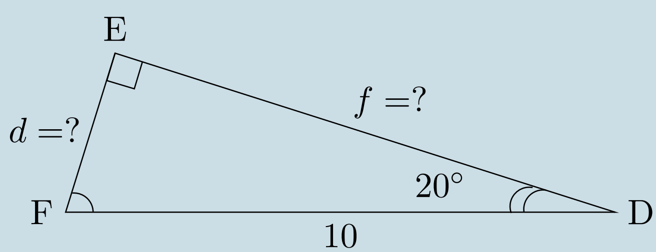
Answer

a = 20.2

b = 16.4

TRY IT 5.2

Find the missing sides. Round your final answer to one decimal place.



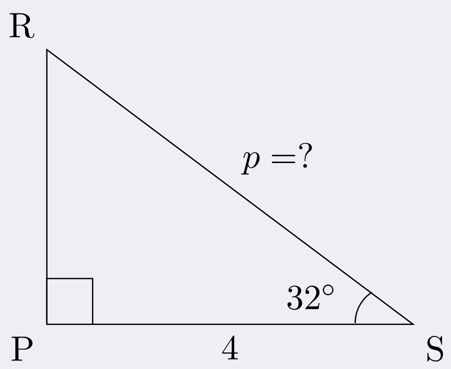
Answer

d = 3.4

f = 9.4

EXAMPLE 6

Find the hypotenuse. Round your final answer to one decimal place.

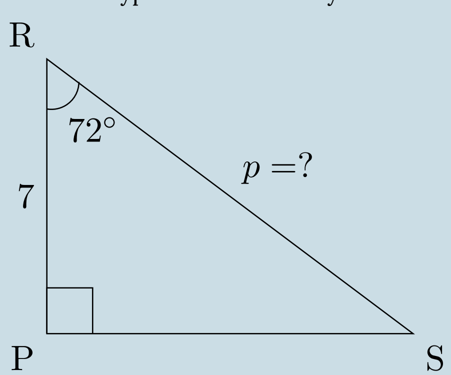


Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	A drawing is given. Angle S is our reference angle, s is opposite side, r = 4 is the adjacent side, and p is the hypotenuse
2. **Identify** what we are looking for.	the hypotenuse
3. **Label** what we are looking for by choosing a variable to represent it.	p=?
4. **Find** the required trigonometric ratio.	$\cos 32° = \frac{4}{p}$
5. **Solve** the ratio using good algebra techniques.	$0.8480 = \frac{4}{p}$ p = 4.7170 Rounding the ratios to 4 decimal places
6. **Check** the answer in the problem and by making sure it makes sense.	$0.8480 \stackrel{?}{=} \frac{4}{4.7170}$ 0.8480 = 0.8480 ✓
7. **Answer** the question with a complete sentence.	The hypotenuse is 4.7 Round my final answer to one decimal place.

TRY IT 6.1

Find the hypotenuse. Round your final answer to one decimal place.

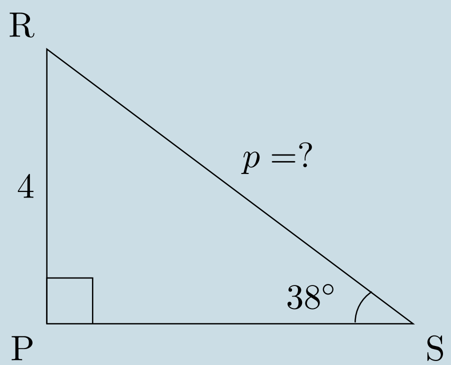


Answer

p = 22.7

TRY IT 6.2

Find the hypotenuse. Round your final answer to one decimal place.



Answer

p = 6.5

Finding Missing Angles of a Right Triangle

Sometimes we have a right triangle with only the sides given. How can we find the missing angles? To find the missing angles, we use the inverse of the trigonometric ratios. The inverse buttons $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ are on your scientific calculator.

EXAMPLE 7

Find the angles. Round your final answer to one decimal place.

a) sin A = 0.5

b) cos B = 0.9735

c) tan C = 2.89358

Solution

Use your calculator and press the 2nd FUNCTION key and then press the SIN, COS, or TAN key

a) $A = \sin^{-1}0.5$

$\angle A$ = 30°

b) $B = \cos^{-1}0.9735$

$\angle B$ = 13.2° Rounded to one decimal place

c) C = $\tan^{-1}$2.89358

$\angle C$ = 70.9° Rounded to one decimal place

TRY IT 7.1

Find the angles. Round your final answer to one decimal place.

a) sin X = 1

b) cos Y = 0.375

c) tan Z = 1.676767

Answer
a) $\angle X$ = 90°

b) $\angle Y$ = 68°

c) $\angle Z$ = 59.2°

TRY IT 7.2

Find the angles. Round your final answer to one decimal place.

a) sin C = 0

b) cos D = 0.95

c) tan F = 6.3333

Answer
a) $\angle C$ = 0°

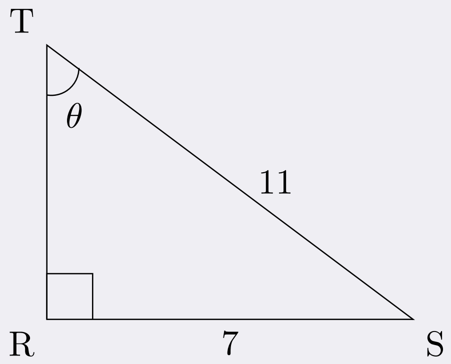
b) $\angle D$ = 18.2°

c) $\angle F$ = 81°

In the example below we have a right triangle with two sides given. Our acute angles are missing. Let us see what the steps are to find the missing angles.

EXAMPLE 8

Find the missing $\angle T$. Round your final answer to one decimal place.

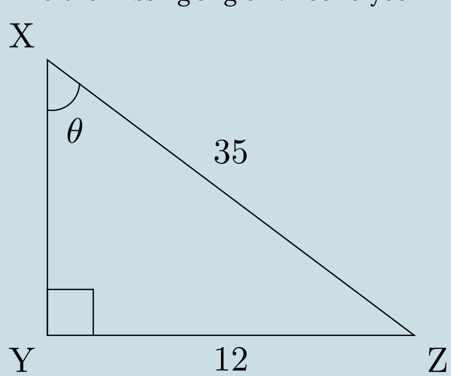


Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	A drawing is given. Angle T is our reference angle, t = 7 is the opposite side, s is adjacent side, and r =11 is the hypotenuse
2. **Identify** what we are looking for.	angle T
3.**Label** what we are looking for by choosing a variable to represent it.	$\angle T$ =?
4. **Find** the required trigonometric ratio.	sin T = $\frac{7}{11}$
5. **Solve** the ratio using good algebra techniques.	sin T = 0.6364 T = $\sin^{-1}$0.6364 $\angle T$ = 39.5239°
6. **Check** the answer in the problem and by making sure it makes sense.	sin 39.5239° $\stackrel{?}{=}$ 0.6364 0.6364 = 0.6364 ✓
7. **Answer** the question with a complete sentence.	The missing angle T is 39.5°.

TRY IT 8.1

Find the missing angle X. Round your final answer to one decimal place.

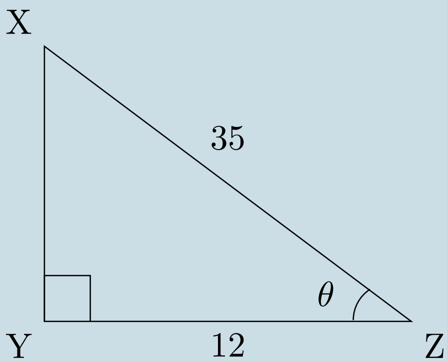


Answer

20.1°

TRY IT 8.2

Find the missing angle Z. Round your final answer to one decimal place.

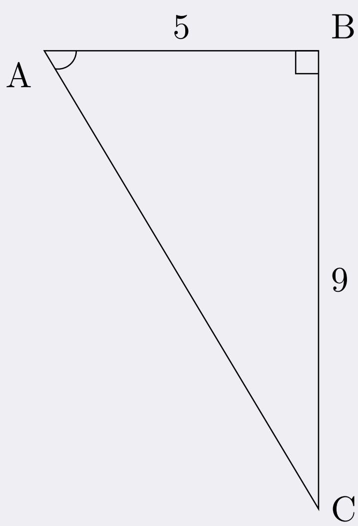


Answer

69.9°

EXAMPLE 9

Find the missing angle A. Round your final answer to one decimal place.

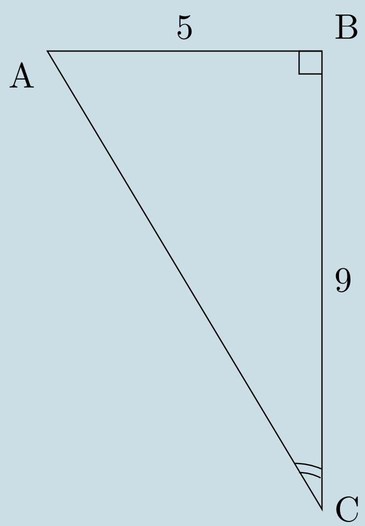


Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	A drawing is given. Angle A is our reference angle, a = 9 is the opposite side, c = 5 is the adjacent side, and b is the hypotenuse
2. **Identify** what we are looking for.	angle A
3.**Label** what we are looking for by choosing a variable to represent it.	$\angle A$ =?
4. **Find** the required trigonometric ratio.	tan A = $\frac{9}{5}$
5. **Solve** the ratio using good algebra techniques.	tan A = 1.8 A = $\tan^{-1}$ 1.8 $\angle A$ = 60.9°
6. **Check** the answer in the problem and by making sure it makes sense.	tan 60.9° $\stackrel{?}{=}$ 1.8 1.8 = 1.8 ✓
7. **Answer** the question with a complete sentence.	The missing angle A is 60.9°.

TRY IT 9.1

Find the missing angle C. Round your final answer to one decimal place.

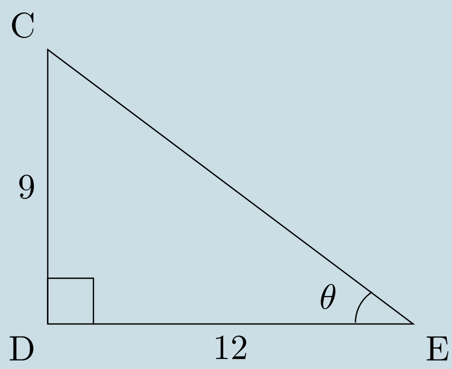


Answer

29.1°

TRY IT 9.2

Find the missing angle E. Round your final answer to one decimal place.



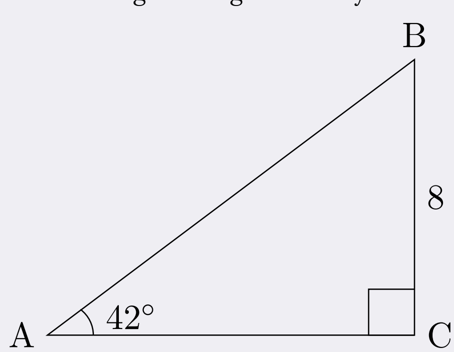
Answer
36.9°

Solving a Right Triangle

From the section before we know that any triangle has three sides and three interior angles. In a right triangle, when all six parts of the triangle are known, we say that the right triangle is solved.

EXAMPLE 10

Solve the right triangle. Round your final answer to one decimal place.



Solution

Since the sum of angles in any triangle is 180°, the measure of angle B can be easy calculated.

$\angle B$ = 180° − 90° − 42°

$\angle B$ = 48°

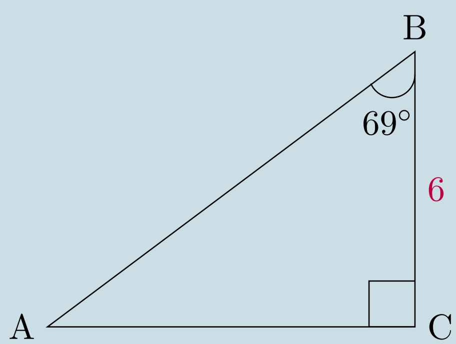
1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	A drawing is given. Angle A is our reference angle, a = 8 is the opposite side, b is the adjacent side, and c is the hypotenuse.	
2. **Identify** what we are looking for.	a) adjacent side	b) hypotenuse
3.**Label** what we are looking for by choosing a variable to represent it.	b = ?	c = ?
4. **Find** the required trigonometric ratio.	tan 42° = $\frac{8}{b}$	sin 42° = $\frac{8}{c}$
5. **Solve** the ratio using good algebra techniques.	0.9004 = $\frac{8}{b}$ 0.9004 b = 8 b = 8.8849	0.6691 = $\frac{8}{c}$ 0.6691 c = 8 c = 11.9563
6. **Check** the answer in the problem and by making sure it makes sense.	tan 42 ° $\stackrel{?}{=} \frac{8}{8.8849}$ 0.9 = 0.9 ✓	sin 42° $\stackrel{?}{=} \frac{8}{11.9563}$ 0.6691 = 0.6691 ✓
7. **Answer** the question with a complete sentence.	The adjacent side is 8.9. Rounded to one decimal place.	The hypotenuse is 12

We solved the right triangle

$\angle A$ = 42°	$\angle B$ = 48°	$\angle C$ = 90°
a = 8	b = 8.9	c = 12

TRY IT 10.1

Solve the right triangle. Round your final answer to one decimal place.

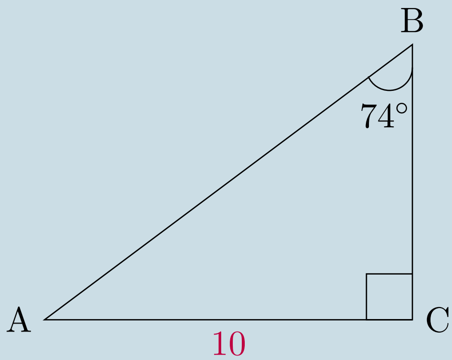


Answer

$\angle A$= 21°	$\angle B$ = 69°	$\angle C$ = 90°
a = 6	b = 15.6	c = 16.7

TRY IT 10.2

Solve the right triangle. Round your final answer to one decimal place.

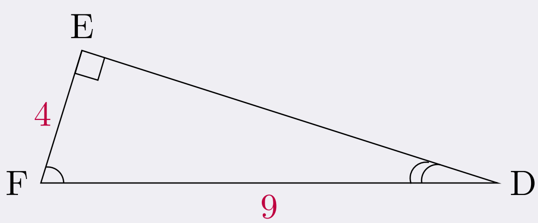


Answer

$\angle A$=16°	$\angle B$ = 74°	$\angle C$ = 90°
a = 2.9	b = 10	c = 10.4

EXAMPLE 11

Solve the right triangle. Round to two decimal places.



Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	A drawing is given. Let angle D be our reference angle, d = 4 is the opposite side, f is the adjacent side, and e = 9 is the hypotenuse	
2. **Identify** what we are looking for.	a) angle D	b) adjacent
3.**Label** what we are looking for by choosing a variable to represent it.	$\angle D$ =?	f = ?
4. **Find** the required trigonometric ratio.	sin D = $\frac{4}{9}$	$4^2 + f^2 = 9^2$
5. **Solve** the ratio using good algebra techniques.	sin D = 0.4444 D = $\sin^{-1}$0.4444 $\angle D$ = 26.3850°	$16 + f^2 = 81$ $f^2 = 81 - 16$ $f^2 = 65$ f = square root of 65 f = 8.06
6. **Check** the answer in the problem and by making sure it makes sense.	sin 26.3850° $\stackrel{?}{=} \frac{4}{9}$ 0.4444 =0.4444 ✓	$4^2 + 8.06^2 \stackrel{?}{=} 9^2$ 81 = 81 ✓
7. **Answer** the question with a complete sentence.	The missing angle D is 26.39°.	The adjacent side is 8.06 Rounded to two decimal places

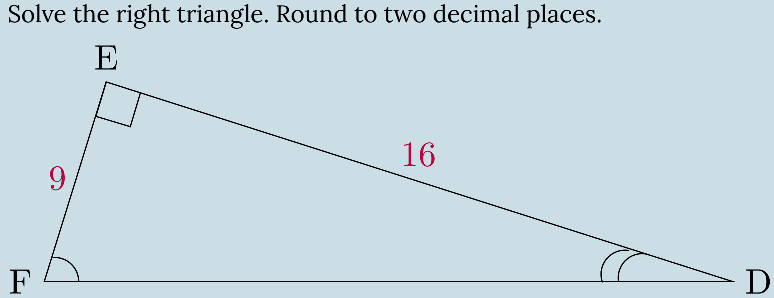
The missing angle F = 180° – 90° – 26.39° = 63.61°

We solved the right triangle

$\angle D$ = 26.39°	$\angle E$ = 90°	$\angle F$ = 63.61°
d = 4	e = 9	f = 8.06

TRY IT 11.1

Solve the right triangle. Round to two decimal places.

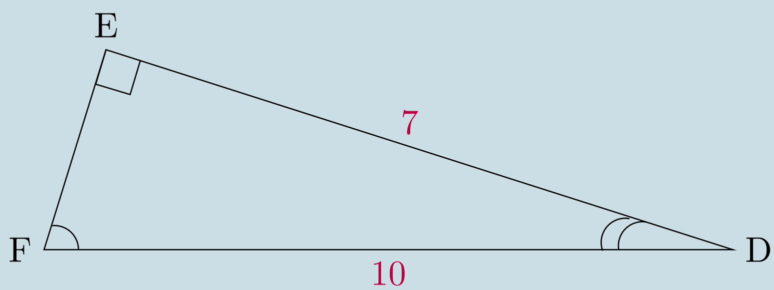


Answer

$\angle D$ = 29.36°	$\angle E$ = 90°	$\angle F$ = 60.64°
d = 9	e = 18.4	f = 16

TRY IT 11.2

Solve the right triangle. Round to one decimal place.



Answer

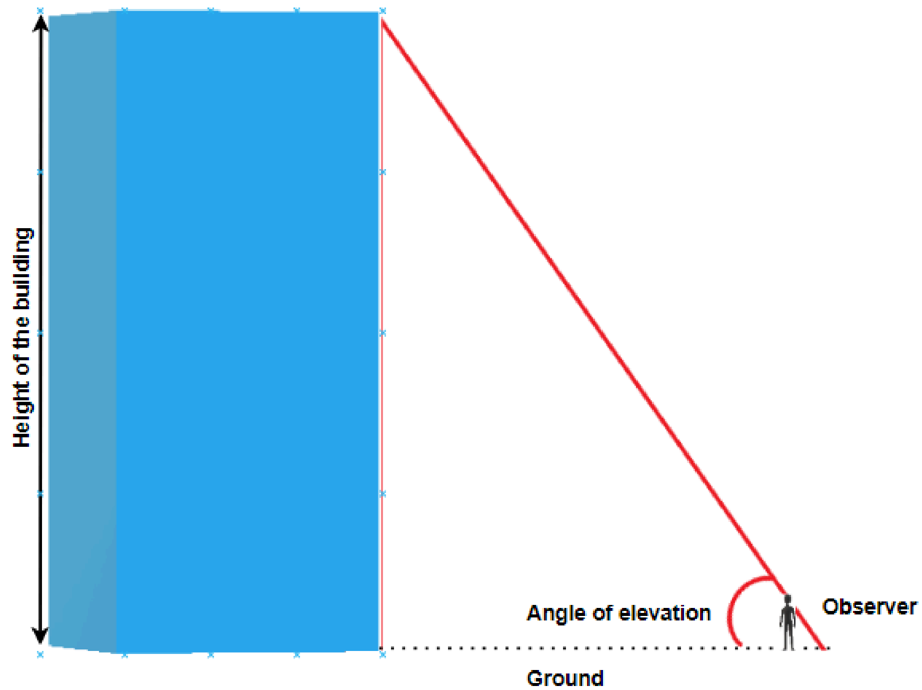
$\angle D$ = 45.6°	$\angle E$ = 90°	$\angle F$ = 44.4°
d = 7.1	e = 10	f = 7

Solve Applications Using Trigonometric Ratios

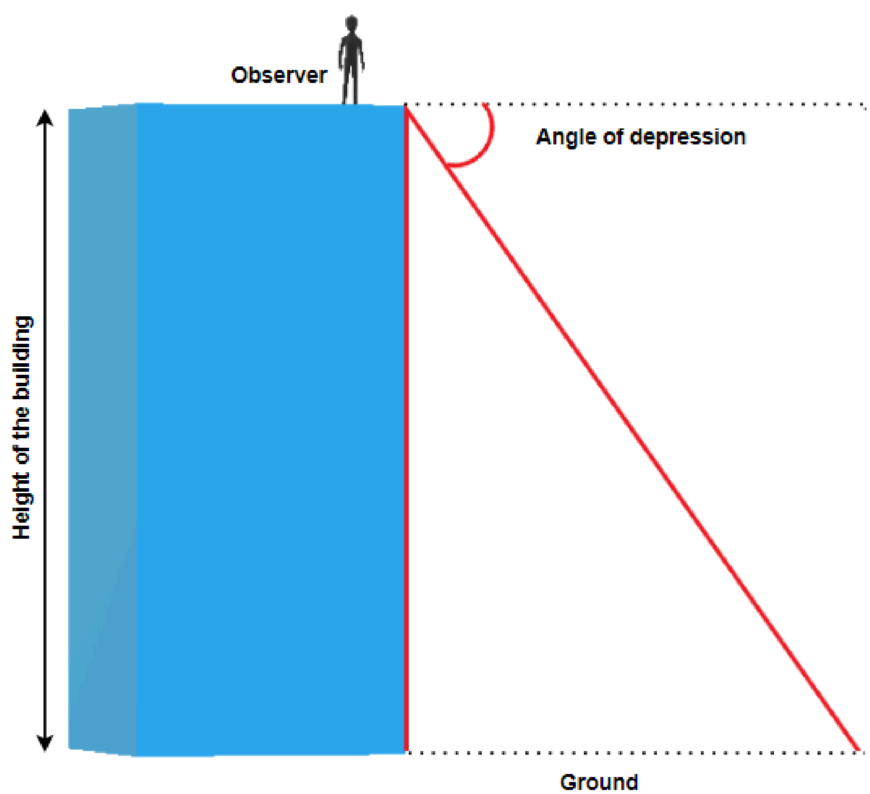
In the previous examples we were able to find missing sides and missing angles of a right triangle. Now, let's use the trigonometric ratios to solve real-life problems.

Many applications of trigonometric ratios involve understanding of an angle of elevation or angle of depression.

The angle of elevation is an angle between the horizontal line (ground) and the observer's line of sight.



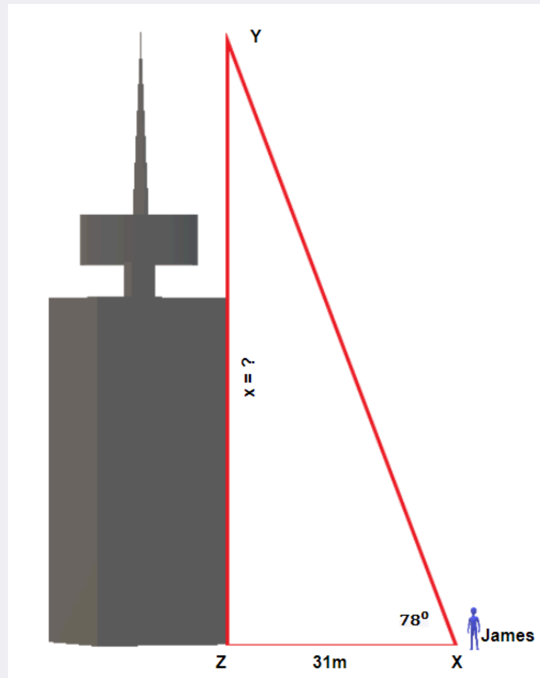
The angle of depression is the angle between horizontal line (that is parallel to the ground) and the observer's line of sight.



EXAMPLE 12

James is standing 31 metres away from the base of the Harbour Centre in Vancouver. He looks up to the top of the building at a 78° angle. How tall is the Harbour Centre?

Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	 Angle X is our reference angle, x is opposite side, y = 31 m is the adjacent side, and z is the hypotenuse.
2. **Identify** what we are looking for.	The opposite side
3.**Label** what we are looking for by choosing a variable to represent it.	x=?
4. **Find** the required trigonometric ratio.	tan 78° = $\frac{x}{31}$
5. **Solve** the ratio using good algebra techniques.	4.7046 = $\frac{x}{31}$ x = 145.8426
6. **Check** the answer in the problem and by making sure it makes sense.	4.7046 $\stackrel{?}{=} \frac{145.8426}{31}$ 4.7046 = 4.7046 ✓
7. **Answer** the question with a complete sentence.	The Harbour Centre is 145.8426 metres or rounded to 146 metres.

TRY IT 12.1

Nicole is standing 75 feet away from the base of the Living Shangri-La, the tallest building in British Columbia. She looks up to the top of the building at a 83.5° angle. How tall is the Living Shangri-La?

Answer

658.3 feet.

TRY IT 12.2

Kelly is standing 23 metres away from the base of the tallest apartment building in Prince George and looks at the top of the building at a 62° angle. How tall is the building?

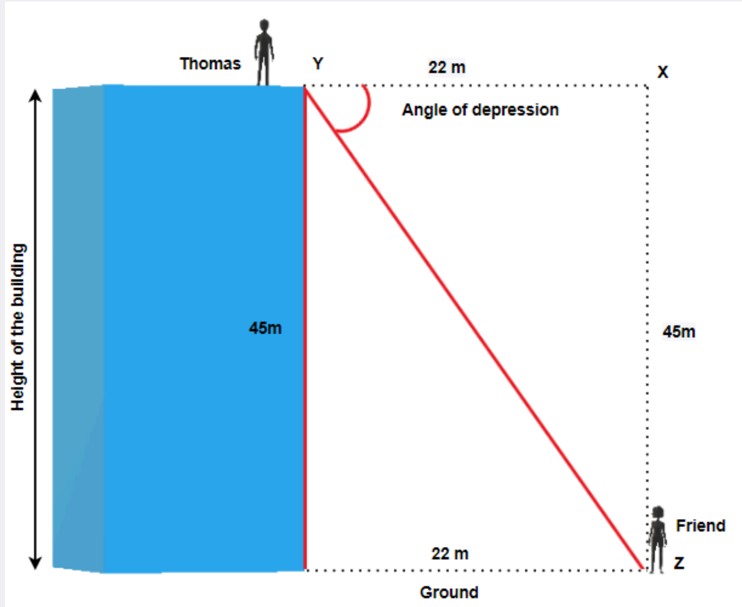
Answer

43.3 metres

EXAMPLE 13

Thomas is standing at the top of the building that is 45 metres high and looks at his friend that is standing on the ground, 22 metres from the base of the building. What is the angle of depression?

Solution

1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.	 Angle Y is our reference angle, y = 45 m is the opposite side, z = 22 m is the adjacent side, and x is the hypotenuse
2. **Identify** what we are looking for.	angle Y
3.**Label** what we are looking for by choosing a variable to represent it.	$\angle Y$ =?
4. **Find** the required trigonometric ratio.	tan Y = $\frac{45}{22}$
5. **Solve** the ratio using good algebra techniques.	tan Y = 2.0455 Y = $\tan^{-1}$2.0455 $\angle Y$ = 63.9470°
6. **Check** the answer in the problem and by making sure it makes sense.	tan 63.9470° $\stackrel{?}{=}$ 2.0455 2.0455 = 2.0455 ✓
7. **Answer** the question with a complete sentence.	The angle of depression is 63.9470° or 64° rounded to one decimal place.

TRY IT 13.1

Hemanth is standing on the top of a cliff 250 feet above the ground and looks at his friend that is standing on the ground, 40 feet from the base of the cliff. What is the angle of depression?

Answer

80.9°

TRY IT 13.2

Klaudia is standing on the ground, 25 metres from the base of the cliff and looks up at her friend on the top of a cliff 100 metres above the ground. What is the angle of elevation?

Answer
76°

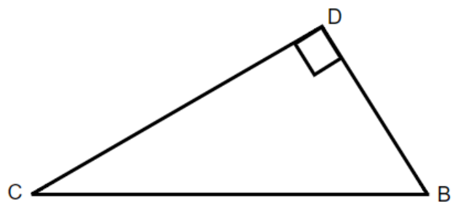
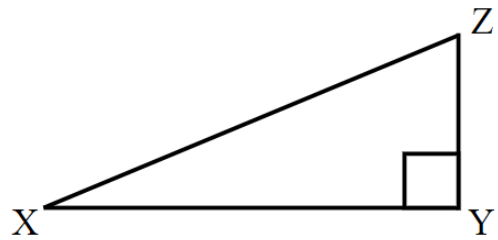
Key Concepts

- Three Basic Trigonometric Ratios: (Where θ is the measure of a reference angle measured in degrees.)
  - sine θ = $\frac{\text{the length of the opposite side}}{\text{the length of the hypotenuse side}}$
  - cosine θ = $\frac{\text{the length of the adjacent side}}{\text{the length of the hypotenuse side}}$
  - tangent θ = $\frac{\text{the length of the opposite side}}{\text{the length of the adjacent side}}$
- **Problem-Solving Strategy for Trigonometry Applications**

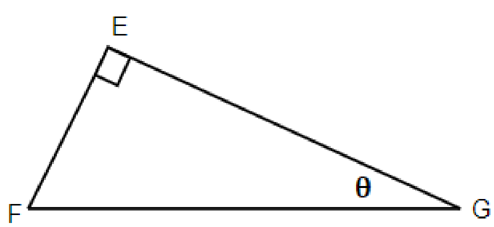
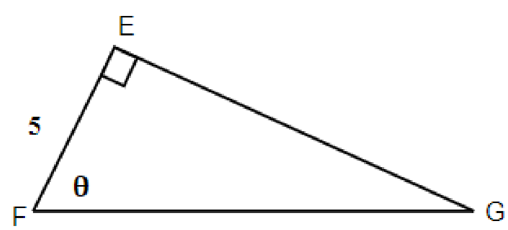
  1. **Read** the problem and make sure all the words and ideas are understood. Draw the right triangle and label the given parts.
  2. **Identify** what we are looking for.
  3. **Label** what we are looking for by choosing a variable to represent it.
  4. **Find** the required trigonometric ratio.
  5. **Solve** the ratio using good algebra techniques.
  6. **Check** the answer by substituting it back into the ratio solved in step 5 and by making sure it makes sense in the context of the problem.
  7. **Answer** the question with a complete sentence.

Practice Makes Perfect

Label the sides of the triangle.

1 	2. 
3. If the reference angle in Question 1 is B, Find the adjacent ?	4. If the reference angle in Question 2 is Z, find the opposite ?

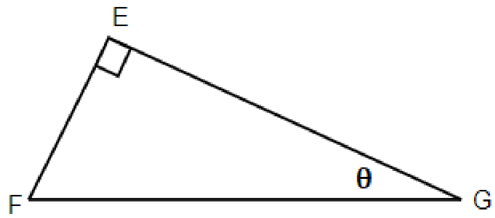
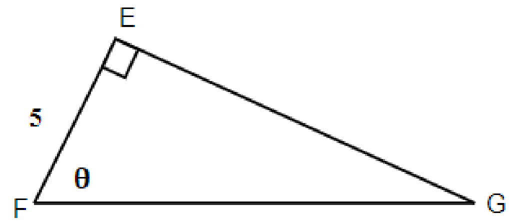
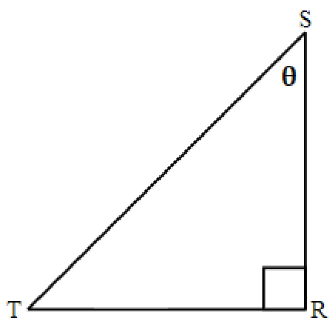
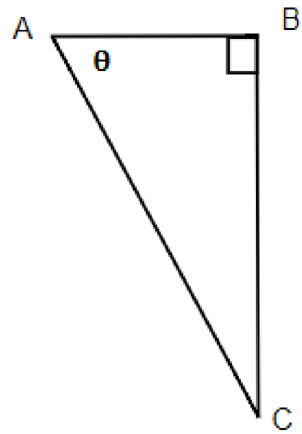
Label the sides of the triangle and find the hypotenuse, opposite and adjacent.

 5.	 6.

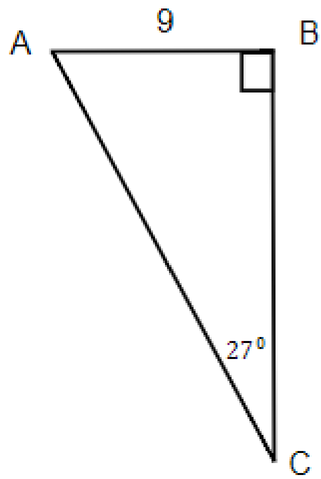
Use your calculator to find the given ratios. Round to four decimal places if necessary:

7. $\sin 47°$	8. $\cos 82°$
9. $\tan 12°$	10. $\sin 30°$

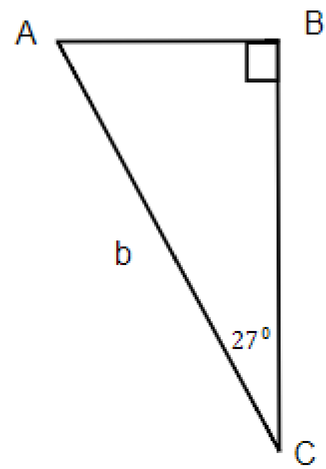
For the given triangles, find the sine, cosine and tangent of the θ.

 11.	 12.
 13.	 14.

For the given triangles, find the missing side. Round it to one decimal place.

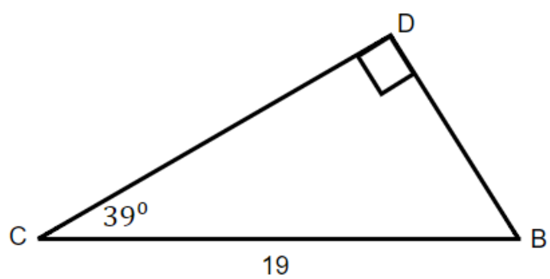


15. Find the hypotenuse.

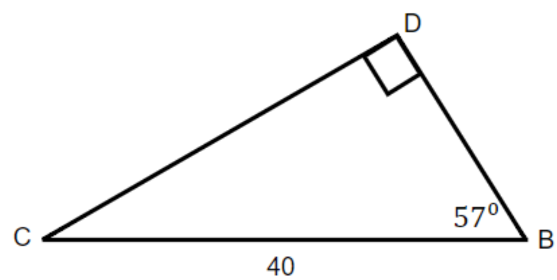


16. Find b.

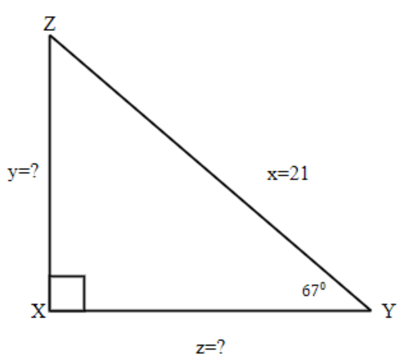
17. Find the opposite.



18. Find the adjacent.



For the given triangles, find the missing sides. Round it to one decimal place.



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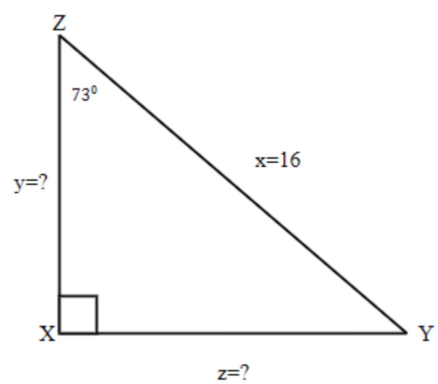

20.

Solve the triangles. Round to one decimal place.

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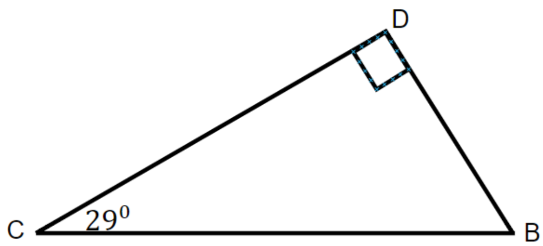

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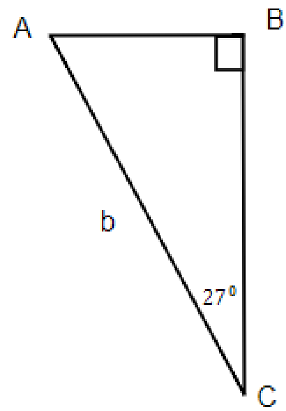

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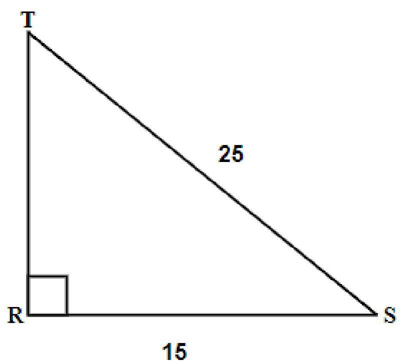

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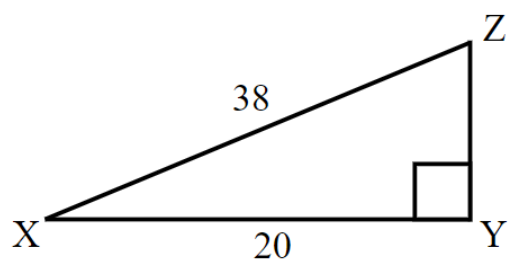

24.
</td>
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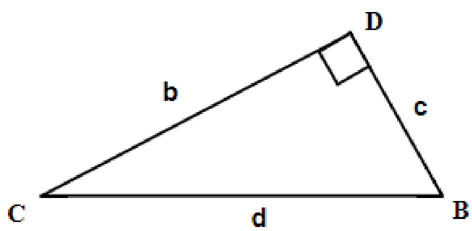
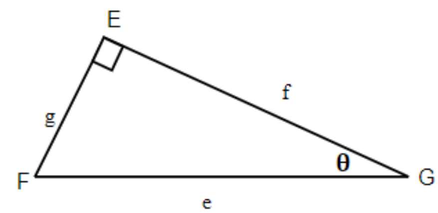
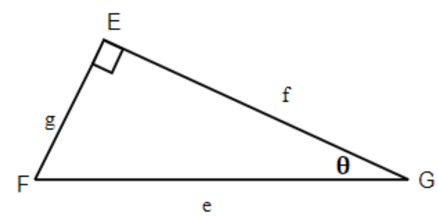
<tr>
<td>25. A surveyor stands 75 metres from the bottom of a tree and looks up at the top of the tree at a 48° angle. How tall is the tree?</td>
<td>26. A tree makes a shadow that is 6 metres long when the angle of elevation to the sun is 52°. How tall is the tree?</td>
</tr>
<tr>
<td>27. A ladder that is 15 feet is leaning against a house and makes a 45° angle with the ground. How far is the base of the ladder from the house?</td>
<td>28. Matt is flying a kite and has let out 100 feet of string. The angle of elevation with the ground is 38°. How high is his kite above the ground?</td>
</tr>
<tr>
<td>29. Marta is flying a kite and has let out 28 metres of string. If the kite is 10 metres above the ground, what is the angle of elevation?</td>
<td>30. An airplane takes off from the ground at the angle of 25°. If the airplane traveled 200 kilometres, how high above the ground is it?</td>
</tr>
</table>

Answers

1. 	3. c	5.  g is opposite , f is adjacent, and e is hypotenuse
7. 0.7314	9. 0.2126	11.  sin θ = $\frac{g}{e}$, cos θ = $\frac{f}{e}$, tan θ = $\frac{g}{f}$
13. sin θ = $\frac{s}{r}$, cos θ = $\frac{t}{r}$, tan θ = $\frac{s}{t}$	15. b = 19.8	17. c = 12
19. y = 19.3, z = 8.2	21. $\angle B$ = 61° $\angle C$ = 29° $\angle D$ = 90° b = 38.5 c = 21.3 d = 44	23. $\angle T$ = 36.9° $\angle R$ = 90° $\angle S$ = 53.1° t = 15 r = 25 s = 20
25. 83.3 m	27. 10.6 ft	29. 20.9°

5.3 Chapter Review

Review Exercises

Use Properties of Angles

In the following exercises, solve using properties of angles.

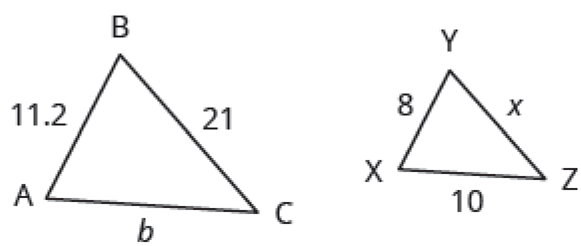
1. What is the supplement of a $48°$ angle?	2. What is the complement of a $61°$ angle?
3. Two angles are complementary. The smaller angle is $24°$ less than the larger angle. Find the measures of both angles.	4. Two angles are supplementary. The larger angle is $45°$ more than the smaller angle. Find the measures of both angles.

Use Properties of Triangles

In the following exercises, solve using properties of triangles.

5. The measures of two angles of a triangle are 22 and 85 degrees. Find the measure of the third angle.	6. One angle of a right triangle measures 41.5 degrees. What is the measure of the other small angle?
7. One angle of a triangle is $30°$ more than the smallest angle. The largest angle is the sum of the other angles. Find the measures of all three angles.	8. One angle of a triangle is twice the measure of the smallest angle. The third angle is $60°$ more than the measure of the smallest angle. Find the measures of all three angles.

In the following exercises, ΔABC is similar to ΔXYZ. Find the length of the indicated side.

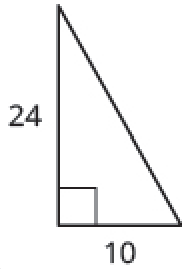


9. side x	10. side b

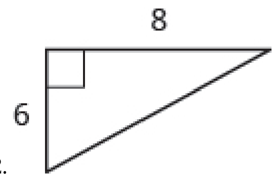
Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

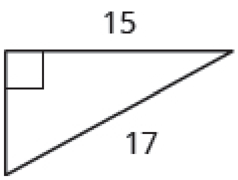
11.



12.



13.



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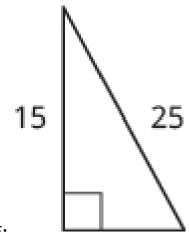
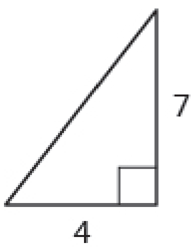
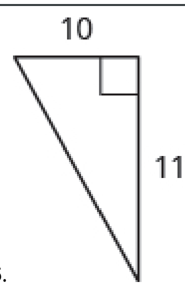


15.

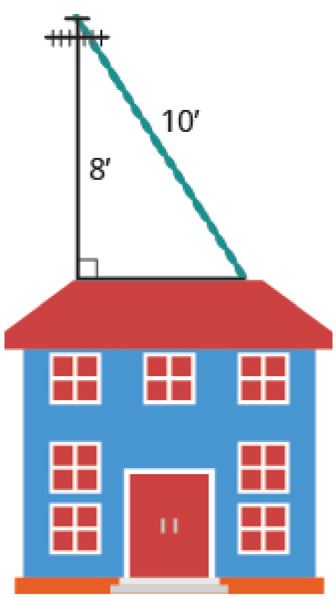


16.



In the following exercises, solve. Approximate to the nearest tenth, if necessary.

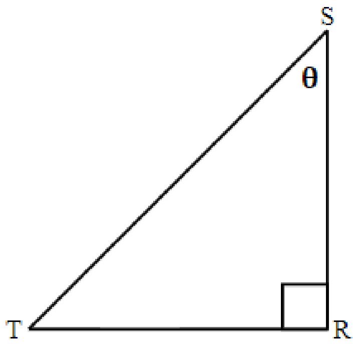
17. Sergio needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is 8 feet tall and Sergio has 10 feet of wire. How far from the base of the antenna can he attach the wire?



18. Seong is building shelving in his garage. The shelves are 36 inches wide and 15 inches tall. He wants to put a diagonal brace across the back to stabilize the shelves, as shown. How long should the brace be?

Find missing side of a right triangle using sine, cosine, or tangent ratios.

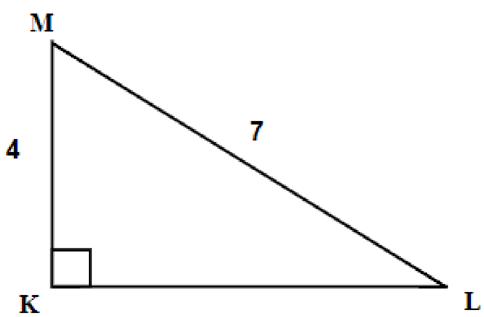
19. Label the triangle and find the sine cosine and tangent of θ.



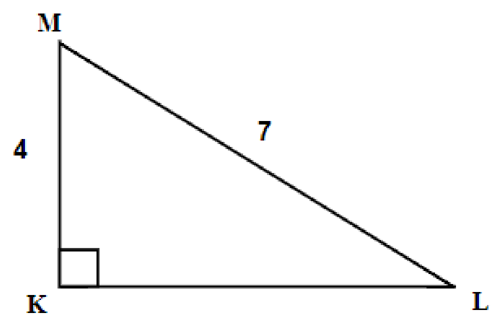
20. If reference angle in above triangle is angle T, label the triangle and find the sine, cosine, and tangent of T.

Find missing angle of a right triangle using sine, cosine, or tangent ratios.

21. Find angle M

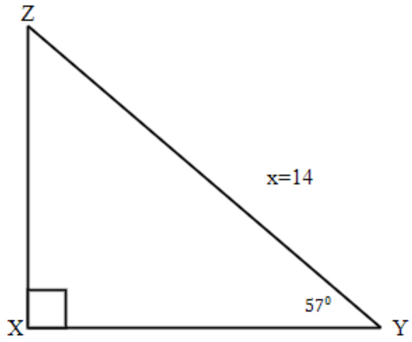


22. Find angle L.

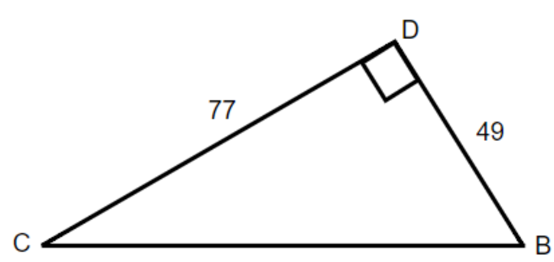


Solve the right triangle.

23. Solve the triangle.

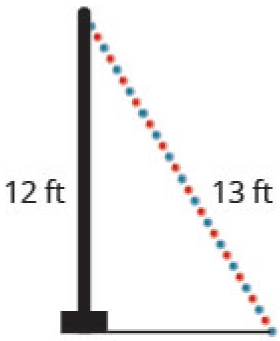


24.



Solve applications using right angle trigonometry.

25. A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display, as shown below. What is the angle that the string of lights makes with the ground?



26. Brian borrowed a 20 foot extension ladder to use when he paints his house. If he sets the base of the ladder 6 feet from the house, as shown below, what is the angle that the ladder makes with the ground?

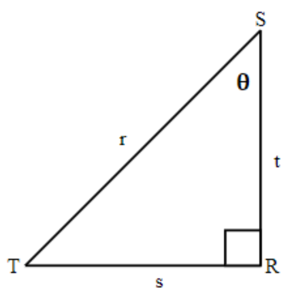
27. John puts the base of a 13-foot ladder five feet from the wall of his house as shown below. What is the angle between the top of the ladder and the house ?

28. The sun is at an angle of elevation of 35°. If Bob casts a shadow that is 6 ft long, how tall is Bob?

29. A 27 foot guy wire to a pole makes an angle of 63.7° with the ground. How high from the ground is the wire attached to the pole?

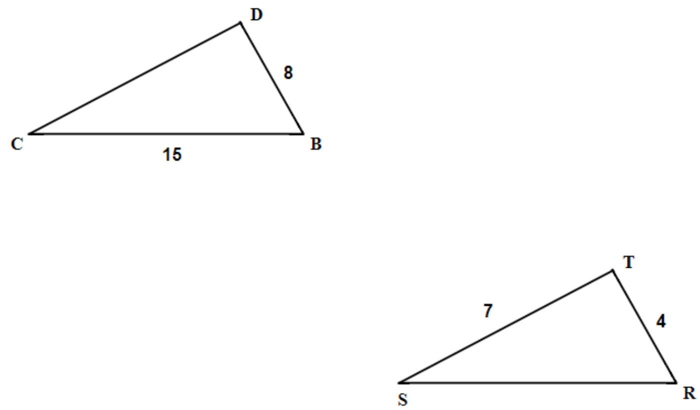
30. A lighthouse is 20 metres tall. If the observer is looking at a boat that is 30 metres away from the base of the lighthouse, what is the angle of depression?

Review Answers

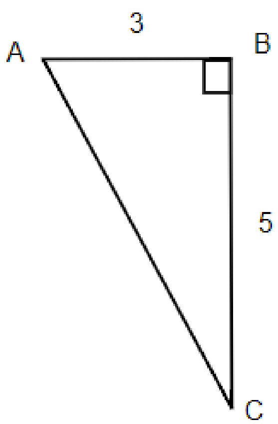
1. 132°	3. 33°, 57°	5. 73°
7. 30°, 60°, 90°	9. 15	11. 26
13. 8	15. 8.1	17. 6 feet
19.  sin θ = $\frac{s}{r}$, cos θ = $\frac{t}{r}$, tan θ = $\frac{s}{t}$	21. 55.2°	23. $\angle X$ = 90° , $\angle Y$ = 57° , $\angle Z$ = 33°x = 14, y = 11.7, z= 7.6
25. 67.4°	27. 22.6°	29. 24

Practice Test

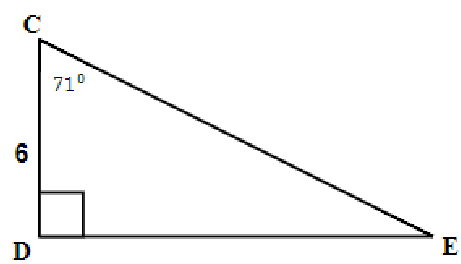
<table>
<tr>
<td>1. What is the supplement of a $\angle 57°$ angle?</td>
<td>2. Two angles are complementary. The smaller angle is $16°$ less than the larger angle. Find the measures of both angles.</td>
</tr>
<tr>
<td>3. The measures of two angles of a triangle are 29 and 75 degrees. Find the measure of the third angle.</td>
<td>4. ΔBCD is similar to ΔSRT. Find the missing sides.



</td>
</tr>
<tr>
<td>5. Use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.



</td>
<td>6. Find the hypotenuse.



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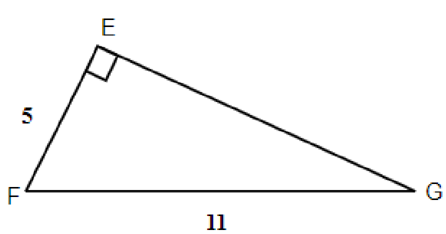
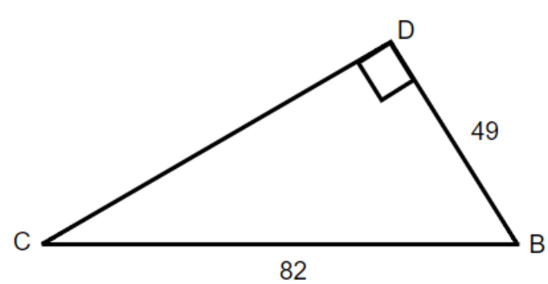
<td>7. Find angle G.



</td>
<td>8. Solve the triangle.



</td>
</tr>
<tr>
<td>9. The sun is at an angle 28°. If Adam casts a shadow that is 7 ft long, how tall is Adam?</td>
<td>10. The road rises 6 metres per every 100 horizontal metres. What is the angle of elevation.</td>
</tr>
</table>

Answers

1. 123°	2. 53°, 37°	3. 76°
4. b = 14, t = 7.5	5. b = 15.3	6. d = 18.4
7. $\angle G$ = 27°	8. $\angle C$ = 36.7°, $\angle B$ = 53.3°, $\angle D$ = 90°, c = 49, b = 65.7, d= 82	9. 5.5 ft
10. 3.4°		