Intermediate Algebra II

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BCCAMPUS
VICTORIA, B.C.

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Contents

Part IV. CHAPTER 6 Polynomials

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CHAPTER 3 Solving First Degree Equations in One Variable

The rocks in this formation must remain perfectly balanced around the centre for the formation to hold its shape.

If we carefully placed more rocks of equal weight on both sides of this formation, it would still balance. Similarly, the expressions in an equation remain balanced when we add the same quantity to both sides of the equation. In this chapter, we will solve equations, remembering that what we do to one side of the equation, we must also do to the other side.

Attributions

3.1 Solve Equations Using the Subtraction and Addition Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that need to be simplified
- Translate an equation and solve
- Translate and solve applications

We are now ready to "get to the good stuff." You have the basics down and are ready to begin one of the most important topics in algebra: solving equations. The applications are limitless and extend to all careers and fields. Also, the skills and techniques you learn here will help improve your critical thinking and problem-solving skills. This is a great benefit of studying mathematics and will be useful in your life in ways you may not see right now.

Solve Equations Using the Subtraction and Addition Properties of Equality

Solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle.

Solution of an Equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

The steps to determine if a value is a solution to an equation are listed here.

HOW TO: Determine whether a number is a solution to an equation.

1. Substitute the number for the variable in the equation.
2. Simplify the expressions on both sides of the equation.

3. Determine whether the resulting equation is true.
   - If it is true, the number is a solution.
   - If it is not true, the number is not a solution.

EXAMPLE 1

Determine whether $y = \frac{3}{4}$ is a solution for $4y + 3 = 8y$.

Solution

	$4y + 3 = 8y$
Substitute $\frac{3}{4}$ for y	$4(\frac{3}{4}) + 3 \stackrel{?}{=} 8(\frac{3}{4})$
Multiply.	$3 + 3 \stackrel{?}{=} 6$
Add.	$6 = 6\checkmark$

Since $y = \frac{3}{4}$ results in a true equation, $\frac{3}{4}$ is a solution to the equation $4y + 3 = 8y$.

TRY IT 1.1

Is $y = \frac{2}{3}$ a solution for $9y + 2 = 6y$?

Show answer
no

TRY IT 1.2

Is $y = \frac{2}{5}$ a solution for $5y - 3 = 10y$?

Show answer
no

In that section,we will model how the Subtraction and Addition Properties work and then we will apply them to solve equations.

Subtraction Property of Equality

For all real numbers a, b, and c, if $a = b$, then $a - c = b - c$.

Addition Property of Equality

For all real numbers a, b, and c, if $a = b$, then $a + c = b + c$.

When you add or subtract the same quantity from both sides of an equation, you still have equality.

We will introduce the Subtraction Property of Equality by modeling equations with envelopes and counters. (Figure .1) models the equation $x + 3 = 8$.

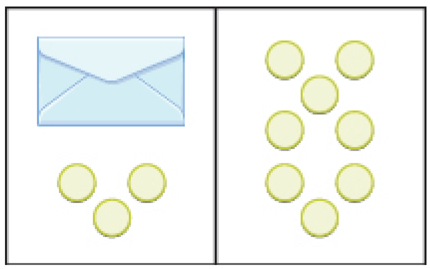

Figure .1

The goal is to isolate the variable on one side of the equation. So we 'took away' 3 from both sides of the equation and found the solution $x = 5$.

Some people picture a balance scale, as in (Figure .2), when they solve equations.

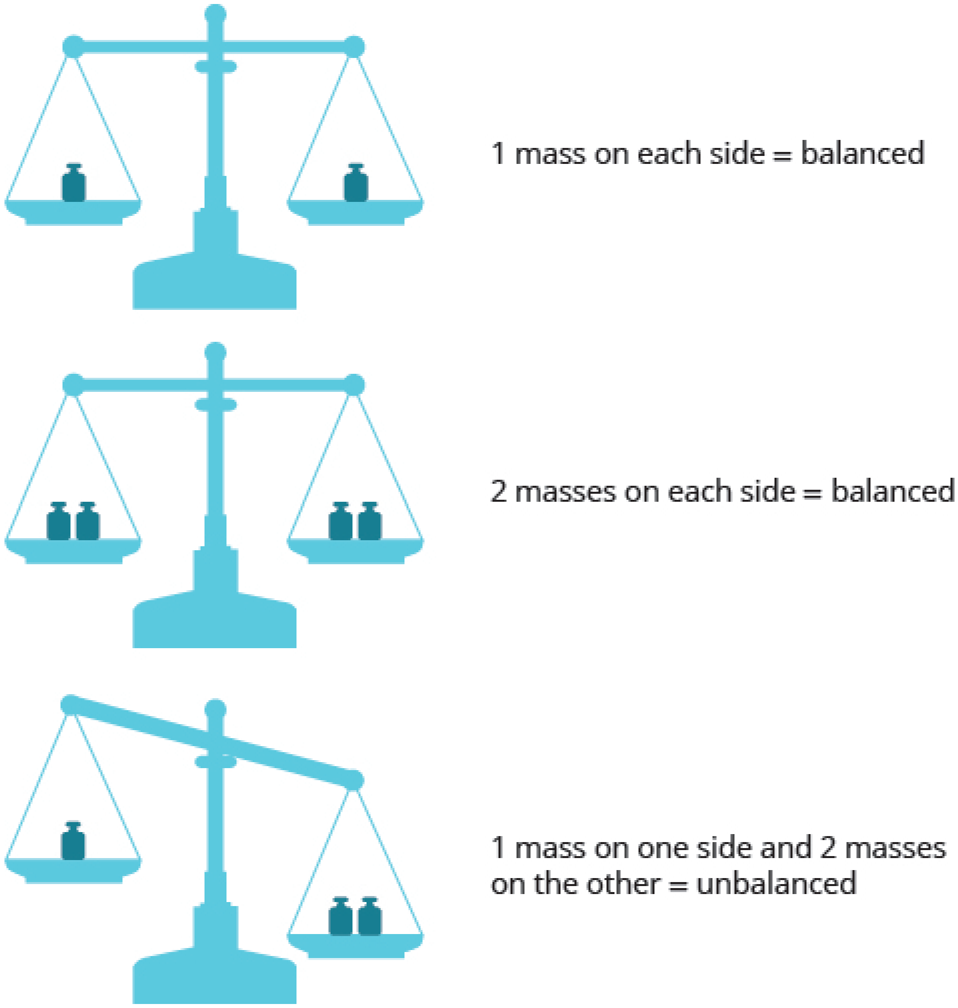

Figure .2

The quantities on both sides of the equal sign in an equation are equal, or balanced. Just as with the balance scale, whatever you do to one side of the equation you must also do to the other to keep it balanced.

Let's see how to use Subtraction and Addition Properties of Equality to solve equations. We need to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

EXAMPLE 2

Solve: $x + 11 = -3$.

Solution

To isolate x, we undo the addition of 11 by using the Subtraction Property of Equality.

		$x+11=-3$
Subtract 11 from each side to “undo” the addition.		$x+11-11=-3-11$
Simplify.		$x=-14$
Check:	$x+11=-3$	
Substitute $x=-14$.	$-14+11\overset{?}{=}-3$	
	$-3=-3\checkmark$	

Since $x=-14$ makes $x+11=-3$ a true statement, we know that it is a solution to the equation.

TRY IT 2.1

Solve: $x+9=-7$.

Show answer
$x=-16$

TRY IT 2.2

Solve: $x+16=-4$.

Show answer
$x=-20$

In the original equation in the previous example, 11 was added to the x, so we subtracted 11 to ‘undo’ the addition. In the next example, we will need to ‘undo’ subtraction by using the Addition Property of Equality.

EXAMPLE 3

Solve: $m+4=-5$.

Solution

		$m+4=-5$
Add 4 to each side to “undo” the subtraction.		$m+4-4=-5-4$
Simplify.		$m=-9$
Check:	$m+4=-5$	
Substitute $m=-1$.	$-9+4\stackrel{?}{=}-5$	
	$-5=-5\checkmark$	
The solution to $m+4=-5$ is $m=-9$.		

TRY IT 3.1

Solve: $n-6=-7$.

Show answer
−1

TRY IT 3.2

Solve: $x-5=-9$.

Show answer
−4

Now let’s solve equations with fractions.

EXAMPLE 4

Solve: $n-\frac{3}{8}=\frac{1}{2}$.

Solution

		$n-\frac{3}{8}=\frac{1}{2}$
Use the Addition Property of Equality.		$n-\frac{3}{8}+\frac{3}{8}=\frac{1}{2}+\frac{3}{8}$
Find the LCD to add the fractions on the right.		$n-\cancel{\frac{3}{8}}+\cancel{\frac{3}{8}}=\frac{4}{8}+\frac{3}{8}$
Simplify		$n=\frac{7}{8}$
Check:	$n-\frac{3}{8}=\frac{1}{2}$	
Substitute $n=\frac{7}{8}$	$\frac{7}{8}-\frac{3}{8}\stackrel{?}{=}\frac{1}{2}$	
Subtract.	$\frac{4}{8}\stackrel{?}{=}\frac{1}{2}$	
Simplify.	$\frac{1}{2}=\frac{1}{2}\checkmark$	
The solution checks.		

Since $n=\frac{7}{8}$ results in a true equation, $\frac{7}{8}$ is a solution of $n-\frac{3}{8}=\frac{1}{2}$

TRY IT 4.1

Solve: $p-\frac{1}{3}=\frac{5}{6}$.

Show answer
$p=\frac{7}{6}$

TRY IT 4.2

Solve: $q-\frac{1}{2}=\frac{1}{6}$.

Show answer
$q=\frac{2}{3}$

Let’s solve equations that contained decimals.

EXAMPLE 5

Solve $a - 3.7 = 4.3$.

Solution

		$a - 3.7 = 4.3$
Use the Addition Property of Equality.		$a - 3.7 + 3.7 = 4.3 + 3.7$
Add.		$a - \cancel{3.7} + \cancel{3.7} = 8$
Check:	$a - 3.7 = 4.3$	
Substitute $a = 8$.	$8 - 3.7 \stackrel{?}{=} 4.3$	
Simplify.	$4.3 = 4.3 \checkmark$	
The solution checks.	Therefore, $a = 8$ is a solution of $a - 3.7 = 4.3$	

TRY IT 5.1

Solve: $b - 2.8 = 3.6$.

Show answer
$b = 6.4$

TRY IT 5.2

Solve: $c - 6.9 = 7.1$.

Show answer
$c = 14$

Solve Equations That Need to Be Simplified

In the examples up to this point, we have been able to isolate the variable with just one operation. Many of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality. You should always simplify as much as possible before trying to isolate the variable.

EXAMPLE 6

Solve: $3x - 7 - 2x - 4 = 1$.

Solution

The left side of the equation has an expression that we should simplify before trying to isolate the variable.

	$3x - 7 - 2x - 4 = 1$
Rearrange the terms, using the Commutative Property of Addition.	$3x - 2x - 7 - 4 = 1$
Combine like terms.	$x - 11 = 1$
Add 11 to both sides to isolate x.	$x - 11 + 11 = 1 + 11$
Simplify.	$x = 12$
Check. Substitute $x = 12$ into the original equation.	$3x - 7 - 2x - 4 = 1$ $3(12) - 7 - 2(12) - 4 = 1$ $36 - 7 - 24 - 4 = 1$ $29 - 24 - 4 = 1$ $5 - 4 = 1$ $1 = 1 \checkmark$

The solution checks.

TRY IT 6.1

Solve: $8y - 4 - 7y - 7 = 4$.

Show answer

$y = 15$

TRY IT 6.2

Solve: $6z + 5 - 5z - 4 = 3$.

Show answer

$z = 2$

EXAMPLE 7

Solve: $3\left(n-4\right)-2n=-3.$

Solution

The left side of the equation has an expression that we should simplify.

	$3(n-4)-2n=-3$
Distribute on the left.	$3n-12-2n=-3$
Use the Commutative Property to rearrange terms.	$3n-2n-12=-3$
Combine like terms.	$n-12=-3$
Isolate *n* using the Addition Property of Equality.	$n-12+12=-3+12$
Simplify.	$n=9$
Check. Substitute $n=9$ into the original equation.	$3(n-4)-2n=-3$ $3(9-4)-2\cdot 9=-3$ $3(5)-18=-3$ $15-18=-3$ $-3=-3\checkmark$
The solution checks.	

TRY IT 7.1

Solve: $5\left(p-3\right)-4p=-10.$

Show answer
$p=5$

TRY IT 7.2

Solve: $4\left(q+2\right)-3q=-8.$

Show answer
$q=-16$

EXAMPLE 8

Solve: $2\left(3k-1\right)-5k=-2-7$.

Solution

Both sides of the equation have expressions that we should simplify before we isolate the variable.

	$2(3k-1)-5k=-2-7$
Distribute on the left, subtract on the right.	$6k-2-5k=-9$
Use the Commutative Property of Addition.	$6k-5k-2=-9$
Combine like terms.	$k-2=-9$
Undo subtraction by using the Addition Property of Equality.	$k-2+2=-9+2$
Simplify.	$k=-7$
Check. Let $k=-7$.	$2(3k-1)-5k=-2-7$ $2(3(-7)-1)-5(-7)\stackrel{?}{=}-2-7$ $2(-21-1)-5(-7)\stackrel{?}{=}-9$ $2(-22)+35\stackrel{?}{=}-9$ $-44+35\stackrel{?}{=}-9$ $-9=-9\checkmark$
The solution checks.	

TRY IT 8.1

Solve: $4\left(2h-3\right)-7h=-6-7$.

Show answer

$h=-1$

TRY IT 8.2

Solve: $2\left(5x+2\right)-9x=-2+7$.

Show answer

$x=1$

Translate an Equation and Solve

Previously, we translated word sentences into equations. The first step is to look for the word (or words) that translate(s) to the equal sign. The list below reminds us of some of the words that translate to the equal sign (=):

- is
- is equal to
- is the same as
- the result is
- gives
- was
- will be

Let’s review the steps we used to translate a sentence into an equation.

HOW TO: Translate a word sentence to an algebraic equation.

1. Locate the “equals” word(s). Translate to an equal sign.
2. Translate the words to the left of the “equals” word(s) into an algebraic expression.
3. Translate the words to the right of the “equals” word(s) into an algebraic expression.

Now we are ready to try an example.

EXAMPLE 9

Translate and solve: five more than x is equal to 26.

Solution

Translate.	Five more than x / is equal to / 26 $x+5 \quad = \quad 26$
Subtract 5 from both sides.	$x+5-5=26-5$
Simplify.	$x=21$
Check: Is 26 five more than 21?	$21+5 \stackrel{?}{=} 26$ $26=26✓$ The solution checks.

TRY IT 9.1

Translate and solve: Eleven more than x is equal to 41.

Show answer
$x + 11 = 41; x = 30$

TRY IT 9.2

Translate and solve: Twelve less than y is equal to 51.

Show answer
$y - 12 = 51; y = 63$

EXAMPLE 10

Translate and solve: The difference of $5p$ and $4p$ is 23.

Solution

Translate.	The difference of 5p and 4p is 23 $5p - 4p = 23$
Simplify.	$p = 23$
Check.	$5p - 4p = 23$ $5(23) - 4(23) \stackrel{?}{=} 23$ $5 \cdot 23 - 4 \cdot 23 \stackrel{?}{=} 23$ $115 - 92 \stackrel{?}{=} 23$ $23 = 23 \checkmark$
The solution checks.	

TRY IT 10.1

Translate and solve: The difference of $4x$ and $3x$ is 14.

Show answer
$4x - 3x = 14; x = 14$

TRY IT 10.2

Translate and solve: The difference of $7a$ and $6a$ is -8.

Show answer
$7a - 6a = -8$; $a = -8$

Translate and Solve Applications

In most of the application problems we solved earlier, we were able to find the quantity we were looking for by simplifying an algebraic expression. Now we will be using equations to solve application problems. We'll start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for.

EXAMPLE 11

The Robles family has two dogs, Buster and Chandler. Together, they weigh 71 pounds. Chandler weighs 28 pounds. How much does Buster weigh?

Solution

Read the problem carefully.	
Identify what you are asked to find, and choose a variable to represent it.	How much does Buster weigh? Let b = Buster's weight
Write a sentence that gives the information to find it.	Buster's weight plus Chandler's weight equals 71 pounds.
We will restate the problem, and then include the given information.	Buster's weight plus 28 equals 71.
Translate the sentence into an equation, using the variable b.	$b+28=71$
Solve the equation using good algebraic techniques.	$b+28-{\color{red}28}=71-{\color{red}28}$ $b+\cancel{28}-\cancel{28}=43$ $b=43$
Check the answer in the problem and make sure it makes sense.	Is 43 pounds a reasonable weight for a dog? Yes. Does Buster's weight plus Chandler's weight equal 71 pounds?
	$43+28\stackrel{?}{=}71$
	$71=71\checkmark$
Write a complete sentence that answers the question, "How much does Buster weigh?"	Buster weighs 43 pounds

TRY IT 11.1

Translate into an algebraic equation and solve: The Pappas family has two cats, Zeus and Athena. Together, they weigh 13 pounds. Zeus weighs 6 pounds. How much does Athena weigh?

Show answer
a + 6 = 13; Athena weighs 7 pounds.

TRY IT 11.2

Translate into an algebraic equation and solve: Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Show answer
26 + *h* = 68; Henry has 42 books.

Devise a Problem-Solving Strategy

1. Read the problem. Make sure you understand all the words and ideas.
2. Identify what you are looking for.
3. Name what you are looking for. Choose a variable to represent that quantity.
4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
5. Solve the equation using good algebra techniques.
6. Check the answer in the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

EXAMPLE 12

Shayla paid $\$24{,}575$ for her new car. This was $\$875$ less than the sticker price. What was the sticker price of the car?

Solution

What are you asked to find?	"What was the sticker price of the car?"
Assign a variable.	Let s = the sticker price of the car.
Write a sentence that gives the information to find it.	$24,575 is $875 less than the sticker price $24,575 is $875 less than s
Translate into an equation.	$24,575 = s - 875$
Solve.	$24,575 + 875 = s - 875 + 875$ $25,450 = s$
Check:	Is $875 less than $25,450 equal to $24,575? $25,450 - 875 \stackrel{?}{=} 24,575$ $24,575 = 24,575\checkmark$
Write a sentence that answers the question.	The sticker price was $25,450.

TRY IT 12.1

Translate into an algebraic equation and solve: Eddie paid $\$19{,}875$ for his new car. This was $\$1{,}025$ less than the sticker price. What was the sticker price of the car?

Show answer
19,875 = s − 1025; the sticker price is $20,900.

TRY IT 12.2

Translate into an algebraic equation and solve: The admission price for the movies during the day is $\$7.75$. This is $\$3.25$ less than the price at night. How much does the movie cost at night?

Show answer
$7.75 = n - 3.25$; the price at night is $11.00.

Key Concepts

- **Determine whether a number is a solution to an equation.**

    1. Substitute the number for the variable in the equation.
    2. Simplify the expressions on both sides of the equation.
    3. Determine whether the resulting equation is true.

    If it is true, the number is a solution.
    If it is not true, the number is not a solution.
- **Subtraction and Addition Properties of Equality**
    - **Subtraction Property of Equality**
      For all real numbers a, b, and c,
      if $a = b$ then $a - c = b - c$.
    - **Addition Property of Equality**
      For all real numbers a, b, and c,
      if $a = b$ then $a + c = b + c$.
- **Translate a word sentence to an algebraic equation.**

    1. Locate the "equals" word(s). Translate to an equal sign.
    2. Translate the words to the left of the "equals" word(s) into an algebraic expression.
    3. Translate the words to the right of the "equals" word(s) into an algebraic expression.
- **Problem-solving strategy**

    1. Read the problem. Make sure you understand all the words and ideas.
    2. Identify what you are looking for.
    3. Name what you are looking for. Choose a variable to represent that quantity.
    4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.

5. Solve the equation using good algebra techniques.
6. Check the answer in the problem and make sure it makes sense.
7. Answer the question with a complete sentence.

Glossary

solution of an equation
A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

Practice Makes Perfect

Solve Equations Using the Subtraction and Addition Properties of Equality

In the following exercises, determine whether the given value is a solution to the equation.

1. Is $y = \frac{1}{3}$ a solution of $4y + 2 = 10y$?	2. Is $x = \frac{3}{4}$ a solution of $5x + 3 = 9x$?
3. Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?	4. Is $v = -\frac{1}{3}$ a solution of $9v - 2 = 3v$?

In the following exercises, solve each equation.

5. $x + 7 = 12$	6. $y + 5 = -6$
7. $b + \frac{1}{4} = \frac{3}{4}$	8. $a + \frac{2}{5} = \frac{4}{5}$
9. $p + 2.4 = -9.3$	10. $m + 7.9 = 11.6$
11. $a - 3 = 7$	12. $m - 8 = -20$
13. $x - \frac{1}{3} = 2$	14. $x - \frac{1}{5} = 4$
15. $y - 3.8 = 10$	16. $y - 7.2 = 5$
17. $x - 15 = -42$	18. $z + 5.2 = -8.5$
19. $q + \frac{3}{4} = \frac{1}{2}$	20. $p - \frac{2}{5} = \frac{2}{3}$

Solve Equations that Need to be Simplified

In the following exercises, solve each equation.

21. $m+6-8=15$	22. $c+3-10=18$
23. $6x+8-5x+16=32$	24. $9x+5-8x+14=20$
25. $-8n-17+9n-4=-41$	26. $-6x-11+7x-5=-16$
27. $4\left(y-2\right)-3y=-6$	28. $3\left(y-5\right)-2y=-7$
29. $5\left(w+2.2\right)-4w=9.3$	30. $8\left(u+1.5\right)-7u=4.9$
31. $-8\left(x-1\right)+9x=-3+9$	32. $-5\left(y-2\right)+6y=-7+4$
33. $2\left(8m+3\right)-15m-4=3-5$	34. $3\left(5n-1\right)-14n+9=1-2$
35. $-\left(k+7\right)+2k+8=7$	36. $-\left(j+2\right)+2j-1=5$
37. $8c-7\left(c-3\right)+4=-16$	38. $6a-5\left(a-2\right)+9=-11$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

39. The sum of x and -5 is 33.	40.Five more than x is equal to 21.
41.Three less than y is -19.	42. Ten less than m is -14.
43. Eight more than p is equal to 52.	44. The sum of y and -3 is 40.
45. The difference of $5c$ and $4c$ is 60.	46. The difference of $9x$ and $8x$ is 17.
47. The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.	48. The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.
49. The sum of $-9m$ and $10m$ is -25.	50. The sum of $-4n$ and $5n$ is -32.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

51.Jeff read a total of 54 pages in his English and Psychology textbooks. He read 41 pages in his English textbook. How many pages did he read in his Psychology textbook?	52. Pilar drove from home to school and then to her aunt's house, a total of 18 miles. The distance from Pilar's house to school is 7 miles. What is the distance from school to her aunt's house?
53. Eva's daughter is 5 years younger than her son. Eva's son is 12 years old. How old is her daughter?	54. Pablo's father is 3 years older than his mother. Pablo's mother is 42 years old. How old is his father?
55. For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday dinner turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?	56. Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?
57. Connor's temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?	58. The nurse reported that Tricia's daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia's daughter weigh at her last checkup?
59. Ron's paycheck this week was $\$17.43$ less than his paycheck last week. His paycheck this week was $\$103.76$. How much was Ron's paycheck last week?	60. Melissa's math book cost $\$22.85$ less than her art book cost. Her math book cost $\$93.75$. How much did her art book cost?

Everyday Math

61.Construction Miguel wants to drill a hole for a $\frac{5}{8}\text{-inch}$ screw. The screw should be $\frac{1}{12}$ inch larger than the hole. Let d equal the size of the hole he should drill. Solve the equation $d+\frac{1}{12}=\frac{5}{8}$ to see what size the hole should be.	**Baking** 62. Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{1}{4}$ cup of sugar and will borrow the rest from her neighbour. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{1}{4}+s=\frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Writing Exercises

63. Write a word sentence that translates the equation $y-18=41$ and then make up an application that uses this equation in its solution.	64. Is -18 a solution to the equation $3x=16-5x?$ How do you know?

Answers

1. yes	3. no	5. $x = 5$
7. $b = \frac{1}{2}$	9. $p = -11.7$	11. $a = 10$
13. $x = \frac{7}{3}$	15. $y = 13.8$	17. $x = -27$
19. $q = -\frac{1}{4}$	21. 17	23. 8
25. −20	27. 2	29. −1.7
31. −2	33. −4	35. 6
37. −41	39. $x + (-5) = 33$; $x = 38$	41. $y - 3 = -19$; $y = -16$
43. $p + 8 = 52$; $p = 44$	45. $5c - 4c = 60$; 60	47. $f - \frac{1}{3} = \frac{1}{12}$; $\frac{5}{12}$
49. $-9m + 10m = -25$; $m = -25$	51. Let p equal the number of pages read in the Psychology book $41 + p = 54$. So $p = 13$. Jeff read 13 pages in his Psychology book.	53. Let d equal the daughter's age. $d = 12 - 5$. Eva's daughter's age is 7 years old.
55. 21 pounds	57. 100.5 degrees	59. $121.19
61. $d = \frac{13}{24}$	63. Answers will vary.	

Attributions

3.2 Solve Equations Using the Division and Multiplication Properties of Equality

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that need to be simplified

Solve Equations Using the Division and Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable. Now we will see how to solve equations that have a variable multiplied by a constant and so will require division to isolate the variable.

Let's look at our puzzle again with the envelopes and counters in (Figure 1).

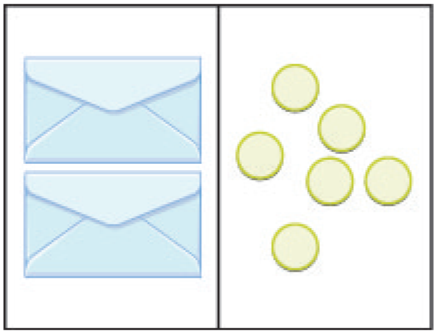

Figure .1

In the illustration there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are "hidden" in the envelopes. So how many counters are in each envelope?

How do we determine the number? We have to separate the counters on the right side into two groups of the same size to correspond with the two envelopes on the left side. The 6 counters divided into 2 equal groups gives 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in (Figure 2)? There are two envelopes, and each contains x counters. Together, the two envelopes must contain a total of 6 counters.

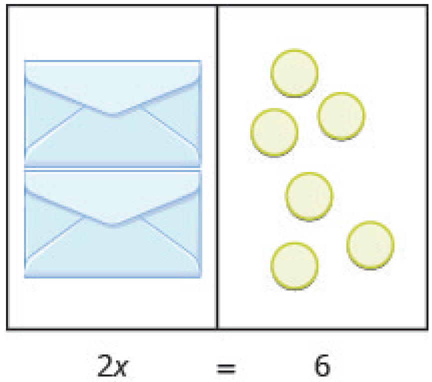

Figure .2

	$2x = 6$
If we divide both sides of the equation by 2, as we did with the envelopes and counters,	$\frac{2x}{2} = \frac{6}{2}$
we get:	$x = 3$

We found that each envelope contains 3 counters. Does this check? We know $2 \times 3 = 6$, so it works! Three counters in each of two envelopes does equal six!

This example leads to the Division Property of Equality.

Division and Multiplication Properties of Equality

Division Property of Equality: For all real numbers a, b, c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Multiplication Property of Equality: For all real numbers a, b, c, if $a = b$, then $ac = bc$.

When you divide or multiply both sides of an equation by the same quantity, you still have equality.

Let's review how these properties of equality can be applied in order to solve equations. Remember, the goal is to 'undo' the operation on the variable. In the example below the variable is multiplied by 4, so we will divide both sides by 4 to 'undo' the multiplication.

EXAMPLE 1

Solve: $4x = -28$.

Solution

We use the Division Property of Equality to divide both sides by 4.

	$4x = -28$
Divide both sides by 4 to undo the multiplication.	$\frac{4x}{4} = \frac{-28}{4}$
Simplify.	$x = -7$
Check your answer. Let $x = -7$.	$4x = -28$ $4(-7) \stackrel{?}{=} -28$ $-28 = -28 \checkmark$

Since this is a true statement, $x = -7$ is a solution to $4x = -28$.

TRY IT 1.1

Solve: $3y = -48$.

Show answer

$y = -16$

TRY IT 1.2

Solve: $4z = -52$.

Show answer

$z = -13$

In the previous example, to ‘undo’ multiplication, we divided. How do you think we ‘undo’ division?

EXAMPLE 2

Solve: $\frac{a}{-7} = -42$.

Solution

Here a is divided by -7. We can multiply both sides by -7 to isolate a.

	$\frac{a}{-7} = -42$
Multiply both sides by -7.	$-7(\frac{a}{-7}) = -7(-42)$ $\frac{-7a}{-7} = 294$
Simplify.	$a = 294$
Check your answer. Let $a = 294$.	$\frac{a}{-7} = -42$ $\frac{294}{-7} \stackrel{?}{=} -42$ $-42 = -42✓$

TRY IT 2.1

Solve: $\frac{b}{-6} = -24$.

Show answer
b = 144

TRY IT 2.2

Solve: $\frac{c}{-8} = -16$.

Show answer
c = 128

EXAMPLE 3

Solve: $-r = 2$.

Solution

Remember $-r$ is equivalent to $-1r$.

	$-r = 2$
Rewrite $-r$ as $-1r$.	$-1r = 2$
Divide both sides by -1.	$\frac{-1r}{-1} = \frac{2}{-1}$
	$r = -2$
Check.	$-r = 2$
Substitute $r = -2$	$-(-2) \stackrel{?}{=} 2$
Simplify.	$2 = 2\checkmark$

We see that there are two other ways to solve $-r = 2$.

We could multiply both sides by -1.

We could take the opposite of both sides.

TRY IT 3.1

Solve: $-k = 8$.

Show answer

$k = -8$

TRY IT 3.2

Solve: $-g = 3$.

Show answer

$g = -3$

EXAMPLE 4

Solve: $\frac{2}{3}x = 18$.

Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{2}{3}$.

	$\frac{2}{3}x = 18$
Multiply by the reciprocal of $\frac{2}{3}$ which is $\frac{3}{2}$	$\frac{3}{2} \cdot \frac{2}{3} = \frac{3}{2} \cdot 18$
Reciprocals multiply to one.	$1x = \frac{3}{2} \cdot \frac{18}{1}$
Multiply.	$x = 27$
Check your answer. Let $x = 27$	$\frac{2}{3} = 18$ $\frac{2}{3} \cdot 27 \stackrel{?}{=} 18$ $18 = 18\checkmark$

Notice that we could have divided both sides of the equation $\frac{2}{3}x = 18$ by $\frac{2}{3}$ to isolate x. While this would work, multiplying by the reciprocal requires fewer steps.

TRY IT 4.1

Solve: $\frac{2}{5}n = 14$.

Show answer
n = 35

TRY IT 4.2

Solve: $\frac{5}{6}y = 15$.

Show answer
y = 18

Solve Equations That Need to be Simplified

Many equations start out more complicated than the ones we've just solved. First, we need to simplify both sides of the equation as much as possible

EXAMPLE 5

Solve: $8x + 9x - 5x = -3 + 15$.

Solution

Start by combining like terms to simplify each side.

	$8x+9x-5x=-3+15$
Combine like terms.	$12x=12$
Divide both sides by 12 to isolate x.	$\frac{12x}{12}=\frac{12}{12}$
Simplify.	$x=1$
Check your answer. Let $x=1$	$8x+9x-5x=-3+15$ $8\cdot 1+9\cdot 1-5\cdot 1\stackrel{?}{=}-3+15$ $8+9-5\stackrel{?}{=}12$ $12=12\checkmark$

TRY IT 5.1

Solve: $7x+6x-4x=-8+26$.

Show answer

$x = 2$

TRY IT 5.2

Solve: $11n-3n-6n=7-17$.

Show answer

$n = -5$

EXAMPLE 6

Solve: $11-20=17y-8y-6y$.

Solution

Simplify each side by combining like terms.

	$11-20=17y-8y-6y$
Simplify each side.	$-9=3y$ *Note
Divide both sides by 3 to isolate y.	$\frac{-9}{3}=\frac{3y}{3}$
Simplify.	$-3=y$
Check your answer. Let $y=-3$	$11-20=17y-8y-6y$ $11-20\stackrel{?}{=}17(-3)-8(-3)-6(-3)$ $11-20\stackrel{?}{=}-51+24+18$ $-9=-9\checkmark$

*Notice that the variable ended up on the right side of the equal sign when we solved the equation. You may prefer to take one more step to write the solution with the variable on the left side of the equal sign.

TRY IT 6.1

Solve: $18-27=15c-9c-3c$.

Show answer
$c=-3$

TRY IT 6.2

Solve: $18-22=12x-x-4x$.

Show answer
$x=-\frac{4}{7}$

EXAMPLE 7

Solve: $-3\left(n-2\right)-6=21$.

Solution

Remember—always simplify each side first.

	$-3(n-2)-6=21$
Distribute.	$-3n+6-6=21$
Simplify.	$-3n+\cancel{6}-\cancel{6}=21$ $-3n=21$
Divide both sides by -3 to isolate n.	$\frac{-3n}{-3}=\frac{21}{-3}$ $n=-7$
Check your answer. Let $n=-7$.	$-3(n-2)-6=21$ $-3(-7-2)-6\stackrel{?}{=}21$ $-3(-9)-6\stackrel{?}{=}21$ $27-6\stackrel{?}{=}21$ $21=21\checkmark$

TRY IT 7.1

Solve: $-4\left(n-2\right)-8=24$.

Show answer

n = −6

TRY IT 7.2

Solve: $-6\left(n-2\right)-12=30$.

Show answer

n = −5

Key Concepts

- **Division and Multiplication Properties of Equality**
  - **Division Property of Equality:** For all real numbers *a*, *b*, *c*, and $c\ne 0$, if $a=b$, then $\frac{a}{c}=\frac{b}{c}$.
  - **Multiplication Property of Equality:** For all real numbers *a*, *b*, *c*, if $a=b$, then $ac=bc$.

Practice Makes Perfect

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation for the variable using the Division Property of Equality and check the solution.

1. $7p = 63$	2. $8x = 32$
3. $-9x = -27$	4. $-5c = 55$
5. $-72 = 12y$	6. $-90 = 6y$
7. $-8m = -56$	8. $-16p = -64$
9. $0.75a = 11.25$	10. $0.25z = 3.25$
11. $4x = 0$	12. $-3x = 0$

In the following exercises, solve each equation for the variable using the Multiplication Property of Equality and check the solution.

13. $\frac{z}{2} = 14$	14. $\frac{x}{4} = 15$
15. $\frac{c}{-3} = -12$	16. $-20 = \frac{q}{-5}$
17. $\frac{q}{6} = -8$	18. $\frac{y}{9} = -6$
19. $-4 = \frac{p}{-20}$	20. $\frac{m}{-12} = 5$
21. $\frac{3}{5}r = 15$	22. $\frac{2}{3}y = 18$
23. $24 = -\frac{3}{4}x$	24. $-\frac{5}{8}w = 40$
25. $-\frac{1}{3}q = -\frac{5}{6}$	26. $-\frac{2}{5} = \frac{1}{10}a$

Solve Equations That Need to be Simplified

In the following exercises, solve the equation.

27. $6y - 3y + 12y = -43 + 28$	28. $8a + 3a - 6a = -17 + 27$
29. $-5m + 7m - 8m = -6 + 36$	30. $-9x - 9x + 2x = 50 - 2$
31. $-18 - 7 = 5t - 9t - 6t$	32. $100 - 16 = 4p - 10p - p$
33. $\frac{5}{12}q + \frac{1}{2}q = 25 - 3$	34. $\frac{7}{8}n - \frac{3}{4}n = 9 + 2$
35. $0.05p - 0.01p = 2 + 0.24$	36. $0.25d + 0.10d = 6 - 0.75$

Everyday Math

37. Teaching Connie's kindergarten class has 24 children. She wants them to get into 4 equal groups. Find the number of children in each group, g, by solving the equation $4g = 24$.	**38. Balloons** Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. Find the number of balloons in each bunch, b, by solving the equation $3b = 18$.
39. Unit price Nishant paid $\$12.96$ for a pack of 12 juice bottles. Find the price of each bottle, b, by solving the equation $12b = 12.96$.	**40. Ticket price** Daria paid $\$36.25$ for 5 children's tickets at the ice skating rink. Find the price of each ticket, p, by solving the equation $5p = 36.25$.
41. Fabric The drill team used 14 yards of fabric to make flags for one-third of the members. Find how much fabric, f, they would need to make flags for the whole team by solving the equation $\frac{1}{3}f = 14$.	**42. Fuel economy** Tania's SUV gets half as many miles per gallon (mpg) as her husband's hybrid car. The SUV gets $18\ \text{mpg}$. Find the miles per gallons, m, of the hybrid car, by solving the equation $\frac{1}{2}m = 18$.

Writing Exercises

43. Emiliano thinks $x = 40$ is the solution to the equation $\frac{1}{2}x = 80$. Explain why he is wrong.	44. Frida started to solve the equation $-3x = 36$ by adding 3 to both sides. Explain why Frida's method will result in the correct solution.

Answers

1. 9	3. 3	5. −6
7. 7	9. 15	11. 0
13. 28	15. 36	17. −48
19. 80	21. 25	23. −32
25. 5/2	27. y = −1	29. m = −5
31. $t = \frac{5}{2}$	33. q = 24	35. p = 56
37. 6 children	39. $1.08	41. 42 yards
43. Answer will vary.		

Attributions

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3.3 Solve Equations with Variables and Constants on Both Sides

Learning Objectives

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides
- Solve equations using a general strategy

Solve an Equation with Constants on Both Sides

You may have noticed that in all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we'll see how to solve equations where the variable terms and/or constant terms are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the variable side, and the other side of the equation to be the constant side. Then, we will use the Subtraction and Addition Properties of Equality, step by step, to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that started with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

EXAMPLE 1

Solve: $4x + 6 = -14$.

Solution

In this equation, the variable is only on the left side. It makes sense to call the left side the variable side. Therefore, the right side will be the constant side. We'll write the labels above the equation to help us remember what goes where.

	$\underset{4x+6=}{\color{red}{variable}}\ \underset{-14}{\color{red}{constant}}$
Since the left side is the variable side, the 6 is out of place. We must "undo" adding 6 by subtracting 6, and to keep the equality we must subtract 6 from both sides. Use the Subtraction Property of Equality.	$4x+6{\color{red}{-6}}=-14{\color{red}{-6}}$
Simplify.	$4x=-20$
Now all the xs are on the left and the constant on the right.	
Use the Division Property of Equality.	$\frac{4x}{\color{red}{4}}=\frac{-20}{\color{red}{4}}$
Simplify.	$x=-5$
Check:	$4x+6=-14$
Let $x=-5$.	$4({\color{red}{-5}})+6\stackrel{?}{=}-14$
	$-20+6\stackrel{?}{=}-14$
	$-14=-14\checkmark$

TRY IT 1.1

Solve: $3x+4=-8$.

Show answer

x = −4

TRY IT 1.2

Solve: $5a+3=-37$.

Show answer

a = −8

EXAMPLE 2

Solve: $2y-7=15$.

Solution

Notice that the variable is only on the left side of the equation, so this will be the variable side and the right side will be the constant side. Since the left side is the variable side, the 7 is out of place. It is subtracted from the $2y$, so to 'undo' subtraction, add 7 to both sides.

	$\overset{variable}{2y-7} = \overset{constant}{15}$
Add 7 to both sides.	$2y-7+7=15+7$
Simplify.	$2y=22$
The variables are now on one side and the constants on the other.	
Divide both sides by 2.	$\frac{2y}{2}=\frac{22}{2}$
Simplify.	$y=11$
Check:	$2y-7=15$
Substitute: $y=11$.	$2\cdot 11-7\overset{?}{=}15$
	$22-7\overset{?}{=}15$
	$15=15\checkmark$

TRY IT 2.1

Solve: $5y-9=16$.

Show answer

y = 5

TRY IT 2.2

Solve: $3m-8=19$.

Show answer

m = 9

Solve an Equation with Variables on Both Sides

What if there are variables on both sides of the equation? We will start like we did above—choosing a variable side and a constant side, and then use the Subtraction and Addition Properties of Equality to collect all variables on one side and all constants on the other side. Remember, what you do to the left side of the equation, you must do to the right side too.

EXAMPLE 3

Solve: $5x = 4x + 7$.

Solution

Here the variable, x, is on both sides, but the constants appear only on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

	$\underset{5x}{\text{variable}} = \underset{4x+7}{\text{constant}}$
We don't want any variables on the right, so subtract the $4x$.	$5x - 4x = 4x - 4x + 7$
Simplify.	$x = 7$
We have all the variables on one side and the constants on the other. We have solved the equation.	
Check:	$5x = 4x + 7$
Substitute 7 for x.	$5 \cdot 7 \overset{?}{=} 4 \cdot 7 + 7$
	$35 \overset{?}{=} 28 + 7$
	$35 = 35\checkmark$

TRY IT 3.1

Solve: $6n = 5n + 10$.

Show answer

n = 10

TRY IT 3.2

Solve: $-6c = -7c + 1$.

Show answer

c = 1

EXAMPLE 4

Solve: $5y - 8 = 7y$.

Solution

The only constant, -8, is on the left side of the equation and variable, y, is on both sides. Let's leave the constant on the left and collect the variables to the right.

	$\underset{constant}{5y - 8} = \underset{variable}{7y}$
Subtract $5y$ from both sides.	$5y - 5y - 8 = 7y - 5y$
Simplify.	$-8 = 2y$
We have the variables on the right and the constants on the left. Divide both sides by 2.	$\frac{-8}{2} = \frac{2y}{2}$
Simplify.	$-4 = y$
Rewrite with the variable on the left.	$y = -4$
Check: Let $y = -4$.	$5y - 8 = 7y$ $5(-4) - 8 \stackrel{?}{=} 7(-4)$ $-20 - 8 \stackrel{?}{=} -28$ $-28 = -28\checkmark$

TRY IT 4.1

Solve: $3p - 14 = 5p$.

Show answer

$p = -7$

TRY IT 4.2

Solve: $8m + 9 = 5m$.

Show answer
$m = -3$

EXAMPLE 5

Solve: $7x = -x + 24$.

Solution

The only constant, 24, is on the right, so let the left side be the variable side.

	$\underset{}{\overset{\text{variable}}{7x}} = \overset{\text{constant}}{-x + 24}$
Remove the $-x$ from the right side by adding x to both sides.	$7x + x = -x + x + 24$
Simplify.	$8x = 24$
All the variables are on the left and the constants are on the right. Divide both sides by 8.	$\frac{8x}{8} = \frac{24}{8}$
Simplify.	$x = 3$
Check: Substitute $x = 3$.	$7x = -x + 24$ $7(3) \overset{?}{=} -(3) + 24$ $21 = 21✓$

TRY IT 5.1

Solve: $12j = -4j + 32$.

Show answer
$j = 2$

TRY IT 5.2

Solve: $8h = -4h + 12$.

Show answer
$h = 1$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables *and* constants on both sides of the equation. As we did before, we'll collect the variable terms to one side and the constants to the other side.

EXAMPLE 6

Solve: $7x + 5 = 6x + 2$.

Solution

Start by choosing which side will be the variable side and which side will be the constant side. The variable terms are $7x$ and $6x$. Since 7 is greater than 6, make the left side the variable side and so the right side will be the constant side.

	$7x + 5 = 6x + 2$
Collect the variable terms to the left side by subtracting $6x$ from both sides.	$7x - 6x + 5 = 6x - 6x + 2$
Simplify.	$x + 5 = 2$
Now, collect the constants to the right side by subtracting 5 from both sides.	$x + 5 - 5 = 2 - 5$
Simplify.	$x = -3$
The solution is $x = -3$.	
Check: Let $x = -3$.	$7x + 5 = 6x + 2$ $7(-3) + 5 \stackrel{?}{=} 6(-3) + 2$ $-21 + 5 \stackrel{?}{=} -18 + 2$ $-16 = -16 \checkmark$

TRY IT 6.1

Solve: $12x + 8 = 6x + 2$.

Show answer

$x = -1$

TRY IT 6.2

Solve: $9y + 4 = 7y + 12$.

Show answer
$y = 4$

We'll summarize the steps we took so you can easily refer to them.

HOW TO: Solve an Equation with Variables and Constants on Both Sides

1. Choose one side to be the variable side and then the other will be the constant side.
2. Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.
3. Collect the constants to the other side, using the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

It is a good idea to make the variable side the one in which the variable has the larger coefficient. This usually makes the arithmetic easier.

EXAMPLE 7

Solve: $6n - 2 = -3n + 7$.

Solution

We have $6n$ on the left and $-3n$ on the right. Since $6 > -3$, make the left side the "variable" side.

	$6n-2=-3n+7$
We don't want variables on the right side—add $3n$ to both sides to leave only constants on the right.	$6n+3n-2=-3n+3n+7$
Combine like terms.	$9n-2=7$
We don't want any constants on the left side, so add 2 to both sides.	$9n-2+2=7+2$
Simplify.	$9n=9$
The variable term is on the left and the constant term is on the right. To get the coefficient of n to be one, divide both sides by 9.	$\frac{9n}{9}=\frac{9}{9}$
Simplify.	$n=1$
Check: Substitute 1 for n.	$6n-2=-3n+7$ $6(1)-2\stackrel{?}{=}-3(1)+7$ $6-2\stackrel{?}{=}-3+7$ $4=4\checkmark$

TRY IT 7.1

Solve: $8q-5=-4q+7$.

Show answer

q = 1

TRY IT 7.2

Solve: $7n-3=n+3$.

Show answer

n = 1

EXAMPLE 8

Solve: $2a-7=5a+8$.

Solution

This equation can be solved following the process in previous examples. But let us also look at an alternate way to solve it.

This equation has $2a$ on the left and $5a$ on the right. Since $5 > 2$, make the right side the variable side and the left side the constant side.

	$2a - 7 = 5a + 8$
Subtract $2a$ from both sides to remove the variable term from the left.	$2a - 2a - 7 = 5a - 2a + 8$
Combine like terms.	$-7 = 3a + 8$
Subtract 8 from both sides to remove the constant from the right.	$-7 - 8 = 3a + 8 - 8$
Simplify.	$-15 = 3a$
Divide both sides by 3 to make 1 the coefficient of a.	$\frac{-15}{3} = \frac{3a}{3}$
Simplify.	$-5 = a$
Check: Let $a = -5$.	$2a - 7 = 5a + 8$ $2(-5) - 7 \stackrel{?}{=} 5(-5) + 8$ $-10 - 7 \stackrel{?}{=} -25 + 8$ $-17 = -17\checkmark$

Note that we could have made the left side the variable side instead of the right side, but it would have led to a negative coefficient on the variable term. While we could work with the negative, there is less chance of error when working with positives. The strategy outlined above helps avoid the negatives!

TRY IT 8.1

Solve: $2a - 2 = 6a + 18$.

Show answer
$a = -5$

TRY IT 8.2

Solve: $4k - 1 = 7k + 17$.

Show answer
$k = -6$

To solve an equation with fractions, we still follow the same steps to get the solution.

EXAMPLE 9

Solve: $\frac{3}{2}x+5=\frac{1}{2}x-3$.

Solution

Since $\frac{3}{2}>\frac{1}{2}$, make the left side the variable side and the right side the constant side.

	$\frac{3}{2}x+5=\frac{1}{2}x-3$
Subtract $\frac{1}{2}x$ from both sides.	$\frac{3}{2}x-\frac{1}{2}x+5=\frac{1}{2}x-\frac{1}{2}x-3$
Combine like terms.	$x+5=-3$
Subtract 5 from both sides.	$x+5-5=-3-5$
Simplify.	$x=-8$
Check: Let $x=-8$.	$\frac{3}{2}x+5=\frac{1}{2}x-3$ $\frac{3}{2}(-8)+5\stackrel{?}{=}\frac{1}{2}(-8)-3$ $-12+5\stackrel{?}{=}-4-3$ $-7=-7\checkmark$

TRY IT 9.1

Solve: $\frac{7}{8}x-12=-\frac{1}{8}x-2$.

Show answer

$x = 10$

TRY IT 9.2

Solve: $\frac{7}{6}y+11=\frac{1}{6}y+8$.

Show answer

$y = -3$

We follow the same steps when the equation has decimals, too.

EXAMPLE 10

Solve: $3.4x + 4 = 1.6x - 5$.

Solution

Since $3.4 > 1.6$, make the left side the variable side and the right side the constant side.

	$3.4x + 4 = 1.6x - 5$
Subtract $1.6x$ from both sides.	$3.4x - 1.6x + 4 = 1.6x - 1.6x - 5$
Combine like terms.	$1.8x + 4 = -5$
Subtract 4 from both sides.	$1.8x + 4 - 4 = -5 - 4$
Simplify.	$1.8x = -9$
Use the Division Property of Equality.	$\frac{1.8x}{1.8} = \frac{-9}{1.8}$
Simplify.	$x = -5$
Check: Let $x = -5$.	$3.4x + 4 = 1.6x - 5$ $3.4(-5) + 4 \stackrel{?}{=} 1.6(-5) - 5$ $-17 + 4 \stackrel{?}{=} -8 - 5$ $-13 = -13✓$

TRY IT 10.1

Solve: $2.8x + 12 = -1.4x - 9$.

Show answer
$x = -5$

TRY IT 10.2

Solve: $3.6y + 8 = 1.2y - 4$.

Show answer
$y = -5$

Solve Equations Using a General Strategy

Each of the first few sections of this chapter has dealt with solving one specific form of a linear equation. It's time now to lay out an overall strategy that can be used to solve *any* linear equation. We call this the *general strategy*. Some equations won't require all the steps to solve, but many will. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

HOW TO: Use a General Strategy for Solving Linear Equations

1. Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms.
2. Collect all the variable terms to one side of the equation. Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term to equal to 1. Use the Multiplication or Division Property of Equality. State the solution to the equation.
5. Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

EXAMPLE 11

Solve: $3\left(x+2\right)=18$.

Solution

	$3(x+2)=18$
Simplify each side of the equation as much as possible. Use the Distributive Property.	$3\cdot x+3\cdot 2=18$ $3x+6=18$
Collect all variable terms on one side of the equation—all xs are already on the left side.	
Collect constant terms on the other side of the equation. Subtract 6 from each side	$3x+6-6=18-6$
Simplify.	$3x=12$
Make the coefficient of the variable term equal to 1. Divide each side by 3.	$\frac{3x}{3}=\frac{12}{3}$
Simplify.	$x=4$
Check: Let $x=4$.	$3(x+2)=18$ $3(4+2)\stackrel{?}{=}18$ $3(6)\stackrel{?}{=}18$ $18=18\checkmark$

TRY IT 11.1

Solve: $5\left(x+3\right)=35$.

Show answer
$x = 4$

TRY IT 11.2

Solve: $6\left(y-4\right)=-18$.

Show answer
$y = 1$

EXAMPLE 12

Solve: $-\left(x+5\right)=7$.

Solution

	$-(x+5)=7$
Simplify each side of the equation as much as possible by distributing. The only x term is on the left side, so all variable terms are on the left side of the equation.	$(-1)(x)+(-1)(5)=7$ $-x-5=7$
Add 5 to both sides to get all constant terms on the right side of the equation.	$-x-5+5=7+5$
Simplify.	$-x=12$
Make the coefficient of the variable term equal to 1 by multiplying (or dividing) both sides by -1.	$(-1)(-x)=(-1)(12)$ OR $\frac{-x}{-1}=\frac{12}{-1}$
Simplify.	$x=-12$
Check: Let $x=-12$.	$-(x+5)=7$ $-(-12+5)\stackrel{?}{=}7$ $-(-7)\stackrel{?}{=}7$ $7=7\checkmark$

TRY IT 12.1

Solve: $-(y+8)=-2$.

Show answer
$y=-6$

TRY IT 12.2

Solve: $-(z+4)=-12$.

Show answer
$z=8$

EXAMPLE 13

Solve: $4(x-2)+5=-3$.

Solution

	$4(x-2)+5=-3$
Simplify each side of the equation as much as possible. Distribute.	$4\cdot x-4\cdot 2+5=-3$ $4x-8+5=-3$
Combine like terms	$4x-3=-3$
The only x is on the left side, so all variable terms are on one side of the equation.	
Add 3 to both sides to get all constant terms on the other side of the equation.	$4x-3+3=-3+3$
Simplify.	$4x=0$
Make the coefficient of the variable term equal to 1 by dividing both sides by 4.	$\frac{4x}{4}=\frac{0}{4}$
Simplify.	$x=0$
Check: Let $x=0$.	$4(x-2)+5=-3$ $4(0-2)+5\stackrel{?}{=}-3$ $4(-2)+5\stackrel{?}{=}-3$ $-8+5\stackrel{?}{=}-3$ $-3=-3\checkmark$

TRY IT 13.1

Solve: $2\left(a-4\right)+3=-1$.

Show answer
a = 2

TRY IT 13.2

Solve: $7\left(n-3\right)-8=-15$.

Show answer
n = 2

EXAMPLE 14

Solve: $8-2\left(3y+5\right)=0$.

Solution

Be careful when distributing the negative.

	$8-2(3y+5)=0$
Simplify—use the Distributive Property.	$8-2\cdot 3y-2\cdot 5=0$ $8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by −6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$
Check: Let $y=-\frac{1}{3}$.	$8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]\stackrel{?}{=}0$ $8-2(-1+5)\stackrel{?}{=}0$ $8-2(4)\stackrel{?}{=}0$ $8-8\stackrel{?}{=}0$ $0=0\checkmark$

TRY IT 14.1

Solve: $12-3\,(4j+3)=-17$.

Show answer
$j=\frac{5}{3}$

TRY IT 14.2

Solve: $-6-8\,(k-2)=-10$.

Show answer
$k=\frac{5}{2}$

EXAMPLE 15

Solve: $3\left(x-2\right)-5=4\left(2x+1\right)+5$.

Solution

	$3(x-2)-5=4(2x+1)+5$
Distribute.	$3\cdot x-3\cdot 2-5=4\cdot 2x+4\cdot 1+5$ $3x-6-5=8x+4+5$
Combine like terms.	$3x-11=8x+9$
Subtract $3x$ to get all the variables on the right since $8>3$.	$3x-3x-11=8x-3x+9$
Simplify.	$-11=5x+9$
Subtract 9 to get the constants on the left.	$-11-9=5x+9-9$
Simplify.	$-20=5x$
Divide by 5.	$\frac{-20}{5}=\frac{5x}{5}$
Simplify.	$-4=x$
Check: Substitute: $-4=x$.	$3(x-2)-5=4(2x+1)+5$ $3(-4-2)-5\stackrel{?}{=}4[2(-4)+1]+5$ $3(-6)-5\stackrel{?}{=}4(-8+1)+5$ $-18-5\stackrel{?}{=}4(-7)+5$ $-23\stackrel{?}{=}-28+5$ $-23=-23\checkmark$

TRY IT 15.1

Solve: $6\left(p-3\right)-7=5\left(4p+3\right)-12$.

Show answer

$p=-2$

TRY IT 15.2

Solve: $8\left(q+1\right)-5=3\left(2q-4\right)-1.$

Show answer
$q=-8$

EXAMPLE 16

Solve: $\frac{1}{2}\left(6x-2\right)=5-x.$

Solution

	$\frac{1}{2}(6x-2)=5-x$
Distribute.	$\frac{1}{2}\cdot 6x-\frac{1}{2}\cdot 2=5-x$ $3x-1=5-x$
Add x to get all the variables on the left.	$3x-1+x=5-x+x$
Simplify.	$4x-1=5$
Add 1 to get constants on the right.	$4x-1+1=5+1$
Simplify.	$4x=6$
Divide by 4.	$\frac{4x}{4}=\frac{6}{4}$
Simplify.	$x=\frac{3}{2}$
Check: Let $x=\frac{3}{2}$.	$\frac{1}{2}(6x-2)=5-x$ $\frac{1}{2}\left(6\cdot\frac{3}{2}-2\right)\stackrel{?}{=}5-\frac{3}{2}$ $\frac{1}{2}(9-2)\stackrel{?}{=}\frac{10}{2}-\frac{3}{2}$ $\frac{1}{2}(7)\stackrel{?}{=}\frac{7}{2}$ $\frac{7}{2}=\frac{7}{2}\checkmark$

TRY IT 16.1

Solve: $\frac{1}{3}\left(6u+3\right)=7-u.$

Show answer
$u=2$

TRY IT 16.2

Solve: $\frac{2}{3}\left(9x-12\right)=8+2x$.

Show answer
$x = 4$

In many applications, we will have to solve equations with decimals. The same general strategy will work for these equations.

EXAMPLE 17

Solve: $0.24\left(100x+5\right)=0.4\left(30x+15\right)$.

Solution

	$0.24(100x+5)=0.4(30x+15)$
Distribute.	$0.24\cdot 100x+0.24\cdot 5=0.4\cdot 30x+0.4\cdot 15$ $24x+1.2=12x+6$
Subtract $12x$ to get all the xs to the left.	$24x+1.2-12x=12x-6-12x$
Simplify.	$12x+1.2=6$
Subtract 1.2 to get the constants to the right.	$12x+1.2-1.2=6-1.2$
Simplify.	$12x=4.8$
Divide.	$\frac{12x}{12}=\frac{4.8}{12}$
Simplify.	$x=0.4$
Check: Let $x=0.4$.	$0.24(100x+5)=0.4(30x+15)$ $0.24(100x(0.4)+5)\stackrel{?}{=}0.4(30(0.4)+15)$ $0.24(40+5)\stackrel{?}{=}0.4(12+15)$ $0.24(45)\stackrel{?}{=}0.4(27)$ $10.8=10.8\checkmark$

TRY IT 17.1

Solve: $0.55\left(100n+8\right)=0.6\left(85n+14\right)$.

Show answer
1

TRY IT 17.2

Solve: $0.15\,(40m - 120) = 0.5\,(60m + 12)$.

Show answer
−1

Key Concepts

- **Solve an equation with variables and constants on both sides**
    1. Choose one side to be the variable side and then the other will be the constant side.
    2. Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.
    3. Collect the constants to the other side, using the Addition or Subtraction Property of Equality.
    4. Make the coefficient of the variable 1, using the Multiplication or Division Property of Equality.
    5. Check the solution by substituting into the original equation.
- **General strategy for solving linear equations**
    1. Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms.
    2. Collect all the variable terms to one side of the equation. Use the Addition or Subtraction Property of Equality.
    3. Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.
    4. Make the coefficient of the variable term to equal to 1. Use the Multiplication or Division Property of Equality. State the solution to the equation.
    5. Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

Practice Makes Perfect

Solve an Equation with Constants on Both Sides

In the following exercises, solve the equation for the variable.

1. $7x - 8 = 34$	2. $6x - 2 = 40$
3. $14y + 7 = 91$	4. $11w + 6 = 93$
5. $4m + 9 = -23$	6. $3a + 8 = -46$
7. $-47 = 6b + 1$	8. $-50 = 7n - 1$
9. $29 = -8x - 3$	10. $25 = -9y + 7$
11. $-14\text{q} - 15 = 13$	12. $-12p - 3 = 15$

Solve an Equation with Variables on Both Sides

In the following exercises, solve the equation for the variable.

13. $9k = 8k - 11$	14. $8z = 7z - 7$
15. $6x + 27 = 9x$	16. $4x + 36 = 10x$
17. $b = -4b - 15$	18. $c = -3c - 20$
19. $7z = 39 - 6z$	20. $5q = 44 - 6q$
21. $8x + \frac{3}{4} = 7x$	22. $3y + \frac{1}{2} = 2y$
23. $-15r - 8 = -11r$	24. $-12a - 8 = -16a$

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the equations for the variable.

25. $4x - 17 = 3x + 2$	26. $6x - 15 = 5x + 3$
27. $21 + 6f = 7f + 14$	28. $26 + 8d = 9d + 11$
29. $8q - 5 = 5q - 20$	30. $3p - 1 = 5p - 33$
31. $9c + 7 = -2c - 37$	32. $4a + 5 = -a - 40$
33. $12x - 17 = -3x + 13$	34. $8y - 30 = -2y + 30$
35. $3y - 4 = 12 - y$	36. $2\text{z} - 4 = 23 - \text{z}$
37. $\frac{4}{3}m - 7 = \frac{1}{3}m - 13$	38. $\frac{5}{4}c - 3 = \frac{1}{4}c - 16$
39. $11 - \frac{1}{4}a = \frac{3}{4}a + 4$	40. $8 - \frac{2}{5}q = \frac{3}{5}q + 6$
41. $\frac{5}{4}a + 15 = \frac{3}{4}a - 5$	42. $\frac{4}{3}n + 9 = \frac{1}{3}n - 9$
43. $\frac{3}{5}p + 2 = \frac{4}{5}p - 1$	44. $\frac{1}{4}y + 7 = \frac{3}{4}y - 3$
45. $13z + 6.45 = 8z + 23.75$	46. $14n + 8.25 = 9n + 19.60$
47. $2.7w - 80 = 1.2w + 10$	48. $2.4w - 100 = 0.8w + 28$
49. $6.6x - 18.9 = 3.4x + 54.7$	50. $5.6r + 13.1 = 3.5r + 57.2$

Solve an Equation Using the General Strategy

In the following exercises, solve the linear equation using the general strategy.

51. $4\left(y+7\right)=64$	52. $5\left(x+3\right)=75$
53. $9=3\left(x-3\right)$	54. $8=4\left(x-3\right)$
55. $14\left(y-6\right)=-42$	56. $20\left(y-8\right)=-60$
57. $-7\left(3n+4\right)=14$	58. $-4\left(2n+1\right)=16$
59. $8\left(3+3\text{p}\right)=0$	60. $3\left(10+5r\right)=0$
61. $\frac{3}{5}\left(10x-5\right)=27$	62. $\frac{2}{3}\left(9c-3\right)=22$
63. $4\left(2.5v-0.6\right)=7.6$	64. $5\left(1.2u-4.8\right)=-12$
65. $0.5\left(16m+34\right)=-15$	66. $0.2\left(30n+50\right)=28$
67. $-\left(t-8\right)=17$	68. $-\left(w-6\right)=24$
69. $8\left(6b-7\right)+23=63$	70. $9\left(3a+5\right)+9=54$
71. $13+2\left(m-4\right)=17$	72. $10+3\left(z+4\right)=19$
73. $-9+6\left(5-k\right)=12$	74. $7+5\left(4-q\right)=12$
75. $18-\left(9r+7\right)=-16$	76. $15-\left(3r+8\right)=28$
77. $18-2\left(y-3\right)=32$	78. $11-4\left(y-8\right)=43$
79. $3\left(4n-1\right)-2=8n+3$	80. $9\left(p-1\right)=6\left(2p-1\right)$
81. $5\left(x-4\right)-4x=14$	82. $9\left(2m-3\right)-8=4m+7$
83. $5+6\left(3s-5\right)=-3+2\left(8s-1\right)$	84. $8\left(x-4\right)-7x=14$
85. $4\left(x-1\right)-8=6\left(3x-2\right)-7$	86. $-12+8\left(x-5\right)=-4+3\left(5x-2\right)$

Everyday Math

Making a fence 87. Jovani has a fence around the rectangular garden in his backyard. The perimeter of the fence is 150 feet. The length is 15 feet more than the width. Find the width, w, by solving the equation $150=2\left(w+15\right)+2w$.	**Concert tickets** 88. At a school concert, the total value of tickets sold was $\$1{,}506$. Student tickets sold for $\$6$ and adult tickets sold for $\$9$. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, s, by solving the equation $6s+9\left(3s-5\right)=1506$.
Coins 89. Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n+0.10\left(2n-1\right)=1.90$.	**Fencing** 90. Micah has 74 feet of fencing to make a rectangular dog pen in his yard. He wants the length to be 25 feet more than the width. Find the length, L, by solving the equation $2L+2\left(L-25\right)=74$.

Writing Exercises

91. When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient as the variable side?	92. Solve the equation $10x + 14 = -2x + 38$, explaining all the steps of your solution.
93. What is the first step you take when solving the equation $3 - 7\left(y - 4\right) = 38$? Explain why this is your first step.	94. Solve the equation $\frac{1}{4}\left(8x + 20\right) = 3x - 4$ explaining all the steps of your solution as in the examples in this section.
95. Using your own words, list the steps in the General Strategy for Solving Linear Equations.	96. Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Answers

1.	6	3.	6	5.	-8	7.	-8
9.	-4	11.	-2	13.	-11	15.	9
17.	-3	19.	3	21.	$-\frac{3}{4}$	23.	-2
25.	19	27.	7	29.	-5	31.	-4
33.	2	35.	4	37.	-6	39.	7
41.	-40	43.	15	45.	3.46	47.	60
49.	23	51.	9	53.	6	55.	3
57.	-2	59.	-1	61.	5	63.	1
65.	-4	67.	-9	69.	2	71.	6
73.	$\frac{3}{2}$	75.	3	77.	-4	79.	2
81.	34	83.	10	85.	$\frac{1}{2}$	87.	30 feet
89.	8 nickels	91.	Answers will vary.	93.	Answers will vary.	95.	Answers will vary.

Attributions

3.4 Solve Equations with Fraction or Decimal Coefficients

Learning Objectives

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Solve Equations with Fraction Coefficients

Let's use the General Strategy for Solving Linear Equations introduced earlier to solve the equation $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$.

Given equation.	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x+\frac{1}{2}-\frac{1}{2}=\frac{1}{4}-\frac{1}{2}$
Simplify the left side.	$\frac{1}{8}x=\frac{1}{4}-\frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x=\frac{1}{4}-\frac{2}{4}$
Subtract. $\frac{1}{8}x=-\frac{1}{4}$	
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1}\cdot\frac{1}{8}x=\frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x=-2$

This method worked fine, but many students don't feel very confident when they see all those fractions. So we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of *all* the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but with no fractions. This process is called *clearing the equation* *of fractions*. Let's solve the same equation again, but this time use the method that clears the fractions.

EXAMPLE 1

Solve: $\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$.

Solution

Find the least common denominator of *all* the fractions in the equation.	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4} \quad \text{LCD}=8$
Multiply both sides of the equation by that LCD, 8. This clears the fractions.	$8(\frac{1}{8}x+\frac{1}{2})=8(\frac{1}{4})$
Use the Distributive Property.	$8\cdot\frac{1}{8}x+8\cdot\frac{1}{2}=8\cdot\frac{1}{4}$
Simplify — and notice, no more fractions!	$x+4=2$
Solve using the General Strategy for Solving Linear Equations.	$x+4-4=2-4$
Simplify.	$x=-2$
Check: Let $x=-2$	$\frac{1}{8}x+\frac{1}{2}=\frac{1}{4}$ $\frac{1}{8}(-2)+\frac{1}{2}\stackrel{?}{=}\frac{1}{4}$ $-\frac{2}{8}+\frac{1}{2}\stackrel{?}{=}\frac{1}{4}$ $-\frac{2}{8}+\frac{4}{8}\stackrel{?}{=}\frac{1}{4}$ $\frac{2}{4}\stackrel{?}{=}\frac{1}{4}$ $\frac{1}{4}=\frac{1}{4}\checkmark$

TRY IT 1.1

Solve: $\frac{1}{4}x+\frac{1}{2}=\frac{5}{8}$.

Show answer

$x=\frac{1}{2}$

TRY IT 1.2

Solve: $\frac{1}{6}y - \frac{1}{3} = \frac{1}{6}$.

Show answer
y = 3

Notice in (Figure) that once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

HOW TO: Solve Equations with Fraction Coefficients by Clearing the Fractions

1. Find the least common denominator of *all* the fractions in the equation.
2. Multiply both sides of the equation by that LCD. This clears the fractions.
3. Solve using the General Strategy for Solving Linear Equations.

EXAMPLE 2

Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the least common denominator of *all* the fractions in the equation.	$7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x, \quad \text{LCD} = 12$
Multiply both sides of the equation by 12.	$12(7) = 12\left(\frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x\right)$
Distribute.	$12 \cdot 7 = 12 \cdot \frac{1}{2}x + 12 \cdot \frac{3}{4}x - 12 \cdot \frac{2}{3}x$
Simplify — and notice, no more fractions!	$84 = 6x + 9x - 8x$
Combine like terms.	$84 = 7x$
Divide by 7.	$\frac{84}{7} = \frac{7x}{7}$

Simplify.	$12 = x$
Check: Let $x = 12$.	$7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$ $7 \stackrel{?}{=} \frac{1}{2}(12) + \frac{3}{4}(12) - \frac{2}{3}(12)$ $7 \stackrel{?}{=} 6 + 9 - 8$ $7 = 7\checkmark$

TRY IT 2.1

Solve: $6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$.

Show answer

$v = 40$

TRY IT 2.2

Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

Show answer

$u = -12$

In the next example, we'll have variables and fractions on both sides of the equation.

EXAMPLE 3

Solve: $x + \frac{1}{3} = \frac{1}{6}x - \frac{1}{2}$.

Solution

Find the LCD of all the fractions in the equation.	$x + \frac{1}{3} = \frac{1}{6}x - \frac{1}{2}, \quad \text{LCD} = 6$
Multiply both sides by the LCD.	$6\left(x + \frac{1}{3}\right) = 6\left(\frac{1}{6}x - \frac{1}{2}\right)$
Distribute.	$6 \cdot x + 6 \cdot \frac{1}{3} = 6 \cdot \frac{1}{6}x - 6 \cdot \frac{1}{2}$
Simplify — no more fractions!	$6x + 2 = x - 3$

Subtract x from both sides.	$6x - x + 2 = x - x - 3$
Simplify.	$5x + 2 = -3$
Subtract 2 from both sides.	$5x + 2 - 2 = -3 - 2$
Simplify.	$5x = -5$
Divide by 5.	$\frac{5x}{5} = \frac{-5}{5}$
Simplify.	$x = -1$
Check: Substitute $x = -1$.	$x + \frac{1}{3} = \frac{1}{6}x - \frac{1}{2}$ $(-1) + \frac{1}{3} \stackrel{?}{=} \frac{1}{6}(-1) - \frac{1}{2}$ $-\frac{3}{3} + \frac{1}{3} \stackrel{?}{=} -\frac{1}{6} - \frac{3}{6}$ $-\frac{2}{3} \stackrel{?}{=} -\frac{4}{6}$ $-\frac{2}{3} = -\frac{2}{3} \checkmark$

TRY IT 3.1

Solve: $a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$.

Show answer

$a = -2$

TRY IT 3.2

Solve: $c + \frac{3}{4} = \frac{1}{2}c - \frac{1}{4}$.

Show answer

$c = -2$

In (Figure), we'll start by using the Distributive Property. This step will clear the fractions right away!

EXAMPLE 4

Solve: $1=\frac{1}{2}\left(4x+2\right)$.

Solution

Given equation.	$1=\frac{1}{2}(4x+2)$
Distribute. $1=\frac{1}{2}\cdot 4x+\frac{1}{2}\cdot 2$	
Simplify. Now there are no fractions to clear!	$1=2x+1$
Subtract 1 from both sides.	$1-1=2x+1-1$
Simplify. $0=2x$	
Divide by 2.	$\frac{0}{2}=\frac{2x}{2}$
Simplify. $0=x$	

Check: Let $x=0$.

$$1=\frac{1}{2}(4x+2)$$

$$1\stackrel{?}{=}\frac{1}{2}(4(0)+2)$$

$$1\stackrel{?}{=}\frac{1}{2}(2)$$

$$1\stackrel{?}{=}\frac{2}{2}$$

$$1=1\checkmark$$

TRY IT 4.1

Solve: $-11=\frac{1}{2}\left(6p+2\right)$.

Show answer

$p = -4$

TRY IT 4.2

Solve: $8 = \frac{1}{3}\left(9q + 6\right)$.

Show answer
$q = 2$

Many times, there will still be fractions, even after distributing.

EXAMPLE 5

Solve: $\frac{1}{2}\left(y - 5\right) = \frac{1}{4}\left(y - 1\right)$.

Solution

Given equation.	$\frac{1}{2}(y-5) = \frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$
Simplify.	$\frac{1}{2}y - \frac{5}{2} = \frac{1}{4}y - \frac{1}{4}$
Multiply by the LCD, 4.	$4\left(\frac{1}{2}y - \frac{5}{2}\right) = 4\left(\frac{1}{4}y - \frac{1}{4}\right)$
Distribute.	$4 \cdot \frac{1}{2}y - 4 \cdot \frac{5}{2} = 4 \cdot \frac{1}{4} - 4 \cdot \frac{1}{4}$
Simplify.	$2y - 10 = y - 1$
Collect the y terms to the left.	$2y - 10 - y = y - 1 - y$
Simplify.	$y - 10 = -1$
Collect the constants to the right.	$y - 10 + 10 = -1 + 10$
Simplify.	$y = 9$
Check: Substitute 9 for y.	

$$\begin{aligned} \frac{1}{2}(y-5) &= \frac{1}{4}(y-1) \\ \frac{1}{2}({\color{red}9}-5) &\stackrel{?}{=} \frac{1}{4}({\color{red}9}-1) \\ \frac{1}{2}(4) &\stackrel{?}{=} \frac{1}{4}(8) \\ 2 &= 2\checkmark \end{aligned}$$

TRY IT 5.1

Solve: $\frac{1}{5}(n+3) = \frac{1}{4}(n+2)$.

Show answer
n = 2

TRY IT 5.2

Solve: $\frac{1}{2}(m-3) = \frac{1}{4}(m-7)$.

Show answer
m = −1

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money and percent. But decimals are really another way to represent fractions. For example, $0.3 = \frac{3}{10}$ and $0.17 = \frac{17}{100}$. So, when we have an equation with decimals, we can use the same process we used to clear fractions—multiply both sides of the equation by the least common denominator.

EXAMPLE 6

Solve: $0.8x - 5 = 7$.

Solution

The only decimal in the equation is 0.8. Since $0.8 = \frac{8}{10}$, the LCD is 10. We can multiply both sides by 10 to clear the decimal.

Given equation.	$0.8x - 5 = 7, \quad {\color{blue}LCD = 10}$

Multiply both sides by the LCD.	$10(0.8x-5)=10(7)$
Distribute.	$10\cdot(0.8x)-10\cdot(5)=10(7)$
Multiply, and notice, no more decimals!	$8x-50=70$
Add 50 to get all constants to the right.	$8x-50+50=70+50$
Simplify.	$8x=120$
Divide both sides by 8.	$\frac{8x}{8}=\frac{120}{8}$
Simplify.	$x=15$
Check: Let $x=15$.	$0.8(15)-5\overset{?}{=}7$ $12-5\overset{?}{=}7$ $7=7\checkmark$

TRY IT 6.1

Solve: $0.6x-1=11$.

Show answer
$x = 20$

TRY IT 6.2

Solve: $1.2x-3=9$.

Show answer
$x = 10$

EXAMPLE 7

Solve: $0.06x+0.02=0.25x-1.5$.

Solution

Look at the decimals and think of the equivalent fractions.

$0.06 = \frac{6}{100}, \quad 0.02 = \frac{2}{100}, \quad 0.25 = \frac{25}{100}, \quad 1.5 = 1\frac{5}{10}$

Notice, the LCD is 100.

By multiplying by the LCD we will clear the decimals.

Given equation.
$0.06x + 0.02 = 0.25x - 1.5$

Multiply both sides by 100.
$100(0.06x + 0.02) = 100(0.25x - 1.5)$

Distribute.
$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$

Multiply, and now no more decimals. $6x + 2 = 25x - 150$

Collect the variables to the right.
$6x - 6x + 2 = 25x - 6x - 150$

Simplify. $2 = 19x - 150$

Collect the constants to the left.
$2 + 150 = 19x - 150 + 150$

Simplify.
$152 = 19x$

Divide by 19. $\frac{152}{19} = \frac{19x}{19}$

Simplify.
$8 = x$

Check: Let $x = 8$.
$0.06(8) + 0.02 = 0.25(8) - 1.5$
$0.48 + 0.02 = 2.00 - 1.5$
$0.50 = 0.50\checkmark$

TRY IT 7.1

Solve: $0.14h + 0.12 = 0.35h - 2.4$.

Show answer
$h = 12$

TRY IT 7.2

Solve: $0.65k - 0.1 = 0.4k - 0.35$.

Show answer
$k = -1$

The next example uses an equation that is typical of the ones we will see in the money applications in the next chapter. Notice that we will distribute the decimal first before we clear all decimals in the equation.

EXAMPLE 8

Solve: $0.25x + 0.05\,(x+3) = 2.85$.

Solution

Given equation.	$0.25x + 0.05(x+3) = 2.85$
Distribute first.	$0.25x + 0.05 \cdot x + 0.05 \cdot 3 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute and collect like terms.	$100 \cdot 0.30x + 100 \cdot 0.15 = 100(2.85)$ $30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$
Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$

Check: Let $x=9$.

$$0.25x+0.05(x+3)$$
$$=2.85$$
$$0.25(9)+0.05(9+3)$$
$$\stackrel{?}{=}2.85$$
$$2.25+0.05(12)$$
$$\stackrel{?}{=}2.85$$
$$2.25+0.60$$
$$\stackrel{?}{=}2.85$$
$$2.85$$
$$=2.85\checkmark$$

TRY IT 8.1

Solve: $0.25n+0.05\left(n+5\right)=2.95$.

Show answer
n = 9

TRY IT 8.2

Solve: $0.10d+0.05\left(d-5\right)=2.15$.

Show answer
d = 16

Key Concepts

- **Solve equations with fraction coefficients by clearing the fractions.**
  1. Find the least common denominator of *all* the fractions in the equation.
  2. Multiply both sides of the equation by that LCD. This clears the fractions.
  3. Solve using the General Strategy for Solving Linear Equations.

Practices Makes Perfect

Solve equations with fraction coefficients

In the following exercises, solve the equation by clearing the fractions.

1. $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$	2. $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$
3. $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$	4. $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$
5. $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$	6. $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$
7. $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$	8. $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$
9. $\frac{1}{4}m - \frac{4}{5}m + \frac{1}{2}m = -1$	10. $\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$
11. $x + \frac{1}{2} = \frac{2}{3}x - \frac{1}{2}$	12. $x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$
13. $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$	14. $\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$
15. $\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$	16. $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$
17. $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$	18. $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$
19. $1 = \frac{1}{6}(12x - 6)$	20. $1 = \frac{1}{5}(15x - 10)$
21. $\frac{1}{4}(p - 7) = \frac{1}{3}(p + 5)$	22. $\frac{1}{5}(q + 3) = \frac{1}{2}(q - 3)$
23. $\frac{1}{2}(x + 4) = \frac{3}{4}$	24. $\frac{1}{3}(x + 5) = \frac{5}{6}$

Solve Equations with Decimal Coefficients

In the following exercises, solve the equation by clearing the decimals.

25. $0.6y+3=9$	26. $0.4y-4=2$
27. $3.6j-2=5.2$	28. $2.1k+3=7.2$
29. $0.4x+0.6=0.5x-1.2$	30. $0.7x+0.4=0.6x+2.4$
31. $0.23x+1.47=0.37x-1.05$	32. $0.48x+1.56=0.58x-0.64$
33. $0.9x-1.25=0.75x+1.75$	34. $1.2x-0.91=0.8x+2.29$
35. $0.05n+0.10\left(n+8\right)=2.15$	36. $0.05n+0.10\left(n+7\right)=3.55$
37. $0.10d+0.25\left(d+5\right)=4.05$	38. $0.10d+0.25\left(d+7\right)=5.25$
39. $0.05\left(q-5\right)+0.25q=3.05$	40. $0.05\left(q-8\right)+0.25q=4.10$

Everyday Math

Coins 41. Taylor has $\$2.00$ in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d+0.01\left(d+2\right)=2$ for d, the number of dimes.	**Stamps 42.** Travis bought $\$9.45$ worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation $0.49s+0.21\left(s-5\right)=9.45$ for s, to find the number of 49-cent stamps Travis bought.

Answers

1. $x=-1$	3. $y=-1$	5. $a=\frac{3}{4}$
7. $x=4$	9. $m=20$	11. $x=-3$
13. $w=\frac{9}{4}$	15. $x=1$	17. $b=12$
19. $x=1$	21. $p=-41$	23. $x=-\frac{5}{2}$
25. $y=10$	27. $j=2$	29. $x=18$
31. $x=18$	33. $x=20$	35. $n=9$
37. $d=8$	39. $q=11$	41. $d=18$

Attributions

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3.5 Use a General Strategy to Solve Linear Equations

Learning Objectives

By the end of this section, you will be able to:

- Solve equations using a general strategy
- Classify equations

Solve Equations Using the General Strategy

Until now we have dealt with solving one specific form of a linear equation. It is time now to lay out one overall strategy that can be used to solve any linear equation. Some equations we solve will not require all these steps to solve, but many will.

Beginning by simplifying each side of the equation makes the remaining steps easier.

EXAMPLE 1. How to Solve Linear Equations Using the General Strategy

Solve: $-6\left(x+3\right)=24$.

Solution

Step 1. Simplify each side of the equation as much as possible.	Use the Distributive Property. Notice that each side of the equation is simplified as much as possible.	$-6(x+3)=24$ $-6\cdot x-6\cdot 3=24$ $-6x-18=24$
Step 2. Collect all variable terms on one side of the equation.	Nothing to do as all the x's are on the left side.	
Step 3. Collect constant terms on the other side of the equation.	To get constants only on the right side, add 18 to each side. Simplify.	$-6x-18+18=24+18$ $-6x=42$
Step 4. Make the coefficient of the variable term to equal to 1.	Divide each side by -6. Simplify	$\frac{-6x}{-6}=\frac{42}{-6}$ $x=-7$
Step 5. Check the solution.	Let $x=-7$	$-6(x+3)=24$ $-6(-7+3)\stackrel{?}{=}24$ $-6(-4)\stackrel{?}{=}24$ $24=24$✓

TRY IT 1.1

Solve: $5\left(x+3\right)=35.$

Show answer
$x=4$

TRY IT 1.2

Solve: $6\left(y-4\right)=-18.$

Show answer
$y=1$

General strategy for solving linear equations.

1. **Simplify each side of the equation as much as possible.**

Use the Distributive Property to remove any parentheses.
Combine like terms.

2. **Collect all the variable terms on one side of the equation.**
Use the Addition or Subtraction Property of Equality.
3. **Collect all the constant terms on the other side of the equation.**
Use the Addition or Subtraction Property of Equality.
4. **Make the coefficient of the variable term to equal to 1.**
Use the Multiplication or Division Property of Equality.
State the solution to the equation.
5. **Check the solution.** Substitute the solution into the original equation to make sure the result is a true statement.

EXAMPLE 2

Solve: $-\,(y+9)=8$.

Solution

Given equation.
$-(y+9)=8$

Simplify each side of the equation by distributing.	$-y-9=8$
The only y term is on the left side, so all variable terms are on the left side of the equation.	
Add 9 to both sides to get all constant terms on the right side of the equation.	$-y-9+9=8+9$
Simplify.	$-y=17$
Rewrite $-y$ as $-1y$.	$-1y=17$
Make the coefficient of the variable term to equal to 1 by dividing both sides by -1.	$\frac{-1y}{-1}=\frac{17}{-1}$
Simplify.	$y=-17$

Check:
Let $y=-17$.

$$-(y+9)=8$$
$$-(-17+9)\stackrel{?}{=}8$$
$$-(-8)\stackrel{?}{=}8$$
$$8=8✓$$

TRY IT 2.1

Solve: $-(y+8)=-2$.

Show answer

$y=-6$

TRY IT 2.2

Solve: $-(z+4)=-12$.

Show answer

$z=8$

EXAMPLE 3

Solve: $5(a-3)+5=-10$.

Solution

Simplify each side of the equation as much as possible.	$5(a-3)+5=-10$
Distribute.	$5\cdot a-5\cdot 3+5=-10$ $5a-15+5=-10$
Combine like terms.	$5a-10=-10$
The only term containing a is on the left side, so all variable terms are on one side of the equation.	
Add 10 to both sides to get all constant terms on the other side of the equation.	$5a-10+10=-10+10$
Simplify.	$5a=0$

Make the coefficient of the variable term to equal to 1 by dividing both sides by 5.	$\frac{5a}{5} = \frac{0}{5}$
Simplify.	$a = 0$
Check: Let $a = 0$	$5(a-3)+5=-10$ $5(0-3)+5 \stackrel{?}{=} -10$ $5(-3)+5 \stackrel{?}{=} -10$ $-15+5 \stackrel{?}{=} -10$ $-10=-10\checkmark$

TRY IT 3.1

Solve: $2\left(m-4\right)+3=-1.$

Show answer

$m=2$

TRY IT 3.2

Solve: $7\left(n-3\right)-8=-15.$

Show answer

$n=2$

EXAMPLE 4

Solve: $\frac{2}{3}\left(6m-3\right)=8-m.$

Solution

Distribute.

Simplify each side of the equation.

$\frac{2}{3}(6m-3)=8-m$

$\frac{2}{3}\cdot 6m-\frac{2}{3}\cdot 3=8-m$

$4m-2=8-m$

Add m to get all the variables on the left side and simplify.	$4m+m-2=8-m+m$ $5m-2=8$
Add 2 to get constants only on the right and simplify.	$5m-2+2=8+2$ $5m=10$
Make the coefficient of the variable term to equal to 1 by dividing both sides by 5.	$\frac{5m}{5}=\frac{10}{5}$
Simplify.	$m=2$
Check: Let $m=2$	$\frac{2}{3}(6m-3)=8-m$ $\frac{2}{3}(6(2)-3)\stackrel{?}{=}8-2$ $\frac{2}{3}(12-3)\stackrel{?}{=}6$ $\frac{2}{3}(9)\stackrel{?}{=}6$ $6=6\checkmark$

TRY IT 4.1

Solve: $\frac{1}{3}\left(6u+3\right)=7-u$.

Show answer
$u=2$

TRY IT 4.2

Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$.

Show answer
$x = 4$

EXAMPLE 5

Solve: $8 - 2(3y + 5) = 0$.

Solution

Distribute Simplify each side of the equation.	$8 - 2(3y + 5) = 0$ $8 - 2 \cdot 3y - 2 \cdot 5 = 0$ $8 - 6y - 10 = 0$
Combine like terms.	$-6y - 2 = 0$
Add 2 to get all the variables on the left side and simplify.	$-6y - 2 + 2 = 0 + 2$ $-6y = 2$
Make the coefficient of the variable term to equal to 1 by dividing both sides by -6.	$\frac{-6y}{-6} = \frac{2}{-6}$
Simplify.	$y = \frac{-1}{3}$
Check: Let $y = \frac{-1}{3}$	$8 - 2(3y + 5) = 0$ $8 - 2\left[3\left(-\frac{1}{3}\right) + 5\right] = 0$ $8 - 2(-1 + 5) \stackrel{?}{=} 0$ $8 - 2(4) \stackrel{?}{=} 0$ $8 - 8 \stackrel{?}{=} 0$ $0 = 0 \checkmark$

TRY IT 5.1

Solve: $12 - 3\left(4j + 3\right) = -17$.

Show answer
$j = \frac{5}{3}$

TRY IT 5.2

Solve: $-6 - 8\left(k - 2\right) = -10$.

Show answer
$k = \frac{5}{2}$

EXAMPLE 6

Solve: $4\left(x - 1\right) - 2 = 5\left(2x + 3\right) + 6$.

Solution

Given equation.	$4(x-1)-2=5(2x+3)+6$
Distribute.	$4 \cdot x - 4 \cdot 1 - 2 = 5 \cdot 2x + 5 \cdot 3 + 6$
Combine like terms.	$4x - 6 = 10x + 21$
Subtract 4x to get the variables only on the right side since 10 > 4.	$4x - 4x - 6 = 10x - 4x + 21$
Simplify.	$-6 = 6x + 21$
Subtract 21 to get the constants on left and simplify.	$-6 - 21 = 6x + 21 - 21$ $-27 = 6x$
Divide by 6 and simplify.	$\frac{-27}{6} = \frac{6x}{6}$ $\frac{-9}{2} = x$
Check. Let $x = \frac{-9}{2}$	

$$4(x-1)-2=5(2x+3)+6$$
$$4\left(-\frac{9}{2}-1\right)-2\stackrel{?}{=}5\left[2\left(-\frac{9}{2}\right)+3\right]+6$$
$$4\left(-\frac{11}{2}\right)-2\stackrel{?}{=}5(-9+3)+6$$
$$-22-2\stackrel{?}{=}5(-6)+6$$
$$-24\stackrel{?}{=}-30+6$$
$$-24=-24\checkmark$$

TRY IT 6.1

Solve: $6\left(p-3\right)-7=5\left(4p+3\right)-12$.

Show answer
$p=-2$

TRY IT 6.2

Solve: $8\left(q+1\right)-5=3\left(2q-4\right)-1$.

Show answer
$q=-8$

EXAMPLE 7

Solve: $10\left[3-8\left(2s-5\right)\right]=15\left(40-5s\right)$.

Solution

Given equation.	$10[3-8(2s-5)]=15(40-5s)$
Distribute and simplify.	$10[3-8\cdot 2s+8\cdot 5]=15\cdot 40-15\cdot 5s$ $10[3-16s+40]=600-75s$ $10(43-16s)=600-75s$

Distribute and combine like terms.	$10 \cdot 43 - 10 \cdot 16s = 600 - 75s$ $430 - 160s = 600 - 75s$
Add $160s$ to get the s's to the right.	$430 - 160s + 160s = 600 - 75s + 160s$
Simplify.	$430 = 600 + 85s$
Subtract 600 to get the constants on left and simplify.	$430 - 600 = 600 + 85s - 600$ $-170 = 85s$
Divide by 85 and simplify.	$\frac{-170}{85} = \frac{85s}{85}$ $-2 = s$
Check: Substitute $s = -2$	$10[3 - 8(2s - 5)] = 15(40 - 5s)$ $10[3 - 8(2(-2) - 5)] \stackrel{?}{=} 15(40 - 5(-2))$ $10[3 - 8(-4 - 5)] \stackrel{?}{=} 15(40 + 10)$ $10[3 - 8(-9)] \stackrel{?}{=} 15(50)$ $10(3 + 72) \stackrel{?}{=} 750$ $10 \cdot 75 \stackrel{?}{=} 750$ $750 = 750 \checkmark$

TRY IT 7.1

Solve: $6\left[4 - 2\left(7y - 1\right)\right] = 8\left(13 - 8y\right)$.

Show answer

$y = -\frac{17}{5}$

TRY IT 7.2

Solve: $12\left[1 - 5\left(4z - 1\right)\right] = 3\left(24 + 11z\right)$.

Show answer

$z = 0$

EXAMPLE 8

Solve: $0.36\,(100n + 5) = 0.6\,(30n + 15)$.

Solution

Given equation.	$0.36(100n + 5) = 0.6(30n + 15)$
Distribute and simplify.	$0.36 \cdot 100n + 0.36 \cdot 5 = 0.6 \cdot 30n + 0.6 \cdot 15$ $36n + 1.8 = 18n + 9$
Subtract $18n$ to get the n's to the left.	$36n + 1.8n - 18n = 18n + 9 - 18n$
Simplify.	$18n + 1.8 = 9$
Subtract 1.8 to get the constants on right and simplify.	$18n + 1.8 - 1.8 = 9 - 1.8$ $18n = 7.2$
Divide by 18 and simplify.	$\frac{18n}{18} = \frac{7.2}{18}$ $n = 0.4$
Check: Substitute $n = 0.4$	$0.36(100n + 5) = 0.6(30n + 15)$ $0.36(100(0.4) + 5) \stackrel{?}{=} 0.6(30(0.4) + 15)$ $0.36(40 + 5) \stackrel{?}{=} 0.6(12 + 15)$ $0.36(45) \stackrel{?}{=} 0.6(27)$ $16.2 = 16.2\checkmark$

TRY IT 8.1

Solve: $0.55\,(100n + 8) = 0.6\,(85n + 14)$.

Show answer

$n = 1$

TRY IT 8.2

Solve: $0.15\,(40m - 120) = 0.5\,(60m + 12)$.

Show answer
$m = -1$

Classify Equations

Consider the equation we solved at the start of the last section, $7x + 8 = -13$. The solution we found was $x = -3$. This means the equation $7x + 8 = -13$ is true when we replace the variable, *x*, with the value -3. We showed this when we checked the solution $x = -3$ and evaluated $7x + 8 = -13$ for $x = -3$.

$$7(-3) + 8 \overset{?}{=} -13$$
$$-21 + 8 \overset{?}{=} -13$$
$$-13 = -13\checkmark$$

If we evaluate $7x + 8$ for a different value of *x*, the left side will not be -13.

The equation $7x + 8 = -13$ is true when we replace the variable, *x*, with the value -3, but not true when we replace *x* with any other value. Whether or not the equation $7x + 8 = -13$ is true depends on the value of the variable. Equations like this are called conditional equations.

All the equations we have solved so far are conditional equations.

Conditional equation

An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

Now let's consider the equation $2y + 6 = 2\,(y + 3)$. Do you recognize that the left side and the right side are equivalent? Let's see what happens when we solve for *y*.

	$2y + 6 = 2(y + 3)$
Distribute.	$2y + 6 = 2 \cdot y + 2 \cdot 3$ $2y + 6 = 2y + 6$
Subtract $2y$ to get the y's to one side.	$2y - 2y + 6 = 2y - 2y + 6$
Simplify—the y's are gone!	$6 = 6$

But $6 = 6$ is true.

This means that the equation $2y + 6 = 2\,(y + 3)$ is true for any value of *y*. We say the solution to the equation is all of the real numbers. An equation that is true for any value of the variable like this is called an identity.

Identity

An equation that is true for any value of the variable is called an **identity**.

The solution of an identity is every real number.

What happens when we solve the equation $5z = 5z - 1$?

	$5z = 5z - 1$
Subtract $5z$ to get the constant alone on the right.	$5z - 5z = 5z - 5z - 1$
Simplify—the z's are gone!	$0 \neq -1$

But $0 \neq 1$.

Solving the equation $5z = 5z - 1$ led to the false statement $0 = -1$. The equation $5z = 5z - 1$ will not be true for any value of z. It has no solution. An equation that has no solution, or that is false for all values of the variable, is called a contradiction.

Contradiction

An equation that is false for all values of the variable is called a contradiction.

A contradiction has no solution.

EXAMPLE 9

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$6(2n-1)+3=2n-8+5(2n+1)$

Solution

Given equation. $$6(2n-1)+3=2n-8+5(2n+1)$$

Distribute and combine like terms.
$$6\cdot 2n-6\cdot 1+3=2n-8+5\cdot 2n+5\cdot 1$$
$$12n-6+3=2n-8+10n+5$$
$$12n-3=12n-3$$

Subtract $12n$ from both sides to get the n's to one side and simplify.

$$12n - 12n - 3 = 12n - 12n - 3$$
$$-3 = -3$$

This equation is a true statement.	The equation is an identity. The solution is every real number.

TRY IT 9.1

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4 + 9\left(3x - 7\right) = -42x - 13 + 23\left(3x - 2\right)$

Show answer

identity; all real numbers

TRY IT 9.2

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$8\left(1 - 3x\right) + 15\left(2x + 7\right) = 2\left(x + 50\right) + 4\left(x + 3\right) + 1$

Show answer

identity; all real numbers

EXAMPLE 10

Classify as a conditional equation, an identity, or a contradiction. Then state the solution.

$10 + 4\left(p - 5\right) = 0$

Solution

Given equation.	$10 + 4(p - 5) = 0$
Distribute and combine like terms.	$10 + 4 \cdot p - 4 \cdot 5 = 0$ $10 + 4p - 20 = 0$ $p - 10 = 0$
Add 10 from both sides.	$4p - 10 + 10 = 0 + 10$
Divide by 4 and simplify.	

$$\frac{4p}{4} = \frac{10}{4}$$
$$p = \frac{5}{2}$$

The equation is true when $p = \frac{5}{2}$	This is a conditional equation. The solution is $p = \frac{5}{2}$.

TRY IT 10.1

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $11\,(q+3) - 5 = 19$

Show answer

conditional equation; $q = -\frac{9}{11}$

TRY IT 10.2

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution: $6 + 14\,(k-8) = 95$

Show answer

conditional equation; $k = \frac{201}{14}$

EXAMPLE 11

Classify the equation as a conditional equation, an identity, or a contradiction. Then state the solution.

$$5m + 3\,(9 + 3m) = 2\,(7m - 11)$$

Solution

Given equation.	$5m + 3(9 + 3m) = 2(7m - 11)$
Distribute.	$5m + 3 \cdot 9 + 3 \cdot 3m = 2 \cdot 7m - 2 \cdot 11$ $5m + 27 + 9m = 14m - 22$
Combine like terms.	$14m + 27 = 14m - 22$

Subtract $14m$ from both sides.	$14m+27{\color{red}-14m}=14m-22{\color{red}-14m}$
Simplify.	$27\neq-22$
But $27\neq-22$.	The equation is a contradiction. It has no solution.

TRY IT 11.1

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$12c+5(5+3c)=3(9c-4)$

Show answer

contradiction; no solution

TRY IT 11.2

Classify the equation as a conditional equation, an identity, or a contradiction and then state the solution:

$4(7d+18)=13(3d-2)-11d$

Show answer

contradiction; no solution

Type of equation – Solution

Type of equation	**What happens when you solve it?**	**Solution**
Conditional Equation	True for one or more values of the variables and false for all other values	One or more values
Identity	**True** for any value of the variable	All real numbers
Contradiction	**False** for all values of the variable	No solution

Key Concepts

- **General Strategy for Solving Linear Equations**

1. Simplify each side of the equation as much as possible.
   Use the Distributive Property to remove any parentheses.
   Combine like terms.
2. Collect all the variable terms on one side of the equation.
   Use the Addition or Subtraction Property of Equality.
3. Collect all the constant terms on the other side of the equation.
   Use the Addition or Subtraction Property of Equality.
4. Make the coefficient of the variable term to equal to 1.
   Use the Multiplication or Division Property of Equality.
   State the solution to the equation.
5. Check the solution.
   Substitute the solution into the original equation.

Glossary

conditional equation
An equation that is true for one or more values of the variable and false for all other values of the variable is a conditional equation.

contradiction
An equation that is false for all values of the variable is called a contradiction. A contradiction has no solution.

identity
An equation that is true for any value of the variable is called an identity. The solution of an identity is all real numbers.

Practice Makes Perfect

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

1. $21\left(y-5\right)=-42$	2. $15\left(y-9\right)=-60$
3. $-16\left(3n+4\right)=32$	4. $-9\left(2n+1\right)=36$
5. $5\left(8+6p\right)=0$	6. $8\left(22+11r\right)=0$
7. $-\left(t-19\right)=28$	8. $-\left(w-12\right)=30$
9. $21+2\left(m-4\right)=25$	10. $32+3\left(z+4\right)=41$
11. $-6+6\left(5-k\right)=15$	12. $51+5\left(4-q\right)=56$
13. $8\left(6t-5\right)-35=-27$	14. $2\left(9s-6\right)-62=16$
15. $-2\left(11-7x\right)+54=4$	16. $3\left(10-2x\right)+54=0$
17. $\frac{3}{5}\left(10x-5\right)=27$	18. $\frac{2}{3}\left(9c-3\right)=22$
19. $\frac{1}{4}\left(20d+12\right)=d+7$	20. $\frac{1}{5}\left(15c+10\right)=c+7$
21. $15-\left(3r+8\right)=28$	22. $18-\left(9r+7\right)=-16$
23. $-3-\left(m-1\right)=13$	24. $5-\left(n-1\right)=19$
25. $18-2\left(y-3\right)=32$	26. $11-4\left(y-8\right)=43$
27. $35-5\left(2w+8\right)=-10$	28. $24-8\left(3v+6\right)=0$
29. $-2\left(a-6\right)=4\left(a-3\right)$	30. $4\left(a-12\right)=3\left(a+5\right)$
31. $5\left(8-r\right)=-2\left(2r-16\right)$	32. $2\left(5-u\right)=-3\left(2u+6\right)$
33. $9\left(2m-3\right)-8=4m+7$	34. $3\left(4n-1\right)-2=8n+3$
35. $-15+4\left(2-5y\right)=-7\left(y-4\right)+4$	36. $12+2\left(5-3y\right)=-9\left(y-1\right)-2$
37. $5\left(x-4\right)-4x=14$	38. $8\left(x-4\right)-7x=14$
39. $-12+8\left(x-5\right)=-4+3\left(5x-2\right)$	40. $5+6\left(3s-5\right)=-3+2\left(8s-1\right)$
41. $7\left(2n-5\right)=8\left(4n-1\right)-9$	42. $4\left(u-1\right)-8=6\left(3u-2\right)-7$
43. $3\left(a-2\right)-\left(a+6\right)=4\left(a-1\right)$	44. $4\left(p-4\right)-\left(p+7\right)=5\left(p-3\right)$
45. $-\left(7m+4\right)-\left(2m-5\right)$ $=14-\left(5m-3\right)$	46. $-\left(9y+5\right)-\left(3y-7\right)=16-\left(4y-2\right)$
47. $5\left[9-2\left(6d-1\right)\right]=11\left(4-10d\right)-139$	48. $4\left[5-8\left(4c-3\right)\right]=12\left(1-13c\right)-8$
49. $3\left[-14+2\left(15k-6\right)\right]=8\left(3-5k\right)-24$	50. $3\left[-9+8\left(4h-3\right)\right]=2\left(5-12h\right)-19$
51. $10\left[5\left(n+1\right)+4\left(n-1\right)\right]$ $=11\left[7\left(5+n\right)-\left(25-3n\right)\right]$	52. $5\left[2\left(m+4\right)+8\left(m-7\right)\right]$ $=2\left[3\left(5+m\right)-\left(21-3m\right)\right]$
53. $4\left(2.5v-0.6\right)=7.6$	54. $5\left(1.2u-4.8\right)=-12$

55. $0.2(p-6)=0.4(p+14)$	56. $0.25(q-6)=0.1(q+18)$
57. $0.5(16m+34)=-15$	58. $0.2(30n+50)=28$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

59. $15y+32=2(10y-7)-5y+46$	60. $23z+19=3(5z-9)+8z+46$
61. $9(a-4)+3(2a+5)=7(3a-4)-6a+7$	62. $5(b-9)+4(3b+9)=6(4b-5)-7b+21$
63. $24(3d-4)+100=52$	64. $18(5j-1)+29=47$
65. $30(2n-1)=5(10n+8)$	66. $22(3m-4)=8(2m+9)$
67. $18u-51=9(4u+5)-6(3u-10)$	68. $7v+42=11(3v+8)-2(13v-1)$
69. $5(p+4)+8(2p-1)=9(3p-5)-6(p-2)$	70. $3(6q-9)+7(q+4)=5(6q+8)-5(q+1)$
71. $9(4k-7)=11(3k+1)+4$	72. $12(6h-1)=8(8h+5)-4$
73. $60(2x-1)=15(8x+5)$	74. $45(3y-2)=9(15y-6)$
75. $36(4m+5)=12(12m+15)$	76. $16(6n+15)=48(2n+5)$
77. $11(8c+5)-8c=2(40c+25)+5$	78. $9(14d+9)+4d=13(10d+6)+3$

Everyday Math

79. **Coins.** Rhonda has \$1.90 in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, *n*, by solving the equation $0.05n+0.10(2n-1)=1.90$.	80. **Fencing.** Micah has 44 feet of fencing to make a dog run in his yard. He wants the length to be 2.5 feet more than the width. Find the length, *L*, by solving the equation $2L+2(L-2.5)=44$.

Writing Exercises

81. Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.	82. Using your own words, list the steps in the general strategy for solving linear equations.
83. Solve the equation $\frac{1}{4}(8x+20)=3x-4$ explaining all the steps of your solution as in the examples in this section.	84. What is the first step you take when solving the equation $3-7(y-4)=38$? Why is this your first step?

Answers

1. $y=3$	3. $n=-2$	5. $p=-\frac{4}{3}$
7. $t=-9$	9. $m=6$	11. $k=\frac{3}{2}$
13. $t=1$	15. $x=-2$	17. $x=5$
19. $d=1$	21. $r=-7$	23. $m=-15$
25. $y=-4$	27. $w=\frac{1}{2}$	29. $a=4$
31. $r=8$	33. $m=3$	35. $y=-3$
37. $x=34$	39. $x=-6$	41. $n=-1$
43. $a=-4$	45. $m=-4$	47. $d=-3$
49. $k=\frac{3}{5}$	51. $n=-5$	53. $v=1$
55. $p=-34$	57. $m=-4$	59. identity; all real numbers
61. identity; all real numbers	63. conditional equation; $d=\frac{2}{3}$	65. conditional equation; $n=7$
67. contradiction; no solution	69. contradiction; no solution	71. conditional equation; $k=26$
73. contradiction; no solution	75. identity; all real numbers	77. identity; all real numbers
79. 8 nickels	81. Answers will vary.	83. Answers will vary.

Attributions

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3.6 Solve a Formula for a Specific Variable

Learning Objectives

By the end of this section, you will be able to:

- Use the Distance, Rate, and Time formula
- Solve a formula for a specific variable

Use the Distance, Rate, and Time Formula

One formula you will use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant rate. Rate is an equivalent word for "speed." The basic idea of rate may already be familiar to you. Do you know what distance you travel if you drive at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the highway.) If you said 120 miles, you already know how to use this formula!

Distance, Rate, and Time

For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula:

$$d = rt \qquad \text{where} \qquad \begin{aligned} d &= \text{distance} \\ r &= \text{rate} \\ t &= \text{time} \end{aligned}$$

We will use the Strategy for Solving Applications that we used earlier in this chapter. When our problem requires a formula, we change Step 4. In place of writing a sentence, we write the appropriate formula. We write the revised steps here for reference.

HOW TO: Solve an application (with a formula).

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.
3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.

5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

You may want to create a mini-chart to summarize the information in the problem. See the chart in this first example.

EXAMPLE 1

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. What distance has he traveled?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	Let *d* = distance.
Step 4. Translate: Write the appropriate formula.	$d = rt$ $d = ?$ $r = 12$ mph $t = 3\frac{1}{2}$ hours
Substitute in the given information.	$d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42$ miles
Step 6. Check Does 42 miles make sense? Jamal rides:	12 miles in 1 hour 24miles in 2 hours $\left.\begin{array}{l}\text{36 miles in 3 hours}\\ \text{48 miles in 4 hours}\end{array}\right\}$ 42 miles in $3\frac{1}{2}$hours seems reasonable
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

TRY IT 1.1

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Show answer
330 miles

TRY IT 1.2

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Show answer
7 miles

EXAMPLE 2

Rey is planning to drive from his house in Saskatoon to visit his grandmother in Winnipeg, a distance of 798 km. If he can drive at a steady rate of 76 km per hour, how many hours will the trip take?

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many hours (time)
Step 3. Name. Choose a variable to represent it.	Let *t* = time.
	d = 798 km r = 76 km/h t = ? hours
Step 4. Translate. Write the appropriate formula.	$d = rt$
Substitute in the given information.	$798 = 76t$
Step 5. Solve the equation.	$t = 10.5$ hours
Step 6. Check. Substitute the numbers into the formula and make sure the result is a true statement.	$d = rt$ $798 \stackrel{?}{=} 76 \cdot 10.5$ $798 = 798$
Step 7. Answer the question with a complete sentence. Rey's trip will take 10.5 hours.	

TRY IT 2.1

Lee wants to drive from Kamloops to his brother's apartment in Banff, a distance of 495 km. If he drives at a steady rate of 90 km/h, how many hours will the trip take?

Show answer
5 1/2 hours

TRY IT 2.2

Yesenia is 168 km from Toronto. If she needs to be in Toronto in 2 hours, at what rate does she need to drive?

Show answer
84 km/h

Solve a Formula for a Specific Variable

You are probably familiar with some geometry formulas. A formula is a mathematical description of the relationship between variables. Formulas are also used in the sciences, such as chemistry, physics, and biology. In medicine they are used for calculations for dispensing medicine or determining body mass index. Spreadsheet programs rely on formulas to make calculations. It is important to be familiar with formulas and be able to manipulate them easily.

In (Example 1) and (Example 2), we used the formula $d = rt$. This formula gives the value of d, distance, when you substitute in the values of $r \text{ and } t$, the rate and time. But in (Example 2), we had to find the value of t. We substituted in values of $d \text{ and } r$ and then used algebra to solve for t. If you had to do this often, you might wonder why there is not a formula that gives the value of t when you substitute in the values of $d \text{ and } r$. We can make a formula like this by solving the formula $d = rt$ for t.

To solve a formula for a specific variable means to isolate that variable on one side of the equals sign with a coefficient of 1. All other variables and constants are on the other side of the equals sign. To see how to solve a formula for a specific variable, we will start with the distance, rate and time formula.

EXAMPLE 3

Solve the formula $d = rt$ for t:

a. when $d = 520$ km and $r = 65$ km/hr
b. in general

Solution

We will write the solutions side-by-side to demonstrate that solving a formula in general uses the same steps as when we have numbers to substitute.

a) when $d = 520$ km and $r = 65$ km/hr		b) in general	
Write the formula.	$d = rt$	Write the formula.	$d = rt$
Substitute.	$520 = 65t$		
Divide, to isolate t.	$\frac{520}{65} = \frac{65t}{65}$	Divide, to isolate t.	$\frac{d}{r} = \frac{rt}{r}$
Simplify.	$8 = t$ $t = 8$ hours	Simplify.	$\frac{d}{r} = t$

We say the formula $t = \frac{d}{r}$ is solved for t.

TRY IT 3.1

Solve the formula $d = rt$ for r:

a) when $d = 180 \text{ km and } t = 4$ hr b) in general

Show answer

a) $r = 45$ km/hr b) $r = \frac{d}{t}$

TRY IT 3.2

Solve the formula $d = rt$ for r:

a) when $d = 780 \text{ km and } t = 12$ hr b) in general

Show answer

a) $r = 65$ km/hr b) $r = \frac{d}{t}$

EXAMPLE 4

Solve the formula $A = \frac{1}{2}bh$ for h:

a) when $A = 90$ and $b = 15$ b) in general

Solution

a) when $A = 90$ and $b = 15$		b) in general	
Write the formula.	$A = \frac{1}{2}bh$	Write the formula.	$A = \frac{1}{2}bh$
Substitute.	$90 = \frac{1}{2} \cdot 15 \cdot h$		
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2}15h$	Clear the fractions.	$2 \cdot A = 2 \cdot \frac{1}{2}bh$
Simplify.	$180 = 15h$	Simplify.	$2A = bh$
Solve for h.	$12 = h$	Solve for h.	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula $h = \frac{2A}{b}$.

TRY IT 4.1

Use the formula $A = \frac{1}{2}bh$ to solve for h:

a) when $A = 170$ and $b = 17$ b) in general

Show answer

a) $h = 20$ b) $h = \frac{2A}{b}$

TRY IT 4.2

Use the formula $A = \frac{1}{2}bh$ to solve for b:

a) when $A = 62$ and $h = 31$ b) in general

Show answer

a) $b = 4$ b) $b = \frac{2A}{h}$

The formula $I = Prt$ is used to calculate simple interest, *I*, for a principal, *P*, invested at rate, *r*, for *t* years.

EXAMPLE 5

Solve the formula $I = Prt$ to find the principal, P:

a) when $I = \$5,600, r = 4\%, t = 7\ years$ b) in general

Solution

a) $I = \$5,600, r = 4\%, t = 7 years$		b) in general	
Write the formula.	$I = Prt$	Write the formula.	$I = Prt$
Substitute.	$5600 = P(0.04)(7)$		
Simplify.	$5600 = P(0.28)$	Simplify.	$I = P(rt)$
Divide, to isolate *P*.	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	Divide, to isolate *P*.	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$	Simplify.	$\frac{I}{rt} = P$
The principal is	$\$20,000$		$P = \frac{I}{rt}$

TRY IT 5.1

Use the formula $I = Prt$ to find the principal, P:

a) when $I = \$2,160, r = 6\%, t = 3\ years$ b) in general

Show answer

a) $12,000 b) $P = \frac{I}{rt}$

TRY IT 5.2

Use the formula $I = Prt$ to find the principal, P:

a) when $I = \$5,400, r = 12\%, t = 5\ years$ b) in general

Show answer

a) $9,000 b) $P = \frac{I}{rt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually *x* and *y*. You might be given an equation that is solved for *y* and need to solve it for *x*, or vice versa. In the following example, we're given an equation with both *x* and *y* on the same side and we'll solve it for *y*.

EXAMPLE 6

Solve the formula $3x + 2y = 18$ for *y*:

a) when $x = 4$ b) in general

Solution

a) when $x = 4$		b) in general	
	$3x + 2y = 18$		$3x + 2y = 18$
Substitute.	$3(4) + 2y = 18$		
Subtract to isolate the y-term.	$12 - 12 + 2y = 18 - 12$ $2y = 6$	Subtract to isolate the y-term.	$3x - 3x + 2y = 18 - 3x$ $2y = 18 - 3x$
Divide.	$\frac{2y}{2} = \frac{6}{2}$	Divide.	$\frac{2y}{2} = \frac{18}{2} - \frac{3x}{2}$
Simplify.	$y = 3$	Simplify.	$y = 9 - \frac{3}{2}x$ or $y = -\frac{3}{2}x + 9$

TRY IT 6.1

Solve the formula $3x + 4y = 10$ for *y*:

a) when $x = \frac{14}{3}$ b) in general

Show answer

a)$y = -1$ b)$y = \frac{10-3x}{4}$

TY IT 6.2

Solve the formula $5x + 2y = 18$ for *y*:

a) when $x = 4$ b) in general

Show answer

a)$y = -1$ b)$y = \frac{18-5x}{2}$

Now we will solve a formula in general without using numbers as a guide.

EXAMPLE 7

Solve the formula $P = a + b + c$ for a.

Solution

We will isolate a on one side of the equation.	$P = a + b + c$
Both b and c are added to a, so we subtract them from both sides of the equation.	$P - b - c = a + b + c - b - c$
Simplify.	$P - b - c = a$ or $a = P - b - c$

TRY IT 7.1

Solve the formula $P = a + b + c$ for b.

Show answer
$b = P - a - c$

TRY IT 7.2

Solve the formula $P = a + b + c$ for c.

Show answer
$c = P - a - b$

EXAMPLE 8

Solve the formula $6x + 5y = 13$ for y.

Solution

	$6x + 5y = 13$
Subtract $6x$ from both sides to isolate the term with y.	$6x - 6x + 5y = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide by 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13-6x}{5}$
Simplify.	$y = \frac{13-6x}{5}$

The fraction is simplified. We cannot divide $13 - 6x$ by 5

TRY IT 8.1

Solve the formula $4x + 7y = 9$ for *y*.

Show answer
$y = \frac{9-4x}{7}$

TRY IT 8.2

Solve the formula $5x + 8y = 1$ for *y*.

Show answer
$y = \frac{1-5x}{8}$

Key Concepts

- **To Solve an Application (with a formula)**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. Write the appropriate formula for the situation. Substitute in the given information.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Distance, Rate and Time**
  For an object moving at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula: $d = rt$ where *d* = distance, *r* = rate, *t* = time.
- **To solve a formula for a specific variable** means to get that variable by itself with a coefficient of 1 on one side of the equation and all other variables and constants on the other side.

Practice Makes Perfect

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

1. Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?	2. Steve drove for $8\frac{1}{2}$ hours at 72 miles per hour. How much distance did he travel?
3. Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?	4. Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?
5. Marta is taking the bus from Abbotsford to Cranbrook. The distance is 774 km and the bus travels at a steady rate of 86 miles per hour. How long will the bus ride be?	6. Connor wants to drive from Vancouver to the Nakusp, a distance of 630 km. If he drives at a steady rate of 90 km/h, how many hours will the trip take?
7. Kareem wants to ride his bike from Golden, BC to Banff, AB. The distance is 140 km. If he rides at a steady rate of 20 km/h, how many hours will the trip take?	8. Aurelia is driving from Calgary to Edmonton at a rate of 85 km/h. The distance is 300 km. To the nearest tenth of an hour, how long will the trip take?
9. Alejandra is driving to Prince George, 450 km away. If she wants to be there in 6 hours, at what rate does she need to drive?	10. Javier is driving to Vernon, 240 km away. If he needs to be in Vernon in 3 hours, at what rate does he need to drive?
11. Philip got a ride with a friend from Calgary to Kelowna, a distance of 890 km. If the trip took 10 hours, how fast was the friend driving?	12. Aisha took the train from Spokane to Seattle. The distance is 280 miles and the trip took 3.5 hours. What was the speed of the train?

Solve a Formula for a Specific Variable

13. Solve for t using $d = rt$ a. when $d = 240$ and $r = 60$ b. in general	14. Solve for t using $d = rt$ a. when $d = 350$ and $r = 70$ b. in general
15. Solve for t using $d = rt$ a. when $d = 175$ and $r = 50$ b. in general	16. Solve for t using $d = rt$ a. when $d = 510$ and $r = 60$ b. in general
17. Solve for r using $d = rt$ a. when $d = 420$ and $t = 6$ b. in general	18. Solve for r using $d = rt$ a. when $d = 204$ and $t = 3$ b. in general
19. Solve for r using $d = rt$ a. when $d = 180$ and $t = 4.5$ b. in general	20. Solve for r using $d = rt$ a. when $d = 160$ and $t = 2.5$ b. in general
21. Solve for h using $A = \frac{1}{2}bh$ a. when $A = 176$ and $b = 22$ b. in general	22. Solve for b using $A = \frac{1}{2}bh$ a. when $A = 126$ and $h = 18$ b. in general
23. Solve for the principal, P using $I = Prt$ for a. $I = \$5,480$, $r = 4\%$, $t = 7$ years b. in general	24. Solve for b using $A = \frac{1}{2}bh$ a. when $A = 65$ and $h = 13$ b. in general.
25. Solve for the time, t using $I = Prt$ for a. $I = \$2,376$, $P = \$9,000$, $r = 4.4\%$ b. in general	26. Solve for the principal, P using $I = Prt$ for a. $I = \$3,950$, $r = 6\%$, $t = 5$ years b. in general
27. Solve the formula $2x + 3y = 12$ for y when a. $x = 3$ b. in general	28. Solve for the time, t for a. $I = \$624$, $P = \$6,000$, $r = 5.2\%$ b. in general
29. Solve the formula $3x - y = 7$ for y when a. $x = -2$ b. in general	30. Solve the formula $5x + 2y = 10$ for y when a. $x = 4$ b. in general

31. Solve $a+b=90$ for a. a when $b=50$ b. a in general	32. Solve the formula $4x+y=5$ for y when a. $x=-3$ b. in general
33. Solve $180=a+b+c$ for a.	34. Solve $a+b=90$ for a.
35. Solve the formula $8x+y=15$ for y.	36. Solve $180=a+b+c$ for c.
37. Solve the formula $-4x+y=-6$ for y.	38. Solve the formula $9x+y=13$ for y.
39. Solve the formula $4x+3y=7$ for y.	40. Solve the formula $-5x+y=-1$ for y.
41. Solve the formula $x-y=-4$ for y.	42. Solve the formula $3x+2y=11$ for y.
43. Solve the formula $P=2L+2W$ for L.	44. Solve the formula $x-y=-3$ for y.
45. Solve the formula $C=\pi d$ for d.	46. Solve the formula $P=2L+2W$ for W.
47. Solve the formula $V=LWH$ for L.	48. Solve the formula $C=\pi d$ for π.
49. Solve the formula $V=LWH$ for H.	

Everyday Math

50. **Converting temperature.** Yon was visiting the United States and he saw that the temperature in Seattle one day was 50^{o} Fahrenheit. Solve for C in the formula $F=\frac{9}{5}C+32$ to find the Celsius temperature.	51. **Converting temperature.** While on a tour in Greece, Tatyana saw that the temperature was 40^{o} Celsius. Solve for F in the formula $C=\frac{5}{9}(F-32)$ to find the Fahrenheit temperature.

Writing Exercises

52. Solve the equation $5x-2y=10$ for x a) when $y=10$ b) in general c) Which solution is easier for you, a) or b)? Why?	53. Solve the equation $2x+3y=6$ for y a) when $x=-3$ b) in general c) Which solution is easier for you, a) or b)? Why?

Answers

1. 290 miles	3. 30 miles	5. 9 hr
7. 7 hr	9. 75 km/hr	11. 89 km/hr
13. a. $t = 4$ b. $t = \frac{d}{r}$	15. a. $t = 3.5$ b. $t = \frac{d}{r}$	17. a. $r = 70$ b. $r = \frac{d}{t}$
19. a. $r = 40$ b. $r = \frac{d}{t}$	21. a. $h = 16$ b. $h = \frac{2A}{b}$	23. a. $P = \$19,571.43$ b. $P = \frac{I}{rt}$
25. a. $t = 6$ b. $t = \frac{I}{Pr}$	27. a. $y = 2$ b. $y = \frac{12-2x}{3}$	29. a. $y = -13$ b. $y = -7 + 3x$ or $y = 3x - 7$
31. $40; a = 90 - b$	33. $a = 180 - b - c$	35. $y = 15 - 8x$
37. $y = 4x - 6$	39. $y = \frac{7-4x}{3}$	41. $y = 4 + x$
43. $L = \frac{P-2W}{2}$	45. $d = \frac{C}{\pi}$	47. $L = \frac{V}{WH}$
49. $H = \frac{V}{LW}$	51. $F = 104^{o}$	53. Answers will vary

Attributions

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3.7 Use a Problem-Solving Strategy

Learning Objectives

By the end of this section, you will be able to:

- Approach word problems with a positive attitude
- Use a problem-solving strategy for word problems
- Solve number problems

Approach Word Problems with a Positive Attitude

“If you think you can… or think you can’t… you’re right.”—Henry Ford

The world is full of word problems! Will my income qualify me to rent that apartment? How much punch do I need to make for the party? What size diamond can I afford to buy my girlfriend? Should I fly or drive to my family reunion?

How much money do I need to fill the car with gas? How much tip should I leave at a restaurant? How many socks should I pack for vacation? What size turkey do I need to buy for Thanksgiving dinner, and then what time do I need to put it in the oven? If my sister and I buy our mother a present, how much does each of us pay?

Now that we can solve equations, we are ready to apply our new skills to word problems. Do you know anyone who has had negative experiences in the past with word problems? Have you ever had thoughts like the student below?

Negative thoughts can be barriers to success.

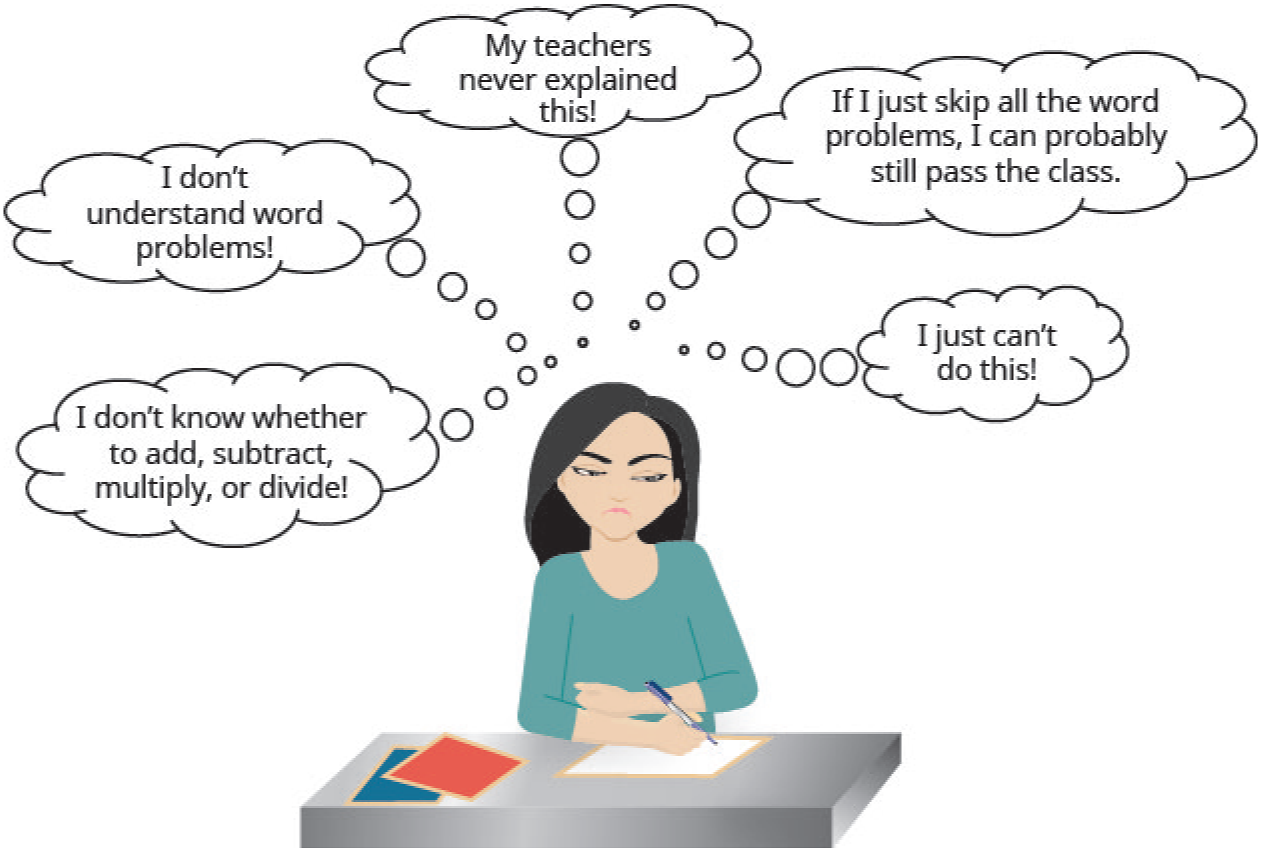

Figure .1

When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. We need to calm our fears and change our negative feelings.

Start with a fresh slate and begin to think positive thoughts. If we take control and believe we can be successful, we will be able to master word problems! Read the positive thoughts in (Figure 2) and say them out loud.

Thinking positive thoughts is a first step towards success.

Figure .2

Think of something, outside of school, that you can do now but couldn't do 3 years ago. Is it driving a car? Snowboarding? Cooking a gourmet meal? Speaking a new language? Your past experiences with word problems happened when you were younger—now you're older and ready to succeed!

Use a Problem-Solving Strategy for Word Problems

We have reviewed translating English phrases into algebraic expressions, using some basic mathematical vocabulary and symbols. We have also translated English sentences into algebraic equations and solved some word problems. The word problems applied math to everyday situations. We restated the situation in one sentence, assigned a variable, and then wrote an equation to solve the problem. This method works as long as the situation is familiar and the math is not too complicated.

Now, we'll expand our strategy so we can use it to successfully solve any word problem. We'll list the strategy here, and then we'll use it to solve some problems. We summarize below an effective strategy for problem solving.

Use a Problem-Solving Strategy to Solve Word Problems.

1. **Read** the problem. Make sure all the words and ideas are understood.
2. **Identify** what we are looking for.

3. **Name** what we are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebraic equation.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

EXAMPLE 1

Pilar bought a purse on sale for $18, which is one-half of the original price. What was the original price of the purse?

Solution

Step 1. Read the problem. Read the problem two or more times if necessary. Look up any unfamiliar words in a dictionary or on the internet.

- *In this problem, is it clear what is being discussed? Is every word familiar?*

Step 2. Identify what you are looking for. Did you ever go into your bedroom to get something and then forget what you were looking for? It's hard to find something if you are not sure what it is! Read the problem again and look for words that tell you what you are looking for!

- *In this problem, the words "what was the original price of the purse" tell us what we need to find.*

Step 3. Name what we are looking for. Choose a variable to represent that quantity. We can use any letter for the variable, but choose one that makes it easy to remember what it represents.

- Let $p =$ the original price of the purse.

Step 4. Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Translate the English sentence into an algebraic equation.

Reread the problem carefully to see how the given information is related. Often, there is one sentence that gives this information, or it may help to write one sentence with all the important information. Look for clue words to help translate the sentence into algebra. Translate the sentence into an equation.

Restate the problem in one sentence with all the important information.	$\underbrace{18}\ \underbrace{\text{is}}\ \underbrace{\text{one-half the original price.}}$
Translate into an equation.	$18 = \frac{1}{2} \cdot p$

Step 5. Solve the equation using good algebraic techniques. Even if you know the solution right away, using good algebraic techniques here will better prepare you to solve problems that do not have obvious answers.

Solve the equation.	$18 = \frac{1}{2}p$
Multiply both sides by 2.	$2 \cdot 18 = 2 \cdot \frac{1}{2}p$
Simplify.	$36 = p$

Step 6. Check the answer in the problem to make sure it makes sense. We solved the equation and found that $p = 36$, which means "the original price" was $36

- *Does $36 make sense in the problem? Yes, because 18 is one-half of 36, and the purse was on sale at half the original price.*

Step 7. Answer the question with a complete sentence. The problem asked "What was the original price of the purse?"

- *The answer to the question is: "The original price of the purse was $36."*

If this were a homework exercise, our work might look like this:

Pilar bought a purse on sale for $18, which is one-half the original price. What was the original price of the purse?

	Let $p =$ the original price.
	18 is one-half the original price.
	$18 = \frac{1}{2}p$
Multiply both sides by 2.	$2 \cdot 18 = 2 \cdot \frac{1}{2}p$
Simplify.	$36 = p$
Check. Is $36 a reasonable price for a purse?	Yes.
Is 18 one half of 36?	$18 \stackrel{?}{=} \frac{1}{2} \cdot 36$
	$18 = 18\checkmark$
	The original price of the purse was $36.

TRY IT 1.1

Joaquin bought a bookcase on sale for $120, which was two-thirds of the original price. What was the original price of the bookcase?

Show answer
$180

TRY IT 1.2

Two-fifths of the songs in Mariel's playlist are country. If there are 16 country songs, what is the total number of songs in the playlist?

Show answer
40

Let's try this approach with another example.

EXAMPLE 2

Ginny and her classmates formed a study group. The number of girls in the study group was three more than twice the number of boys. There were 11 girls in the study group. How many boys were in the study group?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	How many boys were in the study group?
Step 3. Name. Choose a variable to represent the number of boys.	Let b = the number of boys.
Step 4. Translate. Restate the problem in one sentence with all the important information.	The number of girls (11) was three more than twice the number of boys
Translate into an equation.	$11 = 2b + 3$
Step 5. Solve the equation.	$11 = 2b + 3$
Subtract 3 from each side.	$11 - 3 = 2b + 3 - 3$
Simplify.	$8 = 2b$
Divide each side by 2.	$\frac{8}{2} = \frac{2b}{2}$
Simplify.	$4 = b$
Step 6. Check. First, is our answer reasonable?	Yes, having 4 boys in a study group seems OK. The problem says the number of girls was 3 more than twice the number of boys. If there are four boys, does that make eleven girls? Twice 4 boys is 8. Three more than 8 is 11.
Step 7. Answer the question.	There were 4 boys in the study group.

TRY IT 2.1

Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was 3 more than twice the number of notebooks. He bought 7 textbooks. How many notebooks did he buy?

Show answer
2

TRY IT 2.2

Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is eight more than twice the number of crossword puzzles. He completed 22 Sudoku puzzles. How many crossword puzzles did he do?

Show answer
7

Solve Number Problems

Now that we have a problem solving strategy, we will use it on several different types of word problems. The first type we will work on is "number problems." Number problems give some clues about one or more numbers. We use these clues to write an equation. Number problems don't usually arise on an everyday basis, but they provide a good introduction to practicing the problem solving strategy outlined above.

EXAMPLE 3

The difference of a number and six is 13. Find the number.

Solution

Step 1. Read the problem. Are all the words familiar?	
Step 2. Identify what we are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let n = the number.
Step 4. Translate. Remember to look for clue words like "difference… of… and…"	
Restate the problem as one sentence.	$\underbrace{\text{The difference of}}\ \underbrace{\text{the number and 6}}\ \underbrace{\text{is}}\ \underbrace{13}$
Translate into an equation.	$n - 6 = 13$
Step 5. Solve the equation.	$n - 6 = 13$ $n - 6 + 6 = 13 + 6$ $n = 19$
Simplify.	$n = 19$
Step 6. Check.	The difference of 19 and 6 is 13. It checks!
Step 7. Answer the question.	The number is 19.

TRY IT 3.1

The difference of a number and eight is 17. Find the number.

Show answer
25

TRY IT 3.2

The difference of a number and eleven is -7. Find the number.

Show answer
4

EXAMPLE 4

The sum of twice a number and seven is 15. Find the number.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let n = the number.
Step 4. Translate.	
Restate the problem as one sentence.	$\underbrace{\text{The sum of twice a number and 7}}_{2n+7}\ \underbrace{is}_{=}\ \underbrace{15}_{15}$
Translate into an equation.	$2n + 7 = 15$
Step 5. Solve the equation.	$2n + 7 = 15$
Subtract 7 from each side and simplify.	$2n + 7 - 7 = 15 - 7$ $2n = 8$
Divide each side by 2 and simplify.	$\frac{2n}{2} = \frac{8}{2}$ $n = 4$
Step 6. Check.	
Is the sum of twice 4 and 7 equal to 15?	$2 \cdot 4 + 7 \stackrel{?}{=} 15$ $15 = 15 \checkmark$
Step 7. Answer the question.	The number is 4.

You may be now ready to skip some of the steps while solving such equations which is fine to do and just write down as many as you need but remember that if you write all the steps the chances of miscalculations is reduced.

TRY IT 4.1

The sum of four times a number and two is 14. Find the number.

Show answer
3

TRY IT 4.2

The sum of three times a number and seven is 25. Find the number.

Show answer

6

Some number word problems ask us to find two or more numbers. It may be tempting to name them all with different variables, but so far we have only solved equations with one variable. In order to avoid using more than one variable, we will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

EXAMPLE 5

One number is five more than another. The sum of the numbers is 21. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name. We have two numbers to name and need a name for each.	
Choose a variable to represent the first number.	Let $n = 1^{\text{st}}$ number.
What do we know about the second number?	One number is five more than another.
	$n + 5 = 2^{\text{nd}}$ number
Step 4. Translate. Restate the problem as one sentence with all the important information.	The sum of the 1^{st} number and the 2^{nd} number is 21.
Translate into an equation.	$\underbrace{1^{\text{st}}\text{number}}_{n}\ \underbrace{\text{add}}_{+}\ \underbrace{2^{\text{nd}}\text{number}}_{n+5}\ \underbrace{is}_{=}\ \underbrace{21}_{21}$
Substitute the variable expressions.	
Step 5. Solve the equation.	$n + n + 5 = 21$
Combine like terms.	$2n + 5 = 21$
Subtract 5 from both sides and simplify.	$2n = 16$
Divide by 2 and simplify.	$n = 8$ — 1^{st} number
Find the second number, too.	$n + 5$ –2^{nd} number
	$8 + 5$
	13
Step 6. Check.	
Do these numbers check in the problem?	
Is one number 5 more than the other?	$13 \stackrel{?}{=} 8 + 5$
Is thirteen 5 more than 8? Yes.	$13 = 13\checkmark$
Is the sum of the two numbers 21?	$8 + 13 \stackrel{?}{=} 21$
	$21 = 21\checkmark$
Step 7. Answer the question.	The numbers are 8 and 13.

TRY IT 5.1

One number is six more than another. The sum of the numbers is twenty-four. Find the numbers.

Show answer
9, 15

TRY IT 5.2

The sum of two numbers is fifty-eight. One number is four more than the other. Find the numbers.

Show answer
27, 31

EXAMPLE 6

The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	We are looking for two numbers.
Step 3. Name.	
Choose a variable.	Let $n = 1^{\text{st}}$ number.
One number is 4 less than the other.	$n - 4 = 2^{\text{nd}}$ number
Step 4. Translate.	
Write as one sentence.	The sum of the 2 numbers is negative 14.
Translate into an equation.	$\underbrace{1^{\text{st}}\text{number}}_{n}\ \underbrace{\text{add}}_{+}\ \underbrace{2^{\text{nd}}\text{number}}_{n-4}\ \underbrace{is}_{=}\ \underbrace{\text{negative fourteen}}_{-14}$
Step 5. Solve the equation.	$n + n - 4 = -14$
Combine like terms.	$2n - 4 = 14$
Add 4 to each side and simplify.	$2n = -10$
Simplify.	1^{st} number –> $n = -5$
	2^{nd} number–> $n - 4 = -5 - 4 = -9$
Step 6. Check.	
Is −9 four less than −5?	$-5 - 4 \stackrel{?}{=} -9$
	$-9 = -9$✓
Is their sum −14?	$-5 + (-9) \stackrel{?}{=} -14$
	$-14 = -14$✓
Step 7. Answer the question.	The numbers are −5 and −9.

TRY IT 6.1

The sum of two numbers is negative twenty-three. One number is seven less than the other. Find the numbers.

Show answer

$-15, -8$

TRY IT 6.2

The sum of two numbers is -18. One number is 40 more than the other. Find the numbers.

Show answer
$-29, 11$

EXAMPLE 7

One number is ten more than twice another. Their sum is one. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	We are looking for two numbers.
Step 3. Name.	
Choose a variable.	Let $x = 1^{\text{st}}$ number.
One number is 10 more than twice another.	$2x + 10 = 2^{\text{nd}}$ number
Step 4. Translate.	
Restate as one sentence.	Their sum is one.
	The sum of the two numbers is 1.
Translate into an equation.	$\underbrace{1^{\text{st}}\text{number}}_{x}\ \underbrace{\text{add}}_{+}\ \underbrace{2^{\text{nd}}\text{number}}_{2x+10}\ \underbrace{is}_{=}\ \underbrace{1}_{1}$
Step 5. Solve the equation.	
Combine like terms.	$3x + 10 = 1$
Subtract 10 from each side.	$3x = -9$
Divide each side by 3.	1^{st} number –>$x = -3$
	2^{nd} number–> $2x + 10 = 2(-3) + 10 = 4$
Step 6. Check.	
Is ten more than twice −3 equal to 4?	$2(-3) + 10 \stackrel{?}{=} 4$
	$-6 + 10 \stackrel{?}{=} 4$
	$4 = 4$✓
Is their sum 1?	$-3 + 4 \stackrel{?}{=} 1$
	$1 = 1$✓
Step 7. Answer the question.	The numbers are −3 and 4.

TRY IT 7.1

One number is eight more than twice another. Their sum is negative four. Find the numbers.

Show answer

$-4, 0$

TRY IT 7.2

One number is three more than three times another. Their sum is -5. Find the numbers.

Show answer
$-3, -2$

Some number problems involve consecutive integers.*Consecutive integers* are integers that immediately follow each other.

Examples of consecutive integers are:

$$1, 2, 3, 4$$

$$-10, -9, -8, -7$$

$$150, 151, 152, 153$$

Notice that each number is one more than the number preceding it. So if we define the first integer as *n*, the next consecutive integer is $n + 1$. The one after that is one more than $n + 1$, so it is $n + 1 + 1$, which is $n + 2$.

$$\begin{array}{ll} n & 1^{\text{st}} \text{ integer} \\ n+1 & 2^{\text{nd}} \text{ consecutive integer} \\ n+2 & 3^{\text{rd}} \text{ consecutive integer} \ldots \text{ etc.} \end{array}$$

EXAMPLE 8

The sum of two consecutive integers is 47. Find the numbers.

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	two consecutive integers
Step 3. Name each number.	Let $n = 1^{\text{st}}$ integer.
	$n + 1 =$ next consecutive integer
Step 4. Translate.	
Restate as one sentence.	The sum of the integers is 47.
Translate into an equation.	$n + n + 1 = 47$
Step 5. Solve the equation.	$2n + 1 = 47$
Combine like terms.	$2n = 46$
Subtract 1 from each side.	$n = 23$
Divide each side by 2.	1^{st} number –> $n = 23$
	next number –> $n + 1 = 23 + 1 = 24$
Step 6. Check.	$23 + 24 \stackrel{?}{=} 47$ $47 = 47 ✓$
Step 7. Answer the question.	The two consecutive integers are 23 and 24.

TRY IT 8.1

The sum of two consecutive integers is 95. Find the numbers.

Show answer
47, 48

TRY IT 8.2

The sum of two consecutive integers is -31. Find the numbers.

Show answer
$-16, -15$

EXAMPLE 9

Find three consecutive integers whose sum is -42.

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	three consecutive integers
Step 3. Name each of the three numbers.	Let $n = 1^{\text{st}}$ integer.
	$n+1 =$ 2nd consecutive integer
	$n+2 =$ 3rd consecutive integer
Step 4. Translate.	
Restate as one sentence.	The sum of the three integers is −42.
Translate into an equation.	$n + n + 1 + n + 2 = -42$
Step 5. Solve the equation.	$3n + 3 = -42$
Combine like terms. Subtract 3 from each side.	$3n = -45$
Divide each side by 3.	$n = -15$
	1st integer –> $n = -15$
	2nd integer–> $n + 1 = -15 + 1 = -14$
	3rd integer–> $n + 2 = -15 + 2 = -13$
Step 6. Check.	$-13 + (-14) + (-15) \stackrel{?}{=} -42$ $-42 = -42 \checkmark$
Step 7. Answer the question.	The three consecutive integers are −13, −14, and −15.

TRY IT 9.1

Find three consecutive integers whose sum is -96.

Show answer
$-33, -32, -31$

TRY IT 9.2

Find three consecutive integers whose sum is -36.

Show answer
$-13, -12, -11$

Now that we have worked with consecutive integers, we will expand our work to include consecutive even integers and consecutive odd integers. *Consecutive even integers* are even integers that immediately follow one another. Examples of consecutive even integers are:

$18, 20, 22$

$64, 66, 68$

$-12, -10, -8$

Notice each integer is 2 more than the number preceding it. If we call the first one *n*, then the next one is $n + 2$. The next one would be $n + 2 + 2$ or $n + 4$.

n $\quad 1^{\text{st}}$ even integer

$n + 2$ $\quad 2^{\text{nd}}$ consecutive even integer

$n + 4$ $\quad 3^{\text{rd}}$ consecutive even integer . . . etc.

Consecutive odd integers are odd integers that immediately follow one another. Consider the consecutive odd integers 77, 79, and 81

$77, 79, 81$

$n, n + 2, n + 4$

n $\quad 1^{\text{st}}$ odd integer

$n + 2$ $\quad 2^{\text{nd}}$ consecutive odd integer

$n + 4$ $\quad 3^{\text{rd}}$ consecutive odd integer . . . etc.

Does it seem strange to add 2 (an even number) to get from one odd integer to the next? Do you get an odd number or an even number when we add 2 to 3? to 11? to 47?

Whether the problem asks for consecutive even numbers or odd numbers, you don't have to do anything different. The pattern is still the same—to get from one odd or one even integer to the next, add 2

EXAMPLE 10

Find three consecutive even integers whose sum is 84

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	three consecutive even integers
Step 3. Name the integers.	Let $n = 1^{\text{st}}$ even integer. $n + 2 = 2^{\text{nd}}$ consecutive even integer $n + 4 = 3^{\text{rd}}$ consecutive even integer
Step 4. Translate.	
Restate as one sentence.	The sum of the three even integers is 84.
Translate into an equation.	$n + n + 2 + n + 4 \ = \ 84$
Step 5. Solve the equation.	
Combine like terms.	$n + n + 2 + n + 4 \ = \ 84$
Subtract 6 from each side.	$3n + 6 \ = \ 84$
Divide each side by 3.	$3n \ = \ 78$
	$n = 26 \quad 1^{\text{st}}$ integer $n + 2 \quad 2^{\text{nd}}$ integer $26 + 2$ 28 $n + 4 \quad 3^{\text{rd}}$ integer $26 + 4$ 30
Step 6. Check.	$26 + 28 + 30 \stackrel{?}{=} 84$ $84 = 84\checkmark$
Step 7. Answer the question.	The three consecutive integers are 26, 28, and 30.

TRY IT 10.1

Find three consecutive even integers whose sum is 102

Show answer
32, 34, 36

TRY IT 10.2

Find three consecutive even integers whose sum is -24.

Show answer
$-10, -8, -6$

EXAMPLE 11

A married couple together earns $110,000 a year. The wife earns $16,000 less than twice what her husband earns. What does the husband earn?

Solution

Step 1. Read the problem.	
Step 2. Identify what we are looking for.	How much does the husband earn?
Step 3. Name.	
Choose a variable to represent the amount the husband earns.	Let $h =$ the amount the husband earns.
The wife earns $16,000 less than twice that.	$2h - 16,000$ the amount the wife earns.
Step 4. Translate.	Together the husband and wife earn $110,000.
Restate the problem in one sentence with all the important information.	The amount the husband earns plus the amount the wife earns is $110,000
Translate into an equation.	$h + 2h - 16,000 = \$110,000$
Step 5. Solve the equation.	$h + 2h - 16,000 = \$110,000$
Combine like terms.	$3h - 16,000 = \$110,000$
Add 16,000 to both sides and simplify.	3h = 126,000
Divide each side by 3.	h = 42,000
	$42,000 amount husband earns
	2h − 16,000 amount wife earns
	2(42,000) − 16,000
	84,000 − 16,000
	68,000
Step 6. Check.	If the wife earns $68,000 and the husband earns $42,000 is the total $110,000? Yes!
Step 7. Answer the question.	The husband earns $42,000 a year.

TRY IT 11.1

According to the National Automobile Dealers Association, the average cost of a car in 2014 was 28,500. This was 1,500 less than 6 times the cost in 1975. What was the average cost of a car in 1975?

Show answer
5,000

TRY IT 11.2

The Canadian Real Estate Association (CREA) data shows that the median price of new home in the Canada in December 2018 was $470,000. This was $14,000 more than 19 times the price in December 1967. What was the median price of a new home in December 1967?

Show answer
$24,000

Key Concepts

- **Problem-Solving Strategy**
  1. **Read** the problem. Make sure all the words and ideas are understood.
  2. **Identify** what we are looking for.
  3. **Name** what we are looking for. Choose a variable to represent that quantity.
  4. **Translate** into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  5. **Solve** the equation using good algebra techniques.
  6. **Check** the answer in the problem and make sure it makes sense.
  7. **Answer** the question with a complete sentence.
- **Consecutive Integers**
  Consecutive integers are integers that immediately follow each other.
  $$\begin{array}{cc} n & 1^{\text{st}} \text{ integer} \\ n+1 & 2^{\text{nd}} \text{ integer consecutive integer} \\ n+2 & 3^{\text{rd}} \text{ consecutive integer } \ldots \text{ etc.} \end{array}$$
  Consecutive even integers are even integers that immediately follow one another.
  $$\begin{array}{cc} n & 1^{\text{st}} \text{ integer} \\ n+2 & 2^{\text{nd}} \text{ integer consecutive integer} \\ n+4 & 3^{\text{rd}} \text{ consecutive integer } \ldots \text{ etc.} \end{array}$$
  Consecutive odd integers are odd integers that immediately follow one another.

$$\begin{array}{rc} n & 1^{\text{st}} \text{ integer} \\ n+2 & 2^{\text{nd}} \text{ integer consecutive integer} \\ n+4 & 3^{\text{rd}} \text{ consecutive integer . . . etc.} \end{array}$$

Practice Makes Perfect

Use the Approach Word Problems with a Positive Attitude

In the following exercises, prepare the lists described.

1. List five positive thoughts you can say to yourself that will help you approach word problems with a positive attitude. You may want to copy them on a sheet of paper and put it in the front of your notebook, where you can read them often.	2. List five negative thoughts that you have said to yourself in the past that will hinder your progress on word problems. You may want to write each one on a small piece of paper and rip it up to symbolically destroy the negative thoughts.

Use a Problem-Solving Strategy for Word Problems

In the following exercises, solve using the problem solving strategy for word problems. Remember to write a complete sentence to answer each question.

3. Two-thirds of the children in the fourth-grade class are girls. If there are 20 girls, what is the total number of children in the class?	4. Three-fifths of the members of the school choir are women. If there are 24 women, what is the total number of choir members?
5. Zachary has 25 country music CDs, which is one-fifth of his CD collection. How many CDs does Zachary have?	6. One-fourth of the candies in a bag of M&M's are red. If there are 23 red candies, how many candies are in the bag?
7. There are 16 girls in a school club. The number of girls is four more than twice the number of boys. Find the number of boys.	8. There are 18 Cub Scouts in Pack 645. The number of scouts is three more than five times the number of adult leaders. Find the number of adult leaders.
9. Huong is organizing paperback and hardback books for her club's used book sale. The number of paperbacks is 12 less than three times the number of hardbacks. Huong had 162 paperbacks. How many hardback books were there?	10. Jeff is lining up children's and adult bicycles at the bike shop where he works. The number of children's bicycles is nine less than three times the number of adult bicycles. There are 42 adult bicycles. How many children's bicycles are there?
11. Philip pays $1,620 in rent every month. This amount is $120 more than twice what his brother Paul pays for rent. How much does Paul pay for rent?	12. Marc just bought an SUV for $54,000. This is $7,400 less than twice what his wife paid for her car last year. How much did his wife pay for her car?
13. Laurie has $46,000 invested in stocks and bonds. The amount invested in stocks is $8,000 less than three times the amount invested in bonds. How much does Laurie have invested in bonds?	14. Erica earned a total of $50,450 last year from her two jobs. The amount she earned from her job at the store was $1,250 more than three times the amount she earned from her job at the college. How much did she earn from her job at the college?

Solve Number Problems

In the following exercises, solve each number word problem.

15. The sum of a number and eight is 12. Find the number.	16. The sum of a number and nine is 17. Find the number.
17. The difference of a number and 12 is three. Find the number.	18. The difference of a number and eight is four. Find the number.
19. The sum of three times a number and eight is 23. Find the number.	20. The sum of twice a number and six is 14. Find the number.
21.The difference of twice a number and seven is 17. Find the number.	22. The difference of four times a number and seven is 21. Find the number.
23. Three times the sum of a number and nine is 12. Find the number.	24. Six times the sum of a number and eight is 30. Find the number.
25. One number is six more than the other. Their sum is 42. Find the numbers.	26. One number is five more than the other. Their sum is 33. Find the numbers.
27. The sum of two numbers is 20. One number is four less than the other. Find the numbers.	28. The sum of two numbers is 27. One number is seven less than the other. Find the numbers.
29. The sum of two numbers is -45. One number is nine more than the other. Find the numbers.	30. The sum of two numbers is -61. One number is 35 more than the other. Find the numbers.
31. The sum of two numbers is -316. One number is 94 less than the other. Find the numbers.	32. The sum of two numbers is -284. One number is 62 less than the other. Find the numbers.
33. One number is 14 less than another. If their sum is increased by seven, the result is 85. Find the numbers.	34. One number is 11 less than another. If their sum is increased by eight, the result is 71. Find the numbers.
35. One number is five more than another. If their sum is increased by nine, the result is 60. Find the numbers.	36. One number is eight more than another. If their sum is increased by 17, the result is 95. Find the numbers.
37. One number is one more than twice another. Their sum is -5. Find the numbers.	38. One number is six more than five times another. Their sum is six. Find the numbers.
39. The sum of two numbers is 14. One number is two less than three times the other. Find the numbers.	40. The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.
41. The sum of two consecutive integers is 77. Find the integers.	42. The sum of two consecutive integers is 89. Find the integers.
43. The sum of two consecutive integers is -23. Find the integers.	44. The sum of two consecutive integers is -37. Find the integers.
45. The sum of three consecutive integers is 78. Find the integers.	46. The sum of three consecutive integers is 60. Find the integers.
47. Find three consecutive integers whose sum is -36.	48. Find three consecutive integers whose sum is -3.
49. Find three consecutive even integers whose sum is 258.	50. Find three consecutive even integers whose sum is 222.
51. Find three consecutive odd integers whose sum is 171.	52. Find three consecutive odd integers whose sum is 291.

53. Find three consecutive even integers whose sum is -36.	54. Find three consecutive even integers whose sum is -84.
55. Find three consecutive odd integers whose sum is -213.	56. Find three consecutive odd integers whose sum is -267.

Everyday Math

57. **Sale Price.** Patty paid $35 for a purse on sale for $10 off the original price. What was the original price of the purse?	58. **Sale Price.** Travis bought a pair of boots on sale for $25 off the original price. He paid $60 for the boots. What was the original price of the boots?
59. **Buying in Bulk.** Minh spent $6.25 on five sticker books to give his nephews. Find the cost of each sticker book.	60. **Buying in Bulk.** Alicia bought a package of eight peaches for $3.20. Find the cost of each peach.
61. **Price before Sales Tax.** Tom paid $1,166.40 for a new refrigerator, including $86.40 tax. What was the price of the refrigerator?	62. **Price before Sales Tax.** Kenji paid $2,279 for a new living room set, including $129 tax. What was the price of the living room set?

Writing Exercises

63. What has been your past experience solving word problems?	64. When you start to solve a word problem, how do you decide what to let the variable represent?
65. What are consecutive odd integers? Name three consecutive odd integers between 50 and 60.	66. What are consecutive even integers? Name three consecutive even integers between -50 and -40.

Answers

1. Answers will vary	3. 30	5. 125
7. 6	9. 58	11. $750
13. $13,500	15. 4	17. 15
19. 5	21. 12	23. -5
25. 18, 24	27. 8, 12	29. $-18, -27$
31. $-111, -205$	33. 32, 46	35. 23, 28
37. $-2, -3$	39. 4, 10	41. 38, 39
43. $-11, -12$	45. 25, 26, 27	47. $-11, -12, -13$
49. 84, 86, 88	51. 55, 57, 59	53. $-10, -12, -14$
55. $-69, -71, -73$	57. $45	59. $1.25
61. $1080	63. Answers will vary	65. Consecutive odd integers are odd numbers that immediately follow each other. An example of three consecutive odd integers between 50 and 60 would be 51, 53, and 55.

Attributions

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3.8 Chapter Review

Review Exercises

Verify a Solution of an Equation

In the following exercises, determine whether each number is a solution to the equation.

1. $w-8=5,\ w=3$	2. $x+16=31,\ x=15$
3. $4a=72,\ a=18$	4. $-9n=45,\ n=54$

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, solve each equation using the Subtraction Property of Equality.

5. $y+2=-6$	6. $x+7=19$
7. $n+3.6=5.1$	8. $a+\frac{1}{3}=\frac{5}{3}$

In the following exercises, solve each equation using the Addition Property of Equality.

9. $x-9=-4$	10. $u-7=10$
11. $p-4.8=14$	12. $c-\frac{3}{11}=\frac{9}{11}$

In the following exercises, solve each equation.

13. $y+16=-9$	14. $n-12=32$
15. $d-3.9=8.2$	16. $f+\frac{2}{3}=4$

Solve Equations That Require Simplification

In the following exercises, solve each equation.

17. $7x+10-6x+3=5$	18. $y+8-15=-3$
19. $8(3p+5)-23(p-1)=35$	20. $6(n-1)-5n=-14$

Translate to an Equation and Solve

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

21. Four less than n is 13.	22. The sum of -6 and m is 25.

Translate and Solve Applications

In the following exercises, translate into an algebraic equation and solve.

23. Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?	24. Rochelle's daughter is 11 years old. Her son is 3 years younger. How old is her son?
25. Elissa earned $152.84 this week, which was $21.65 more than she earned last week. How much did she earn last week?	26. Peter paid $9.75 to go to the movies, which was $46.25 less than he paid to go to a concert. How much did he pay for the concert?

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the division and multiplication properties of equality and check the solution.

27. $13a=-65$	28. $8x=72$
29. $-y=4$	30. $0.25p=5.25$
31. $\frac{y}{-10}=30$	32. $\frac{n}{6}=18$
33. $\frac{5}{8}u=\frac{15}{16}$	34. $36=\frac{3}{4}x$
35. $\frac{c}{9}=36$	36. $-18m=-72$
37. $\frac{11}{12}=\frac{2}{3}y$	38. $0.45x=6.75$

Solve Equations That Require Simplification

In the following exercises, solve each equation requiring simplification.

39. $24x+8x-11x=-7-14$	40. $5r-3r+9r=35-2$
41. $-9\left(d-2\right)-15=-24$	42. $\frac{11}{12}n-\frac{5}{6}n=9-5$

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

43. The quotient of *b* and and 9 is -27.	44. 143 is the product of -11 and *y*.
45. The difference of *s* and one-twelfth is one fourth.	46. The sum of *q* and one-fourth is one.

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

47. Janet gets paid \$24 per hour. She heard that this is $\frac{3}{4}$ of what Adam is paid. How much is Adam paid per hour?	48. Ray paid \$21 for 12 tickets at the county fair. What was the price of each ticket?

Solve an Equation with Constants on Both Sides

In the following exercises, solve the following equations with constants on both sides.

49. $10w-5=65$	50. $8p+7=47$
51. $32=-4-9n$	52. $3x+19=-47$

Solve an Equation with Variables on Both Sides

In the following exercises, solve the following equations with variables on both sides.

53. $5a+21=2a$	54. $7y=6y-13$
55. $4x-\frac{3}{8}=3x$	56. $k=-6k-35$

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the following equations with variables and constants on both sides.

57. $5n-20=-7n-80$	58. $12x-9=3x+45$
59. $\frac{5}{8}c-4=\frac{3}{8}c+4$	60. $4u+16=-19-u$

Solve Equations Using the General Strategy for Solving Linear Equations

In the following exercises, solve each linear equation.

61. $9\left(2p-5\right)=72$	62. $6\left(x+6\right)=24$
63. $8+3\left(n-9\right)=17$	64. - $\left(s+4\right)=18$
65. $\frac{1}{3}\left(6m+21\right)=m-7$	66. $23-3\left(y-7\right)=8$
67. $0.25\left(q-8\right)=0.1\left(q+7\right)$	68. $4\left(3.5y+0.25\right)=365$
69. $5+7\left(2-5x\right)=2\left(9x+1\right)$ $-\left(13x-57\right)$	70. $8\left(r-2\right)=6\left(r+10\right)$
71. $2\left[-16+5\left(8k-6\right)\right]=8\left(3-4k\right)-32$	72. $\left(9n+5\right)-\left(3n-7\right)=20-\left(4n-2\right)$

Classify Equations

In the following exercises, classify each equation as a conditional equation, an identity, or a contradiction and then state the solution.

73. $9u+32=15\left(u-4\right)-3\left(2u+21\right)$	74. $17y-3\left(4-2y\right)=11\left(y-1\right)+12y-1$
75. $21\left(c-1\right)-19\left(c+1\right)=2\left(c-20\right)$	76. $-8\left(7m+4\right)=-6\left(8m+9\right)$

Solve Equations with Fraction Coefficients

In the following exercises, solve each equation with fraction coefficients.

77. $\frac{1}{3}x+\frac{1}{5}x=8$	78. $\frac{2}{5}n-\frac{1}{10}=\frac{7}{10}$
79. $\frac{1}{2}\left(k-3\right)=\frac{1}{3}\left(k+16\right)$	80. $\frac{3}{4}a-\frac{1}{3}=\frac{1}{2}a-\frac{5}{6}$
81. $\frac{5y-1}{3}+4=\frac{-8y+4}{6}$	82. $\frac{3x-2}{5}=\frac{3x+4}{8}$

Solve Equations with Decimal Coefficients

In the following exercises, solve each equation with decimal coefficients.

83. $0.36u + 2.55 = 0.41u + 6.8$	84. $0.8x - 0.3 = 0.7x + 0.2$

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

85. Mallory is taking the bus from Edmonton to North Battleford. The distance is 300 miles and the bus travels at a steady rate of 60 miles per hour. How long will the bus ride be?	86. Natalie drove for $7\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?
87. Link rode his bike at a steady rate of 15 miles per hour for $2\frac{1}{2}$ hours. How much distance did he travel?	88. Aaron's friend drove him from Williams Lake to Kamloops. The distance is 187 miles and the trip took 2.75 hours. How fast was Aaron's friend driving?

Solve a Formula for a Specific Variable

In the following exercises, solve.

89. Use the formula. $d = rt$ to solve for r a) when when $d = 451$ and $t = 5.5$ b) in general	90. Use the formula. $d = rt$ to solve for t a) when $d = 510$ and $r = 60$ b) in general
91. Use the formula $A = \frac{1}{2}bh$ to solve for h a) when $A = 153$ and $b = 18$ b) in general	92. Use the formula $A = \frac{1}{2}bh$ to solve for b a) when $A = 390$ and $h = 26$ b) in general
93. Solve the formula $4x + 3y = 6$ for y a) when $x = -2$ b) in general	94. Use the formula $I = Prt$ to solve for the principal, P for a) $I = \$2,501$, $r = 4.1\%$, $t = 5$ years b) in general
95. Solve the formula $V = LWH$ for H.	96. Solve $180 = a + b + c$ for c.

Everyday Math

97. Describe how you have used two topics from this chapter in your life outside of your math class during the past month.

Review Exercises Answers

1. no	3. yes
5. $y=-8$	7. $n=1.5$
9. $x=5$	11. $p=18.8$
13. $y=-25$	15. $d=12.1$
17. $x=-8$	19. $p=-28$
21. $n-4=13; n=17$	23. 161 pounds
25. $131.19	27. $a=-5$
29. $y=-4$	31. $y=-300$
33. $u=\frac{3}{2}$	35. $c=324$
37. $y=\frac{11}{8}$	39. $x=-1$
41. $d=3$	43. $\frac{b}{9}=-27; b=-243$
45. $s-\frac{1}{12}=\frac{1}{4}; s=\frac{1}{3}$	47. $32
49. $w=7$	51. $n=-4$
53. $a=-7$	55. $x=\frac{3}{8}$
57. $n=-5$	59. $c=32$
61. $p=\frac{13}{2}$	63. $n=12$
65. $m=-14$	67. $q=18$
69. $x=-1$	71. $k=\frac{3}{4}$
73. contradiction; no solution	75. identity; all real numbers
77. $x=15$	79. $k=41$
81. $y=-1$	83. $u=-85$
85. 5 hours	87. 37.5 miles
89. a)$r=82\text{ mph}$; b)$r=\frac{D}{t}$	91. a)$h=17$b)$h=\frac{2A}{b}$
93. a)$y=\frac{14}{3}$b)$y=\frac{6-4x}{3}$	95. $H=\frac{V}{LW}$

Practice Test

Determine whether each number is a solution to the equation $3x+5=20$.

1. a) 5 b) $\frac{23}{5}$

In the following exercises, solve each equation.

2. $n-18=31$	3. $9c=144$
4. $4y-8=16$	5. $-8x-15+9x-1=-21$
6. $-15a=120$	7. $\frac{2}{3}x=6$
8. $x-3.8=8.2$	9. $10y=-5y-60$
10. $8n-2=6n-12$	11. $9m-2-4m-m=42-8$
12. $-5\left(2x-1\right)=45$	13. - $\left(d-9\right)=23$
14. $\frac{1}{4}\left(12m-28\right)=6-2\left(3m-1\right)$	15. $2\left(6x-5\right)-8=-22$
16.$8\left(3a-5\right)-7\left(4a-3\right)=20-3a$	17. $\frac{1}{4}p-\frac{1}{3}=\frac{1}{2}$
18. $0.1d+0.25\left(d+8\right)=4.1$	19. $14n-3\left(4n+5\right)=-9+2\left(n-8\right)$
20. $9\left(3u-2\right)-4\left[6-8\left(u-1\right)\right]$ $=3\left(u-2\right)$	21. Solve the formula $x-2y=5$ for *y* a) when $x=-3$ b) in general
22. Samuel paid \$25.82 for gas this week, which was \$3.47 less than he paid last week. How much had he paid last week?	

Practice Test Answers

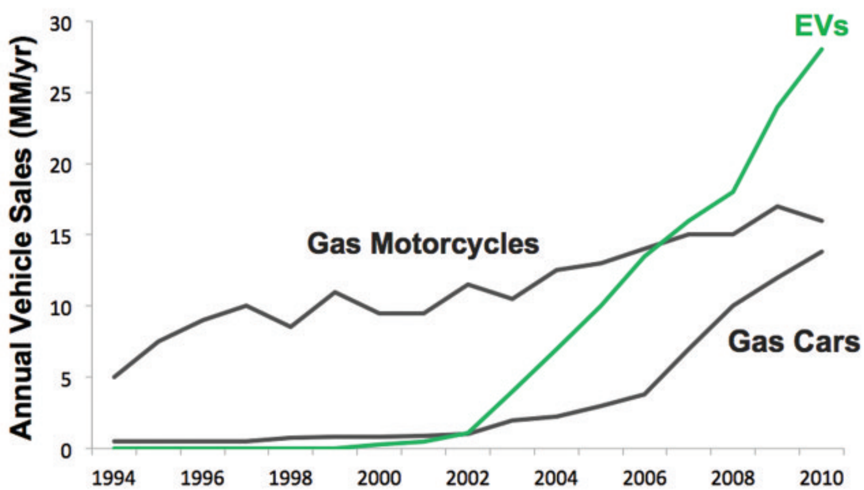
1. a) yes b) no	2. $n = 49$
3. $c = 16$	4. $y = 6$
5. $x = -5$	6. $a = -8$
7. $x = 9$	8. $x = 12$
9. $y = -4$	10. $n = -5$
11. $m = 9$	12. $x = -4$
13. $d = -14$	14. $m = \frac{5}{3}$
15. $x = -\frac{1}{3}$	16. $a = -39$
17. $p = \frac{10}{3}$	18. $d = 6$
19. contradiction; no solution	20. $u = \frac{17}{14}$
21. a) $y = 4$ b) $y = \frac{5-x}{2}$	22. $\$29.29$

Attributions

II

CHAPTER 4 Linear Equations and Graphing

This graph illustrates the annual vehicle sales of gas motorcycles, gas cars, and electric vehicles from 1994 to 2010. It is a line graph with *x*– and *y*-axes, one of the most common types of graphs. (credit: Steve Jurvetson, Flickr)



Graphs are found in all areas of our lives—from commercials showing you which cell phone carrier provides the best coverage, to bank statements and news articles, to the boardroom of major corporations. In this chapter, we will study the rectangular coordinate system, which is the basis for most consumer graphs. We will look at linear graphs, slopes of lines, and equations of lines.

Attributions

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4.1 Use the Rectangular Coordinate System

Learning Objectives

By the end of this section, you will be able to:

- Plot points in a rectangular coordinate system
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to a linear equation in two variables

Plot Points on a Rectangular Coordinate System

Just like maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a **rectangular coordinate system**. The rectangular coordinate system is also called the *xy*-plane or the 'coordinate plane.'

The horizontal number line is called the *x-axis*. The vertical number line is called the *y-axis.* The *x*-axis and the *y*-axis together form the rectangular coordinate system. These axes divide a plane into four regions, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See (Figure 1).

'Quadrant' has the root 'quad,' which means 'four.'

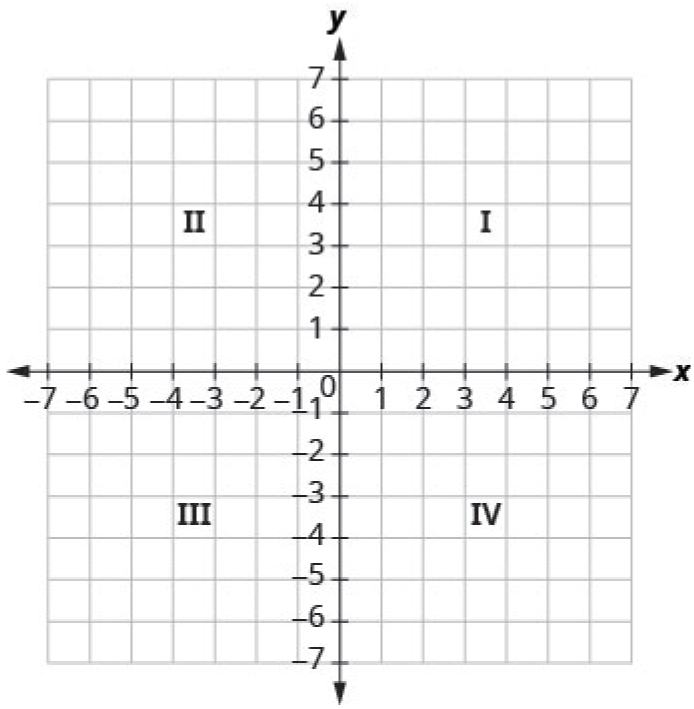

Figure .1

In the rectangular coordinate system, every point is represented by an *ordered pair*. The first number in the ordered pair is the ***x*-coordinate** of the point, and the second number is the ***y*-coordinate** of the point.

> **Ordered pair**
>
> An ordered pair, (x, y), gives the coordinates of a point in a rectangular coordinate system.
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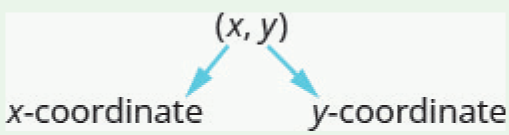
> 
>
>
> The first number is the x-coordinate.
>
> The second number is the y-coordinate.

The phrase 'ordered pair' means the order is important. What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point $(0, 0)$ has a special name. It is called the **origin**.

> **The origin**
>
> The point $(0, 0)$ is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

We use the coordinates to locate a point on the *xy*-plane. Let's plot the point $(1, 3)$ as an example. First, locate 1 on the *x*-axis and lightly sketch a vertical line through $x = 1$. Then, locate 3 on the *y*-axis and sketch a horizontal line through $y = 3$. Now, find the point where these two lines meet—that is the point with coordinates $(1, 3)$.

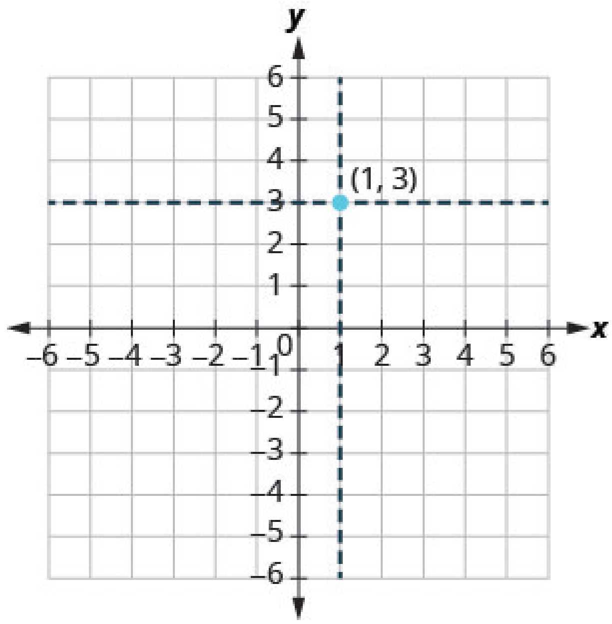


Figure .2

Notice that the vertical line through $x = 1$ and the horizontal line through $y = 3$ are not part of the graph. We just used them to help us locate the point $(1, 3)$.

EXAMPLE 1

Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located:

A $(-5, 4)$ B $(-3, -4)$ C $(2, -3)$ D $(-2, 3)$ E $\left(3, \frac{5}{2}\right)$.

Solution

The first number of the coordinate pair is the *x*-coordinate, and the second number is the *y*-coordinate.

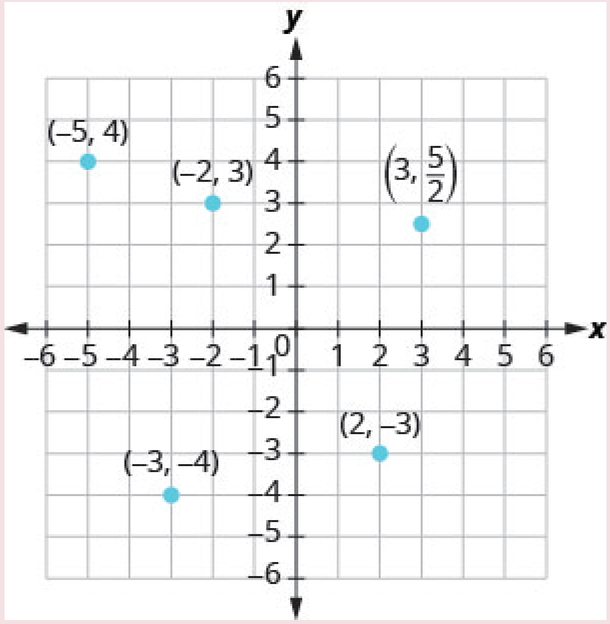
A. Since $x = -5$, the point is to the left of the *y*-axis. Also, since $y = 4$, the point is above the *x*-axis. The point $(-5, 4)$ is in Quadrant II.

B. Since $x = -3$, the point is to the left of the *y*-axis. Also, since $y = -4$, the point is below the *x*-axis. The point $(-3, -4)$ is in Quadrant III.

C. Since $x = 2$, the point is to the right of the *y*-axis. Since $y = -3$, the point is below the *x*-axis. The point $(2, -3)$ is in Quadrant lV.

D. Since $x = -2$, the point is to the left of the *y*-axis. Since $y = 3$, the point is above the *x*-axis. The point $(-2, 3)$ is in Quadrant II.

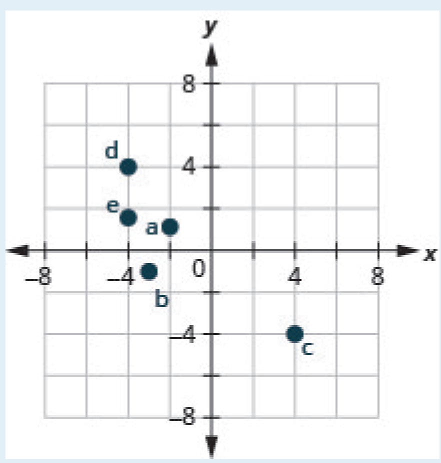
E. Since $x = 3$, the point is to the right of the *y*-axis. Since $y = \frac{5}{2}$, the point is above the *x*-axis. (It may be helpful to write $\frac{5}{2}$ as a mixed number or decimal.) The point $\left(3, \frac{5}{2}\right)$ is in Quadrant I.



TRY IT 1.1

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located: A $(-2,1)$ B $(-3,-1)$ C $(4,-4)$ D $(-4,4)$ E $\left(-4,\frac{3}{2}\right)$.

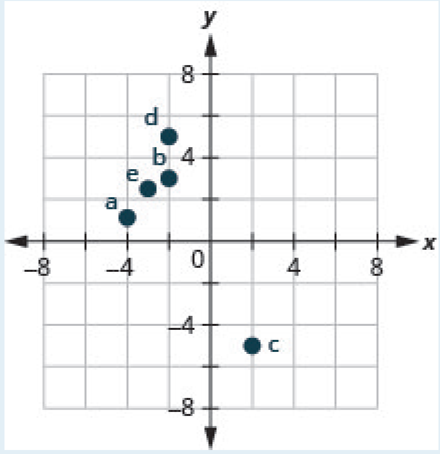
Show answer



TRY IT 1.2

Plot each point in a rectangular coordinate system and identify the quadrant in which the point is located: A $(-4,1)$ B $(-2,3)$ C $(2,-5)$ D $(-2,5)$ E $\left(-3,\frac{5}{2}\right)$

Show answer



How do the signs affect the location of the points? You may have noticed some patterns as you graphed the points in the previous example.

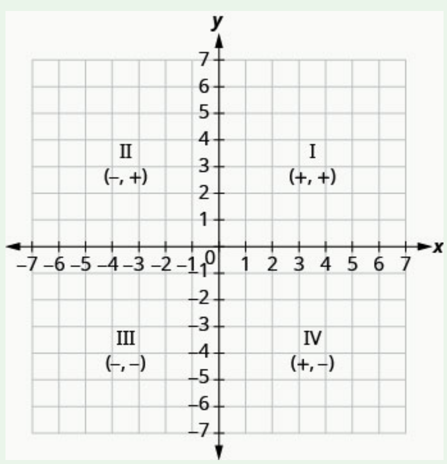
For the point in (Figure 2) in Quadrant IV, what do you notice about the signs of the coordinates? What about the signs of the coordinates of points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point $(-2, 5)$ is located? In which quadrant is $(2, -5)$ located?

Quadrants

We can summarize sign patterns of the quadrants in this way.

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x, y)	(x, y)	(x, y)	(x, y)
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$



What if one coordinate is zero as shown in (Figure 3)? Where is the point $(0, 4)$ located? Where is the point $(-2, 0)$ located?

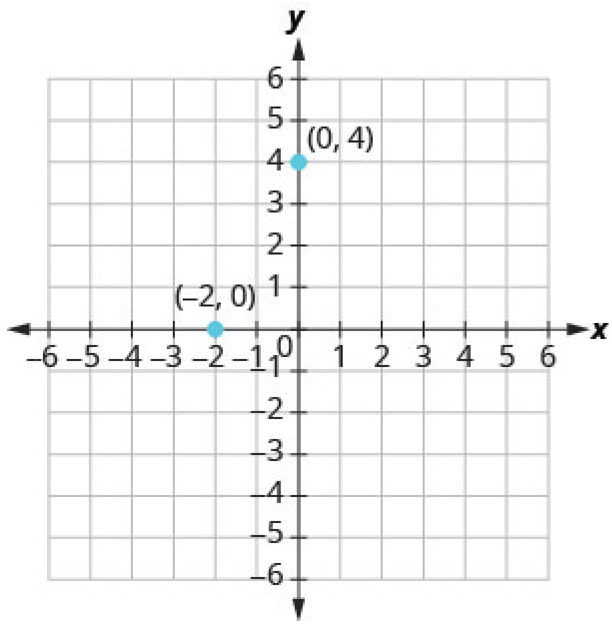


Figure .3

The point $(0, 4)$ is on the *y*-axis and the point $(-2, 0)$ is on the *x*-axis.

Points on the axes

Points with a *y*-coordinate equal to 0 are on the *x*-axis, and have coordinates $(a, 0)$.

Points with an *x*-coordinate equal to 0 are on the *y*-axis, and have coordinates $(0, b)$.

EXAMPLE 2

Plot each point:A $(0, 5)$ B $(4, 0)$ C $(-3, 0)$ D $(0, 0)$ E $(0, -1)$.

Solution

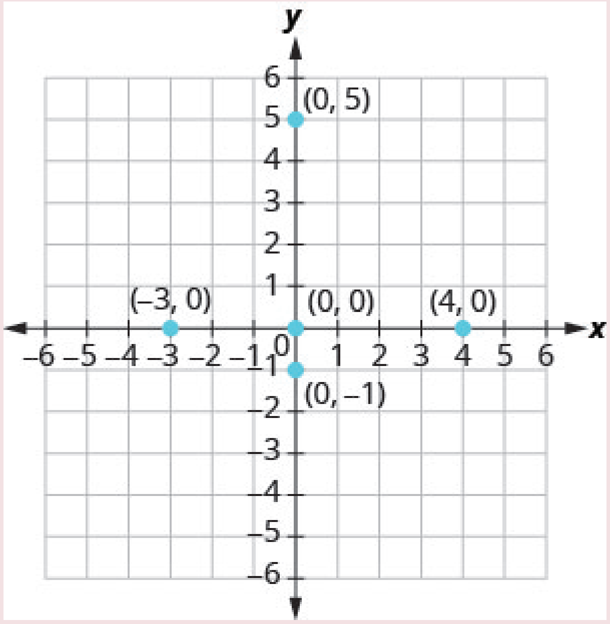
A. Since $x = 0$, the point whose coordinates are $(0, 5)$ is on the *y*-axis.

B. Since $y = 0$, the point whose coordinates are $(4, 0)$ is on the *x*-axis.

C. Since $y = 0$, the point whose coordinates are $(-3, 0)$ is on the *x*-axis.

D. Since $x = 0$ and $y = 0$, the point whose coordinates are $(0, 0)$ is the origin.

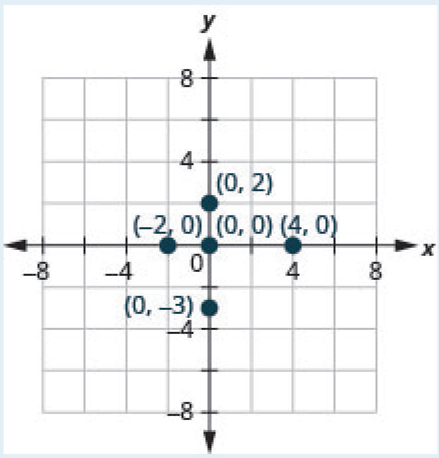
E. Since $x = 0$, the point whose coordinates are $(0, -1)$ is on the *y*-axis.



TRY IT 2.1

Plot each point: A $(4, 0)$ B $(-2, 0)$ C $(0, 0)$ D $(0, 2)$ E $(0, -3)$.

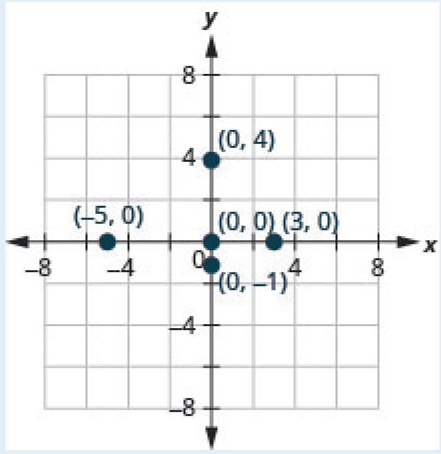
Show answer



TRY IT 2.2

Plot each point: A $(-5, 0)$ B $(3, 0)$ C $(0, 0)$ D $(0, -1)$ E $(0, 4)$.

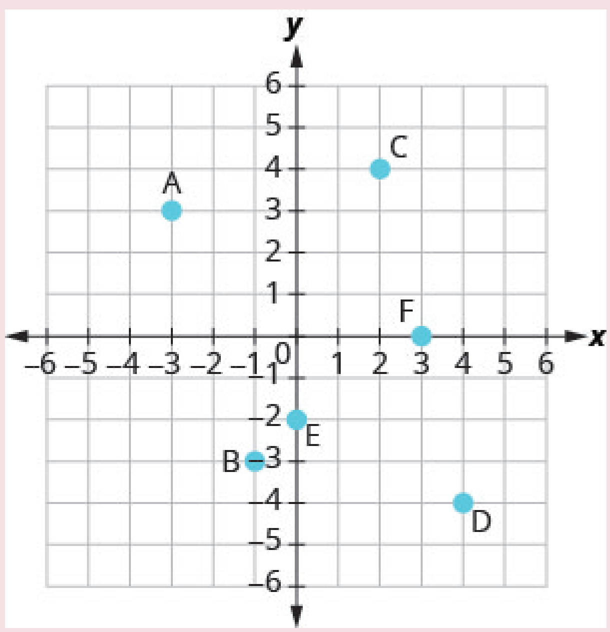
Show answer



In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the *x*-coordinate of a point on a graph, read the number on the *x*-axis directly above or below the point. To identify the *y*-coordinate of a point, read the number on the *y*-axis directly to the left or right of the point. Remember, when you write the ordered pair use the correct order, (x, y).

EXAMPLE 3

Name the ordered pair of each point shown in the rectangular coordinate system.



Solution

Point A is above -3 on the *x*-axis, so the *x*-coordinate of the point is -3.

- The point is to the left of 3 on the *y*-axis, so the *y*-coordinate of the point is 3.

- The coordinates of the point are $(-3, 3)$.

Point B is below -1 on the *x*-axis, so the *x*-coordinate of the point is -1.

- The point is to the left of -3 on the *y*-axis, so the *y*-coordinate of the point is -3.
- The coordinates of the point are $(-1, -3)$.

Point C is above 2 on the *x*-axis, so the *x*-coordinate of the point is 2

- The point is to the right of 4 on the *y*-axis, so the *y*-coordinate of the point is 4.
- The coordinates of the point are $(2, 4)$.

Point D is below 4 on the *x*-axis, so the *x*-coordinate of the point is 4

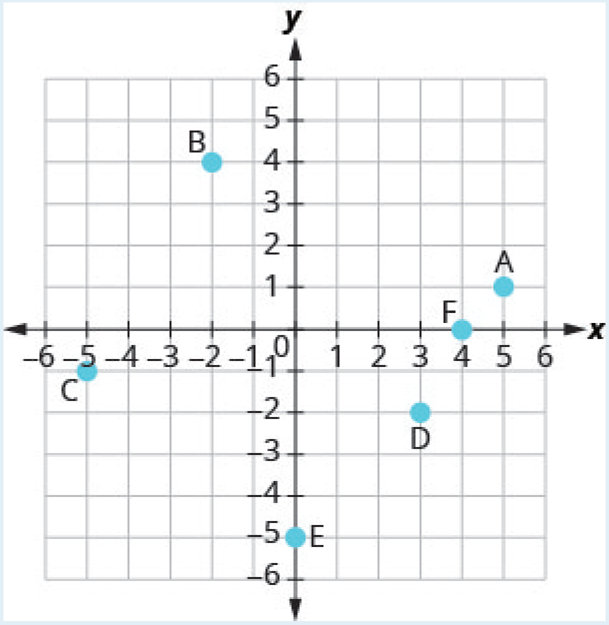
- The point is to the right of -4 on the *y*-axis, so the *y*-coordinate of the point is -4.
- The coordinates of the point are $(4, -4)$.

Point E is on the *y*-axis at $y = -2$. The coordinates of point E are $(0, -2)$.

Point F is on the *x*-axis at $x = 3$. The coordinates of point F are $(3, 0)$.

TRY IT 3.1

Name the ordered pair of each point shown in the rectangular coordinate system.

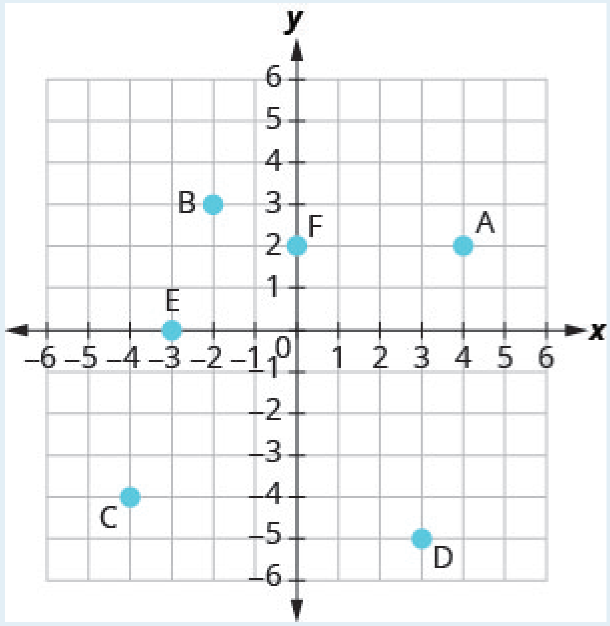


Show answer

A: $(5, 1)$ B: $(-2, 4)$ C: $(-5, -1)$ D: $(3, -2)$ E: $(0, -5)$ F: $(4, 0)$

TRY IT 3.2

Name the ordered pair of each point shown in the rectangular coordinate system.



Show answer
A: $(4, 2)$ B: $(-2, 3)$ C: $(-4, -4)$ D: $(3, -5)$ E: $(-3, 0)$ F: $(0, 2)$

Verify Solutions to an Equation in Two Variables

Up to now, all the equations you have solved were equations with just one variable. In almost every case, when you solved the equation you got exactly one solution. The process of solving an equation ended with a statement like $x = 4$. (Then, you checked the solution by substituting back into the equation.)

Here's an example of an equation in one variable, and its one solution.

$$\begin{aligned} 3x + 5 &= 17 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

But equations can have more than one variable. Equations with two variables may be of the form $Ax + By = C$. Equations of this form are called **linear equations in two variables**.

Linear equation

An equation of the form $Ax + By = C$, where A and B are not both zero, is called a linear equation **in two variables**.

Notice the word *line* in **linear**. Here is an example of a linear equation in two variables, x and y.

$$Ax + By = C$$
$$x + 4y = 8$$
$$A = 1,\ B = 4,\ C = 8$$

The equation $y = -3x + 5$ is also a linear equation. But it does not appear to be in the form $Ax + By = C$. We can use the Addition Property of Equality and rewrite it in $Ax + By = C$ form.

	$y = -3x + 5$
Add to both sides.	$y + 3x = -3x + 5 + 3x$
Simplify.	$y + 3x = 5$
Use the Commutative Property to put it in $Ax + By = C$ form.	$3x + y = 5$

By rewriting $y = -3x + 5$ as $3x + y = 5$, we can easily see that it is a linear equation in two variables because it is of the form $Ax + By = C$. When an equation is in the form $Ax + By = C$, we say it is in *standard form*.

Standard Form of Linear Equation

A linear equation is in standard form when it is written $Ax + By = C$.

Most people prefer to have A, B, and C be integers and $A \geq 0$ when writing a linear equation in standard form, although it is not strictly necessary.

Linear equations have infinitely many solutions. For every number that is substituted for x there is a corresponding y value. This pair of values is a *solution* to the linear equation and is represented by the ordered pair (x, y). When we substitute these values of x and y into the equation, the result is a true statement, because the value on the left side is equal to the value on the right side.

Solution of a Linear Equation in Two Variables

An ordered pair (x, y) is a **solution** of the linear equation $Ax + By = C$, if the equation is a true statement when the *x*– and *y*-values of the ordered pair are substituted into the equation.

EXAMPLE 4

Determine which ordered pairs are solutions to the equation $x + 4y = 8$.

A $(0, 2)$ B $(2, -4)$ C $(-4, 3)$

Solution

Substitute the x- and y-values from each ordered pair into the equation and determine if the result is a true statement.

(a)	(b)	(c)
(0, 2)	(2, –4)	(–4, 3)
$x = 0, y = 2$	$x = 2, y = -4$	$x = -4, y = 3$
$x + 4y = 8$	$x + 4y = 8$	$x + 4y = 8$
$0 + 4 \cdot 2 \stackrel{?}{=} 8$	$2 + 4(-4) \stackrel{?}{=} 8$	$-4 + 4 \cdot 3 \stackrel{?}{=} 8$
$0 + 8 \stackrel{?}{=} 8$	$2 + (-16) \stackrel{?}{=} 8$	$-4 + 12 \stackrel{?}{=} 8$
$8 = 8$ ✓	$-14 \neq 8$	$8 = 8$ ✓
(0, 2) is a solution.	(2, –4) is not a solution.	(–4, 3) is a solution.

TRY IT 4.1

Which of the following ordered pairs are solutions to $2x + 3y = 6$?
A $(3, 0)$ B $(2, 0)$ C $(6, -2)$

Show answer
A, C

TRY IT 4.2

Which of the following ordered pairs are solutions to the equation $4x - y = 8$? A $(0, 8)$ B $(2, 0)$ C $(1, -4)$

Show answer
B, C

EXAMPLE 5

Which of the following ordered pairs are solutions to the equation $y = 5x - 1$?

A $(0, -1)$ B $(1, 4)$ C $(-2, -7)$

Solution

Substitute the *x*– and *y*-values from each ordered pair into the equation and determine if it results in a true statement.

(a)	(b)	(c)
(0, –1)	(1, 4)	(–2, –7)
$x = 0, y = -1$	$x = 1, y = 4$	$x = -2, y = -7$
$y = 5x - 1$	$y = 5x - 1$	$y = 5x - 1$
$-1 \stackrel{?}{=} 5(0) - 1$	$4 \stackrel{?}{=} 5(1) - 1$	$-7 \stackrel{?}{=} 5(-2) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$4 \stackrel{?}{=} 5 - 1$	$-7 \stackrel{?}{=} -10 - 1$
$-1 = -1$ ✓	$4 = 4$ ✓	$-7 \neq -11$
(0, –1) is a solution.	(1, 4) is a solution.	(–2, –7) is not a solution.

TRY IT 5.1

Which of the following ordered pairs are solutions to the equation $y = 4x - 3$? A $(0, 3)$ B $(1, 1)$ C $(-1, -1)$

Show answer
B

TRY IT 5.2

Which of the following ordered pairs are solutions to the equation $y = -2x + 6$? A $(0, 6)$ B $(1, 4)$ C $(-2, -2)$

Show answer
A, B

Complete a Table of Solutions to a Linear Equation in Two Variables

In the examples above, we substituted the *x*– and *y*-values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do you find the ordered pairs if they are not given? It's easier than you might think—you can just pick a value for x and then solve the equation for y. Or, pick a value for y and then solve for x.

We'll start by looking at the solutions to the equation $y = 5x - 1$ that we found in (Example 5). We can summarize this information in a table of solutions, as shown in (Table 1).

Table 1

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$

To find a third solution, we'll let $x = 2$ and solve for y.

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$
Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

The ordered pair $(2, 9)$ is a solution to $y = 5x - 1$. We will add it to (Table 2).

Table 2

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$
2	9	$(2, 9)$

We can find more solutions to the equation by substituting in any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are infinitely many solutions of this equation.

EXAMPLE 6

Complete the table to find three solutions to the equation $y = 4x - 2$.

$y = 4x - 2$		
x	y	(x, y)
0		
-1		
2		

Solution
Substitute $x = 0$, $x = -1$, and $x = 2$ into $y = 4x - 2$.

$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = 0 - 2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in the table below.

$y = 4x - 2$		
x	y	(x, y)
0	-2	$(0, -2)$
-1	-6	$(-1, -6)$
2	6	$(2, 6)$

TRY IT 6.1

Complete the table to find three solutions to this equation: $y = 3x - 1$.

$y = 3x - 1$		
x	y	(x, y)
0		
-1		
2		

Show answer

$y = 3x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
-1	-4	$(-1, -4)$
2	5	$(2, 5)$

TRY IT 6.2

Complete the table to find three solutions to this equation: $y = 6x + 1$.

$y = 6x + 1$		
x	y	(x, y)
0		
1		
-2		

Show answer

$y = 6x + 1$		
x	y	(x, y)
0	1	$(0, 1)$
1	7	$(1, 7)$
-2	-11	$(-2, -11)$

EXAMPLE 7

Complete the table to find three solutions to the equation $5x - 4y = 20$.

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution

Substitute the given value into the equation $5x - 4y = 20$ and solve for the other variable. Then, fill in the values in the table.

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in the table below.

$5x - 4y = 20$		
x	y	(x, y)
0	-5	$(0, -5)$
4	0	$(4, 0)$
8	5	$(8, 5)$

TRY IT 7.1

Complete the table to find three solutions to this equation: $2x - 5y = 20$.

$2x - 5y = 20$		
x	y	(x, y)
0		
	0	
-5		

Show answer

$2x - 5y = 20$		
x	y	(x, y)
0	-4	$(0, -4)$
10	0	$(10, 0)$
-5	-6	$(-5, -6)$

TRY IT 7.2

Complete the table to find three solutions to this equation: $3x - 4y = 12$.

$3x - 4y = 12$		
x	y	(x, y)
0		
	0	
-4		

Show answer

$3x - 4y = 12$		
x	y	(x, y)
0	-3	$(0, -3)$
4	0	$(4, 0)$
-4	-6	$(-4, -6)$

Find Solutions to a Linear Equation

To find a solution to a linear equation, you really can pick *any* number you want to substitute into the equation for x or y. But since you'll need to use that number to solve for the other variable it's a good idea to choose a number that's easy to work with.

When the equation is in *y*-form, with the *y* by itself on one side of the equation, it is usually easier to choose values of x and then solve for y.

EXAMPLE 8

Find three solutions to the equation $y = -3x + 2$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in *y*-form, it will be easier to substitute in values of x. Let's pick $x = 0$, $x = 1$, and $x = -1$.

	$x = 0$	$x = 1$	$x = -1$
Substitute the value into the equation.	$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$
Simplify.	$y = -3 \cdot 0 + 2$	$y = -3 \cdot 1 + 2$	$y = -3(-1) + 2$
Simplify.	$y = 0 + 2$	$y = -3 + 2$	$y = 3 + 2$
Write the ordered pair.	$y = 2$	$y = -1$	$y = 5$
Check.	(0, 2)	(1, −1)	(−1, 5)
	$y = -3x + 2$	$y = -3x + 2$	$y = -3x + 2$
	$2 \stackrel{?}{=} -3 \cdot 0 + 2$	$-1 \stackrel{?}{=} -3 \cdot 1 + 2$	$5 \stackrel{?}{=} -3(-1) + 2$
	$2 \stackrel{?}{=} 0 + 2$	$-1 \stackrel{?}{=} -3 + 2$	$5 \stackrel{?}{=} 3 + 2$
	$2 = 2\checkmark$	$-1 = -1\checkmark$	$5 = 5\checkmark$

So, $(0, 2)$, $(1, -1)$ and $(-1, 5)$ are all solutions to $y = -3x + 2$. We show them in table below.

$y = -3x + 2$		
x	y	(x, y)
0	2	$(0, 2)$
1	-1	$(1, -1)$
-1	5	$(-1, 5)$

TRY IT 8.1

Find three solutions to this equation: $y = -2x + 3$.

Show answer
Answers will vary.

TRY IT 8.2

Find three solutions to this equation: $y = -4x + 1$.

Show answer
Answers will vary

We have seen how using zero as one value of x makes finding the value of y easy. When an equation is in standard form, with both the x and y on the same side of the equation, it is usually easier to first find one solution when $x = 0$ find a second solution when $y = 0$, and then find a third solution.

EXAMPLE 9

Find three solutions to the equation $3x + 2y = 6$.

Solution

We can substitute any value we want for x or any value for y. Since the equation is in standard form, let's pick first $x = 0$, then $y = 0$, and then find a third point.

	$x = 0$	$y = 0$	$x = 1$
	$3x + 2y = 6$	$3x + 2y = 6$	$3x + 2y = 6$
Substitute the value into the equation.	$3(0) + 2y = 6$	$3x + 2(0) = 6$	$3(1) + 2y = 6$
Simplify.	$0 + 2y = 6$	$3x + 0 = 6$	$3 + 2y = 6$
Solve.	$2y = 6$	$3x = 6$	$2y = 3$
	$y = 3$	$x = 2$	$y = \frac{3}{2}$
Write the ordered pair.	(0, 3)	(2, 0)	$\left(1, \frac{3}{2}\right)$
Check.	$3x + 2y = 6$	$3x + 2y = 6$	$3x + 2y = 6$
	$3 \cdot 0 + 2 \cdot 3 \stackrel{?}{=} 6$	$3 \cdot 2 + 2 \cdot 0 \stackrel{?}{=} 6$	$3 \cdot 1 + 2 \cdot \frac{3}{2} \stackrel{?}{=} 6$
	$0 + 6 \stackrel{?}{=} 6$	$6 + 0 \stackrel{?}{=} 6$	$3 + 3 \stackrel{?}{=} 6$
	$6 = 6$✓	$6 = 6$✓	$6 = 6$✓

So $(0, 3)$, $(2, 0)$, and $\left(1, \frac{3}{2}\right)$ are all solutions to the equation $3x + 2y = 6$. We can list these three solutions in the table below.

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$(1, \frac{3}{2})$

EXAMPLE 9.1

Find three solutions to the equation $2x + 3y = 6$.

Show answer
Answers will vary.

TRY IT 9.2

Find three solutions to the equation $4x + 2y = 8$.

Show answer
Answers will vary.

Glossary

linear equation
A linear equation is of the form $Ax + By = C$, where A and B are not both zero, is called a linear equation in two variables.

ordered pair
An ordered pair (x, y) gives the coordinates of a point in a rectangular coordinate system.

origin
The point $(0, 0)$ is called the origin. It is the point where the *x*-axis and *y*-axis intersect.

quadrant
The *x*-axis and the *y*-axis divide a plane into four regions, called quadrants.

rectangular coordinate system
A grid system is used in algebra to show a relationship between two variables; also called the *xy*-plane or the ‘coordinate plane.’

***x*-coordinate**
The first number in an ordered pair (x, y).

***y*-coordinate**
The second number in an ordered pair (x, y).

Practice Makes Perfect

Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

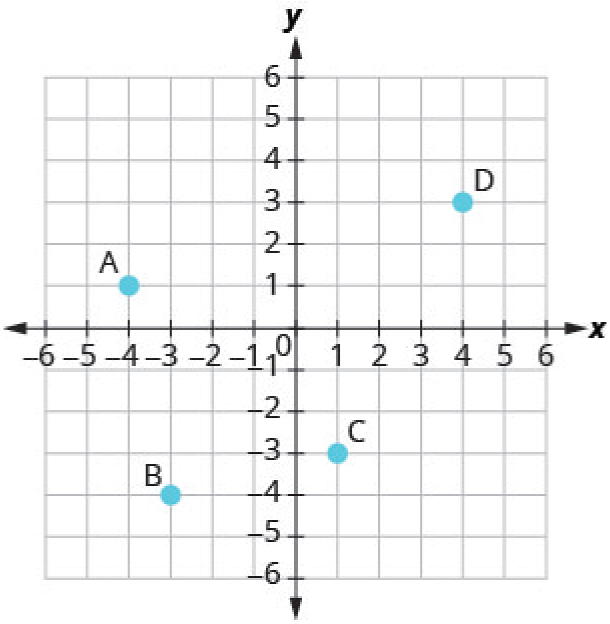
1.A $(-4, 2)$ B $(-1, -2)$ C $(3, -5)$ D $(-3, 5)$ E $\left(\frac{5}{3}, 2\right)$	2. A $(-2, -3)$ B $(3, -3)$ C $(-4, 1)$ D $(4, -1)$ E $\left(\frac{3}{2}, 1\right)$
3. A $(3, -1)$ B $(-3, 1)$ C $(-2, 2)$ D $(-4, -3)$ E $\left(1, \frac{14}{5}\right)$	4. A $(-1, 1)$ B $(-2, -1)$ C $(2, 1)$ D $(1, -4)$ E $\left(3, \frac{7}{2}\right)$

In the following exercises, plot each point in a rectangular coordinate system.

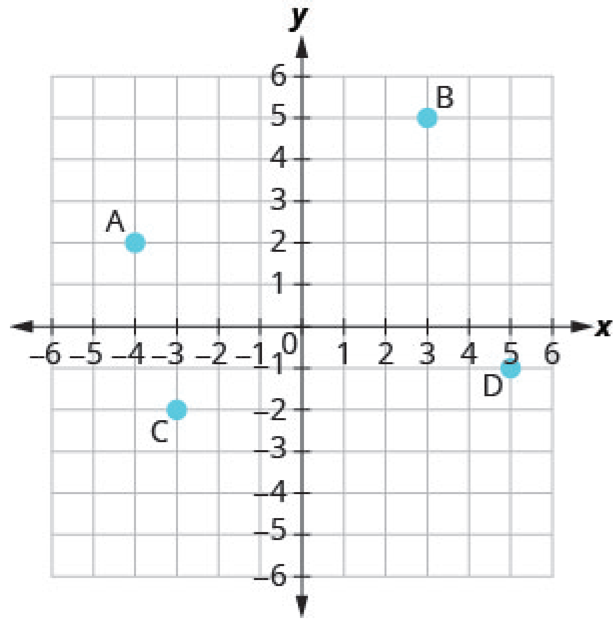
5. A $(-2, 0)$ B $(-3, 0)$ C $(0, 0)$ D $(0, 4)$ E $(0, 2)$	6. A $(0, 1)$ B $(0, -4)$ C $(-1, 0)$ D $(0, 0)$ E $(5, 0)$
7. A $(0, 0)$ B $(0, -3)$ C $(-4, 0)$ D $(1, 0)$ E $(0, -2)$	8. A $(-3, 0)$ B $(0, 5)$ C $(0, -2)$ D $(2, 0)$ E $(0, 0)$

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

9.



10.



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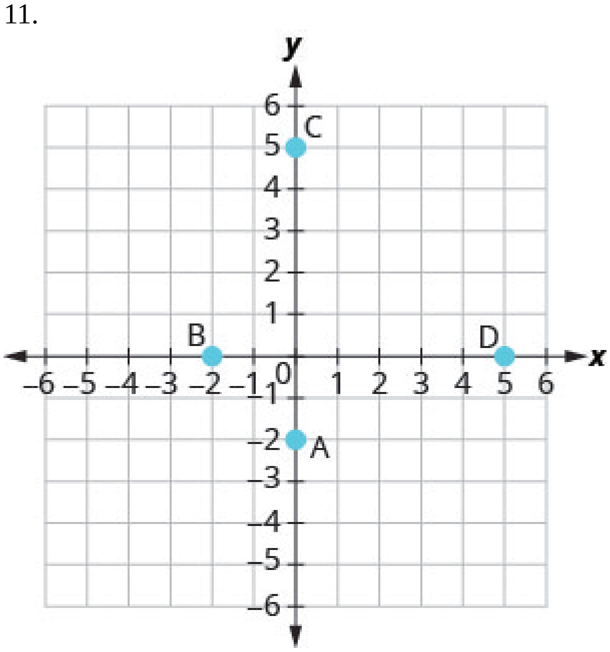

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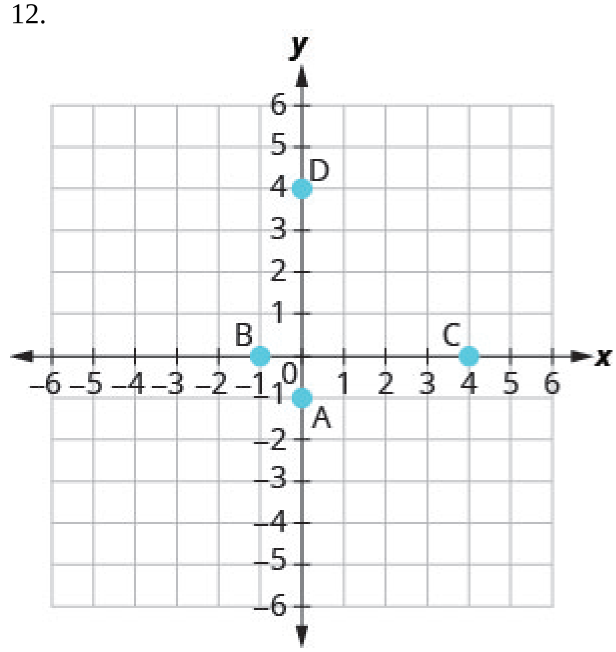


Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

13. $2x + y = 6$ A $(1, 4)$ B $(3, 0)$ C $(2, 3)$	14. $x + 3y = 9$ A $(0, 3)$ B $(6, 1)$ C $(-3, -3)$
15. $4x - 2y = 8$ A $(3, 2)$ B $(1, 4)$ C $(0, -4)$	16. $3x - 2y = 12$ A $(4, 0)$ B $(2, -3)$ C $(1, 6)$
17. $y = 4x + 3$ A $(4, 3)$ B $(-1, -1)$ C $\left(\frac{1}{2}, 5\right)$	18. $y = 2x - 5$ A $(0, -5)$ B $(2, 1)$ C $\left(\frac{1}{2}, -4\right)$
19. $y = \frac{1}{2}x - 1$ A $(2, 0)$ B $(-6, -4)$ C $(-4, -1)$	20. $y = \frac{1}{3}x + 1$ A $(-3, 0)$ B $(9, 4)$ C $(-6, -1)$

Complete a Table of Solutions to a Linear Equation

In the following exercises, complete the table to find solutions to each linear equation.

21. $y = 2x - 4$

x	y	(x, y)
0		
2		
-1		

22. $y = 3x - 1$

x	y	(x, y)
0		
2		
-1		

23. $y = -x + 5$

x	y	(x, y)
0		
3		
-2		

24. $y = -x + 2$

x	y	(x, y)
0		
3		
-2		

25. $y = \frac{1}{3}x + 1$

x	y	(x, y)
0		
3		
6		

26. $y = \frac{1}{2}x + 4$

x	y	(x, y)
0		
2		
4		

27. $y = -\frac{3}{2}x - 2$

x	y	(x, y)
0		
2		
-2		

28. $y = -\frac{2}{3}x - 1$

x	y	(x, y)
0		
3		
-3		

29. $x + 3y = 6$

x	y	(x, y)
0		
3		
	0	

30. $x + 2y = 8$

x	y	(x, y)
0		
4		
	0	

31. $2x - 5y = 10$

x	y	(x, y)
0		
10		
	0	

32. $3x - 4y = 12$

x	y	(x, y)
0		
8		
	0	

Find Solutions to a Linear Equation

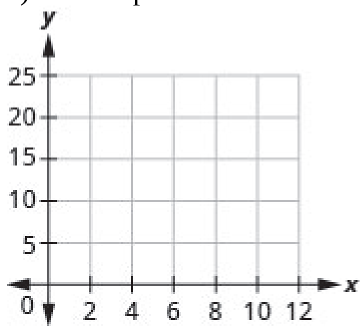
In the following exercises, find three solutions to each linear equation.

33. $y = 5x - 8$	34. $y = 3x - 9$
35. $y = -4x + 5$	36. $y = -2x + 7$
37. $x + y = 8$	38. $x + y = 6$
39. $x + y = -2$	40. $x + y = -1$
41. $3x + y = 5$	42. $2x + y = 3$
43. $4x - y = 8$	44. $5x - y = 10$
45. $2x + 4y = 8$	46. $3x + 2y = 6$
47. $5x - 2y = 10$	48. $4x - 3y = 12$

Everyday Math

49. Weight of a baby. Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

a) Plot the points on a coordinate plane.

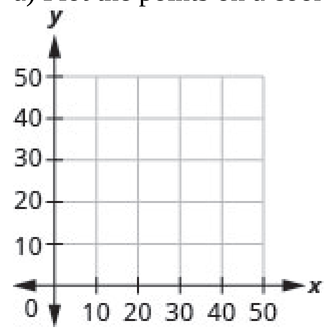


b) Why is only Quadrant I needed?

Age x	**Weight** y	(x, y)
0	7	(0, 7)
2	11	(2, 11)
4	15	(4, 15)
6	16	(6, 16)
8	19	(8, 19)
10	20	(10, 20)
12	21	(12, 21)

50. Weight of a child. Latresha recorded her son's height and weight every year. His height, in inches, and weight, in pounds, are listed in the table below, and shown as an ordered pair in the third column.

a) Plot the points on a coordinate plane.



b) Why is only Quadrant I needed?

Height x	**Weight** y	(x, y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)
37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

Writing Exercises

51. Explain in words how you plot the point $(4, -2)$ in a rectangular coordinate system.

52. How do you determine if an ordered pair is a solution to a given equation?

53. Is the point $(-3, 0)$ on the *x*-axis or *y*-axis? How do you know?

54. Is the point $(0, 8)$ on the *x*-axis or *y*-axis? How do you know?

Answers

<table>
<tr><td>1.</td><td>A graph plotting the points a (negative 4, 2), b (negative 1, negative 2), c (3, negative 5), d (negative 3, 5), e (5 thirds, 2).</td><td>3.</td><td>A graph plotting the points a (3, negative 1), b (negative 3, 1), c (negative 2, 2), d (negative 4, negative 3), e (1, 14 fifths).</td><td>5.</td><td>A graph plotting the points a (negative 2, 0), b (negative 3, (0, 0), d (0, 4), e (0, 3).</td></tr>
<tr><td>7.</td><td>A graph plotting the points a (0, 0), b (0, negative 3), c (negative 4, 0), d (1, 0), e (0, negative 2).</td><td>9.</td><td>A: $(-4,1)$ B: $(-3,-4)$ C: $(1,-3)$ D: $(4,3)$</td><td>11.</td><td>A: $(0,-2)$ B: $(-2,0)$ C</td></tr>
<tr><td>13.</td><td>A, B</td><td>15.</td><td>A, C</td><td>17.</td><td>B, C</td></tr>
<tr><td>21.</td><td><table><tr><td>x</td><td>y</td><td>(x,y)</td></tr><tr><td>0</td><td>-4</td><td>$(0,-4)$</td></tr><tr><td>2</td><td>0</td><td>$(2,0)$</td></tr><tr><td>-1</td><td>-6</td><td>$(-1,-6)$</td></tr></table></td><td>23.</td><td><table><tr><td>x</td><td>y</td><td>(x,y)</td></tr><tr><td>0</td><td>5</td><td>$(0,5)$</td></tr><tr><td>3</td><td>2</td><td>$(3,2)$</td></tr><tr><td>-2</td><td>7</td><td>$(-2,7)$</td></tr></table></td><td>25.</td><td><table><tr><td>x</td><td>y</td><td>(x,y)</td></tr><tr><td>0</td><td>1</td><td>$(0,1)$</td></tr><tr><td>3</td><td>2</td><td>$(3,2)$</td></tr><tr><td>6</td><td>3</td><td>$(6,3)$</td></tr></table></td></tr>
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<table>
<tr>
<td>27.</td>
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<table>
<tr><td>x</td><td>y</td><td>(x, y)</td></tr>
<tr><td>0</td><td>-2</td><td>$(0, -2)$</td></tr>
<tr><td>2</td><td>-5</td><td>$(2, -5)$</td></tr>
<tr><td>-2</td><td>1</td><td>$(-2, 1)$</td></tr>
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</td>
<td>29.</td>
<td>
<table>
<tr><td>x</td><td>y</td><td>(x, y)</td></tr>
<tr><td>0</td><td>2</td><td>$(0, 2)$</td></tr>
<tr><td>3</td><td>1</td><td>$(3, 1)$</td></tr>
<tr><td>6</td><td>0</td><td>$(6, 0)$</td></tr>
</table>
</td>
<td>31.</td>
<td>
<table>
<tr><td>x</td><td>y</td><td>(x, y)</td></tr>
<tr><td>0</td><td>-2</td><td>$(0, -2)$</td></tr>
<tr><td>10</td><td>2</td><td>$(10, 2)$</td></tr>
<tr><td>5</td><td>0</td><td>$(5, 0)$</td></tr>
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<tr>
<td>33.
35.
3
7.
3
9.</td>
<td>Answers will vary.
Answers will vary.
Answers will vary.
Answers will vary.</td>
<td>41.
43.
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5.
4
7.</td>
<td>Answers will vary.
Answers will vary.
Answers will vary.
Answers will vary.</td>
<td>49.</td>
<td>a)
A graph that plots the po 11), (4, 15), (6, 16), (8, 19), (12, 21).</td>
</tr>
<tr>
<td>51.</td>
<td>Answers will vary.</td>
<td>53.</td>
<td>Answers will vary.</td>
<td></td>
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</table>

Attributions

4.2 Graph Linear Equations in Two Variables

Learning Objectives

By the end of this section, you will be able to:

- Recognize the relationship between the solutions of an equation and its graph.
- Graph a linear equation by plotting points.
- Graph vertical and horizontal lines.

Recognize the Relationship Between the Solutions of an Equation and its Graph

In the previous section, we found several solutions to the equation $3x + 2y = 6$. They are listed in the table below. So, the ordered pairs $(0, 3)$, $(2, 0)$, and $\left(1, \frac{3}{2}\right)$ are some solutions to the equation $3x + 2y = 6$. We can plot these solutions in the rectangular coordinate system as shown in (Figure 1).

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$\left(1, \frac{3}{2}\right)$

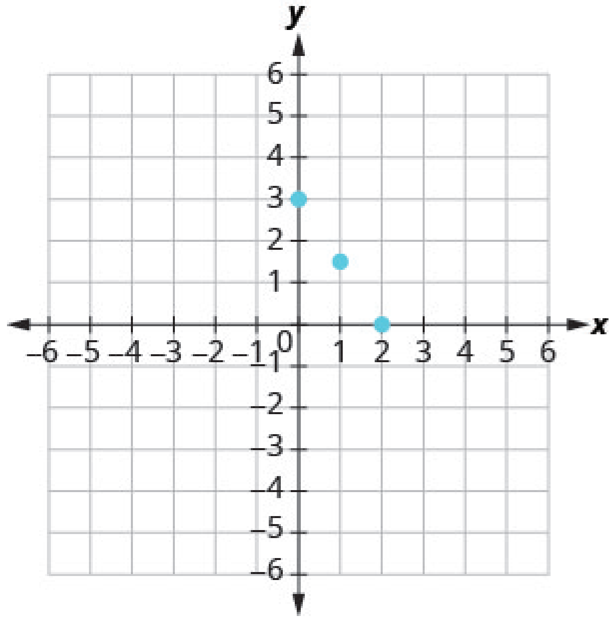


Figure .1

Notice how the points line up perfectly? We connect the points with a line to get the graph of the equation $3x + 2y = 6$. See (Figure 2). Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.

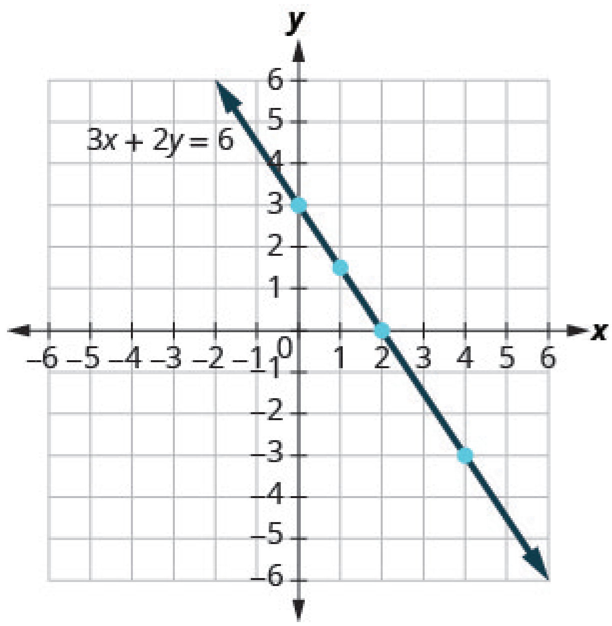


Figure .2

Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points *not* on the line are not solutions.

Notice that the point whose coordinates are $(-2, 6)$ is on the line shown in (Figure 3). If you substitute $x = -2$ and $y = 6$ into the equation, you find that it is a solution to the equation.

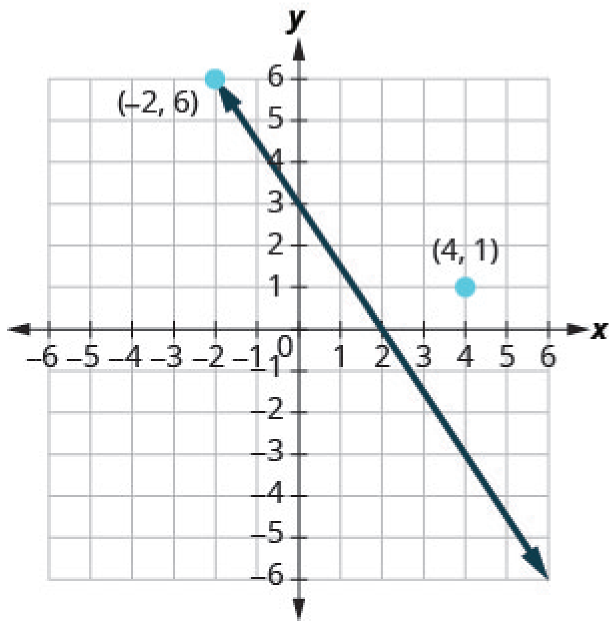


Figure .3

$$\text{Test } (-2, 6)$$
$$3x + 2y = 6$$
$$3(-2) + 2(6) = 6$$
$$-6 + 12 = 6$$
$$6 = 6 \checkmark$$

So the point $(-2, 6)$ is a solution to the equation $3x + 2y = 6$. (The phrase "the point whose coordinates are $(-2, 6)$" is often shortened to "the point $(-2, 6)$.")

$$\text{What about } (4, 1)\,?$$
$$3x + 2y = 6$$
$$3 \cdot 4 + 2 \cdot 1 = 6$$
$$12 + 2 \stackrel{?}{=} 6$$
$$14 \neq 6$$

So $(4, 1)$ is not a solution to the equation $3x + 2y = 6$. Therefore, the point $(4, 1)$ is not on the line. See (Figure 2). This is an example of the saying, "A picture is worth a thousand words." The line shows you *all* the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

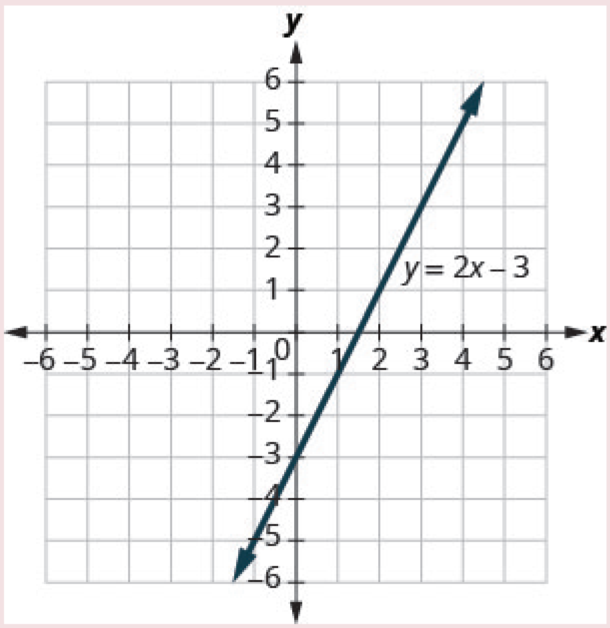
Graph of a linear equation

The graph of a linear equation $Ax + By = C$ is a line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

EXAMPLE 1

The graph of $y = 2x - 3$ is shown.



For each ordered pair, decide:

a) Is the ordered pair a solution to the equation?
b) Is the point on the line?

A $(0, -3)$ B $(3, 3)$ C $(2, -3)$ D $(-1, -5)$

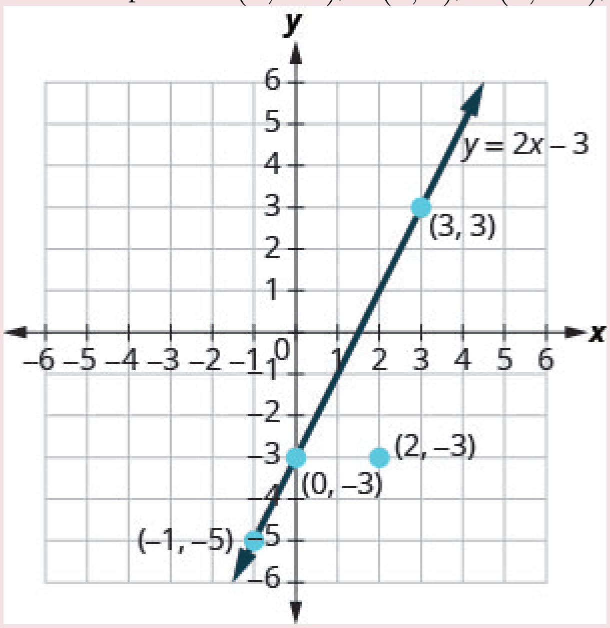
Solution

Substitute the *x*– and *y*– values into the equation to check if the ordered pair is a solution to the equation.

a.

A: $(0, -3)$	B: $(3, 3)$	C: $(2, -3)$	D: $(-1, -5)$
$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$	$y = 2x - 3$
$-3 \stackrel{?}{=} 2(0) - 3$	$3 \stackrel{?}{=} 2(3) - 3$	$-3 \stackrel{?}{=} 2(2) - 3$	$-5 \stackrel{?}{=} 2(-1) - 3$
$-3 = -3$ ✓	$3 = 3$ ✓	$-3 \neq 1$	$-5 = -5$ ✓
(0, –3) is a solution.	(3, 3) is a solution.	(2, –3) is not a solution.	(–1, –5) is a solution.

b. Plot the points A $(0,-3)$, B $(3,3)$, C $(2,-3)$, and D $(-1,-5)$.



The points $(0,-3)$, $(3,3)$, and $(-1,-5)$ are on the line $y=2x-3$, and the point $(2,-3)$ is not on the line.

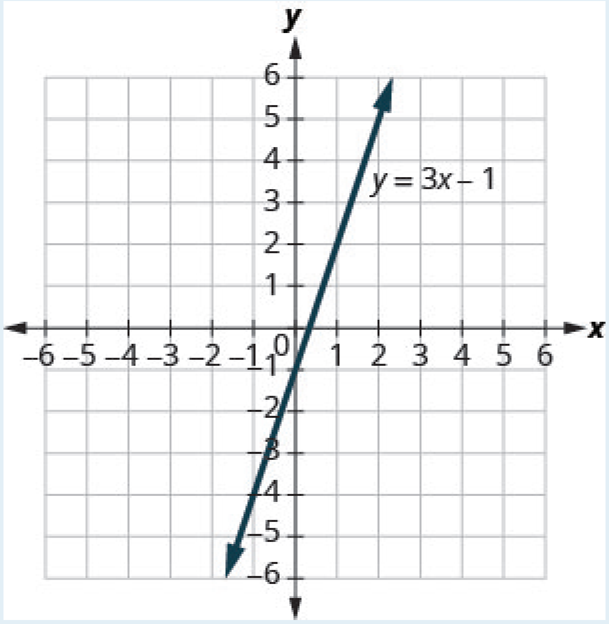
The points that are solutions to $y=2x-3$ are on the line, but the point that is not a solution is not on the line.

TRY IT 1.1

Use the graph of $y=3x-1$ to decide whether each ordered pair is:

- a solution to the equation.
- on the line.

a) $(0,-1)$ b) $(2,5)$



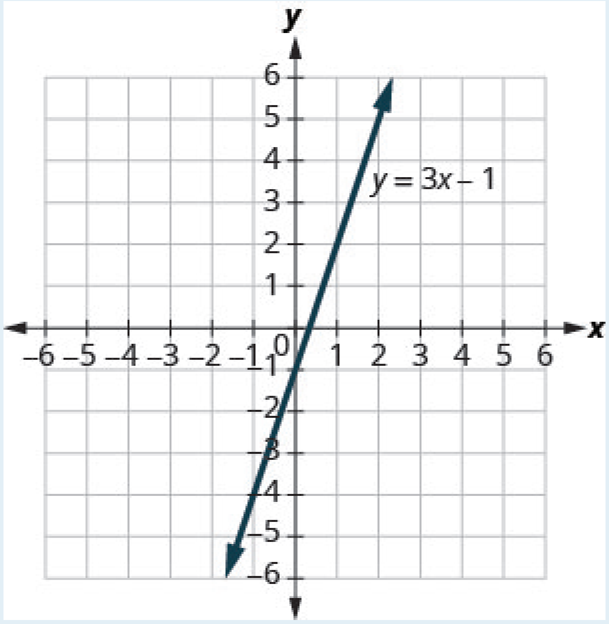
Show answer
a) yes, yes b) yes, yes

TRY IT 1.2

Use graph of $y = 3x - 1$ to decide whether each ordered pair is:

- a solution to the equation
- on the line

a) $(3, -1)$ b) $(-1, -4)$



Show answer
a) no, no b) yes, yes

Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The method we used to graph $3x + 2y = 6$ is called plotting points, or the Point–Plotting Method.

EXAMPLE 2

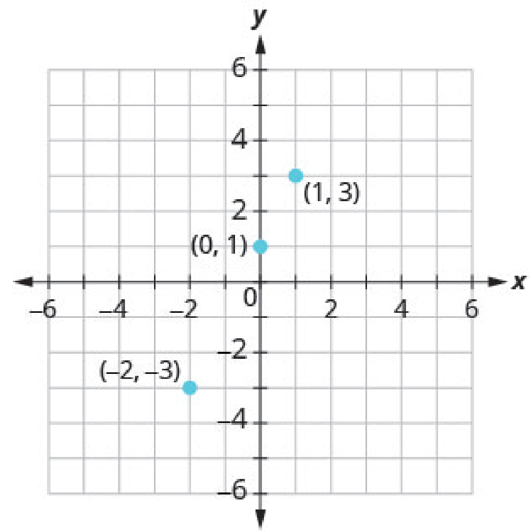
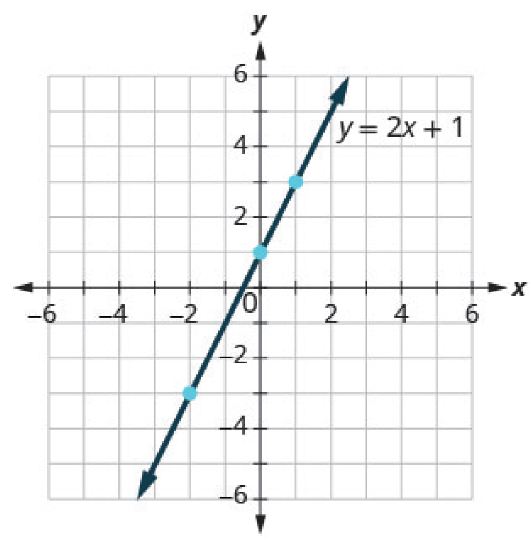
How To Graph an Equation By Plotting Points

Graph the equation $y = 2x + 1$ by plotting points.

Solution

Step 1. Find three points whose coordinates are solutions to the equation.	You can choose any values for x or y. In this case, since y is isolated on the left side of the equation, it is easier to choose values for x.	$y = 2x + 1$ $x = 0$ $y = 2x + 1$ $y = 2 \cdot 0 + 1$ $y = 0 + 1$ $y = 1$ $x = 1$ $y = 2x + 1$ $y = 2 \cdot 1 + 1$ $y = 2 + 1$ $y = 3$ $x = -2$ $y = 2x + 1$ $y = 2(-2) + 1$ $y = -4 + 1$ $y = -3$
Organize the solutions in a table.	Put the three solutions in a table.	see table below

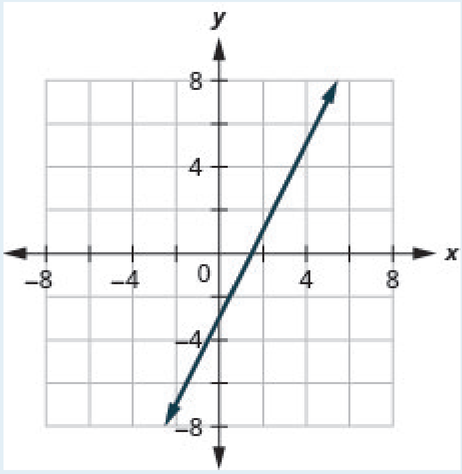
$y = 2x + 1$		
x	y	(x, y)
0	1	(0, 1)
1	3	(1, 3)
–2	–3	(–2, –3)

Step 2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!	Plot: (0, 1), (1, 3), (–2, –3). Do the points line up? Yes, the points line up.	
Step 3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.	This line is the graph of $y = 2x + 1$.	

TRY IT 2.1

Graph the equation by plotting points: $y = 2x - 3$.

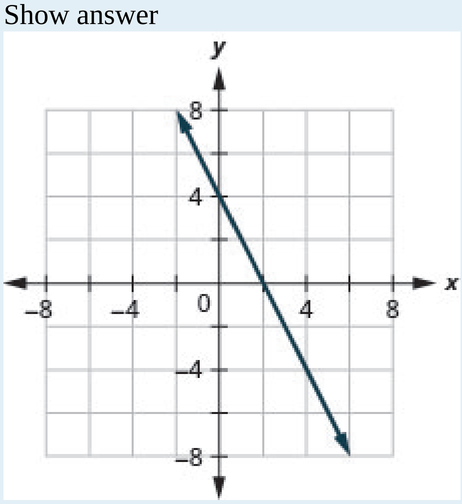
Show answer



TRY IT 2.2

Graph the equation by plotting points: $y = -2x + 4$.

Show answer



HOW TO: Graph a linear equation by plotting points.

The steps to take when graphing a linear equation by plotting points are summarized below.

1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.
3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you only plot two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line.

If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. Look at the difference between part (a) and part (b) in (Figure 4).

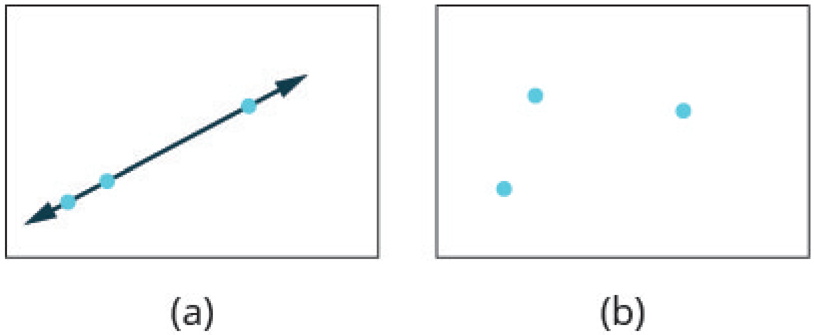


Figure .4

Let's do another example. This time, we'll show the last two steps all on one grid.

EXAMPLE 3

Graph the equation $y = -3x$.

Solution

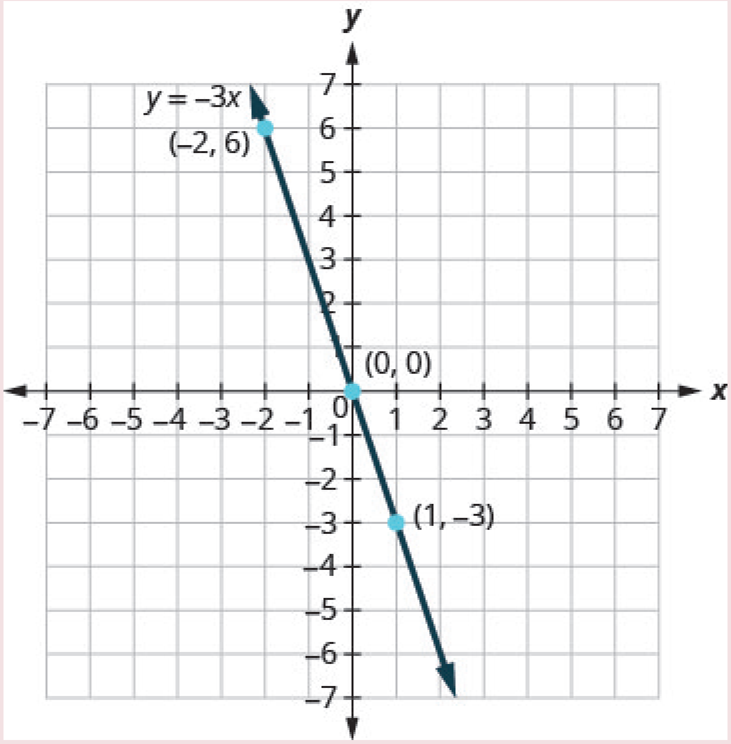
Find three points that are solutions to the equation. Here, again, it's easier to choose values for x. Do you see why?

$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3x$	$y = -3x$
$y = -3 \cdot 0$	$y = -3 \cdot 1$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$

We list the points in the table below.

$y = -3x$		
x	y	(x, y)
0	0	$(0, 0)$
1	-3	$(1, -3)$
-2	6	$(-2, 6)$

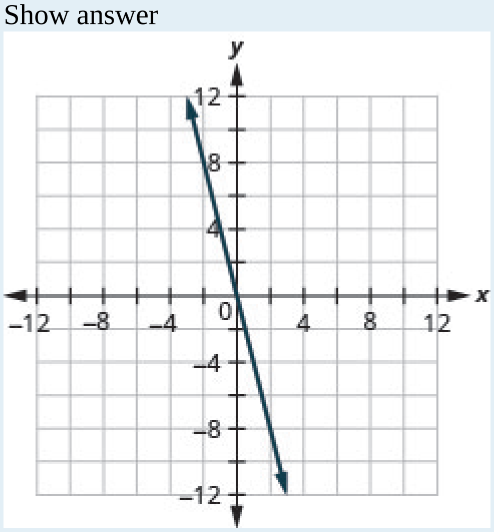
Plot the points, check that they line up, and draw the line.



TRY IT 3.1

Graph the equation by plotting points: $y = -4x$.

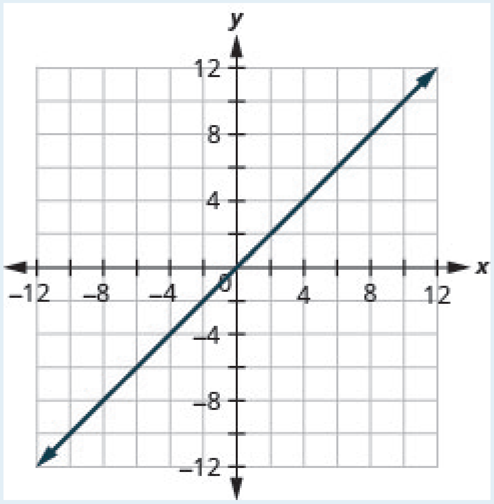
Show answer



EXAMPLE 3.2

Graph the equation by plotting points: $y = x$.

Show answer



When an equation includes a fraction as the coefficient of x, we can still substitute any numbers for x. But the math is easier if we make 'good' choices for the values of x. This way we will avoid fraction answers, which are hard to graph precisely.

EXAMPLE 4

Graph the equation $y = \frac{1}{2}x + 3$.

Solution

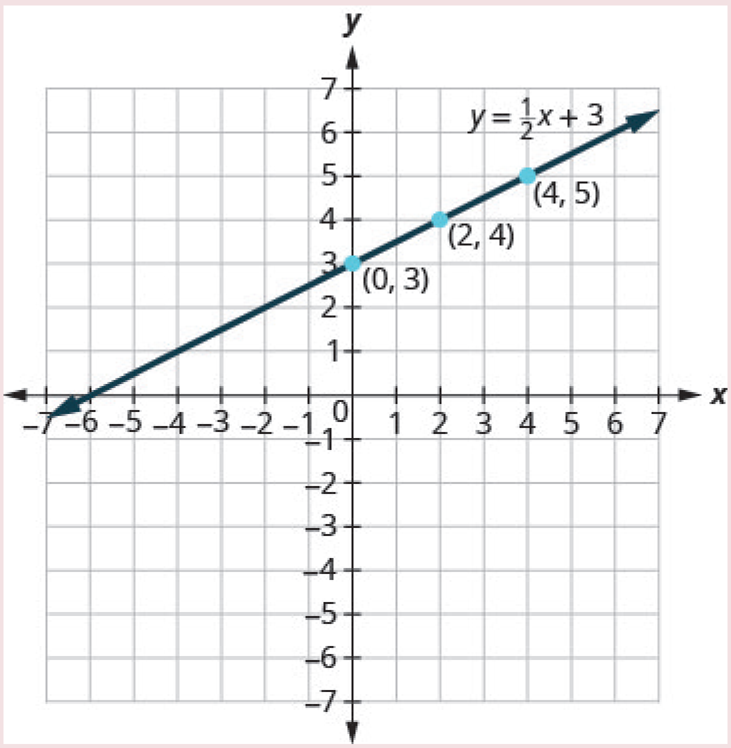
Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x, we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices. Why are multiples of 2 a good choice for values of x?

$x = 0$	$x = 2$	$x = 4$
$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$	$y = \frac{1}{2}x + 3$
$y = \frac{1}{2}(0) + 3$	$y = \frac{1}{2}(2) + 3$	$y = \frac{1}{2}(4) + 3$
$y = 0 + 3$	$y = 1 + 3$	$y = 2 + 3$
$y = 3$	$y = 4$	$y = 5$

The points are shown in the table below.

$y = \frac{1}{2}x + 3$		
x	y	(x, y)
0	3	$(0, 3)$
2	4	$(2, 4)$
4	5	$(4, 5)$

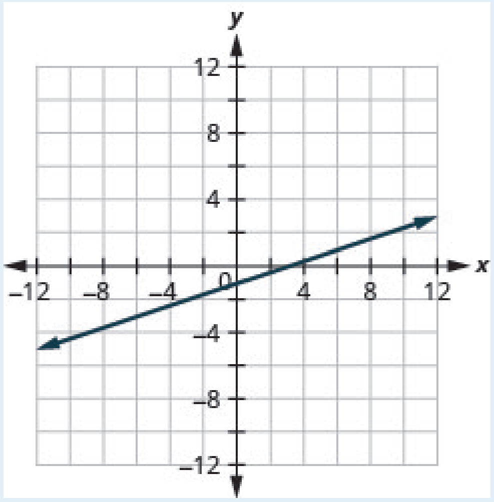
Plot the points, check that they line up, and draw the line.



TRY IT 4. 1

Graph the equation $y = \frac{1}{3}x - 1$.

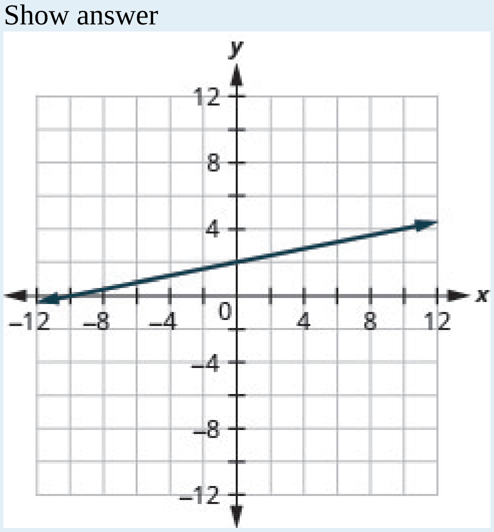
Show answer



TRY IT 4.2

Graph the equation $y = \frac{1}{4}x + 2$.

Show answer



So far, all the equations we graphed had y given in terms of x. Now we'll graph an equation with x and y on the same side. Let's see what happens in the equation $2x + y = 3$. If $y = 0$ what is the value of x?

$$y = 0$$
$$2x + y = 3$$
$$2x + 0 = 3$$
$$2x = 3$$
$$x = \frac{3}{2}$$
$$\left(\frac{3}{2}, 0\right)$$

This point has a fraction for the *x*– coordinate and, while we could graph this point, it is hard to be precise graphing fractions. Remember in the example $y = \frac{1}{2}x + 3$, we carefully chose values for x so as not to graph fractions at all. If we solve the equation $2x + y = 3$ for y, it will be easier to find three solutions to the equation.

$$\begin{aligned} 2x + y &= 3 \\ y &= -2x + 3 \end{aligned}$$

The solutions for $x = 0$, $x = 1$, and $x = -1$ are shown in the table below. The graph is shown in (Figure 5).

$2x + y = 3$		
x	y	(x, y)
0	3	$(0, 3)$
1	1	$(1, 1)$
-1	5	$(-1, 5)$

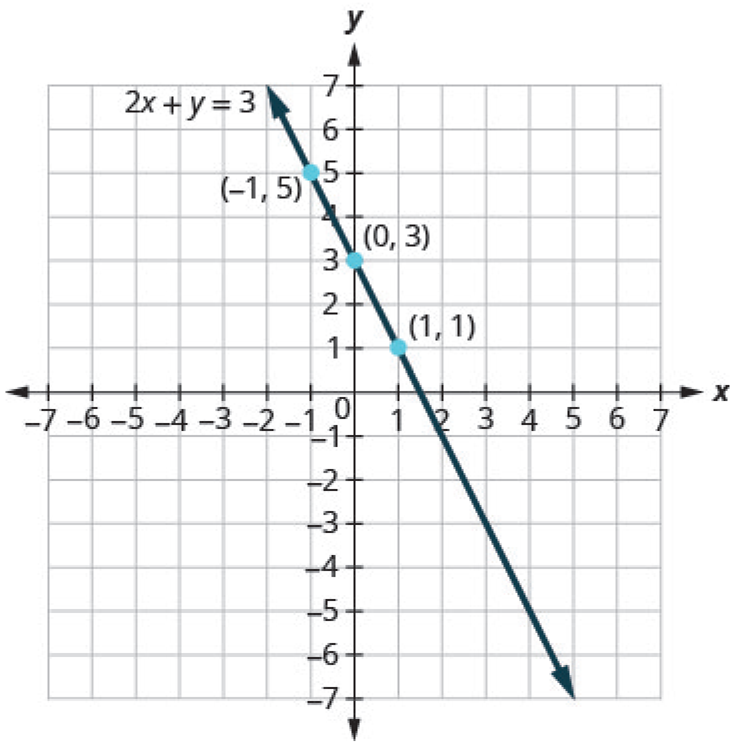


Figure .5

Can you locate the point $\left(\frac{3}{2}, 0\right)$, which we found by letting $y = 0$, on the line?

EXAMPLE 5

Graph the equation $3x + y = -1$.

Solution

Find three points that are solutions to the equation.	$3x + y \quad = \quad -1$
First, solve the equation for y.	$y \quad = \quad -3x - 1$

We'll let x be 0, 1, and -1 to find 3 points. The ordered pairs are shown in the table below. Plot the points, check that they line up, and draw the line. See (Figure 6).

$3x + y = -1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	-4	$(1, -4)$
-1	2	$(-1, 2)$

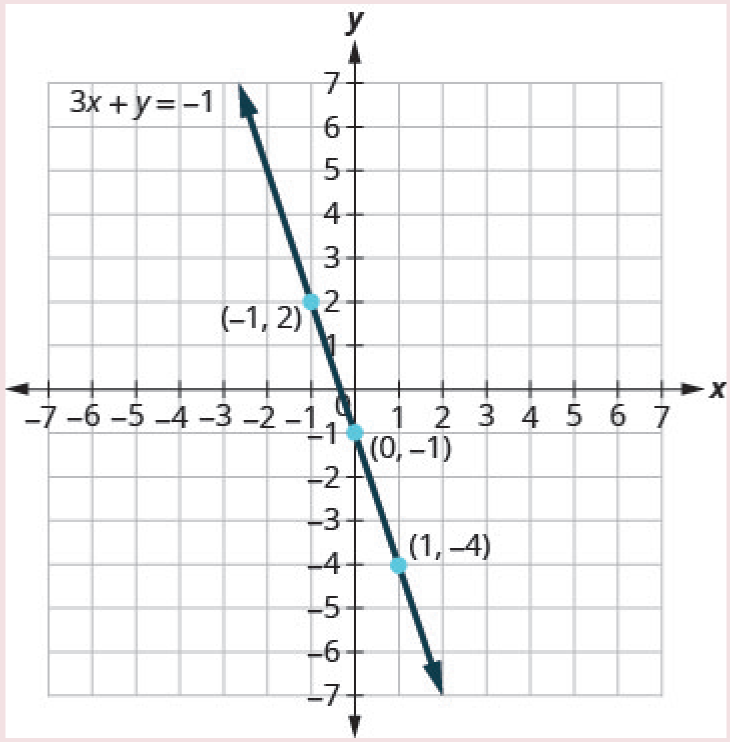
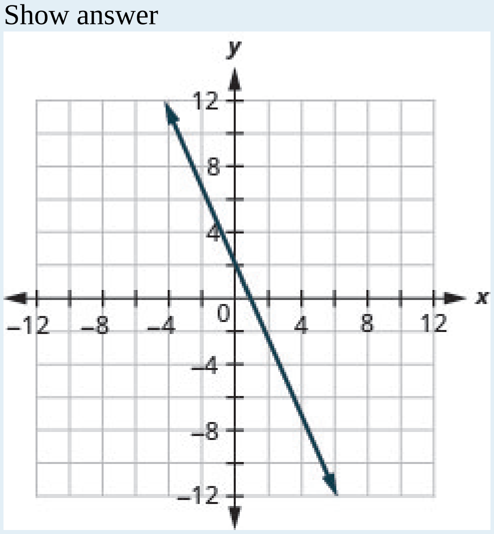


Figure .6

EXAMPLE 5.1

Graph the equation $2x + y = 2$.

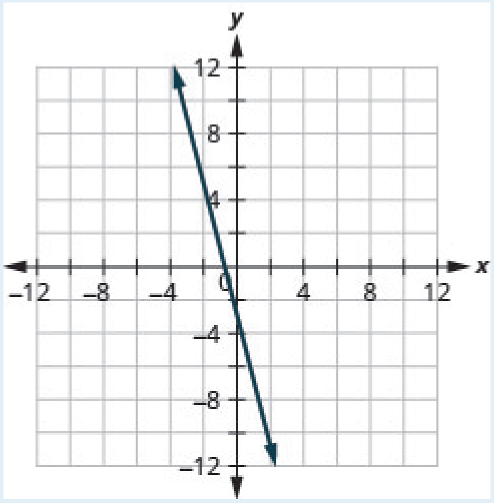
Show answer



TRY IT 5.2

Graph the equation $4x + y = -3$.

Show answer



If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the *x*– and *y*-axis are the same, the graphs match!

The equation in (Example 5) was written in standard form, with both x and y on the same side. We solved that equation for y in just one step. But for other equations in standard form it is not that easy to solve for y, so we will leave them in standard form. We can still find a first point to plot by letting $x = 0$ and solving for y. We can plot a second point by letting $y = 0$ and then solving for x. Then we will plot a third point by using some other value for x or y.

EXAMPLE 6

Graph the equation $2x - 3y = 6$.

Solution

Find three points that are solutions to the equation.	$2x - 3y = 6$
First, let $x = 0$.	$2(0) - 3y = 6$
Solve for y.	$-3y = 6$ $y = -2$
Now let $y = 0$.	$2x - 3(0) = 6$
Solve for x.	$2x = 6$ $x = 3$
We need a third point. Remember, we can choose any value for x or y. We'll let $x = 6$.	$2(6) - 3y = 6$
Solve for y.	$12 - 3y = 6$ $-3y = -6$ $y = 2$

We list the ordered pairs in the table below. Plot the points, check that they line up, and draw the line. See (Figure 7).

$2x - 3y = 6$		
x	y	(x, y)
0	-2	$(0, -2)$
3	0	$(3, 0)$
6	2	$(6, 2)$

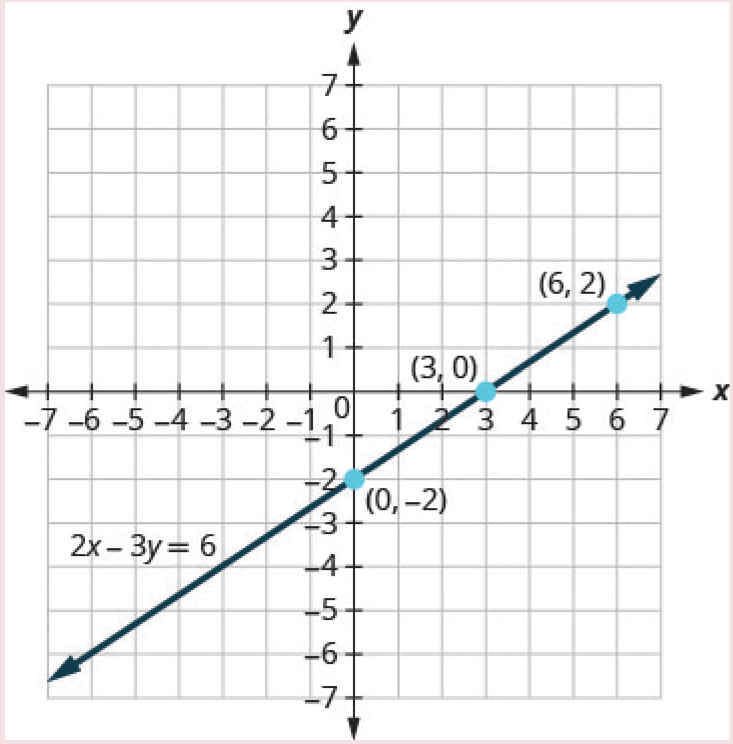
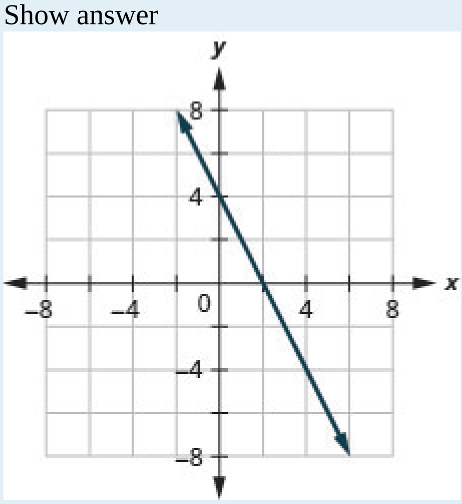


Figure .7

TRY IT 6.1

Graph the equation $4x + 2y = 8$.

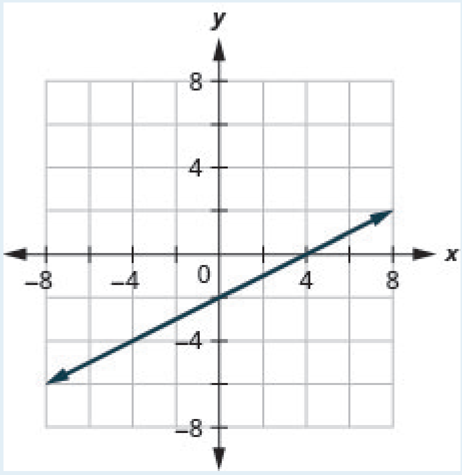
Show answer



TRY IT 6.2

Graph the equation $2x - 4y = 8$.

Show answer



Graph Vertical and Horizontal Lines

Can we graph an equation with only one variable? Just x and no y, or just y without an x? How will we make a table of values to get the points to plot?

Let's consider the equation $x = -3$. This equation has only one variable, x. The equation says that x is *always* equal to -3, so its value does not depend on y. No matter what y is, the value of x is always -3.

So to make a table of values, write -3 in for all the x values. Then choose any values for y. Since x does not depend on y, you can choose any numbers you like. But to fit the points on our coordinate graph, we'll use 1, 2, and 3 for the *y*-coordinates. See the table below.

$x = -3$		
x	y	(x, y)
-3	1	$(-3, 1)$
-3	2	$(-3, 2)$
-3	3	$(-3, 3)$

Plot the points from the table and connect them with a straight line. Notice in (Figure 8) that we have graphed a *vertical line*.

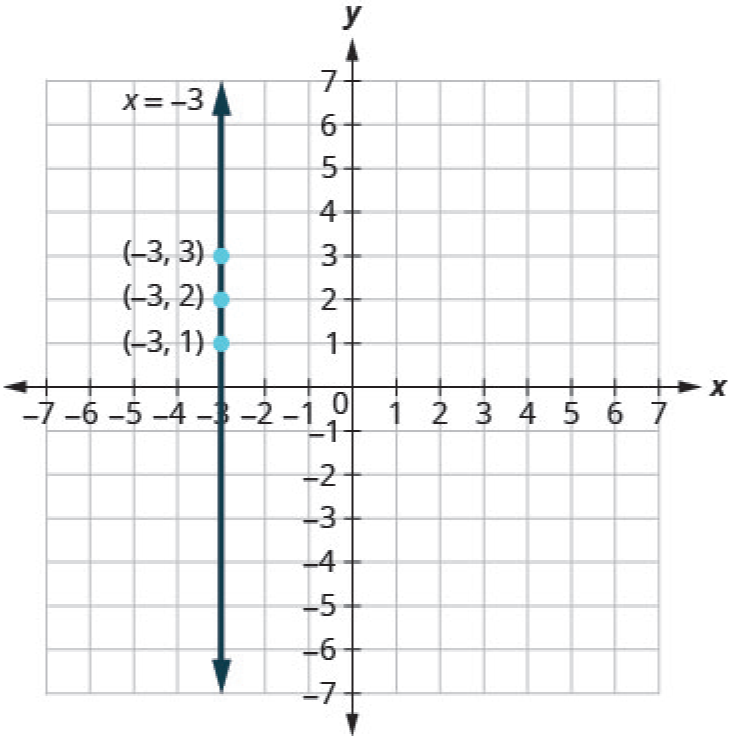


Figure .8

Vertical line

A vertical line is the graph of an equation of the form $x = a$.

The line passes through the *x*-axis at $(a, 0)$.

EXAMPLE 7

Graph the equation $x = 2$.

Solution

The equation has only one variable, x, and x is always equal to 2. We create the table below where x is always 2 and then put in any values for y. The graph is a vertical line passing through the *x*-axis at 2. See (Figure 9).

$x = 2$		
x	y	(x, y)
2	1	$(2, 1)$
2	2	$(2, 2)$
2	3	$(2, 3)$

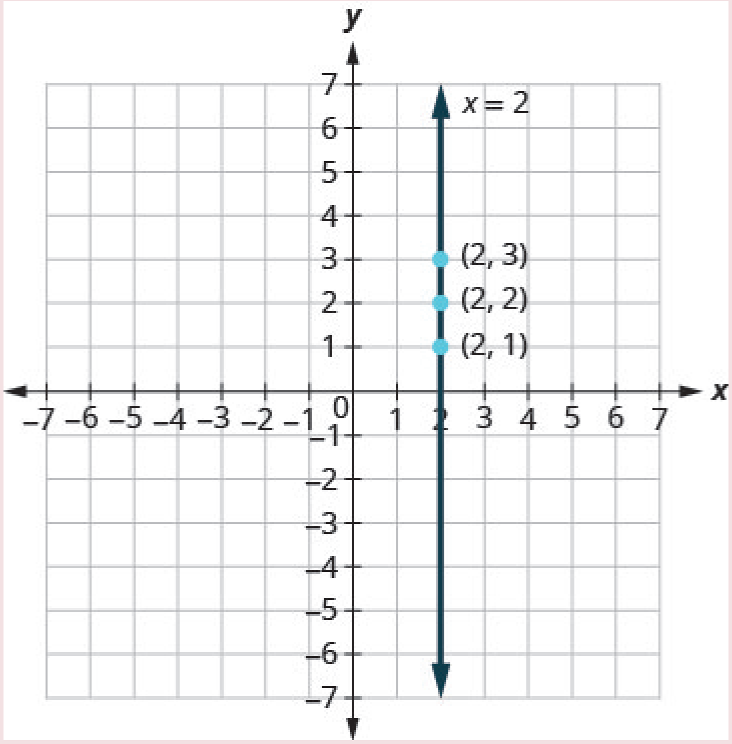
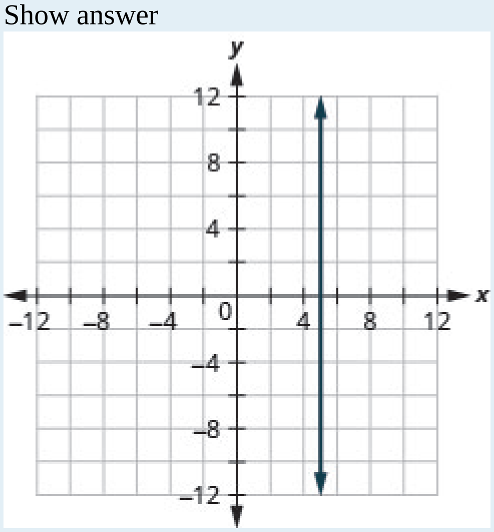


Figure .9

TRY IT 7.1

Graph the equation $x = 5$.

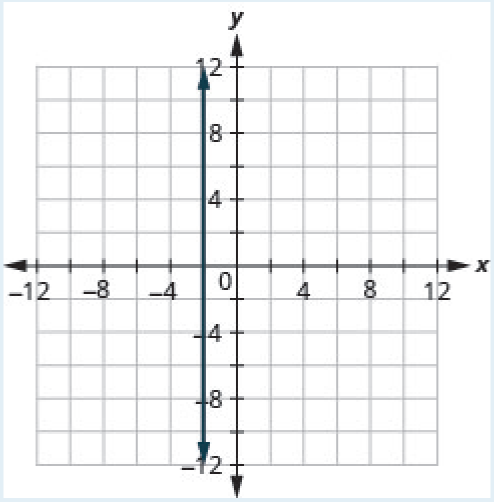
Show answer



TRY IT 7.2

Graph the equation $x = -2$.

Show answer



What if the equation has y but no x? Let's graph the equation $y = 4$. This time the *y*– value is a constant, so in this equation, y does not depend on x. Fill in 4 for all the y's in the table below and then choose any values for x. We'll use 0, 2, and 4 for the *x*-coordinates.

$y = 4$		
x	y	(x, y)
0	4	$(0, 4)$
2	4	$(2, 4)$
4	4	$(4, 4)$

The graph is a horizontal line passing through the *y*-axis at 4. See (Figure 10).

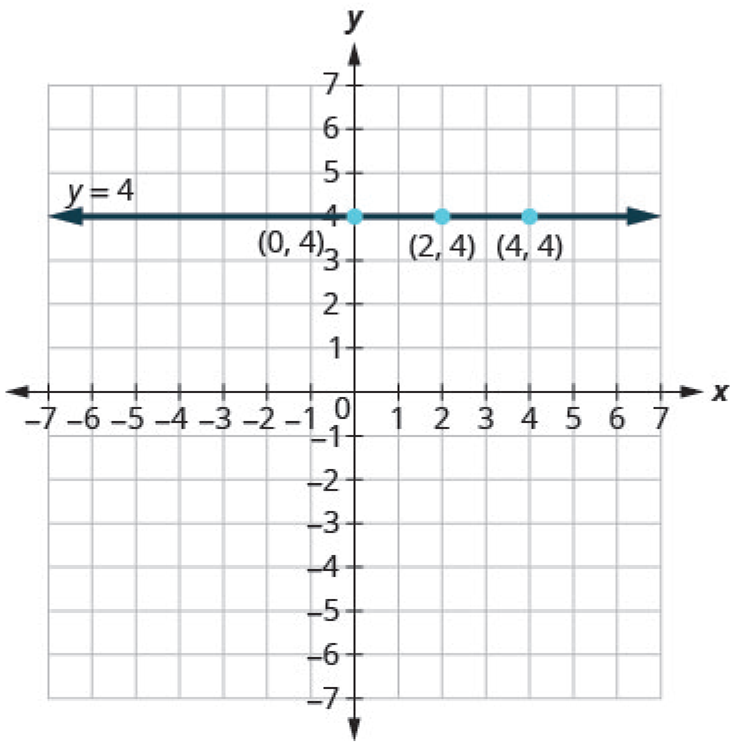


Figure .10

Horizontal line

A horizontal line is the graph of an equation of the form $y = b$.

The line passes through the *y*-axis at $(0, b)$.

EXAMPLE 8

Graph the equation $y = -1$.

Solution

The equation $y = -1$ has only one variable, y. The value of y is constant. All the ordered pairs in the table below have the same *y*-coordinate. The graph is a horizontal line passing through the *y*-axis at -1, as shown in (Figure 11).

$y = -1$		
x	y	(x, y)
0	-1	$(0, -1)$
3	-1	$(3, -1)$
-3	-1	$(-3, -1)$

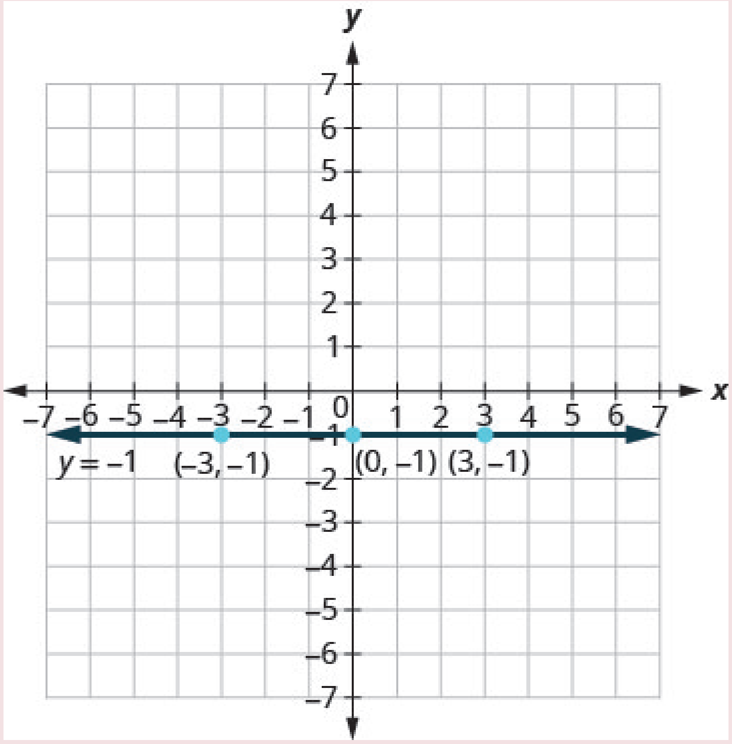
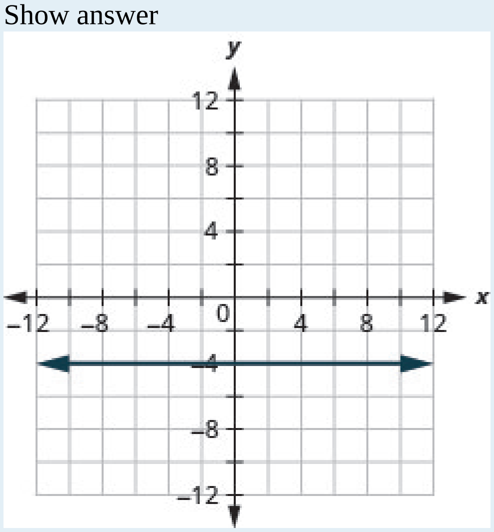


Figure .11

TRY IT 8.1

Graph the equation $y = -4$.

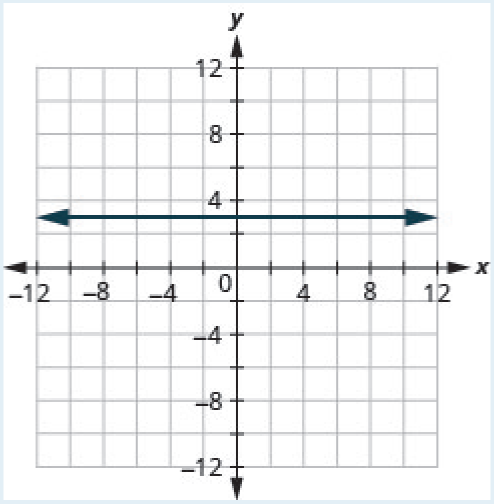
Show answer



TRY IT 8.2

Graph the equation $y = 3$.

Show answer



The equations for vertical and horizontal lines look very similar to equations like $y = 4x$. What is the difference between the equations $y = 4x$ and $y = 4$?

The equation $y = 4x$ has both x and y. The value of y depends on the value of x. The *y*-coordinate changes according to the value of x. The equation $y = 4$ has only one variable. The value of y is constant. The *y*-coordinate is always 4. It does not depend on the value of x. See the table below.

$y = 4x$				$y = 4$		
x	y	(x, y)		x	y	(x, y)
0	0	$(0, 0)$		0	4	$(0, 4)$
1	4	$(1, 4)$		1	4	$(1, 4)$
2	8	$(2, 8)$		2	4	$(2, 4)$

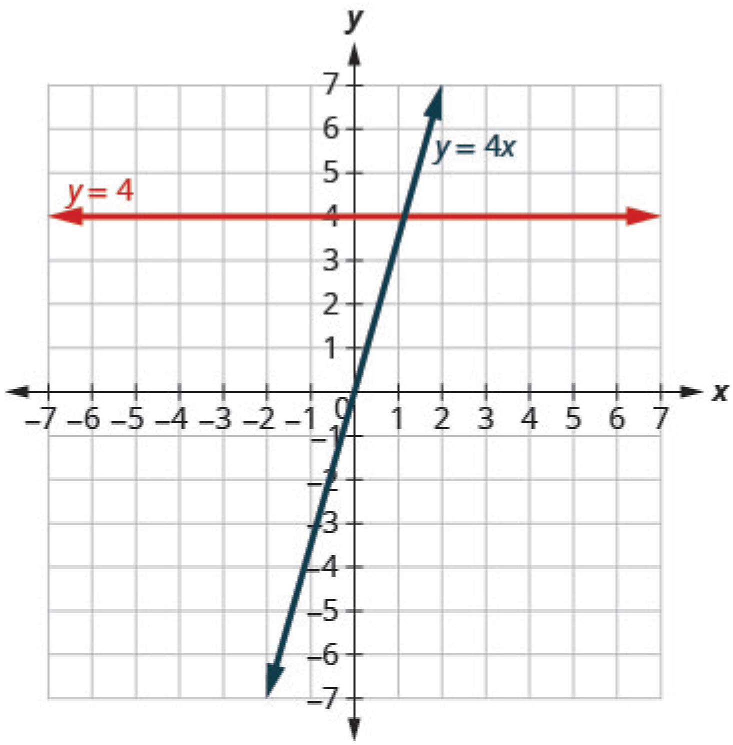


Figure .12

Notice, in (Figure 12), the equation $y = 4x$ gives a slanted line, while $y = 4$ gives a horizontal line.

EXAMPLE 9

Graph $y = -3x$ and $y = -3$ in the same rectangular coordinate system.

Solution

Notice that the first equation has the variable x, while the second does not. See the table below. The two graphs are shown in (Figure 13).

$y = -3x$				$y = -3$		
x	y	(x, y)		x	y	(x, y)
0	0	$(0, 0)$		0	-3	$(0, -3)$
1	-3	$(1, -3)$		1	-3	$(1, -3)$
2	-6	$(2, -6)$		2	-3	$(2, -3)$

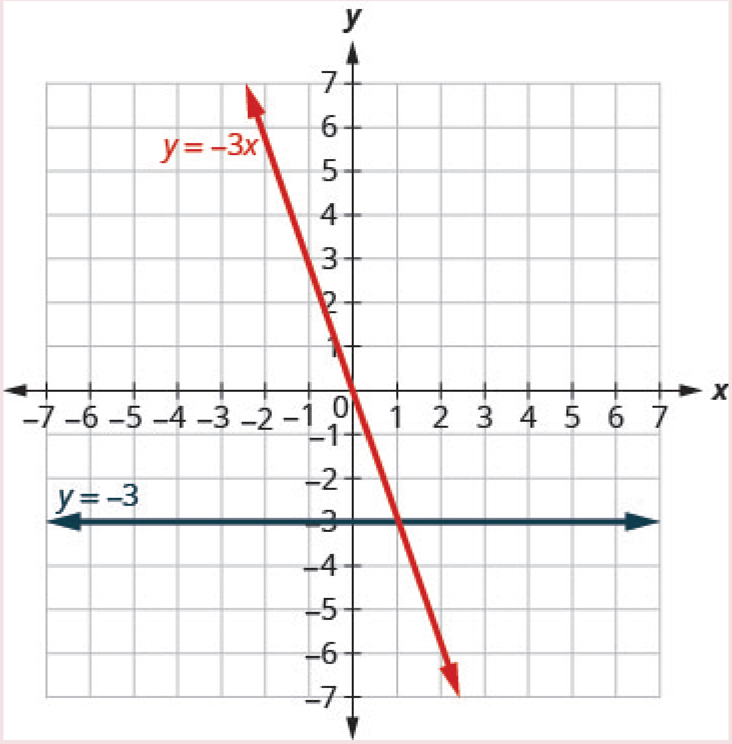
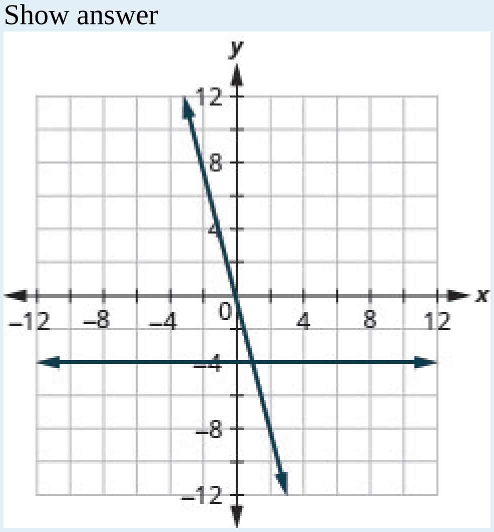


Figure .13

TRY IT 9.1

Graph $y = -4x$ and $y = -4$ in the same rectangular coordinate system.

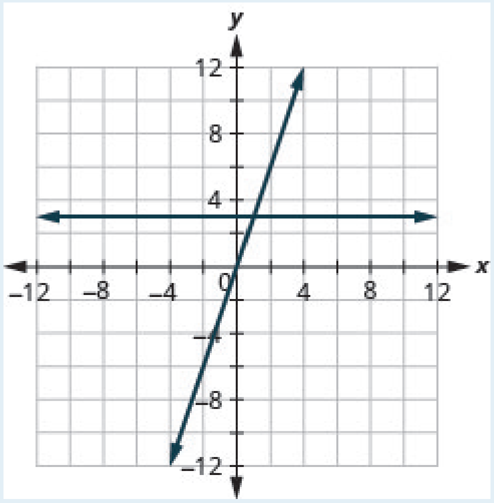
Show answer



TRY IT 9.2

Graph $y = 3$ and $y = 3x$ in the same rectangular coordinate system.

Show answer



Key Concepts

- **Graph a Linear Equation by Plotting Points**
    1. Find three points whose coordinates are solutions to the equation. Organize them in a table.
    2. Plot the points in a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work!
    3. Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line.

Glossary

graph of a linear equation
The graph of a linear equation $Ax + By = C$ is a straight line. Every point on the line is a solution of the equation. Every solution of this equation is a point on this line.

horizontal line
A horizontal line is the graph of an equation of the form $y = b$. The line passes through the *y*-axis at $(0, b)$.

vertical line
A vertical line is the graph of an equation of the form $x = a$. The line passes through the *x*-axis at $(a, 0)$.

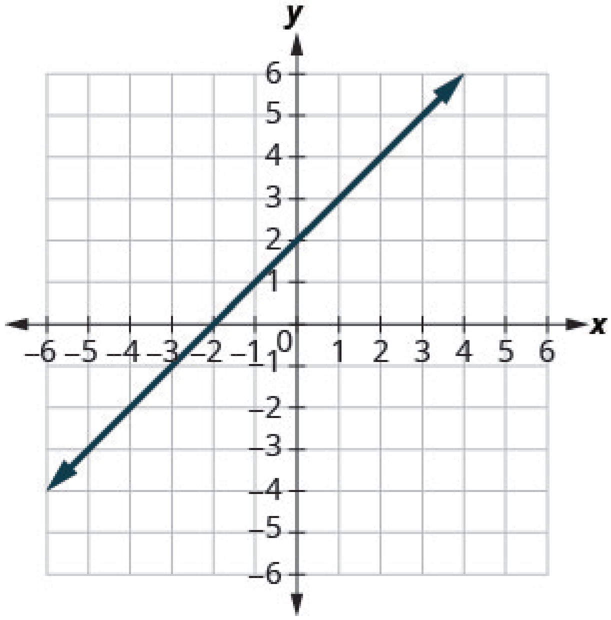
Practice Makes Perfect

Recognize the Relationship Between the Solutions of an Equation and its Graph

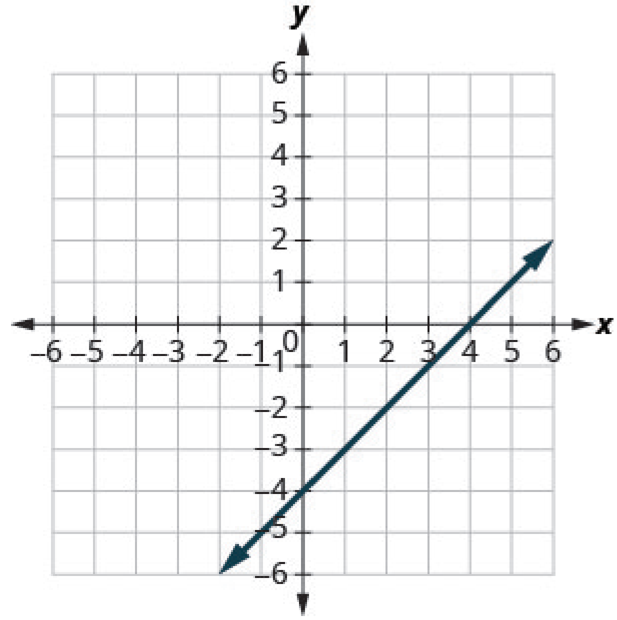
In the following exercises, for each ordered pair, decide:

a) Is the ordered pair a solution to the equation? b) Is the point on the line?

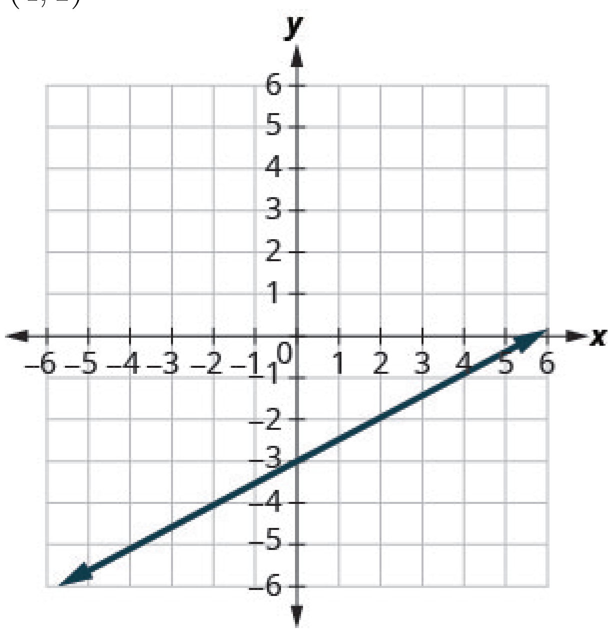
1. $y = x + 2$
a) $(0, 2)$
b) $(1, 2)$
c) $(-1, 1)$
d) $(-3, -1)$



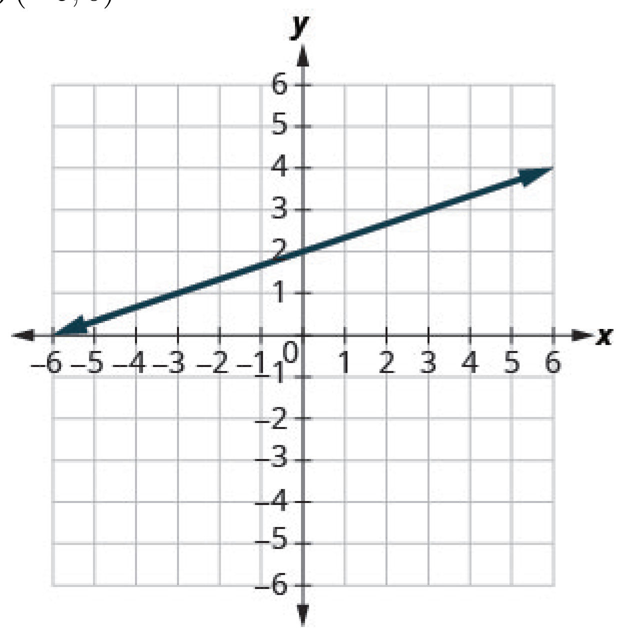
2. $y = x - 4$
a) $(0, -4)$
b) $(3, -1)$
c) $(2, 2)$
d) $(1, -5)$



3. $y = \frac{1}{2}x - 3$
a) $(0, -3)$
b) $(2, -2)$
c) $(-2, -4)$
d) $(4, 1)$



4. $y = \frac{1}{3}x + 2$
a) $(0, 2)$
b) $(3, 3)$
c) $(-3, 2)$
d) $(-6, 0)$



Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

5. $y = 3x - 1$	6. $y = 2x + 3$
7. $y = -3x + 3$	8. $y = -3x + 1$
9. $y = x + 2$	10. $y = x - 3$
11. $y = -x - 3$	12. $y = -x - 2$
13. $y = 2x$	14. $y = 3x$
15. $y = 3x$	16. $y = -2x$
17. $y = \frac{1}{2}x + 2$	18. $y = \frac{1}{3}x - 1$
19. $y = \frac{4}{3}x - 5$	20. $y = \frac{3}{2}x - 3$
21. $y = -\frac{2}{5}x + 1$	22. $y = -\frac{4}{5}x - 1$
23. $y = -\frac{3}{2}x + 2$	24. $y = -\frac{5}{3}x + 4$
25. $x + y = 6$	26. $x + y = 4$
27. $x + y = -3$	28. $x + y = -3$
29. $x - y = 2$	30. $x - y = 1$
31. $x - y = -1$	32. $x - y = -3$
33. $3x + y = 7$	34. $5x + y = 6$
35. $2x + y = -3$	36. $4x + y = -5$
37. $\frac{1}{3}x + y = 2$	38. $\frac{1}{2}x + y = 3$
39. $\frac{2}{5}x + y = -4$	40. $\frac{3}{4}x - y = 6$
41. $2x + 3y = 12$	42. $4x + 2y = 12$
43. $3x - 4y = 12$	44. $2x - 5y = 10$
45. $x - 6y = 3$	46. $x - 4y = 2$
47. $3x + y = 2$	48. $3x + 5y = 5$

Graph Vertical and Horizontal Lines

In the following exercises, graph each equation.

49. $x = 4$	50. $x = 3$
51. $x = -2$	52. $x = -5$
53. $y = 3$	54. $y = 1$
55. $y = -5$	56. $y = -2$
57. $x = \frac{7}{3}$	58. $x = \frac{5}{4}$
59. $y = -\frac{15}{4}$	60. $y = -\frac{5}{3}$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

61. $y = 2x$ and $y = 2$	62. $y = 5x$ and $y = 5$
63. $y = -\frac{1}{2}x$ and $y = -\frac{1}{2}$	64. $y = -\frac{1}{3}x$ and $y = -\frac{1}{3}$

Mixed Practice

In the following exercises, graph each equation.

65. $y = 4x$	66. $y = 2x$
67. $y = -\frac{1}{2}x + 3$	68. $y = \frac{1}{4}x - 2$
69. $y = -x$	70. $y = x$
71. $x - y = 3$	72. $x + y = -5$
73. $4x + y = 2$	74. $2x + y = 6$
75. $y = -1$	76. $y = 5$
77. $2x + 6y = 12$	78. $5x + 2y = 10$
79. $x = 3$	80. $x = -4$

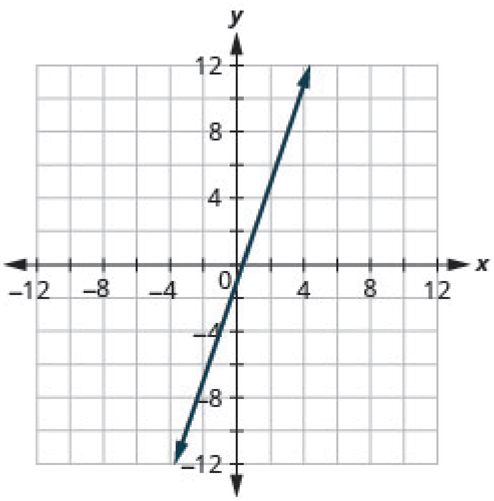
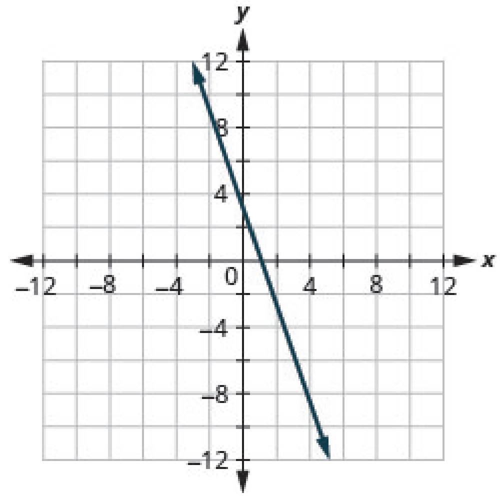
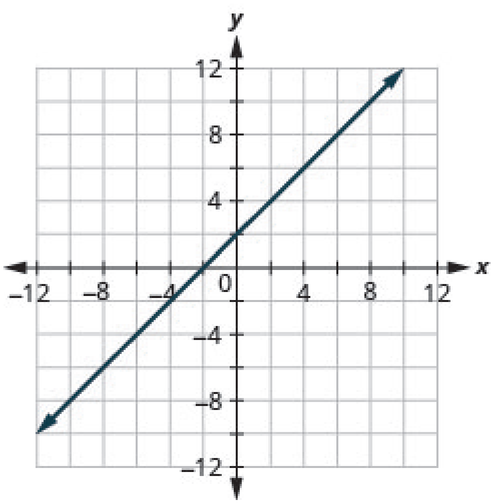
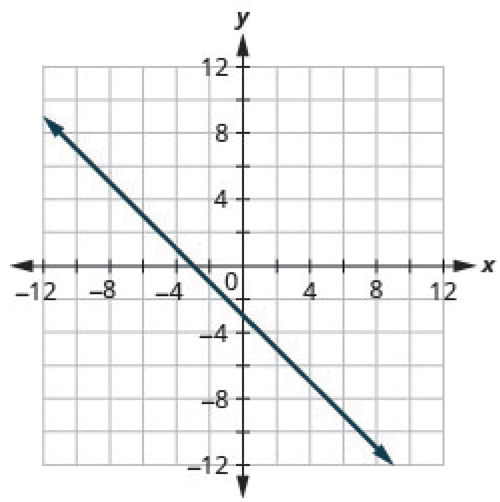
Everyday Math

81. **Motor home cost.** The Stonechilds rented a motor home for one week to go on vacation. It cost them \$594 plus \$0.32 per mile to rent the motor home, so the linear equation $y = 594 + 0.32x$ gives the cost, y, for driving x miles. Calculate the rental cost for driving 400, 800, and 1200 miles, and then graph the line.	82. **Weekly earnings.** At the art gallery where he works, Archisma gets paid \$200 per week plus 15% of the sales he makes, so the equation $y = 200 + 0.15x$ gives the amount, y, he earns for selling x dollars of artwork. Calculate the amount Archisma earns for selling \$900, \$1600, and \$2000, and then graph the line.

Writing Exercises

83. Explain how you would choose three *x*– values to make a table to graph the line $y = \frac{1}{5}x - 2$.	84. What is the difference between the equations of a vertical and a horizontal line?

Answers

1. a) yes; no b) no; no c) yes; yes d) yes; yes	3. a) yes; yes b) yes; yes c) yes; yes d) no; no
5. 	7. 
9. 	11. 

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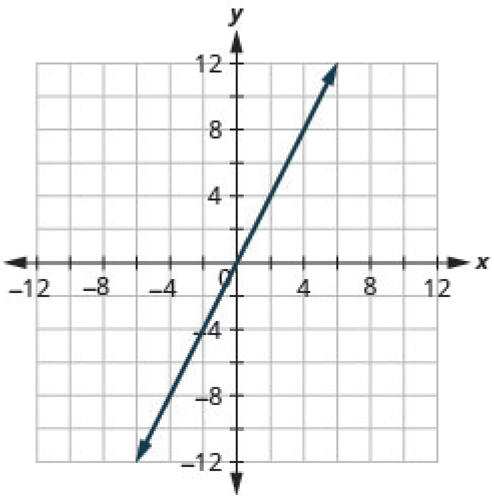
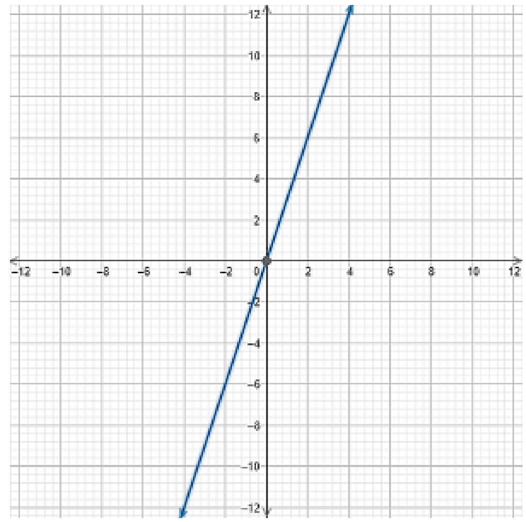


15. Click on the graph to see a larger version



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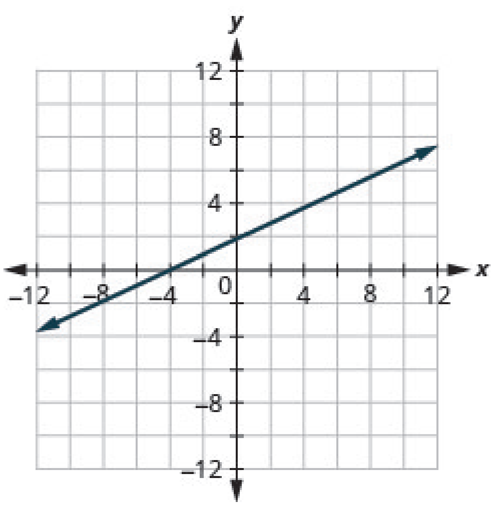

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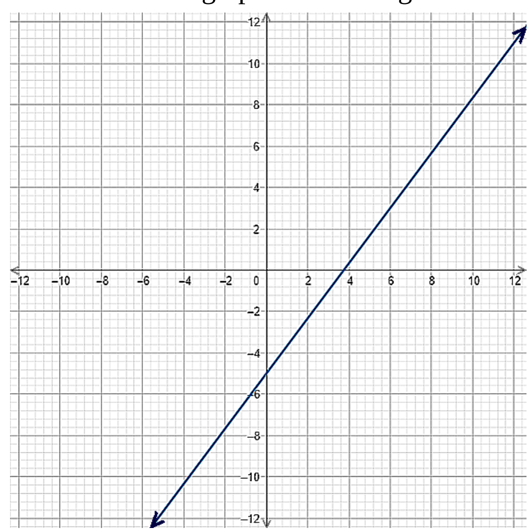

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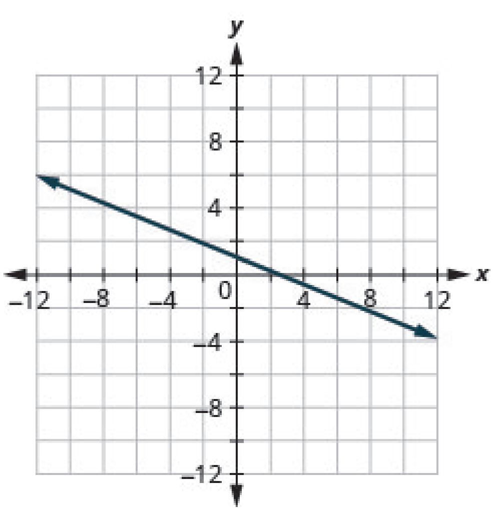

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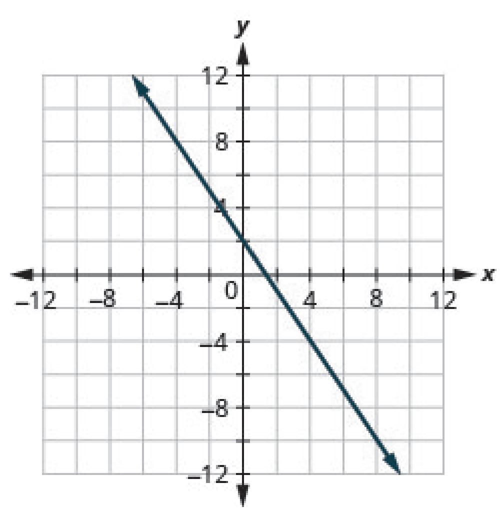

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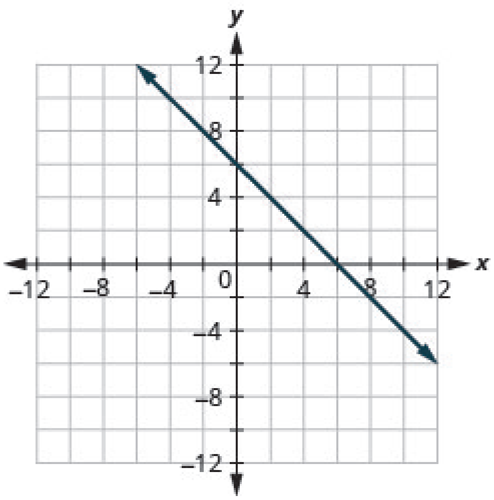

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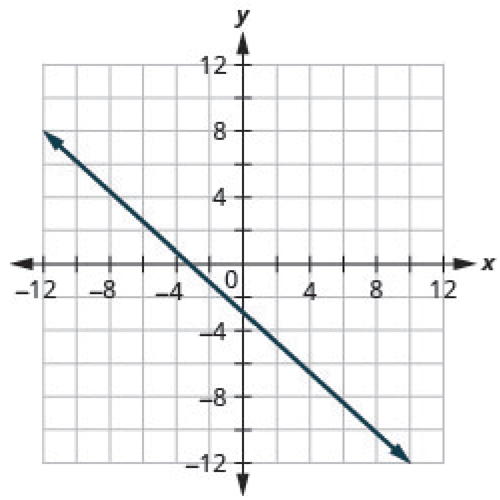

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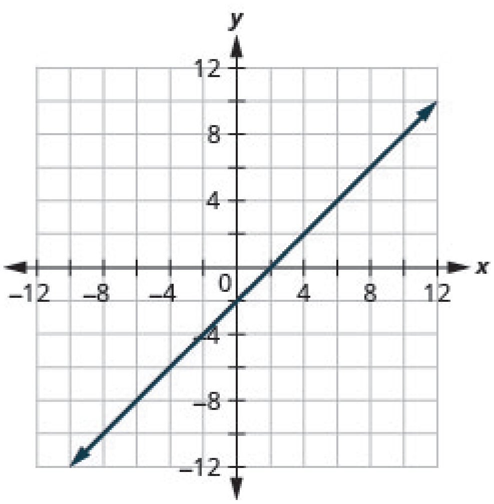

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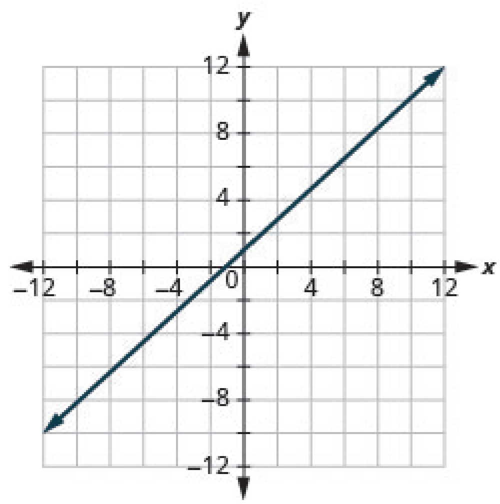
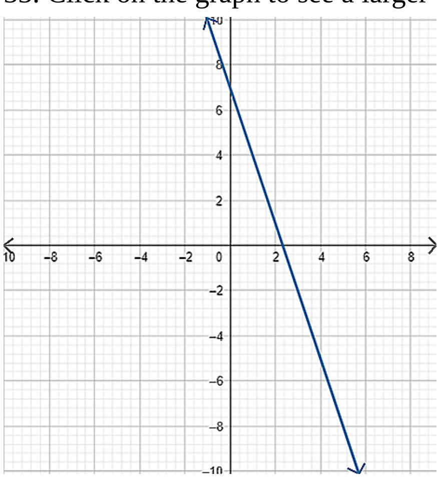


33. Click on the graph to see a larger version



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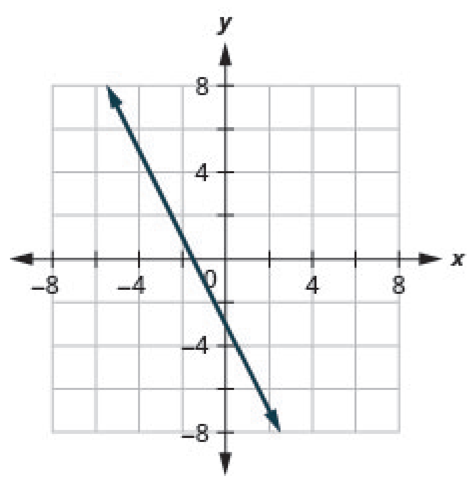

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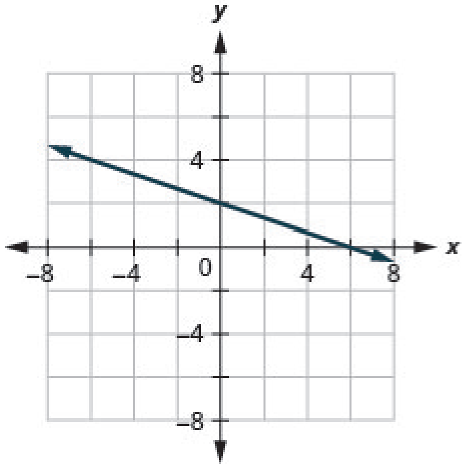
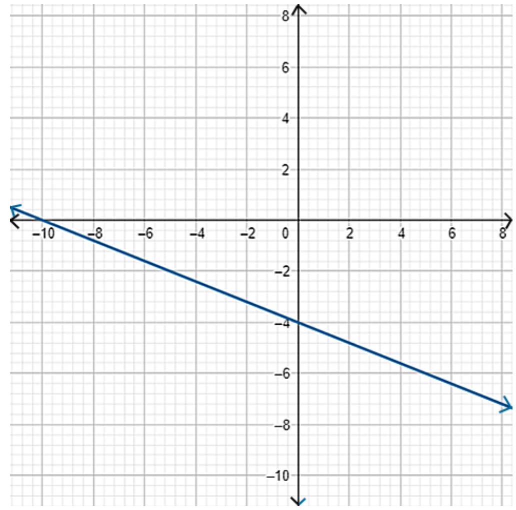


39. Click on the graph to see a larger version



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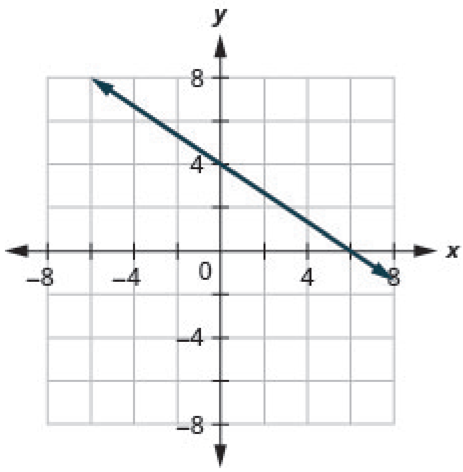

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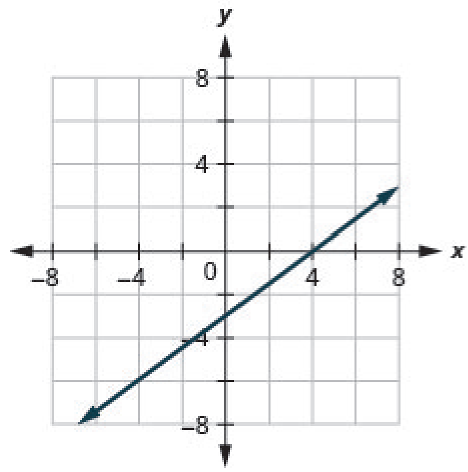

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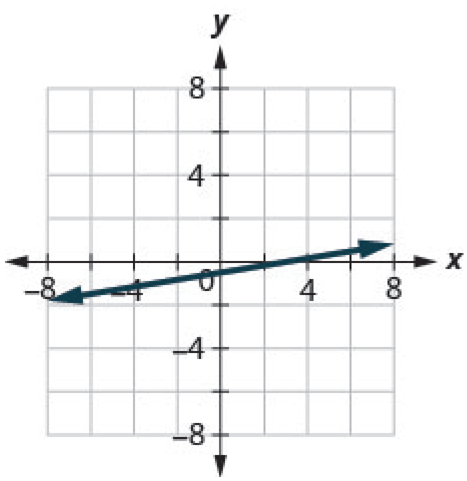

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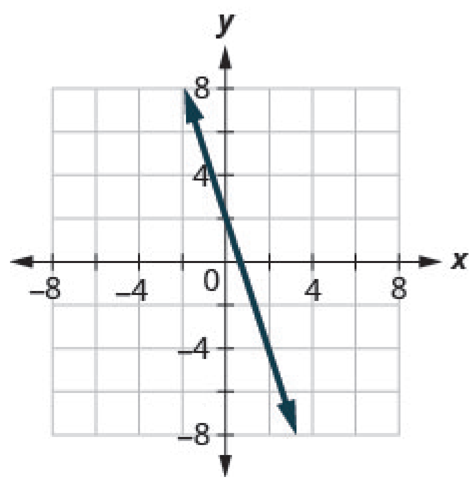

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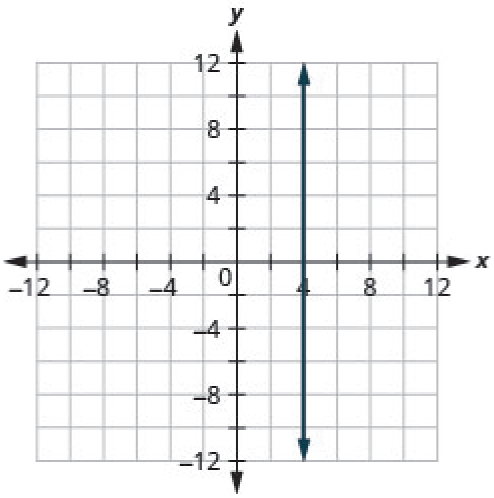
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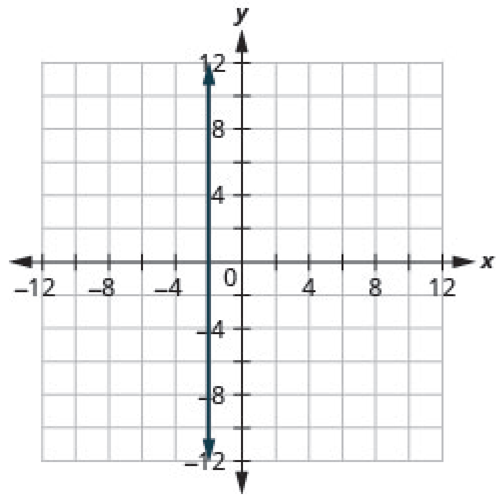
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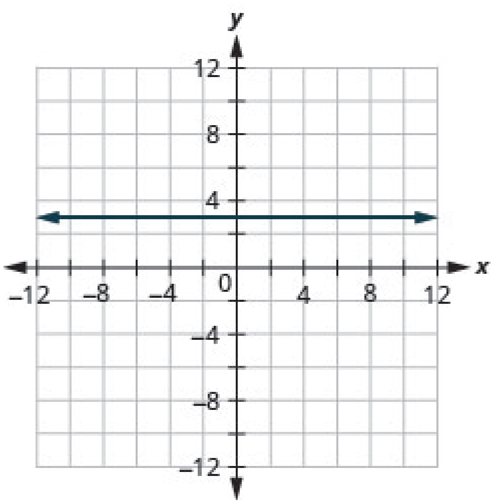
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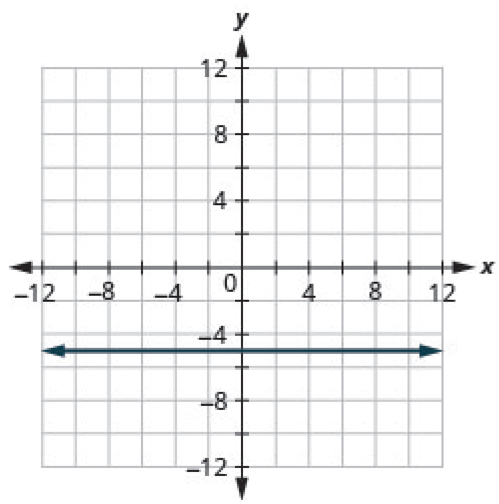
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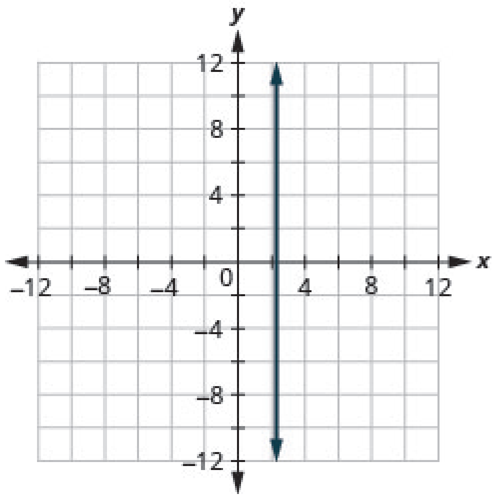
57.



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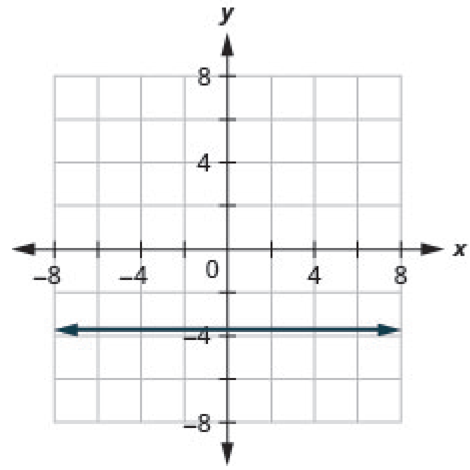

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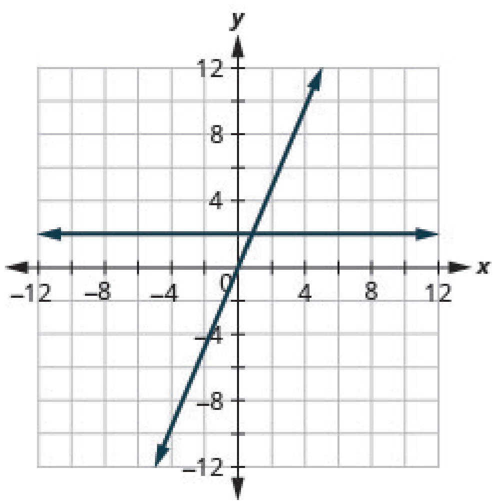

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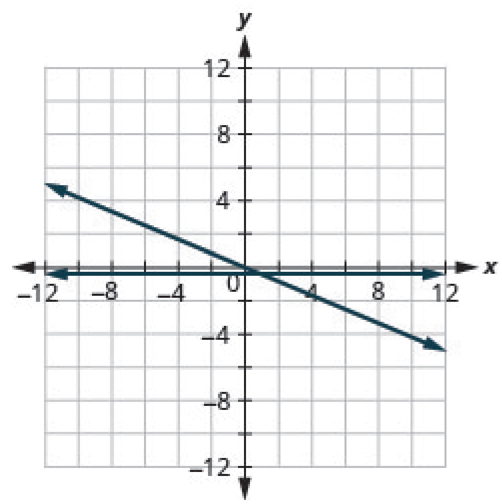

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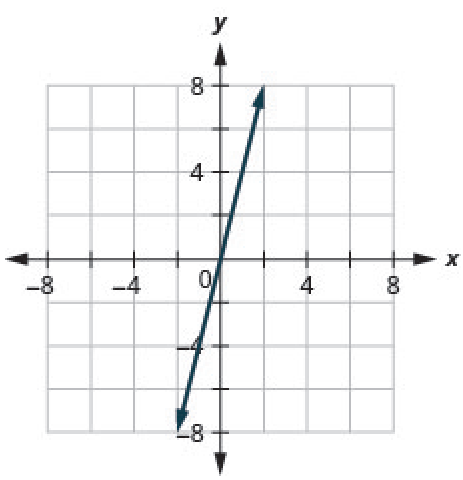

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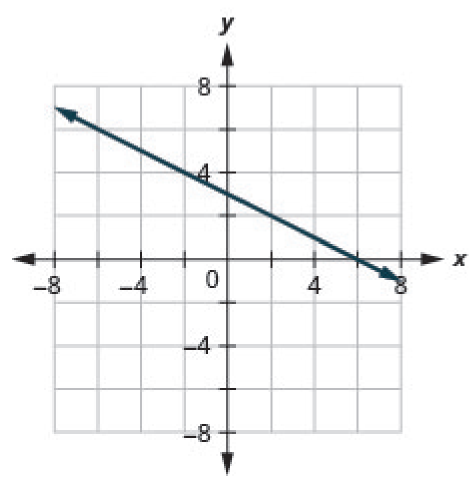

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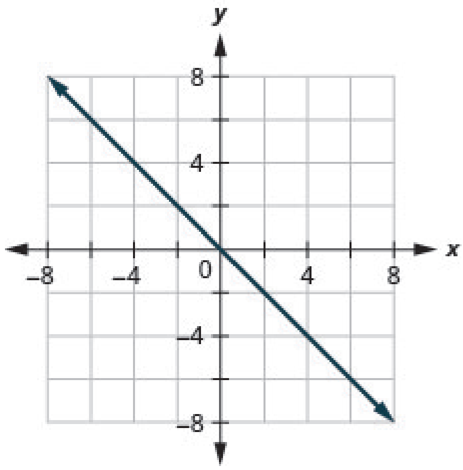

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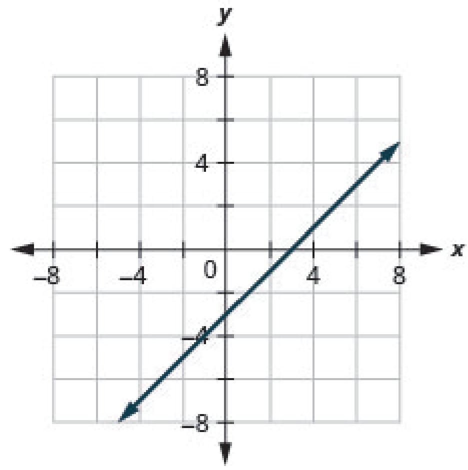

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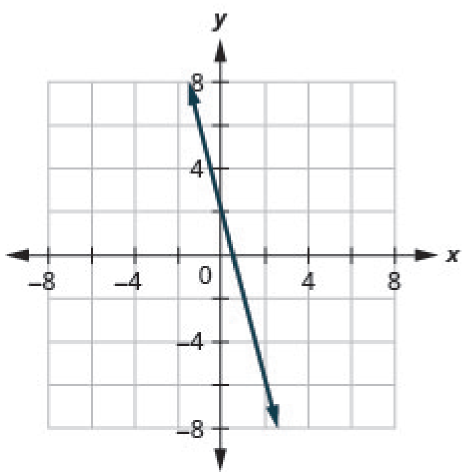

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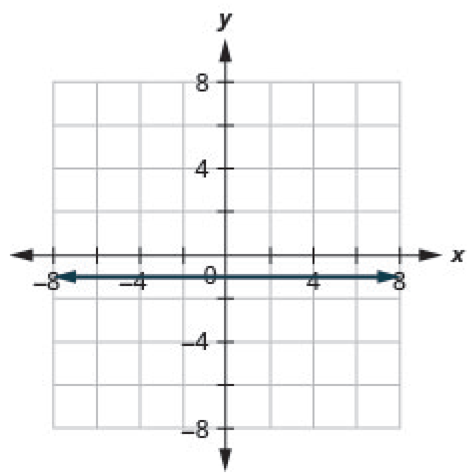

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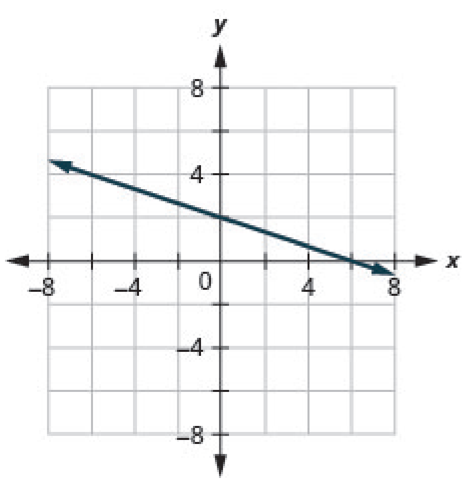

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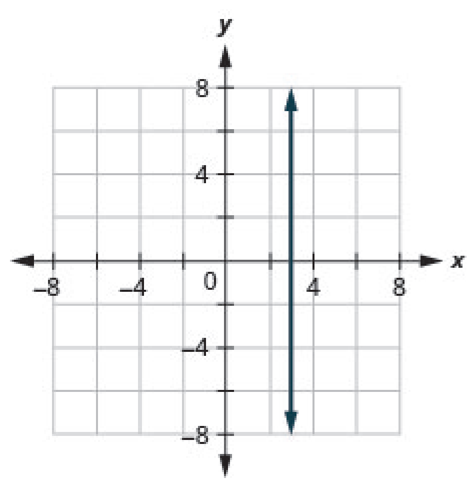
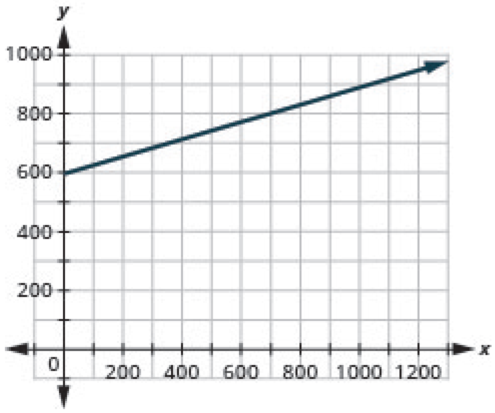


81. \$722, \$850, \$978 	83. Answers will vary.

Attributions

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4.3 Graph with Intercepts

Learning Objectives

By the end of this section, you will be able to:

- Identify the x– and y– intercepts on a graph
- Find the x– and y– intercepts from an equation of a line
- Graph a line using the intercepts

Identify the *x*– and *y*– Intercepts on a Graph

Every linear equation can be represented by a unique line that shows all the solutions of the equation. We have seen that when graphing a line by plotting points, you can use any three solutions to graph. This means that two people graphing the line might use different sets of three points.

At first glance, their two lines might not appear to be the same, since they would have different points labeled. But if all the work was done correctly, the lines should be exactly the same. One way to recognize that they are indeed the same line is to look at where the line crosses the *x*– axis and the *y*– axis. These points are called the *intercepts* of the line.

Intercepts of a line

The points where a line crosses the *x*– axis and the *y*– axis are called the intercepts of a line.

Let's look at the graphs of the lines in (Figure 1).

Examples of graphs crossing the x-negative axis.

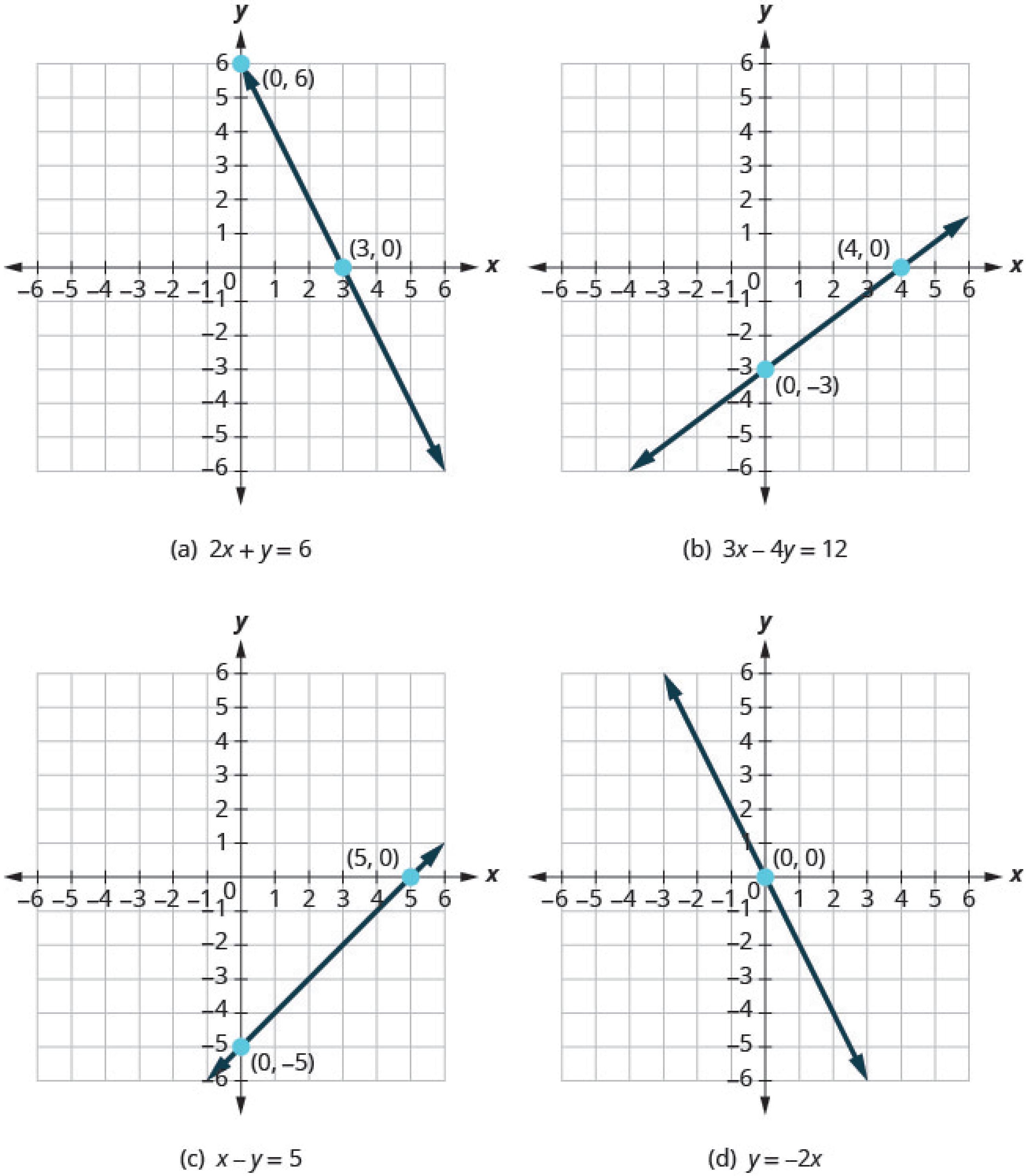


Figure .1

First, notice where each of these lines crosses the x negative axis. See (Figure 1).

Figure	**The line crosses the *x*– axis at:**	**Ordered pair of this point**
Figure (a)	3	$(3, 0)$
Figure (b)	4	$(4, 0)$
Figure (c)	5	$(5, 0)$
Figure (d)	0	$(0, 0)$

Do you see a pattern?

For each row, the *y*– coordinate of the point where the line crosses the *x*– axis is zero. The point where the line crosses the *x*– axis has the form $(a, 0)$ and is called the *x*– intercept of a line. The *x*– intercept occurs when y is zero.

Now, let's look at the points where these lines cross the *y*– axis. See the table below.

Figure	**The line crosses the *y*-axis at:**	**Ordered pair for this point**
Figure (a)	6	$(0, 6)$
Figure (b)	-3	$(0, -3)$
Figure (c)	-5	$(0, -5)$
Figure (d)	0	$(0, 0)$

What is the pattern here?

In each row, the *x*– coordinate of the point where the line crosses the *y*– axis is zero. The point where the line crosses the *y*– axis has the form $(0, b)$ and is called the *y*- *intercept* of the line. The *y*– intercept occurs when x is zero.

x– intercept and *y*– intercept of a line

The *x*– intercept is the point $(a, 0)$ where the line crosses the *x*– axis.

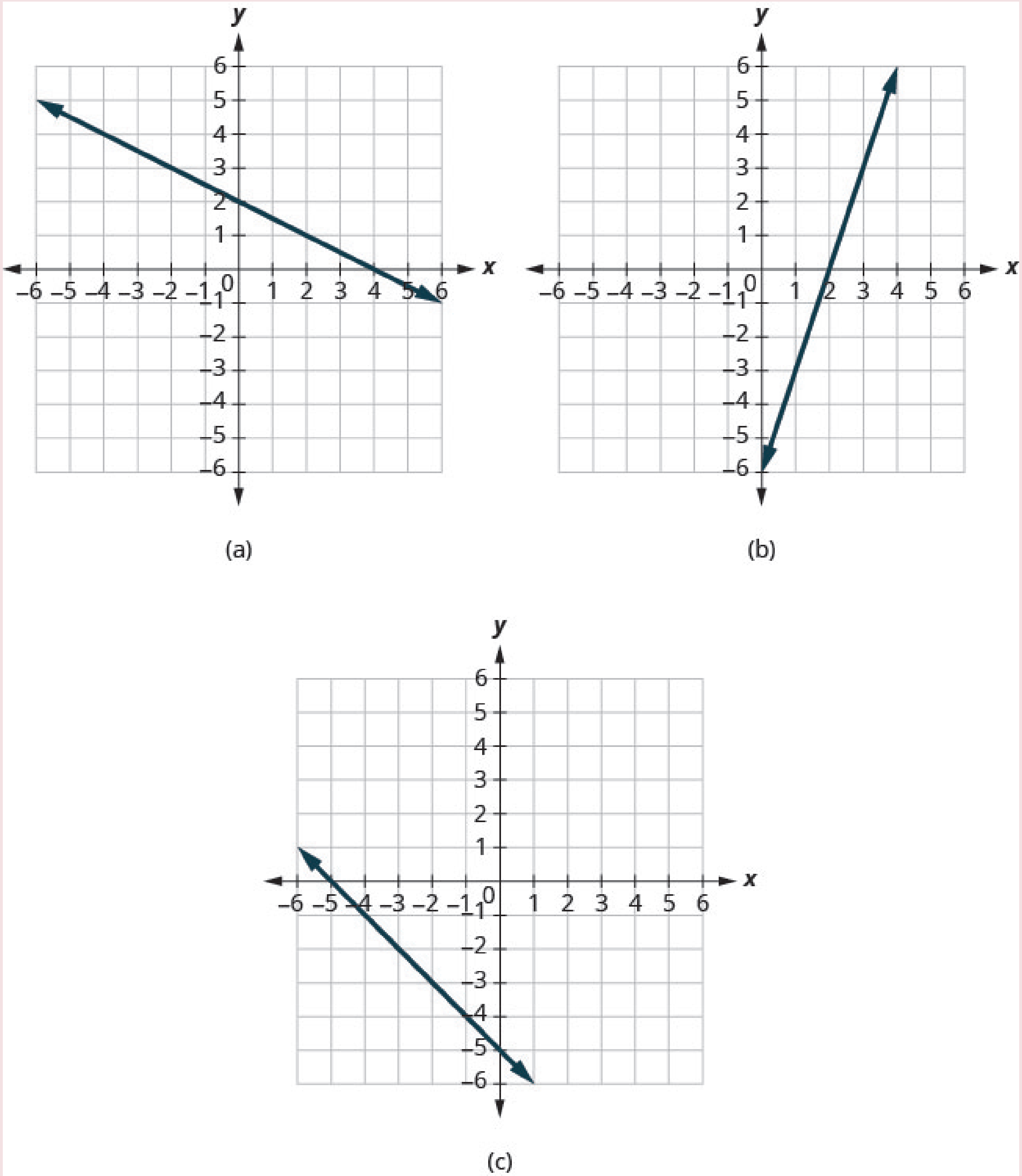
The *y*– intercept is the point $(0, b)$ where the line crosses the *y*– axis.

- The *x*-intercept occurs when *y* is zero.
- The *y*-intercept occurs when *x* is zero.

x	*y*
a	0
0	*b*

EXAMPLE 1

Find the *x*– and *y*– intercepts on each graph.



Solution

a. The graph crosses the *x*– axis at the point $(4, 0)$. The *x*– intercept is $(4, 0)$.

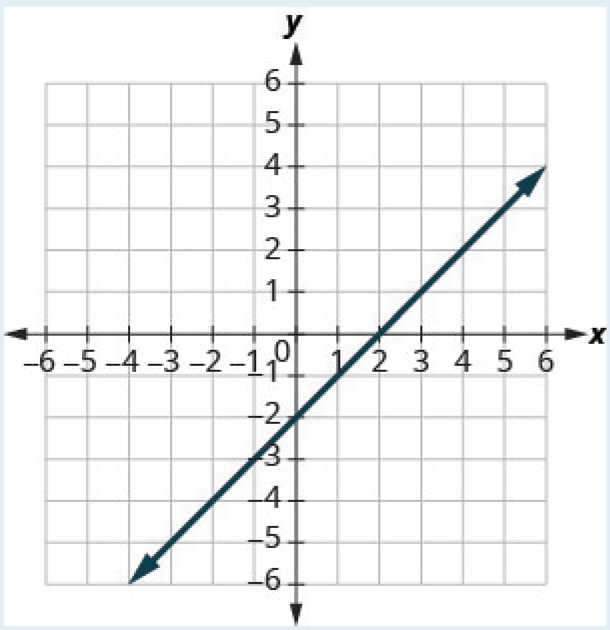
The graph crosses the *y*– axis at the point $(0, 2)$. The *y*– intercept is $(0, 2)$.

b. The graph crosses the *x*– axis at the point $(2, 0)$. The *x*– intercept is $(2, 0)$
The graph crosses the *y*– axis at the point $(0, -6)$. The *y*– intercept is $(0, -6)$.

c. The graph crosses the *x*– axis at the point $(-5, 0)$. The *x*– intercept is $(-5, 0)$.
The graph crosses the *y*– axis at the point $(0, -5)$. The *y*– intercept is $(0, -5)$.

TRY IT 1.1

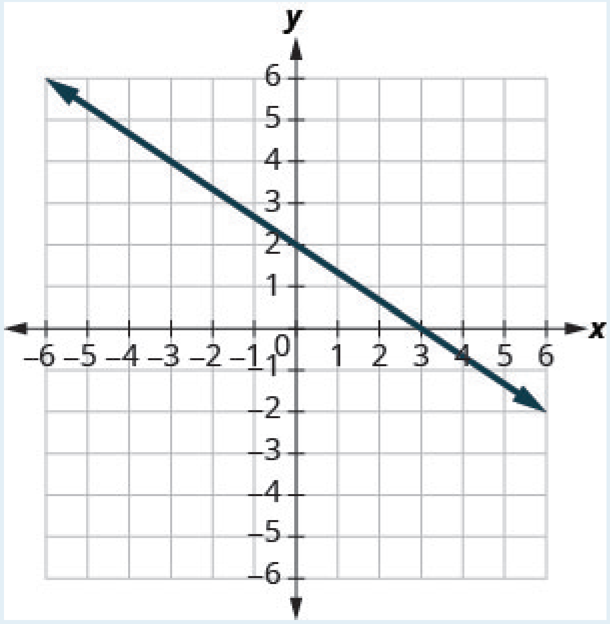
Find the *x*– and *y*– intercepts on the graph.



Show answer
x– intercept: $(2, 0)$; *y*– intercept: $(0, -2)$

TRY IT 1.2

Find the *x*– and *y*– intercepts on the graph.



Show answer
x– intercept: $(3, 0)$, *y*– intercept: $(0, 2)$

Find the *x*– and *y*– Intercepts from an Equation of a Line

Recognizing that the *x*– intercept occurs when *y* is zero and that the *y*– intercept occurs when *x* is zero, gives us a method to find the intercepts of a line from its equation. To find the *x*– intercept, let $y = 0$ and solve for *x*. To find the *y*– intercept, let $x = 0$ and solve for *y*.

Find the *x*– and *y*– intercepts from the equation of a line

Use the equation of the line. To find:

- the *x*– intercept of the line, let $y = 0$ and solve for x.
- the *y*– intercept of the line, let $x = 0$ and solve for y.

EXAMPLE 2

Find the intercepts of $2x + y = 6$.

Solution
We will let $y = 0$ to find the *x*– intercept, and let $x = 0$ to find the *y*– intercept. We will fill in the table, which reminds us of what we need to find.

$2x+y=6$		
x	y	
	0	x-intercept
0		y-intercept

To find the *x*– intercept, let $y=0$.

	$2x+y=6$
Let *y* = 0.	$2x+0=6$
Simplify.	$2x=6$
	$x=3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let *x* = 0.	
	$2x+y=6$
Let *x* = 0.	$2\cdot 0+y=6$
Simplify.	$0+y=6$
	$y=6$
The *y*-intercept is	(0, 6)

The intercepts are the points $(3,0)$ and $(0,6)$ as shown in the following table.

$2x+y=6$	
x	y
3	0
0	6

TRY 2.1

Find the intercepts of $3x + y = 12$.

Show answer

x– intercept: $(4, 0)$, *y*– intercept: $(0, 12)$

TRY IT 2.2

Find the intercepts of $x + 4y = 8$.

Show answer

x– intercept: $(8, 0)$, *y*– intercept: $(0, 2)$

EXAMPLE 3

Find the intercepts of $4x - 3y = 12$.

Solution

To find the *x*-intercept, let *y* = 0.	
	$4x - 3y = 12$
Let *y* = 0.	$4x - 3 \cdot 0 = 12$
Simplify.	$4x - 0 = 12$
	$4x = 12$
	$x = 3$
The *x*-intercept is	(3, 0)
To find the *y*-intercept, let *x* = 0.	
	$4x - 3y = 12$
Let *x* = 0.	$4 \cdot 0 - 3y = 12$
Simplify.	$0 - 3y = 12$
	$-3y = 12$
	$y = -4$
The *y*-intercept is	(0, −4)

The intercepts are the points (3, 0) and (0, −4) as shown in the following table.

$4x - 3y = 12$	
x	y
3	0
0	-4

TRY IT 3.1

Find the intercepts of $3x - 4y = 12$.

Show answer
x– intercept: $(4, 0)$, *y*– intercept: $(0, -3)$

TRY IT 3.2

Find the intercepts of $2x - 4y = 8$.

Show answer
x– intercept: $(4, 0)$, *y*– intercept: $(0, -2)$

Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you need to find three points whose coordinates are solutions to the equation. You can use the *x*– and *y*– intercepts as two of your three points. Find the intercepts, and then find a third point to ensure accuracy. Make sure the points line up—then draw the line. This method is often the quickest way to graph a line.

EXAMPLE 4

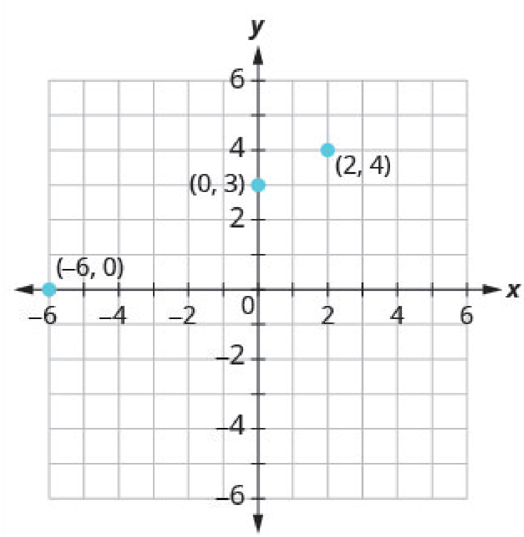
How to Graph a Line Using Intercepts

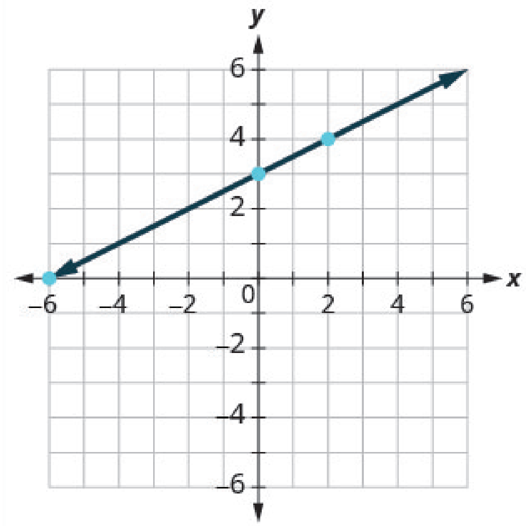
Graph $-x + 2y = 6$ using the intercepts.

Solution

<table>
<tr>
<td>Step 1. Find the x- and y-intercepts of the line.

Let $y = 0$ and solve for x.
Let $x = 0$ and solve for y.</td>
<td>Find the x-intercept.

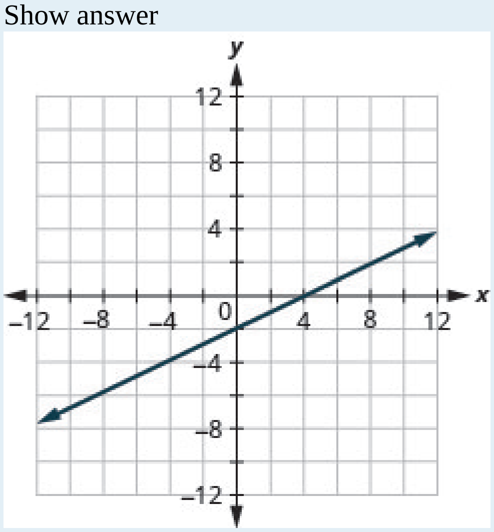
Find the y-intercept.</td>
<td>Let $y = 0$
$-x + 2y = 6$
$-x + 2(0) = 6$
$-x = 6$
$x = -6$
The x-intercept is $(-6, 0)$.
Let $x = 0$
$-x + 2y = 6$
$-0 + 2y = 6$
$2y = 6$
$y = 3$
The y-intercept is $(0, 3)$.</td>
</tr>
<tr>
<td>Step 2. Find another solution to the equation.</td>
<td>We'll use $x = 2$.</td>
<td>Let $x = 2$
$-x + 2y = 6$
$-2 + 2y = 6$
$2y = 8$
$y = 4$
A third point is $(2, 4)$.</td>
</tr>
<tr>
<td>Step 3. Plot the three points. Check that the points line up.</td>
<td><table><tr><th>x</th><th>y</th><th>(x, y)</th></tr><tr><td>–6</td><td>0</td><td>(–6, 0)</td></tr><tr><td>0</td><td>3</td><td>(0, 3)</td></tr><tr><td>2</td><td>4</td><td>(2, 4)</td></tr></table></td>
<td></td>
</tr>
</table>

Step 4. Draw the line.	See the graph.	

TRY IT 4.1

Graph $x - 2y = 4$ using the intercepts.

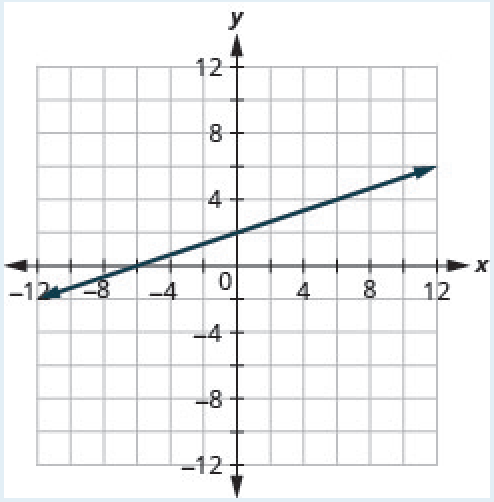
Show answer



TRY IT 4.2

Graph $-x + 3y = 6$ using the intercepts.

Show answer



HOW TO: Graph a linear equation using the intercepts

The steps to graph a linear equation using the intercepts are summarized below.

1. Find the *x*– and *y*– intercepts of the line.
    - Let $y = 0$ and solve for x
    - Let $x = 0$ and solve for y.
2. Find a third solution to the equation.
3. Plot the three points and check that they line up.
4. Draw the line.

EXAMPLE 5

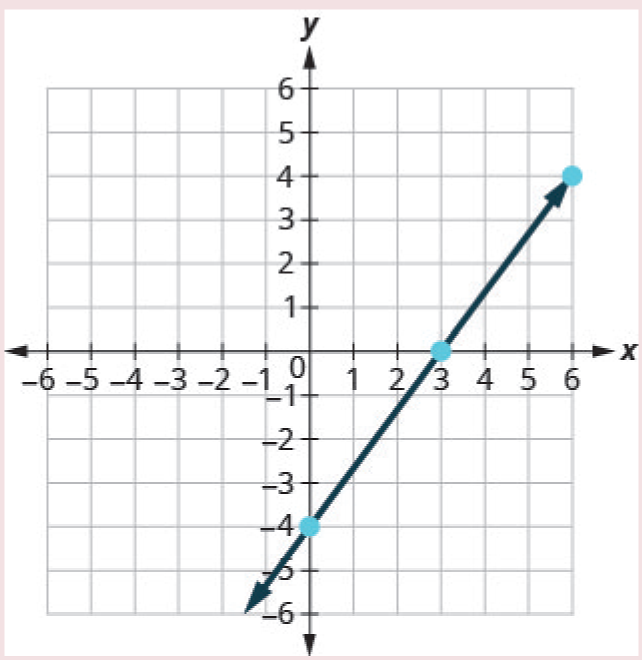
Graph $4x - 3y = 12$ using the intercepts.

Solution
Find the intercepts and a third point.

x-intercept, let $y=0$	y-intercept, let $x=0$	third point, let $y=4$
$4x-3y=12$	$4x-3y=12$	$4x-3y=12$
$4x-3(0)=12$	$4(0)-3y=12$	$4x-3(4)=12$
$4x=12$	$-3y=12$	$4x-12=12$
$x=3$	$y=-4$	$4x=24$
		$x=6$

We list the points in following table and show the graph below.

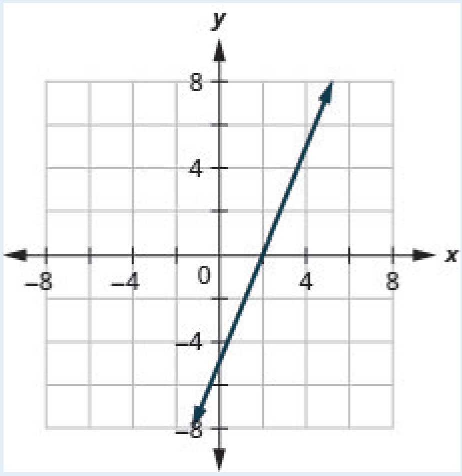
$4x-3y=12$		
x	y	(x,y)
3	0	$(3,0)$
0	-4	$(0,-4)$
6	4	$(6,4)$



TRY IT 5.1

Graph $5x-2y=10$ using the intercepts.

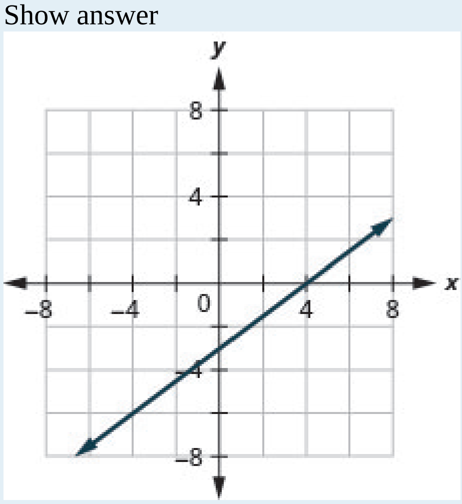
Show answer



TRY IT 5.2

Graph $3x-4y=12$ using the intercepts.

Show answer



EXAMPLE 6

Graph $y=5x$ using the intercepts.

Solution

x-intercept	*y*-intercept
Let $y = 0$.	Let $x = 0$.
$y = 5x$	$y = 5x$
$0 = 5x$	$y = 5 \cdot 0$
$0 = x$	$y = 0$
$(0, 0)$	$(0, 0)$

This line has only one intercept. It is the point $(0, 0)$.

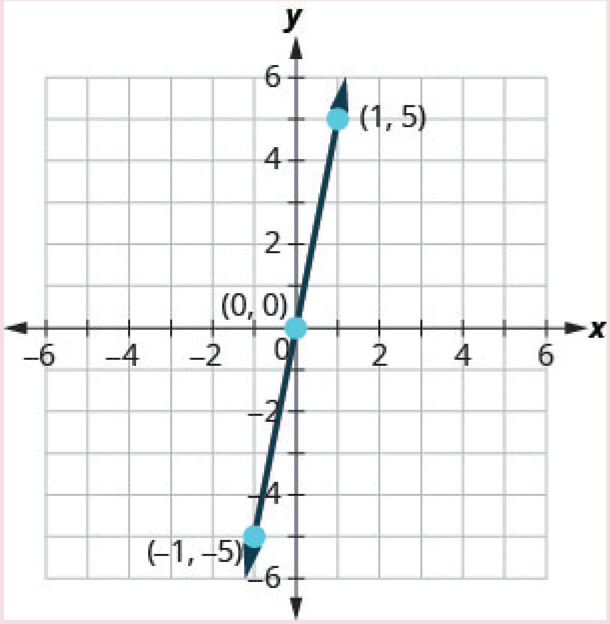
To ensure accuracy we need to plot three points. Since the *x*– and *y*– intercepts are the same point, we need *two* more points to graph the line.

Let $x = 1$.	Let $x = -1$.
$y = 5x$	$y = 5x$
$y = 5 \cdot 1$	$y = 5(-1)$
$y = 5$	$y = -5$

See following table..

$y = 5x$		
x	y	(x, y)
0	0	$(0, 0)$
1	5	$(1, 5)$
-1	-5	$(-1, -5)$

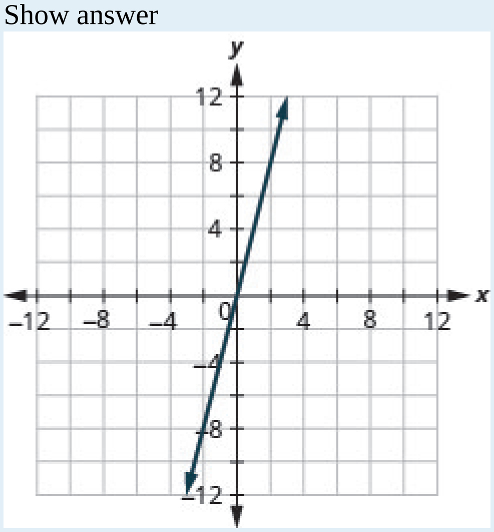
Plot the three points, check that they line up, and draw the line.



TRY IT 6.1

Graph $y = 4x$ using the intercepts.

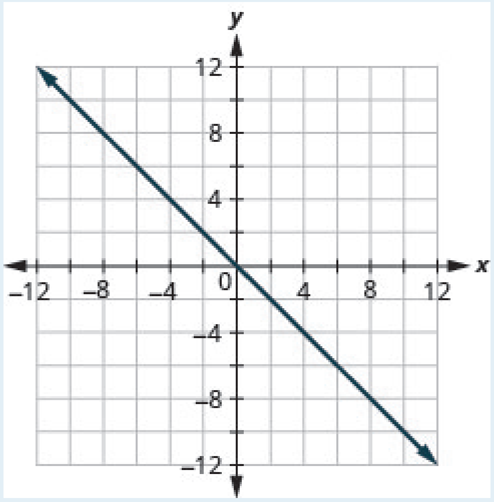
Show answer



TRY IT 6.2

Graph $y = -x$ the intercepts.

Show answer



Key Concepts

- **Find the *x*– and *y*– Intercepts from the Equation of a Line**
    - Use the equation of the line to find the *x*– intercept of the line, let $y = 0$ and solve for *x*.
    - Use the equation of the line to find the *y*– intercept of the line, let $x = 0$ and solve for *y*.
- **Graph a Linear Equation using the Intercepts**

    1. Find the *x*– and *y*– intercepts of the line.
       Let $y = 0$ and solve for *x*.
       Let $x = 0$ and solve for *y*.
    2. Find a third solution to the equation.
    3. Plot the three points and then check that they line up.
    4. Draw the line.

- **Strategy for Choosing the Most Convenient Method to Graph a Line:**
    - Consider the form of the equation.
    - If it only has one variable, it is a vertical or horizontal line.
      $x = a$ is a vertical line passing through the *x*– axis at a
      $y = b$ is a horizontal line passing through the *y*– axis at b.
    - If *y* is isolated on one side of the equation, graph by plotting points.
    - Choose any three values for *x* and then solve for the corresponding *y*– values.
    - If the equation is of the form $ax + by = c$, find the intercepts. Find the *x*– and *y*–

intercepts and then a third point.

Glossary

intercepts of a line
The points where a line crosses the *x*– axis and the *y*– axis are called the intercepts of the line.

***x*– intercept**
The point $(a, 0)$ where the line crosses the *x*– axis; the *x*– intercept occurs when y is zero.

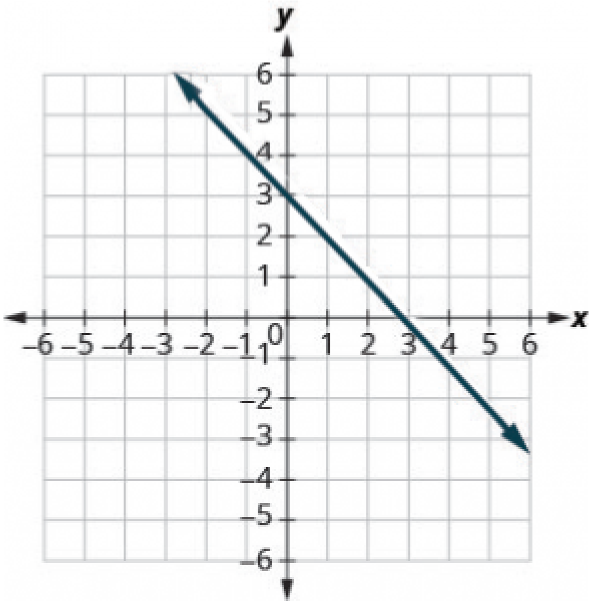
y-intercept
The point $(0, b)$ where the line crosses the *y*– axis; the *y*– intercept occurs when x is zero.

Practice Makes Perfect

Identify the *x*– and *y*– Intercepts on a Graph

In the following exercises, find the *x*– and *y*– intercepts on each graph.

1.


y
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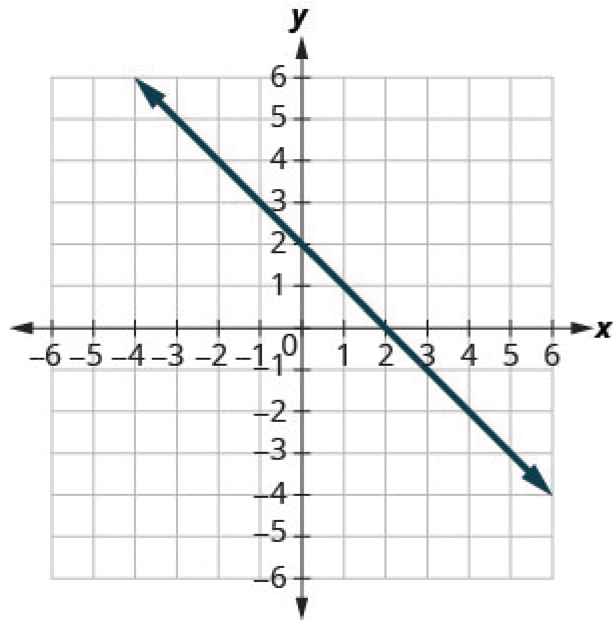
2.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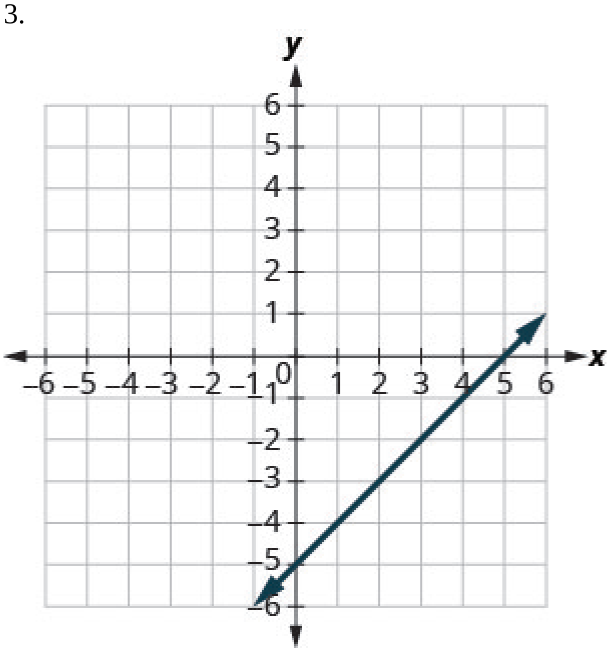
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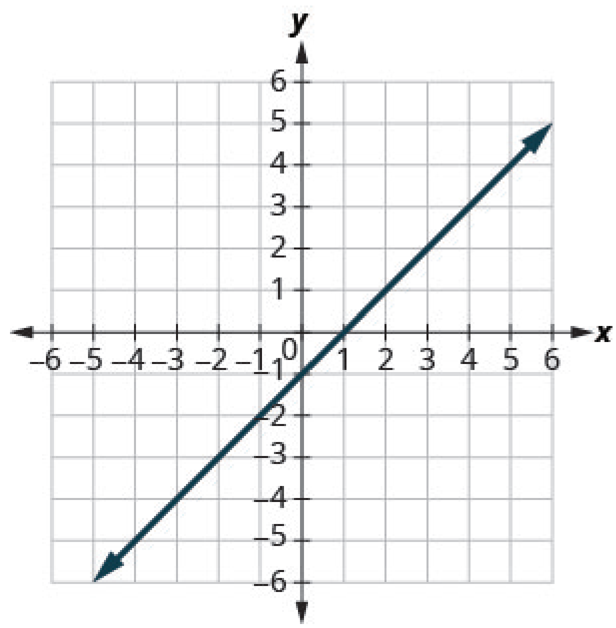
4.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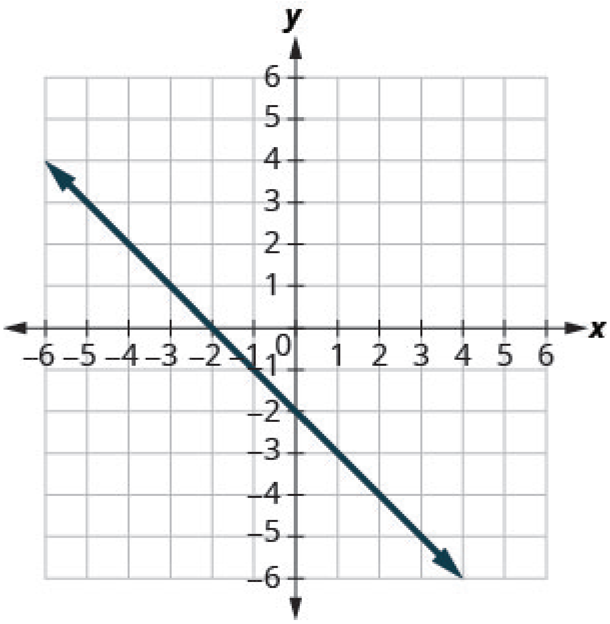
5.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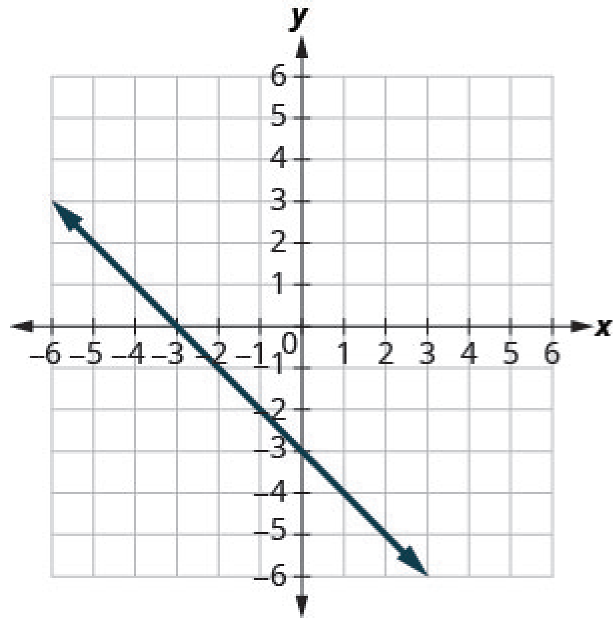
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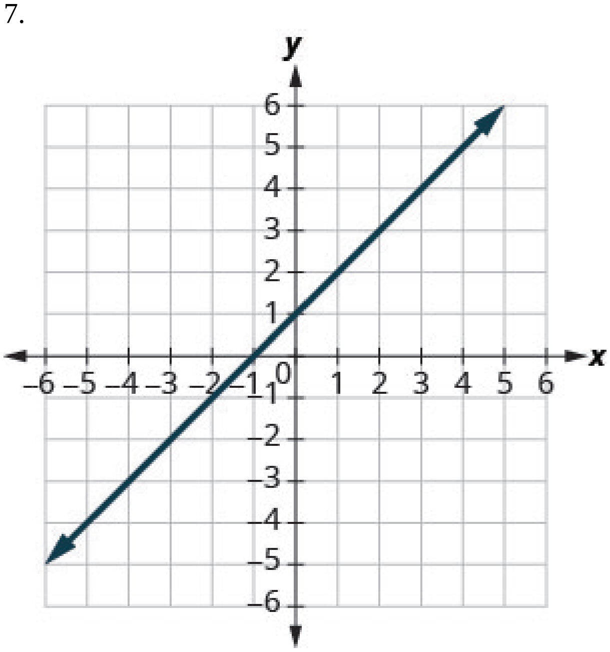
7.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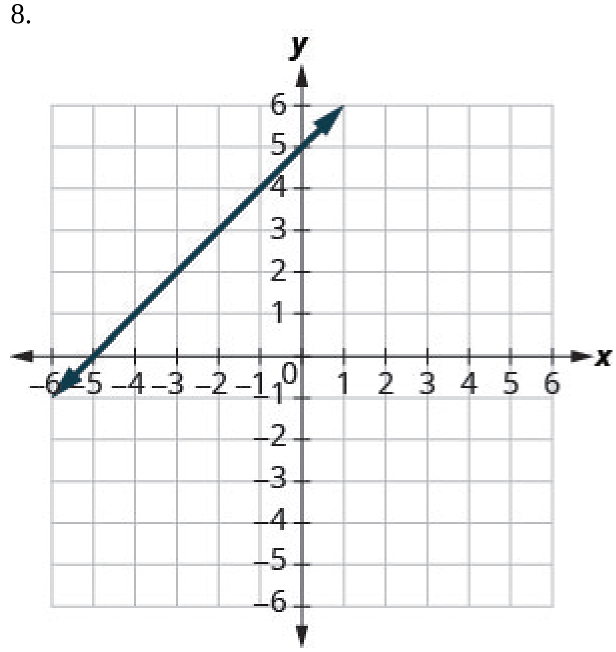
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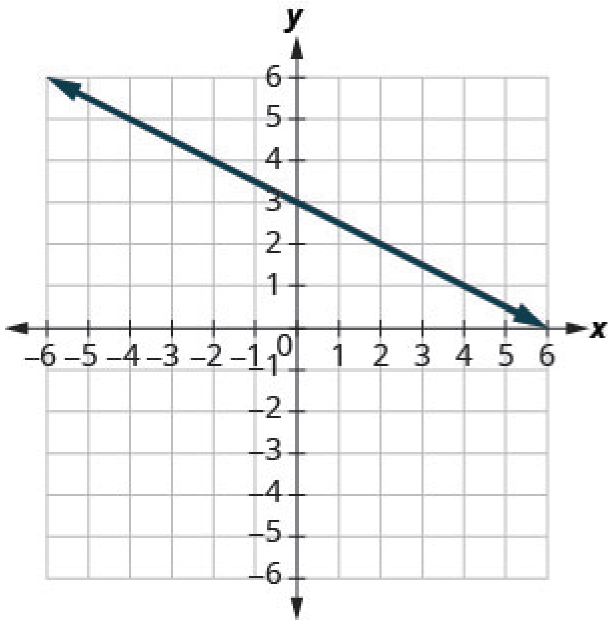

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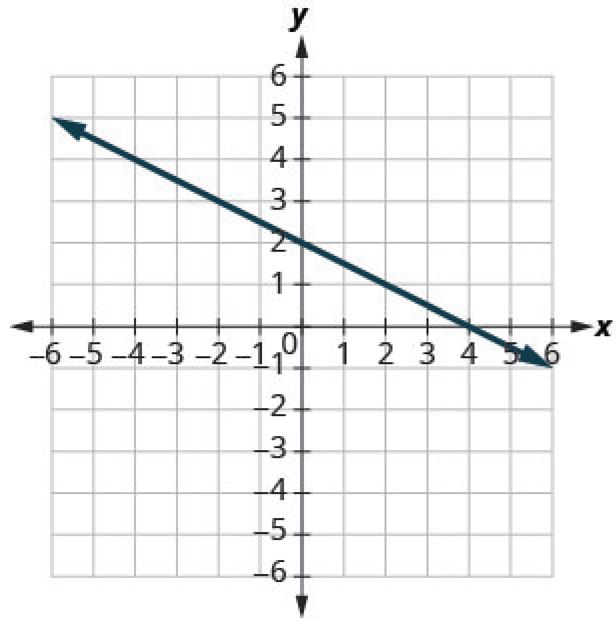

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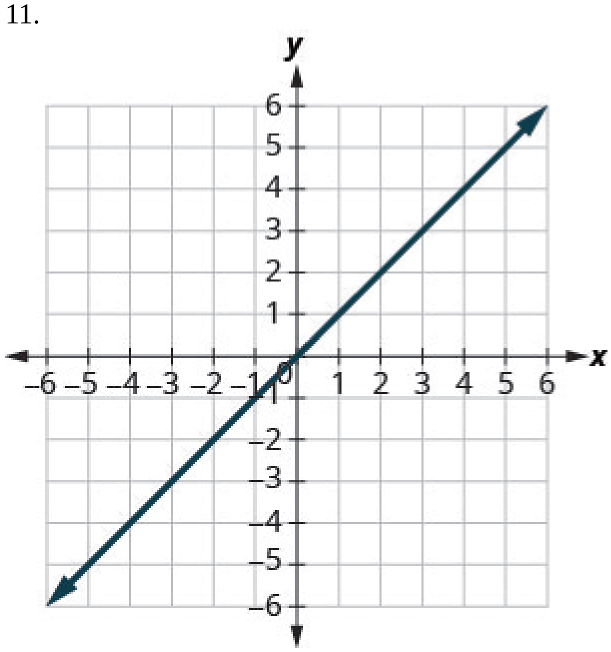

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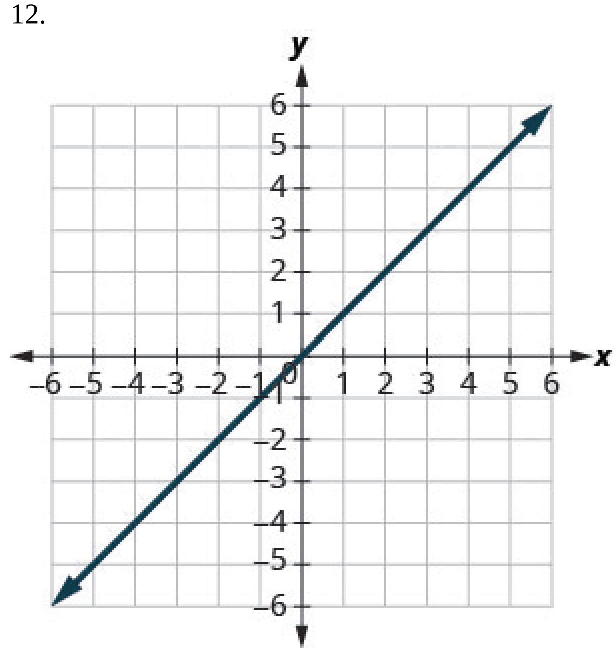


Find the *x*– and *y*– Intercepts from an Equation of a Line

In the following exercises, find the intercepts for each equation.

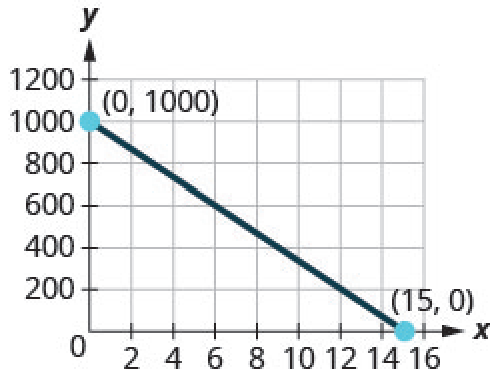
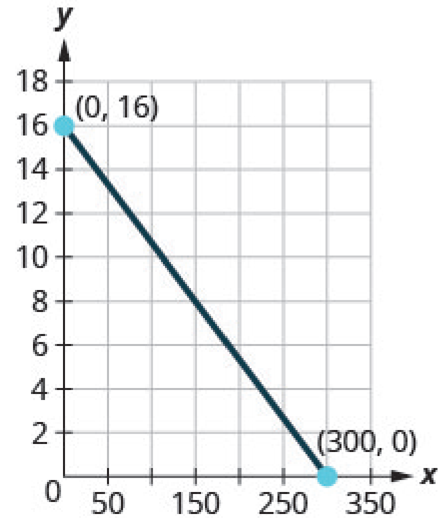
13. $x + y = 4$	14. $x + y = 3$
15. $x + y = -2$	17. $x - y = 5$
18. $x - y = 1$	19. $x - y = -3$
20. $x - y = -4$	21. $x + 2y = 8$
22. $x + 2y = 10$	23. $3x + y = 6$
24. $3x + y = 9$	25. $x - 3y = 12$
25. $x - 3y = 12$	27. $4x - y = 8$
28. $5x - y = 5$	29. $5y + 2x = 10$
30. $2x + 3y = 6$	31. $3x - 2y = 12$
32. $3x - 5y = 30$	33. $y = \frac{1}{3}x + 1$
34. $y = \frac{1}{4}x - 1$	35. $y = \frac{1}{5}x + 2$
36. $y = \frac{1}{3}x + 4$	37. $y = 3x$
38. $y = -2x$	39. $y = -4x$
40. $y = 5x$	

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

41. $-x+5y=10$	42. $-x+4y=8$
43. $x+2y=4$	44. $x+2y=6$
45. $x+y=2$	46. $x+y=5$
47. $x+y=-3$	48. $x+y=-1$
49. $x-y=1$	49. $x-y=1$
51. $x-y=-4$	52. $x-y=-3$
53. $4x+y=4$	54. $3x+y=3$
55. $2x+4y=12$	56. $3x+2y=12$
57. $3x-2y=6$	58. $5x-2y=10$
59. $2x-5y=-20$	60. $3x-4y=-12$
61. $3x-y=-6$	62. $2x-y=-8$
63. $y=\frac{3}{2}x-3$	64. $y=-4x$
65. $y=x$	66. $y=3x$

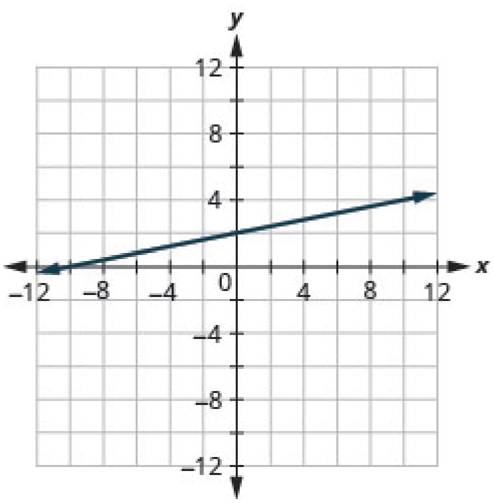
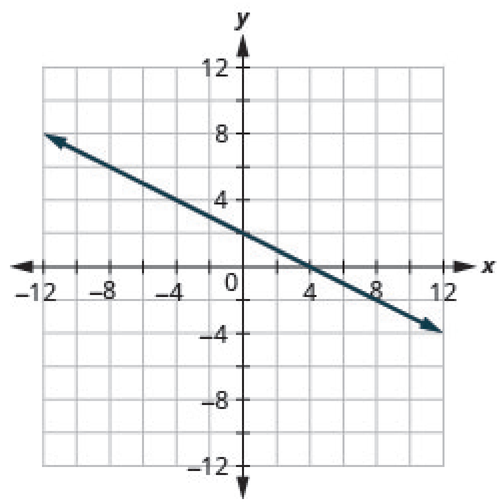
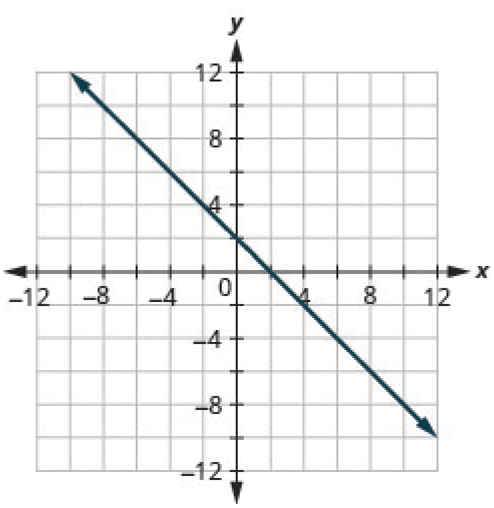
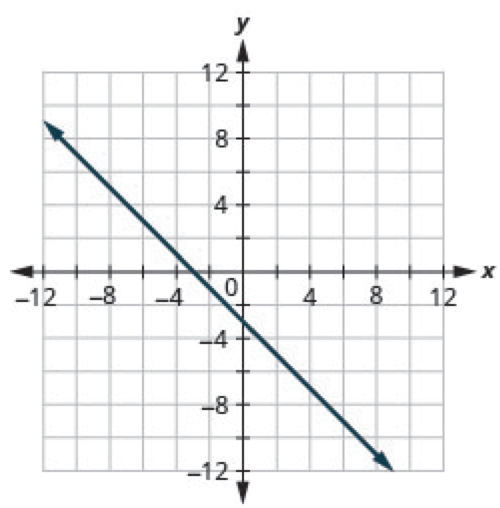
Everyday Math

67. **Road trip.** Damien is driving from Thunder Bay to Montreal, a distance of 1000 miles. The *x*– axis on the graph below shows the time in hours since Damien left Thunder Bay. The *y*– axis represents the distance he has left to drive.  1. a) Find the *x*– and *y*– intercepts. 2. b) Explain what the *x*– and *y*– intercepts mean for Damien.	68. **Road trip.** Jenna filled up the gas tank of her truck and headed out on a road trip. The *x*– axis on the graph below shows the number of miles Jenna drove since filling up. The *y*– axis represents the number of gallons of gas in the truck's gas tank.  1. a) Find the *x*– and *y*– intercepts. 2. b) Explain what the *x*– and *y*– intercepts mean for Ozzie.

Writing Exercises

69. How do you find the *x*– intercept of the graph of $3x - 2y = 6$?	70. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $4x + y = -4$? Why?
71. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = \frac{2}{3}x - 2$? Why?	72. Do you prefer to use the method of plotting points or the method using the intercepts to graph the equation $y = 6$? Why?

Answers

1. $(3,0),(0,3)$	3. $(5,0),(0,-5)$
5. $(-2,0),(0,-2)$	7. $(-1,0),(0,1)$
9. $(6,0),(0,3)$	11. $(0,0)$
13. $(4,0),(0,4)$	15. $(-2,0),(0,-2)$
17. $(5,0),(0,-5)$	19. $(-3,0),\quad(0,3)$
21. $(8,0),(0,4)$	23. $(2,0),(0,6)$
25. $(12,0),(0,-4)$	27. $(2,0),(0,-8)$
29. $(5,0),(0,2)$	31. $(4,0),(0,-6)$
33. $(-3,0),(0,1)$	35. $(-10,0),(0,2)$
37. $(0,0)$	39. $(0,0)$
41. 	43. 
45. 	47. 

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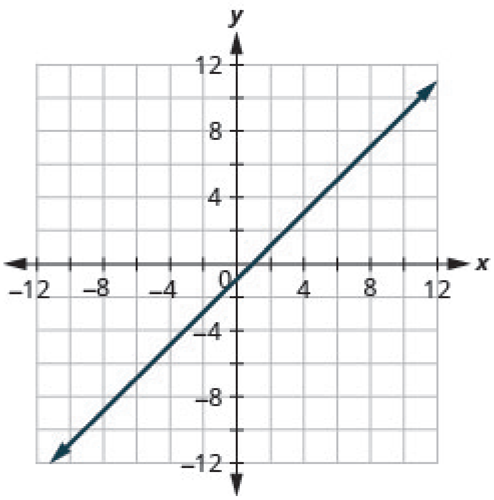

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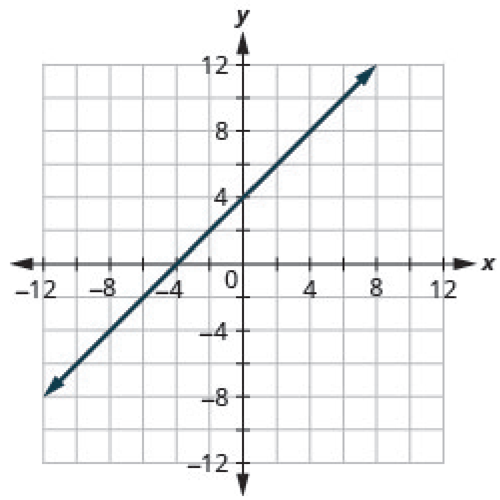

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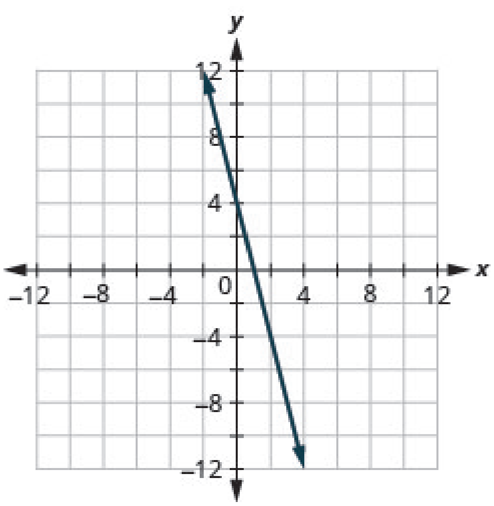

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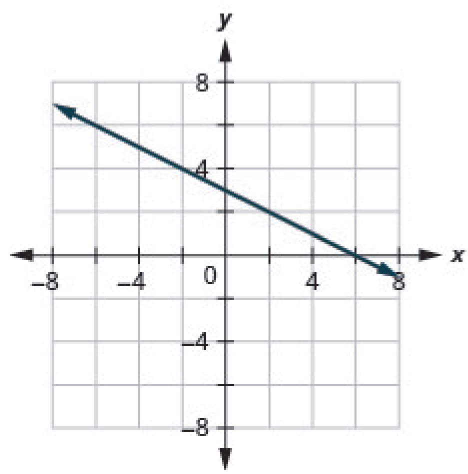

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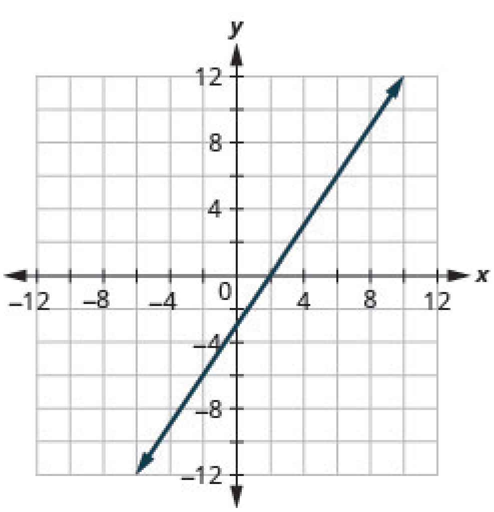
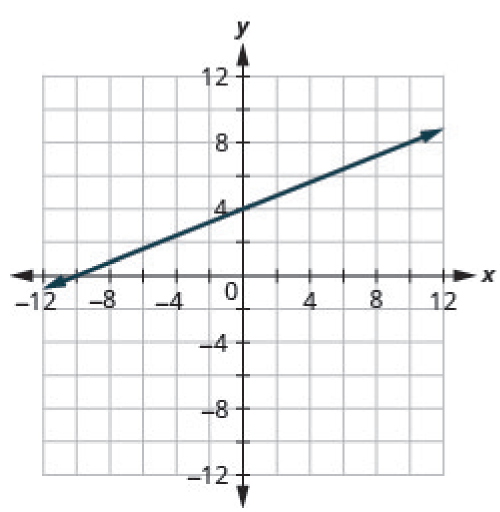
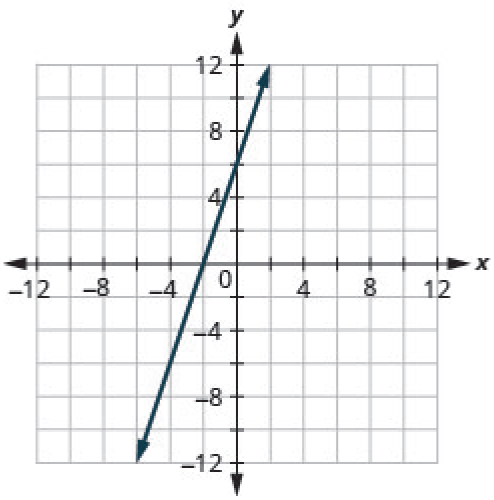
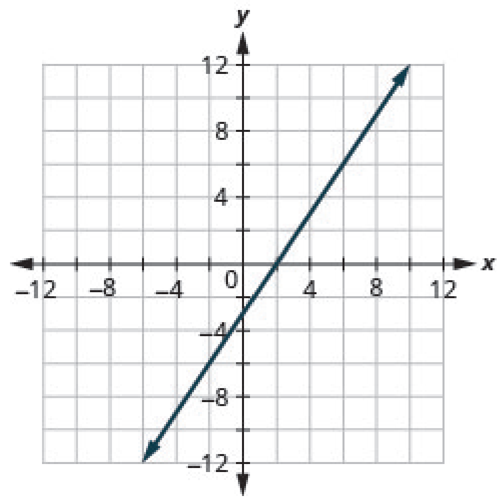
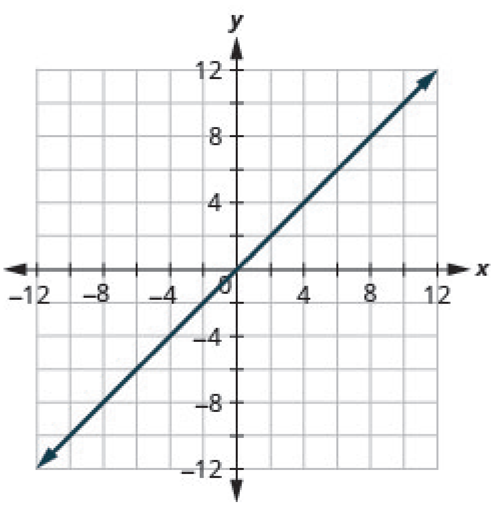


59.



61. 	63. 
65. 	67. a)$(0, 1000)$, $(15, 0)$ b) At $(0, 1000)$, he has been gone 0 hours and has 1000 miles left. At $(15, 0)$, he has been gone 15 hours and has 0 miles left to go.
69. Answers will vary.	71. Answers will vary.

Attributions

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4.4 Understand Slope of a Line

Learning Objectives

By the end of this section, you will be able to:

- Use geoboards to model slope
- Use $m = \frac{rise}{run}$ to find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

When you graph linear equations, you may notice that some lines tilt up as they go from left to right and some lines tilt down. Some lines are very steep and some lines are flatter. What determines whether a line tilts up or down or if it is steep or flat?

In mathematics, the 'tilt' of a line is called the *slope* of the line. The concept of slope has many applications in the real world. The pitch of a roof, grade of a highway, and a ramp for a wheelchair are some examples where you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

In this section, we will explore the concept of slope.

Use Geoboards to Model Slope

A geoboard is a board with a grid of pegs on it. Using rubber bands on a geoboard gives us a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line.

Doing the Manipulative Mathematics activity "Exploring Slope" will help you develop a better understanding of the slope of a line. (Graph paper can be used instead of a geoboard, if needed.)

We'll start by stretching a rubber band between two pegs as shown in (Figure 1).

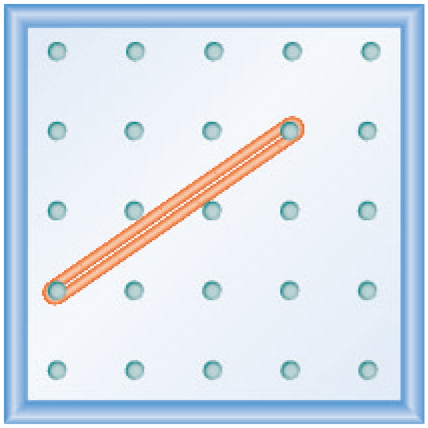


Figure .1

Doesn't it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle, as shown in (Figure 2)

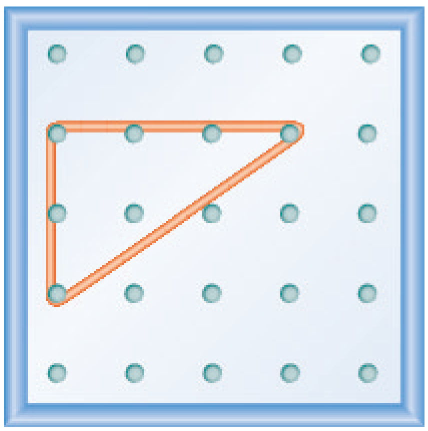


Figure .2

We carefully make a 90º angle around the third peg, so one of the newly formed lines is vertical and the other is horizontal.

To find the slope of the line, we measure the distance along the vertical and horizontal sides of the triangle. The vertical distance is called the rise and the horizontal distance is called the run, as shown in (Figure 3).

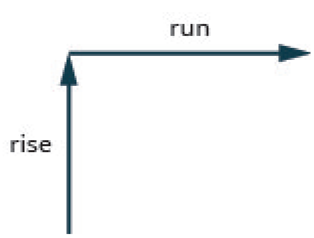


Figure .3

If our geoboard and rubber band look just like the one shown in (Figure 4), the rise is 2. The rubber band goes up 2 units. (Each space is one unit.)

The rise on this geoboard is 2, as the rubber band goes up two units.

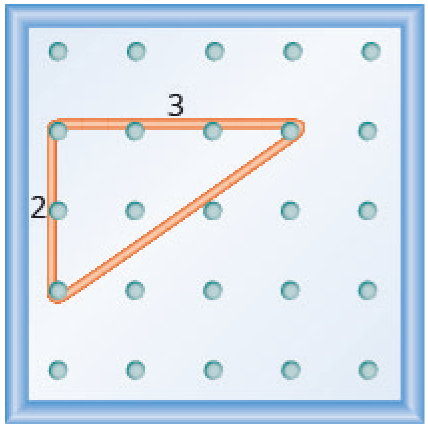


Figure .4

What is the run?

The rubber band goes across 3 units. The run is 3 (see (Figure 4)).

The slope of a line is the ratio of the rise to the run. In mathematics, it is always referred to with the letter m.

Slope of a line

The slope of a line of a line is $m = \frac{\text{rise}}{\text{run}}$.

The rise measures the vertical change and the run measures the horizontal change between two points on the line.

What is the slope of the line on the geoboard in (Figure 4)?

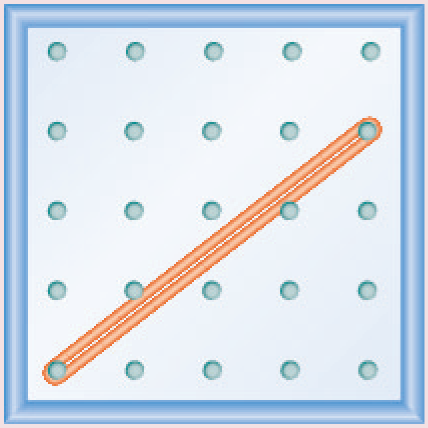
$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{2}{3}$$

The line has slope $\frac{2}{3}$. This means that the line rises 2 units for every 3 units of run.

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. If the rise goes up it is positive and if it goes down it is negative. The run will go from left to right and be positive.

EXAMPLE 1

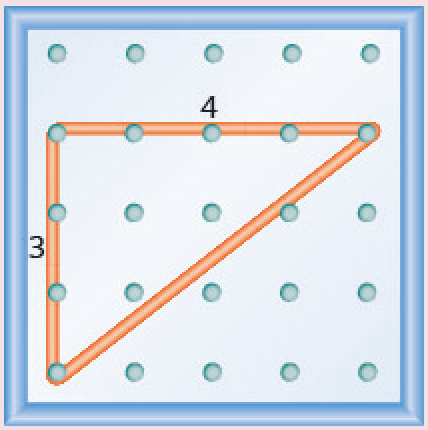
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \dfrac{\text{rise}}{\text{run}}$.

Start at the left peg and count the spaces up and to the right to reach the second peg.

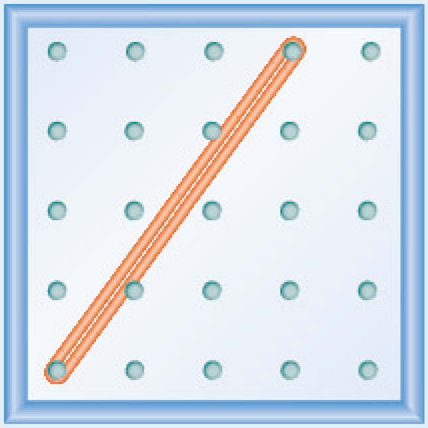


The rise is 3.	$m = \dfrac{3}{\text{run}}$
The run is 4.	$m = \dfrac{3}{4}$
	The slope is $\dfrac{3}{4}$.

This means that the line rises 3 units for every 4 units of run.

TRY IT 1.1

What is the slope of the line on the geoboard shown?

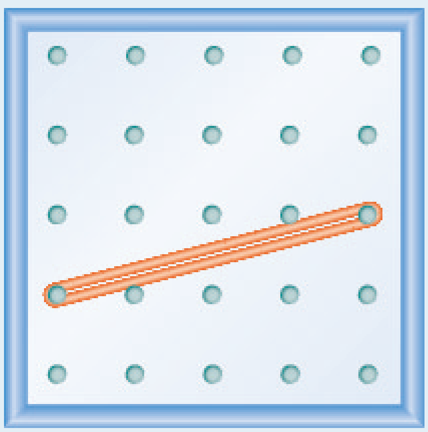


Show answer

$\frac{4}{3}$

TRY IT 1.2

What is the slope of the line on the geoboard shown?

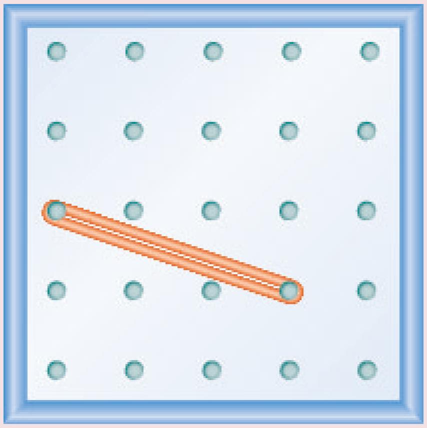


Show answer

$\frac{1}{4}$

EXAMPLE 2

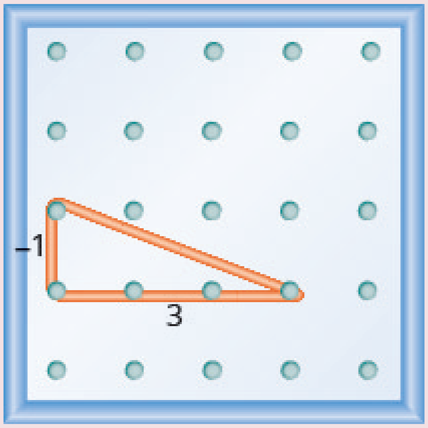
What is the slope of the line on the geoboard shown?



Solution

Use the definition of slope: $m = \frac{\text{rise}}{\text{run}}$.

Start at the left peg and count the units down and to the right to reach the second peg.

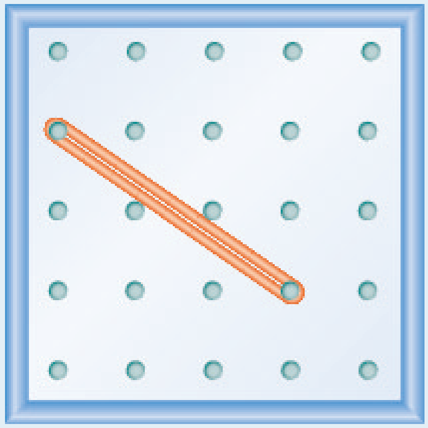


The rise is −1.	$= \frac{-1}{\text{run}}$
The run is 3.	$m = \frac{-1}{3}$ $m = -\frac{1}{3}$
	The slope is $-\frac{1}{3}$.

This means that the line drops 1 unit for every 3 units of run.

TRY IT 2.1

What is the slope of the line on the geoboard?

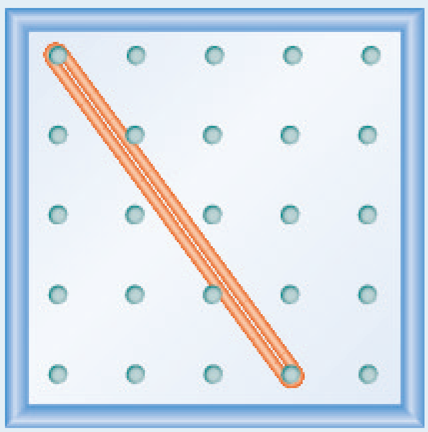


Show answer

$-\frac{2}{3}$

TRY IT 2.2

What is the slope of the line on the geoboard?



Show answer

$-\frac{4}{3}$

Notice that in (Example 1) the slope is positive and in (Example 2) the slope is negative. Do you notice any difference in the two lines shown in (Figure 5a) and (Figure 5b)?

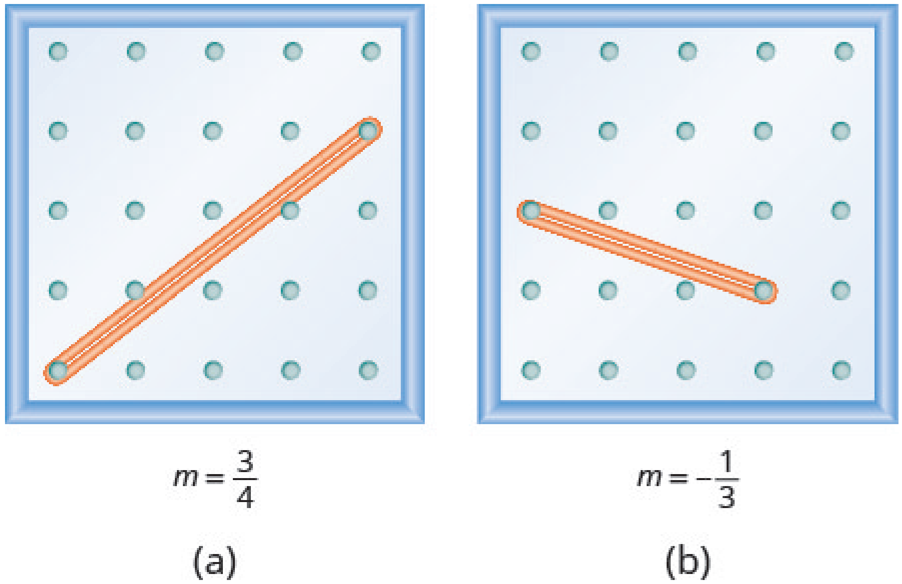


Figure .5 (a) (b)

Positive and negative slopes

We 'read' a line from left to right just like we read words in English. As you read from left to right, the line in (Figure 5a) is going up; it has positive slope. The line in (Figure 5b) is going down; it has negative slope.

EXAMPLE 3

Use a geoboard to model a line with slope $\frac{1}{2}$.

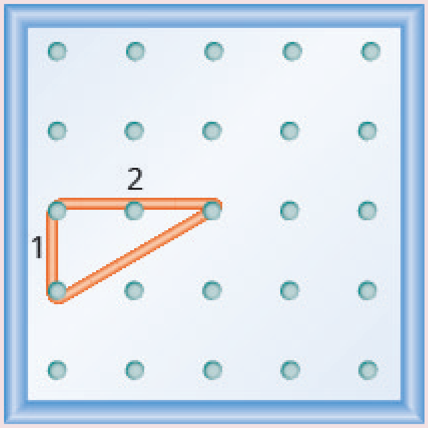
Solution
To model a line on a geoboard, we need the rise and the run.

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{1}{2}$.	$\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

So, the rise is 1 and the run is 2

Start at a peg in the lower left of the geoboard.

Stretch the rubber band up 1 unit, and then right 2 units.

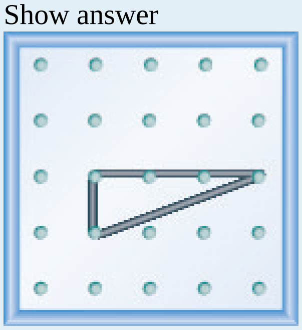


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{1}{2}$.

TRY IT 3.1

Model the slope $m = \frac{1}{3}$. Draw a picture to show your results.

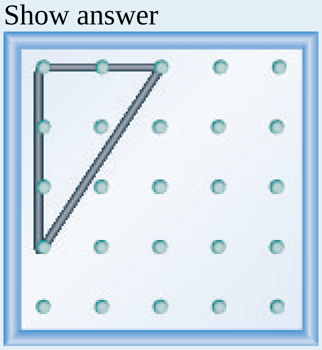
Show answer



TRY IT 3.2

Model the slope $m = \frac{3}{2}$. Draw a picture to show your results.

Show answer



EXAMPLE 4

Use a geoboard to model a line with slope $\frac{-1}{4}$.

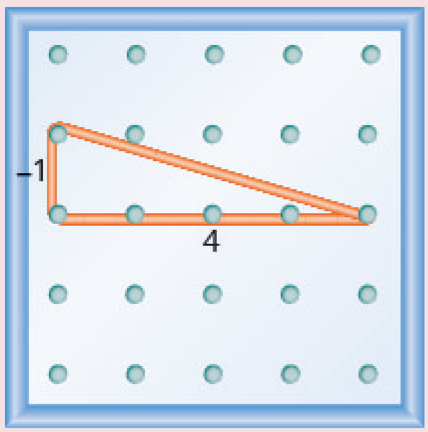
Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $\frac{-1}{4}$.	$\frac{-1}{4} = \frac{\text{rise}}{\text{run}}$

So, the rise is -1 and the run is 4

Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down.

We stretch the rubber band down 1 unit, then go to the right 4 units, as shown.

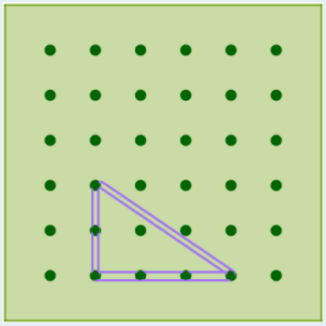


The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $\frac{-1}{4}$.

TRY IT 4.1

Model the slope $m = \frac{-2}{3}$. Draw a picture to show your results.

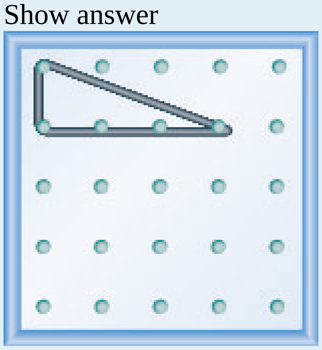
Show answer



TRY IT 4.2

Model the slope $m = \dfrac{-1}{3}$. Draw a picture to show your results.

Show answer



Use $m = \dfrac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

Now, we'll look at some graphs on the xy-coordinate plane and see how to find their slopes. The method will be very similar to what we just modeled on our geoboards.

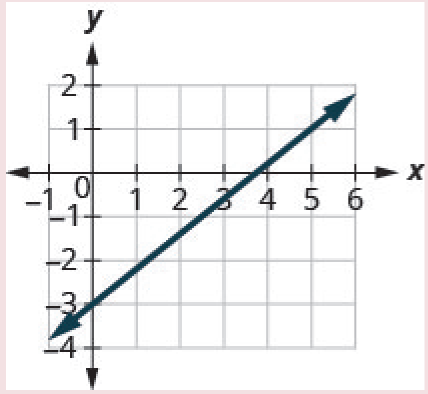
To find the slope, we must count out the rise and the run. But where do we start?

We locate two points on the line whose coordinates are integers. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

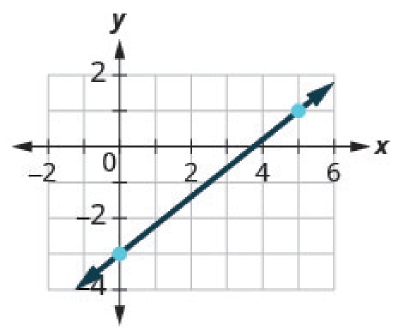
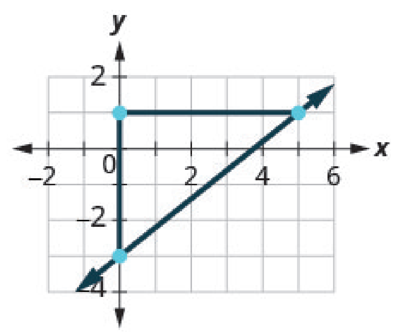
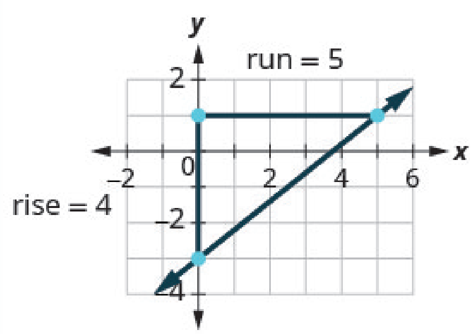
EXAMPLE 5

How to Use $m = \dfrac{\text{rise}}{\text{run}}$ to Find the Slope of a Line from its Graph

Find the slope of the line shown.



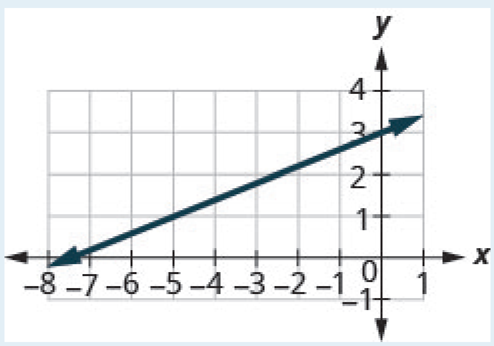
Solution

Step 1. Locate two points on the graph whose coordinates are integers.	Mark (0, –3) and (5, 1).	
Step 2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.	Starting at (0, –3), sketch a right triangle to (5, 1).	
Step 3. Count the rise and the run on the legs of the triangle.	Count the rise. Count the run.	 The rise is 4. The run is 5.

Step 4. Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$	Use the slope formula. Substitute the values of the rise and run.	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{4}{5}$ The slope of the line is $\frac{4}{5}$. This means that y increases 4 units as x increases 5 units.

TRY IT 5.1

Find the slope of the line shown.

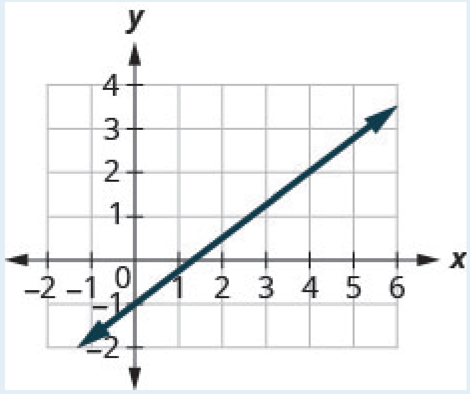


Show answer

$\frac{2}{5}$

TRY IT 5.2

Find the slope of the line shown.



Show answer

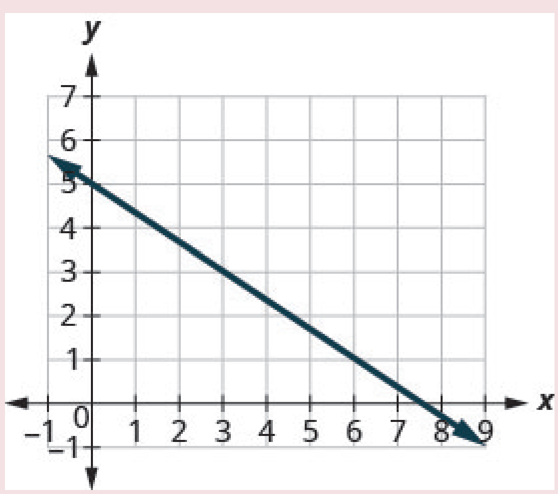
$\frac{3}{4}$

HOW TO: Find the slope of a line from its graph using $m = \frac{rise}{run}$.

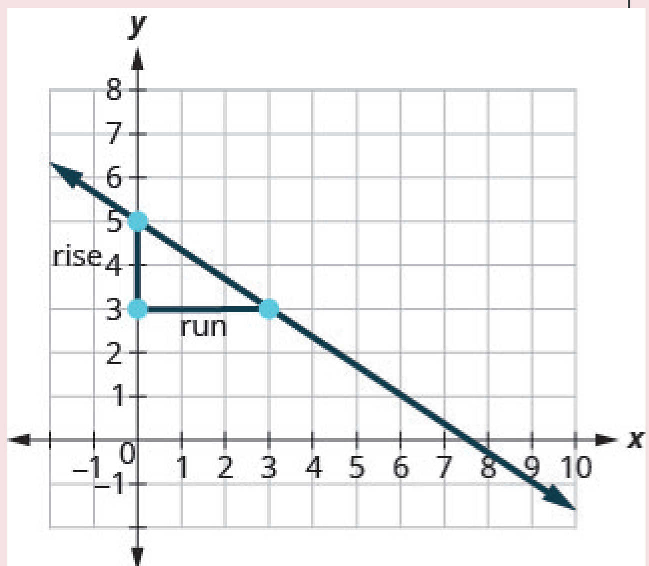
1. Locate two points on the line whose coordinates are integers.
2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
3. Count the rise and the run on the legs of the triangle.
4. Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$.

EXAMPLE 6

Find the slope of the line shown.

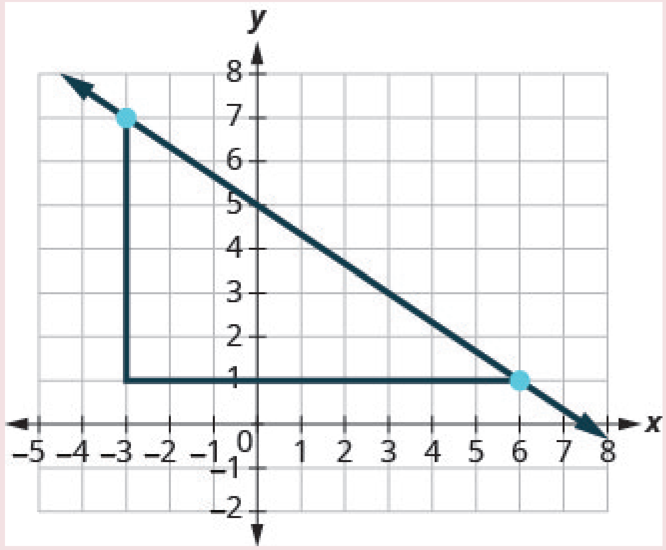


Solution

Locate two points on the graph whose coordinates are integers.	$(0, 5)$ and $(3, 3)$
Which point is on the left?	$(0, 5)$
Starting at $(0, 5)$, sketch a right triangle to $(3, 3)$.	
Count the rise—it is negative.	The rise is -2.
Count the run.	The run is 3.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-2}{3}$
Simplify.	$m = -\frac{2}{3}$
	The slope of the line is $-\frac{2}{3}$.

So y increases by 3 units as x decreases by 2 units.

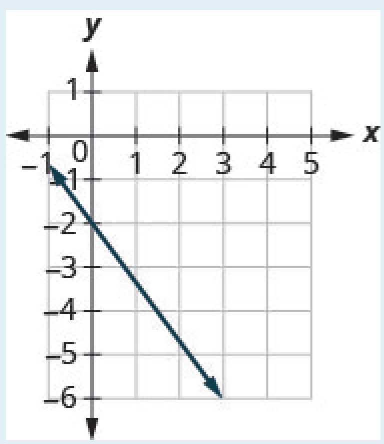
What if we used the points $(-3, 7)$ and $(6, 1)$ to find the slope of the line?



The rise would be -6 and the run would be 9. Then $m = \frac{-6}{9}$, and that simplifies to $m = -\frac{2}{3}$.
Remember, it does not matter which points you use—the slope of the line is always the same.

TRY IT 6.1

Find the slope of the line shown.

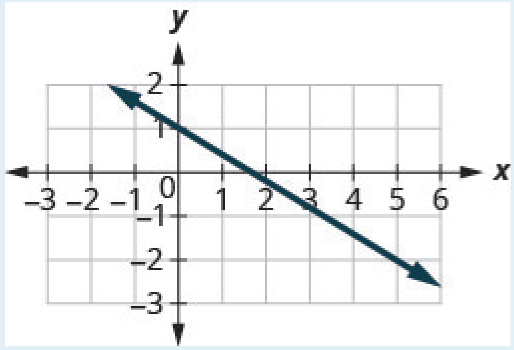


Show answer

$-\frac{4}{3}$

TRY IT 6.2

Find the slope of the line shown.



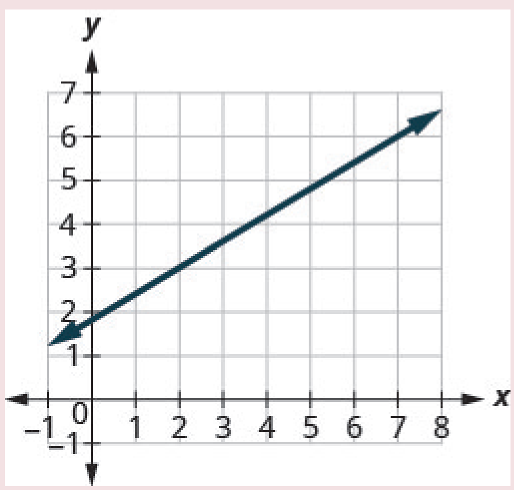
Show answer

$-\frac{3}{5}$

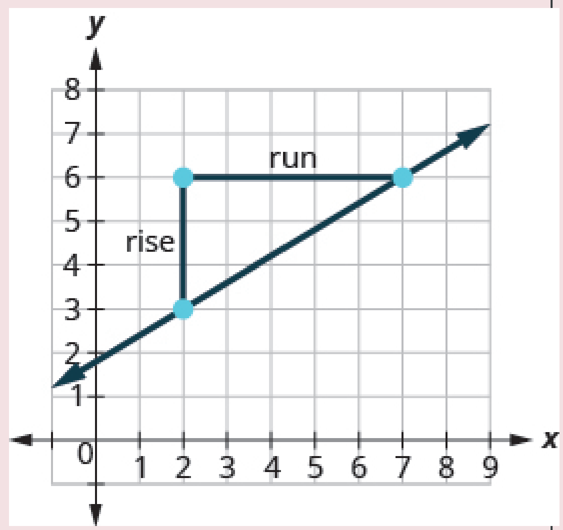
In the last two examples, the lines had *y*-intercepts with integer values, so it was convenient to use the *y*-intercept as one of the points to find the slope. In the next example, the *y*-intercept is a fraction. Instead of using that point, we'll look for two other points whose coordinates are integers. This will make the slope calculations easier.

EXAMPLE 7

Find the slope of the line shown.



Solution

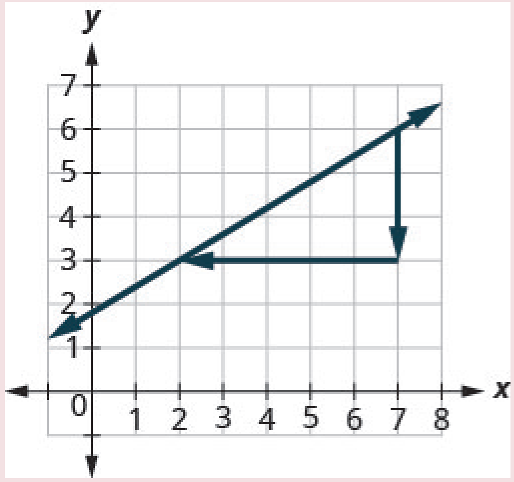
Locate two points on the graph whose coordinates are integers.	$(2,3)$ and $(7,6)$
Which point is on the left?	$(2,3)$
Starting at $(2,3)$, sketch a right triangle to $(7,6)$.	
Count the rise.	The rise is 3.
Count the run.	The run is 5.
Use the slope formula.	$m=\frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m=\dfrac{3}{5}$
	The slope of the line is $\dfrac{3}{5}$.

This means that y increases 5 units as x increases 3 units.

When we used geoboards to introduce the concept of slope, we said that we would always start with the point on the left and count the rise and the run to get to the point on the right. That way the run was always positive and the rise determined whether the slope was positive or negative.

What would happen if we started with the point on the right?

Let's use the points $(2,3)$ and $(7,6)$ again, but now we'll start at $(7,6)$.

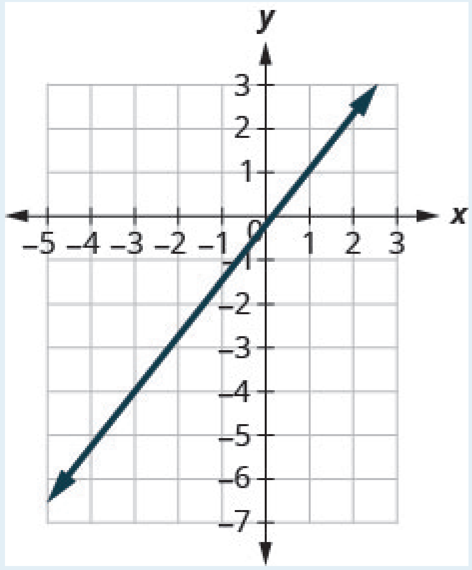


Count the rise.	The rise is -3.
Count the run. It goes from right to left, so it is negative.	The run is -5.
Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values of the rise and run.	$m = \frac{-3}{-5} = \frac{3}{5}$
	The slope of the line is $\frac{3}{5}$.

It does not matter where you start—the slope of the line is always the same.

TRY IT 7.1

Find the slope of the line shown.

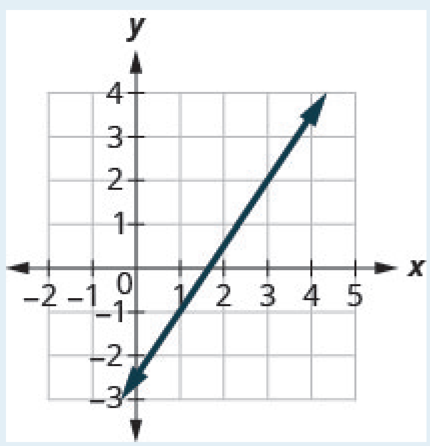


Show answer

$\frac{5}{4}$

EXAMPLE 7.2

Find the slope of the line shown.



Show answer

$\frac{3}{2}$

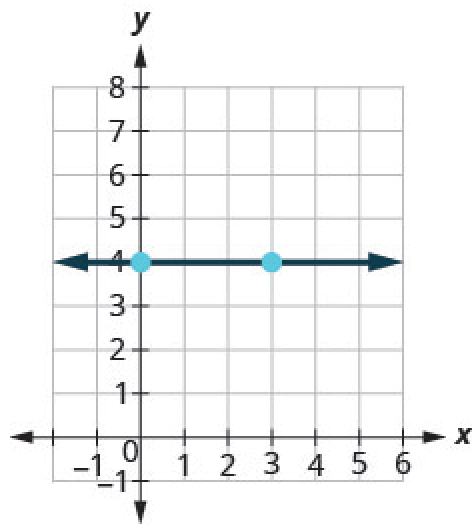
Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

Horizontal line $\mathbf{y = b}$ **Vertical line** $\mathbf{x = a}$

y-coordinates are the same. x-coordinates are the same.

So how do we find the slope of the horizontal line $y = 4$? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens when we do this.



What is the rise?	The rise is 0.
Count the run.	The run is 3.
What is the slope?	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{0}{3}$ $m = 0$
	The slope of the horizontal line $y = 4$ is 0.

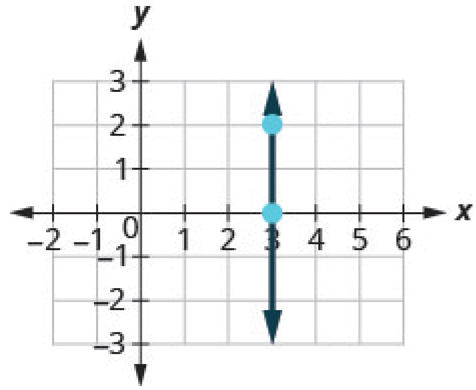
All horizontal lines have slope 0. When the *y*-coordinates are the same, the rise is 0.

Slope of a horizontal line

The slope of a horizontal line, $y = b$, is 0.

The floor of your room is horizontal. Its slope is 0. If you carefully placed a ball on the floor, it would not roll away.

Now, we'll consider a vertical line, the line.



What is the rise?	The rise is 2.
Count the run.	The run is 0.
What is the slope?	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is not defined. So we say that the slope of the vertical line $x = 3$ is undefined.

The slope of any vertical line is undefined. When the *x*-coordinates of a line are all the same, the run is 0.

Slope of a vertical line

The slope of a vertical line, $x = a$, is undefined.

EXAMPLE 8

Find the slope of each line:

a) $x = 8$ b) $y = -5$.

Solution

a) $x = 8$

This is a vertical line.

Its slope is undefined.

b) $y = -5$

This is a horizontal line.

It has slope 0.

TRY IT 8.1

Find the slope of the line: $x = -4$.

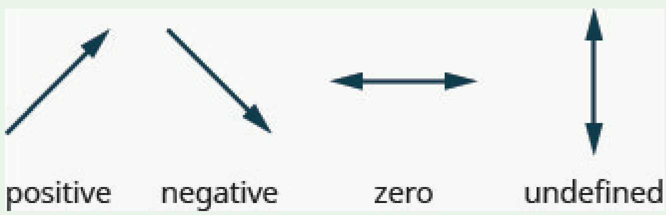
Show answer
undefined

TRY 8.2

Find the slope of the line: $y = 7$.

Show answer
0

Quick guide to the slopes of lines



Remember, we 'read' a line from left to right, just like we read written words in English.

Use the Slope Formula to find the Slope of a Line Between Two Points

Sometimes we'll need to find the slope of a line between two points when we don't have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but as we'll see, there is a way to find the slope without graphing. Before we get to it, we need to introduce some algebraic notation.

We have seen that an ordered pair (x, y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x, y) be used to represent two different points? Mathematicians use subscripts to distinguish the points.

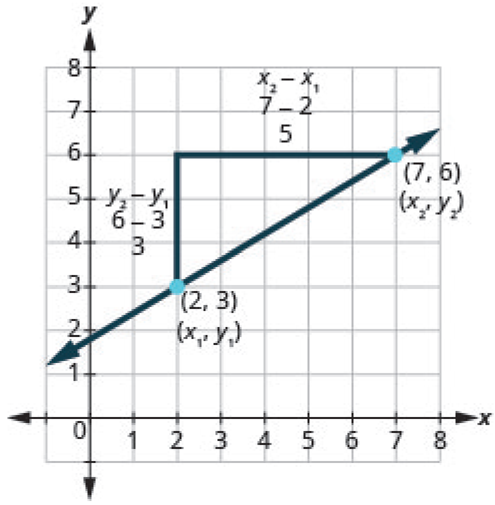
(x_1, y_1) read ' x sub 1, y sub 1'
(x_2, y_2) read ' x sub 2, y sub 2'

The use of subscripts in math is very much like the use of last name initials in elementary school. Maybe you remember Laura C. and Laura M. in your third grade class?

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point.

If we had more than two points, we could use (x_3, y_3), (x_4, y_4), and so on.

Let's see how the rise and run relate to the coordinates of the two points by taking another look at the slope of the line between the points $(2, 3)$ and $(7, 6)$.



Since we have two points, we will use subscript notation, $\begin{pmatrix} x_1, & y_1 \\ 2, & 3 \end{pmatrix}\begin{pmatrix} x_2, & y_2 \\ 7, & 6 \end{pmatrix}$.

On the graph, we counted the rise of 3 and the run of 5

Notice that the rise of 3 can be found by subtracting the *y*-coordinates 6 and 3

$3 = 6 - 3$

And the run of 5 can be found by subtracting the *x*-coordinates 7 and 2

$5 = 7 - 2$

We know $m = \frac{\text{rise}}{\text{run}}$. So $m = \frac{3}{5}$.

We rewrite the rise and run by putting in the coordinates $m = \frac{6-3}{7-2}$.

But 6 is y_2, the *y*-coordinate of the second point and 3 is y_1, the *y*-coordinate of the first point.

So we can rewrite the slope using subscript notation. $m = \frac{y_2 - y_1}{7-2}$

Also, 7 is x_2, the *x*-coordinate of the second point and 2 is x_1, the *x*-coordinate of the first point.

So, again, we rewrite the slope using subscript notation. $m = \frac{y_2 - y_1}{x_2 - x_1}$

We've shown that $m = \frac{y_2 - y_1}{x_2 - x_1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

Slope formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

This is the slope formula.

The slope is:

$$\begin{array}{r} \text{y of the second point minus y of the first point} \\ \text{over} \\ \text{x of the second point minus x of the first point.} \end{array}$$

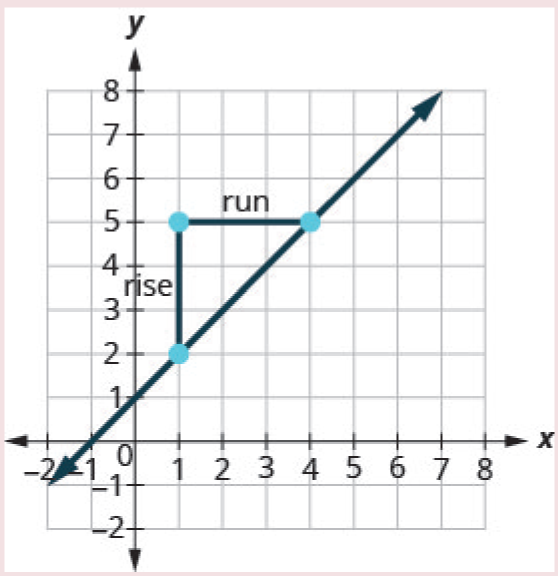
EXAMPLE 9

Use the slope formula to find the slope of the line between the points $(1, 2)$ and $(4, 5)$.

Solution

We'll call $(1, 2)$ point #1 and $(4, 5)$ point #2.	$\begin{pmatrix} x_1, & y_1 \\ 1, & 2 \end{pmatrix} \begin{pmatrix} x_2, & y_2 \\ 4, & 5 \end{pmatrix}.$
Use the slope formula.	$m = \dfrac{y_2 - y_1}{x_2 - x_1}.$
Substitute the values.	
y of the second point minus y of the first point	$m = \dfrac{5 - 2}{x_2 - x_1}.$
x of the second point minus x of the first point	$m = \dfrac{5 - 2}{4 - 1}.$
Simplify the numerator and the denominator.	$m = \dfrac{3}{3}.$
Simplify.	$m = 1.$

Let's confirm this by counting out the slope on a graph using $m = \dfrac{\text{rise}}{\text{run}}$.



It doesn't matter which point you call point #1 and which one you call point #2. The slope will be the same. Try the calculation yourself.

TRY IT 9.1

Use the slope formula to find the slope of the line through the points: $(8,5)$ and $(6,3)$.

Show answer
1

TRY IT 9.2

Use the slope formula to find the slope of the line through the points: $(1,5)$ and $(5,9)$.

Show answer
1

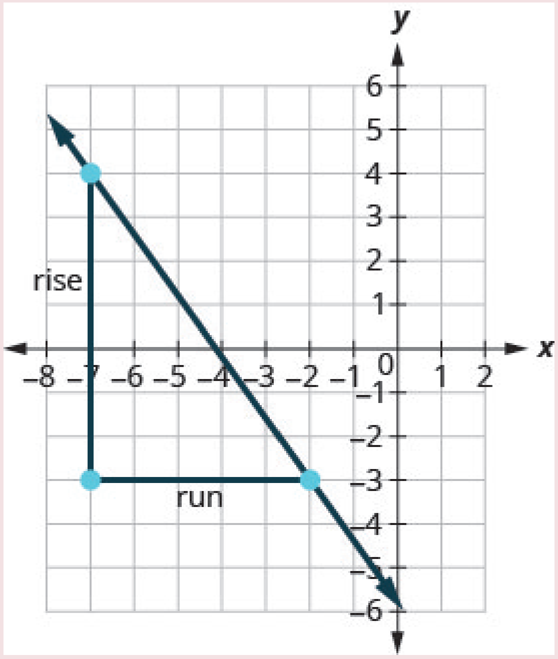
EXAMPLE 10

Use the slope formula to find the slope of the line through the points $(-2,-3)$ and $(-7,4)$.

Solution

We'll call $(-2,-3)$ point #1 and $(-7,4)$ point #2.	$\begin{pmatrix} x_1, & y_1 \\ -2, & -3 \end{pmatrix} \begin{pmatrix} x_2, & y_2 \\ -7, & 4 \end{pmatrix}.$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}.$
Substitute the values.	
y of the second point minus y of the first point	$m = \frac{4-(-3)}{x_2 - x_1}.$
x of the second point minus x of the first point	$m = \frac{4-(-3)}{-7-(-2)}.$
Simplify.	$m = \frac{7}{-5}$ or $m = -\frac{7}{5}$

Let's verify this slope on the graph shown.



$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{7}{-5}$$
$$or$$
$$m = -\frac{7}{5} \checkmark$$

TRY IT 10.1

Use the slope formula to find the slope of the line through the points: $(-3, 4)$ and $(2, -1)$.

Show answer
-1

TRY IT 10.2

Use the slope formula to find the slope of the line through the pair of points: $(-2, 6)$ and $(-3, -4)$.

Show answer
10

Graph a Line Given a Point and the Slope

Up to now, in this chapter, we have graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

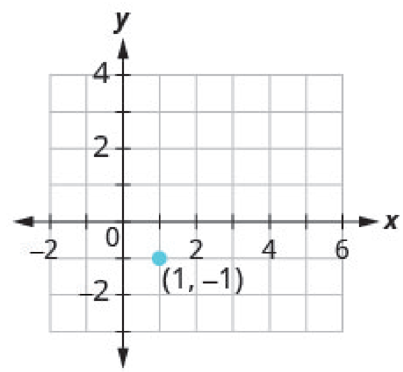
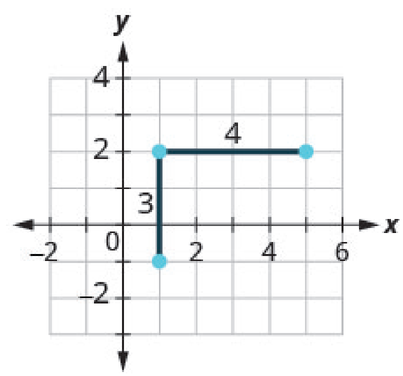
One other method we can use to graph lines is called the point–slope method. We will use this method when we know one point and the slope of the line. We will start by plotting the point and then use the definition of slope to draw the graph of the line.

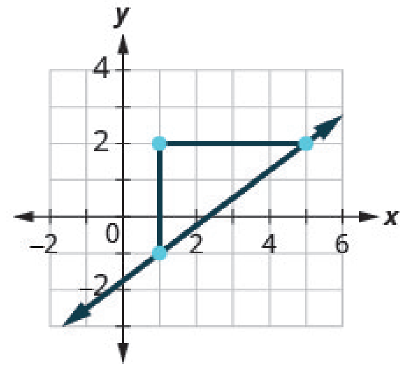
EXAMPLE 11

How To Graph a Line Given a Point and The Slope

Graph the line passing through the point $(1, -1)$ whose slope is $m = \frac{3}{4}$.

Solution

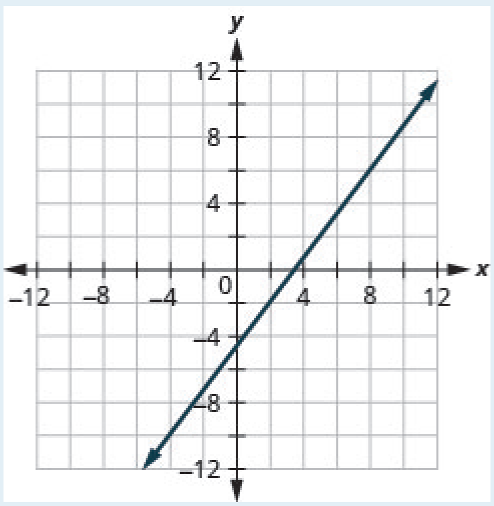
Step 1. Plot the given point.	Plot (1, –1).	
Step 2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = \frac{3}{4}$ $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ $\text{rise} = 3$ $\text{run} = 4$
Step 3. Starting at the given point, count out the rise and run to mark the second point.	Start at (1, –1) and count the rise and the run. Up 3 units, right 4 units.	

Step 4. Connect the points with a line.	Connect the two points with a line.	

EXAMPLE 11.1

Graph the line passing through the point $(2, -2)$ with the slope $m = \frac{4}{3}$.

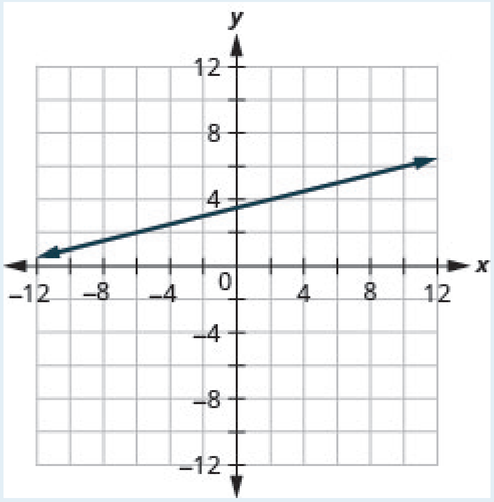
Show answer



TRY IT 11.2

Graph the line passing through the point $(-2, 3)$ with the slope $m = \frac{1}{4}$.

Show answer



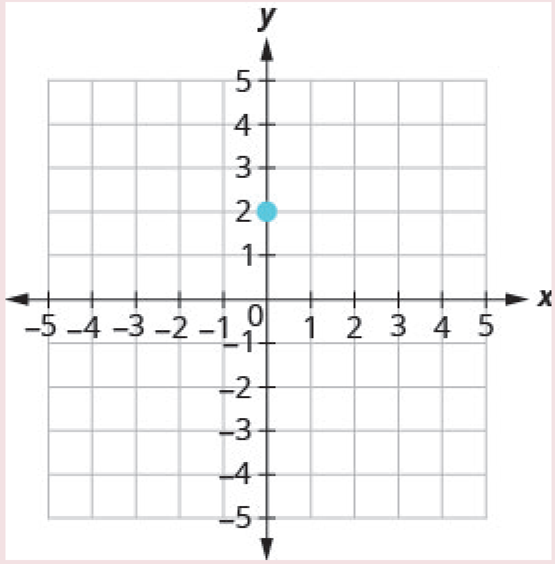
Graph a line given a point and the slope.

1. Plot the given point.
2. Use the slope formula $m = \dfrac{\text{rise}}{\text{run}}$ to identify the rise and the run.
3. Starting at the given point, count out the rise and run to mark the second point.
4. Connect the points with a line.

EXAMPLE 12

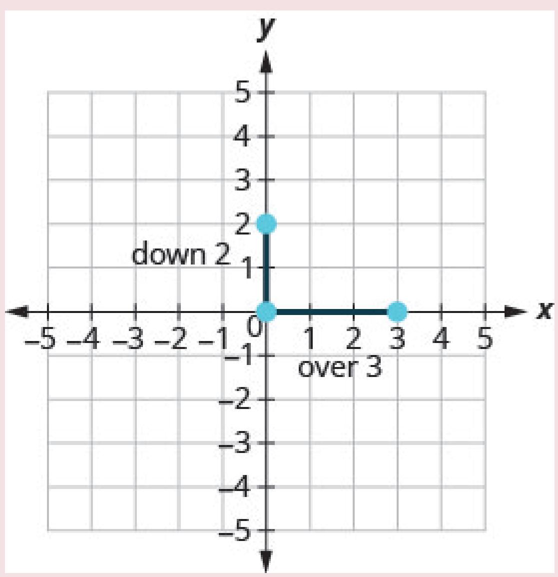
Graph the line with *y*-intercept 2 whose slope is $m = -\dfrac{2}{3}$.

Solution
Plot the given point, the *y*-intercept, $(0, 2)$.

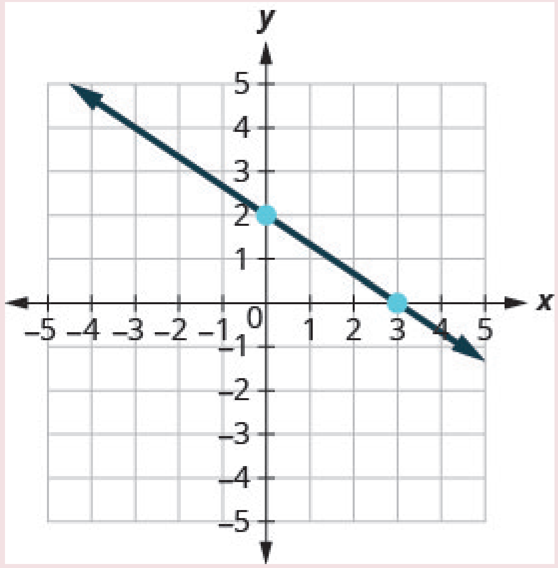


Identify the rise and the run.	$m=-\frac{2}{3}$
	$\frac{\text{rise}}{\text{run}}=\frac{-2}{3}$
	$\text{rise}=-2$
	$\text{run}=3$

Count the rise and the run. Mark the second point.



Connect the two points with a line.

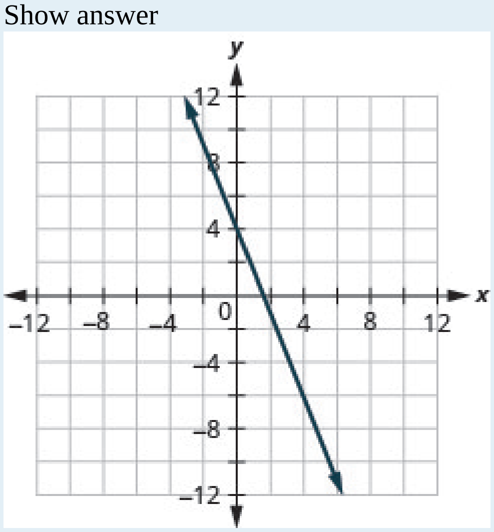


You can check your work by finding a third point. Since the slope is $m=-\frac{2}{3}$, it can be written as $m=\frac{2}{-3}$. Go back to $(0,2)$ and count out the rise, 2, and the run, -3.

TRY IT 12.1

Graph the line with the *y*-intercept 4 and slope $m=-\frac{5}{2}$.

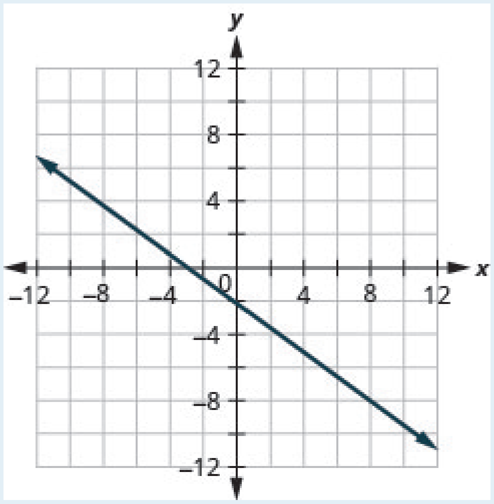
Show answer



TRY IT 12.2

Graph the line with the *x*-intercept -3 and slope $m = -\frac{3}{4}$.

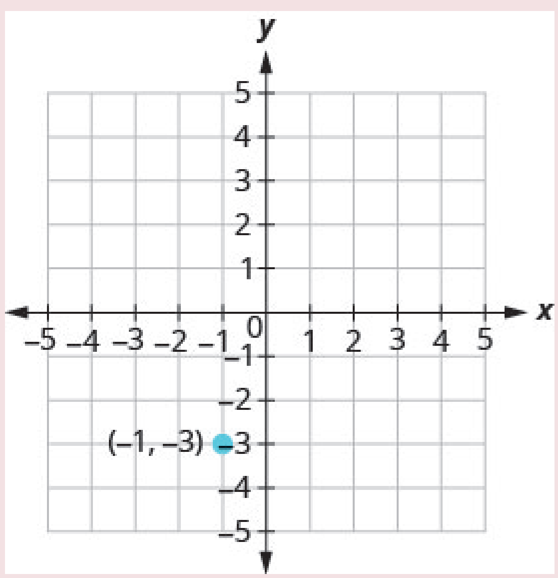
Show answer



EXAMPLE 13

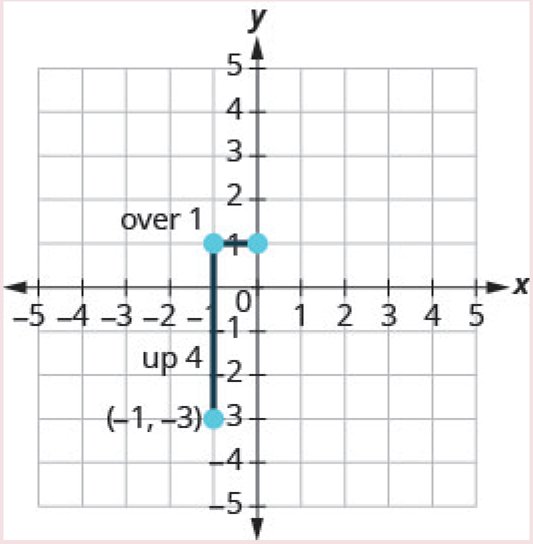
Graph the line passing through the point $(-1, -3)$ whose slope is $m = 4$.

Solution
Plot the given point.

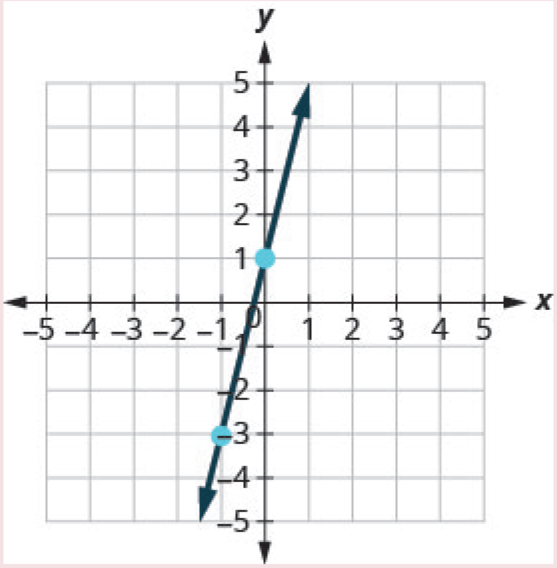


Identify the rise and the run.	$m = 4$
Write 4 as a fraction.	$\frac{\text{rise}}{\text{run}} = \frac{4}{1}$
	$\text{rise} = 4, \text{run} = 1$

Count the rise and run and mark the second point.



Connect the two points with a line.

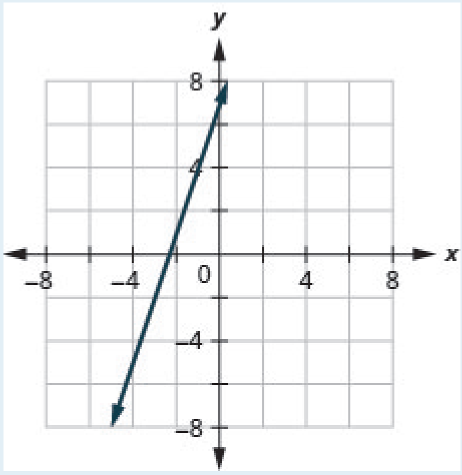


You can check your work by finding a third point. Since the slope is $m = 4$, it can be written as $m = \frac{-4}{-1}$. Go back to $(-1, -3)$ and count out the rise, -4, and the run, -1.

TRY IT 13.1

Graph the line with the point $(-2, 1)$ and slope $m = 3$.

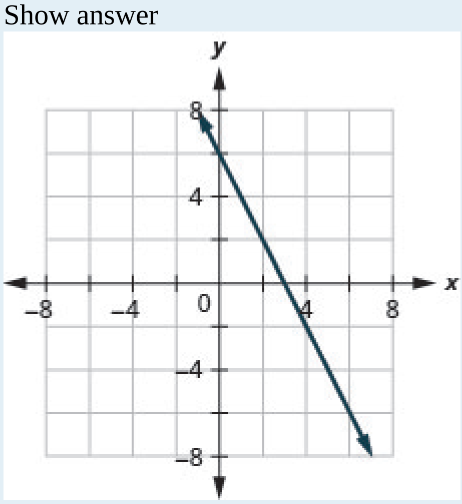
Show answer



EXAMPLE 13.2

Graph the line with the point $(4, -2)$ and slope $m = -2$.

Show answer

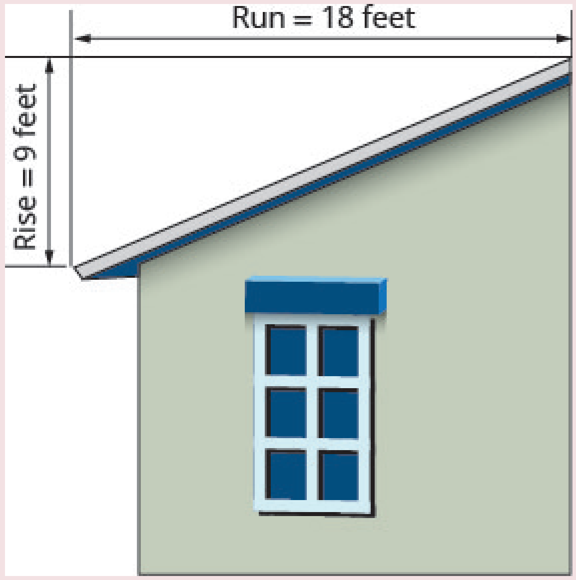


Solve Slope Applications

At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

EXAMPLE 14

The ‘pitch’ of a building’s roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values for rise and run.	$m = \frac{9}{18}$
Simplify.	$m = \frac{1}{2}$
The slope of the roof is $\frac{1}{2}$.	
	The roof rises 1 foot for every 2 feet of horizontal run.

TRY IT 14.1

Use (Example 14), substituting the rise = 14 and run = 24

Show answer

$\frac{7}{12}$

TRY IT 14.2

Use (Example 14), substituting rise = 15 and run = 36

Show answer

$\frac{5}{12}$

EXAMPLE 15

Have you ever thought about the sewage pipes going from your house to the street? They must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?

Solution

Use the slope formula. 1 foot = 12 inches	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{-\frac{1}{4}\text{inch}}{1\ \text{foot}}$ $m = \frac{-\frac{1}{4}\text{inch}}{12\ \text{inches}}$
Simplify .	$m = -\frac{1}{48}$ (because $-\frac{1}{4} \div 12 = -\frac{1}{4} \times \frac{1}{12} = -\frac{1}{48}$)
	The slope of the pipe is $-\frac{1}{48}$.

The pipe drops 1 inch for every 48 inches of horizontal run.

TRY IT 15.1

Find the slope of a pipe that slopes down $\frac{1}{3}$ inch per foot (1 foot = 12 inches).

Show answer

$-\frac{1}{36}$

TRY IT 15.2

Find the slope of a pipe that slopes down $\frac{3}{4}$ inch per yard (1 yard = 36 inches).

Show answer

$-\frac{1}{48}$

Access these online resources for additional instruction and practice with understanding slope of a line.

- Practice Slope with a Virtual Geoboard
- Explore Area and Perimeter with a Geoboard

Key Concepts

- **Find the Slope of a Line from its Graph using** $m = \frac{\text{rise}}{\text{run}}$
    1. Locate two points on the line whose coordinates are integers.
    2. Starting with the point on the left, sketch a right triangle, going from the first point to the second point.
    3. Count the rise and the run on the legs of the triangle.
    4. Take the ratio of rise to run to find the slope.
- **Graph a Line Given a Point and the Slope**
    1. Plot the given point.
    2. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
    3. Starting at the given point, count out the rise and run to mark the second point.
    4. Connect the points with a line.
- **Slope of a Horizontal Line**
    - The slope of a horizontal line, $y = b$, is 0.
- **Slope of a vertical line**
    - The slope of a vertical line, $x = a$, is undefined

Glossary

geoboard
A geoboard is a board with a grid of pegs on it.

negative slope
A negative slope of a line goes down as you read from left to right.

positive slope
A positive slope of a line goes up as you read from left to right.

rise
The rise of a line is its vertical change.

run
The run of a line is its horizontal change.

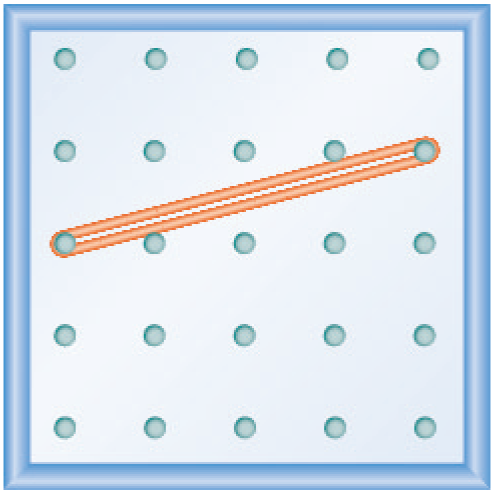
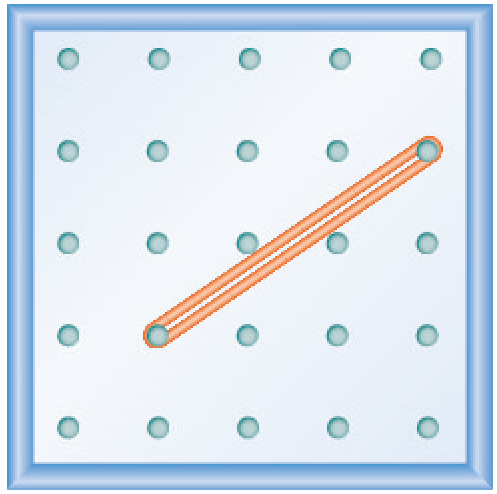
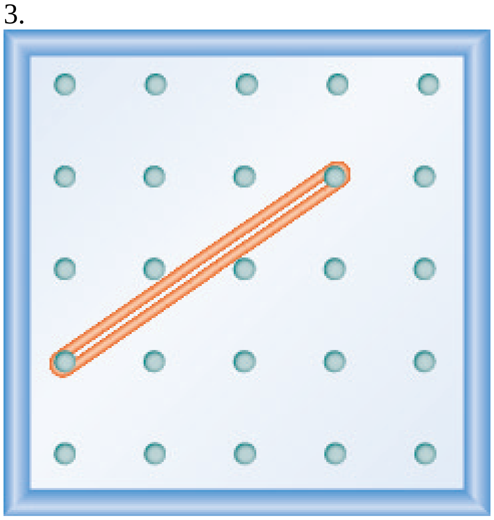
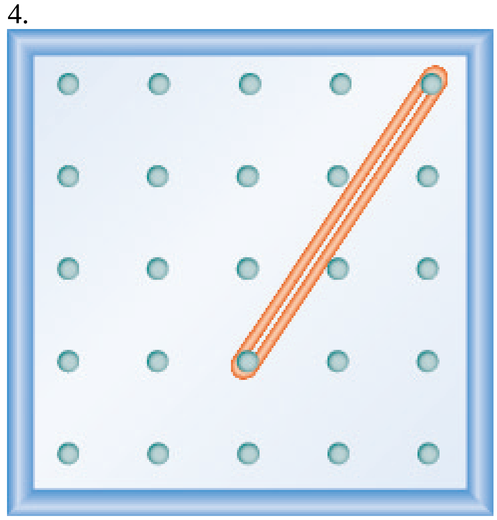
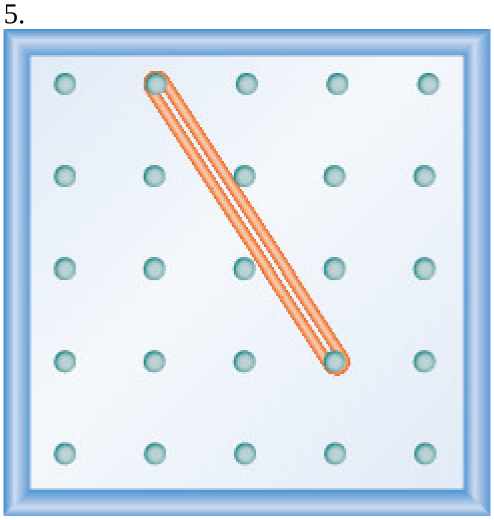
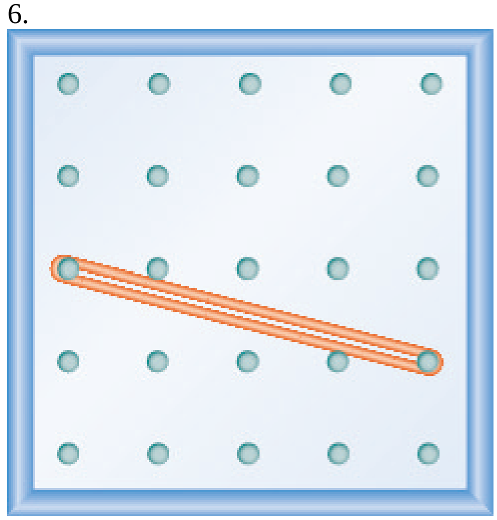
slope formula
The slope of the line between two points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

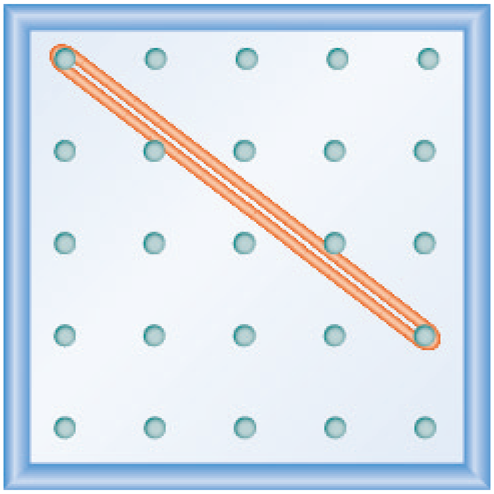
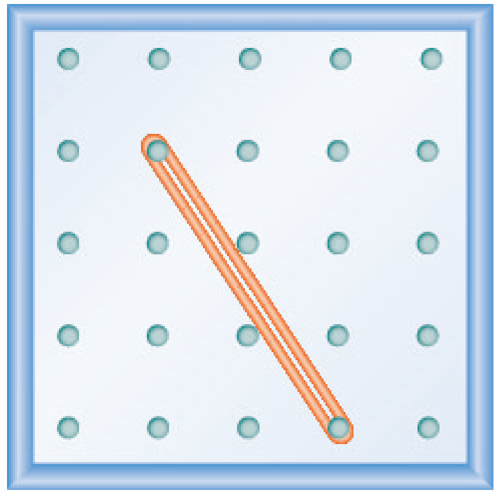
slope of a line
The slope of a line is $m = \frac{\text{rise}}{\text{run}}$. The rise measures the vertical change and the run measures the horizontal change.

Practice Makes Perfect

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

1. 	2. 
3. 	4. 
5. 	6. 

7. 	8. 

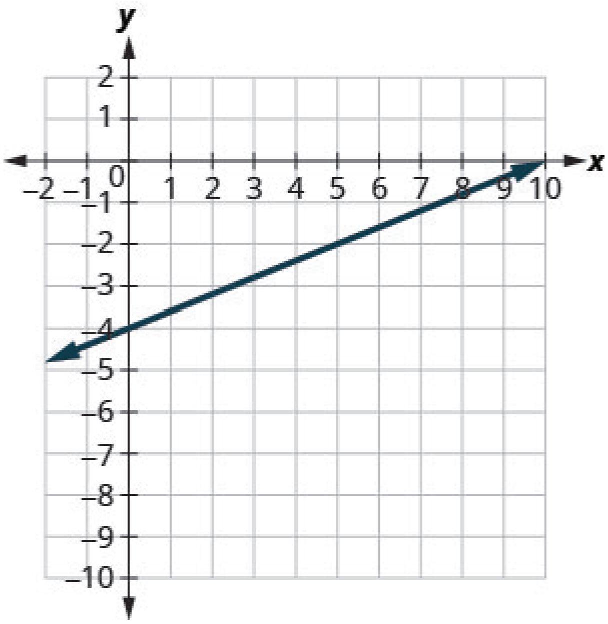
In the following exercises, model each slope. Draw a picture to show your results.

9. $\frac{2}{3}$	10. $\frac{3}{4}$
11. $\frac{1}{4}$	12. $\frac{4}{3}$
13. $-\frac{1}{2}$	14. $-\frac{3}{4}$
15. $-\frac{2}{3}$	16. $-\frac{3}{2}$

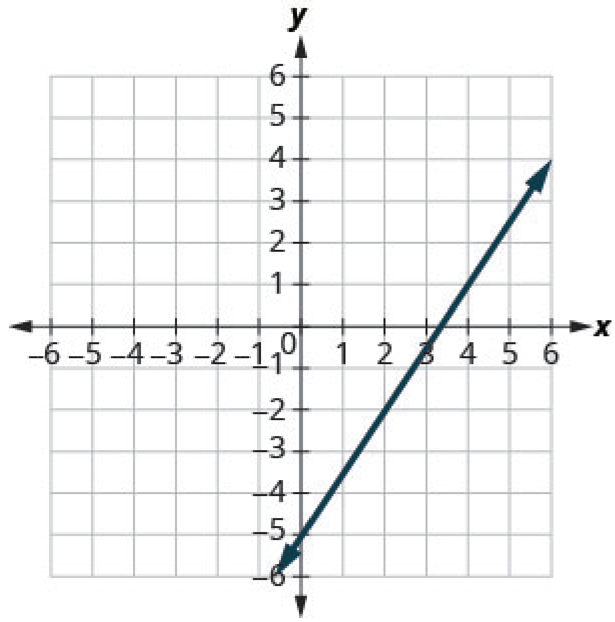
Use $m = \frac{\text{rise}}{\text{run}}$ to find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

17.


y
x
2
1
−2 −1 0 1 2 3 4 5 6 7 8 9 10
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10

18.


y
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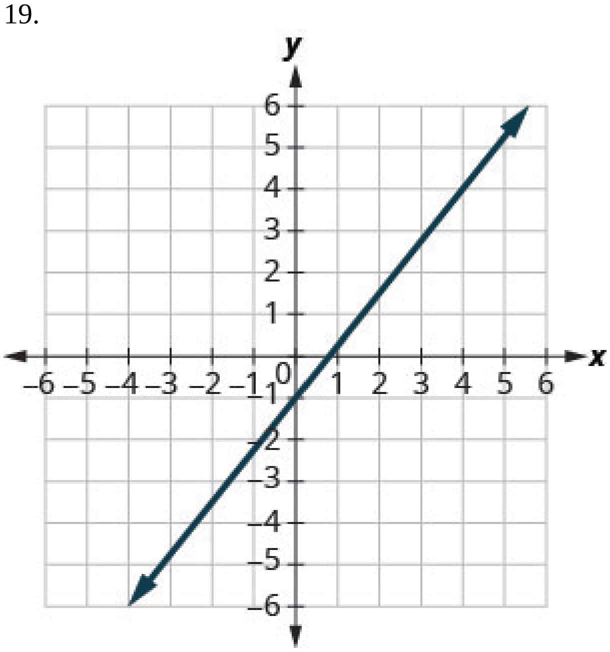
19.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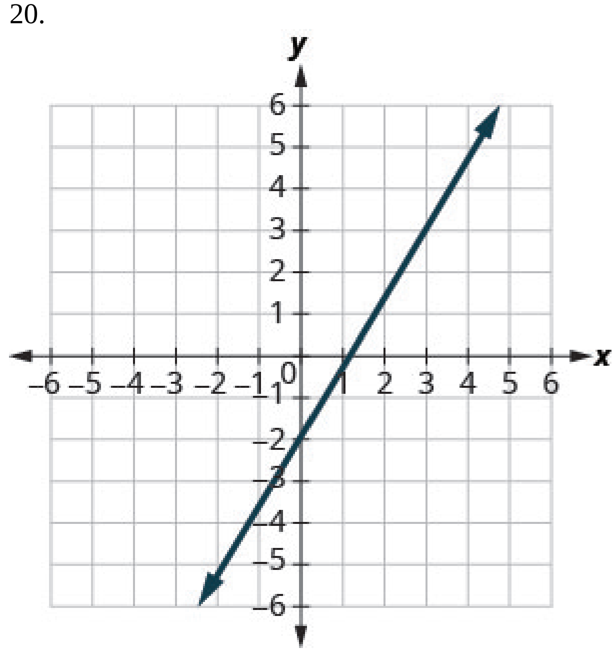
20.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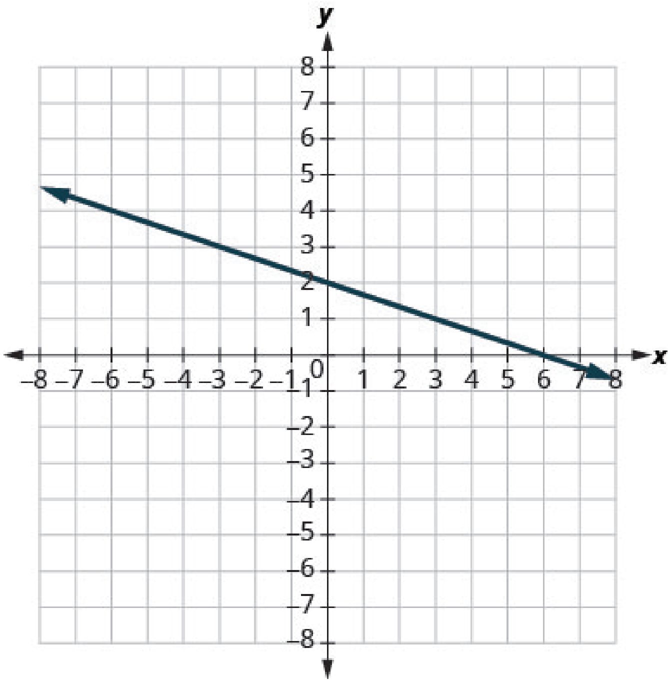
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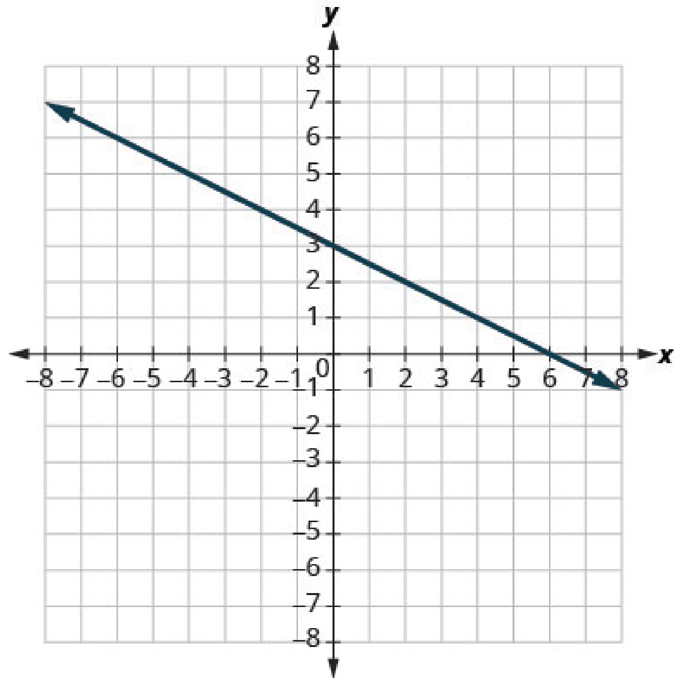
22.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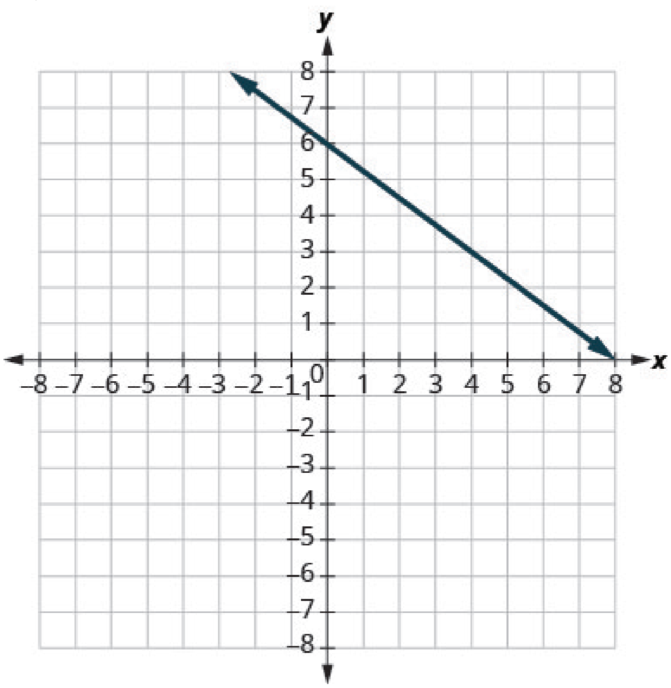
23.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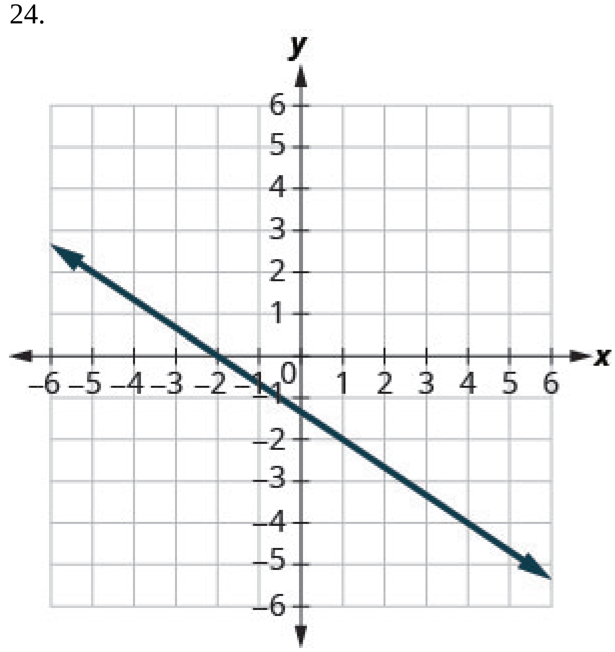
24.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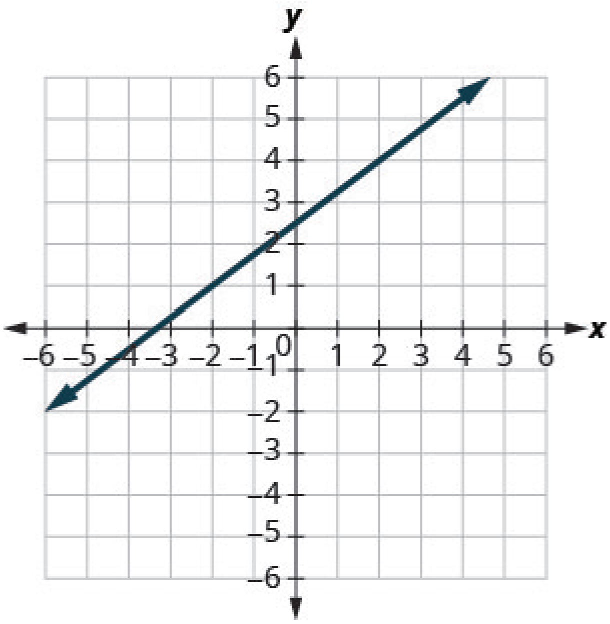
25.



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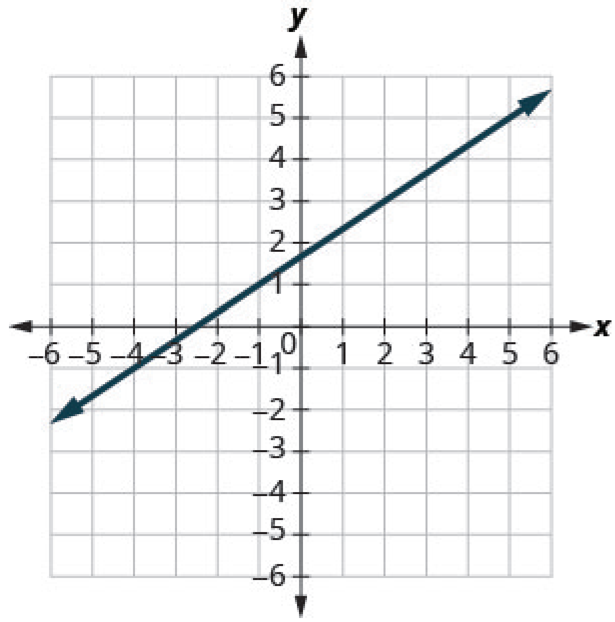

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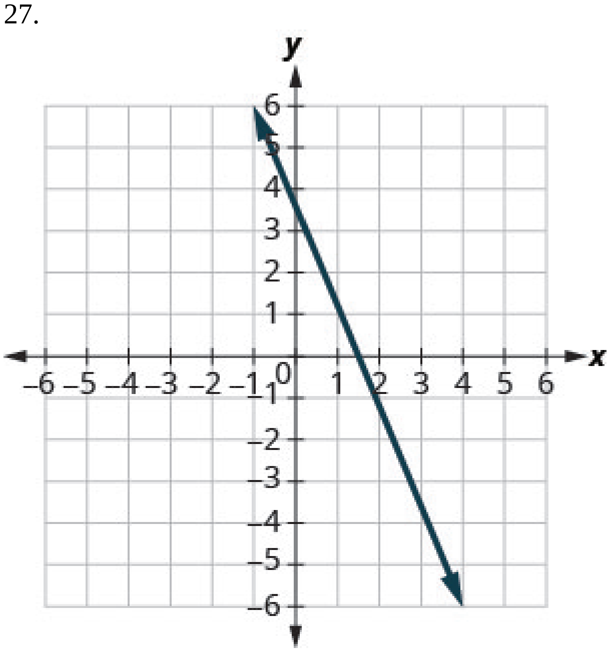

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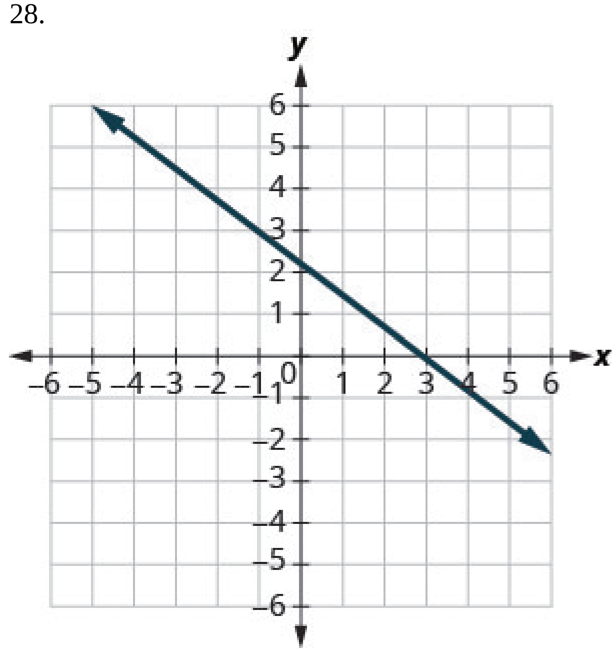

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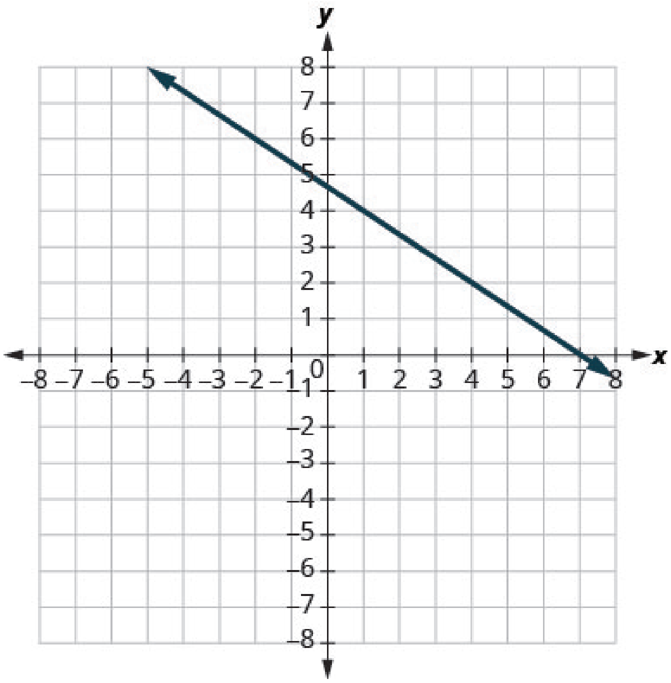

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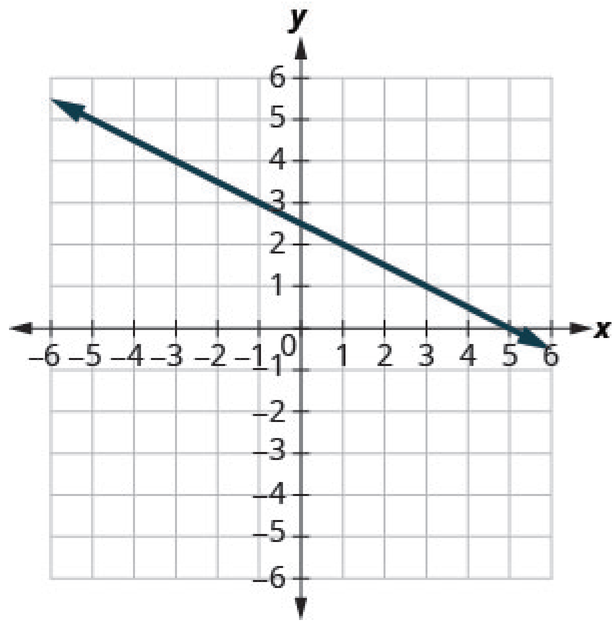

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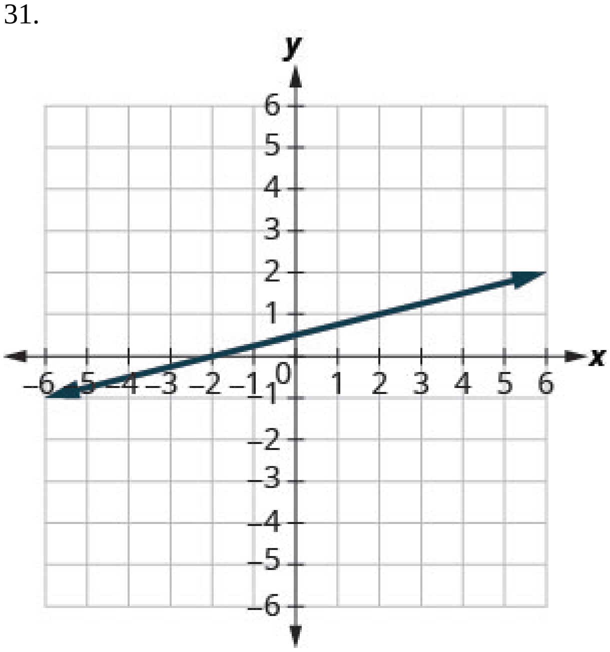

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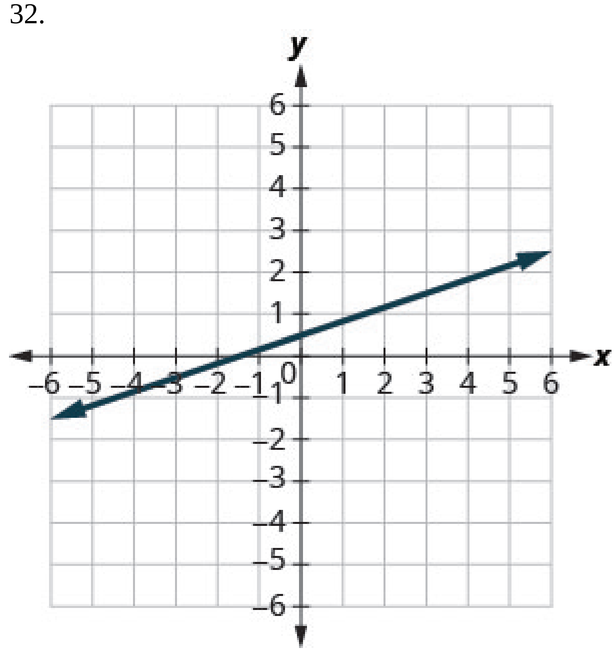


Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

33. $y=3$	34. $y=1$
35. $x=4$	36. $x=2$
37. $y=-2$	38. $y=-3$
39. $x=-5$	40. $x=-4$

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

41. $(1,4),(3,9)$	42. $(2,3),(5,7)$
43. $(0,3),(4,6)$	44. $(0,1),(5,4)$
45. $(2,5),(4,0)$	46. $(3,6),(8,0)$
47. $(-3,3),(4,-5)$	48. $(-2,4),(3,-1)$
49. $(-1,-2),(2,5)$	50. $(-2,-1),(6,5)$
51. $(4,-5),(1,-2)$	52. $(3,-6),(2,-2)$

Graph a Line Given a Point and the Slope

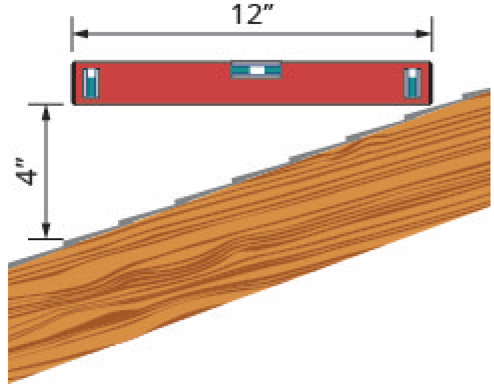
In the following exercises, graph each line with the given point and slope.

53. $(1,-2); m=\frac{3}{4}$	54. $(1,-1); m=\frac{2}{3}$
55. $(2,5); m=-\frac{1}{3}$	56. $(1,4); m=-\frac{1}{2}$
57. $(-3,4); m=-\frac{3}{2}$	58. $(-2,5); m=-\frac{5}{4}$
59. $(-1,-4); m=\frac{4}{3}$	60.$(-3,-5); m=\frac{3}{2}$
61. *y*-intercept 3; $m=-\frac{2}{5}$	62. *y*-intercept 5; $m=-\frac{4}{3}$
63. *x*-intercept -2; $m=\frac{3}{4}$	64. *x*-intercept -1; $m=\frac{1}{5}$
65. $(-3,3); m=2$	66. $(-4,2); m=4$
67. $(1,5); m=-3$	68. $(2,5); m=3$

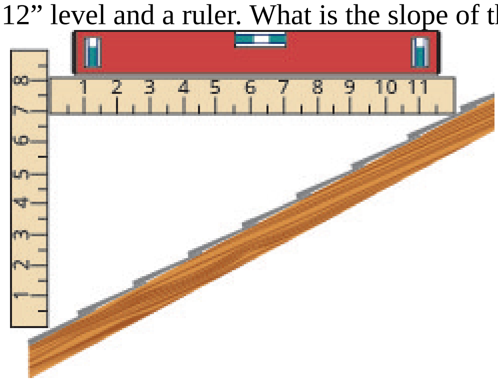
Everyday Math

69. **Slope of a roof.** An easy way to determine the slope of a roof is to set one end of a 12 inch level on the roof surface and hold it level. Then take a tape measure or ruler and measure from the other end of the level down to the roof surface. This will give you the slope of the roof. Builders, sometimes, refer to this as pitch and state it as an "x 12 pitch" meaning $\frac{x}{12}$, where x is the measurement from the roof to the level—the rise. It is also sometimes stated as an "x-in-12 pitch".

1. a) What is the slope of the roof in this picture?
2. b) What is the pitch in construction terms?



70. The slope of the roof shown here is measured with a 12" level and a ruler. What is the slope of this roof?



71. **Road grade.** A local road has a grade of 6%. The grade of a road is its slope expressed as a percent. Find the slope of the road as a fraction and then simplify. What rise and run would reflect this slope or grade?

72. **Highway grade.** A local road rises 2 feet for every 50 feet of highway.
a) What is the slope of the highway?
b) The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

73. **Wheelchair ramp.** The rules for wheelchair ramps require a maximum 1-inch rise for a 12-inch run.
a) How long must the ramp be to accommodate a 24-inch rise to the door?
b) Create a model of this ramp.

74. **Wheelchair ramp.** A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend a ramp.
a) How long must a ramp be to easily accommodate a 24-inch rise to the door?
b) Create a model of this ramp.

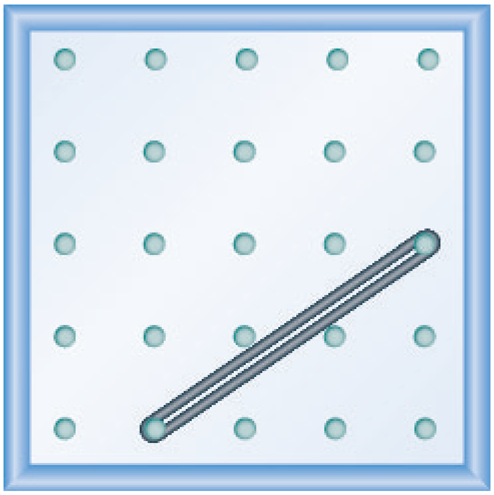
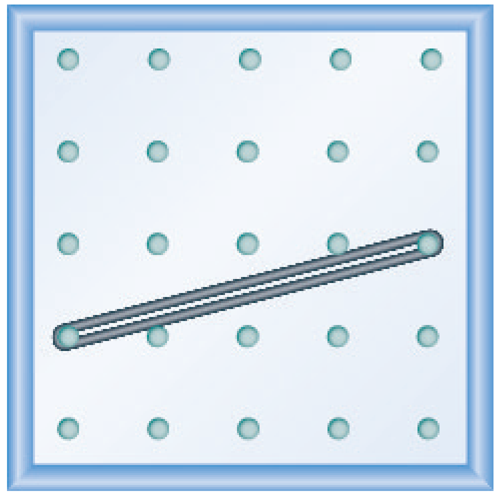
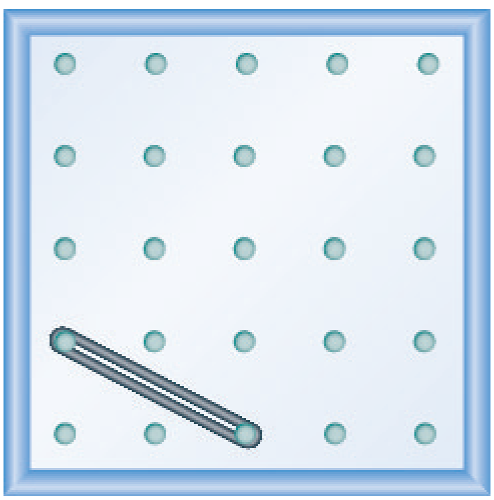
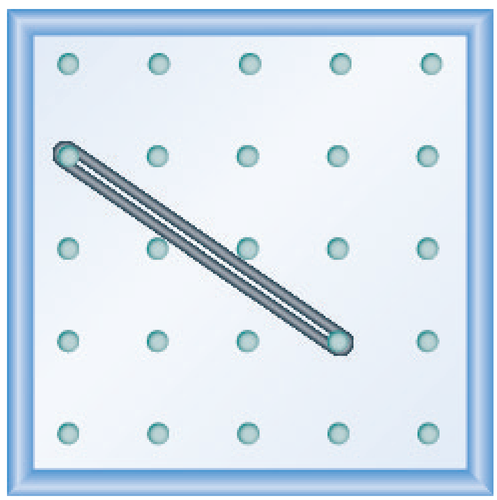
Writing Exercises

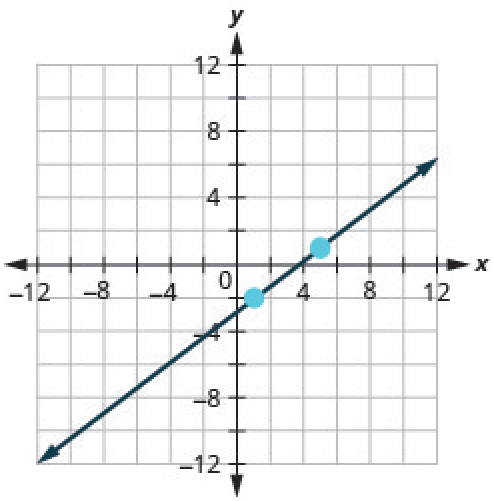
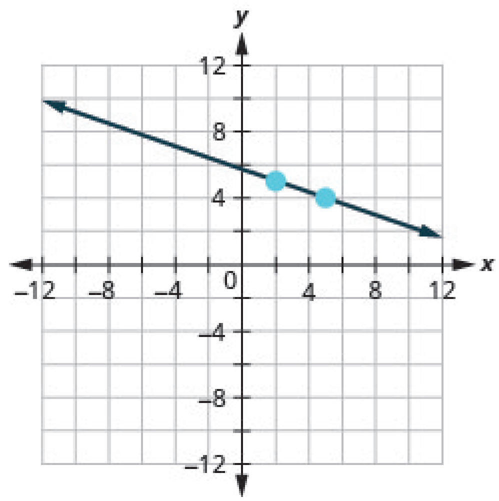
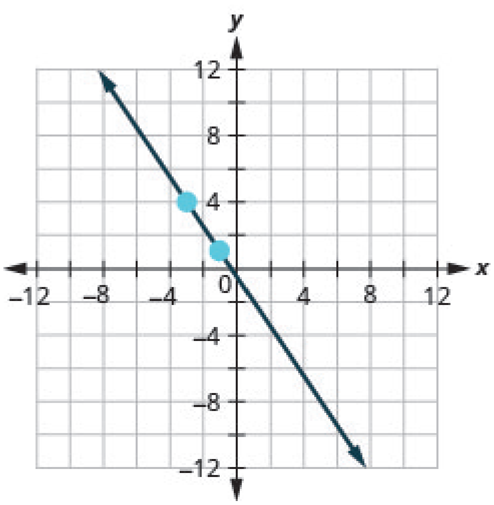
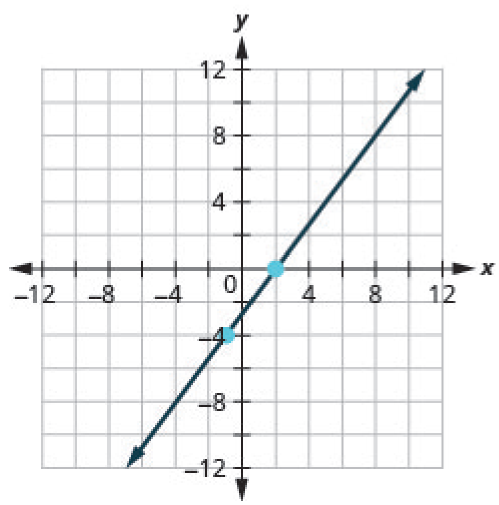
75. What does the sign of the slope tell you about a line?

76. How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

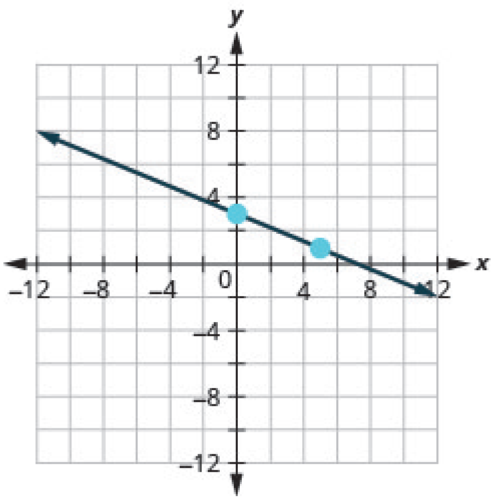
77. Why is the slope of a vertical line "undefined"?

Answers

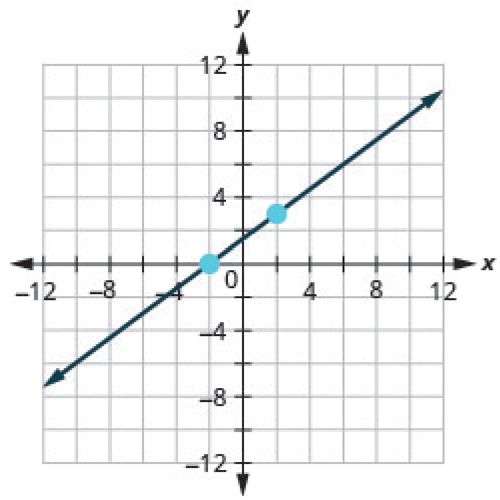
1. $\frac{1}{4}$	3. $\frac{2}{3}$
5. $\frac{-3}{2} = -\frac{3}{2}$	7. $-\frac{3}{4}$
9. 	11. 
13. 	15. 
17. $\frac{2}{5}$	19. $\frac{5}{4}$
21. $-\frac{1}{3}$	23. $-\frac{3}{4}$
25. $\frac{3}{4}$	27. $-\frac{7}{3}$
29. $-\frac{2}{3}$	31. $\frac{1}{4}$

33. 0	35. undefined
37. 0	39. undefined
41. $\frac{5}{2}$	43. $\frac{3}{4}$
45. $-\frac{5}{2}$	47. $-\frac{8}{7}$
49. $\frac{7}{3}$	51. -1
53. 	55. 
57. 	59. 

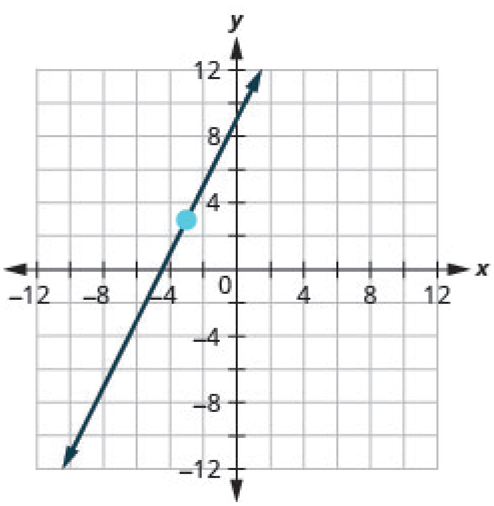
61.



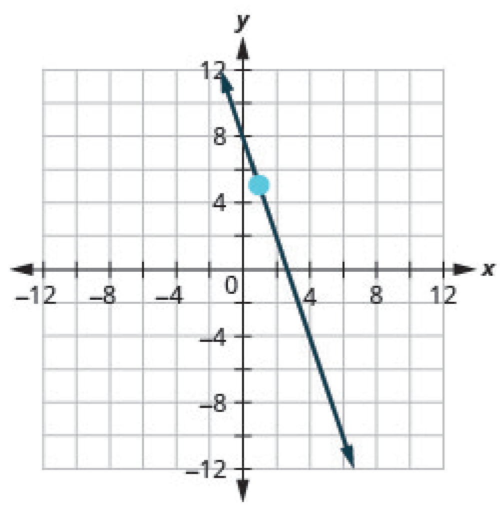
63.



65.



67.



69. a) $\frac{1}{3}$ b) 4 12 pitch or 4-in-12 pitch	71. $\frac{3}{50}$; rise = 3, run = 50
73. a) 288 inches (24 feet) b) Models will vary.	75. When the slope is a positive number the line goes up from left to right. When the slope is a negative number the line goes down from left to right.
77. A vertical line has 0 run and since division by 0 is undefined the slope is undefined.	

Attributions

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4.5 Use the Slope–Intercept Form of an Equation of a Line

Learning Objectives

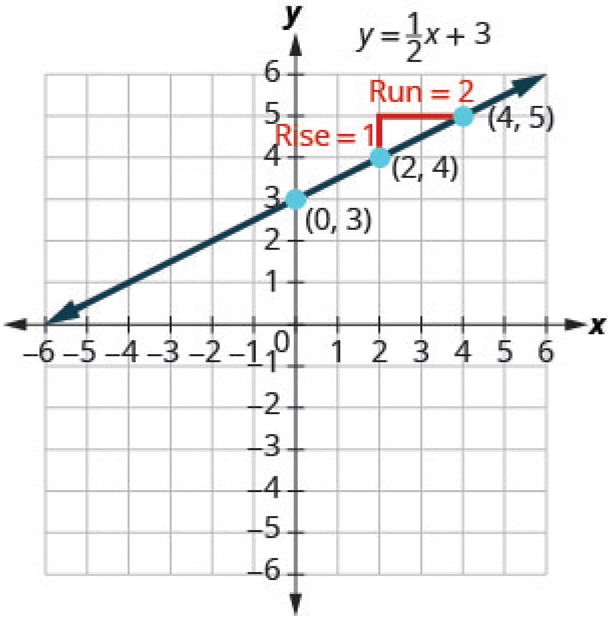
By the end of this section, you will be able to:

- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In Graph Linear Equations in Two Variables, we graphed the line of the equation $y = \frac{1}{2}x + 3$ by plotting points. See (Figure). Let's find the slope of this line.



The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{1}{2}$$

What is the *y*-intercept of the line? The *y*-intercept is where the line crosses the *y*-axis, so *y*-intercept is $(0, 3)$. The equation of this line is:

$$y = \frac{1}{2}x + 3$$

Notice, the line has:

slope $m = \frac{1}{2}$ and *y*-intercept (0, 3)

When a linear equation is solved for y, the coefficient of the x term is the slope and the constant term is the *y*-coordinate of the *y*-intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$m = \frac{1}{2}$; *y*-intercept is (0, 3)

$$y = \frac{1}{2}x + 3$$

$$y = mx + b$$

Slope-intercept form of an equation of a line

The slope–intercept form of an equation of a line with slope m and *y*-intercept, $(0, b)$ is,

$$y = mx + b$$

Sometimes the slope–intercept form is called the "*y*-form."

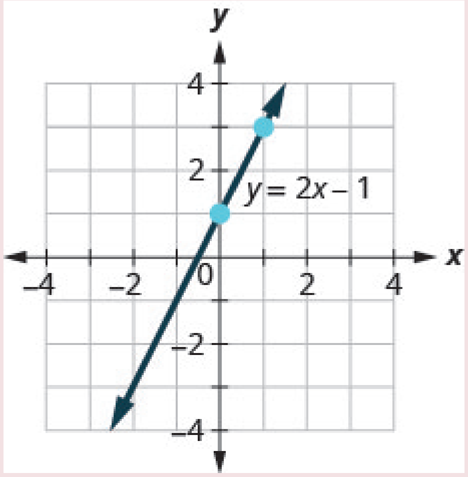
EXAMPLE 1

Use the graph to find the slope and *y*-intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Solution

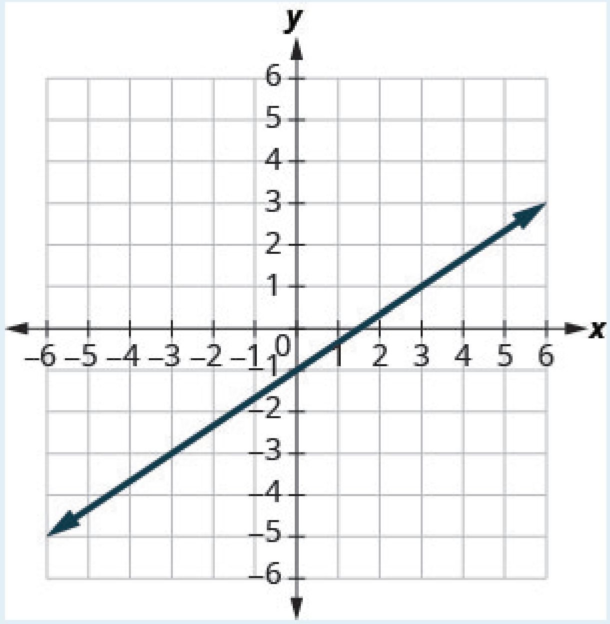
To find the slope of the line, we need to choose two points on the line. We'll use the points $(0, 1)$ and $(1, 3)$.

	
Find the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{1}$
	$m = 2$
Find the *y*-intercept of the line.	The *y*-intercept is the point (0, 1).
We found slope $m = 2$ and *y*-intercept (0, 1).	$y = 2x + 1$ $y = mx + b$

The slope is the same as the coefficient of x and the *y*-coordinate of the *y*-intercept is the same as the constant term.

TRY IT 1.1

Use the graph to find the slope and *y*-intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.

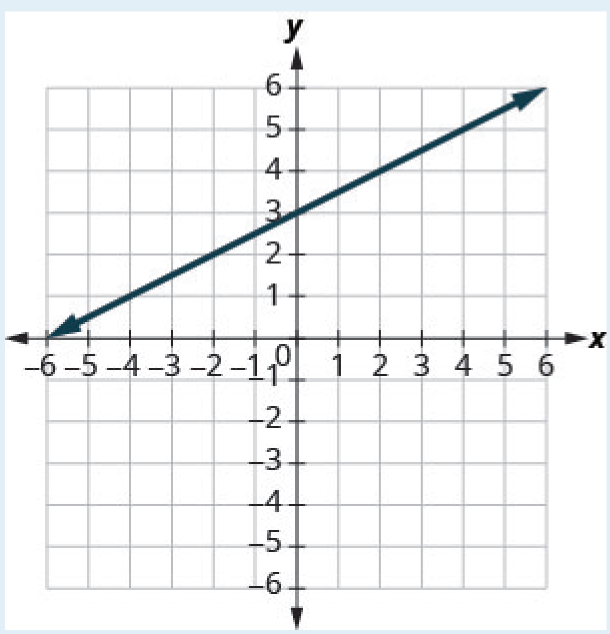


Show answer

slope $m = \dfrac{2}{3}$ and *y*-intercept $(0, -1)$

TRY IT 1.2

Use the graph to find the slope and *y*-intercept of the line $y = \dfrac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.



Show answer

slope $m = \dfrac{1}{2}$ and *y*-intercept $(0, 3)$

Identify the Slope and *y*-Intercept From an Equation of a Line

In Understand Slope of a Line, we graphed a line using the slope and a point. When we are given an equation in slope–intercept form, we can use the *y*-intercept as the point, and then count out the slope from there. Let's practice finding the values of the slope and *y*-intercept from the equation of a line.

EXAMPLE 2

Identify the slope and *y*-intercept of the line with equation $y = -3x + 5$.

Solution
We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the *y*-intercept.	*y*-intercept is $(0, 5)$

TRY IT 2.1

Identify the slope and *y*-intercept of the line $y = \dfrac{2}{5}x - 1$.

Show answer
$\dfrac{2}{5}; (0, -1)$

TRY IT 2.2

Identify the slope and *y*-intercept of the line $y = -\dfrac{4}{3}x + 1$.

Show answer
$-\dfrac{4}{3}; (0, 1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y.

EXAMPLE 3

Identify the slope and *y*-intercept of the line with equation $x + 2y = 6$.

Solution

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for$y$.

Solve for *y*.	$x + 2y = 6$
Subtract *x* from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x+6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
(Remember:$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$)	
Simplify.	$y = -\frac{1}{2}x + 3$
Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the *y*-intercept.	*y*-intercept is (0, 3)

TRY IT 3.1

Identify the slope and *y*-intercept of the line $x + 4y = 8$.

Show answer

$-\frac{1}{4}; (0, 2)$

TRY IT 3.2

Identify the slope and *y*-intercept of the line $3x + 2y = 12$.

Show answer

$-\frac{3}{2}; (0, 6)$

Graph a Line Using its Slope and Intercept

Now that we know how to find the slope and *y*-intercept of a line from its equation, we can graph the line by plotting the *y*-intercept and then using the slope to find another point.

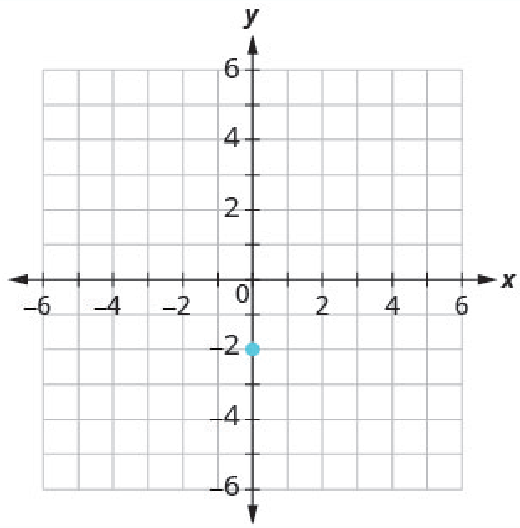
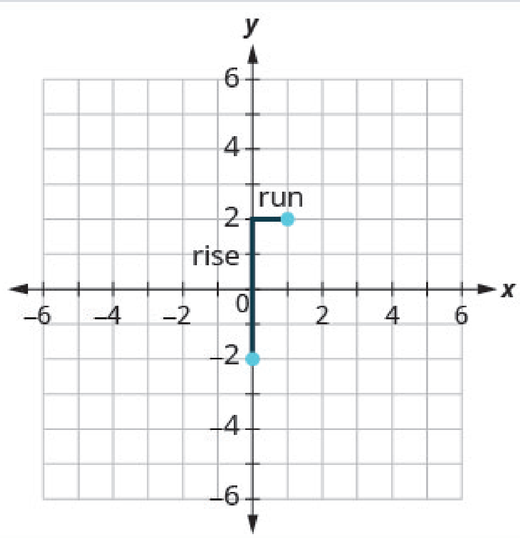
EXAMPLE 4

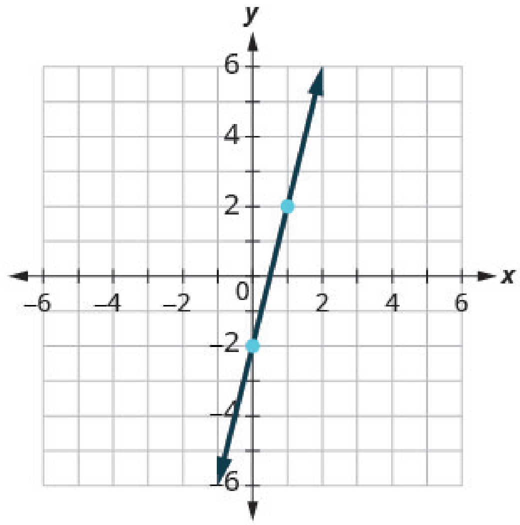
How to Graph a Line Using its Slope and Intercept

Graph the line of the equation $y = 4x - 2$ using its slope and *y*-intercept.

Solution

Step 1. Find the slope–intercept form of the equation.	This equation is in slope–intercept form.	$y = 4x - 2$
Step 2. Identify the slope and *y*-intercept.	Use $y = mx + b$ Find the slope. Find the *y*-intercept.	$y = mx + b$ $y = 4x + (-2)$ $m = 4$ $b = -2, (0, -2)$

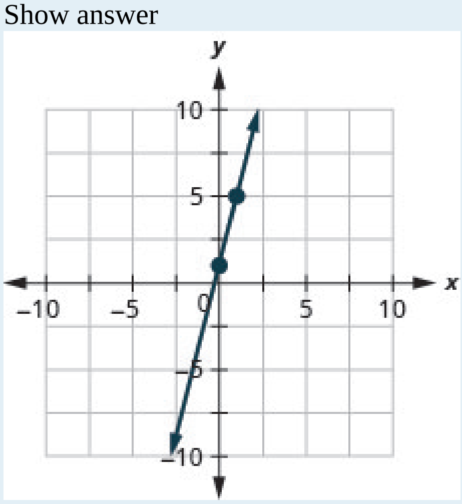
Step 3. Plot the *y*-intercept.	Plot (0, –2).	
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.	Identify the rise and the run.	$m = 4$ $\frac{\text{rise}}{\text{run}} = \frac{4}{1}$ rise = 4 run = 1
Step 5. Starting at the *y*-intercept, count out the rise and run to mark the second point.	Start at (0, –2) and count the rise and the run. Up 4, right 1.	

Step 6. Connect the points with a line.	Connect the two points with a line.	

TRY IT 4.1

Graph the line of the equation $y = 4x + 1$ using its slope and *y*-intercept.

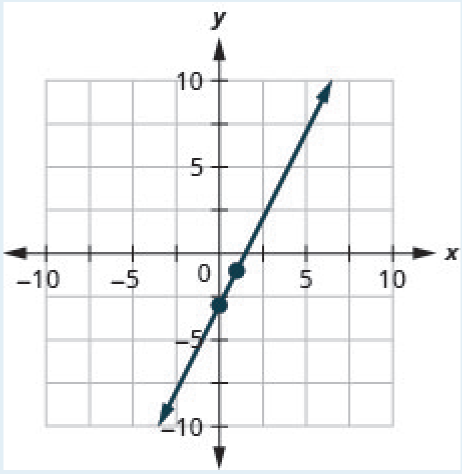
Show answer



TRY IT 4.2

Graph the line of the equation $y = 2x - 3$ using its slope and *y*-intercept.

Show answer



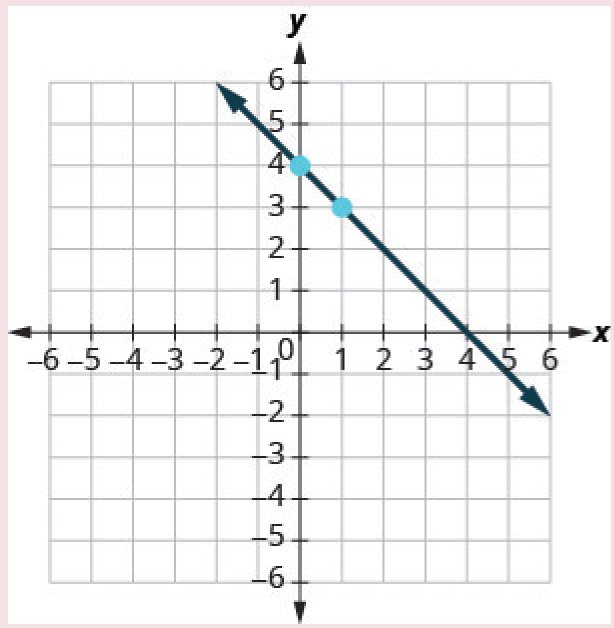
HOW TO: Graph a line using its slope and *y*-intercept

1. Find the slope-intercept form of the equation of the line.
2. Identify the slope and *y*-intercept.
3. Plot the *y*-intercept.
4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.
5. Starting at the *y*-intercept, count out the rise and run to mark the second point.
6. Connect the points with a line.

EXAMPLE 5

Graph the line of the equation $y = -x + 4$ using its slope and *y*-intercept.

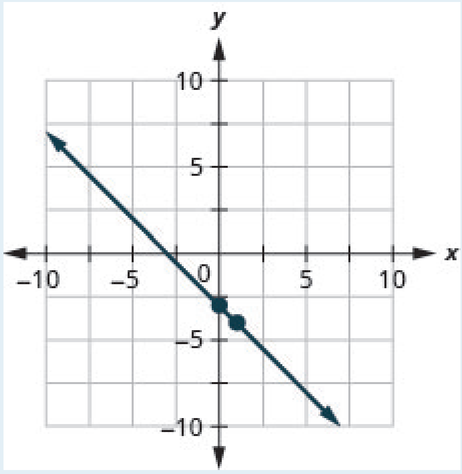
Solution

	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and *y*-intercept.	$m = -1$
	y-intercept is (0, 4)
Plot the *y*-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise −1, run 1
Draw the line.	
To check your work, you can find another point on the line and make sure it is a solution of the equation. In the graph we see the line goes through (4, 0).	
Check. $y = -x + 4$ $0 \stackrel{?}{=} -4 + 4$ $0 = 0 \checkmark$	

TRY IT 5.1

Graph the line of the equation $y = -x - 3$ using its slope and *y*-intercept.

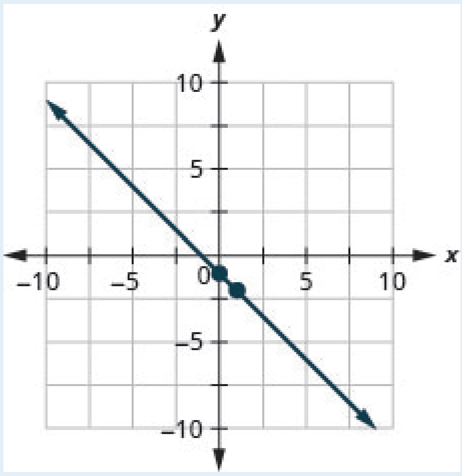
Show answer



TRY IT 5.2

Graph the line of the equation $y = -x - 1$ using its slope and *y*-intercept.

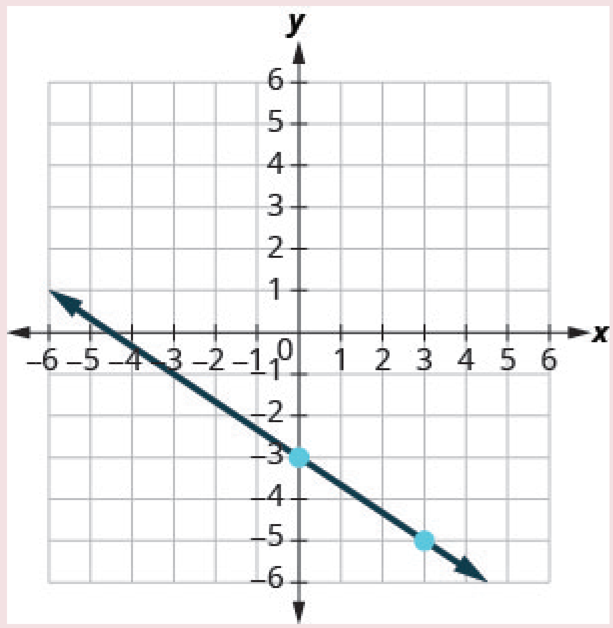
Show answer



EXAMPLE 6

Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and *y*-intercept.

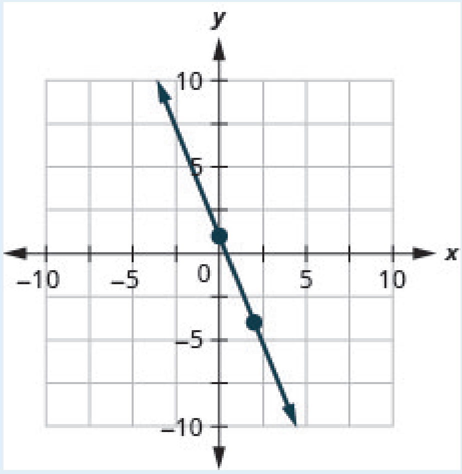
Solution

	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and *y*-intercept.	$m = -\frac{2}{3}$; *y*-intercept is (0, −3)
Plot the *y*-intercept.	See graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	
Draw the line.	

TRY IT 6.1

Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and *y*-intercept.

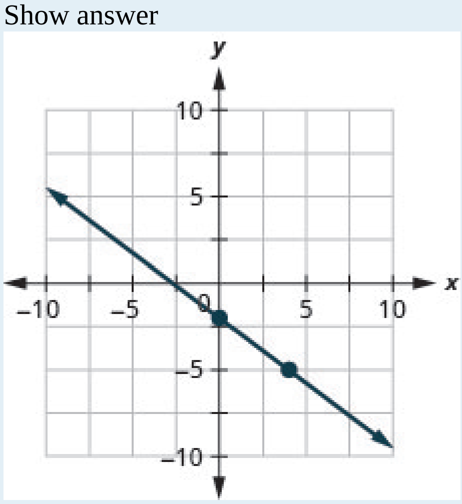
Show answer



TRY IT 6.2

Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and *y*-intercept.

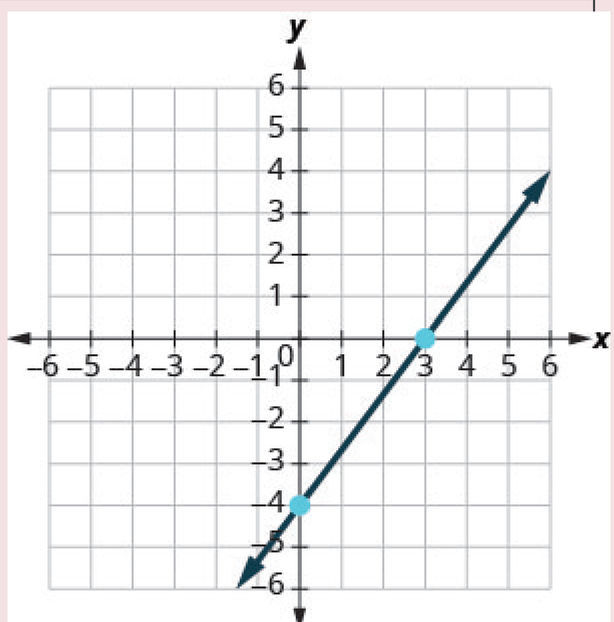
Show answer



EXAMPLE 7

Graph the line of the equation $4x - 3y = 12$ using its slope and *y*-intercept.

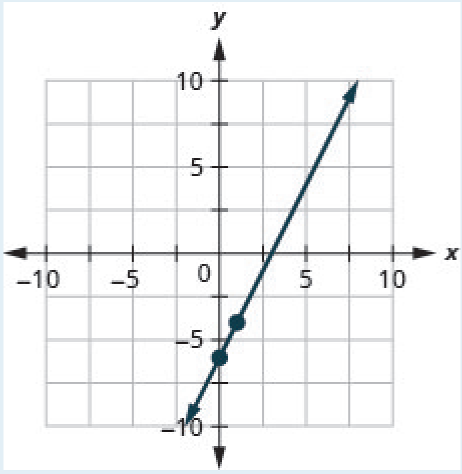
Solution

	$4x - 3y = 12$
Find the slope–intercept form of the equation.	$-3y = -4x + 12$
	$-\frac{3y}{3} = \frac{-4x + 12}{-3}$
The equation is now in slope–intercept form.	$y = \frac{4}{3}x - 4$
Identify the slope and *y*-intercept.	$m = \frac{4}{3}$
	y-intercept is (0, −4)
Plot the *y*-intercept.	See graph below.
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

TRY IT 7.1

Graph the line of the equation $2x - y = 6$ using its slope and *y*-intercept.

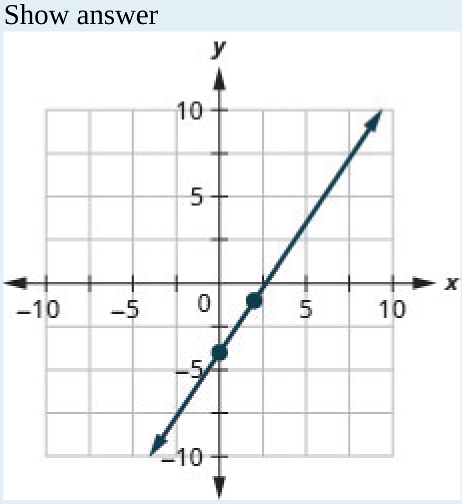
Show answer



TRY IT 7.2

Graph the line of the equation $3x - 2y = 8$ using its slope and *y*-intercept.

Show answer



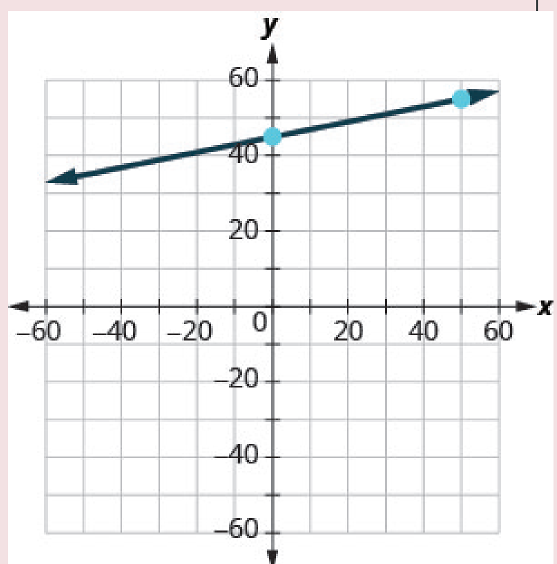
We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

EXAMPLE 8

Graph the line of the equation $y = 0.2x + 45$ using its slope and *y*-intercept.

Solution

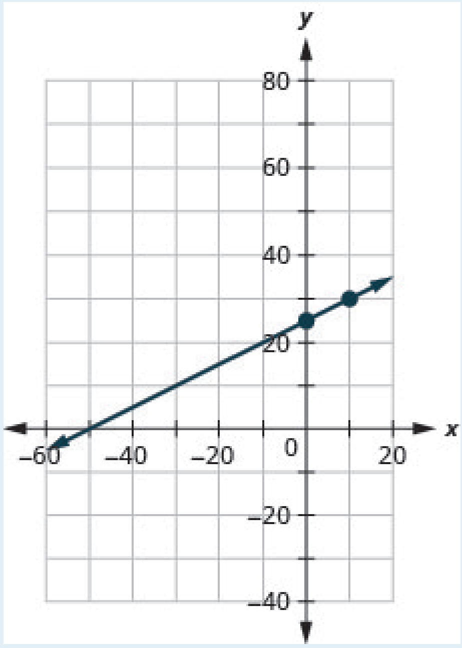
We'll use a grid with the axes going from about -80 to 80.

	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and *y*-intercept.	$m = 0.2$
	The *y*-intercept is (0, 45)
Plot the *y*-intercept.	See graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

TRY IT 8.1

Graph the line of the equation $y = 0.5x + 25$ using its slope and *y*-intercept.

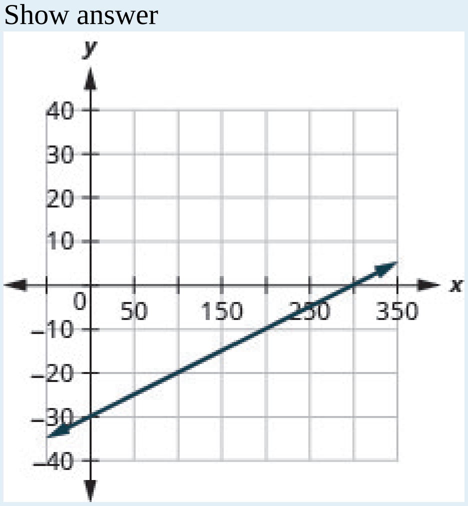
Show answer



TRY IT 8.2

Graph the line of the equation $y = 0.1x - 30$ using its slope and *y*-intercept.

Show answer



Now that we have graphed lines by using the slope and *y*-intercept, let’s summarize all the methods we have used to graph lines. See (Figure).

Methods to graph lines

Methods to Graph Lines			
Point Plotting	**Slope–Intercept**	**Intercepts**	**Recognize Vertical and Horizontal Lines**
x | y	$y = mx + b$	x | y ; 0 | ; | 0	
Find three points. Plot the points, make sure they line up, then draw the line.	Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope–intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in sections 4.3, 4.4, and earlier in this section. Let’s look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

	Equation	**Method**
#1	$x = 2$	Vertical line
#2	$y = 4$	Horizontal line
#3	$-x + 2y = 6$	Intercepts
#4	$4x - 3y = 12$	Intercepts
#5	$y = 4x - 2$	Slope-intercept
#6	$y = -x + 4$	Slope-intercept

Equations #1 and #2 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the *x*-intercept and $x = 0$ to find the *y*-intercept, and then found a third point by choosing another value for x or y.

Equations #5 and #6 are written in slope–intercept form. After identifying the slope and *y*-intercept from the equation we used them to graph the line.

This leads to the following strategy.

Strategy for choosing the most convenient method to graph a line

Consider the form of the equation.

- If it only has one variable, it is a vertical or horizontal line.
  - $x = a$ is a vertical line passing through the *x*-axis at a.
  - $y = b$ is a horizontal line passing through the *y*-axis at b.
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
  - Identify the slope and *y*-intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
  - Find the *x*– and *y*-intercepts, a third point, and then graph.

EXAMPLE 9

Determine the most convenient method to graph each line.

a) $y = -6$ b)$5x - 3y = 15$ c) $x = 7$ d) $y = \frac{2}{5}x - 1$.

Solution

a. $y = -6$
This equation has only one variable,y. Its graph is a horizontal line crossing the *y*-axis at -6.

b. $5x - 3y = 15$
This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.

c. $x = 7$
There is only one variable, x. The graph is a vertical line crossing the *x*-axis at 7.

d. $y = \frac{2}{5}x - 1$
Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and *y*-intercept.

TRY IT 9.1

Determine the most convenient method to graph each line: a) $3x + 2y = 12$ b) $y = 4$ c) $y = \frac{1}{5}x - 4$ d) $x = -7$.

Show answer

a) intercepts b) horizontal line c) slope–intercept d) vertical line

TRY IT 9.2

Determine the most convenient method to graph each line: a) $x = 6$ b) $y = -\frac{3}{4}x + 1$ c) $y = -8$ d) $4x - 3y = -1$.

Show answer

a) vertical line b) slope–intercept c) horizontal line d) intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually when a linear equation models a real-world situation, different letters are used for the variables, instead of *x* and *y*. The variable names remind us of what quantities are being measured.

EXAMPLE 10

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C, on the Celsius scale to temperatures, F, on the Fahrenheit scale.

a) Find the Fahrenheit temperature for a Celsius temperature of 0.
b) Find the Fahrenheit temperature for a Celsius temperature of 20.
c) Interpret the slope and *F*-intercept of the equation.
d) Graph the equation.

Solution

a) Find the Fahrenheit temperature for a Celsius temperature of 0. Find F when $C = 0$. Simplify.	$F = \frac{9}{5}C + 32$ $F = \frac{9}{5}(0) + 32$ $F = 32$
b) Find the Fahrenheit temperature for a Celsius temperature of 20. Find F when $C = 20$. Simplify. Simplify.	$F = \frac{9}{5}C + 32$ $F = \frac{9}{5}(20) + 32$ $F = 36 + 32$ $F = 68$

c) Interpret the slope and *F*-intercept of the equation.

Even though this equation uses $F$and C, it is still in slope–intercept form.

$$y = mx + b$$

$$F = mC + b$$

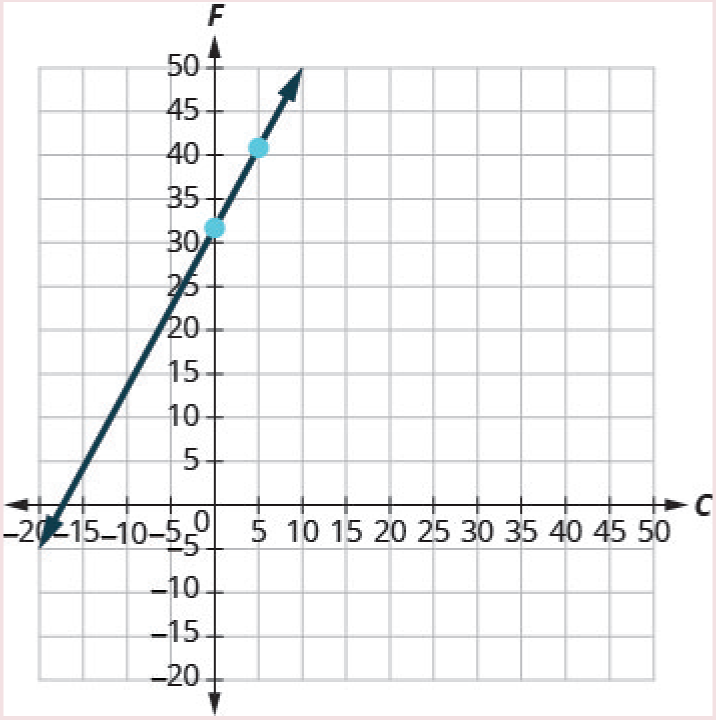
$$F = \frac{9}{5}C + 32$$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (*F*) increases 9 degrees when the temperature Celsius (*C*) increases 5 degrees.

The *F*-intercept means that when the temperature is 0° on the Celsius scale, it is 32° on the Fahrenheit scale.

d) Graph the equation.

We'll need to use a larger scale than our usual. Start at the *F*-intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See (Figure).



TRY IT 10.1

The equation $h = 2s + 50$ is used to estimate a woman's height in inches, *h*, based on her shoe size, *s*.

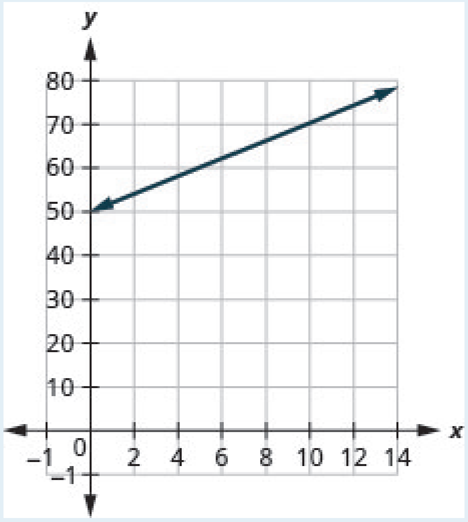
a) Estimate the height of a child who wears women's shoe size 0.
b) Estimate the height of a woman with shoe size 8.
c) Interpret the slope and *h*-intercept of the equation.
d) Graph the equation.

Show answer

a. 50 inches

b. 66 inches

c. The slope, 2, means that the height, *h*, increases by 2 inches when the shoe size, *s*, increases by 1. The *h*-intercept means that when the shoe size is 0, the height is 50 inches.

d. 

TRY IT 10.2

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, *T*, based on the number of cricket chirps, *n*, in one minute.

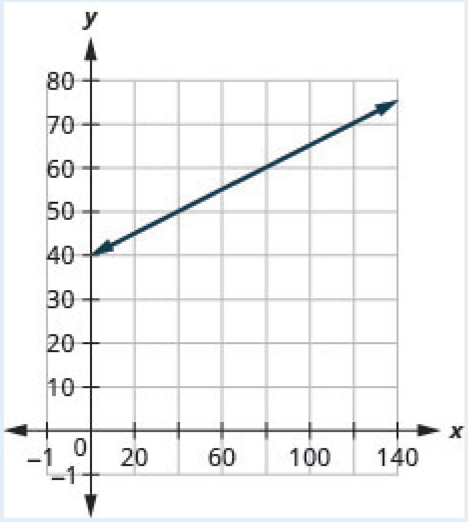
a) Estimate the temperature when there are no chirps.
b) Estimate the temperature when the number of chirps in one minute is 100.
c) Interpret the slope and *T*-intercept of the equation.
d) Graph the equation.

Show answer

a. 40 degrees

b. 65 degrees

c. The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (*F*) increases 1 degree when the number of chirps, *n*, increases by 4. The *T*-intercept means that when the number of chirps is 0, the temperature is 40°.

d. 

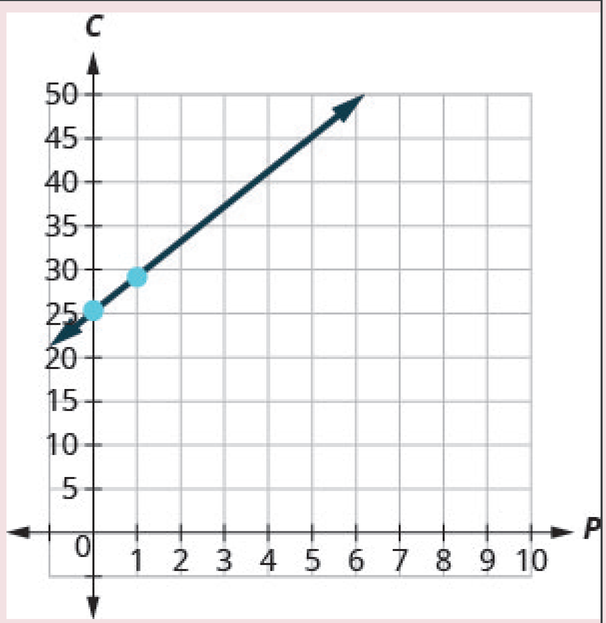
The cost of running some types business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labour needed to produce each item.

EXAMPLE 11

Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, *C*, in dollars and the number of pizzas, *p*, that she sells.

a) Find Stella's cost for a week when she sells no pizzas.
b) Find the cost for a week when she sells 15 pizzas.
c) Interpret the slope and *C*-intercept of the equation.
d) Graph the equation.

Solution

a) Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is $25 when she sells no pizzas.
b) Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are $85 when she sells 15 pizzas.
c) Interpret the slope and C-intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by $4 for each pizza Stella sells. The C-intercept means that even when Stella sells no pizzas, her costs for the week are $25.
d) Graph the equation. We'll need to use a larger scale than our usual. Start at the C-intercept (0, 25) then count out the rise of 4 and the run of 1 to get a second point.	

TRY IT 11.1

Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, *C*, in dollars and the number of miles, *m*, that he drives.

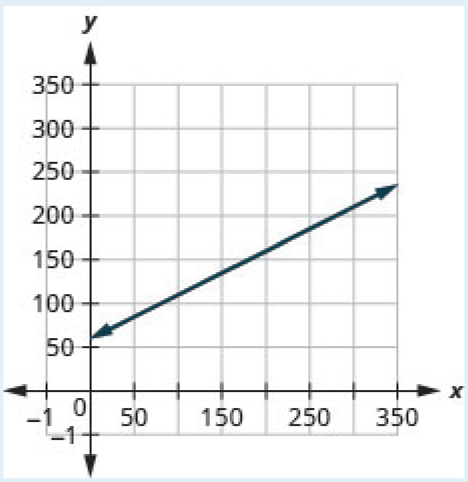
a) Find Sam's cost for a week when he drives 0 miles.
b) Find the cost for a week when he drives 250 miles.
c) Interpret the slope and *C*-intercept of the equation.
d) Graph the equation.

Show answer

a. $60

b. $185

c. The slope, 0.5, means that the weekly cost, *C*, increases by $0.50 when the number of miles driven, *n*, increases by 1. The *C*-intercept means that when the number of miles driven is 0, the weekly cost is $60

d. 

TRY IT 11.2

Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, *C*, in dollars and the number of wedding invitations, *n*, that she writes.

a) Find Loreen's cost for a week when she writes no invitations.
b) Find the cost for a week when she writes 75 invitations.
c) Interpret the slope and *C*-intercept of the equation.
d) Graph the equation.

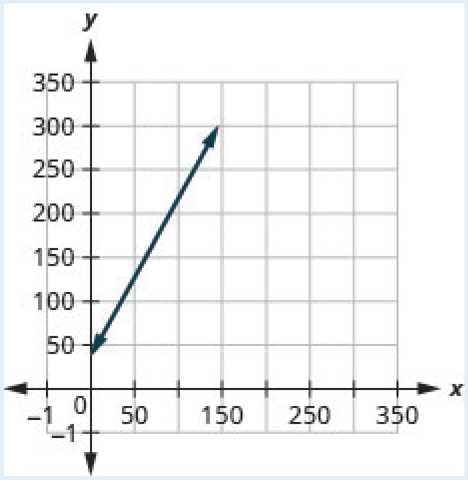
Show answer

a. $35

b. $170

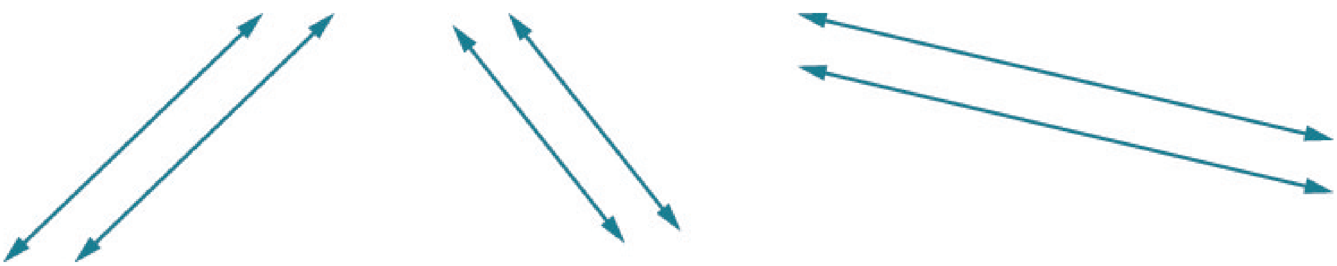
c. The slope, 1.8, means that the weekly cost, C, increases by $1.80 when the number of invita-

tions, *n*, increases by 1.80.
The *C*-intercept means that when the number of invitations is 0, the weekly cost is $35.;

d. 

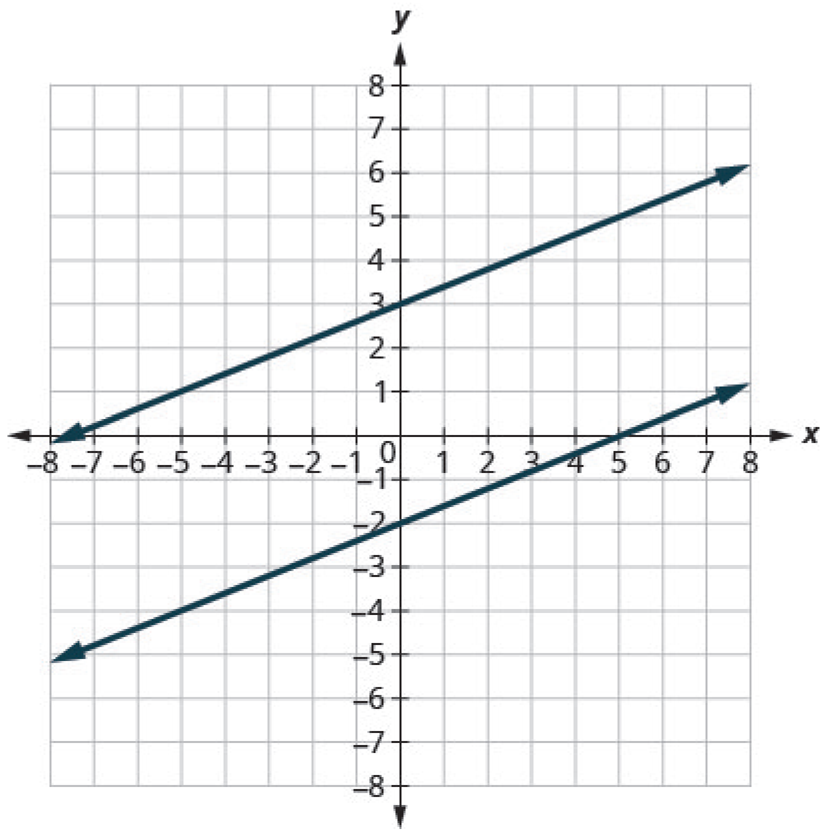
Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.



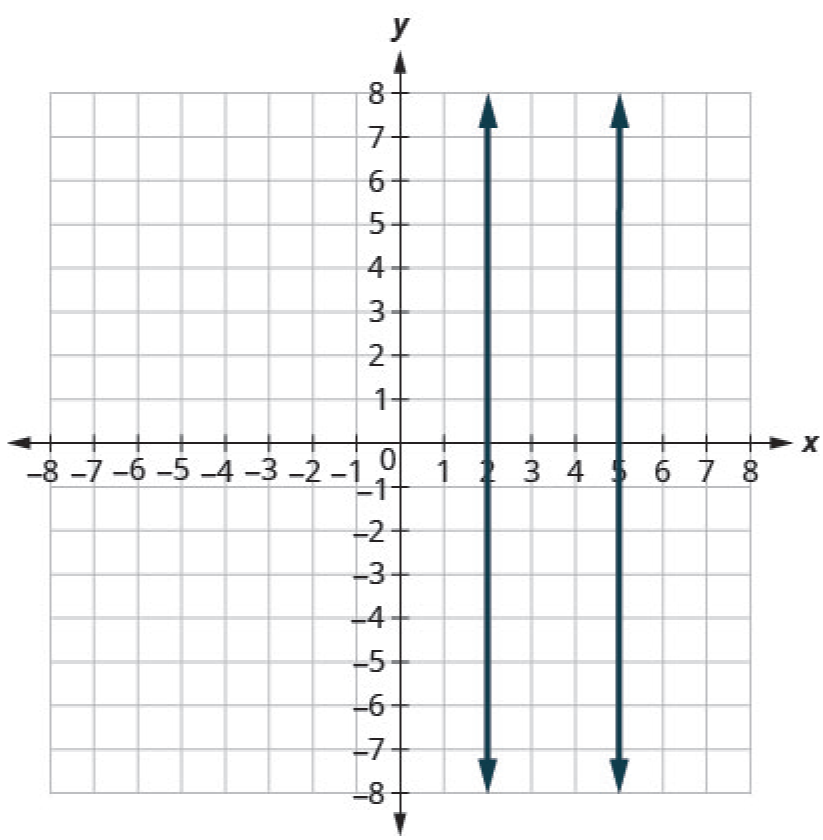
We say this more formally in terms of the rectangular coordinate system. Two lines that have the same slope and different *y*-intercepts are called parallel lines. See (Figure).

Verify that both lines have the same slope, $m = \frac{2}{5}$, and different *y*-intercepts.



What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different *x*-intercepts are parallel. See (Figure).

Vertical lines with different *x*-intercepts are parallel.



Parallel lines

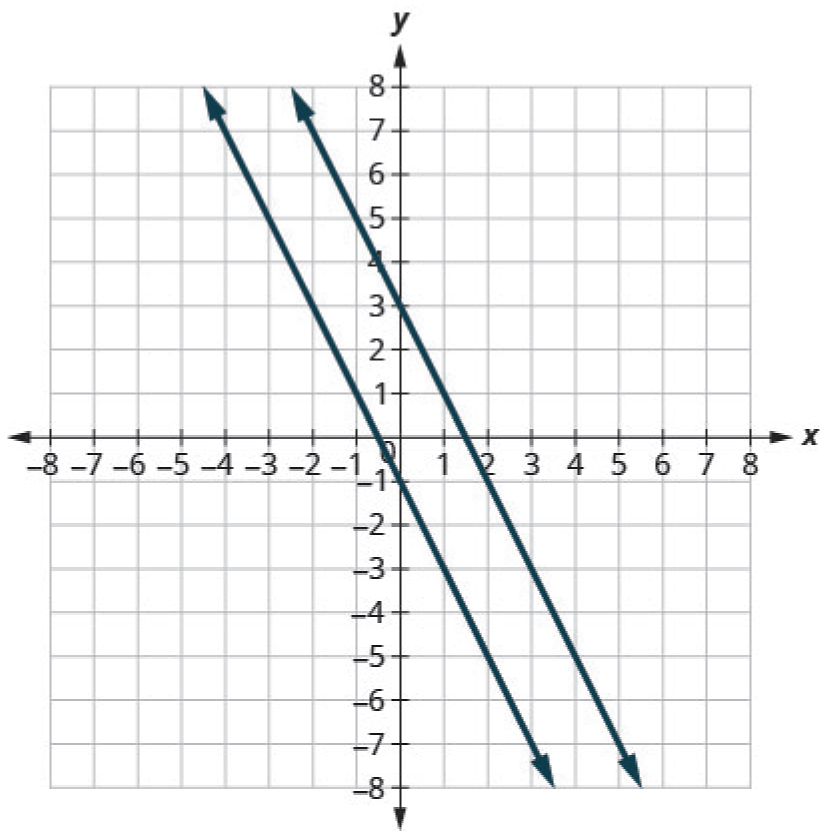
Parallel lines are lines in the same plane that do not intersect.

- Parallel lines have the same slope and different *y*-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then$m_1 = m_2$.
- Parallel vertical lines have different *x*-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope–intercept form: $y = -2x + 3$. We solve the second equation for y:

$$\begin{aligned} 2x + y &= -1 \\ y &= -2x - 1 \end{aligned}$$

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the *y*-intercept of each line?

$$\begin{aligned} y &= mx + b & \quad y &= mx + b \\ y &= -2x + 3 & \quad y &= -2x - 1 \\ m &= -2 & \quad m &= -2 \\ b &= 3, (0,\ 3) & \quad b &= -1, (0,\ \text{-1}) \end{aligned}$$

The slopes of the lines are the same and the *y*-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different *y*-intercepts, we can now just look at the slope–intercept form of the equations of lines and decide if the lines are parallel.

EXAMPLE 12

Use slopes and *y*-intercepts to determine if the lines $3x - 2y = 6$ and $y = \frac{3}{2}x + 1$ are parallel.

Solution

Solve the first equation for y.	$\begin{aligned} 3x - 2y &= 6 \\ -2y &= -3x + 6 \\ \frac{-2y}{-2} &= \frac{-3x + 6}{-2} \end{aligned}$	and	$y = \frac{3}{2}x + 1$
The equation is now in slope-intercept form.	$y = \frac{3}{2}x - 3$		
The equation of the second line is already in slope-intercept form.			$y = \frac{3}{2}x + 1$
Identify the slope and y-intercept of both lines.	$y = \frac{3}{2}x - 3$ $y = mx + b$ $m = \frac{3}{2}$		$y = \frac{3}{2}x + 1$ $y = mx + b$ $m = \frac{3}{2}$
	y-intercept is (0, −3)		y-intercept is (0, 1)

The lines have the same slope and different *y*-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

TRY IT 12.1

Use slopes and *y*-intercepts to determine if the lines $2x + 5y = 5$ and $y = -\frac{2}{5}x - 4$ are parallel.

Show answer
parallel

TRY IT 12.2

Use slopes and *y*-intercepts to determine if the lines $4x - 3y = 6$ and $y = \frac{4}{3}x - 1$ are parallel.

Show answer
parallel

EXAMPLE 13

Use slopes and *y*-intercepts to determine if the lines $y = -4$ and $y = 3$ are parallel.

Solution

	$y = -4$ $y = 0x - 4$	and	$y = 3$ $y = 0x + 3$
Write each equation in slope-intercept form.	$y = 0x - 4$		$y = 0x + 3$
Since there is no x term we write $0x$.	$y = mx + b$		$y = mx + b$
Identify the slope and y-intercept of both lines.	$m = 0$		$m = 0$
	y-intercept is (0, 4)		y-intercept is (0, 3)

The lines have the same slope and different *y*-intercepts and so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the *y*-axis at $y = -4$ and at $y = 3$, we know the *y*-intercepts are $(0, -4)$ and $(0, 3)$. The lines have the same slope and different *y*-intercepts and so they are parallel.

TRY IT 13.1

Use slopes and *y*-intercepts to determine if the lines $y = 8$ and $y = -6$ are parallel.

Show answer
parallel

TRY IT 13.2

Use slopes and *y*-intercepts to determine if the lines $y = 1$ and $y = -5$ are parallel.

Show answer
parallel

EXAMPLE 14

Use slopes and *y*-intercepts to determine if the lines $x = -2$ and $x = -5$ are parallel.

Solution
$x = -2$ and $x = -5$
Since there is no$y$, the equations cannot be put in slope–intercept form. But we recognize them as equa-

tions of vertical lines. Their *x*-intercepts are -2 and -5. Since their *x*-intercepts are different, the vertical lines are parallel.

TRY IT 14.1

Use slopes and *y*-intercepts to determine if the lines $x = 1$ and $x = -5$ are parallel.

Show answer
parallel

TRY IT 14.2

Use slopes and *y*-intercepts to determine if the lines $x = 8$ and $x = -6$ are parallel.

Show answer
parallel

EXAMPLE 15

Use slopes and *y*-intercepts to determine if the lines $y = 2x - 3$ and $-6x + 3y = -9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution

	$y=2x-3$	and	$-6x+3y=-9$
The first equation is already in slope-intercept form.	$y=2x-3$		
Solve the second equation for y.			$\begin{aligned} -6x+3y &= -9 \\ 3y &= 6x-9 \\ \frac{3y}{3} &= \frac{6x-9}{3} \\ y &= 2x-3 \end{aligned}$
The second equation is now in slope-intercept form.	$y=2x-3$		
Identify the slope and y-intercept of both lines.	$y=2x-3$ $y=mx+b$ $m=2$		$y=2x-3$ $y=mx+b$ $m=2$
	y-intercept is (0, -3)		y-intercept is (0, -3)

The lines have the same slope, but they also have the same *y*-intercepts. Their equations represent the same line. They are not parallel; they are the same line.

TRY IT 15.1

Use slopes and *y*-intercepts to determine if the lines $y=-\frac{1}{2}x-1$ and $x+2y=-2$ are parallel.

Show answer
not parallel; same line

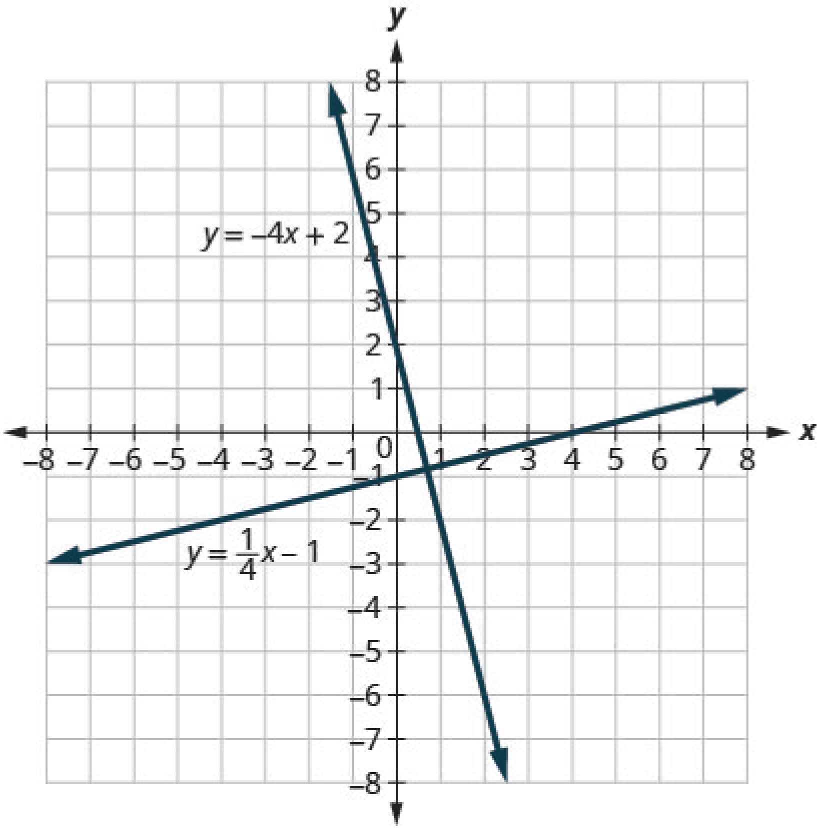
TRY IT 15.2

Use slopes and *y*-intercepts to determine if the lines $y=\frac{3}{4}x-3$ and $3x-4y=12$ are parallel.

Show answer
not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y=\frac{1}{4}x-1$ and $y=-4x+2$, shown in (Figure).



These lines lie in the same plane and intersect in right angles. We call these lines perpendicular.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y=\frac{1}{4}x-1$ rises, so its slope is positive. The line $y=-4x+2$ drops from left to right, so it has a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1=\frac{1}{4}$, and the slope of the second line, $m_2=-4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1.

$$m_1\cdot m_2$$
$$\frac{1}{4}(-4)$$
$$-1$$

This is always true for perpendicular lines and leads us to this definition.

Perpendicular lines

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

$m_1 \cdot m_2 = -1$ and $m_1 = \dfrac{-1}{m_2}$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope–intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope–intercept form of the equation, and then see if the slopes are negative reciprocals. If the product of the slopes is -1, the lines are perpendicular. Perpendicular lines may have the same *y*-intercepts.

EXAMPLE 16

Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution

The first equation is already in slope-intercept form.	$y = -5x - 4$	
Solve the second equation for y.	$\begin{aligned} x - 5y &= 5 \\ -5y &= -x + 5 \\ \frac{-5y}{-5} &= \frac{-x+5}{-5} \\ y &= \frac{1}{5}x - 1 \end{aligned}$	
Identify the slope of each line.	$\begin{aligned} y &= -5x - 4 \\ y &= mx + b \\ m_1 &= -5 \end{aligned}$	$\begin{aligned} y &= \frac{1}{5}x - 1 \\ y &= mx + b \\ m_2 &= \frac{1}{5} \end{aligned}$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

$m_1 \cdot m_2$

$-5\left(\frac{1}{5}\right)$

$-1\checkmark$

TRY IT 16.1

Use slopes to determine if the lines $y = -3x + 2$ and $x - 3y = 4$ are perpendicular.

Show answer
perpendicular

TRY IT 16.2

Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Show answer
perpendicular

EXAMPLE 17

Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Solution

Solve the equations for y.	$$\begin{aligned} 7x + 2y &= 3 \\ 2y &= -7x + 3 \\ \frac{2y}{2} &= \frac{-7x + 3}{2} \\ y &= -\frac{7}{2}x + \frac{3}{2} \end{aligned}$$	$$\begin{aligned} 2x + 7y &= 5 \\ 7y &= -2x + 5 \\ \frac{7y}{7} &= \frac{-2x + 5}{7} \\ y &= -\frac{2}{7}x + \frac{5}{7} \end{aligned}$$
Identify the slope of each line.	$$\begin{aligned} y &= mx + b \\ m_1 &= -\frac{7}{2} \end{aligned}$$	$$\begin{aligned} y &= mx + b \\ m_2 &= -\frac{2}{7} \end{aligned}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

TRY IT 17.1

Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Show answer
not perpendicular

TRY IT 17.2

Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Show answer
not perpendicular

Access this online resource for additional instruction and practice with graphs.

- Explore the Relation Between a Graph and the Slope–Intercept Form of an Equation of a Line

Key Concepts

- **The slope–intercept form of an equation of a line with slope** m and *y*-intercept, $(0, b)$ is, $y = mx + b$.
- **Graph a Line Using its Slope and *y*-Intercept**
  1. Find the slope-intercept form of the equation of the line.
  2. Identify the slope and *y*-intercept.
  3. Plot the *y*-intercept.
  4. Use the slope formula $m = \dfrac{\text{rise}}{\text{run}}$ to identify the rise and the run.
  5. Starting at the *y*-intercept, count out the rise and run to mark the second point.
  6. Connect the points with a line.
- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
  - If it only has one variable, it is a vertical or horizontal line.
    $x = a$ is a vertical line passing through the *x*-axis at a.
    $y = b$ is a horizontal line passing through the *y*-axis at b.
  - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and *y*-intercept.
    Identify the slope and *y*-intercept and then graph.
  - If the equation is of the form $Ax + By = C$, find the intercepts.
    Find the *x*– and *y*-intercepts, a third point, and then graph.
- **Parallel lines are lines in the same plane that do not intersect.**
  - Parallel lines have the same slope and different *y*-intercepts.
  - If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
  - Parallel vertical lines have different *x*-intercepts.
- **Perpendicular lines are lines in the same plane that form a right angle.**
  - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and

$$m_1 = \frac{-1}{m_2}.$$

- Vertical lines and horizontal lines are always perpendicular to each other.

Glossary

parallel lines
Lines in the same plane that do not intersect.

perpendicular lines
Lines in the same plane that form a right angle.

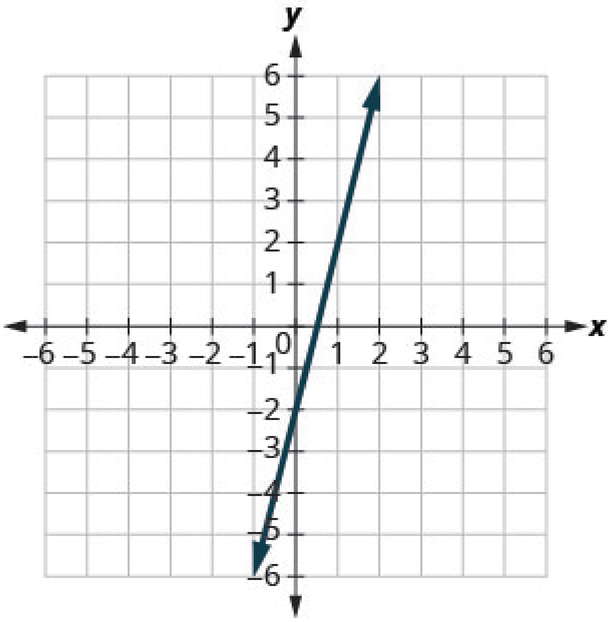
slope-intercept form of an equation of a line
The slope–intercept form of an equation of a line with slope and *y*-intercept, is,
.

Practice Makes Perfect

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

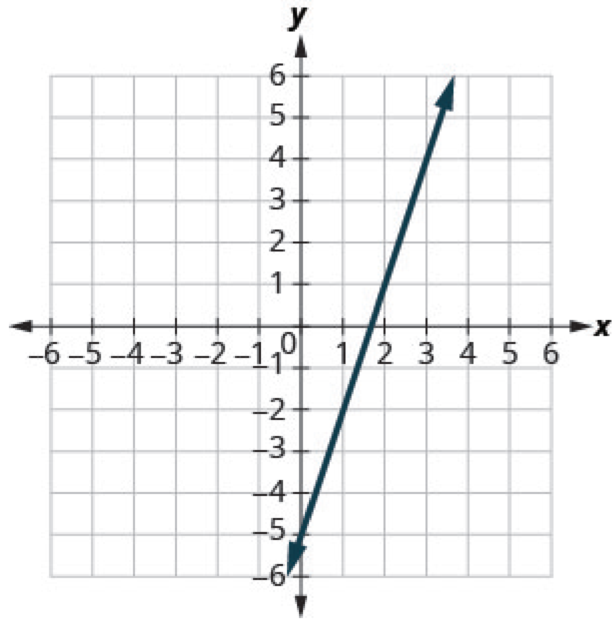
In the following exercises, use the graph to find the slope and y-intercept of each line. Compare the values to the equation $y = mx + b$.

1.



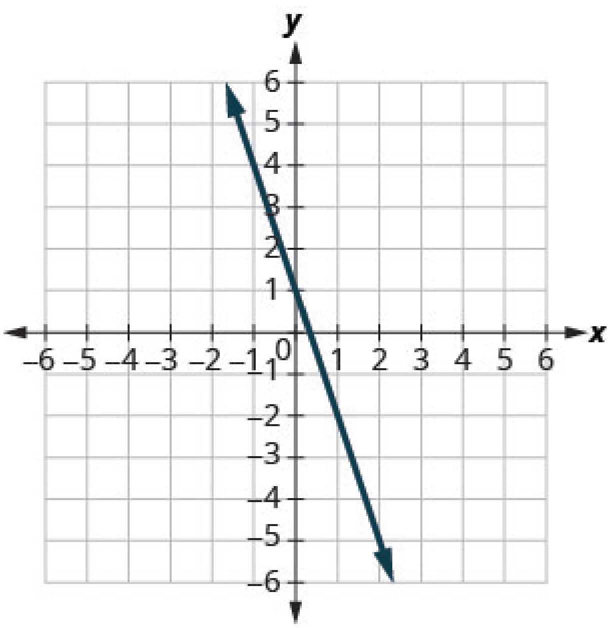
$y = 4x - 2$

2.



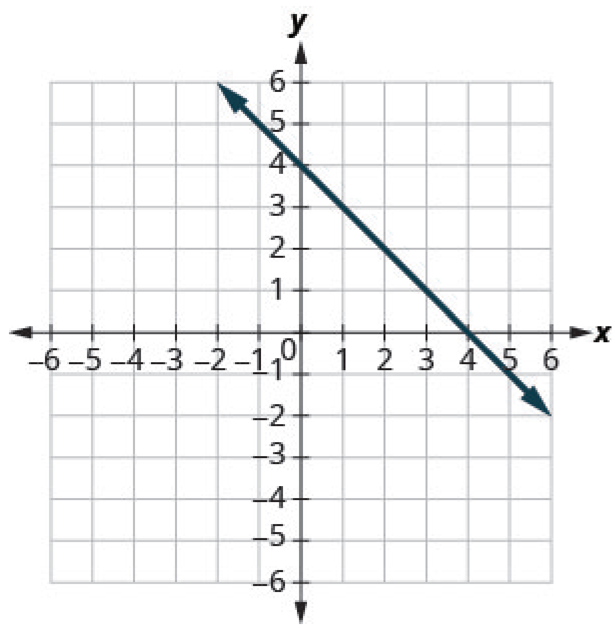
$y = 3x - 5$

3.



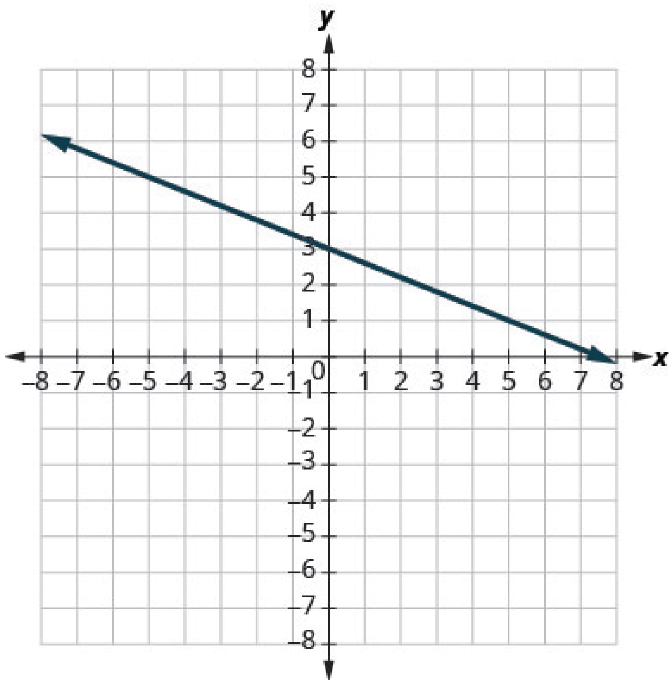
$y = -3x + 1$

4.



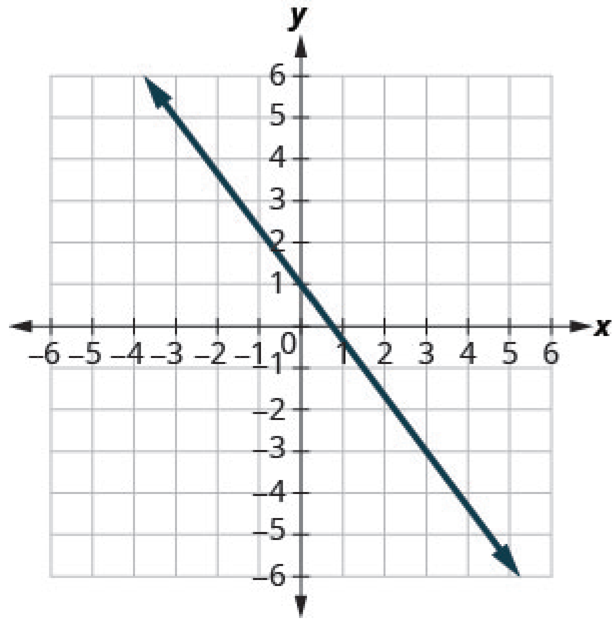
$y = -x + 4$

5.



$y = -\frac{2}{5}x + 3$

6.



$y = -\frac{4}{3}x + 1$

Identify the Slope and y-Intercept From an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

7. $y = -9x + 7$	8. $y = -7x + 3$
9. $y = 4x - 10$	10.$y = 6x - 8$
11. $4x + y = 8$	12. $3x + y = 5$
13. $8x + 3y = 12$	14. $6x + 4y = 12$
15. $7x - 3y = 9$	16. $5x - 2y = 6$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

17. $y = x + 4$	18. $y = x + 3$
19. $y = 2x - 3$	20. $y = 3x - 1$
21. $y = -x + 3$	22. $y = -x + 2$
23. $y = -x - 2$	24. $y = -x - 4$
25. $y = -\frac{2}{5}x - 3$	26. $y = -\frac{3}{4}x - 1$
27. $y = -\frac{2}{3}x + 1$	28. $y = -\frac{3}{5}x + 2$
29. $4x - 3y = 6$	30. $3x - 4y = 8$
31. $y = 0.3x + 25$	32. $y = 0.1x + 15$

Choose the Most Convenient Method to Graph a Line

In the following exercises, determine the most convenient method to graph each line.

33. $y = 4$	34. $x = 2$
35. $x = -3$	36. $y = 5$
37. $y = -3x + 4$	38. $y = -3x + 4$
39. $x - y = 1$	40. $x - y = 5$
41. $y = \frac{4}{5}x - 3$	42. $y = \frac{2}{3}x - 1$
43. $y = -1$	44. $y = -3$
45. $2x - 5y = -10$	46. $3x - 2y = -12$
47. $y = -\frac{1}{3}x + 5$	48. $y = -\frac{1}{4} + 3$

Graph and Interpret Applications of Slope–Intercept

49. The equation $P = 28 + 2.54w$ models the relation between the amount of Randy's monthly water bill payment, *P*, in dollars, and the number of units of water, *w*, used.

a. Find the payment for a month when Randy used 0 units of water.
b. Find the payment for a month when Randy used 15 units of water.
c. Interpret the slope and *P*-intercept of the equation.
d. Graph the equation.

50. The equation $P = 31 + 1.75w$ models the relation between the amount of Tuyet's monthly water bill payment, *P*, in dollars, and the number of units of water, *w*, used.

a. Find Tuyet's payment for a month when 0 units of water are used.
b. Find Tuyet's payment for a month when 12 units of water are used.
c. Interpret the slope and *P*-intercept of the equation.
d. Graph the equation.

51. Janelle is planning to rent a car while on vacation. The equation $C = 0.32m + 15$ models the relation between the cost in dollars, *C*, per day and the number of miles, *m*, she drives in one day.

a. Find the cost if Janelle drives the car 0 miles one day.
b. Find the cost on a day when Janelle drives the car 400 miles.
c. Interpret the slope and *C*–intercept of the equation.
d. Graph the equation.

52. Bruce drives his car for his job. The equation $R = 0.575m + 42$ models the relation between the amount in dollars, *R*, that he is reimbursed and the number of miles, *m*, he drives in one day.

a. Find the amount Bruce is reimbursed on a day when he drives 0 miles.
b. Find the amount Bruce is reimbursed on a day when he drives 220 miles.
c. Interpret the slope and *R*-intercept of the equation.
d. Graph the equation.

53. Patel's weekly salary includes a base pay plus commission on his sales. The equation $S = 750 + 0.09c$ models the relation between his weekly salary, *S*, in dollars and the amount of his sales, *c*, in dollars.

a. Find Patel's salary for a week when his sales were 0.
b. Find Patel's salary for a week when his sales were 18,540.
c. Interpret the slope and *S*-intercept of the equation.
d. Graph the equation.

54. Cherie works in retail and her weekly salary includes commission for the amount she sells. The equation $S = 400 + 0.15c$ models the relation between her weekly salary, *S*, in dollars and the amount of her sales, *c*, in dollars.

a. Find Cherie's salary for a week when her sales were 0.
b. Find Cherie's salary for a week when her sales were 3600.
c. Interpret the slope and *S*–intercept of the equation.
d. Graph the equation.

55. Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, *C* of the banquet and the number of guests, *g*. a. Find the cost if the number of guests is 50. b. Find the cost if the number of guests is 100. c. Interpret the slope and *C*–intercept of the equation. d. Graph the equation.	56. Costa is planning a lunch banquet. The equation $C = 450 + 28g$ models the relation between the cost in dollars, *C*, of the banquet and the number of guests, *g*. a. Find the cost if the number of guests is 40. b. Find the cost if the number of guests is 80. c. Interpret the slope and *C*-intercept of the equation. d. Graph the equation.

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

57. $y = \frac{2}{3}x - 1;\ \ 2x - 3y = -2$	58. $y = \frac{3}{4}x - 3;\ \ 3x - 4y = -2$
59. $3x - 4y = -2;\ \ y = \frac{3}{4}x - 3$	60. $2x - 5y = -3;\ \ y = \frac{2}{5}x + 1$
61. $6x - 3y = -9;\ \ 2x - y = 3$	62. $2x - 4y = 6;\ \ x - 2y = -3$
63. $8x + 6y = 6;\ \ 12x + 9y = 12$	64. $4x + 2y = 6;\ \ 6x + 3y = 3$
65. $x = 7;\ \ x = -8$	66. $x = 5;\ \ x = -6$
67. $x = -3;\ \ x = -2$	68. $x = -4;\ \ x = -1$
69. $y = 5;\ \ y = 1$	70. $y = 2;\ \ y = 6$
71. $y = -1;\ \ y = 2$	72. $y = -4;\ \ y = 3$
73. $4x + 4y = -8;\ \ x + y = 2$	74. $x - y = 2;\ \ 2x - 2y = -8$
75. $5x - 2y = 11;\ \ 5x - y = 7$	76. $x - 3y = -6;\ \ 2x - 6y = 12$
77. $4x - 8y = 16;\ \ x - 2y = -4$	78. $3x - 6y = 12;\ \ 6x - 3y = 3$
79. $x - 5y = 10;\ \ 5x - y = -10$	80. $9x - 3y = 6;\ \ 3x - y = 12$
81. $9x - 5y = 4;\ \ 5x + 9y = -1$	82. $7x - 4y = 8;\ \ 4x + 7y = 14$

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

83. $x - 4y = 8; 4x + y = 2$	84. $3x - 2y = 8; 2x + 3y = 6$
85. $2x + 3y = 5; 3x - 2y = 7$	86. $2x + 5y = 3; 5x - 2y = 6$
87. $3x - 4y = 8; 4x - 3y = 6$	88. $3x - 2y = 1; 2x - 3y = 2$
89. $2x + 4y = 3; 6x + 3y = 2$	90. $5x + 2y = 6; 2x + 5y = 8$
91. $2x - 6y = 4; 12x + 4y = 9$	92. $4x - 2y = 5; 3x + 6y = 8$
93. $8x - 2y = 7; 3x + 12y = 9$	94. $6x - 4y = 5; 8x + 12y = 3$

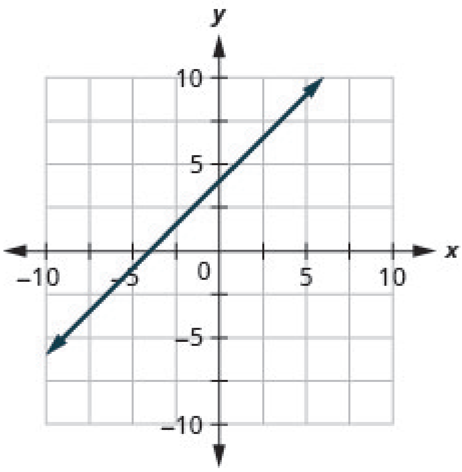
Everyday Math

95. The equation $n = 4T - 160$ is used to estimate the number of cricket chirps, *n*, in one minute based on the temperature in degrees Fahrenheit, *T*. a. Explain what the slope of the equation means. b. Explain what the *n*–intercept of the equation means. Is this a realistic situation?	96. The equation $C = \frac{5}{9}F - 17.8$ can be used to convert temperatures *F*, on the Fahrenheit scale to temperatures, *C*, on the Celsius scale. a. Explain what the slope of the equation means. b. Explain what the *C*–intercept of the equation means.
97. Why are all horizontal lines parallel?	98. Explain in your own words how to decide which method to use to graph a line.

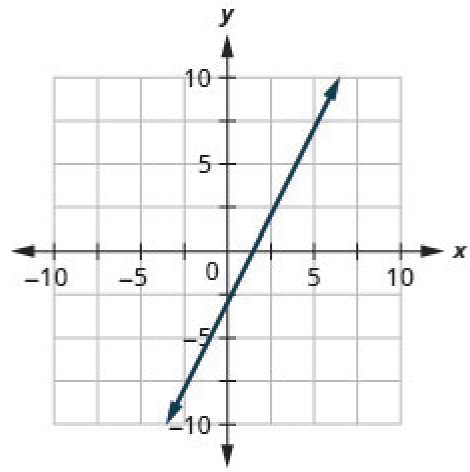
Answers

1. slope $m = 4$ and *y*-intercept $(0, -2)$	3. slope $m = -3$ and *y*-intercept $(0, 1)$
5. slope $m = -\frac{2}{5}$ and *y*-intercept $(0, 3)$	7. -9; $(0, 7)$
9. 4; $(0, -10)$	11. -4; $(0, 8)$
13. $-\frac{8}{3}$; $(0, 4)$	15. $\frac{7}{3}$; $(0, -3)$

17.



19.



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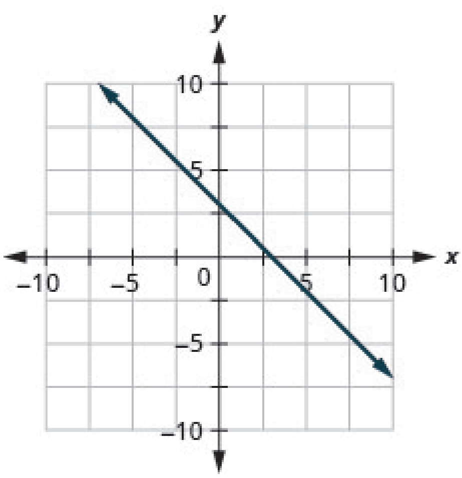

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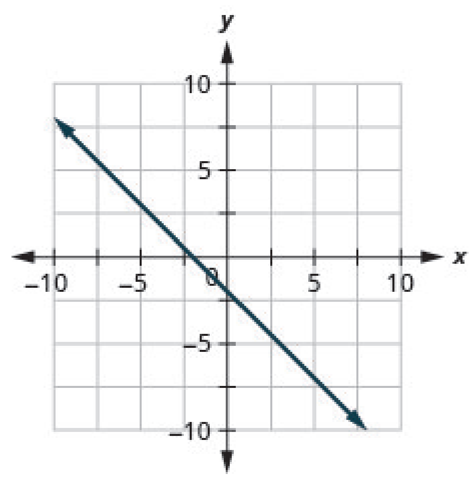
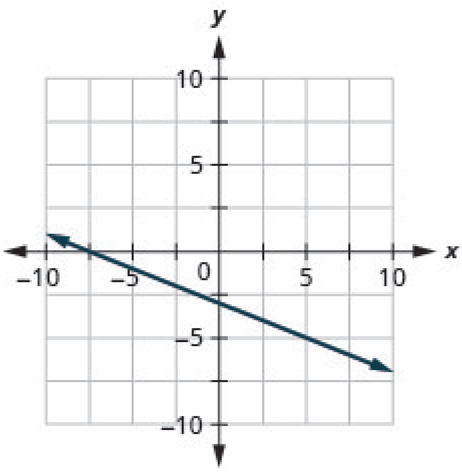
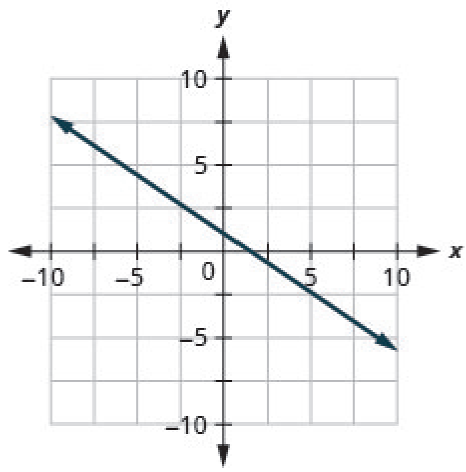
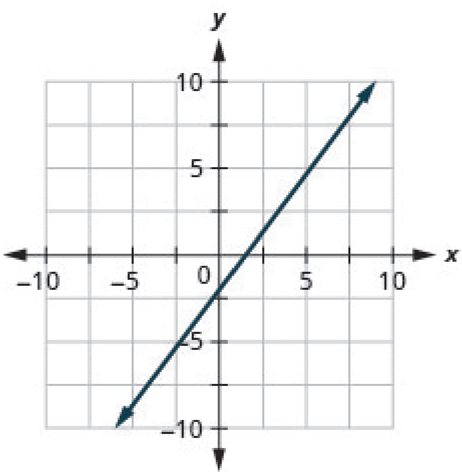
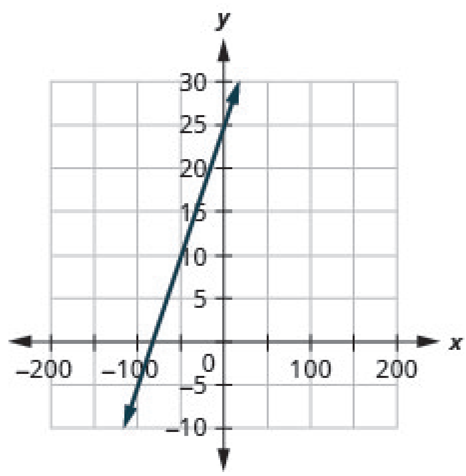
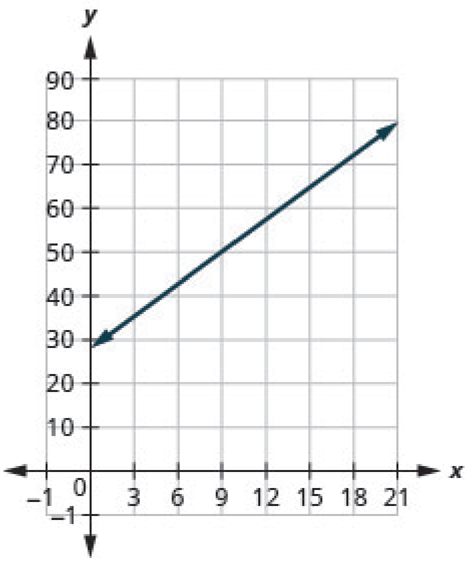
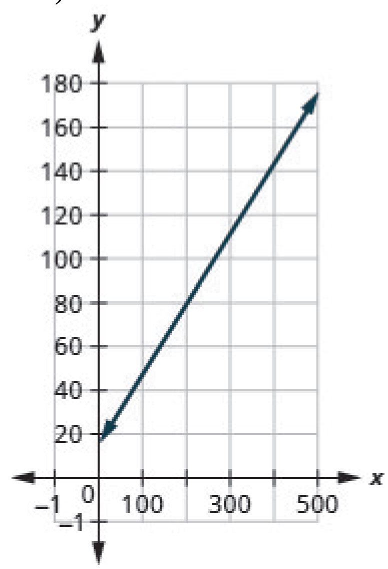
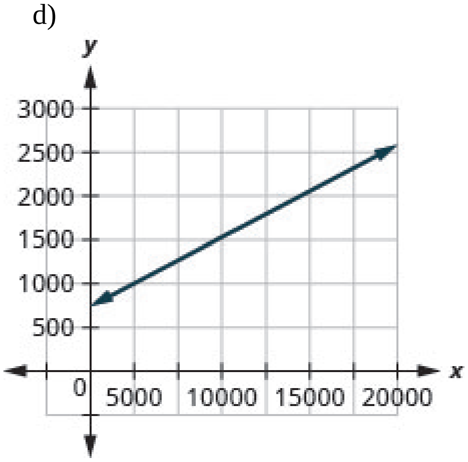
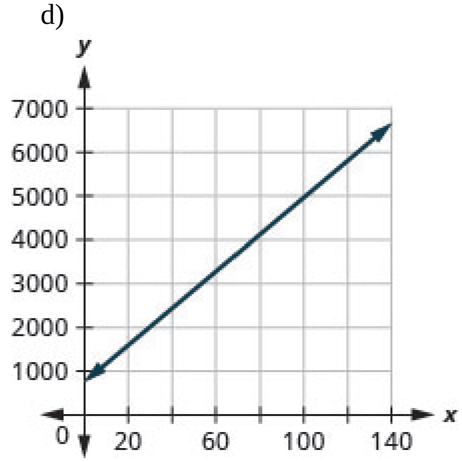


25. 	27. 
29. 	31. 
33. horizontal line	35. vertical line
37. slope–intercept	39. intercepts
41. slope–intercept	43. horizontal line
45. intercepts	47. slope–intercept

<table>
<tr>
<td>49.
a) $28
b) $66.10
c) The slope, 2.54, means that Randy’s payment, P, increases by $2.54 when the number of units of water he used, w, increases by 1. The P–intercept means that if the number units of water Randy used was 0, the payment would be $28.
d)
</td>
<td>51.
a) $15
b) $143
c) The slope, 0.32, means that the cost, C, increases by $0.32 when the number of miles driven, m, increases by 1. The C-intercept means that if Janelle drives 0 miles one day, the cost would be $15.
d)
</td>
</tr>
<tr>
<td>53.
a) $750
b) $2418.60
c) The slope, 0.09, means that Patel’s salary, S, increases by $0.09 for every $1 increase in his sales. The S-intercept means that when his sales are $0, his salary is $750.
d)
</td>
<td>55.
a) $2850
b) $4950
c) The slope, 42, means that the cost, C, increases by $42 for when the number of guests increases by 1. The C-intercept means that when the number of guests is 0, the cost would be $750.
d)
</td>
</tr>
<tr><td>57. parallel</td><td>59. parallel</td></tr>
<tr><td>61. parallel</td><td>63. parallel</td></tr>
<tr><td>65. parallel</td><td>67. parallel</td></tr>
</table>

69. parallel	71. parallel
73. not parallel	75. not parallel
77. not parallel	79. not parallel
81. not parallel	83. perpendicular
85. perpendicular	87. not perpendicular
89. not perpendicular	91. perpendicular
93. perpendicular	95. a) For every increase of one degree Fahrenheit, the number of chirps increases by four. b) There would be -160 chirps when the Fahrenheit temperature is $0°$. (Notice that this does not make sense; this model cannot be used for all possible temperatures.)
97. Answers will vary.	

Attributions

4.6 Find the Equation of a Line

Learning Objectives

By the end of this section, you will be able to:

- Find an equation of the line given the slope and y-intercept
- Find an equation of the line given the slope and a point
- Find an equation of the line given two points
- Find an equation of a line parallel to a given line
- Find an equation of a line perpendicular to a given line

How do online retailers know that 'you may also like' a particular item based on something you just ordered? How can economists know how a rise in the minimum wage will affect the unemployment rate? How do medical researchers create drugs to target cancer cells? How can traffic engineers predict the effect on your commuting time of an increase or decrease in gas prices? It's all mathematics.

You are at an exciting point in your mathematical journey as the mathematics you are studying has interesting applications in the real world.

The physical sciences, social sciences, and the business world are full of situations that can be modeled with linear equations relating two variables. Data is collected and graphed. If the data points appear to form a straight line, an equation of that line can be used to predict the value of one variable based on the value of the other variable.

To create a mathematical model of a linear relation between two variables, we must be able to find the equation of the line. In this section we will look at several ways to write the equation of a line. The specific method we use will be determined by what information we are given.

Find an Equation of the Line Given the Slope and y-Intercept

We can easily determine the slope and intercept of a line if the equation was written in slope–intercept form, $y = mx + b$. Now, we will do the reverse—we will start with the slope and *y*-intercept and use them to find the equation of the line.

EXAMPLE 1

Find an equation of a line with slope -7 and *y*-intercept $(0, -1)$.

Solution

Since we are given the slope and *y*-intercept of the line, we can substitute the needed values into the slope–intercept form, $y = mx + b$.

Name the slope.	$m = -7$
Name the *y*-intercept.	*y*-intercept $(0, -1)$
Substitute the values into $y = mx + b$.	$y = mx + b$
	$y = -7x + (-1)$
	$y = -7x + -1$

TRY IT 1.1

Find an equation of a line with slope $\frac{2}{5}$ and *y*-intercept $(0, 4)$.

Show answer

$y = \frac{2}{5}x + 4$

TRY IT 1.2

Find an equation of a line with slope -1 and *y*-intercept $(0, -3)$.

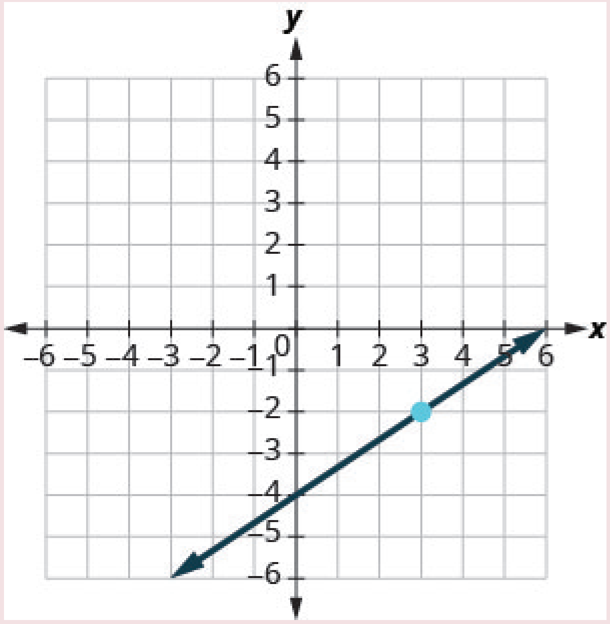
Show answer

$y = -x - 3$

Sometimes, the slope and intercept need to be determined from the graph.

EXAMPLE 2

Find the equation of the line shown.



Solution

We need to find the slope and *y*-intercept of the line from the graph so we can substitute the needed values into the slope–intercept form, $y = mx + b$.

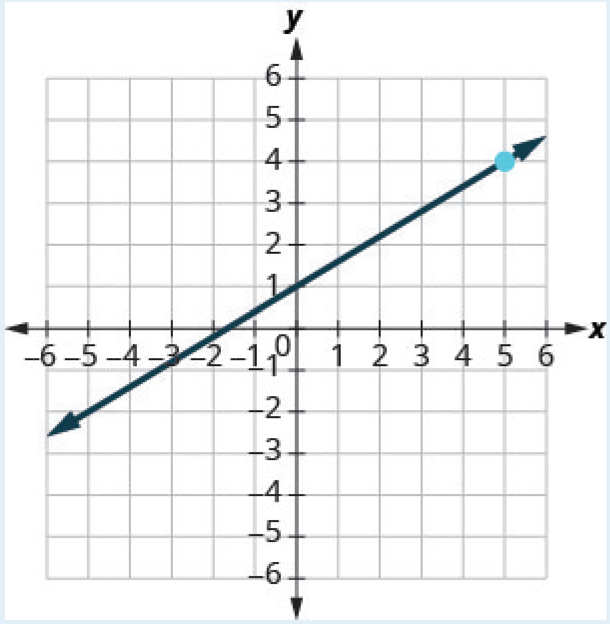
To find the slope, we choose two points on the graph.

The *y*-intercept is $(0, -4)$ and the graph passes through $(3, -2)$.

Find the slope by counting the rise and run.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{3}$
Find the *y*-intercept.	*y*-intercept $(0, -4)$
Substitute the values into $y = mx + b$.	$y = mx + b$
	$y = \frac{2}{3}x - 4$

TRY IT 2.1

Find the equation of the line shown in the graph.

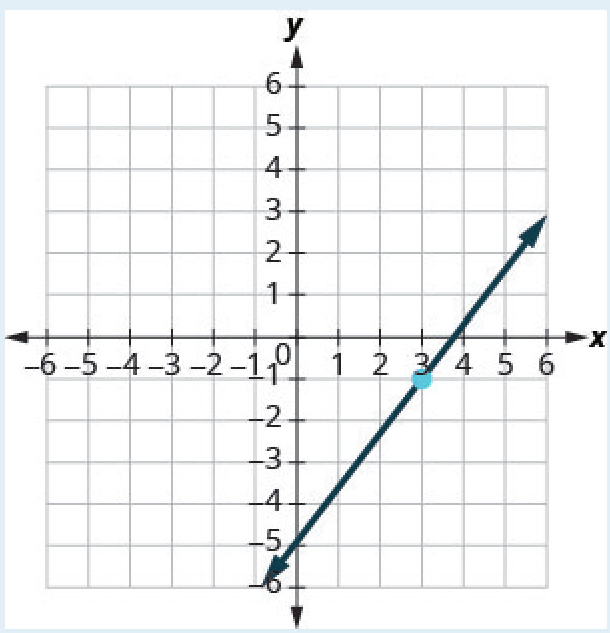


Show answer

$$y = \frac{3}{5}x + 1$$

TRY IT 2.2

Find the equation of the line shown in the graph.



Show answer

$$y = \frac{4}{3}x - 5$$

Find an Equation of the Line Given the Slope and a Point

Finding an equation of a line using the slope–intercept form of the equation works well when you are given the slope and *y*-intercept or when you read them off a graph. But what happens when you have another point instead of the *y*-intercept?

We are going to use the slope formula to derive another form of an equation of the line. Suppose we have a line that has slope m and that contains some specific point (x_1, y_1) and some other point, which we will just call (x, y). We can write the slope of this line and then change it to a different form.

	$m = \frac{y - y_1}{x - x_1}$
Multiply both sides of the equation by $x - x_1$.	$m(x - x_1) = \left(\frac{y - y_1}{x - x_1}\right)(x - x_1)$
Simplify.	$m(x - x_1) = y - y_1$
Rewrite the equation with the y terms on the left.	$y - y_1 = m(x - x_1)$

This format is called the point–slope form of an equation of a line.

Point–slope form of an equation of a line

The point–slope form of an equation of a line with slope m and containing the point (x_1, y_1) is

$y - y_1 = m(x - x_1)$

We can use the point–slope form of an equation to find an equation of a line when we are given the slope and one point. Then we will rewrite the equation in slope–intercept form. Most applications of linear equations use the the slope–intercept form.

EXAMPLE 3

Find an Equation of a Line Given the Slope and a Point

Find an equation of a line with slope $m = \frac{2}{5}$ that contains the point $(10, 3)$. Write the equation in slope–intercept form.

Solution

Step 1. Identify the slope.	The slope is given.	$m = \frac{2}{5}$

Step 2. Identify the point.	The point is given.	$\begin{pmatrix} x_1 & y_1 \\ 10, & 3 \end{pmatrix}$
Step 3. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.		$y - y_1 = m(x - x_1)$ $y - 3 = \frac{2}{5}(x - 10)$
	Simplify.	$y - 3 = \frac{2}{5}x - 4$
Step 4. Write the equation in slope–intercept form.		$y = \frac{2}{5}x - 1$

TRY IT 3.1

Find an equation of a line with slope $m = \frac{5}{6}$ and containing the point $(6, 3)$.

Show answer

$y = \frac{5}{6}x - 2$

TRY IT 3.2

Find an equation of a line with slope $m = \frac{2}{3}$ and containing thepoint $(9, 2)$.

Show answer

$y = \frac{2}{3}x - 4$

HOW TO: Find an equation of a line given the slope and a point

1. Identify the slope.
2. Identify the point.

3. Substitute the values into the point-slope form, $y - y_1 = m\left(x - x_1\right)$.
4. Write the equation in slope–intercept form.

EXAMPLE 4

Find an equation of a line with slope $m = -\frac{1}{3}$ that contains the point $(6, -4)$. Write the equation in slope–intercept form.

Solution

Since we are given a point and the slope of the line, we can substitute the needed values into the point–slope form, $y - y_1 = m\left(x - x_1\right)$.

Identify the slope.	$m = -\frac{1}{3}$
Identify the point.	$\left(\overset{x_1}{6}, \overset{y_1}{-4}\right)$
Substitute the values into $y - y_1 = m\left(x - x_1\right)$.	$y - y_1 = m(x - x_1)$
	$y - (-4) = -\frac{1}{3}(x - 6)$
Simplify.	$y + 4 = -\frac{1}{3}x + 2$
Write in slope–intercept form.	$y = -\frac{1}{3}x - 2$

TRY IT 4.1

Find an equation of a line with slope $m = -\frac{2}{5}$ and containing the point $(10, -5)$.

Show answer

$y = -\frac{2}{5}x - 1$

TRY IT 4.2

Find an equation of a line with slope $m = -\frac{3}{4}$, and containing the point $(4, -7)$.

Show answer

$y = -\frac{3}{4}x - 4$

EXAMPLE 5

Find an equation of a horizontal line that contains the point $(-1, 2)$. Write the equation in slope–intercept form.

Solution

Every horizontal line has slope 0. We can substitute the slope and points into the point–slope form, $y - y_1 = m\left(x - x_1\right)$.

Identify the slope.	$m = 0$
Identify the point.	$\begin{pmatrix} x_1 & y_1 \\ -1, & 2 \end{pmatrix}$
Substitute the values into $y - y_1 = m\left(x - x_1\right)$.	$y - y_1 = m(x - x_1)$
	$y - 2 = 0(x - (-1))$
Simplify.	$y - 2 = 0(x + 1)$
	$y - 2 = 0$
	$y = 2$
Write in slope–intercept form.	It is in *y*-form, but could be written $y = 0x + 2$.

Did we end up with the form of a horizontal line, $y = a$?

TRY IT 5.1

Find an equation of a horizontal line containing the point $(-3, 8)$.

Show answer

$y = 8$

TRY IT 5.2

Find an equation of a horizontal line containing the point $(-1, 4)$.

Show answer
$y = 4$

Find an Equation of the Line Given Two Points

When real-world data is collected, a linear model can be created from two data points. In the next example we'll see how to find an equation of a line when just two points are given.

We have two options so far for finding an equation of a line: slope–intercept or point–slope. Since we will know two points, it will make more sense to use the point–slope form.

But then we need the slope. Can we find the slope with just two points? Yes. Then, once we have the slope, we can use it and one of the given points to find the equation.

EXAMPLE 6

Find an Equation of a Line Given Two Points

Find an equation of a line that contains the points $(5, 4)$ and $(3, 6)$. Write the equation in slope–intercept form.

Solution

Step 1. Find the slope using the given points.	To use the point-slope form, we first find the slope.	$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6 - 4}{3 - 5}$ $m = \frac{2}{-2}$ $m = -1$
Step 2. Choose one point.	Choose either point.	$\begin{pmatrix} x_1 & y_1 \\ 5, & 4 \end{pmatrix}$
Step 3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 4 = -1(x - 5)$ $y - 4 = -1x + 5$

Step 4. Write the equation in slope–intercept form.		$y = -1x + 9$

Use the point $(3, 6)$ and see that you get the same equation.

TRY IT 6.1

Find an equation of a line containing the points $(3, 1)$ and $(5, 6)$.

Show answer

$y = \frac{5}{2}x - \frac{13}{2}$

TRY IT 6.2

Find an equation of a line containing the points $(1, 4)$ and $(6, 2)$.

Show answer

$y = -\frac{2}{5}x + \frac{22}{5}$

HOW TO: Find an equation of a line given two points

1. Find the slope using the given points.
2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m\left(x - x_1\right)$.
4. Write the equation in slope–intercept form.

EXAMPLE 7

Find an equation of a line that contains the points $(-3, -1)$ and $(2, -2)$. Write the equation in slope–intercept form.

Solution

Since we have two points, we will find an equation of the line using the point–slope form. The first step will be to find the slope.

Find the slope of the line through (−3, −1) and (2, −2).	$m=\frac{y_2-y_1}{x_2-x_1}$
	$m=\frac{-2-(-1)}{2-(-3)}$
	$m=\frac{-1}{5}$
	$m=-\frac{1}{5}$
Choose either point.	$\begin{pmatrix} x_1 & y_1 \\ 2, & -2 \end{pmatrix}$
Substitute the values into $y-y_1=m\left(x-x_1\right)$.	$y-y_1=m(x-x_1)$
	$y-(-2)=-\frac{1}{5}(x-2)$
	$y+2=-\frac{1}{5}x+\frac{2}{5}$
Write in slope–intercept form.	$y=-\frac{1}{5}x-\frac{8}{5}$

TRY IT 7.1

Find an equation of a line containing the points $(-2,-4)$ and $(1,-3)$.

Show answer

$y=\frac{1}{3}x-\frac{10}{3}$

TRY IT 7.2

Find an equation of a line containing the points $(-4,-3)$ and $(1,-5)$.

Show answer

$y=-\frac{2}{5}x-\frac{23}{5}$

EXAMPLE 8

Find an equation of a line that contains the points $(-2, 4)$ and $(-2, -3)$. Write the equation in slope–intercept form.

Solution
Again, the first step will be to find the slope.

Find the slope of the line through (-2,4) and (-2,-3).	$m = \frac{y_2 - y_1}{x_2 - x_1}$
	$m = \frac{-3 - 4}{-2 - (-2)}$
	$m = \frac{-7}{0}$
	The slope is undefined.

This tells us it is a vertical line. Both of our points have an *x*-coordinate of -2. So our equation of the line is $x = -2$. Since there is no y, we cannot write it in slope–intercept form.

You may want to sketch a graph using the two given points. Does the graph agree with our conclusion that this is a vertical line?

TRY IT 8.1

Find an equation of a line containing the points $(5, 1)$ and $(5, -4)$.

Show answer
$x = 5$

TRY IT 8.2

Find an equation of a line containing the points $(-4, 4)$ and $(-4, 3)$.

Show answer
$x = -4$

We have seen that we can use either the slope–intercept form or the point–slope form to find an equation of a line. Which form we use will depend on the information we are given. This is summarized in the following table.

To Write an Equation of a Line

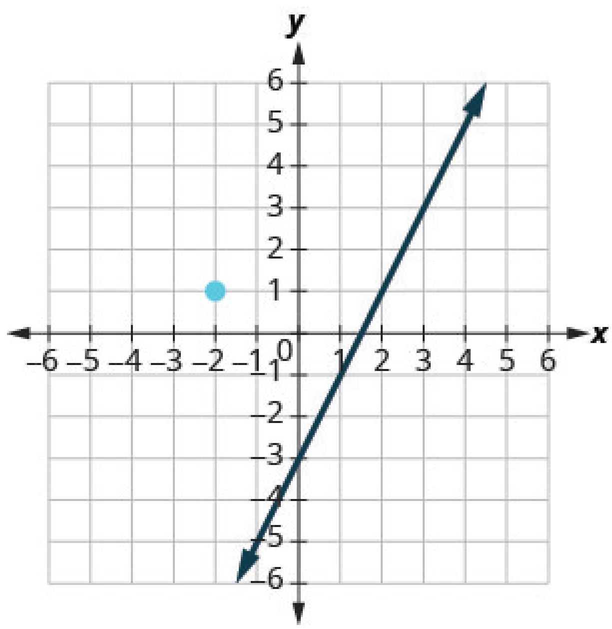
If given:	**Use:**	**Form:**
Slope and *y*-intercept	slope–intercept	$y = mx + b$
Slope and a point	point–slope	$y - y_1 = m(x - x_1)$
Two points	point–slope	$y - y_1 = m(x - x_1)$

Find an Equation of a Line Parallel to a Given Line

Suppose we need to find an equation of a line that passes through a specific point and is parallel to a given line. We can use the fact that parallel lines have the same slope. So we will have a point and the slope—just what we need to use the point–slope equation.

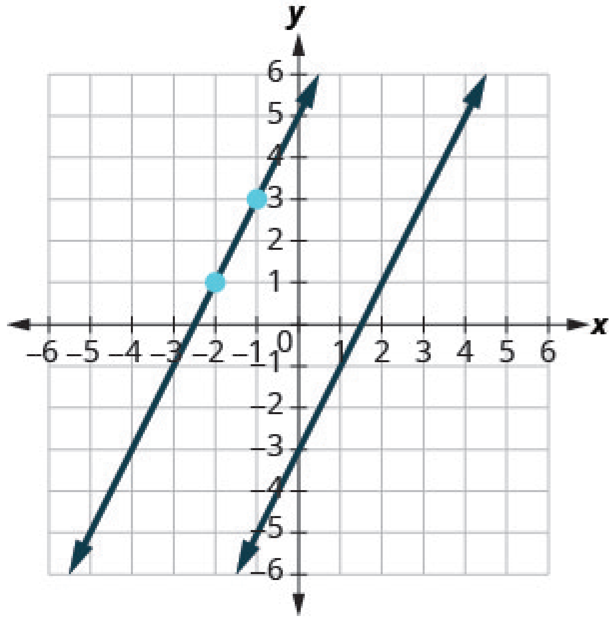
First let's look at this graphically.

The graph shows the graph of $y = 2x - 3$. We want to graph a line parallel to this line and passing through the point $(-2, 1)$.



We know that parallel lines have the same slope. So the second line will have the same slope as $y = 2x - 3$. That slope is$m_{\|} = 2$. We'll use the notation $m_{\|}$ to represent the slope of a line parallel to a line with slope m. (Notice that the subscript $_{\|}$ looks like two parallel lines.)

The second line will pass through $(-2, 1)$ and have $m = 2$. To graph the line, we start at$(-2, 1)$ and count out the rise and run. With $m = 2$ (or $m = \frac{2}{1}$), we count out the rise 2 and the run 1. We draw the line.



Do the lines appear parallel? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically.

We can use either the slope–intercept form or the point–slope form to find an equation of a line. Here we know one point and can find the slope. So we will use the point–slope form.

EXAMPLE 9

How to Find an Equation of a Line Parallel to a Given Line

Find an equation of a line parallel to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the parallel line.	Parallel lines have the same slope.	$m_{\parallel} = 2$
Step 3. Identify the point.	The given point is, (–2, 1).	$\begin{pmatrix} x_1, & y_1 \\ -2, & 1 \end{pmatrix}$

Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = 2(x - (-2))$ $y - 1 = 2(x + 2)$ $y - 1 = 2x + 4$
Step 5. Write the equation in slope–intercept form.		$y = 2x + 5$

Does this equation make sense? What is the *y*-intercept of the line? What is the slope?

TRY IT 9.1

Find an equation of a line parallel to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope–intercept form.

Show answer

$y = 3x - 10$

TRY IT 9.2

Find an equation of a line parallel to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Show answer

$y = \frac{1}{2}x + 1$

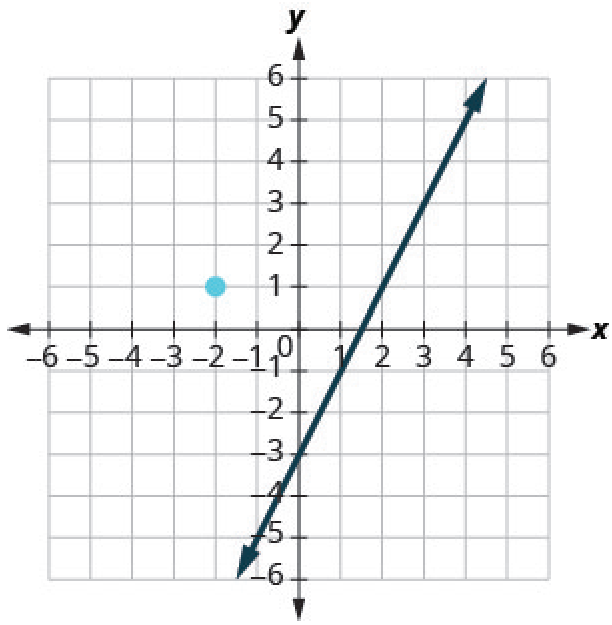
HOW TO: Find an equation of a line parallel to a given line

1. Find the slope of the given line.
2. Find the slope of the parallel line.
3. Identify the point.
4. Substitute the values into the point–slope form, $y - y_1 = m\left(x - x_1\right)$.
5. Write the equation in slope–intercept form.

Find an Equation of a Line Perpendicular to a Given Line

Now, let's consider perpendicular lines. Suppose we need to find a line passing through a specific point and which is perpendicular to a given line. We can use the fact that perpendicular lines have slopes that are negative reciprocals. We will again use the point–slope equation, like we did with parallel lines.

The graph shows the graph of $y = 2x - 3$. Now, we want to graph a line perpendicular to this line and passing through $(-2, 1)$.

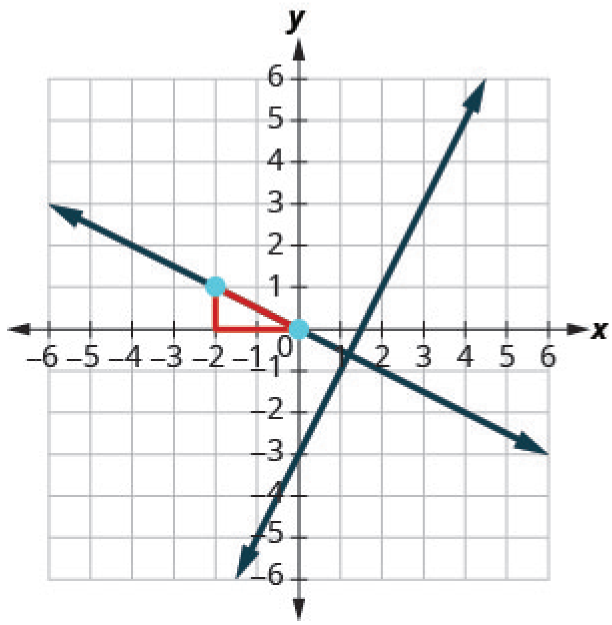


We know that perpendicular lines have slopes that are negative reciprocals. We'll use the notation $m_\perp$ to represent the slope of a line perpendicular to a line with slope m. (Notice that the subscript $\perp$ looks like the right angles made by two perpendicular lines.)

$$\begin{array}{ll} y = 2x - 3 & \text{perpendicular line} \\ m = 2 & m_\perp = -\frac{1}{2} \end{array}$$

We now know the perpendicular line will pass through $(-2, 1)$ with $m_\perp = -\frac{1}{2}$.

To graph the line, we will start at $(-2, 1)$ and count out the rise -1 and the run 2. Then we draw the line.



Do the lines appear perpendicular? Does the second line pass through $(-2, 1)$?

Now, let's see how to do this algebraically. We can use either the slope–intercept form or the point–slope form to find an equation of a line. In this example we know one point, and can find the slope, so we will use the point–slope form.

EXAMPLE 10

How to Find an Equation of a Line Perpendicular to a Given Line

Find an equation of a line perpendicular to $y = 2x - 3$ that contains the point $(-2, 1)$. Write the equation in slope–intercept form.

Solution

Step 1. Find the slope of the given line.	The line is in slope–intercept form, $y = 2x - 3$.	$m = 2$
Step 2. Find the slope of the perpendicular line.	The slopes of perpendicular lines are negative reciprocals.	$m_{\perp} = -\frac{1}{2}$
Step 3. Identify the point.	The given point is, (–2, 1)	$\begin{pmatrix} x_1, & y_1 \\ -2, & 1 \end{pmatrix}$

Step 4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.	Simplify.	$y - y_1 = m(x - x_1)$ $y - 1 = -\frac{1}{2}(x - (-2))$ $y - 1 = -\frac{1}{2}(x + 2)$ $y - 1 = -\frac{1}{2}x - 1$
Step 5. Write the equation in slope–intercept form.		$y = -\frac{1}{2}x$

TRY IT 10.1

Find an equation of a line perpendicular to the line $y = 3x + 1$ that contains the point $(4, 2)$. Write the equation in slope–intercept form.

Show answer

$y = -\frac{1}{3}x + \frac{10}{3}$

TRY IT 10.2

Find an equation of a line perpendicular to the line $y = \frac{1}{2}x - 3$ that contains the point $(6, 4)$.

Show answer

$y = -2x + 16$

HOW TO: Find an equation of a line perpendicular to a given line

1. Find the slope of the given line.
2. Find the slope of the perpendicular line.
3. Identify the point.

4. Substitute the values into the point–slope form, $y - y_1 = m(x - x_1)$.
5. Write the equation in slope–intercept form.

EXAMPLE 11

Find an equation of a line perpendicular to $x = 5$ that contains the point $(3, -2)$. Write the equation in slope–intercept form.

Solution

Again, since we know one point, the point–slope option seems more promising than the slope–intercept option. We need the slope to use this form, and we know the new line will be perpendicular to $x = 5$. This line is vertical, so its perpendicular will be horizontal. This tells us the $m_\perp = 0$.

Identify the point.	$(3, -2)$
Identify the slope of the perpendicular line.	$m_\perp = 0$
Substitute the values into $y - y_1 = m(x - x_1)$.	$y - y_1 = m(x - x_1)$ $y - (-2) = 0(x - 3)$ $y + 2 = 0$
Simplify.	$y = -2$

Sketch the graph of both lines. Do they appear to be perpendicular?

TRY IT 11.1

Find an equation of a line that is perpendicular to the line $x = 4$ that contains the point $(4, -5)$. Write the equation in slope–intercept form.

Show answer

$y = -5$

TRY IT 11.2

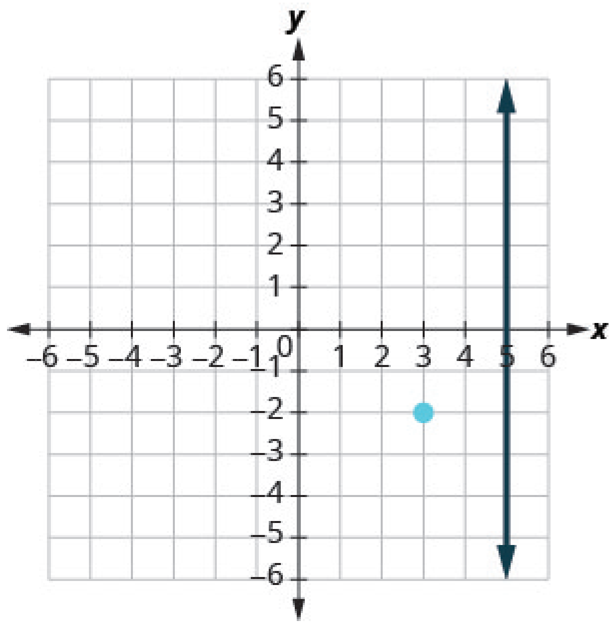
Find an equation of a line that is perpendicular to the line $x = 2$ that contains the point $(2, -1)$. Write the equation in slope–intercept form.

Show answer

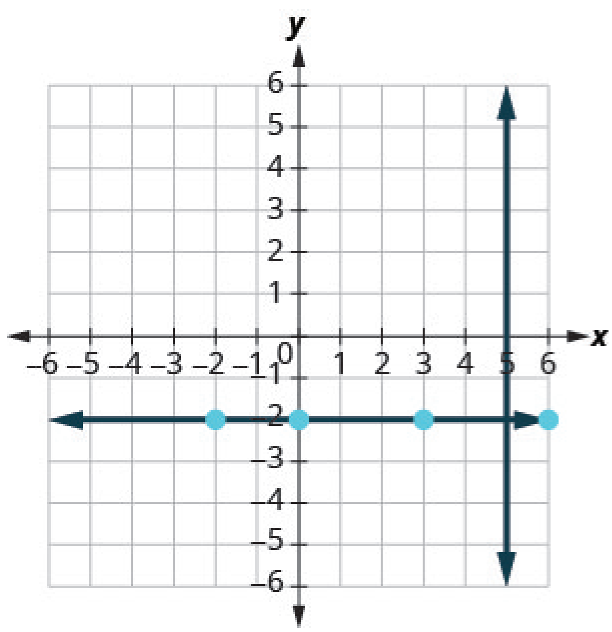
$y = -1$

In (Example 11), we used the point–slope form to find the equation. We could have looked at this in a different way.

We want to find a line that is perpendicular to $x = 5$ that contains the point $(3, -2)$. The graph shows us the line$x = 5$ and the point $(3, -2)$.



We know every line perpendicular to a vertical line is horizontal, so we will sketch the horizontal line through $(3, -2)$.



Do the lines appear perpendicular?

If we look at a few points on this horizontal line, we notice they all have *y*-coordinates of -2. So, the equation of the line perpendicular to the vertical line $x = 5$ is $y = -2$.

EXAMPLE 12

Find an equation of a line that is perpendicular to $y = -4$ that contains the point $(-4, 2)$.

Write the equation in slope–intercept form.

Solution

The line $y = -4$ is a horizontal line. Any line perpendicular to it must be vertical, in the form $x = a$. Since the perpendicular line is vertical and passes through $(-4, 2)$, every point on it has an *x*-coordinate of -4. The equation of the perpendicular line is $x = -4$. You may want to sketch the lines. Do they appear perpendicular?

TRY IT 12.1

Find an equation of a line that is perpendicular to the line $y = 1$ that contains the point $(-5, 1)$. Write the equation in slope–intercept form.

Show answer

$x = -5$

TRY IT 12.1

Find an equation of a line that is perpendicular to the line $y = -5$ that contains the point $(-4, -5)$.

Show answer

$x = -4$

Access this online resource for additional instruction and practice with finding the equation of a line.

- Use the Point-Slope Form of an Equation of a Line

Key Concepts

- **To Find an Equation of a Line Given the Slope and a Point**
  1. Identify the slope.
  2. Identify the point.
  3. Substitute the values into the point-slope form, $y - y_1 = m(x - x_1)$.
  4. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Given Two Points**
  1. Find the slope using the given points.

2. Choose one point.
3. Substitute the values into the point-slope form, $y - y_1 = m\,(x - x_1)$.
4. Write the equation in slope-intercept form.

- **To Write and Equation of a Line**
  - If given slope and *y*-intercept, use slope–intercept form $y = mx + b$.
  - If given slope and a point, use point–slope form $y - y_1 = m\,(x - x_1)$.
  - If given two points, use point–slope form $y - y_1 = m\,(x - x_1)$.
- **To Find an Equation of a Line Parallel to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the parallel line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y - y_1 = m\,(x - x_1)$.
  5. Write the equation in slope-intercept form.
- **To Find an Equation of a Line Perpendicular to a Given Line**
  1. Find the slope of the given line.
  2. Find the slope of the perpendicular line.
  3. Identify the point.
  4. Substitute the values into the point-slope form, $y - y_1 = m\,(x - x_1)$.
  5. Write the equation in slope-intercept form.

Glossary

point–slope form
The point–slope form of an equation of a line with slope m and containing the point (x_1, y_1) is $y - y_1 = m\,(x - x_1)$.

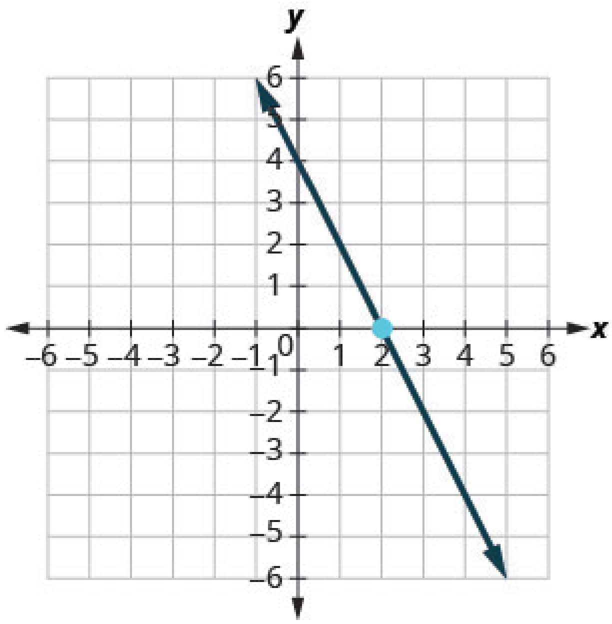
Practice Makes Perfect

Find an Equation of the Line Given the Slope and *y*-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

1. slope 4 and *y*-intercept $(0, 1)$	2. slope 3 and *y*-intercept $(0, 5)$
3. slope 8 and *y*-intercept $(0, -6)$	4. slope 6 and *y*-intercept $(0, -4)$
5. slope -1 and *y*-intercept $(0, 7)$	6. slope -1 and *y*-intercept $(0, 3)$
7. slope -3 and *y*-intercept $(0, -1)$	8. slope -2 and *y*-intercept $(0, -3)$
9. slope $\frac{1}{5}$ and *y*-intercept $(0, -5)$	10. slope $\frac{3}{5}$ and *y*-intercept $(0, -1)$
11. slope $-\frac{2}{3}$ and *y*-intercept $(0, -3)$	12. slope $-\frac{3}{4}$ and *y*-intercept $(0, -2)$
13. slope 0 and *y*-intercept $(0, 2)$	14. slope 0 and *y*-intercept $(0, -1)$
15. slope -4 and *y*-intercept $(0, 0)$	16. slope -3 and *y*-intercept $(0, 0)$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.



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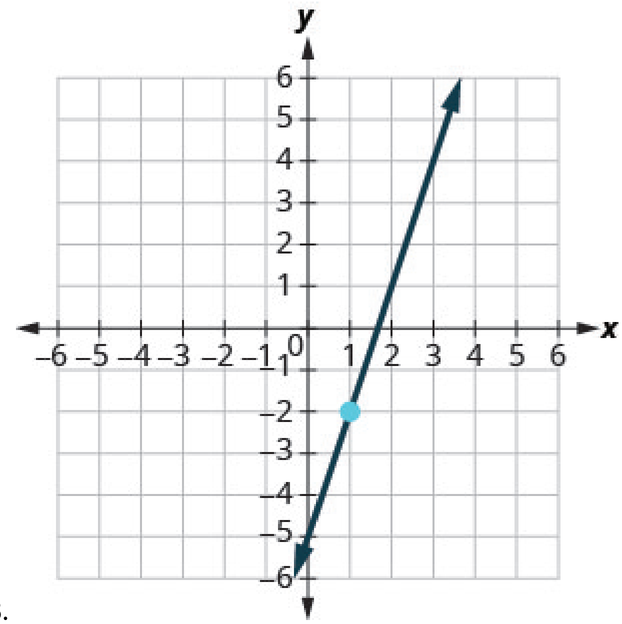

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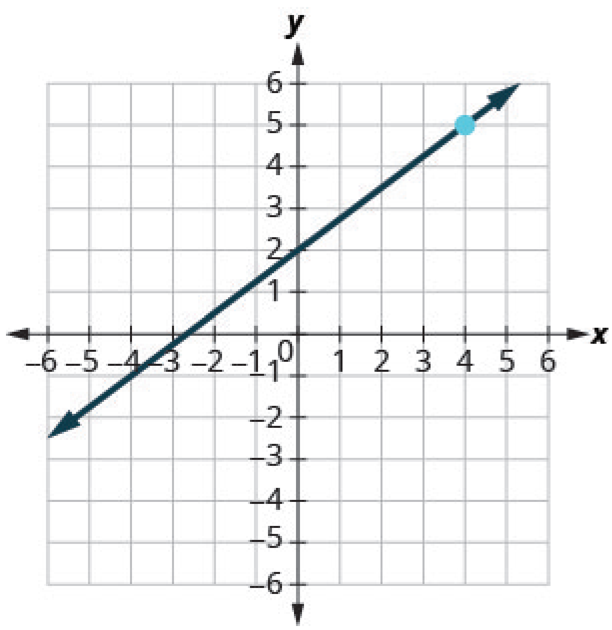

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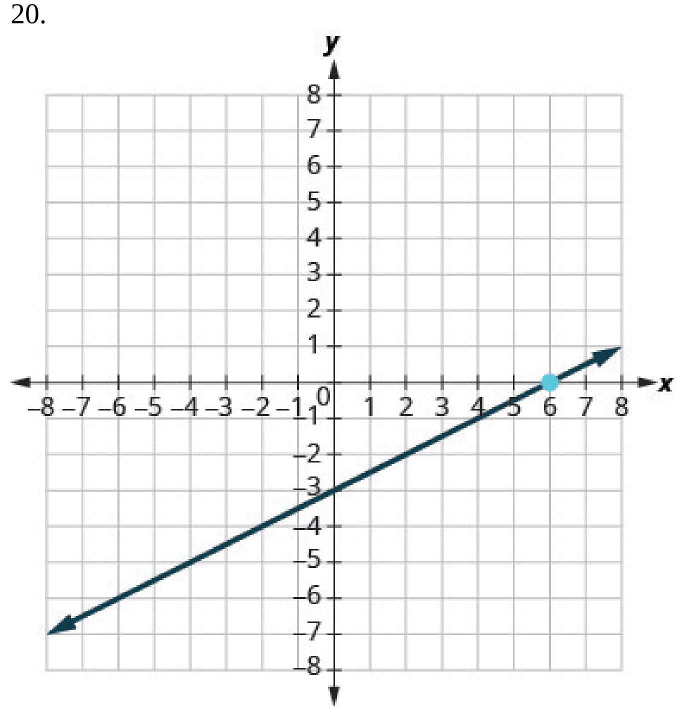

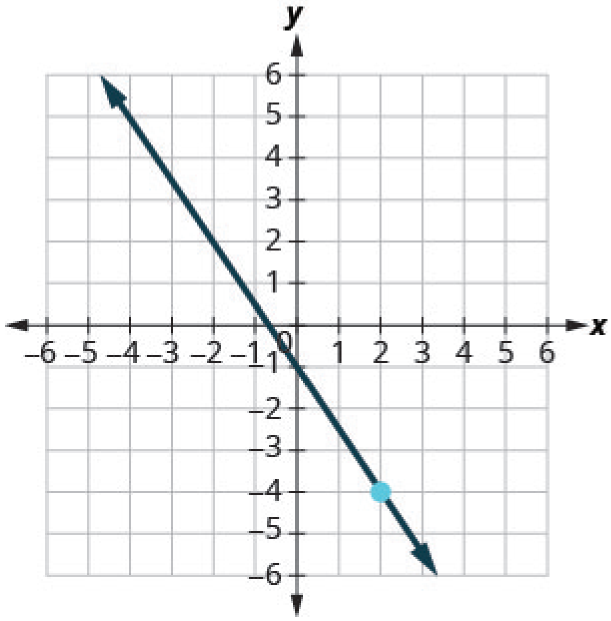

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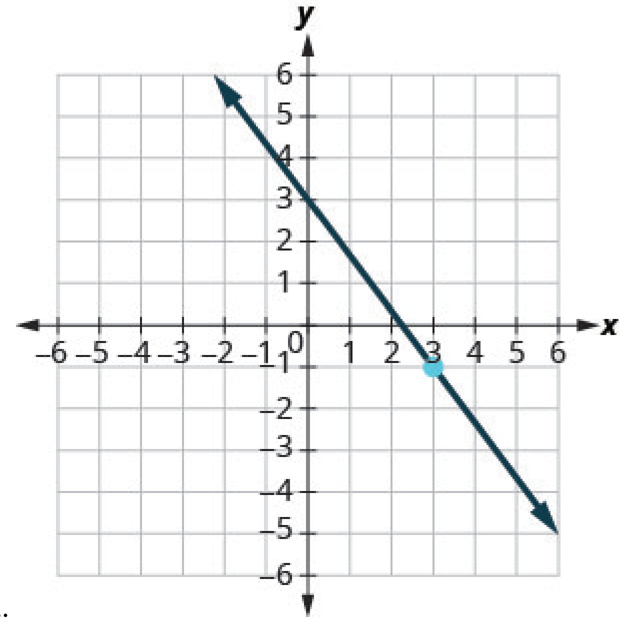

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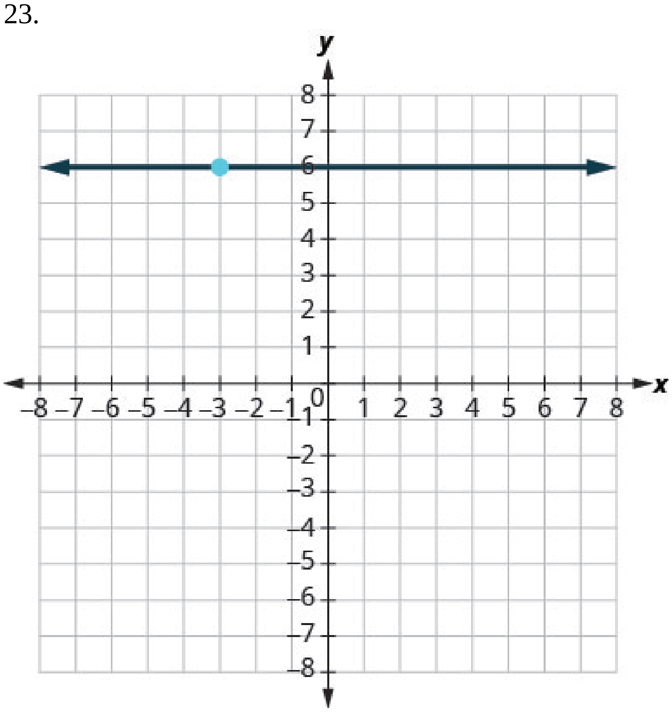

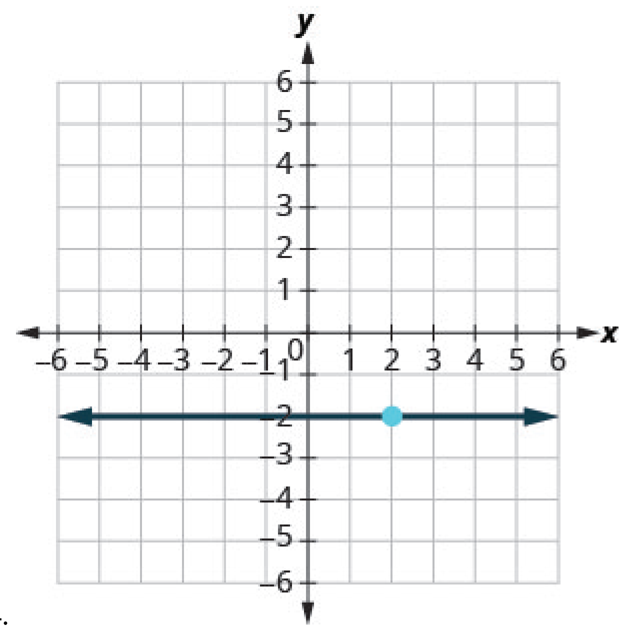

24.

Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

25. $m = \frac{3}{8}$, point $(8, 2)$	26. $m = \frac{5}{8}$, point $(8, 3)$
27. $m = \frac{5}{6}$, point $(6, 7)$	28. $m = \frac{1}{6}$, point $(6, 1)$
29. $m = -\frac{3}{5}$, point $(10, -5)$	30. $m = -\frac{3}{4}$, point $(8, -5)$
31. $m = -\frac{1}{3}$, point $(-9, -8)$	32. $m = -\frac{1}{4}$, point $(-12, -6)$
33. Horizontal line containing $(-1, 4)$	34. Horizontal line containing $(-2, 5)$
35. Horizontal line containing $(-1, -7)$	36. Horizontal line containing $(-2, -3)$
37. $m = -\frac{5}{2}$, point $(-8, -2)$	38. $m = -\frac{3}{2}$, point $(-4, -3)$
39. $m = -4$, point $(-2, -3)$	40. $m = -7$, point $(-1, -3)$
41. Horizontal line containing $(4, -8)$	42. Horizontal line containing $(2, -3)$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

43. $(3, 1)$ and $(2, 5)$	44. $(2, 6)$ and $(5, 3)$
45. $(2, 7)$ and $(3, 8)$	46. $(4, 3)$ and $(8, 1)$
47. $(-5, -3)$ and $(4, -6)$	48. $(-3, -4)$ and $(5 - 2)$
49. $(-2, 8)$ and $(-4, -6)$	50. $(-1, 3)$ and $(-6, -7)$
51. $(3, -2)$ and $(-4, 4)$	52. $(6, -4)$ and $(-2, 5)$
53. $(0, -2)$ and $(-5, -3)$	54. $(0, 4)$ and $(2, -3)$
55. $(4, 2)$ and $(4, -3)$	56. $(7, 2)$ and $(7, -2)$
57. $(-2, 1)$ and $(-2, -4)$	58. $(-7, -1)$ and $(-7, -4)$
59. $(6, 2)$ and $(-3, 2)$	60. $(6, 1)$ and $(0, 1)$
61. $(-6, -3)$ and $(-1, -3)$	62. $(3, -4)$ and $(5, -4)$
63. $(0, 0)$ and $(1, 4)$	64. $(4, 3)$ and $(8, 0)$
65. $(-3, 0)$ and $(-7, -2)$	66. $(-2, -3)$ and $(-5, -6)$
67. $(3, 5)$ and $(-7, 5)$	68. $(8, -1)$ and $(8, -5)$

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

69. line $y = 3x + 4$, point $(2, 5)$	70. line $y = 4x + 2$, point $(1, 2)$
71. line $y = -3x - 1$, point $(2, -3)$	72. line $y = -2x - 3$, point $(-1, 3)$
73. line $2x - y = 6$, point $(3, 0)$	74. line $3x - y = 4$, point $(3, 1)$
75. line $2x + 3y = 6$, point $(0, 5)$	76. line $4x + 3y = 6$, point $(0, -3)$
77. line $x = -4$, point $(-3, -5)$	78. line $x = -3$, point $(-2, -1)$
79. line $x - 6 = 0$, point $(4, -3)$	80. line $x - 2 = 0$, point $(1, -2)$
81. line $y = 1$, point $(3, -4)$	82. line $y = 5$, point $(2, -2)$
83. line $y + 7 = 0$, point $(1, -1)$	84. line $y + 2 = 0$, point $(3, -3)$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

85. line $y = -x + 5$, point $(3, 3)$	86. line $y = -2x + 3$, point $(2, 2)$
87. line $y = \frac{2}{3}x - 4$, point $(2, -4)$	88. line $y = \frac{3}{4}x - 2$, point $(-3, 4)$
89. line $4x - 3y = 5$, point $(-3, 2)$	90. line $2x - 3y = 8$, point $(4, -1)$
91. line $4x + 5y = -3$, point $(0, 0)$	92. line $2x + 5y = 6$, point $(0, 0)$
93. line $y - 6 = 0$, point $(-5, -3)$	94. line $y - 3 = 0$, point $(-2, -4)$
95. line *y*-axis, point $(2, 1)$	96. line *y*-axis, point $(3, 4)$

Mixed Practice

In the following exercises, find the equation of each line. Write the equation in slope–intercept form.

97. Containing the points $(2, 7)$ and $(3, 8)$	98. Containing the points $(4, 3)$ and $(8, 1)$
99. $m = \frac{5}{6}$, containing point $(6, 7)$	100. $m = \frac{1}{6}$, containing point $(6, 1)$
101. Parallel to the line $2x + 3y = 6$, containing point $(0, 5)$	102. Parallel to the line $4x + 3y = 6$, containing point $(0, -3)$
103. $m = -\frac{3}{5}$, containing point $(10, -5)$	104. $m = -\frac{3}{4}$, containing point $(8, -5)$
105. Perpendicular to the line *y*-axis, point $(-6, 2)$	106. Perpendicular to the line $y - 1 = 0$, point $(-2, 6)$
107. Containing the points $(-2, 0)$ and $(-3, -2)$	108. Containing the points $(4, 3)$ and $(8, 1)$
109. Parallel to the line $x = -4$, containing point $(-3, -5)$	110. Parallel to the line $x = -3$, containing point $(-2, -1)$
111. Containing the points $(-5, -3)$ and $(4, -6)$	112. Containing the points $(-3, -4)$ and $(2, -5)$
113. Perpendicular to the line $4x + 3y = 1$, containing point $(0, 0)$	114. Perpendicular to the line $x - 2y = 5$, containing point $(-2, 2)$

Everyday Math

115. **Fuel consumption.** The city mpg, x, and highway mpg, y, of two cars are given by the points $(29, 40)$ and$(19, 28)$. Find a linear equation that models the relationship between city mpg and highway mpg.	116. **Cholesterol.** The age, x, and LDL cholesterol level, y, of two men are given by the points $(18, 68)$ and $(27, 122)$. Find a linear equation that models the relationship between age and LDL cholesterol level.

Writing Exercises

117. Explain in your own words why the slopes of two perpendicular lines must have opposite signs.	118. Why are all horizontal lines parallel?

Answers

1. $y = 4x + 1$	3. $y = 8x - 6$
5. $y = -x + 7$	7. $y = -3x - 1$
9. $y = \frac{1}{5}x - 5$	11. $y = -\frac{2}{3}x - 3$
13. $y = 2$	15. $y = -4x$
17. $y = -2x + 4$	19. $y = \frac{3}{4}x + 2$
21. $y = -\dfrac{3}{2}x - 1$	23. $y = 6$
25. $y = \frac{3}{8}x - 1$	27. $y = \frac{5}{6}x + 2$
29. $y = -\frac{3}{5}x + 1$	31. $y = -\frac{1}{3}x - 11$
33. $y = 4$	35. $y = -7$
37. $y = -\frac{5}{2}x - 22$	39. $y = -4x - 11$
41. $y = -8$	43. $y = -4x + 13$
45. $y = x + 5$	47. $y = -\frac{1}{3}x - \frac{14}{3}$
49. $y = 7x + 22$	51. $y = -\frac{6}{7}x + \frac{4}{7}$
53. $y = \frac{1}{5}x - 2$	55. $x = 4$
57. $x = -2$	59. $y = 2$
61. $y = -3$	63. $y = 4x$
65. $y = \frac{1}{2}x + \frac{3}{2}$	67. $y = 5$
69. $y = 3x - 1$	71. $y = -3x + 3$
73. $y = 2x - 6$	75. $y = -\frac{2}{3}x + 5$
77. $x = -3$	79. $x = 4$
81. $y = -4$	83. $y = -1$
85. $y = x$	87. $y = -\frac{3}{2}x - 1$
89. $y = -\frac{3}{4}x - \frac{1}{4}$	91. $y = \frac{5}{4}x$
93. $x = -5$	95. $y = 1$
97. $y = x + 5$	99. $y = \frac{5}{6}x + 2$
101. $y = -\frac{2}{3}x + 5$	103. $y = -\frac{3}{5}x + 1$

105. $y=2$	107. $y=2x+4$
109. $x=-3$	111. $y=-\frac{1}{3}x-\frac{14}{3}$
113. $y=\frac{3}{4}x$	115. $y=1.2x+5.2$
117. Answers will vary.	

Attributions

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4.7 Chapter Review

Review Exercises

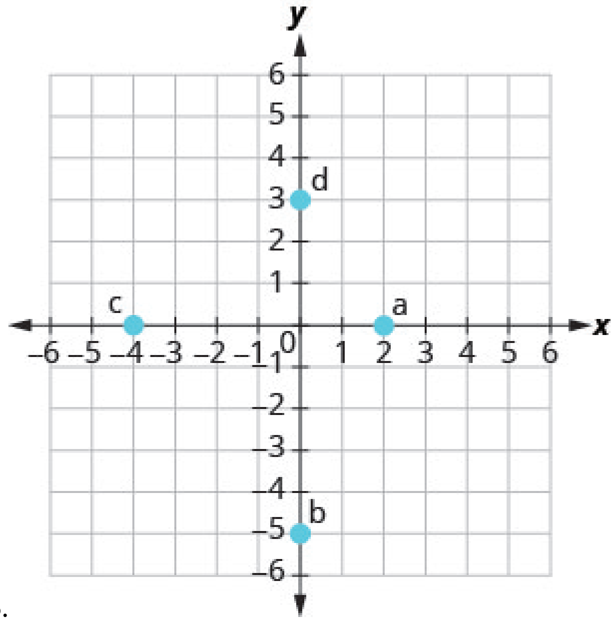
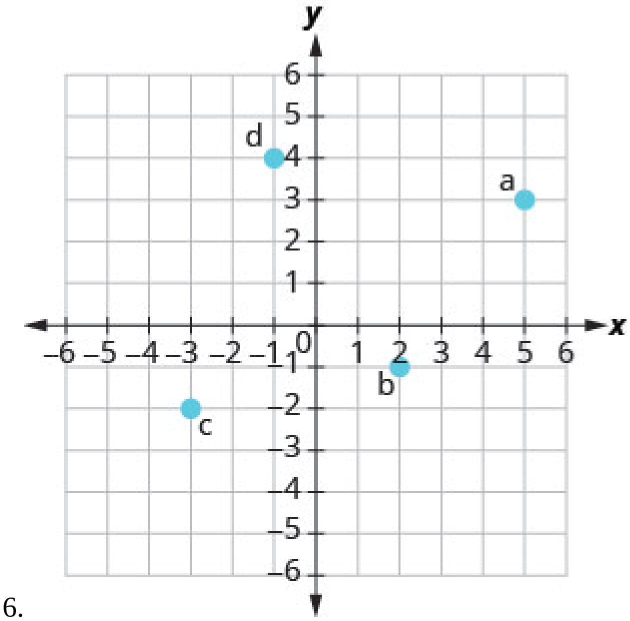
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

1. a) $(4, 3)$ b) $(-4, 3)$ c) $(-4, -3)$ d) $(4, -3)$	2. a) $(-1, -5)$ b) $(-3, 4)$ c) $(2, -3)$ d) $\left(1, \frac{5}{2}\right)$
3. a) $\left(2, \frac{3}{2}\right)$ b) $\left(3, \frac{4}{3}\right)$ c) $\left(\frac{1}{3}, -4\right)$ d) $\left(\frac{1}{2}, -5\right)$	4. a) $(-2, 0)$ b) $(0, -4)$ c) $(0, 5)$ d) $(3, 0)$

Identify Points on a Graph

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

 5.	 6.

Verify Solutions to an Equation in Two Variables

In the following exercises, which ordered pairs are solutions to the given equations?

7. $y = 6x - 2$ a) $(1, 4)$ b) $\left(\frac{1}{3}, 0\right)$ c) $(6, -2)$	8. $5x + y = 10$ a) $(5, 1)$ b) $(2, 0)$ c) $(4, -10)$

Complete a Table of Solutions to a Linear Equation in Two Variables

In the following exercises, complete the table to find solutions to each linear equation.

9. $y = -\frac{1}{2}x + 3$

x	y	(x, y)
0		
4		
-2		

10. $y = 4x - 1$

x	y	(x, y)
0		
1		
-2		

11. $3x + 2y = 6$

x	y	(x, y)
0		
	0	
-2		

12. $x + 2y = 5$

x	y	(x, y)
	0	
1		
-1		

Find Solutions to a Linear Equation in Two Variables

In the following exercises, find three solutions to each linear equation.

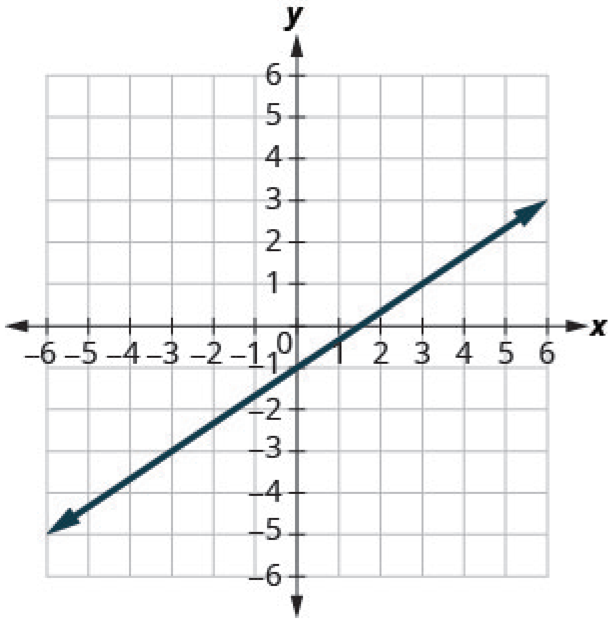
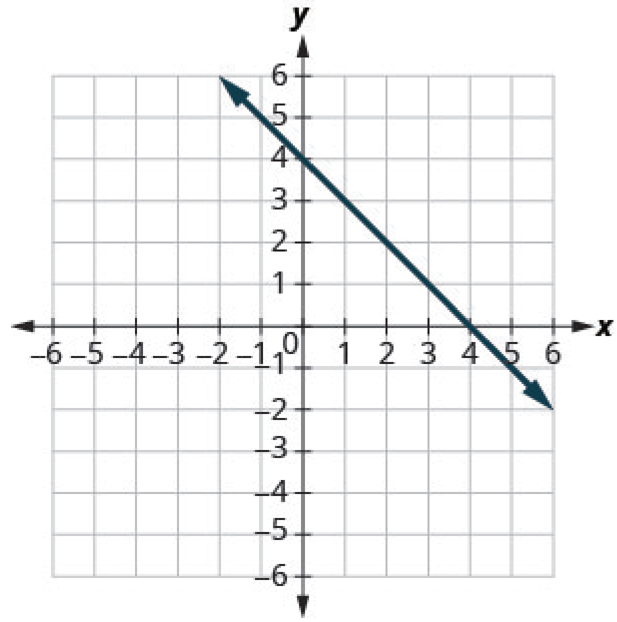
13. $x + y = -4$	14. $x + y = 3$
15. $y = -x - 1$	16. $y = 3x + 1$

Recognize the Relation Between the Solutions of an Equation and its Graph

In the following exercises, for each ordered pair, decide:

a) Is the ordered pair a solution to the equation?

b) Is the point on the line?

17. $y = \frac{2}{3}x - 1$ $(0, -1)$ (3, 1) $(-3, -3)$ (6, 4) 	18. $y =$ -$x + 4$ $(0, 4)(-1, 3)$ $(2, 2)(-2, 6)$ 

Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

19. $y = -3x$	20. $y = 4x - 3$
21. $x - y = 6$	22. $y = \frac{1}{2}x + 3$
23. $3x - 2y = 6$	24. $2x + y = 7$

Graph Vertical and Horizontal lines

In the following exercises, graph each equation.

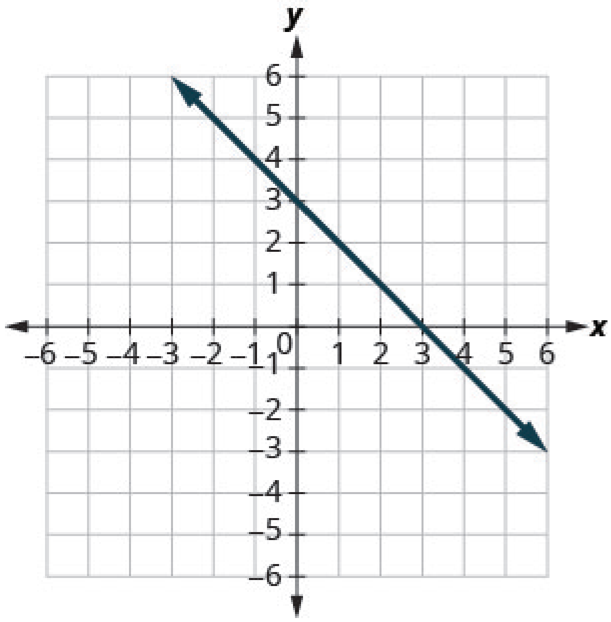
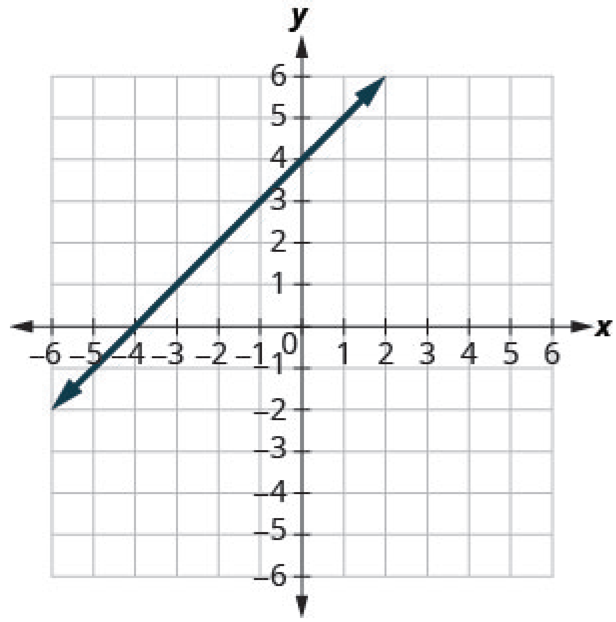
25. $x = 3$	26. $y = -2$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

27. $y = \frac{4}{3}x$ and $y = \frac{4}{3}$	28. $y = -2x$ and $y = -2$

Identify the *x*– and *y*-Intercepts on a Graph

In the following exercises, find the *x*– and *y*-intercepts.

29. 	30. 

Find the *x*– and *y*-Intercepts from an Equation of a Line

In the following exercises, find the intercepts of each equation.

31. $x - y = -1$	32. $x + y = 5$
33. $2x + 3y = 12$	34. $x + 2y = 6$
35. $y = 3x$	36. $y = \frac{3}{4}x - 12$

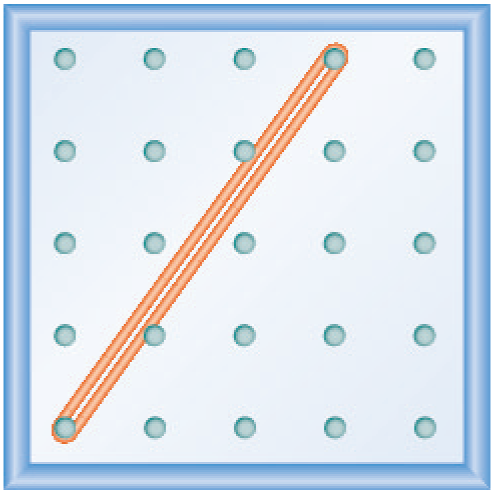
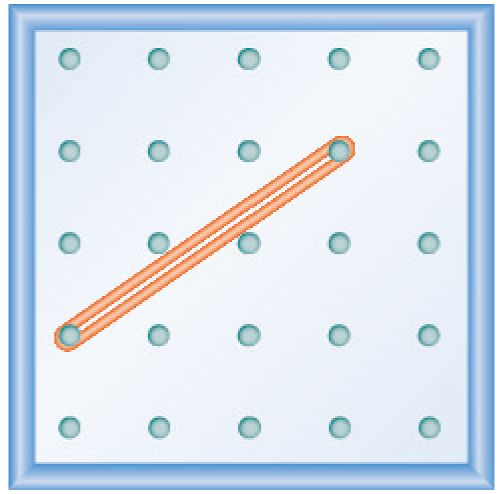
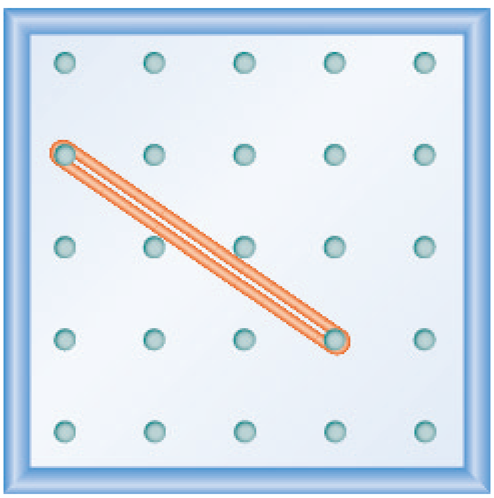
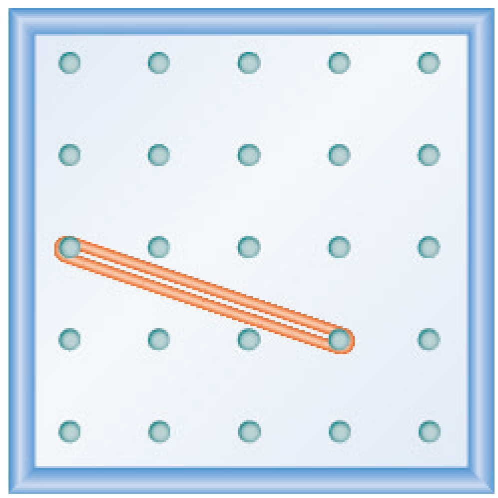
Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

37. $-x + 3y = 3$	38. $x + y = -2$
39. $x - y = 4$	40. $2x - y = 5$
41. $2x - 4y = 8$	42. $y = 2x$

Use Geoboards to Model Slope

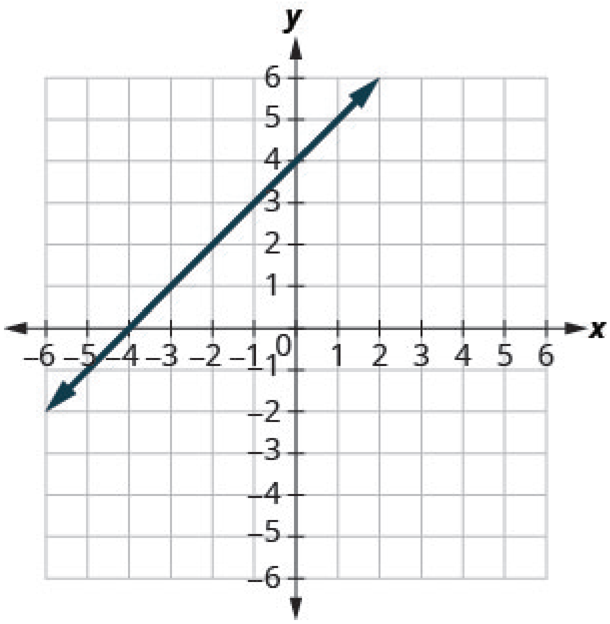
In the following exercises, find the slope modeled on each geoboard.

43. 	44. 
45. 	46. 

In the following exercises, model each slope. Draw a picture to show your results.

47. $\frac{3}{2}$	48. $\frac{1}{3}$
49. $-\frac{1}{2}$	50. $-\frac{2}{3}$

In the following exercises, find the slope of each line shown. Use $m = \frac{\text{rise}}{\text{run}}$ to find the slope of a line from its graph.



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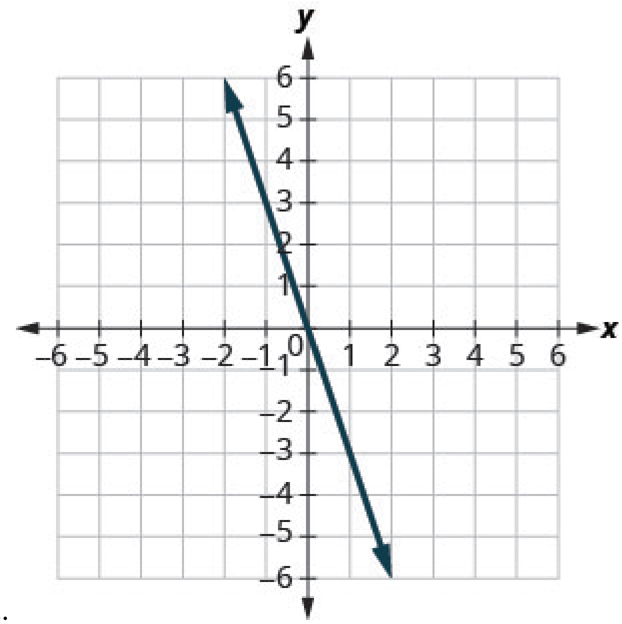

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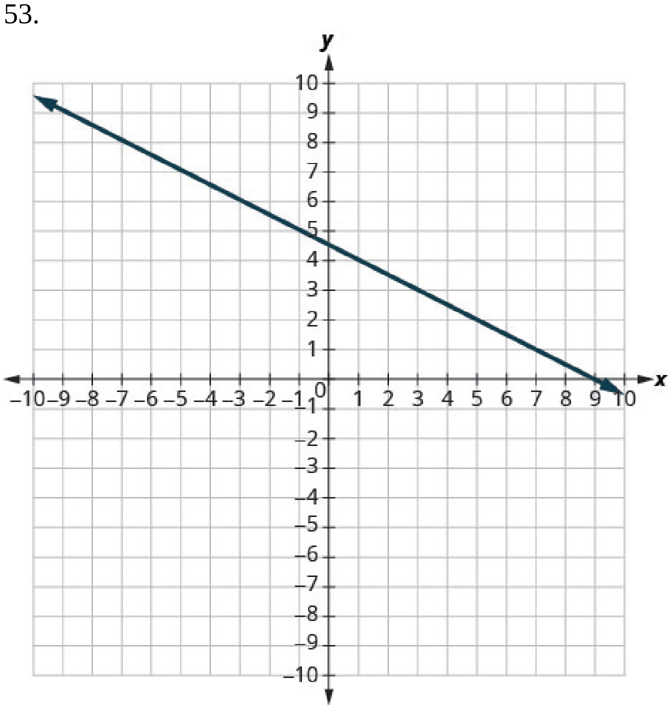

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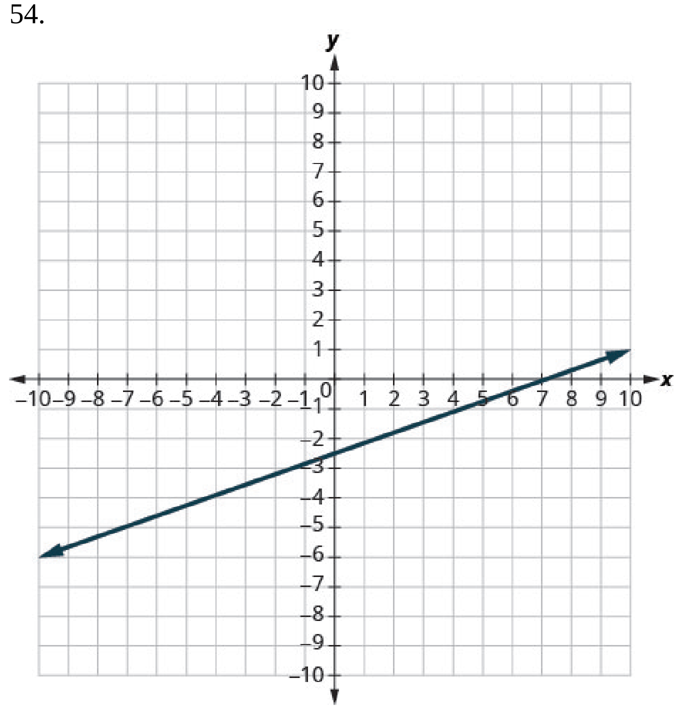


Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

55. $x = 5$	56. $y = 2$
57. $y = -1$	58. $x = -3$

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

59. $(3,5),(4,-1)$	60. $(-1,-1),(0,5)$
61. $(2,1),(4,6)$	62. $(-5,-2),(3,2)$

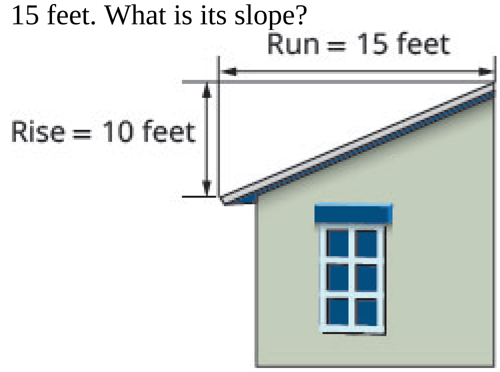
Graph a Line Given a Point and the Slope

In the following exercises, graph each line with the given point and slope.

63. $(-3,4); m=-\frac{1}{3}$	64. $(2,-2); m=\frac{5}{2}$
65. *y*-intercept 1; $m=-\frac{3}{4}$	66. *x*-intercept -4; $m=3$

Solve Slope Applications

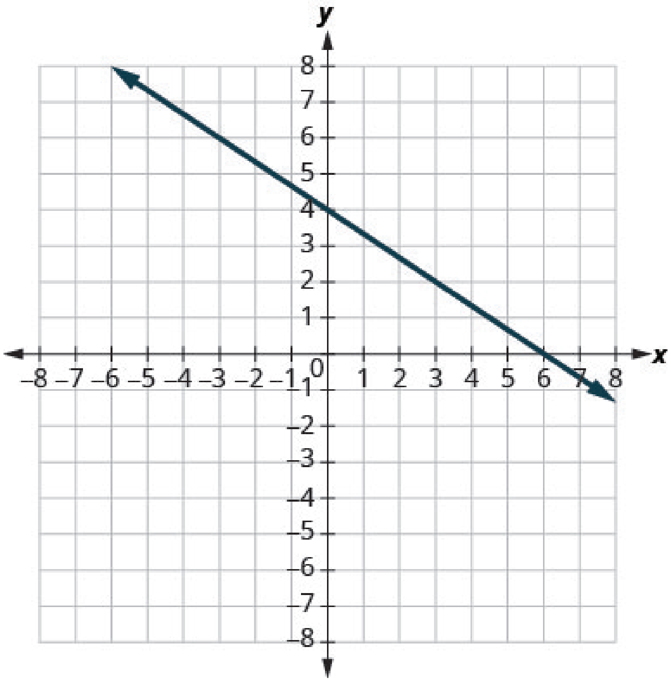
In the following exercises, solve these slope applications.

67. A mountain road rises 50 feet for a 500-foot run. What is its slope?	68. The roof pictured below has a rise of 10 feet and a run of 15 feet. What is its slope? 

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

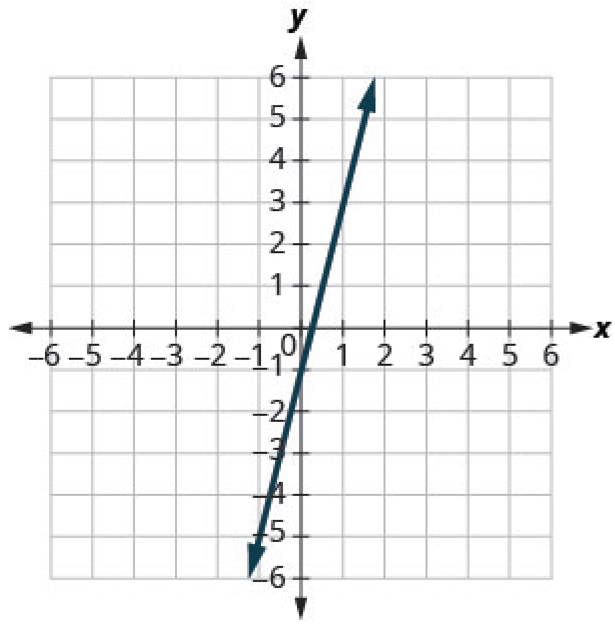
In the following exercises, use the graph to find the slope and *y*-intercept of each line. Compare the values to the equation $y=mx+b$.

69.



$y = -\frac{2}{3}x + 4$

70.



$y = 4x - 1$

Identify the Slope and y-Intercept from an Equation of a Line

In the following exercises, identify the slope and *y*-intercept of each line.

71. $y = \frac{5}{3}x - 6$	72. $y = -4x + 9$
73. $4x - 5y = 8$	74. $5x + y = 10$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and *y*-intercept.

75. $y =$ -$x - 1$	76. $y = 2x + 3$
77. $4x - 3y = 12$	78. $y = -\frac{2}{5}x + 3$

In the following exercises, determine the most convenient method to graph each line.

79. $y=-3$	80. $x=5$
81. $x-y=2$	82. $2x+y=5$
83. $y=\frac{3}{4}x-1$	84. $y=x+2$

Graph and Interpret Applications of Slope–Intercept

85. Marjorie teaches piano. The equation $P=35h-250$ models the relation between her weekly profit, *P*, in dollars and the number of student lessons, *s*, that she teaches. a. Find Marjorie's profit for a week when she teaches no student lessons. b. Find the profit for a week when she teaches 20 student lessons. c. Interpret the slope and *P*–intercept of the equation. d. Graph the equation.	86. Katherine is a private chef. The equation $C=6.5m+42$ models the relation between her weekly cost, *C*, in dollars and the number of meals, *m*, that she serves. a. Find Katherine's cost for a week when she serves no meals. b. Find the cost for a week when she serves 14 meals. c. Interpret the slope and *C*-intercept of the equation. d. Graph the equation.

Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

87. $2x-y=8;\ x-2y=4$	88. $4x-3y=-1;\ y=\frac{4}{3}x-3$

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

89. $3x-2y=5;2x+3y=6$	90. $y=5x-1;10x+2y=0$

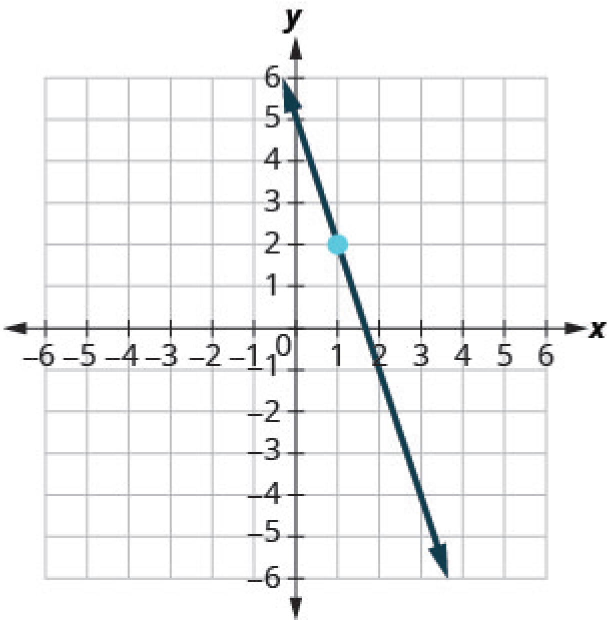
Find an Equation of the Line Given the Slope and *y*-Intercept

In the following exercises, find the equation of a line with given slope and y-intercept. Write the equation in slope–intercept form.

91. slope -5 and $y\text{-intercept }(0,-3)$	92. slope $\frac{1}{3}$ and $y\text{-intercept }(0,-6)$
93. slope -2 and $y\text{-intercept }(0,0)$	94. slope 0 and $y\text{-intercept }(0,4)$

In the following exercises, find the equation of the line shown in each graph. Write the equation in slope–intercept form.

95.



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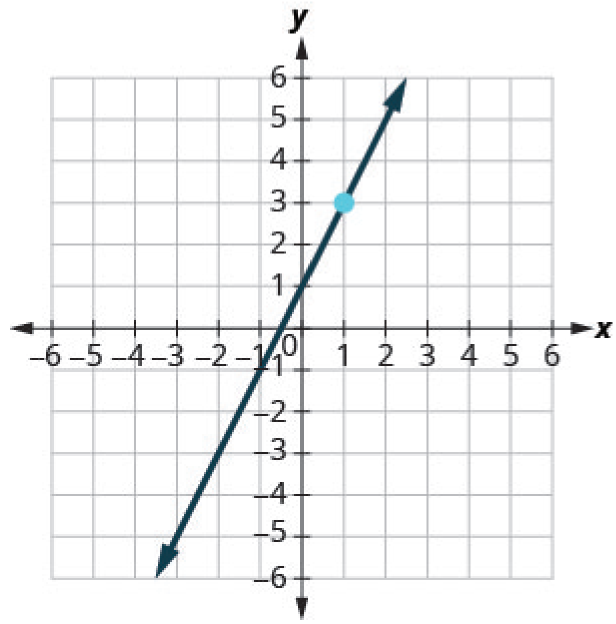

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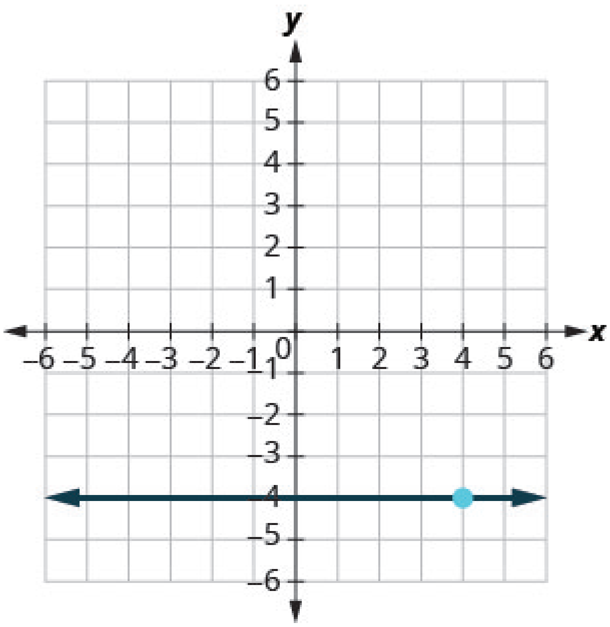

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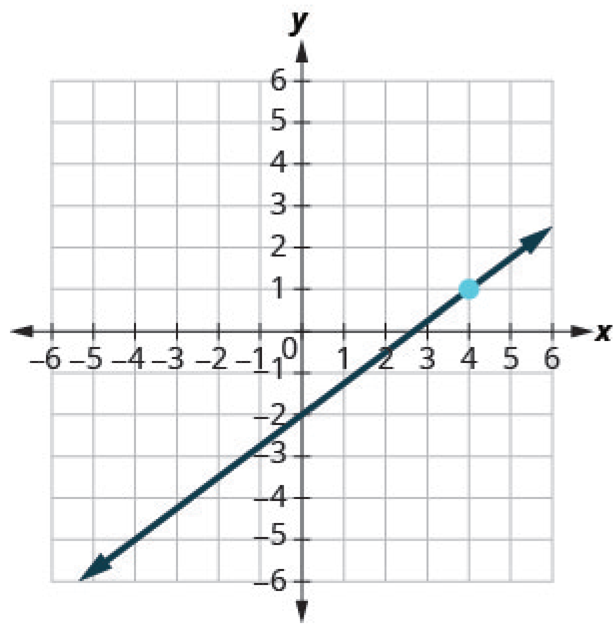


Find an Equation of the Line Given the Slope and a Point

In the following exercises, find the equation of a line with given slope and containing the given point. Write the equation in slope–intercept form.

99. $m=\frac{3}{5}$, point $(10,6)$	100. $m=-\frac{1}{4}$, point $(-8,3)$
101. $m=-2$, point $(-1,-3)$	102. Horizontal line containing $(-2,7)$

Find an Equation of the Line Given Two Points

In the following exercises, find the equation of a line containing the given points. Write the equation in slope–intercept form.

103. $(7,1)$ and $(5,0)$	104. $(2,10)$ and $(-2,-2)$
105. $(5,2)$ and $(-1,2)$	106. $(3,8)$ and $(3,-4)$.

Find an Equation of a Line Parallel to a Given Line

In the following exercises, find an equation of a line parallel to the given line and contains the given point. Write the equation in slope–intercept form.

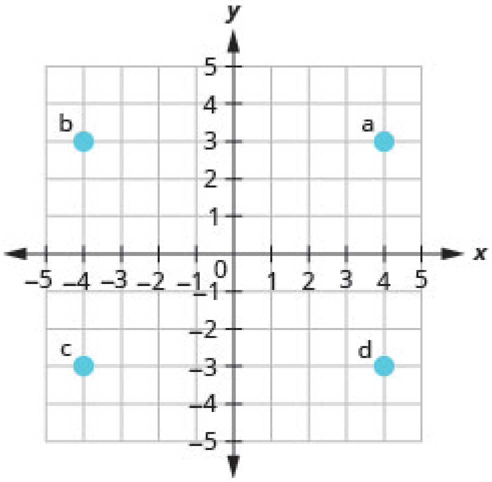
107. line $2x+5y=-10$, point $(10,4)$	108. line $y=-3x+6$, point $(1,-5)$
109. line $y=-5$, point $(-4,3)$	110. line $x=4$, point $(-2,-1)$

Find an Equation of a Line Perpendicular to a Given Line

In the following exercises, find an equation of a line perpendicular to the given line and contains the given point. Write the equation in slope–intercept form.

111. line $2x-3y=9$, point $(-4,0)$	112. line $y=-\frac{4}{5}x+2$, point $(8,9)$
113. line $x=-5$ point $(2,1)$	114. line $y=3$, point $(-1,-3)$

Review Answers



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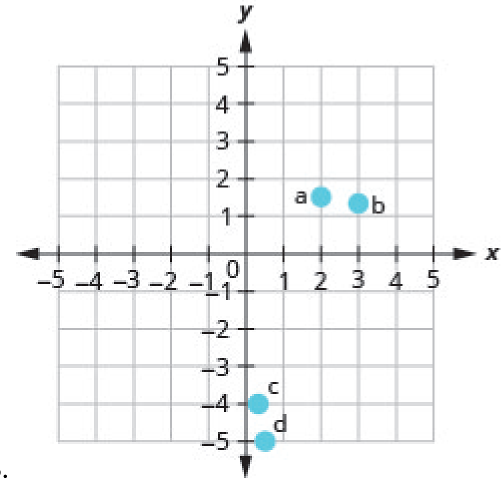


3.

5. a) $(2,0)$ b) $(0,-5)$ c) (-4.0) d) $(0,3)$

7. a, b

9.

x	y	(x,y)
0	3	$(0,3)$
4	1	(4, 1)
-2	4	$(-2,4)$

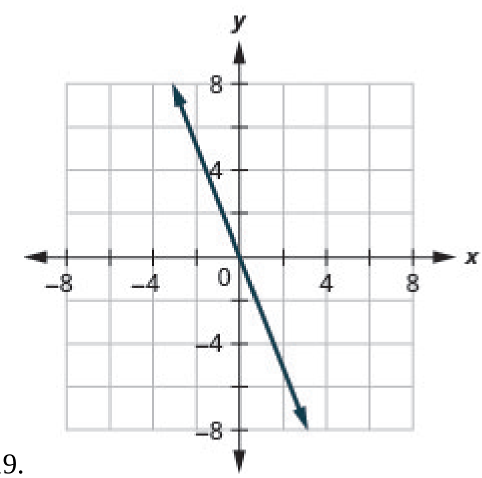
11.

x	y	(x,y)
0	-3	$(0,-3)$
2	0	$(2,0)$
-2	-6	$(-2,-6)$

13. Answers will vary.

15. Answers will vary.

17. a) yes; yes b) yes; no



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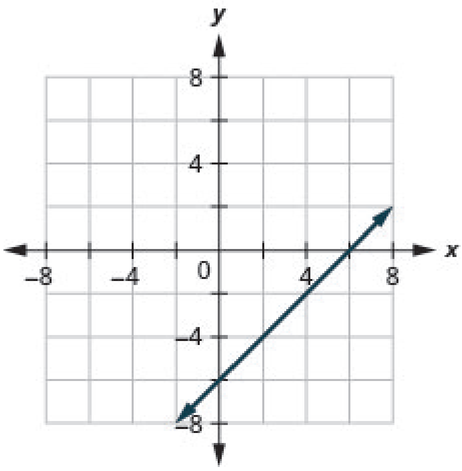

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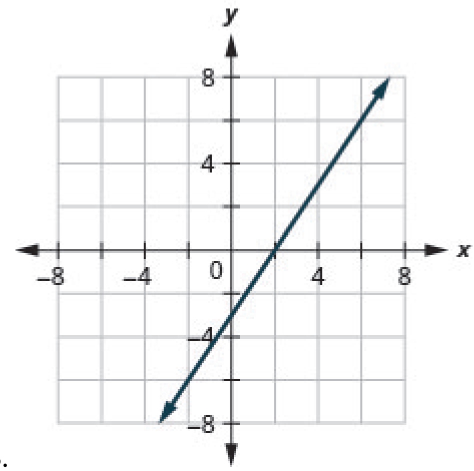

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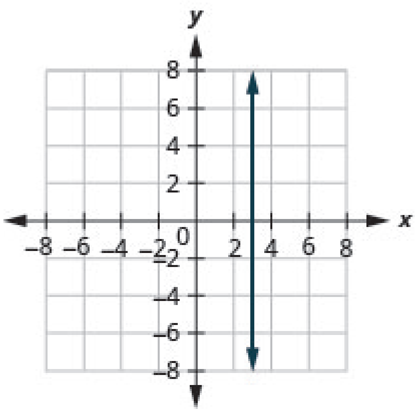

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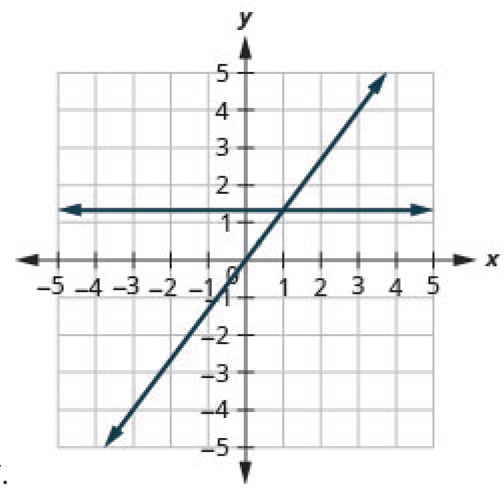
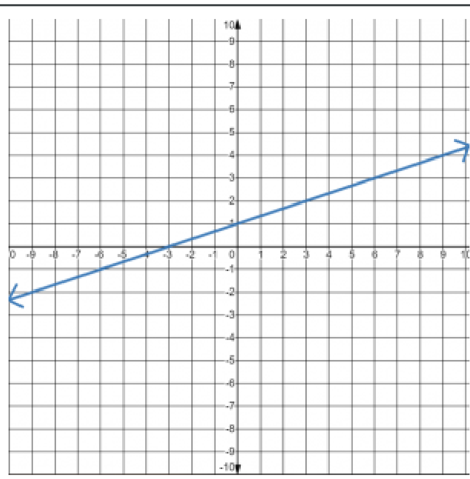


27.

29. $(3,0), (0,3)$	31. $(-1,0), (0,1)$
33. $(6,0), (0,4)$	35. $(0,0)$



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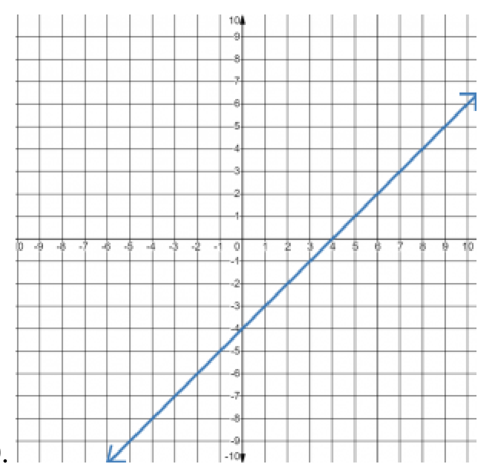
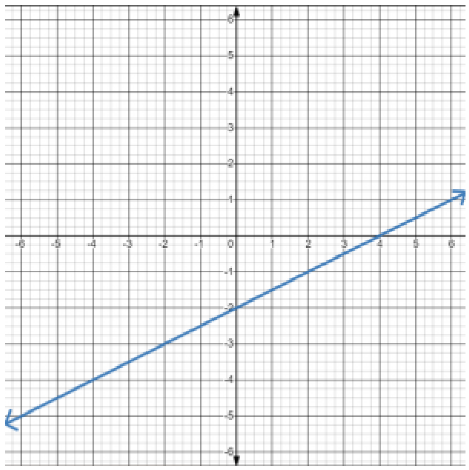
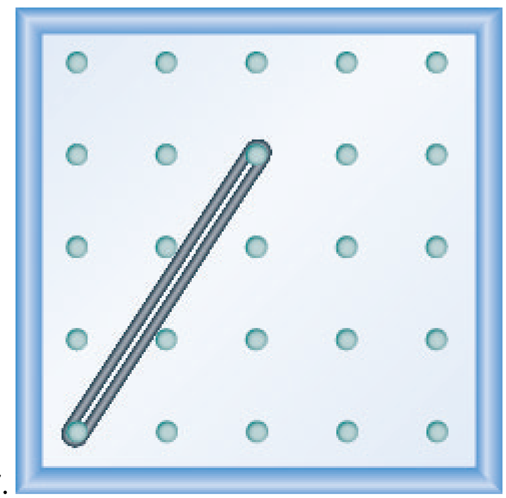
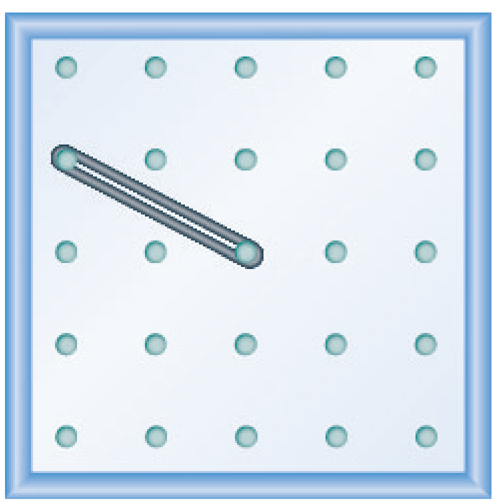
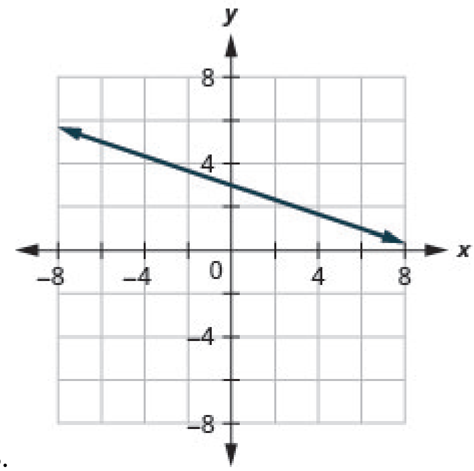
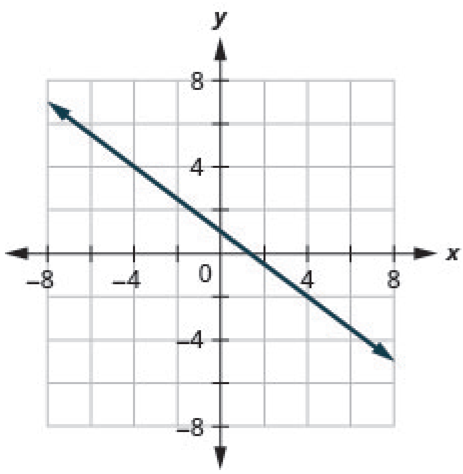
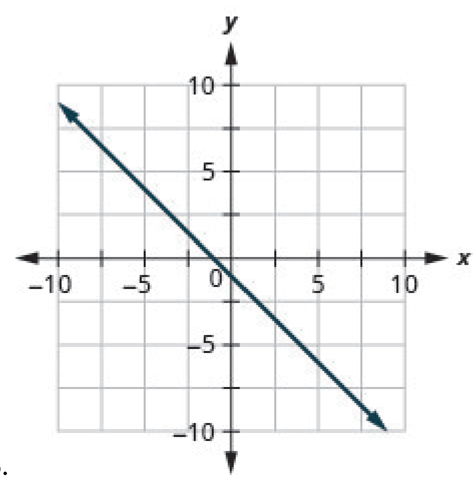
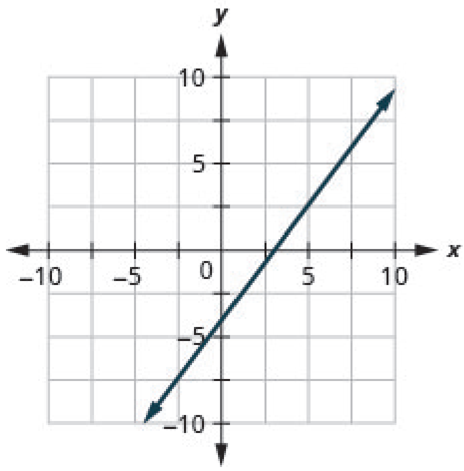
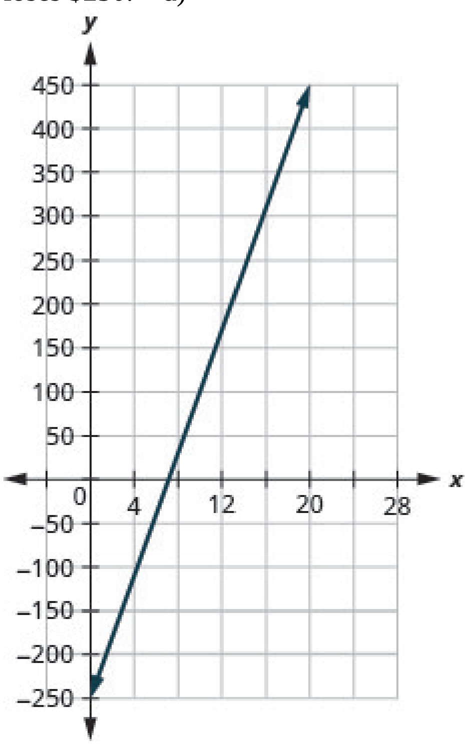


39.

41. 	43. $\frac{4}{3}$
45. $-\frac{2}{3}$	47. 
49. 	51. 1
53. $-\frac{1}{2}$	55. undefined
57. 0	59. -6

61. $\frac{5}{2}$	 63.
 65.	67. $\frac{1}{10}$
69. slope $m = -\frac{2}{3}$ and *y*-intercept $(0, 4)$	71. $\frac{5}{3}$; $(0, -6)$
73. $\frac{4}{5}$; $\left(0, -\frac{8}{5}\right)$	 75.

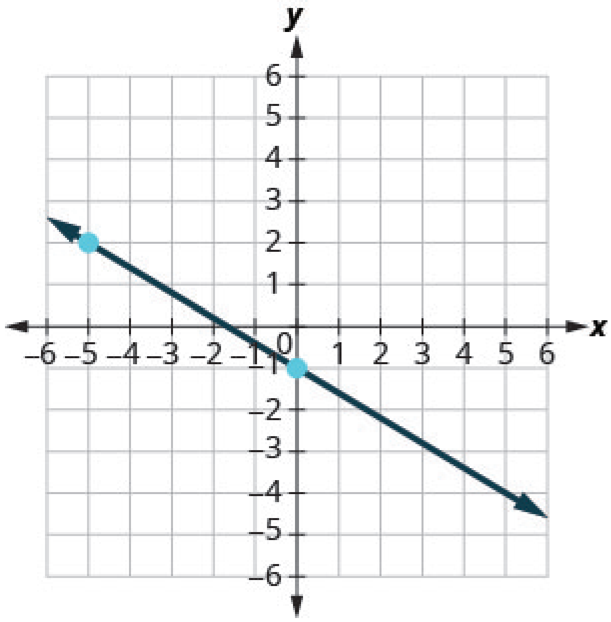
77. 	79. horizontal line
81. intercepts	83. plotting points
85. a) −250 b) 450 c) The slope, 35, means that Marjorie's weekly profit, *P*, increases by $35 for each additional student lesson she teaches. The *P*–intercept means that when the number of lessons is 0, Marjorie loses $250. d) 	87. not parallel
89. perpendicular	91. $y=-5x-3$
93. $y=-2x$	95. $y=-3x+5$
97. $y=-4$	99. $y=\frac{3}{5}x$
101. $y=-2x-5$	103. $y=\frac{1}{2}x-\frac{5}{2}$

105. $y=2$	107. $y=-\frac{2}{5}x+8$
109. $y=3$	111. $y=-\frac{3}{2}x-6$
113. $y=1$	

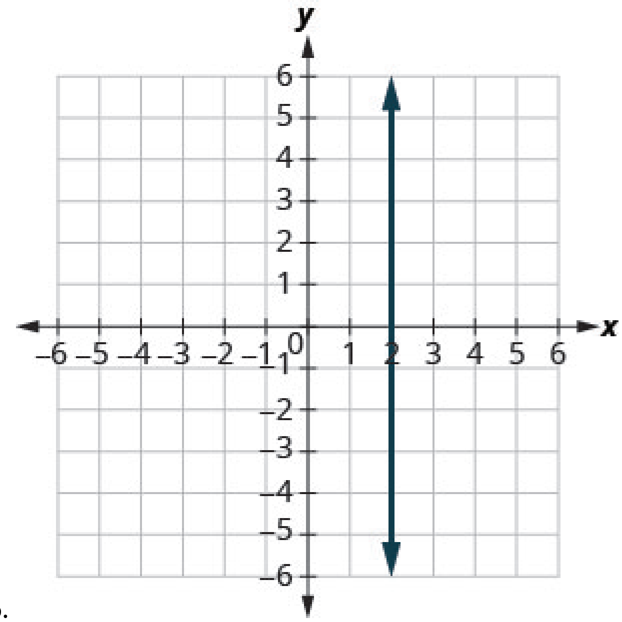
Practice Test

1. Plot each point in a rectangular coordinate system. a) $(2,5)$ b) $(-1,-3)$ c) $(0,2)$ d) $\left(-4,\frac{3}{2}\right)$ e) $(5,0)$	2. Which of the given ordered pairs are solutions to the equation $3x-y=6$? a) $(3,3)$ b) $(2,0)$ c) $(4,-6)$
3. Find three solutions to the linear equation $y=-2x-4$.	4. Find the *x*– and *y*-intercepts of the equation $4x-3y=12$.

Find the slope of each line shown.



5.



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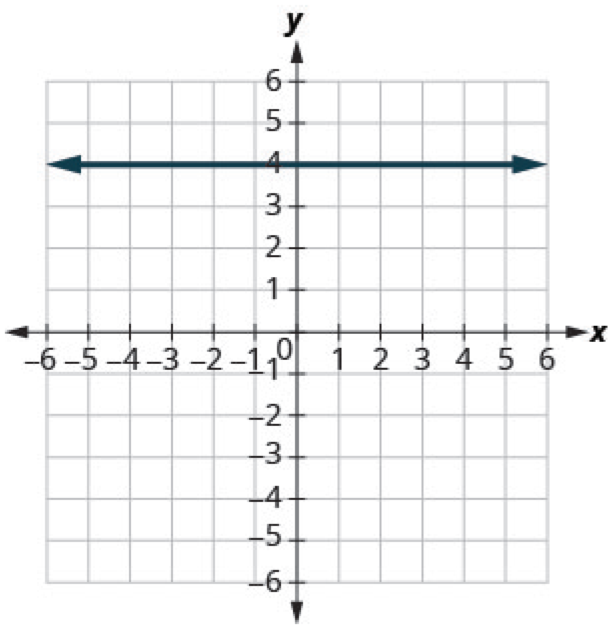


7.

8. Find the slope of the line between the points $(5, 2)$ and $(-1, -4)$.	9. Graph the line with slope $\frac{1}{2}$ containing the point $(-3, -4)$.

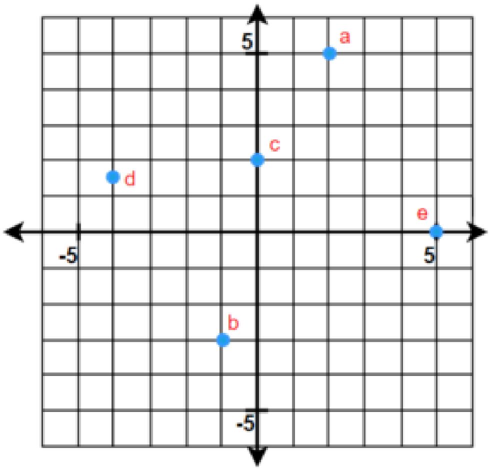
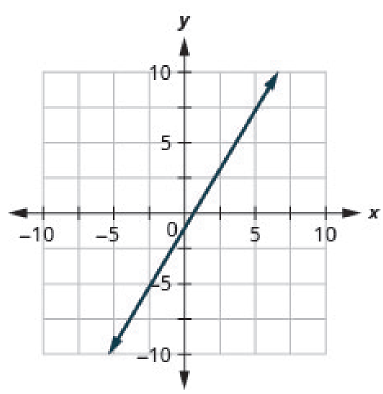
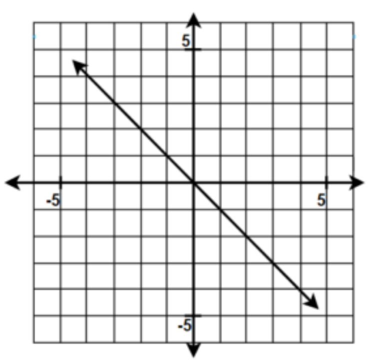
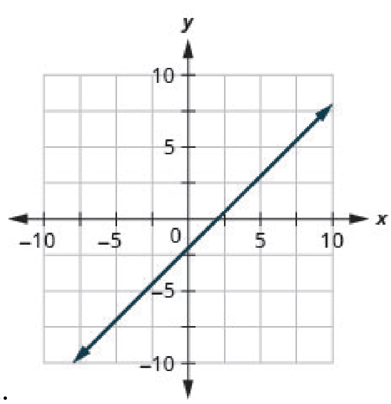
Graph the line for each of the following equations

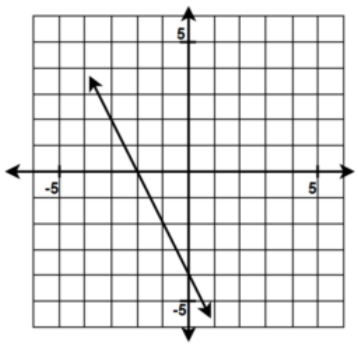
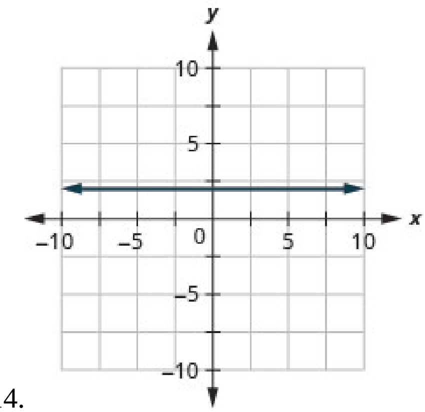
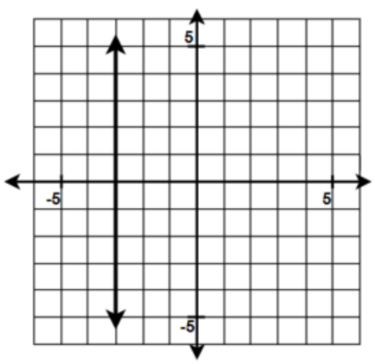
10. $y = \frac{5}{3}x - 1$	11. $y = -x$
12. $x - y = 2$	13. $4x + 2y = -8$
14. $y = 2$	15. $x = -3$

Find the equation of each line. Write the equation in slope–intercept form.

16. slope $-\frac{3}{4}$ and *y*-intercept $(0, -2)$	17. $m = 2$, point $(-3, -1)$
18. containing $(10, 1)$ and $(6, -1)$	19. parallel to the line $y = -\frac{2}{3}x - 1$, containing the point $(-3, 8)$
20. perpendicular to the line $y = \frac{5}{4}x + 2$, containing the point $(-10, 3)$	

Practice Test Answers

 1.	2. a) yes b) yes c) no
3. Answer may vary	4. $(3, 0)$, $(0, -4)$
5. m = $\frac{-3}{5}$	6. undefined
7. m = 0	8. 1
9. y = $\frac{1}{2}$x – $\frac{5}{2}$	 10.
 11.	 12.

13. 	14. 
15. 	16. $y = -\frac{3}{4}x - 2$
17.$y = 2x + 5$	18. $y = \frac{1}{2}x - 4$
19. $y = -\frac{2}{3}x + 6$	20. $y = -\frac{4}{5}x - 5$

Attributions

This chapter has been adapted from “Review Exercises” and “Practice Test” in Chapter 4 of *Elementary Algebra* (OpenStax) by Lynn Marecek and MaryAnne Anthony-Smith, which is under a CC BY 4.0 Licence. Adapted by Izabela Mazur. See the Copyright page for more information.

III

CHAPTER 5 Powers, Roots, and Scientific Notation

Square roots are used to determine the time it would take for a stone falling from the edge of this cliff to hit the land below.

Suppose a stone falls from the edge of a cliff. The number of feet the stone has dropped after t seconds can be found by multiplying 16 times the square of t. But to calculate the number of seconds it would take the stone to hit the land below, we need to use a square root. In this chapter, we will introduce and apply the properties of exponents and square roots, and scientific notation.

5.1 Use Multiplication Properties of Exponents

Learning Objectives

By the end of this section, you will be able to:

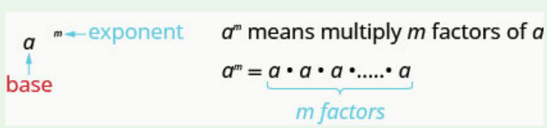
- Simplify expressions with exponents
- Simplify expressions using the Product Property for Exponents
- Simplify expressions using the Power Property for Exponents
- Simplify expressions using the Product to a Power Property
- Simplify expressions by applying several properties
- Multiply monomials

Simplify Expressions with Exponents

Remember that an exponent indicates repeated multiplication of the same quantity. For example, 2^4 means to multiply 2 by itself 4 times, so 2^4 means $2 \cdot 2 \cdot 2 \cdot 2$

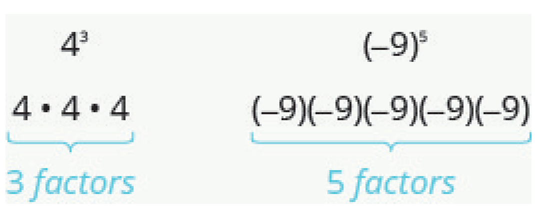
Let's review the vocabulary for expressions with exponents.

Exponential Notation



This is read a to the m^{th} power.

In the expression a^m, the *exponent* m tells us how many times we use the *base* a as a factor.



Before we begin working with variable expressions containing exponents, let's simplify a few expressions involving only numbers.

EXAMPLE 1

Simplify: a) 4^3 b) 7^1 c) $\left(\frac{5}{6}\right)^2$ d) $(0.63)^2$.

Solution

a)	4^3
Multiply three factors of 4.	$4 \cdot 4 \cdot 4$
Simplify.	64
b)	7^1
Multiply one factor of 7.	7
c)	$\left(\frac{5}{6}\right)^2$
Multiply two factors.	$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)$
Simplify.	$\frac{25}{36}$
d)	$(0.63)^2$
Multiply two factors.	$(0.63)(0.63)$
Simplify.	0.3969

TRY IT 1.1

Simplify: a) 6^3 b) 15^1 c) $\left(\frac{3}{7}\right)^2$ d) $(0.43)^2$.

Show answer

a) 216 b) 15 c) $\frac{9}{49}$ d) 0.1849

TRY IT 1.2

Simplify: a) 2^5 b) 21^1 c) $\left(\frac{2}{5}\right)^3$ d) $(0.218)^2$.

Show answer

a) 32b) 21 c) $\frac{8}{125}$ d) 0.047524

EXAMPLE 2

Simplify: a) $(-5)^4$ b) -5^4.

Solution

a)	$(-5)^4$
Multiply four factors of -5.	$(-5)(-5)(-5)(-5)$
Simplify.	625
b)	-5^4
Multiply four factors of 5.	-(5 · 5 · 5 · 5)
Simplify.	-625

TRY IT 2.1

Simplify: a) $(-3)^4$ b) -3^4.

Show answer
a) 81 b) -81

TRY IT 2.2

Simplify: a) $(-13)^2$ b) -13^2.

Show answer
a) 169 b) -169

Notice the similarities and differences in (Example 2) a) and (Example 2) b)! Why are the answers different? As we follow the order of operations in part a) the parentheses tell us to raise the (-5) to the 4th power. In part b) we raise just the 5 to the 4th power and then take the opposite.

Simplify Expressions Using the Product Property for Exponents

You have seen that when you combine like terms by adding and subtracting, you need to have the same base with the same exponent. But when you multiply and divide, the exponents may be different, and sometimes the bases may be different, too.

We'll derive the properties of exponents by looking for patterns in several examples.

First, we will look at an example that leads to the Product Property.

	$x^2 \quad \bullet \quad x^3$
What does this mean? How many factors altogether?	$\underbrace{x \bullet x}_{2\ factors} \bullet \underbrace{x \bullet x \bullet x}_{3\ factors}$ 5 factors
So, we have	x^5
Notice that 5 is the sum of the exponents, 2 and 3.	$x^2 \bullet x^3$ is x^{2+3}, or x^5

We write:

$$x^2 \cdot x^3$$
$$x^{2+3}$$
$$x^5$$

The base stayed the same and we added the exponents. This leads to the **Product Property for Exponents**.

Product Property for Exponents

If a is a real number, and m and n are counting numbers, then

$a^m \cdot a^n = a^{m+n}$

To multiply with like bases, add the exponents.

An example with numbers helps to verify this property.

$$\begin{aligned} 2^2 \cdot 2^3 &\stackrel{?}{=} 2^{2+3} \\ 4 \cdot 8 &\stackrel{?}{=} 2^5 \\ 32 &= 32\checkmark \end{aligned}$$

EXAMPLE 3

Simplify: $y^5 \cdot y^6$.

Solution

	$y^5 \cdot y^6$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	y^{5+6}
Simplify.	y^{11}

TRY IT 3.1

Simplify: $b^9 \cdot b^8$.

Show answer
b^{17}

TRY IT 3.2

Simplify: $x^{12} \cdot x^4$.

Show answer
x^{16}

EXAMPLE 4

Simplify: a) $2^5 \cdot 2^9$ b) $3 \cdot 3^4$.

Solution

a.

	$2^5 \cdot 2^9$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	2^{5+9}
Simplify.	2^{14}

b.

	$3^1 \cdot 3^4$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	3^{1+4}
Simplify.	3^5

TRY IT 4.1

Simplify: a) $5 \cdot 5^5$ b) $4^9 \cdot 4^9$.

Show answer
a) 5^6 b) 4^{18}

TRY IT 4.2

Simplify: a) $7^6 \cdot 7^8$ b) $10 \cdot 10^{10}$.

Show answer
a) 7^{14} b) 10^{11}

EXAMPLE 5

Simplify: a) $a^7 \cdot a$ b) $x^{27} \cdot x^{13}$.

Solution

a.

	$a^7 \cdot a$
Rewrite, a = a^1.	$a^7 \cdot a^1$
Use the product property, $a^m \cdot a^n = a^{m+n}$.	a^{7+1}
Simplify.	a^8

b.

	$x^{27} \cdot x^{13}$
Notice, the bases are the same, so add the exponents.	x^{27+13}
Simplify.	x^{40}

TRY IT 5.1

Simplify: a) $p^5 \cdot p$ b) $y^{14} \cdot y^{29}$.

Show answer
a) p^6 b) y^{43}

TRY IT 5.2

Simplify: a) $z \cdot z^7$ b) $b^{15} \cdot b^{34}$.

Show answer
a) z^8 b) b^{49}

We can extend the Product Property for Exponents to more than two factors.

EXAMPLE 6

Simplify: $d^4 \cdot d^5 \cdot d^2$.

Solution

	$d^4 \cdot d^5 \cdot d^2$
Add the exponents, since bases are the same.	d^{4+5+2}
Simplify.	d^{11}

TRY IT 6.1

Simplify: $x^6 \cdot x^4 \cdot x^8$.

Show answer
x^{18}

TRY IT 6.2

Simplify: $b^5 \cdot b^9 \cdot b^5$.

Show answer
b^{19}

Simplify Expressions Using the Power Property for Exponents

Now let's look at an exponential expression that contains a power raised to a power. See if you can discover a general property.

	$(x^2)^3$
What does this mean? How many factors altogether?	$x^2 \cdot x^2 \cdot x^2$ $x \cdot x \cdot x \cdot x \cdot x \cdot x$ 2 factors 2 factors 2 factors 6 factors
So we have	x^6
Notice that 6 is the product of the exponents, 2 and 3.	$(x^2)^3$ is $x^{2\cdot 3}$ or x^6

We write:

$$\left(x^2\right)^3$$
$$x^{2\cdot 3}$$
$$x^6$$

We multiplied the exponents. This leads to the Power Property for Exponents.

Power Property for Exponents

If a is a real number, and m and n are whole numbers, then

$$(a^m)^n = a^{m\cdot n}$$

To raise a power to a power, multiply the exponents.

An example with numbers helps to verify this property.

$$\begin{aligned}(2^2)^3 &\stackrel{?}{=} 2^{2\cdot 3}\\ 4^3 &\stackrel{?}{=} 2^6\\ 64 &= 64\checkmark\end{aligned}$$

EXAMPLE 7

Simplify: a) $\left(y^5\right)^9$ b) $\left(4^4\right)^7$.

Solution

a)

	$(y^5)^9$
Use the power property, $(a^m)^n = a^{m\cdot n}$.	$y^{5\cdot 9}$
Simplify.	y^{45}

b)

	$(4^4)^7$
Use the power property.	$4^{4\cdot7}$
Simplify.	4^{28}

TRY IT 7.1

Simplify: a) $\left(b^7\right)^5$ b) $\left(5^4\right)^3$.

Show answer
a) b^{35} b) 5^{12}

TRY IT 7.2

Simplify: a) $\left(z^6\right)^9$ b) $\left(3^7\right)^7$.

Show answer
a) z^{54} b) 3^{49}

Simplify Expressions Using the Product to a Power Property

We will now look at an expression containing a product that is raised to a power. Can you find this pattern?

	$(2x)^3$
What does this mean?	$2x \cdot 2x \cdot 2x$
We group the like factors together.	$2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$
How many factors of 2 and of x?	$2^3 \cdot x^3$

Notice that each factor was raised to the power and $(2x)^3$ is $2^3 \cdot x^3$.

We write:	$(2x)^3$
	$2^3 \cdot x^3$

The exponent applies to each of the factors! This leads to the Product to a Power Property for Exponents.

Product to a Power Property for Exponents

If a and b are real numbers and m is a whole number, then

$(ab)^m = a^m b^m$

To raise a product to a power, raise each factor to that power.

An example with numbers helps to verify this property:

$$\begin{aligned} (2 \cdot 3)^2 &\stackrel{?}{=} 2^2 \cdot 3^2 \\ 6^2 &\stackrel{?}{=} 4 \cdot 9 \\ 36 &= 36\checkmark \end{aligned}$$

EXAMPLE 8

Simplify: a) $(-9d)^2$ b) $(3mn)^3$.

Solution

a.

	$(-9d)^2$
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(-9)^2 d^2$
Simplify.	$81d^2$

b.

	$(3mn)^3$
Use Power of a Product Property, $(ab)^m = a^m b^m$.	$(3)^3 m^3 n^3$
Simplify.	$27m^3n^3$

TRY IT 8.1

Simplify: a) $(-12y)^2$ b) $(2wx)^5$.

Show answer

a) $144y^2$ b) $32w^5x^5$

TRY IT 8.2

Simplify: a) $(5wx)^3$ b) $(-3y)^3$.

Show answer
a) $125w^3x^3$ b) $-27y^3$

Simplify Expressions by Applying Several Properties

We now have three properties for multiplying expressions with exponents. Let's summarize them and then we'll do some examples that use more than one of the properties.

Properties of Exponents

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$

All exponent properties hold true for any real numbers$m$ and n. Right now, we only use whole number exponents.

EXAMPLE 9

Simplify: a) $\left(y^3\right)^6\left(y^5\right)^4$ b) $\left(-6x^4y^5\right)^2$.

Solution

a)	$\left(y^3\right)^6\left(y^5\right)^4$
Use the Power Property.	$y^{18}\cdot y^{20}$
Add the exponents.	y^{38}
b)	$\left(-6x^4y^5\right)^2$
Use the Product to a Power Property.	$(-6)^2\left(x^4\right)^2\left(y^5\right)^2$
Use the Power Property.	$(-6)^2$
Simplify.	$36x^8y^{10}$

TRY IT 9.1

Simplify: a) $\left(a^4\right)^5\left(a^7\right)^4$ b) $\left(-2c^4d^2\right)^3$.

Show answer
a) a^{48} b) $-8c^{12}d^6$

TRY IT 9.2

Simplify: a) $\left(-3x^6y^7\right)^4$ b) $\left(q^4\right)^5\left(q^3\right)^3$.

Show answer
a) $81x^{24}y^{28}$ b) q^{29}

EXAMPLE 10

Simplify: a) $(5m)^2\left(3m^3\right)$ b) $\left(3x^2y\right)^4\left(2xy^2\right)^3$.

Solution

a)	$(5m)^2\left(3m^3\right)$
Raise $5m$ to the second power.	$5^2m^2\cdot 3m^3$
Simplify.	$25m^2\cdot 3m^3$
Use the Commutative Property.	$25\cdot 3\cdot m^2\cdot m^3$
Multiply the constants and add the exponents.	$75m^5$
b)	$\left(3x^2y\right)^4\left(2xy^2\right)^3$
Use the Product to a Power Property.	$\left(3^4x^8y^4\right)\left(2^3x^3y^6\right)$
Simplify.	$\left(81x^8y^4\right)\left(8x^3y^6\right)$
Use the Commutative Property.	$81\cdot 8\cdot x^8\cdot x^3\cdot y^4\cdot y^6$
Multiply the constants and add the exponents.	$648x^{11}y^{10}$

TRY IT 10.1

Simplify: a) $(5n)^2\left(3n^{10}\right)$ b) $\left(c^4d^2\right)^5\left(3cd^5\right)^4$.

Show answer
a) $75n^{12}$ b) $81c^{24}d^{30}$

TRY IT 10.2

Simplify: a) $\left(a^3b^2\right)^6\left(4ab^3\right)^4$ b) $(2x)^3\left(5x^7\right)$.

Show answer
a) $256a^{22}b^{24}$ b) $40x^{10}$

Multiply Monomials

A term in algebra is a constant or the product of a constant and one or more variables. When it is of the form ax^m, where a is a constant and m is a whole number, it is called a monomial. Some examples of monomial are 8, $-2x^2$, $4y^3$, and $11z^7$.

Monomials

A monomial is a term of the form ax^m, where a is a constant and m is a positive whole number.

Since a monomial is an algebraic expression,we can use the properties of exponents to multiply monomials.

EXAMPLE 11

Multiply: $\left(3x^2\right)\left(-4x^3\right)$.

Solution

	$\left(3x^2\right)\left(-4x^3\right)$
Use the Commutative Property to rearrange the terms.	$3\cdot(-4)\cdot x^2\cdot x^3$
Multiply.	$-12x^5$

TRY IT 11.1

Multiply: $\left(5y^7\right)\left(-7y^4\right)$.

Show answer
$-35y^{11}$

TRY IT 11.2

Multiply: $\left(-6b^4\right)\left(-9b^5\right)$.

Show answer
$54b^9$

EXAMPLE 12

Multiply: $\left(\frac{5}{6}x^3y\right)\left(12xy^2\right)$.

Solution

	$\left(\frac{5}{6}x^3y\right)(12xy^2)$
Use the Commutative Property to rearrange the terms.	$\frac{5}{6}\cdot 12\cdot x^3\cdot x\cdot y\cdot y^2$
Multiply.	$10x^4y^3$

TRY IT 12.1

Multiply: $\left(\frac{2}{5}a^4b^3\right)(15ab^3)$.

Show answer
$6a^5b^6$

TRY IT 12.2

Multiply: $\left(\frac{2}{3}r^5s\right)(12r^6s^7)$.

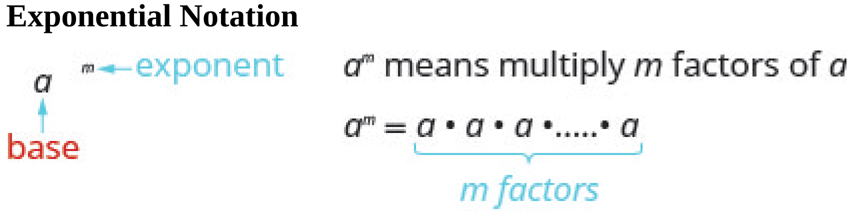
Show answer
$8r^{11}s^8$

Additional Online Resources

- [Multiplication Properties of Exponents]

Key Concepts

- **Exponential Notation**



- **Properties of Exponents**

- If a, b are real numbers and m, n are whole numbers, then

Product Property	$a^m \cdot a^n$	$= a^{m+n}$
Power Property	$(a^m)^n$	$= a^{m \cdot n}$
Product to a Power	$(ab)^m$	$= a^m b^m$

Practice Makes Perfect

Simplify Expressions with Exponents

In the following exercises, simplify each expression with exponents.

1. a) 3^5 b) 9^1 c) $\left(\frac{1}{3}\right)^2$ d) $(0.2)^4$	2. a) 10^4 b) 17^1 c) $\left(\frac{2}{9}\right)^2$ d) $(0.5)^3$
3. a) 2^6 b) 14^1 c) $\left(\frac{2}{5}\right)^3$ d) $(0.7)^2$	4. a) 8^3 b) 8^1 c) $\left(\frac{3}{4}\right)^3$ d) $(0.4)^3$
5. a) $(-6)^4$ b) -6^4	6. a) $(-2)^6$ b) -2^6
7. a) $-\left(\frac{1}{4}\right)^4$ b) $\left(-\frac{1}{4}\right)^4$	8. a) $-\left(\frac{2}{3}\right)^2$ b) $\left(-\frac{2}{3}\right)^2$
9. a) -0.5^2 b) $(-0.5)^2$	10. a) -0.1^4 b) $(-0.1)^4$

Simplify Expressions Using the Product Property for Exponents

In the following exercises, simplify each expression using the Product Property for Exponents.

11. $d^3 \cdot d^6$	12. $x^4 \cdot x^2$
13. $n^{19} \cdot n^{12}$	14. $q^{27} \cdot q^{15}$
15. a) $4^5 \cdot 4^9$ b) $8^9 \cdot 8$	16. a) $3^{10} \cdot 3^6$ b) $5 \cdot 5^4$
17. a) $y \cdot y^3$ b) $z^{25} \cdot z^8$	18. a) $w^2 \cdot w^4$ b) $v^{44} \cdot v^{53}$
19. $w \cdot w^2 \cdot w^3$	20. $y \cdot y^3 \cdot y^5$
21. $a^4 \cdot a^3 \cdot a^9$	22. $c^5 \cdot c^{11} \cdot c^2$
23. $m^x \cdot m^3$	24. $n^y \cdot n^2$
25. $y^a \cdot y^b$	26. $x^p \cdot x^q$

Simplify Expressions Using the Power Property for Exponents

In the following exercises, simplify each expression using the Power Property for Exponents.

27. a) $\left(m^4\right)^2$ b) $\left(10^3\right)^6$	28. a) $\left(b^2\right)^7$ b) $\left(3^8\right)^2$
29. a) $\left(y^3\right)^x$ b) $(5^x)^y$	30. a) $\left(x^2\right)^y$ b) $(7^a)^b$

Simplify Expressions Using the Product to a Power Property

In the following exercises, simplify each expression using the Product to a Power Property.

31. a) $(6a)^2$ b) $(3xy)^2$	32. a) $(5x)^2$ b) $(4ab)^2$
33. a) $(-4m)^3$ b) $(5ab)^3$	34. a) $(-7n)^3$ b) $(3xyz)^4$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify each expression.

35. a) $\left(y^2\right)^4 \cdot \left(y^3\right)^2$ b) $\left(10a^2b\right)^3$	36. a) $\left(w^4\right)^3 \cdot \left(w^5\right)^2$ b) $\left(2xy^4\right)^5$
37. a) $\left(-2r^3s^2\right)^4$ b) $\left(m^5\right)^3 \cdot \left(m^9\right)^4$	38. a) $\left(-10q^2p^4\right)^3$ b) $\left(n^3\right)^{10} \cdot \left(n^5\right)^2$
39. a) $(3x)^2\,(5x)$ b) $\left(5t^2\right)^3(3t)^2$	40. a) $(2y)^3\,(6y)$ b) $\left(10k^4\right)^3\left(5k^6\right)^2$
41. a) $(5a)^2(2a)^3$ b) $\left(\frac{1}{2}y^2\right)^3\left(\frac{2}{3}y\right)^2$	42. a) $(4b)^2(3b)^3$ b) $\left(\frac{1}{2}j^2\right)^5\left(\frac{2}{5}j^3\right)^2$
43. a) $\left(\frac{2}{5}x^2y\right)^3$ b) $\left(\frac{8}{9}xy^4\right)^2$	44. a) $\left(2r^2\right)^3(4r)^2$ b) $\left(3x^3\right)^3\left(x^5\right)^4$
45. a) $\left(m^2n\right)^2\left(2mn^5\right)^4$ b) $\left(3pq^4\right)^2\left(6p^6q\right)^2$	

Multiply Monomials

In the following exercises, multiply the terms.

46. $\left(6y^7\right)\left(-3y^4\right)$	47. $\left(-10x^5\right)\left(-3x^3\right)$
48. $\left(-8u^6\right)\left(-9u\right)$	49. $\left(-6c^4\right)\left(-12c\right)$
50. $\left(\frac{1}{5}f^8\right)\left(20f^3\right)$	51. $\left(\frac{1}{4}d^5\right)\left(36d^2\right)$
52. $\left(4a^3b\right)\left(9a^2b^6\right)$	53. $\left(6m^4n^3\right)\left(7mn^5\right)$
54. $\left(\frac{4}{7}rs^2\right)\left(14rs^3\right)$	55. $\left(\frac{5}{8}x^3y\right)\left(24x^5y\right)$
56. $\left(\frac{2}{3}x^2y\right)\left(\frac{3}{4}xy^2\right)$	57. $\left(\frac{5}{7}xy^3\right)\left(\frac{49}{125}x^5y^2\right)$

Mixed Practice

In the following exercises, simplify each expression.

58. $\left(x^2\right)^4 \cdot \left(x^3\right)^2$	59. $\left(y^4\right)^3 \cdot \left(y^5\right)^2$
60. $\left(a^2\right)^6 \cdot \left(a^3\right)^8$	61. $\left(b^7\right)^5 \cdot \left(b^2\right)^6$
62. $\left(2m^6\right)^3$	63. $\left(3y^2\right)^4$
64. $\left(10x^2y\right)^3$	65. $\left(2mn^4\right)^5$
66. $\left(-2a^3b^2\right)^4$	67. $\left(-10u^2v^4\right)^3$
68. $\left(\frac{2}{3}x^2y\right)^3$	69. $\left(\frac{7}{9}pq^4\right)^2$
70. $\left(8a^3\right)^2(2a)^4$	71. $\left(5r^2\right)^3(3r)^2$
72. $\left(10p^4\right)^3\left(5p^6\right)^2$	73. $\left(4x^3\right)^3\left(2x^5\right)^4$
74. $\left(\frac{1}{2}x^2y^3\right)^4\left(4x^5y^3\right)^2$	75. $\left(\frac{1}{3}m^3n^2\right)^4\left(9m^8n^3\right)^2$
76. $\left(3m^2n\right)^2\left(2mn^5\right)^4$	77. $\left(2pq^4\right)^3\left(5p^6q\right)^2$

Everyday Math

78. **Email** Kate emails a flyer to ten of her friends and tells them to forward it to ten of their friends, who forward it to ten of their friends, and so on. The number of people who receive the email on the second round is 10^2, on the third round is 10^3, as shown in the table below. How many people will receive the email on the sixth round? Simplify the expression to show the number of people who receive the email.

Round	Number of people
1	10
2	10^2
3	10^3
…	…
6	?

79. **Salary** Jamal's boss gives him a 3% raise every year on his birthday. This means that each year, Jamal's salary is 1.03 times his last year's salary. If his original salary was $35,000, his salary after 1 year was $\$35,000\,(1.03)$, after 2 years was $\$35,000(1.03)^2$, after 3 years was $\$35,000(1.03)^3$, as shown in the table below. What will Jamal's salary be after 10 years? Simplify the expression, to show Jamal's salary in dollars.

Year	Salary
1	$\$35,000\,(1.03)$
2	$\$35,000(1.03)^2$
3	$\$35,000(1.03)^3$
…	…
10	?

80. **Clearance** A department store is clearing out merchandise in order to make room for new inventory. The plan is to mark down items by 30% each week. This means that each week the cost of an item is 70% of the previous week's cost. If the original cost of a sofa was $1,000, the cost for the first week would be $\$1,000\,(0.70)$ and the cost of the item during the second week would be $\$1,000(0.70)^2$. Complete the table shown below. What will be the cost of the sofa during the fifth week? Simplify the expression, to show the cost in dollars.

Week	Cost
1	$\$1,000\,(0.70)$
2	$\$1,000(0.70)^2$
3	
…	…
5	?

81. **Depreciation** Once a new car is driven away from the dealer, it begins to lose value. Each year, a car loses 10% of its value. This means that each year the value of a car is 90% of the previous year's value. If a new car was purchased for ?20,000, the value at the end of the first year would be $\$20,000\,(0.90)$ and the value of the car after the end of the second year would be $\$20,000(0.90)^2$. Complete the table shown below. What will be the value of the car at the end of the eighth year? Simplify the expression, to show the value in dollars.

Week	Cost
1	$\$20,000\,(0.90)$
2	$\$20,000(0.90)^2$
3	
4	…
8	?

Writing Exercises

82. Use the Product Property for Exponents to explain why $x \cdot x = x^2$.	83. Explain why $-5^3 = (-5)^3$ but $-5^4 \neq (-5)^4$.
84. Jorge thinks $\left(\frac{1}{2}\right)^2$ is 1. What is wrong with his reasoning?	85. Explain why $x^3 \cdot x^5$ is x^8, and not x^{15}

Answers

1. a) 243 b) 9 c) $\frac{1}{9}$ d) 0.0016	3. a) 64 b) 14 c) $\frac{8}{125}$ d) 0.49
5. a) 1,296 b) $-1,296$	7. a) $-\frac{1}{256}$ b) $\frac{1}{256}$
9. a) -0.25 b) 0.25	11. d^9
13. n^{31}	15. a) 4^{14} b) 8^{10}
17. a) y^4 b) z^{33}	19. w^6
21. a^{16}	23. m^{x+3}
25. y^{a+b}	27. a) m^8 b) 10^{18}
29. a) y^{3x} b) 5^{xy}	31. a) $36a^2$ b) $9x^2y^2$
33. a) $-64m^3$ b) $125a^3b^3$	35. a) y^{14} b) $1000a^6b^3$
37. a) $16r^{12}s^8$ b) m^{51}	39. a) $45x^3$ b) $1,125t^8$
41. a) $200a^5$ b) $\frac{1}{18}y^8$	43. a) $\frac{8}{125}x^6y^3$ b) $\frac{64}{81}x^2y^8$
45. a) $16m^8n^{22}$ b) $324p^{14}q^{10}$	47. $30x^8$
49. $72c^5$	51. $9d^7$
53. $42m^5n^8$	55. $15x^8y^2$
57. $\frac{7}{25}x^6y^5$	59. y^{22}
61. b^{47}	63. $81y^8$
65. $32m^5n^{20}$	67. $-1,000u^6v^{12}$
69. $\frac{49}{81}p^2q^8$	71. $1,125r^8$
73. $1,024x^{29}$	75. $m^{28}n^{14}$
77. $200p^{15}q^{14}$	79. $47,037.07
81. $8,609.34	83. and 85. Answers will vary.

Attributions

5.2 Use Quotient Property of Exponents

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions using the Quotient Property for Exponents
- Simplify expressions with zero exponents
- Simplify expressions using the quotient to a Power Property
- Simplify expressions by applying several properties

Simplify Expressions Using the Quotient Property for Exponents

Earlier in this chapter, we developed the properties of exponents for multiplication. We summarize these properties below.

Summary of Exponent Properties for Multiplication

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$

Now we will look at the exponent properties for division. A quick memory refresher may help before we get started. You have learned to simplify fractions by dividing out common factors from the numerator and denominator using the Equivalent Fractions Property. This property will also help you work with algebraic fractions—which are also quotients.

Equivalent Fractions Property

If $a, b,$ and c are whole numbers where $b \neq 0, c \neq 0,$

then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$ and $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$

As before, we'll try to discover a property by looking at some examples.

Consider	$\frac{x^5}{x^2}$	and	$\frac{x^2}{x^3}$
What do they mean?	$\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$		$\frac{x \cdot x}{x \cdot x \cdot x}$
Use the Equivalent Fractions Property.	$\frac{\not{x} \cdot \not{x} \cdot x \cdot x \cdot x}{\not{x} \cdot \not{x}}$		$\frac{\not{x} \cdot \not{x} \cdot 1}{\not{x} \cdot \not{x} \cdot x}$
Simplify.	x^3		$\frac{1}{x}$

Notice, in each case the bases were the same and we subtracted exponents.

When the larger exponent was in the numerator, we were left with factors in the numerator.

When the larger exponent was in the denominator, we were left with factors in the denominator—notice the numerator of 1

We write:

$$\begin{array}{cc} \frac{x^5}{x^2} & \frac{x^2}{x^3} \\ x^{5-2} & \frac{1}{x^{3-2}} \\ x^3 & \frac{1}{x} \end{array}$$

This leads to the *Quotient Property for Exponents*.

Quotient Property for Exponents

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$\frac{a^m}{a^n} = a^{m-n}, m > n$ and $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$

A couple of examples with numbers may help to verify this property.

$$\begin{array}{rlrl} \frac{3^4}{3^2} & = 3^{4-2} & \frac{5^2}{5^3} & = \frac{1}{5^{3-2}} \\ \frac{81}{9} & = 3^2 & \frac{25}{125} & = \frac{1}{5^1} \\ 9 & = 9\checkmark & \frac{1}{5} & = \frac{1}{5}\checkmark \end{array}$$

EXAMPLE 1

Simplify: a) $\frac{x^9}{x^7}$ b) $\frac{3^{10}}{3^2}$.

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

a.

Since 9 > 7, there are more factors of *x* in the numerator.	$\frac{x^9}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	x^{9-7}
Simplify.	x^2

b.

Since 10 > 2, there are more factors of *x* in the numerator.	$\frac{3^{10}}{3^2}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	3^{10-2}
Simplify.	3^8

Notice that when the larger exponent is in the numerator, we are left with factors in the numerator.

TRY IT 1.1

Simplify: a) $\frac{x^{15}}{x^{10}}$ b) $\frac{6^{14}}{6^5}$.

Show answer

a) x^5 b) 6^9

TRY IT 1.2

Simplify: a) $\frac{y^{43}}{y^{37}}$ b) $\frac{10^{15}}{10^7}$.

Show answer

a) y^6 b) 10^8

EXAMPLE 2

Simplify: a) $\frac{b^8}{b^{12}}$ b) $\frac{7^3}{7^5}$.

Solution

To simplify an expression with a quotient, we need to first compare the exponents in the numerator and denominator.

a.

Since 12 > 8, there are more factors of *b* in the denominator.	$\frac{b^8}{b^{12}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{b^{12-8}}$
Simplify.	$\frac{1}{b^4}$

b.

Since 5 > 3, there are more factors of 3 in the denominator.	$\frac{7^3}{7^5}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{7^{5-3}}$
Simplify.	$\frac{1}{7^2}$
Simplify.	$\frac{1}{49}$

Notice that when the larger exponent is in the denominator, we are left with factors in the denominator.

TRY IT 2.1

Simplify: a) $\frac{x^{18}}{x^{22}}$ b) $\frac{12^{15}}{12^{30}}$.

Show answer

a) $\frac{1}{x^4}$ b) $\frac{1}{12^{15}}$

TRY IT 2.2

Simplify: a) $\frac{m^7}{m^{15}}$ b) $\frac{9^8}{9^{19}}$.

Show answer

a) $\frac{1}{m^8}$ b) $\frac{1}{9^{11}}$

Notice the difference in the two previous examples:

- If we start with more factors in the numerator, we will end up with factors in the numerator.
- If we start with more factors in the denominator, we will end up with factors in the denominator.

The first step in simplifying an expression using the Quotient Property for Exponents is to determine whether the exponent is larger in the numerator or the denominator.

EXAMPLE 3

Simplify: a) $\frac{a^5}{a^9}$ b) $\frac{x^{11}}{x^7}$.

Solution

a. Is the exponent of a larger in the numerator or denominator? Since 9 > 5, there are more a's in the denominator and so we will end up with factors in the denominator.

	$\frac{a^5}{a^9}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{a^{9-5}}$
Simplify.	$\frac{1}{a^4}$

b. Notice there are more factors of x in the numerator, since 11 > 7. So we will end up with factors in the numerator.

	$\frac{x^{11}}{x^7}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	x^{11-7}
Simplify.	x^4

TRY IT 3.1

Simplify: a) $\frac{b^{19}}{b^{11}}$ b) $\frac{z^5}{z^{11}}$.

Show answer

a) b^8 b) $\frac{1}{z^6}$

TRY IT 3.2

Simplify: a) $\frac{p^9}{p^{17}}$ b) $\frac{w^{13}}{w^9}$.

Show answer

a) $\frac{1}{p^8}$ b) w^4

Simplify Expressions with an Exponent of Zero

A special case of the Quotient Property is when the exponents of the numerator and denominator are equal, such as an expression like $\frac{a^m}{a^m}$. From your earlier work with fractions, you know that:

$$\frac{2}{2} = 1 \quad \frac{17}{17} = 1 \quad \frac{-43}{-43} = 1$$

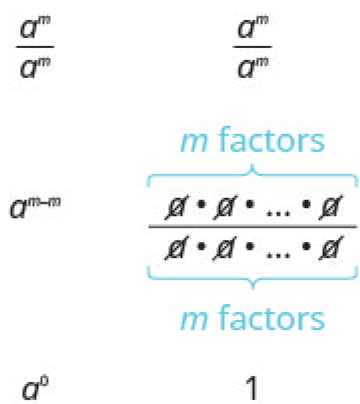
In words, a number divided by itself is 1. So, $\frac{x}{x} = 1$, for any x $(x \neq 0)$, since any number divided by itself is 1

The Quotient Property for Exponents shows us how to simplify $\frac{a^m}{a^n}$ when $m > n$ and when $n < m$ by subtracting exponents. What if $m = n$?

Consider $\frac{8}{8}$, which we know is 1

	$\frac{8}{8} = 1$
Write 8 as 2^3.	$\frac{2^3}{2^3} = 1$
Subtract exponents.	$2^{3-3} = 1$
Simplify.	$2^0 = 1$

Now we will simplify $\frac{a^m}{a^m}$ in two ways to lead us to the definition of the zero exponent. In general, for $a \neq 0$:



We see $\frac{a^m}{a^m}$ simplifies to a^0 and to 1. So $a^0 = 1$.

Zero Exponent

If a is a non-zero number, then $a^0 = 1$.

Any nonzero number raised to the zero power is 1

In this text, we assume any variable that we raise to the zero power is not zero.

EXAMPLE 4

Simplify: a) 9^0 b) n^0.

Solution
The definition says any non-zero number raised to the zero power is 1

a) Use the definition of the zero exponent.	9^0 $= 1$
b) Use the definition of the zero exponent.	n^0 $= 1$

TRY IT 4.1

Simplify: a) 15^0 b) m^0.

Show answer

a) 1 b) 1

TRY IT 4.2

Simplify: a) k^0 b) 29^0.

Show answer

a) 1 b) 1

Now that we have defined the zero exponent, we can expand all the Properties of Exponents to include whole number exponents.

What about raising an expression to the zero power? Let's look at $(2x)^0$. We can use the product to a power rule to rewrite this expression.

	$(2x)^0$
Use the product to a power rule.	2^0x^0
Use the zero exponent property.	$1 \cdot 1$
Simplify.	1

This tells us that any nonzero expression raised to the zero power is one.

EXAMPLE 5

Simplify: a) $(5b)^0$ b) $\left(-4a^2b\right)^0$.

Solution

a)	$(5b)^0$
Use the definition of the zero exponent.	1
b)	$\left(-4a^2b\right)^0$
Use the definition of the zero exponent.	1

TRY IT 5.1

Simplify: a) $(11z)^0$ b) $\left(-11pq^3\right)^0$.

Show answer
a) 1 b) 1

TRY IT 5.2

Simplify: a) $(-6d)^0$ b) $\left(-8m^2n^3\right)^0$.

Show answer
a) 1 b) 1

Simplify Expressions Using the Quotient to a Power Property

Now we will look at an example that will lead us to the Quotient to a Power Property.

	$\left(\frac{x}{y}\right)^3$
This means:	$\frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y}$
Multiply the fractions.	$\frac{x \cdot x \cdot x}{y \cdot y \cdot y}$
Write with exponents.	$\frac{x^3}{y^3}$

Notice that the exponent applies to both the numerator and the denominator.

We write:	$\left(\frac{x}{y}\right)^3$
	$\frac{x^3}{y^3}$

This leads to the *Quotient to a Power Property for Exponents*.

Quotient to a Power Property for Exponents

If a and b are real numbers, $b \neq 0$, and m is a counting number, then

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

To raise a fraction to a power, raise the numerator and denominator to that power.

An example with numbers may help you understand this property:

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$$
$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$
$$\frac{8}{27} = \frac{8}{27} \checkmark$$

EXAMPLE 6

Simplify: a) $\left(\frac{3}{7}\right)^2$ b) $\left(\frac{b}{3}\right)^4$ c) $\left(\frac{k}{j}\right)^3$.

Solution

a)

	$\left(\frac{3}{7}\right)^2$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{3^2}{7^2}$
Simplify.	$\frac{9}{49}$

b)

	$\left(\frac{b}{3}\right)^4$
Use the Quotient Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{b^4}{3^4}$
Simplify.	$\frac{b^4}{81}$

c)

	$\left(\frac{k}{j}\right)^3$
Raise the numerator and denominator to the third power.	$\frac{k^3}{j^3}$

TRY IT 6.1

Simplify: a) $\left(\frac{5}{8}\right)^2$ b) $\left(\frac{p}{10}\right)^4$ c) $\left(\frac{m}{n}\right)^7$.

Show answer

a) $\frac{25}{64}$ b) $\frac{p^4}{10,000}$ c) $\frac{m^7}{n^7}$

TRY IT 6.2

Simplify: a) $\left(\frac{1}{3}\right)^3$ b) $\left(\frac{-2}{q}\right)^3$ c) $\left(\frac{w}{x}\right)^4$.

Show answer

a) $\frac{1}{27}$ b) $\frac{-8}{q^3}$ c) $\frac{w^4}{x^4}$

Simplify Expressions by Applying Several Properties

We'll now summarize all the properties of exponents so they are all together to refer to as we simplify expressions using several properties. Notice that they are now defined for whole number exponents.

Summary of Exponent Properties

If a and b are real numbers, and m and n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0, m > n$ $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, a \neq 0, n > m$
Zero Exponent Definition	$a^o = 1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

EXAMPLE 7

Simplify: $\frac{\left(y^4\right)^2}{y^6}$.

Solution

	$\frac{\left(y^4\right)^2}{y^6}$
Multiply the exponents in the numerator.	$\frac{y^8}{y^6}$
Subtract the exponents.	y^2

TRY IT 7.1

Simplify: $\dfrac{\left(m^5\right)^4}{m^7}$.

Show answer
m^{13}

TRY IT 7.2

Simplify: $\dfrac{\left(k^2\right)^6}{k^7}$.

Show answer
k^5

EXAMPLE 8

Simplify: $\dfrac{b^{12}}{(b^2)^6}$.

Solution

	$\dfrac{b^{12}}{(b^2)^6}$
Multiply the exponents in the numerator.	$\dfrac{b^{12}}{b^{12}}$
Subtract the exponents.	b^0
Simplify.	1

TRY IT 8.1

Simplify: $\dfrac{n^{12}}{(n^3)^4}$.

Show answer
1

TRY IT 8.2

Simplify: $\frac{x^{15}}{(x^3)^5}$.

Show answer
1

EXAMPLE 9

Simplify: $\left(\frac{y^9}{y^4}\right)^2$.

Solution

	$\left(\frac{y^9}{y^4}\right)^2$
Remember parentheses come before exponents. Notice the bases are the same, so we can simplify inside the parentheses. Subtract the exponents.	$(y^5)^2$
Multiply the exponents.	y^{10}

TRY IT 9.1

Simplify: $\left(\frac{r^5}{r^3}\right)^4$.

Show answer
r^8

TRY IT 9.2

Simplify: $\left(\frac{v^6}{v^4}\right)^3$.

Show answer
v^6

EXAMPLE 10

Simplify: $\left(\frac{j^2}{k^3}\right)^4$.

Solution
Here we cannot simplify inside the parentheses first, since the bases are not the same.

	$\left(\frac{j^2}{k^3}\right)^4$
Raise the numerator and denominator to the third power using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{j^{2\cdot 4}}{k^{3\cdot 4}}$
Use the Power Property and simplify.	$\frac{j^8}{k^{12}}$

TRY IT 10.1

Simplify: $\left(\frac{a^3}{b^2}\right)^4$.

Show answer
$\frac{a^{12}}{b^8}$

TRY IT 10.2

Simplify: $\left(\frac{q^7}{r^5}\right)^3$.

Show answer
$\frac{q^{21}}{r^{15}}$

EXAMPLE 11

Simplify: $\left(\frac{2m^2}{5n}\right)^4$.

Solution

	$\left(\frac{2m^2}{5n}\right)^4$
Raise the numberator and denominator to the fourth power, using the Quotient to a Power Property, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.	$\frac{\left(2m^2\right)^4}{(5n)^4}$
Raise each factor to the fourth power.	$\frac{\left(2m^2\right)^4}{(5n)^4}$
Use the Power Property and simplify.	$\frac{16m^8}{625n^4}$

TRY IT 11.1

Simplify: $\left(\frac{7x^3}{9y}\right)^2$.

Show answer

$\frac{49x^6}{81y^2}$

TRY IT 11.2

Simplify: $\left(\frac{3x^4}{7y}\right)^2$.

Show answer

$\frac{9x^8}{49y^2}$

EXAMPLE 12

Simplify: $\frac{\left(x^3\right)^4\left(x^2\right)^5}{\left(x^6\right)^5}$.

Solution

	$\frac{\left(x^3\right)^4\left(x^2\right)^5}{\left(x^6\right)^5}$
Use the Power Property, $\left(a^m\right)^n = a^{m\cdot n}$.	$\frac{\left(x^{12}\right)\left(x^{10}\right)}{\left(x^{30}\right)}$
Add the exponents in the numerator.	$\frac{x^{22}}{x^{30}}$
Use the Quotient Property, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\frac{1}{x^8}$

TRY IT 12.1

Simplify: $\frac{\left(a^2\right)^3\left(a^2\right)^4}{\left(a^4\right)^5}$.

Show answer

$\frac{1}{a^6}$

TRY IT 12.2

Simplify: $\frac{\left(p^3\right)^4\left(p^5\right)^3}{\left(p^7\right)^6}$.

Show answer

$\frac{1}{p^{15}}$

EXAMPLE 13

Simplify: $\dfrac{\left(10p^3\right)^2}{(5p)^3\left(2p^5\right)^4}$.

Solution

	$\dfrac{\left(10p^3\right)^2}{(5p)^3\left(2p^5\right)^4}$
Use the Product to a Power Property, $(ab)^m = a^m b^m$.	$\dfrac{(10)^2\left(p^3\right)^2}{(5)^3(p)^3(2)^4\left(p^5\right)^4}$
Use the Power Property, $\left(a^m\right)^n = a^{m \cdot n}$.	$\dfrac{100p^6}{125p^3 \cdot 16p^{20}}$
Add the exponents in the denominator.	$\dfrac{100p^6}{125 \cdot 16p^{23}}$
Use the Quotient Property, $\dfrac{a^m}{a^n} = \frac{1}{a^{n-m}}$.	$\dfrac{100}{125 \cdot 16p^{17}}$
Simplify.	$\dfrac{1}{20p^{17}}$

TRY IT 13.1

Simplify: $\dfrac{\left(3r^3\right)^2\left(r^3\right)^7}{\left(r^3\right)^3}$.

Show answer
$9r^{18}$

TRY IT 13.2

Simplify: $\dfrac{\left(2x^4\right)^5}{\left(4x^3\right)^2\left(x^3\right)^5}$.

Show answer

$\dfrac{2}{x}$

Divide Monomials

You have now been introduced to all the properties of exponents and used them to simplify expressions. Next, you'll see how to use these properties to divide monomials. Later, you'll use them to divide polynomials.

EXAMPLE 14

Find the quotient: $56x^7 \div 8x^3$.

Solution

	$56x^7 \div 8x^3$
Rewrite as a fraction.	$\dfrac{56x^7}{8x^3}$
Use fraction multiplication.	$\dfrac{56}{8} \cdot \dfrac{x^7}{x^3}$
Simplify and use the Quotient Property.	$7x^4$

TRY IT 14.1

Find the quotient: $42y^9 \div 6y^3$.

Show answer
$7y^6$

TRY IT 14.2

Find the quotient: $48z^8 \div 8z^2$.

Show answer
$6z^6$

EXAMPLE 15

Find the quotient: $\dfrac{45a^2b^3}{-5ab^5}$.

Solution

	$\dfrac{45a^2b^3}{-5ab^5}$
Use fraction multiplication.	$\dfrac{45}{-5} \cdot \dfrac{a^2}{a} \cdot \dfrac{b^3}{b^5}$
Simplify and use the Quotient Property.	$-9 \cdot a \cdot \dfrac{1}{b^2}$
Multiply.	$-\dfrac{9a}{b^2}$

TRY IT 15.1

Find the quotient: $\dfrac{-72a^7b^3}{8a^{12}b^4}$.

Show answer

$-\dfrac{9}{a^5b}$

TRY IT 15.2

Find the quotient: $\dfrac{-63c^8d^3}{7c^{12}d^2}$.

Show answer

$\dfrac{-9d}{c^4}$

EXAMPLE 16

Find the quotient: $\dfrac{24a^5b^3}{48ab^4}$.

Solution

	$\frac{24a^5b^3}{48ab^4}$
Use fraction multiplication.	$\frac{24}{48} \cdot \frac{a^5}{a} \cdot \frac{b^3}{b^4}$
Simplify and use the Quotient Property.	$\frac{1}{2} \cdot a^4 \cdot \frac{1}{b}$
Multiply.	$\frac{a^4}{2b}$

TRY IT 16.1

Find the quotient: $\frac{16a^7b^6}{24ab^8}$.

Show answer

$\frac{2a^6}{3b^2}$

TRY IT 16.2

Find the quotient: $\frac{27p^4q^7}{-45p^{12}q}$.

Show answer

$-\frac{3q^6}{5p^8}$

Once you become familiar with the process and have practiced it step by step several times, you may be able to simplify a fraction in one step.

EXAMPLE 17

Find the quotient: $\frac{14x^7y^{12}}{21x^{11}y^6}$.

Solution

Be very careful to simplify $\frac{14}{21}$ by dividing out a common factor, and to simplify the variables by subtracting their exponents.

	$\frac{14x^7y^{12}}{21x^{11}y^6}$
Simplify and use the Quotient Property.	$\frac{2y^6}{3x^4}$

TRY IT 17.1

Find the quotient: $\frac{28x^5y^{14}}{49x^9y^{12}}$.

Show answer

$\frac{4y^2}{7x^4}$

TRY IT 17.2

Find the quotient: $\frac{30m^5n^{11}}{48m^{10}n^{14}}$.

Show answer

$\frac{5}{8m^5n^3}$

In all examples so far, there was no work to do in the numerator or denominator before simplifying the fraction. In the next example, we'll first find the product of two monomials in the numerator before we simplify the fraction. This follows the order of operations. Remember, a fraction bar is a grouping symbol.

EXAMPLE 18

Find the quotient: $\frac{(6x^2y^3)(5x^3y^2)}{(3x^4y^5)}$.

Solution

	$\dfrac{\left(6x^2y^3\right)\left(5x^3y^2\right)}{\left(3x^4y^5\right)}$
Simplify the numerator.	$\dfrac{30x^5y^5}{3x^4y^5}$
Simplify.	$10x$

TRY IT 18.1

Find the quotient: $\dfrac{\left(6a^4b^5\right)\left(4a^2b^5\right)}{12a^5b^8}$.

Show answer
$2ab^2$

TRY IT 18.2

Find the quotient: $\dfrac{\left(-12x^6y^9\right)\left(-4x^5y^8\right)}{-12x^{10}y^{12}}$.

Show answer
$-4xy^5$

Additional Online Resources

- Rational Expressions
- Dividing Monomials
- Dividing Monomials 2

Key Concepts

- **Quotient Property for Exponents:**
    - If a is a real number, $a \neq 0$, and m, n are whole numbers, then:
      $\dfrac{a^m}{a^n} = a^{m-n}, m > n$ and $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}, n > m$

- **Zero Exponent**
  - If a is a non-zero number, then $a^0 = 1$.
- **Quotient to a Power Property for Exponents**:
  - If a and b are real numbers, $b \neq 0$, and m is a counting number, then: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
  - To raise a fraction to a power, raise the numerator and denominator to that power.
- **Summary of Exponent Properties**
  - If a, b are real numbers and m, n are whole numbers, then

Product Property	$a^m \cdot a^n = a^{m+n}$
Power Property	$(a^m)^n = a^{m \cdot n}$
Product to a Power	$(ab)^m = a^m b^m$
Quotient Property	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0, m > n$ $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}, a \neq 0, n > m$
Zero Exponent Definition	$a^o = 1, a \neq 0$
Quotient to a Power Property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Practice Makes Perfect

Simplify Expressions Using the Quotient Property for Exponents

In the following exercises, simplify.

1. a) $\frac{x^{18}}{x^3}$ b) $\frac{5^{12}}{5^3}$	2. a) $\frac{y^{20}}{y^{10}}$ b) $\frac{7^{16}}{7^2}$
3. a) $\frac{p^{21}}{p^7}$ b) $\frac{4^{16}}{4^4}$	4. a) $\frac{u^{24}}{u^3}$ b) $\frac{9^{15}}{9^5}$
5. a) $\frac{q^{18}}{q^{36}}$ b) $\frac{10^2}{10^3}$	6. a) $\frac{t^{10}}{t^{40}}$ b) $\frac{8^3}{8^5}$
7. a) $\frac{b}{b^9}$ b) $\frac{4}{4^6}$	8. a) $\frac{x}{x^7}$ b) $\frac{10}{10^3}$

Simplify Expressions with Zero Exponents

In the following exercises, simplify.

9. a) 20^0 b) b^0	10. a) 13^0 b) k^0
11. a) -27^0 b) $-\left(27^0\right)$	12. a) -15^0 b) $-\left(15^0\right)$
13. a) $(25x)^0$ b) $25x^0$	14. a) $(6y)^0$ b) $6y^0$
15. a) $(12x)^0$ b) $\left(-56p^4q^3\right)^0$	16. a) $7y^0(17y)^0$ b) $\left(-93c^7d^{15}\right)^0$
17. a) $12n^0-18m^0$ b) $(12n)^0-(18m)^0$	18. a) $15r^0-22s^0$ b) $(15r)^0-(22s)^0$

Simplify Expressions Using the Quotient to a Power Property

In the following exercises, simplify.

19. a) $\left(\frac{3}{4}\right)^3$ b) $\left(\frac{p}{2}\right)^5$ c) $\left(\frac{x}{y}\right)^6$	20. a) $\left(\frac{2}{5}\right)^2$ b) $\left(\frac{x}{3}\right)^4$ c) $\left(\frac{a}{b}\right)^5$
21. a) $\left(\frac{a}{3b}\right)^4$ b) $\left(\frac{5}{4m}\right)^2$	22. a) $\left(\frac{x}{2y}\right)^3$ b) $\left(\frac{10}{3q}\right)^4$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify.

23. $\frac{\left(a^2\right)^3}{a^4}$	24. $\frac{\left(p^3\right)^4}{p^5}$
25. $\frac{\left(y^3\right)^4}{y^{10}}$	26. $\frac{\left(x^4\right)^5}{x^{15}}$
27. $\frac{u^6}{(u^3)^2}$	28. $\frac{v^{20}}{(v^4)^5}$
29. $\frac{m^{12}}{(m^8)^3}$	30. $\frac{n^8}{(n^6)^4}$
31. $\left(\frac{p^9}{p^3}\right)^5$	32. $\left(\frac{q^8}{q^2}\right)^3$
33. $\left(\frac{r^2}{r^6}\right)^3$	34. $\left(\frac{m^4}{m^7}\right)^4$
35. $\left(\frac{p}{r^{11}}\right)^2$	36. $\left(\frac{a}{b^6}\right)^3$
37. $\left(\frac{w^5}{x^3}\right)^8$	38. $\left(\frac{y^4}{z^{10}}\right)^5$
39. $\left(\frac{2j^3}{3k}\right)^4$	40. $\left(\frac{3m^5}{5n}\right)^3$
41. $\left(\frac{3c^2}{4d^6}\right)^3$	42. $\left(\frac{5u^7}{2v^3}\right)^4$
43. $\left(\frac{k^2k^8}{k^3}\right)^2$	44. $\left(\frac{j^2j^5}{j^4}\right)^3$
45. $\frac{\left(t^2\right)^5\left(t^4\right)^2}{(t^3)^7}$	46. $\frac{\left(q^3\right)^6\left(q^2\right)^3}{(q^4)^8}$
47. $\frac{\left(-2p^2\right)^4\left(3p^4\right)^2}{(-6p^3)^2}$	48. $\frac{\left(-2k^3\right)^2\left(6k^2\right)^4}{(9k^4)^2}$
49. $\frac{\left(-4m^3\right)^2\left(5m^4\right)^3}{(-10m^6)^3}$	50. $\frac{\left(-10n^2\right)^3\left(4n^5\right)^2}{(2n^8)^2}$

Divide Monomials

In the following exercises, divide the monomials.

51. $56b^8 \div 7b^2$	52. $63v^{10} \div 9v^2$
53. $-88y^{15} \div 8y^3$	54. $-72u^{12} \div 12u^4$
55. $\frac{45a^6b^8}{-15a^{10}b^2}$	56. $\frac{54x^9y^3}{-18x^6y^{15}}$
57. $\frac{15r^4s^9}{18r^9s^2}$	58. $\frac{20m^8n^4}{30m^5n^9}$
59. $\frac{18a^4b^8}{-27a^9b^5}$	60. $\frac{45x^5y^9}{-60x^8y^6}$
61. $\frac{64q^{11}r^9s^3}{48q^6r^8s^5}$	62. $\frac{65a^{10}b^8c^5}{42a^7b^6c^8}$
63. $\frac{\left(10m^5n^4\right)\left(5m^3n^6\right)}{25m^7n^5}$	64. $\frac{\left(-18p^4q^7\right)\left(-6p^3q^8\right)}{-36p^{12}q^{10}}$
65. $\frac{\left(6a^4b^3\right)\left(4ab^5\right)}{(12a^2b)(a^3b)}$	66. $\frac{\left(4u^2v^5\right)\left(15u^3v\right)}{(12u^3v)(u^4v)}$

Mixed Practice

67. a) $24a^5 + 2a^5$ b) $24a^5 - 2a^5$ c) $24a^5 \cdot 2a^5$ d) $24a^5 \div 2a^5$	68. a) $15n^{10} + 3n^{10}$ b) $15n^{10} - 3n^{10}$ c) $15n^{10} \cdot 3n^{10}$ d) $15n^{10} \div 3n^{10}$
69. a) $p^4 \cdot p^6$ b) $\left(p^4\right)^6$	70. a) $q^5 \cdot q^3$ b) $\left(q^5\right)^3$
71. a) $\frac{y^3}{y}$ b) $\frac{y}{y^3}$	72. a) $\frac{z^6}{z^5}$ b) $\frac{z^5}{z^6}$
73. $\left(8x^5\right)(9x) \div 6x^3$	74. $(4y)\left(12y^7\right) \div 8y^2$
75. $\frac{27a^7}{3a^3} + \frac{54a^9}{9a^5}$	76. $\frac{32c^{11}}{4c^5} + \frac{42c^9}{6c^3}$
77. $\frac{32y^5}{8y^2} - \frac{60y^{10}}{5y^7}$	78. $\frac{48x^6}{6x^4} - \frac{35x^9}{7x^7}$
79. $\frac{63r^6s^3}{9r^4s^2} - \frac{72r^2s^2}{6s}$	80. $\frac{56y^4z^5}{7y^3z^3} - \frac{45y^2z^2}{5y}$

Everyday Math

81. **Memory** One megabyte is approximately 10^6 bytes. One gigabyte is approximately 10^9 bytes. How many megabytes are in one gigabyte?	82. **Memory** One gigabyte is approximately 10^9 bytes. One terabyte is approximately 10^{12} bytes. How many gigabytes are in one terabyte?

Writing Exercises

83. Jennifer thinks the quotient $\frac{a^{24}}{a^6}$ simplifies to a^4. What is wrong with her reasoning?	84. Maurice simplifies the quotient $\frac{d^7}{d}$ by writing $\frac{\cancel{d}^7}{\cancel{d}} = 7$. What is wrong with his reasoning?
85. When Drake simplified -3^0 and $(-3)^0$ he got the same answer. Explain how using the Order of Operations correctly gives different answers.	86. Robert thinks x^0 simplifies to 0. What would you say to convince Robert he is wrong?

Answers

1. a) x^{15} b) 5^9	3. a) p^{14} b) 4^{12}	5. $\frac{1}{q^{18}}$ b) $\frac{1}{10}$
7. a) $\frac{1}{b^8}$ b) $\frac{1}{4^5}$	9. a) 1 b) 1	11. a) -1 b) -1
13. a) 1 b) 25	15. a) 1 b) 1	17. a) -6 b) 0
19. a) $\frac{27}{64}$ b) $\frac{p^5}{32}$ c) $\frac{x^6}{y^6}$	21. a) $\frac{a^4}{81b^4}$ b) $\frac{25}{16m^2}$	23. a^2
25. $\frac{1}{y^4}$	27. 1	29. $\frac{1}{m^{12}}$
31. p^{30}	33. $\frac{1}{r^{12}}$	35. $\frac{p2}{r^{22}}$
37. $\frac{w^{40}}{x^{24}}$	39. $\frac{16j^{12}}{81k^4}$	41. $\frac{27c^6}{64d^{18}}$
43. k^{14}	45. $\frac{1}{t^3}$	47. $4p^{10}$
49. -2	51. $8b^6$	53. $-11u^{12}$
55. $-\frac{3b^6}{a^4}$	57. $\frac{3s^7}{5r^5}$	59. $-\frac{2b^3}{3a^5}$
61. $\frac{4q^5r}{3s^2}$	63. $2mn^5$	65. $2b^6$
67. a) $26a^5$ b) $22a^5$ c) $48a^{10}$ d) 12	69. a) p^{10} b) p^{24}	71. a) y^2 b) $\frac{1}{y^2}$
73. $12x^3$	75. $15a^4$	77. $-8y^3$
81. 1000	83. Answers will vary.	85. Answers will vary.

Attributions

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5.3 Integer Exponents and Scientific Notation

Learning Objectives

By the end of this section, you will be able to:

- Use the definition of a negative exponent
- Simplify expressions with integer exponents
- Convert from decimal notation to scientific notation
- Convert scientific notation to decimal form
- Multiply and divide using scientific notation

Use the Definition of a Negative Exponent

We saw that the Quotient Property for Exponents introduced earlier in this chapter, has two forms depending on whether the exponent is larger in the numerator or the denominator.

Quotient Property for Exponents

If a is a real number, $a \neq 0$, and m and n are whole numbers, then

$$\frac{a^m}{a^n} = a^{m-n}, m > n \quad \text{and} \quad \frac{a^m}{a^n} = \frac{1}{a^{n-m}}, n > m$$

What if we just subtract exponents regardless of which is larger?

Let's consider $\frac{x^2}{x^5}$.

We subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{x^2}{x^5}$$

$$x^{2-5}$$

$$x^{-3}$$

We can also simplify $\frac{x^2}{x^5}$ by dividing out common factors:

$$\frac{\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x}$$

$$\frac{1}{x^3}$$

This implies that $x^{-3} = \frac{1}{x^3}$ and it leads us to the definition of a *negative exponent*.

Negative Exponent

If n is an integer and $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$.

The negative exponent tells us we can re-write the expression by taking the reciprocal of the base and then changing the sign of the exponent.

Any expression that has negative exponents is not considered to be in simplest form. We will use the definition of a negative exponent and other properties of exponents to write the expression with only positive exponents.

For example, if after simplifying an expression we end up with the expression x^{-3}, we will take one more step and write $\frac{1}{x^3}$. The answer is considered to be in simplest form when it has only positive exponents.

EXAMPLE 1

Simplify: a) 4^{-2} b) 10^{-3}.

Solution

a)	4^{-2}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{4^2}$
Simplify.	$\frac{1}{16}$
b)	10^{-3}
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{10^3}$
Simplify.	$\frac{1}{1000}$

TRY IT 1.1

Simplify: a) 2^{-3} b) 10^{-7}.

Show answer

a) $\frac{1}{8}$ b) $\frac{1}{10^7}$

TRY IT 1.2

Simplify: a) 3^{-2} b) 10^{-4}.

Show answer

a) $\frac{1}{9}$ b) $\frac{1}{10{,}000}$

In (Example 1) we raised an integer to a negative exponent. What happens when we raise a fraction to a negative exponent? We'll start by looking at what happens to a fraction whose numerator is one and whose denominator is an integer raised to a negative exponent.

	$\frac{1}{a^{-n}}$
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{\frac{1}{a^n}}$
Simplify the complex fraction.	$1 \cdot \frac{a^n}{1}$
Multiply.	a^n

This leads to the Property of Negative Exponents.

Property of Negative Exponents

If n is an integer and $a \neq 0$, then $\frac{1}{a^{-n}} = a^n$.

EXAMPLE 2

Simplify: a) $\frac{1}{y^{-4}}$ b) $\frac{1}{3^{-2}}$.

Solution

a)	$\frac{1}{y^{-4}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}} = a^n$.	y^4
b)	$\frac{1}{3^{-2}}$
Use the property of a negative exponent, $\frac{1}{a^{-n}} = a^n$.	3^2
Simplify.	9

TRY IT 2.1

Simplify: a) $\frac{1}{p^{-8}}$ b) $\frac{1}{4^{-3}}$.

Show answer

a) p^8 b) 64

TRY IT 2.2

Simplify: a) $\frac{1}{q^{-7}}$ b) $\frac{1}{2^{-4}}$.

Show answer

a) q^7 b) 16

Suppose now we have a fraction raised to a negative exponent. Let's use our definition of negative exponents to lead us to a new property.

	$\left(\frac{3}{4}\right)^{-2}$
Use the definition of a negative exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{\left(\frac{3}{4}\right)^2}$
Simplify the denominator.	$\frac{1}{\frac{9}{16}}$
Simplify the complex fraction.	$\frac{16}{9}$
But we know that $\frac{16}{9}$ is $\left(\frac{4}{3}\right)^2$.	
This tells us that:	$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$

To get from the original fraction raised to a negative exponent to the final result, we took the reciprocal of the base—the fraction—and changed the sign of the exponent.

This leads us to the *Quotient to a Negative Power Property*.

Quotient to a Negative Exponent Property

If a and b are real numbers, $a \neq 0, b \neq 0$, and n is an integer, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$.

EXAMPLE 3

Simplify: a) $\left(\frac{5}{7}\right)^{-2}$ b) $\left(-\frac{2x}{y}\right)^{-3}$.

Solution

a)	$\left(\frac{5}{7}\right)^{-2}$
Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$.	
Take the reciprocal of the fraction and change the sign of the exponent.	$\left(\frac{7}{5}\right)^{2}$
Simplify.	$\frac{49}{25}$
b)	$\left(-\frac{2x}{y}\right)^{-3}$
Use the Quotient to a Negative Exponent Property, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$.	
Take the reciprocal of the fraction and change the sign of the exponent.	$\left(-\frac{y}{2x}\right)^{3}$
Simplify.	$-\frac{y^3}{8x^3}$

TRY IT 3.1

Simplify: a) $\left(\frac{2}{3}\right)^{-4}$ b) $\left(-\frac{6m}{n}\right)^{-2}$.

Show answer

a) $\frac{81}{16}$ b) $\frac{n^2}{36m^2}$

TRY IT 3.2

Simplify: a) $\left(\frac{3}{5}\right)^{-3}$ b) $\left(-\frac{a}{2b}\right)^{-4}$.

Show answer

a) $\frac{125}{27}$ b) $\frac{16b^4}{a^4}$

When simplifying an expression with exponents, we must be careful to correctly identify the base.

EXAMPLE 4

Simplify: a) $(-3)^{-2}$ b) -3^{-2} c) $\left(-\frac{1}{3}\right)^{-2}$ d) $-\left(\frac{1}{3}\right)^{-2}$.

Solution

a) Here the exponent applies to the base -3.	$(-3)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\frac{1}{(-3)^2}$
Simplify.	$\frac{1}{9}$
b) The expression -3^{-2} means "find the opposite of 3^{-2}." Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.	-3^{-2}
Rewrite as a product with -1.	$-1 \cdot 3^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \frac{1}{3^2}$
Simplify.	$-\frac{1}{9}$
c) Here the exponent applies to the base $\left(-\frac{1}{3}\right)$.	$\left(-\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$\left(-\frac{3}{1}\right)^{2}$
Simplify.	9
d) The expression $-\left(\frac{1}{3}\right)^{-2}$ means "find the opposite of $\left(\frac{1}{3}\right)^{-2}$." Here the exponent applies to the base $\left(\frac{1}{3}\right)$.	
Rewrite as a product with -1.	$-1 \cdot \left(\frac{1}{3}\right)^{-2}$
Take the reciprocal of the base and change the sign of the exponent.	$-1 \cdot \left(\frac{3}{1}\right)^{2}$
Simplify.	-9

TRY IT 4.1

Simplify: a) $(-5)^{-2}$ b) -5^{-2} c) $\left(-\frac{1}{5}\right)^{-2}$ d) $-\left(\frac{1}{5}\right)^{-2}$.

Show answer

a) $\frac{1}{25}$ b) $-\frac{1}{25}$ c) 25 d) -25

TRY IT 4.2

Simplify: a) $(-7)^{-2}$ b) -7^{-2}, c) $\left(-\frac{1}{7}\right)^{-2}$ d) $-\left(\frac{1}{7}\right)^{-2}$.

Show answer

a) $\frac{1}{49}$ b) $-\frac{1}{49}$ c) 49 d) -49

We must be careful to follow the Order of Operations. In the next example, parts (a) and (b) look similar, but the results are different.

EXAMPLE 5

Simplify: a) $4 \cdot 2^{-1}$ b) $(4 \cdot 2)^{-1}$.

Solution

a) Do exponents before multiplication.	$4 \cdot 2^{-1}$
Use $a^{-n} = \frac{1}{a^n}$.	$4 \cdot \frac{1}{2^1}$
Simplify.	2
b)	$(4 \cdot 2)^{-1}$
Simplify inside the parentheses first.	$(8)^{-1}$
Use $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{8^1}$
Simplify.	$\frac{1}{8}$

TRY IT 5.1

Simplify: a) $6 \cdot 3^{-1}$ b) $(6 \cdot 3)^{-1}$.

Show answer

a) 2 b) $\frac{1}{18}$

TRY IT 5.2

Simplify: a) $8 \cdot 2^{-2}$ b) $(8 \cdot 2)^{-2}$.

Show answer
a) 2 b) $\frac{1}{256}$

When a variable is raised to a negative exponent, we apply the definition the same way we did with numbers. We will assume all variables are non-zero.

EXAMPLE 6

Simplify: a) x^{-6} b) $\left(u^4\right)^{-3}$.

Solution

a)	x^{-6}
Use the definition of a negative exponent $a^{-n} = \frac{1}{a^n}$	$\frac{1}{x^6}$
b)	$\left(u^4\right)^{-3}$
Use the definition of a negative exponent $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{(u^4)^3}$
Simplify.	$\frac{1}{u^{12}}$

TRY IT 6.1

Simplify: a) y^{-7} b) $\left(z^3\right)^{-5}$

Show answer
a) $\frac{1}{y^7}$ b) $\frac{1}{z^{15}}$

TRY IT 6.2

Simplify: a) p^{-9} b) $\left(q^4\right)^{-6}$.

Show answer
a) $\frac{1}{p^9}$ b) $\frac{1}{q^{24}}$

When there is a product and an exponent we have to be careful to apply the exponent to the correct quantity. According to the Order of Operations, we simplify expressions in parentheses before applying exponents. We'll see how this works in the next example.

EXAMPLE 7

Simplify: a) $5y^{-1}$ b) $(5y)^{-1}$ c) $(-5y)^{-1}$.

Solution

a) Notice the exponent applies to just the base.	$5y^{-1}$
Take the reciprocal of y and change the sign of the exponent.	$5 \cdot \frac{1}{y^1}$
Simplify.	$\frac{5}{y}$
b) Here the parentheses make the exponent apply to the base.	$(5y)^{-1}$
Take the reciprocal of $5y$ and change the sign of the exponent.	$\frac{1}{(5y)^1}$
Simplify.	$\frac{1}{5y}$
c) The base here is $-5y$.	$(-5y)^{-1}$
Take the reciprocal of $-5y$ and change the sign of the exponent.	$\frac{1}{(-5y)^1}$
Simplify.	$\frac{1}{-5y}$
Use $\frac{a}{-b} = -\frac{a}{b}$	$-\frac{1}{5y}$

TRY IT 7.1

Simplify: a) $8p^{-1}$ b) $(8p)^{-1}$ c) $(-8p)^{-1}$.

Show answer
a) $\frac{8}{p}$ b) $\frac{1}{8p}$ c) $-\frac{1}{8p}$

TRY IT 7.2

Simplify: a) $11q^{-1}$ b) $(11q)^{-1}$ c) $(-11q)^{-1}$.

Show answer
a) $\frac{11}{q}$ b) $\frac{1}{11q}$ c) $-\frac{1}{11q}$

With negative exponents, the Quotient Rule needs only one form $\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$. When the exponent in the denominator is larger than the exponent in the numerator, the exponent of the quotient will be negative.

Simplify Expressions with Integer Exponents

All of the exponent properties we developed earlier in the chapter with whole number exponents apply to integer exponents, too. We restate them here for reference.

Summary of Exponent Properties

If a and b are real numbers, and m and n are integers, then

$$\begin{array}{ccc}
\text{Product Property} & a^m \cdot a^n = & a^{m+n} \\
\text{Power Property} & (a^m)^n = & a^{m \cdot n} \\
\text{Product to a Power} & (ab)^m = & a^m b^m \\
\text{Quotient Property} & \frac{a^m}{a^n} = & a^{m-n}, a \neq 0 \\
\text{Zero Exponent Property} & a^0 = & 1, a \neq 0 \\
\text{Quotient to a Power Property} & \left(\frac{a}{b}\right)^m = & \frac{a^m}{b^m},\ b \neq 0 \\
\text{Properties of Negative Exponents} & a^{-n} = & \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n \\
\text{Quotient to a Negative Exponent} & \left(\frac{a}{b}\right)^{-n} = & \left(\frac{b}{a}\right)^n
\end{array}$$

EXAMPLE 8

Simplify: a) $x^{-4} \cdot x^6$ b) $y^{-6} \cdot y^4$ c) $z^{-5} \cdot z^{-3}$.

Solution

a.

	$x^{-4} \cdot x^{6}$
Use the Product Property, $a^m \cdot a^n = a^{m+n}$.	x^{-4+6}
Simplify	x^2

b.

	$y^{-6} \cdot y^{4}$
Notice the same bases, so add the exponents.	y^{-6+4}
Simplify.	y^{-2}
Use the definition of a negative exponent, $\frac{1}{a^n}$.	$\frac{1}{y^2}$

c.

	$z^{-5} \cdot z^{-3}$
Add the exponents, since the bases are the same.	z^{-5-3}
Simplify.	z^{-8}
Take the reciprocal and change the sign of the exponent, using the definition of a negative exponent.	$\frac{1}{z^8}$

TRY IT 8.1

Simplify: a) $x^{-3} \cdot x^{7}$ b) $y^{-7} \cdot y^{2}$ c) $z^{-4} \cdot z^{-5}$.

Show answer

a) x^4 b) $\frac{1}{y^5}$ c) $\frac{1}{z^9}$

TRY IT 8.2

Simplify: a) $a^{-1} \cdot a^{6}$ b) $b^{-8} \cdot b^{4}$ c) $c^{-8} \cdot c^{-7}$.

Show answer

a) a^5 b) $\frac{1}{b^4}$ c) $\frac{1}{c^{15}}$

In the next two examples, we'll start by using the Commutative Property to group the same variables together. This makes it easier to identify the like bases before using the Product Property.

EXAMPLE 9

Simplify: $\left(m^4n^{-3}\right)\left(m^{-5}n^{-2}\right)$.

Solution

	$\left(m^4n^{-3}\right)\left(m^{-5}n^{-2}\right)$
Use the Commutative Property to get like bases together.	$m^4m^{-5}\cdot n^{-2}n^{-3}$
Add the exponents for each base.	$m^{-1}\cdot n^{-5}$
Take the reciprocals and change the signs of the exponents.	$\frac{1}{m^1}\cdot\frac{1}{n^5}$
Simplify.	$\frac{1}{mn^5}$

TRY IT 9.1

Simplify: $\left(p^6q^{-2}\right)\left(p^{-9}q^{-1}\right)$.

Show answer

$\frac{1}{p^3q^3}$

TRY IT 9.2

Simplify: $\left(r^5s^{-3}\right)\left(r^{-7}s^{-5}\right)$.

Show answer

$\frac{1}{r^2s^8}$

If the monomials have numerical coefficients, we multiply the coefficients, just like we did earlier.

EXAMPLE 10

Simplify: $\left(2x^{-6}y^8\right)\left(-5x^5y^{-3}\right)$.

Solution

	$\left(2x^{-6}y^{8}\right)\left(-5x^{5}y^{-3}\right)$
Rewrite with the like bases together.	$2\left(-5\right)\cdot\left(x^{-6}x^{5}\right)\cdot\left(y^{8}y^{-3}\right)$
Multiply the coefficients and add the exponents of each variable.	$-10\cdot x^{-1}\cdot y^{5}$
Use the definition of a negative exponent, $a^{-n}=\frac{1}{a^{n}}$.	$-10\cdot\frac{1}{x^{1}}\cdot y^{5}$
Simplify.	$\frac{-10y^{5}}{x}$

TRY IT 10.1

Simplify: $\left(3u^{-5}v^{7}\right)\left(-4u^{4}v^{-2}\right)$.

Show answer
$-\frac{12v^{5}}{u}$

TRY IT 10.2

Simplify: $\left(-6c^{-6}d^{4}\right)\left(-5c^{-2}d^{-1}\right)$.

Show answer
$\frac{30d^{3}}{c^{8}}$

In the next two examples, we'll use the Power Property and the Product to a Power Property.

EXAMPLE 11

Simplify: $\left(6k^{3}\right)^{-2}$.

Solution

	$\left(6k^3\right)^{-2}$
Use the product to a Power Property, $(ab)^m = a^m b^m$.	$(6)^{-2}\left(k^3\right)^{-2}$
Use the Power Property, $\left(a^m\right)^n = a^{m\cdot n}$.	$6^{-2}k^{-6}$
Use the Definition of a Negative Exponent, $a^{-n} = \frac{1}{a^n}$.	$\frac{1}{6^2}\cdot\frac{1}{k^6}$
Simplify.	$\frac{1}{36k^6}$

TRY IT 11.1

Simplify: $\left(-4x^4\right)^{-2}$.

Show answer

$\frac{1}{16x^8}$

TRY IT 11.2

Simplify: $\left(2b^3\right)^{-4}$.

Show answer

$\frac{1}{16b^{12}}$

EXAMPLE 12

Simplify: $\left(5x^{-3}\right)^2$.

Solution

	$\left(5x^{-3}\right)^2$
Use the Product to a Power Property, $(ab)^m = a^m b^m$.	$5^2\left(x^{-3}\right)^2$
Simplify and multiply the exponents of x using the Power Property, $\left(a^m\right)^n = a^{m \cdot n}$.	$25 \cdot x^{-6}$
Rewrite by using the Definition of a Negative Exponent, $a^{-n} = \frac{1}{a^n}$.	$25 \cdot \frac{1}{x^6}$
Simplify.	$\frac{25}{x^6}$

TRY IT 12.1

Simplify: $\left(8a^{-4}\right)^2$.

Show answer
$\frac{64}{a^8}$

TRY IT 12.2

Simplify: $\left(2c^{-4}\right)^3$.

Show answer
$\frac{8}{c^{12}}$

To simplify a fraction, we use the Quotient Property and subtract the exponents.

EXAMPLE 13

Simplify: $\frac{r^5}{r^{-4}}$.

Solution

	$\frac{r^5}{r^{-4}}$
Use the Quotient Property, $\frac{a^m}{a^n} = a^{m-n}$.	$r^{5-(-4)}$
Simplify.	r^9

TRY IT 13.1

Simplify: $\frac{x^8}{x^{-3}}$.

Show answer
x^{11}

TRY IT 13.2

Simplify: $\frac{y^8}{y^{-6}}$.

Show answer
y^{14}

Convert from Decimal Notation to Scientific Notation

Remember working with place value for whole numbers and decimals? Our number system is based on powers of 10. We use tens, hundreds, thousands, and so on. Our decimal numbers are also based on powers of tens—tenths, hundredths, thousandths, and so on. Consider the numbers 4,000 and 0.004. We know that 4,000 means $4 \times 1,000$ and 0.004 means $4 \times \frac{1}{1{,}000}$.

If we write the 1000 as a power of ten in exponential form, we can rewrite these numbers in this way:

$4,000$	0.004
$4 \times 1,000$	$4 \times \frac{1}{1{,}000}$
4×10^3	$4 \times \frac{1}{10^3}$
	4×10^{-3}

In *scientific notation*, a number is expressed as the product of a coefficient and an exponential expression with a base of 10 and an integer power. The coefficient is a decimal number greater than or equal to 1 but less than 10, and the power of 10 is always an **integer**.

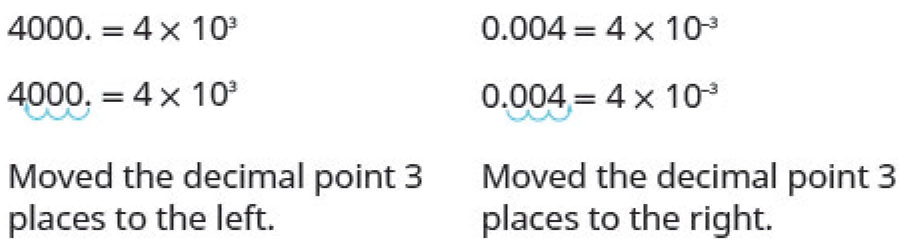
Scientific Notation

A number is expressed in scientific notation when it is of the form

$a \times 10^n$ where $1 \leq a < 10$ and n is an integer

It is customary in scientific notation to use as the $\times$ multiplication sign, even though we avoid using this sign elsewhere in algebra.

If we look at what happened to the decimal point, we can see a method to easily convert from decimal notation to scientific notation.



In both cases, the decimal was moved 3 places to get the coefficient between 1 and 10

The power of 10 is positive when the number is larger than 1: $4,000 = 4 \times 10^3$

The power of 10 is negative when the number is between 0 and 1: $0.004 = 4 \times 10^{-3}$

EXAMPLE 14

How to Convert from Decimal Notation to Scientific Notation

Write in scientific notation: 37,000.

Solution

Step 1. Identify the decimal point's current position in the number: In this case, the decimal point is after the last digit, so it's located at the end of the number.	$37,000$.
Step2. Count the number of places you need to move the decimal point to make the number between 1 and 10: In this case, you need to move the decimal point 4 places to the left.	
Step 3. Write the number as a decimal between 1 and 10, followed by the multiplication symbol$(\times)$ and 10 raised to the power of the number of places the decimal point moved: ◦ Decimal between 1 and 10: **3.7** (move the decimal point from 37000 four places to the left) ◦ Power of 10: 10^4	3.7×10^4
Step 4. Check: Check to see if your answer makes sense.	10^4 is 10,000 and $10,000 \times 3.7$ will be $37,000$.

TRY IT 14.1

Write in scientific notation: $96,000$.

Show answer

9.6×10^4

TRY IT 14.2

Write in scientific notation: $48,300$.

Show answer

4.83×10^4

HOW TO: Convert from decimal notation to scientific notation

1. Move the decimal point so that the first number is greater than or equal to 1 but less than 10.
2. Count the number of decimal places, *n*, that the decimal point was moved.
3. Write the number as a product with a power of 10.

If the original number is:

- greater than 1, the power of 10 will be 10^n.
- between 0 and 1, the power of 10 will be 10^{-n}.

4. Check.

EXAMPLE 15

Write in scientific notation: 0.0052.

Solution
The original number, 0.0052, is between 0 and 1 so we will have a negative power of 10

	0.0052
Move the decimal point to get 5.2, a number between 1 and 10.	
Count the number of decimal places the point was moved.	3 places
Write as a product with a power of 10.	5.2×10^{-3}
Check.	5.2×10^{-3} $= 5.2 \times \frac{1}{10^3}$ $= 5.2 \times \frac{1}{1000}$ $= 5.2 \times 0.001$ $= 0.0052$✓

TRY IT 15.1

Write in scientific notation: 0.0078.

Show answer
7.8×10^{-3}

TRY IT 15.2

Write in scientific notation: 0.0129.

Show answer
1.29×10^{-2}

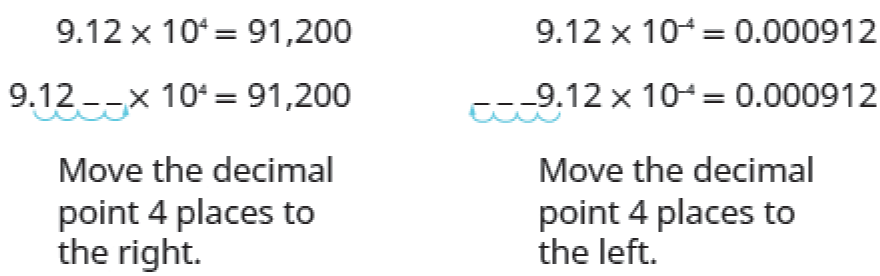
Convert Scientific Notation to Decimal Form

How can we convert from scientific notation to decimal form? Let's look at two numbers written in scientific notation and see.

$$9.12 \times 10^4 \qquad\qquad 9.12 \times 10^{-4}$$
$$9.12 \times 10,000 \qquad\qquad 9.12 \times 0.0001$$
$$91,200 \qquad\qquad 0.000912$$

If we look at the location of the decimal point, we can see an easy method to convert a number from scientific notation to decimal form.

$$9.12 \times 10^4 = 91,200 \qquad\qquad 9.12 \times 10^{-4} = 0.000912$$



In both cases the decimal point moved 4 places. When the exponent was positive, the decimal moved to the right. When the exponent was negative, the decimal point moved to the left.

EXAMPLE 16

How to Convert Scientific Notation to Decimal Form

Convert to decimal form: 6.2×10^3.

Solution

Step 1. Determine the exponent, n on the factor 10.	The exponent is 3.	6.2×10^3
Step 2. Move the decimal n places, adding zeros if needed. If the exponent is positive, move the decimal point n places to the right. If the exponent is negative, move the decimal point $\lvert n \rvert$ places to the left.	The exponent is positive, so move the decimal point 3 places to the right. We need to add 2 zeros as placeholders.	 $= 6,200$
Step 3. Check to see if your answer makes sense.	10^3 is 1000 and 1000 times 6.2 will be $6,200$.	$6.2 \times 10^3 = 6,200$✓

TRY IT 16.1

Convert to decimal form: 1.3×10^3.

Show answer
1,300

TRY IT 16.2

Convert to decimal form: 9.25×10^4.

Show answer
92,500

The steps are summarized below.

HOW TO: Convert scientific notation to decimal form.

To convert scientific notation to decimal form:

1. Identify the exponent, n, that represents the power of 10 in the scientific notation.
2. Move the decimal n places, adding zeros if needed.
    - If the exponent is positive, move the decimal point n places to the right.
    - If the exponent is negative, move the decimal point $|n|$ places to the left.
3. Check.

EXAMPLE 17

Convert to decimal form: 8.9×10^{-2}.

Solution

	8.9×10^{-2}
Identify the exponent, n, that represents the power of 10 in the scientific notation.	The exponent is -2
Since the exponent is negative, move the decimal point 2 places to the left.	
Add zeros as needed for placeholders.	$8.9 \times 10^{-2} = 0.089$

TRY IT 17.1

Convert to decimal form: 1.2×10^{-4}.

Show answer
0.00012

TRY IT 17.2

Convert to decimal form: 7.5×10^{-2}.

Show answer
0.075

Multiply and Divide Using Scientific Notation

Astronomers use very large numbers to describe distances in the universe and ages of stars and planets. Chemists use very small numbers to describe the size of an atom or the charge on an electron. When scientists perform calculations with very large or very small numbers, they use scientific notation. Scientific notation provides a way for the calculations to be done without writing a lot of zeros. We will see how the Properties of Exponents are used to multiply and divide numbers in scientific notation.

EXAMPLE 18

Multiply. Write answers in decimal form: $\left(4 \times 10^5\right)\left(2 \times 10^{-7}\right)$.

Solution

	$\left(4 \times 10^5\right)\left(2 \times 10^{-7}\right)$
Use the Commutative Property to rearrange the factors.	$4 \cdot 2 \cdot 10^5 \cdot 10^{-7}$
Multiply.	8×10^{-2}
Change to decimal form by moving the decimal two places left.	0.08

TRY IT 18.1

Multiply $\left(3 \times 10^6\right)\left(2 \times 10^{-8}\right)$. Write answers in decimal form.

Show answer

0.06

TRY IT 18.2

Multiply $\left(3 \times 10^{-2}\right)\left(3 \times 10^{-1}\right)$. Write answers in decimal form.

Show answer
0.009

EXAMPLE 19

Divide. Write answers in decimal form: $\frac{9 \times 10^{3}}{3 \times 10^{-2}}$.

Solution

	$\frac{9\times10^{3}}{3\times10^{-2}}$
Rewrite as the product of two fractions.	$\frac{9}{3} \times \frac{10^{3}}{10^{-2}}$
Divide.	3×10^{5}
Change to decimal form by moving the decimal five places right.	$300,000$

TRY IT 19.1

Divide $\frac{8 \times 10^{4}}{2 \times 10^{-1}}$. Write answers in decimal form.

Show answer
400,000

TRY IT 19.2

Divide $\frac{8 \times 10^{2}}{4 \times 10^{-2}}$. Write answers in decimal form.

Show answer
20,000

Access these online resources for additional instruction and practice with integer exponents and scientific notation:

- Negative Exponents
- Scientific Notation
- Scientific Notation 2

Key Concepts

- **Property of Negative Exponents**
  - If n is a positive integer and $a \neq 0$, then $\frac{1}{a^{-n}} = a^n$
- **Quotient to a Negative Exponent**
  - If a, b are real numbers, $b \neq 0$ and n is an integer , then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- **To convert a decimal to scientific notation:**
  1. Move the decimal point so that the coefficient is greater than or equal to 1 but less than 10.
  2. Count the number of decimal places, n, that the decimal point was moved.
  3. Write the number as a product with a power of 10. If the original number is:
     - greater than 1, the power of 10 will be 10^n
     - between 0 and 1, the power of 10 will be 10^{-n}
  4. Check.
- **To convert scientific notation to decimal form:**
  1. Identify the exponent, denoted as n, that represents the power of 10 in the scientific notation.
  2. Move the decimal n places, adding zeros if needed.
     - If the exponent is positive, move the decimal point n places to the right.
     - If the exponent is negative, move the decimal point $|n|$ places to the left.
  3. Check

Practice Makes Perfect

Use the Definition of a Negative Exponent

In the following exercises, simplify.

1. a) 3^{-4} b) 10^{-2}	2. a) 4^{-2} b) 10^{-3}
3. a) 2^{-8} b) 10^{-2}	4. a) 5^{-3} b) 10^{-5}
5. a) $\frac{1}{c^{-5}}$ b) $\frac{1}{5^{-2}}$	6. a) $\frac{1}{c^{-5}}$ b) $\frac{1}{3^{-2}}$
7. a) $\frac{1}{t^{-9}}$ b) $\frac{1}{10^{-4}}$	8. a) $\frac{1}{q^{-10}}$ b) $\frac{1}{10^{-3}}$
9. a) $\left(\frac{3}{10}\right)^{-2}$ b) $\left(-\frac{2}{cd}\right)^{-3}$	10. a) $\left(\frac{5}{8}\right)^{-2}$ b) $\left(-\frac{3m}{n}\right)^{-2}$
11. a) $\left(\frac{7}{2}\right)^{-3}$ b) $\left(-\frac{3}{xy^2}\right)^{-3}$	12. a) $\left(\frac{4}{9}\right)^{-3}$ b) $\left(-\frac{u^2}{2v}\right)^{-5}$
13. a) $(-7)^{-2}$ b) -7^{-2} c) $\left(-\frac{1}{7}\right)^{-2}$ d) $-\left(\frac{1}{7}\right)^{-2}$	14. a) $(-5)^{-2}$ b) -5^{-2} c) $\left(-\frac{1}{5}\right)^{-2}$ d) $-\left(\frac{1}{5}\right)^{-2}$
15. a) -5^{-3} b) $\left(-\frac{1}{5}\right)^{-3}$ c) $-\left(\frac{1}{5}\right)^{-3}$ d) $(-5)^{-3}$	16. a) -3^{-3} b) $\left(-\frac{1}{3}\right)^{-3}$ c) $\left(\frac{1}{3}\right)^{-3}$ d) $(-3)^{-3}$
17. a) $2 \cdot 5^{-1}$ b) $(2 \cdot 5)^{-1}$	18. a) $3 \cdot 5^{-1}$ b) $(3 \cdot 5)^{-1}$
19. a) $3 \cdot 4^{-2}$ b) $(3 \cdot 4)^{-2}$	20. a) $4 \cdot 5^{-2}$ b) $(4 \cdot 5)^{-2}$
21. a) b^{-5} b) $\left(k^2\right)^{-5}$	22. a) m^{-4} b) $\left(x^3\right)^{-4}$
23. a) s^{-8} b) $\left(a^9\right)^{-10}$	24. a) p^{-10} b) $\left(q^6\right)^{-8}$
25. a) $6r^{-1}$ b) $(6r)^{-1}$ c) $(-6r)^{-1}$	26. a) $7n^{-1}$ b) $(7n)^{-1}$ c) $(-7n)^{-1}$
27. a) $(2q)^{-4}$ b) $2q^{-4}$ c) $-2q^{-4}$	28. a) $(3p)^{-2}$ b) $3p^{-2}$ c) $-3p^{-2}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

29. a) $s^3 \cdot s^{-7}$ b) $q^{-8} \cdot q^3$ c) $y^{-2} \cdot y^{-5}$	30. a) $b^4 b^{-8}$ b) $r^{-2} r^5$ c) $x^{-7} x^{-3}$
31. a) $y^5 \cdot y^{-5}$ b) $y \cdot y^5$ c) $y \cdot y^{-5}$	32. a) $a^3 \cdot a^{-3}$ b) $a \cdot a^3$ c) $a \cdot a^{-3}$
33. $x^4 \cdot x^{-2} \cdot x^{-3}$	34. $\left(w^4 x^{-5}\right)\left(w^{-2} x^{-4}\right)$
35. $\left(m^3 n^{-3}\right)\left(m^{-5} n^{-1}\right)$	36. $\left(uv^{-2}\right)\left(u^{-5} v^{-3}\right)$
37. $\left(pq^{-4}\right)\left(p^{-6} q^{-3}\right)$	38. $\left(-6c^{-3} d^9\right)\left(2c^4 d^{-5}\right)$
39. $\left(-2j^{-5} k^8\right)\left(7j^2 k^{-3}\right)$	40. $\left(-4r^{-2} s^{-8}\right)\left(9r^4 s^3\right)$
41. $\left(-5m^4 n^6\right)\left(8m^{-5} n^{-3}\right)$	42. $\left(5x^2\right)^{-2}$
43. $\left(4y^3\right)^{-3}$	44. $\left(3z^{-3}\right)^2$
45. $\left(2p^{-5}\right)^2$	46. $\frac{t^9}{t^{-3}}$
47. $\frac{n^5}{n^{-2}}$	48. $\frac{x^{-7}}{x^{-3}}$
49. $\frac{y^{-5}}{y^{-10}}$	

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

50. 57,000	51. 340,000
52. 8,750,000	53. 1,290,000
54. 0.026	55. 0.041
56. 0.00000871	57. 0.00000103

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

58. 5.2×10^{2}	59. 8.3×10^{2}
60. 7.5×10^{6}	61. 1.6×10^{10}
62. 2.5×10^{-2}	63. 3.8×10^{-2}
64. 4.13×10^{-5}	65. 1.93×10^{-5}

Multiply and Divide Using Scientific Notation

In the following exercises, multiply. Write your answer in decimal form.

66. $\left(3 \times 10^{-5}\right)\left(3 \times 10^{9}\right)$	67. $\left(2 \times 10^{2}\right)\left(1 \times 10^{-4}\right)$
68. $\left(7.1 \times 10^{-2}\right)\left(2.4 \times 10^{-4}\right)$	69. $\left(3.5 \times 10^{-4}\right)\left(1.6 \times 10^{-2}\right)$

In the following exercises, divide. Write your answer in decimal form.

70. $\frac{7 \times 10^{-3}}{1 \times 10^{-7}}$	71. $\frac{5 \times 10^{-2}}{1 \times 10^{-10}}$
72. $\frac{6 \times 10^{4}}{3 \times 10^{-2}}$	73. $\frac{8 \times 10^{6}}{4 \times 10^{-1}}$

Everyday Math

74. The population of the United States on July 1, 2010 was about 34,000,000. Write the number in scientific notation.	75. The population of the world on July 1, 2010 was more than 6,850,000,000. Write the number in scientific notation
76. The average width of a human hair is 0.0018 centimetres. Write the number in scientific notation.	77. The probability of winning the 2010 Megamillions lottery was about 0.0000000057. Write the number in scientific notation.
78. In 2010, the number of Facebook users each day who changed their status to 'engaged' was 2×10^4. Convert this number to decimal form.	79. At the start of 2012, the US federal budget had a deficit of more than $?1.5 \times 10^{13}$. Convert this number to decimal form.
80. The concentration of carbon dioxide in the atmosphere is 3.9×10^{-4}. Convert this number to decimal form.	81. The width of a proton is 1×10^{-5} of the width of an atom. Convert this number to decimal form.
82. **Health care costs** The Centers for Medicare and Medicaid projects that American consumers will spend more than $4 trillion on health care by 2017 a. Write 4 trillion in decimal notation. b. Write 4 trillion in scientific notation.	83. **Coin production** In 1942, the U.S. Mint produced 154,500,000 nickels. Write 154,500,000 in scientific notation.
84. **Distance** The distance between Earth and one of the brightest stars in the night star is 33.7 light years. One light year is about 6,000,000,000,000 (6 trillion), miles. a) Write the number of miles in one light year in scientific notation. b)Use scientific notation to find the distance between Earth and the star in miles. Write the answer in scientific notation.	85. **Debt** At the end of fiscal year 2019 the gross Canadian federal government debt was estimated to be approximately $685,450,000,000 ($685.45 billion), according to the Federal Budget. The population of Canada was approximately 37,590,000 people at the end of fiscal year 2019 a) Write the debt in scientific notation. b) Write the population in scientific notation. c) Find the amount of debt per person by using scientific notation to divide the debt by the population. Write the answer in scientific notation.

Writing Exercises.

86. a) Explain the meaning of the exponent in the expression 2^3. b) Explain the meaning of the exponent in the expression 2^{-3}.	87. When you convert a number from decimal notation to scientific notation, how do you know if the exponent will be positive or negative?

Answers

1. a) $\frac{1}{81}$ b) $\frac{1}{100}$	3. a) $\frac{1}{256}$ b) $\frac{1}{100}$	5. a) c^5 b) 25
7. a) t^9 b) 10000	9. a) $\frac{100}{9}$ b) $-\frac{c^3d^3}{8}$	11. a) $\frac{8}{343}$ b) $-\frac{x^3y^6}{27}$
13. a) $\frac{1}{49}$ b) $-\frac{1}{49}$c) 49 d) -49	15. a) $-\frac{1}{125}$ b) -125 c) -125d) $-\frac{1}{125}$	17. a) $\frac{2}{5}$ b) $\frac{1}{10}$
19. a) $\frac{3}{16}$ b) $\frac{1}{144}$	21. a)$\frac{1}{b^5}$ b) $\frac{1}{k^{10}}$	23. a) $\frac{1}{s^8}$ b) $\frac{1}{a^{90}}$
25. a) $\frac{6}{r}$ b) $\frac{1}{6r}$c)$-\frac{1}{6r}$	27. a)$\frac{1}{16q^4}$ b) $\frac{2}{q^4}$ c) $-\frac{2}{q^4}$	29. a) $\frac{1}{s^4}$ b) $\frac{1}{q^5}$ c) $\frac{1}{y^7}$
31. a) 1 b) y^6 c) $\frac{1}{y^4}$	33. $\frac{1}{x}$	35. $\frac{1}{m^2n^4}$
37. $\frac{1}{p^5q^7}$	39. $-\frac{14k^5}{j^3}$	41. $-\frac{40n^3}{m}$
43. $\frac{1}{64y^9}$	45. $\frac{4}{p^{10}}$	47. n^7
49. y^5	51. 3.4×10^5	53. 1.29×10^6
55. 4.1×10^{-2}	57. 1.03×10^{-6}	59. 830
61. 16,000,000,000	63. 0.038	65. 0.0000193
67. 0.02	69. 5.6×10^{-6}	71. 500,000,000
73. 20,000,000	75. 6.85×10^9.	77. 5.7×10^{-9}
79. 15,000,000,000,000	81. 0.00001	83. 1.545×10^8
85. a. 6.8545×10^{11} b. 3.759×10^7 c. 1.82349×10^4		87. Answers will vary

Attributions

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5.4 Simplify and Use Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Simplify expressions with square roots
- Estimate square roots
- Approximate square roots
- Simplify variable expressions with square roots
- Use square roots in applications

Simplify Expressions with Square Roots

To start this section, we need to review some important vocabulary and notation.

Remember that when a number n is multiplied by itself, we can write this as n^2, which we read aloud as “n squared.” For example, 8^2 is read as “8 squared.”

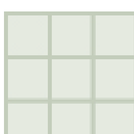
We call 64 the *square* of 8 because $8^2 = 64$. Similarly, 121 is the square of 11, because $11^2 = 121$.

Square of a Number

If $n^2 = m$, then m is the square of n.

Modeling Squares

Do you know why we use the word *square*? If we construct a square with three tiles on each side, the total number of tiles would be nine.



This is why we say that the square of three is nine.

$3^2 = 9$

The number 9 is called a perfect square because it is the square of a whole number.

The chart shows the squares of the counting numbers 1 through 15. You can refer to it to help you identify the perfect squares.

Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Perfect Squares

A perfect square is the square of a whole number.

What happens when you square a negative number?

$$\begin{aligned}(-8)^2 &= (-8)(-8)\\ &= 64\end{aligned}$$

When we multiply two negative numbers, the product is always positive. So, the square of a negative number is always positive.

The chart shows the squares of the negative integers from -1 to -15.

Number	n	–1	–2	–3	–4	–5	–6	–7	–8	–9	–10	–11	–12	–13	–14	–15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Did you notice that these squares are the same as the squares of the positive numbers?

Square Roots

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We can also say that 10 is a square root of 100.

Square Root of a Number

A number whose square is m is called a square root of m.

If $n^2 = m$, then n is a square root of m.

Notice $(-10)^2 = 100$ also, so -10 is also a square root of 100. Therefore, both 10 and -10 are square roots of 100.

So, every positive number has two square roots: one positive and one negative.

What if we only want the positive square root of a positive number? The *radical sign*, $\sqrt{\ }$, stands for the positive square root. The positive square root is also called the principal square root.

Square Root Notation

$\sqrt{m}$ is read as "the square root of m."

If $m = n^2$, then $\sqrt{m} = n$ for n ≥ 0.

We can also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

The chart shows the square roots of the first 15 perfect square numbers.

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

EXAMPLE 1

Simplify: a) $\sqrt{25}$ b) $\sqrt{121}$.

Solution

a)	
	$\sqrt{25}$
Since $5^2 = 25$	5

b)	
	$\sqrt{121}$
Since $11^2 = 121$	11

TRY IT 1.1

Simplify: a) $\sqrt{36}$ b) $\sqrt{169}$.

Show answer

a. 6

b. 13

TRY IT 1.2

Simplify: a) $\sqrt{16}$ b) $\sqrt{196}$.

Show answer

a. 4
b. 14

Every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$.

EXAMPLE 2

Simplify. a) $-\sqrt{9}$ b) $-\sqrt{144}$.

Solution

a)	
	$-\sqrt{9}$
The negative is in front of the radical sign.	-3

b)	
	$-\sqrt{144}$
The negative is in front of the radical sign.	-12

TRY IT 2.1

Simplify: a) $-\sqrt{4}$ b) $-\sqrt{225}$.

Show answer

a. −2
b. −15

TRY IT 2.2

Simplify: a) $-\sqrt{81}$ b) $-\sqrt{64}$.

Show answer

a. −9
b. −8

Square Root of a Negative Number

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25?
$(\quad)^2 = -25?$

None of the numbers that we have dealt with so far have a square that is -25. Why? Any positive number squared is positive, and any negative number squared is also positive. In the next chapter we will see that all the numbers we work with are called the real numbers. So we say there is no real number equal to $\sqrt{-25}$. If we are asked to find the square root of any negative number, we say that the solution is not a real number.

EXAMPLE 3

Simplify: a) $\sqrt{-169}$ b) $-\sqrt{121}$.

Solution
a) There is no real number whose square is -169. Therefore, $\sqrt{-169}$ is not a real number.

b) The negative is in front of the radical sign, so we find the opposite of the square root of 121.

	$-\sqrt{121}$
The negative is in front of the radical.	-11

TRY IT 3.1

Simplify: a) $\sqrt{-196}$ b) $-\sqrt{81}$.

Show answer

a. not a real number
b. -9

TRY IT 3.2

Simplify: a) $\sqrt{-49}$ b) $-\sqrt{121}$.

Show answer

a. not a real number

b. -11

Square Roots and the Order of Operations

When using the order of operations to simplify an expression that has square roots, we treat the radical sign as a grouping symbol. We simplify any expressions under the radical sign before performing other operations.

EXAMPLE 4

Simplify: a) $\sqrt{25}+\sqrt{144}$ b) $\sqrt{25+144}$.

Solution

a) Use the order of operations.	
	$\sqrt{25}+\sqrt{144}$
Simplify each radical.	$5+12$
Add.	17

b) Use the order of operations.	
	$\sqrt{25+144}$
Add under the radical sign.	$\sqrt{169}$
Simplify.	13

TRY IT 4.1

Simplify: a) $\sqrt{9}+\sqrt{16}$ b) $\sqrt{9+16}$.

Show answer

a. 7

b. 5

TRY IT 4.2

Simplify: a) $\sqrt{64+225}$ b) $\sqrt{64}+\sqrt{225}$.

Show answer

a. 17

b. 23

Notice the different answers in parts a) and b) of (Example 4). It is important to follow the order of operations correctly. In a), we took each square root first and then added them. In b), we added under the radical sign first and then found the square root.

Estimate Square Roots

So far we have only worked with square roots of perfect squares. The square roots of other numbers are not whole numbers.

Number	Square root
4	$\sqrt{4}=2$
5	$\sqrt{5}$
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	$\sqrt{9}=3$

We might conclude that the square roots of numbers between 4 and 9 will be between 2 and 3, and they will not be whole numbers. Based on the pattern in the table above, we could say that $\sqrt{5}$ is between 2 and 3. Using inequality symbols, we write

$$2<\sqrt{5}<3$$

EXAMPLE 5

Estimate $\sqrt{60}$ between two consecutive whole numbers.

Solution

Think of the perfect squares closest to 60. Make a small table of these perfect squares and their squares roots.

Number	Square root
36	6
49	7
64	8
81	9

60

$\sqrt{60}$

Locate 60 between two consecutive perfect squares.	$49 < 60 < 64$
$\sqrt{60}$ is between their square roots.	$7 < \sqrt{60} < 8$

TRY IT 5.1

Estimate $\sqrt{38}$ between two consecutive whole numbers.

Show answer
$6 < \sqrt{38} < 7$

TRY IT 5.2

Estimate $\sqrt{84}$ between two consecutive whole numbers.

Show answer
$9 < \sqrt{84} < 10$

Approximate Square Roots with a Calculator

The square roots of numbers that are not perfect squares are not whole numbers, they are irrational numbers. Its decimal form does not stop and does not repeat. Are irrational numbers real numbers? Yes, they are. When we put together the irrational numbers and rational numbers, we get the set of real numbers.

Let's see how we can use calculator to find the approximate square roots of those irrational numbers.

There are mathematical methods to approximate square roots, but it is much more convenient to use a calculator to find square roots. Find the $\sqrt{}$ or $\sqrt{x}$ key on your calculator. You will to use this key to approximate square roots. When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact number. It is an approximation, to the number of digits shown on your calculator's display. The symbol for an approximation is $\approx$ and it is read *approximately*.

Suppose your calculator has a 10-digit display. Using it to find the square root of 5 will give 2.236067977. This is the approximate square root of 5. When we report the answer, we should use the "approximately equal to" sign instead of an equal sign.
$\sqrt{5} \approx 2.236067978$. The square root of 5 is the example of irrational number and its approximation displays nine digits after the decimal place.

You will seldom use this many digits for applications in algebra. So, if you wanted to round $\sqrt{5}$ to two decimal places, you would write
$\sqrt{5} \approx 2.24$

How do we know these values are approximations and not the exact values? Look at what happens when we square them.

$$\begin{aligned} 2.236067978^2 &= 5.000000002 \\ 2.24^2 &= 5.0176 \end{aligned}$$

The squares are close, but not exactly equal, to 5.

EXAMPLE 6

Round $\sqrt{17}$ to two decimal places using a calculator.

Solution

	$\sqrt{17}$
Use the calculator square root key.	4.123105626
Round to two decimal places.	4.12
	$\sqrt{17} \approx 4.12$

TRY IT 6.1

Round $\sqrt{11}$ to two decimal places.

Show answer
≈ 3.32

TRY IT 6.2

Round $\sqrt{13}$ to two decimal places.

Show answer
≈ 3.61

Simplify Variable Expressions with Square Roots

Expressions with square root that we have looked at so far have not had any variables. What happens when we have to find a square root of a variable expression?

Consider $\sqrt{9x^2}$, where $x \geq 0$. Can you think of an expression whose square is $9x^2$?

$$(?)^2 = 9x^2$$

$$(3x)^2 = 9x^2 \quad \text{so } \sqrt{9x^2} = 3x$$

When we use a variable in a square root expression, for our work, we will assume that the variable represents a non-negative number. In every example and exercise that follows, each variable in a square root expression is greater than or equal to zero.

EXAMPLE 7

Simplify: $\sqrt{x^2}$ where $x \geq 0$.

Solution

Think about what we would have to square to get x^2. Algebraically, $(?)^2 = x^2$

	$\sqrt{x^2}$
Since $(x)^2 = x^2$	x

TRY IT 7.1

Simplify: $\sqrt{y^2}$ where $y \geq 0$.

Show answer

y

TRY IT 7.2

Simplify: $\sqrt{m^2}$ where $m \geq 0$.

Show answer

m

EXAMPLE 8

Simplify: $\sqrt{16x^2}$ where $x \geq 0$.

Solution

	$\sqrt{16x^2}$
Since $(4x)^2 = 16x^2$	$4x$

TRY IT 8.1

Simplify: $\sqrt{64x^2}$ where $x \geq 0$.

Show answer
8*x*

TRY IT 8.2

Simplify: $\sqrt{169y^2}$ where $y \geq 0$.

Show answer
13*y*

EXAMPLE 9

Simplify: $-\sqrt{81y^2}$ where $y \geq 0$.

Solution

	$-\sqrt{81y^2}$
Since $(9y)^2 = 81y^2$	$-9y$

TRY IT 9.1

Simplify: $-\sqrt{121y^2}$ where $y \geq 0$.

Show answer
−11*y*

TRY IT 9.2

Simplify: $-\sqrt{100p^2}$ where $p \geq 0$.

Show answer
$-10p$

EXAMPLE 10

Simplify: $\sqrt{36x^2y^2}$ where $x \geq 0, y \geq 0$.

Solution

	$\sqrt{36x^2y^2}$
Since $(6xy)^2 = 36x^2y^2$	$6xy$

TRY IT 10.1

Simplify: $\sqrt{100a^2b^2}$ where $a \geq 0, b \geq 0$.

Show answer
$10ab$

TRY IT 10.2

Simplify: $\sqrt{225m^2n^2}$ where $m \geq 0, n \geq 0$.

Show answer
$15mn$

Use Square Roots in Applications

As you progress through your college courses, you'll encounter several applications of square roots. Once again, if we use our strategy for applications, it will give us a plan for finding the answer!

HOW TO: Use a strategy for applications with square roots.

1. Identify what you are asked to find.
2. Write a phrase that gives the information to find it.
3. Translate the phrase to an expression.
4. Simplify the expression.
5. Write a complete sentence that answers the question.

Square Roots and Area

We have solved applications with area before. If we were given the length of the sides of a square, we could find its area by squaring the length of its sides. Now we can find the length of the sides of a square if we are given the area, by finding the square root of the area.

If the area of the square is A square units, the length of a side is $\sqrt{A}$ units. See the table below.

Area (square units)	**Length of side (units)**
9	$\sqrt{9} = 3$
144	$\sqrt{144} = 12$
A	$\sqrt{A}$

EXAMPLE 11

Mike and Lychelle want to make a square patio. They have enough concrete for an area of 200 square feet. To the nearest tenth of a foot, how long can a side of their square patio be?

Solution

We know the area of the square is 200 square feet and want to find the length of the side. If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.

What are you asked to find?	The length of each side of a square patio
Write a phrase.	The length of a side
Translate to an expression.	$\sqrt{A}$
Evaluate $\sqrt{A}$ when $A = 200$.	$\sqrt{200}$
Use your calculator.	$14.142135...$
Round to one decimal place.	14.1 feet
Write a sentence.	Each side of the patio should be 14.1 feet.

TRY IT 11.1

Katie wants to plant a square lawn in her front yard. She has enough sod to cover an area of 370 square feet. To the nearest tenth of a foot, how long can a side of her square lawn be?

Show answer
19.2 feet

TRY IT 11.2

Sergio wants to make a square mosaic as an inlay for a table he is building. He has enough tile to cover an area of 2704 square centimetres. How long can a side of his mosaic be?

Show answer
52 centimetres

Square Roots and Gravity

Another application of square roots involves gravity. On Earth, if an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$. For example, if an object is dropped from a height of 64 feet, we can find the time it takes to reach the ground by evaluating $\frac{\sqrt{64}}{4}$.

	$\frac{\sqrt{64}}{4}$
Take the square root of 64.	$\frac{8}{4}$
Simplify the fraction.	2

It would take 2 seconds for an object dropped from a height of 64 feet to reach the ground.

EXAMPLE 12

Christy dropped her sunglasses from a bridge 400 feet above a river. How many seconds does it take for the sunglasses to reach the river?

Solution

What are you asked to find?	The number of seconds it takes for the sunglasses to reach the river
Write a phrase.	The time it will take to reach the river
Translate to an expression.	$\frac{\sqrt{h}}{4}$
Evaluate $\frac{\sqrt{h}}{4}$ when $h = 400$.	$\frac{\sqrt{400}}{4}$
Find the square root of 400.	$\frac{20}{4}$
Simplify.	5
Write a sentence.	It will take 5 seconds for the sunglasses to reach the river.

TRY IT 12.1

A helicopter drops a rescue package from a height of 1296 feet. How many seconds does it take for the package to reach the ground?

Show answer
9 seconds

TRY IT 12.2

A window washer drops a squeegee from a platform 196 feet above the sidewalk. How many seconds does it take for the squeegee to reach the sidewalk?

Show answer
3.5 seconds

Square Roots and Accident Investigations

Police officers investigating car accidents measure the length of the skid marks on the pavement. Then they use square roots to determine the speed, in miles per hour, a car was going before applying the brakes. According to some formulas, if the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$.

EXAMPLE 13

After a car accident, the skid marks for one car measured 190 feet. To the nearest tenth, what was the speed of the car (in mph) before the brakes were applied?

Solution

What are you asked to find?	The speed of the car before the brakes were applied
Write a phrase.	The speed of the car
Translate to an expression.	$\sqrt{24d}$
Evaluate $\sqrt{24d}$ when $d = 190.$	$\sqrt{24 \cdot 190}$
Multiply.	$\sqrt{4,560}$
Use your calculator.	$67.527772...$
Round to tenths.	67.5
Write a sentence.	The speed of the car was approximately 67.5 miles per hour.

TRY IT 13.1

An accident investigator measured the skid marks of a car and found their length was 76 feet. To the nearest tenth, what was the speed of the car before the brakes were applied?

Show answer
42.7 mph

TRY IT 13.2

The skid marks of a vehicle involved in an accident were 122 feet long. To the nearest tenth, how fast had the vehicle been going before the brakes were applied?

Show answer
54.1 mph

The *Links to Literacy* activity “Sea Squares” will provide you with another view of the topics covered in this section.
ACCESS ADDITIONAL ONLINE RESOURCES

- Introduction to Square Roots

- Estimating Square Roots with a Calculator

Key Concepts

- **Square Root Notation** $\sqrt{m}$ is read 'the square root of m'
  If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$. 

- **Use a strategy for applications with square roots.**
    - Identify what you are asked to find.
    - Write a phrase that gives the information to find it.
    - Translate the phrase to an expression.
    - Simplify the expression.
    - Write a complete sentence that answers the question.

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.

1. $\sqrt{36}$	2. $\sqrt{4}$
3. $\sqrt{64}$	4. $\sqrt{144}$
5. $-\sqrt{4}$	6. $-\sqrt{100}$
7. $-\sqrt{1}$	8. $-\sqrt{121}$
9. $\sqrt{-121}$	10. $\sqrt{-36}$
11. $\sqrt{-9}$	12. $\sqrt{-49}$
13. $\sqrt{9+16}$	14. $\sqrt{25+144}$
15. $\sqrt{9}+\sqrt{16}$	16. $\sqrt{25}+\sqrt{144}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

17. $\sqrt{70}$	18. $\sqrt{55}$
19. $\sqrt{200}$	20. $\sqrt{172}$

Approximate Square Roots with a Calculator

In the following exercises, use a calculator to approximate each square root and round to two decimal places.

21. $\sqrt{19}$	22. $\sqrt{21}$
23. $\sqrt{53}$	24. $\sqrt{47}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

25. $\sqrt{y^2}$	26. $\sqrt{b^2}$
27. $\sqrt{49x^2}$	28. $\sqrt{100y^2}$
29. $-\sqrt{64a^2}$	30. $-\sqrt{25x^2}$
31. $\sqrt{144x^2y^2}$	32. $\sqrt{196a^2b^2}$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

33. **Landscaping** Reid wants to have a square garden plot in his backyard. He has enough compost to cover an area of 75 square feet. How long can a side of his garden be? *Note: If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.*	34. **Landscaping** Tasha wants to make a square patio in her yard. She has enough concrete to pave an area of 130 square feet. How long can a side of her patio be? *Note: If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.*
35. **Gravity** An airplane dropped a flare from a height of $1,024$ feet above a lake. How many seconds did it take for the flare to reach the water? *Note: If an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$*	36. **Gravity** A hang glider dropped his cell phone from a height of 350 feet. How many seconds did it take for the cell phone to reach the ground? *Note: If an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$*
37. **Gravity** A construction worker dropped a hammer while building the Grand Canyon skywalk, $4,000$ feet above the Colorado River. How many seconds did it take for the hammer to reach the river? *Note: If an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$*	38. **Accident investigation** The skid marks from a car involved in an accident measured 54 feet. What was the speed of the car before the brakes were applied? *Note: If the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$*
39. **Accident investigation** The skid marks from a car involved in an accident measured 216 feet. What was the speed of the car before the brakes were applied? *Note: If the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$*	40. **Accident investigation** An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 175 feet. What was the speed of the vehicle before the brakes were applied? *Note: If the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$*
41. **Accident investigation** An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 117 feet. What was the speed of the vehicle before the brakes were applied? *Note: If the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$*	

Everyday Math

42. **Decorating** Denise wants to install a square accent of designer tiles in her new shower. She can afford to buy 625 square centimetres of the designer tiles. How long can a side of the accent be?	43. **Decorating** Morris wants to have a square mosaic inlaid in his new patio. His budget allows for $2,025$ tiles. Each tile is square with an area of one square inch. How long can a side of the mosaic be?

Writing Exercises

44. Why is there no real number equal to $\sqrt{-64}$?	45. What is the difference between 9^2 and $\sqrt{9}$?

Answers

1. 6	3. 8	5. -2
7. -1	9. not a real number	11. not a real number
13. 5	15. 7	17. $8 < \sqrt{70} < 9$
19. $14 < \sqrt{200} < 15$	21. 4.36	23. 7.28
25. y	27. $7x$	29. $-8a$
31. $12xy$	33. 8.7 feet	35. 8 seconds
37. 15.8 seconds	39. 72 mph	41. 53.0 mph
43. 45 inches	45. Answers will vary. 9^2 reads: "nine squared" and means nine times itself. The expression $\sqrt{9}$ reads: "the square root of nine" which gives us the number such that if it were multiplied by itself would give you the number inside of the square root.	

Attributions

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5.5 Simplify Square Roots

Learning Objectives

By the end of this section, you will be able to:

- Use the Product Property to simplify square roots
- Use the Quotient Property to simplify square roots

In the last section, we estimated the square root of a number between two consecutive whole numbers. We can say that $\sqrt{50}$ is between 7 and 8. This is fairly easy to do when the numbers are small enough that we can use in (Simplify and Use Square Roots).

But what if we want to estimate $\sqrt{500}$? If we simplify the square root first, we'll be able to estimate it easily. There are other reasons, too, to simplify square roots as you'll see later in this chapter.

A square root is considered *simplified* if its radicand contains no perfect square factors.

Simplified Square Root

$\sqrt{a}$ is considered simplified if a has no perfect square factors.

So $\sqrt{31}$ is simplified. But $\sqrt{32}$ is not simplified, because 16 is a perfect square factor of 32

Use the Product Property to Simplify Square Roots

The properties we will use to simplify expressions with square roots are similar to the properties of exponents. We know that $(ab)^m = a^m b^m$. The corresponding property of square roots says that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Product Property of Square Roots

If *a*, *b* are non-negative real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

We use the Product Property of Square Roots to remove all perfect square factors from a radical. We will show how to do this in (Example 1).

EXAMPLE 1

How To Use the Product Property to Simplify a Square Root

Simplify: $\sqrt{50}$.

Solution

Step 1. Find the largest perfect square factor of the radicand.	25 is the largest perfect square factor of 50.	$\sqrt{50}$
Rewrite the radicand as a product using the perfect square factor.	$50 = 25 \cdot 2$	$\sqrt{25 \cdot 2}$
Step 2. Use the product rule to rewrite the radical as the product of two radicals.		$\sqrt{25} \cdot \sqrt{2}$
Step 3. Simplify the square root of the perfect square.		$5\sqrt{2}$

TRY IT 1.1

Simplify: $\sqrt{48}$.

Show answer

$4\sqrt{3}$

TRY IT 1.2

Simplify: $\sqrt{45}$.

Show answer

$3\sqrt{5}$

Notice in the previous example that the simplified form of $\sqrt{50}$ is $5\sqrt{2}$, which is the product of an integer and a square root. We always write the integer in front of the square root.

HOW TO: Simplify a square root using the product property.

1. Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect-square factor.
2. Use the product rule to rewrite the radical as the product of two radicals.
3. Simplify the square root of the perfect square.

EXAMPLE 2

Simplify: $\sqrt{500}$.

Solution

	$\sqrt{500}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{100 \cdot 5}$
Rewrite the radical as the product of two radicals.	$\sqrt{100} \cdot \sqrt{5}$
Simplify.	$10\sqrt{5}$

TRY IT 2.1

Simplify: $\sqrt{288}$.

Show answer
$12\sqrt{2}$

TRY IT 2.2

Simplify: $\sqrt{432}$.

Show answer
$12\sqrt{3}$

We could use the simplified form $10\sqrt{5}$ to estimate $\sqrt{500}$. We know 5 is between 2 and 3, and $\sqrt{500}$ is $10\sqrt{5}$. So $\sqrt{500}$ is between 20 and 30.

The next example is much like the previous examples, but with variables.

EXAMPLE 3

Simplify: $\sqrt{x^3}$ where $x \geq 0$.

Solution

	$\sqrt{x^3}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{x^2 \cdot x}$
Rewrite the radical as the product of two radicals.	$\sqrt{x^2} \cdot \sqrt{x}$
Simplify.	$x\sqrt{x}$

TRY IT 3.1

Simplify: $\sqrt{b^5}$ where $b \geq 0$.

Show answer
$b^2\sqrt{b}$

TRY IT 3.2

Simplify: $\sqrt{p^9}$ where $p \geq 0$.

Show answer
$p^4\sqrt{p}$

We follow the same procedure when there is a coefficient in the radical, too.

EXAMPLE 4

Simplify: $\sqrt{25y^5}$ where $y \geq 0$

Solution

	$\sqrt{25y^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{25y^4 \cdot y}$
Rewrite the radical as the product of two radicals.	$\sqrt{25y^4} \cdot \sqrt{y}$
Simplify.	$5y^2\sqrt{y}$

TRY IT 4.1

Simplify: $\sqrt{16x^7}$ where $x \geq 0$.

Show answer
$4x^3\sqrt{x}$

TRY IT 4.2

Simplify: $\sqrt{49v^9}$ where $v \geq 0$.

Show answer
$7v^4\sqrt{v}$

In the next example both the constant and the variable have perfect square factors.

EXAMPLE 5

Simplify: $\sqrt{72n^7}$ where $n \geq 0$.

Solution

	$\sqrt{72n^7}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{36n^6 \cdot 2n}$
Rewrite the radical as the product of two radicals.	$\sqrt{36n^6} \cdot \sqrt{2n}$
Simplify.	$6n^3\sqrt{2n}$

TRY IT 5.1

Simplify: $\sqrt{32y^5}$ where $y \geq 0$.

Show answer
$4y^2\sqrt{2y}$

TRY IT 5.2

Simplify: $\sqrt{75a^9}$ $a \geq 0$.

Show answer
$5a^4\sqrt{3a}$

EXAMPLE 6

Simplify: $\sqrt{63u^3v^5}$ where $u \geq 0$ and $v \geq 0$.

Solution

	$\sqrt{63u^3v^5}$
Rewrite the radicand as a product using the largest perfect square factor.	$\sqrt{9u^2v^4 \cdot 7uv}$
Rewrite the radical as the product of two radicals.	$\sqrt{9u^2v^4} \cdot \sqrt{7uv}$
Simplify.	$3uv^2\sqrt{7uv}$

TRY IT 6.1

Simplify: $\sqrt{98a^7b^5}$ where $a \geq 0$ and $b \geq 0$.

Show answer
$7a^3b^2\sqrt{2ab}$

TRY IT 6.2

Simplify: $\sqrt{180m^9n^{11}}$ where $n \geq 0$.

Show answer
$6m^4n^5\sqrt{5mn}$

We have seen how to use the Order of Operations to simplify some expressions with radicals. To simplify $\sqrt{25} + \sqrt{144}$ we must simplify each square root separately first, then add to get the sum of 17

The expression $\sqrt{17} + \sqrt{7}$ cannot be simplified—to begin we'd need to simplify each square root, but neither 17 nor 7 contains a perfect square factor.

In the next example, we have the sum of an integer and a square root. We simplify the square root but cannot add the resulting expression to the integer.

EXAMPLE 7

Simplify: $3+\sqrt{32}$.

Solution

	$3+\sqrt{32}$
Rewrite the radicand as a product using the largest perfect square factor.	$3+\sqrt{16\cdot 2}$
Rewrite the radical as the product of two radicals.	$3+\sqrt{16}\cdot\sqrt{2}$
Simplify.	$3+4\sqrt{2}$

The terms are not like and so we cannot add them. Trying to add an integer and a radical is like trying to add an integer and a variable—they are not like terms!

TRY IT 7.1

Simplify: $5+\sqrt{75}$.

Show answer
$5+5\sqrt{3}$

TRY IT 7.2

Simplify: $2+\sqrt{98}$.

Show answer
$2+7\sqrt{2}$

The next example includes a fraction with a radical in the numerator. Remember that in order to simplify a fraction you need a common factor in the numerator and denominator.

EXAMPLE 8

Simplify: $\frac{4-\sqrt{48}}{2}$.

Solution

	$\frac{4-\sqrt{48}}{2}$
Rewrite the radicand as a product using the largest perfect square factor.	$\frac{4-\sqrt{16\cdot 3}}{2}$
Rewrite the radical as the product of two radicals.	$\frac{4-\sqrt{16}\cdot\sqrt{3}}{2}$
Simplify.	$\frac{4-4\sqrt{3}}{2}$
Factor the common factor from the numerator.	$\frac{4\left(1-\sqrt{3}\right)}{2}$
Remove the common factor, 2, from the numerator and denominator.	$\frac{2\cdot 2\left(1-\sqrt{3}\right)}{2}$ *because* $4=2\cdot 2$ $\frac{\cancel{2}\cdot 2\left(1-\sqrt{3}\right)}{\cancel{2}}$
Simplify.	$2\left(1-\sqrt{3}\right)$

TRY IT 8.1

Simplify: $\frac{10-\sqrt{75}}{5}$.

Show answer

$2-\sqrt{3}$

TRY IT 8.2

Simplify: $\frac{6-\sqrt{45}}{3}$.

Show answer

$2-\sqrt{5}$

Use the Quotient Property to Simplify Square Roots

Whenever you have to simplify a square root, the first step you should take is to determine whether the radicand is a perfect square. A *perfect square fraction* is a fraction in which both the numerator and the denominator are perfect squares.

EXAMPLE 9

Simplify: $\sqrt{\frac{9}{64}}$.

Solution

	$\sqrt{\frac{9}{64}}$
Since $\left(\frac{3}{8}\right)^2 = \frac{9}{64}$	$\frac{3}{8}$

TRY IT 9.1

Simplify: $\sqrt{\frac{25}{16}}$.

Show answer
$\frac{5}{4}$

TRY IT 9.2

Simplify: $\sqrt{\frac{49}{81}}$.

Show answer
$\frac{7}{9}$

If the numerator and denominator have any common factors, remove them. You may find a perfect square fraction!

EXAMPLE 10

Simplify: $\sqrt{\frac{45}{80}}$.

Solution

	$\sqrt{\frac{45}{80}}$
Simplify inside the radical first. Rewrite showing the common factors of the numerator and denominator.	$\sqrt{\frac{5 \cdot 9}{5 \cdot 16}}$
Simplify the fraction by removing common factors.	$\sqrt{\frac{9}{16}}$
Simplify $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$	$\frac{3}{4}$

TRY IT 10.1

Simplify: $\sqrt{\frac{75}{48}}$.

Show answer
$\frac{5}{4}$

TRY IT 10.2

Simplify: $\sqrt{\frac{98}{162}}$.

Show answer
$\frac{7}{9}$

In the last example, our first step was to simplify the fraction under the radical by removing common factors. In the next example we will use the Quotient Property to simplify under the radical. We divide the like bases by subtracting their exponents, $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$.

EXAMPLE 11

Simplify: $\sqrt{\frac{m^6}{m^4}}$ where $m > 0$.

Solution

	$\sqrt{\frac{m^6}{m^4}}$
Simplify the fraction inside the radical first. Divide the like bases by subtracting the exponents.	$\sqrt{m^2}$
Simplify.	m

TRY IT 11.1

Simplify: $\sqrt{\frac{a^8}{a^6}}$ where $a > 0$.

Show answer

a

TRY IT 11.2

Simplify: $\sqrt{\frac{x^{14}}{x^{10}}}$ where $x > 0$.

Show answer

x^2

EXAMPLE 12

Simplify: $\sqrt{\frac{48p^7}{3p^3}}$ where $p > 0$.

Solution

	$\sqrt{\frac{48p^7}{3p^3}}$
Simplify the fraction inside the radical first.	$\sqrt{16p^4}$
Simplify.	$4p^2$

TRY IT 12.1

Simplify: $\sqrt{\frac{75x^5}{3x}}$ where $x > 0$.

Show answer
$5x^2$

TRY IT 12.2

Simplify: $\sqrt{\frac{72z^{12}}{2z^{10}}}$ where $z > 0$.

Show answer
$6z$

Remember the Quotient to a Power Property? It said we could raise a fraction to a power by raising the numerator and denominator to the power separately.
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$
We can use a similar property to simplify a square root of a fraction. After removing all common factors from the numerator and denominator, if the fraction is not a perfect square we simplify the numerator and denominator separately.

Quotient Property of Square Roots

If *a*, *b* are non-negative real numbers and $b \neq 0$, then
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

EXAMPLE 13

Simplify: $\sqrt{\frac{21}{64}}$.

Solution

	$\sqrt{\frac{21}{64}}$
We cannot simplify the fraction inside the radical. Rewrite using the quotient property.	$\frac{\sqrt{21}}{\sqrt{64}}$
Simplify the square root of 64. The numerator cannot be simplified.	$\frac{\sqrt{21}}{8}$

TRY IT 13.1

Simplify: $\sqrt{\frac{19}{49}}$.

Show answer

$\frac{\sqrt{19}}{7}$

TRY IT 13.2

Simplify: $\sqrt{\frac{28}{81}}$.

Show answer

$\frac{2\sqrt{7}}{9}$

EXAMPLE 14

How to Use the Quotient Property to Simplify a Square Root

Simplify: $\sqrt{\frac{27m^3}{196}}$ where $m \geq 0$.

Solution

Step 1. Simplify the fraction in the radicand, if possible.	$\frac{27m^3}{196}$ cannot be simplified.	$\sqrt{\frac{27m^3}{196}}$
Step 2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.	We rewrite $\sqrt{\frac{27m^3}{196}}$ as the quotient of $\sqrt{27m^3}$ and $\sqrt{196}$.	$\frac{\sqrt{27m^3}}{\sqrt{196}}$
Step 3. Simplify the radicals in the numerator and the denominator.	We know that $27m^3 = 9m^2 \cdot 3m$ and $9m^2$ and 196 are perfect squares.	$\frac{\sqrt{9m^2} \cdot \sqrt{3m}}{\sqrt{196}}$ $\frac{3m\sqrt{3m}}{14}$

TRY IT 14.1

Simplify: $\sqrt{\frac{24p^3}{49}}$ where $p \geq 0$.

Show answer

$\frac{2p\sqrt{6p}}{7}$

TRY IT 14.2

Simplify: $\sqrt{\frac{48x^5}{100}}$ where $x \geq 0$.

Show answer
$\frac{2x^2\sqrt{3x}}{5}$

HOW TO: Simplify a square root using the quotient property.

1. Simplify the fraction in the radicand, if possible.
2. Use the Quotient Property to rewrite the radical as the quotient of two radicals.
3. Simplify the radicals in the numerator and the denominator.

EXAMPLE 15

Simplify: $\sqrt{\frac{45x^5}{y^4}}$ where $x \geq 0$ and $y > 0$.

Solution

	$\sqrt{\frac{45x^5}{y^4}}$
We cannot simplify the fraction in the radicand. Rewrite using the Quotient Property.	$\frac{\sqrt{45x^5}}{\sqrt{y^4}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{9x^4}\cdot\sqrt{5x}}{y^2}$
Simplify.	$\frac{3x^2\sqrt{5x}}{y^2}$

TRY IT 15.1

Simplify: $\sqrt{\frac{80m^3}{n^6}}$ where $m \geq 0$ and $n > 0$.

Show answer

$\frac{4m\sqrt{5m}}{n^3}$

TRY IT 15.2

Simplify: $\sqrt{\frac{54u^7}{v^8}}$ where $u \geq 0$ and $v > 0$.

Show answer
$\frac{3u^3\sqrt{6u}}{v^4}$

Be sure to simplify the fraction in the radicand first, if possible.

EXAMPLE 16

Simplify: $\sqrt{\frac{81d^9}{25d^4}}$ where $d > 0$.

Solution

	$\sqrt{\frac{81d^9}{25d^4}}$
Simplify the fraction in the radicand.	$\sqrt{\frac{81d^5}{25}}$
Rewrite using the Quotient Property.	$\frac{\sqrt{81d^5}}{\sqrt{25}}$
Simplify the radicals in the numerator and the denominator.	$\frac{\sqrt{81d^4} \cdot \sqrt{d}}{5}$
Simplify.	$\frac{9d^2\sqrt{d}}{5}$

TRY IT 16.1

Simplify: $\sqrt{\frac{64x^7}{9x^3}}$ where $x > 0$.

Show answer
$\frac{8x^2}{3}$

TRY IT 16.2

Simplify: $\sqrt{\dfrac{16a^9}{100a^5}}$ where $a > 0$.

Show answer
$\dfrac{2a^2}{5}$

EXAMPLE 17

Simplify: $\sqrt{\dfrac{18p^5q^7}{32pq^2}}$ where $p > 0$ and $q > 0$.

Solution

	$\sqrt{\dfrac{18p^5q^7}{32pq^2}}$
Simplify the fraction in the radicand, if possible.	$\sqrt{\dfrac{9p^4q^5}{16}}$
Rewrite using the Quotient Property.	$\dfrac{\sqrt{9p^4q^5}}{\sqrt{16}}$
Simplify the radicals in the numerator and the denominator.	$\dfrac{\sqrt{9p^4q^4}\cdot\sqrt{q}}{4}$
Simplify.	$\dfrac{3p^2q^2\sqrt{q}}{4}$

TRY IT 17.1

Simplify: $\sqrt{\dfrac{50x^5y^3}{72x^4y}}$ where $x > 0$ and $y > 0$.

Show answer
$\dfrac{5y\sqrt{x}}{6}$

TRY IT 17.2

Simplify: $\sqrt{\dfrac{48m^7n^2}{125m^5n^9}}$ where $m > 0$ and $n > 0$.

Show answer

$\frac{4m\sqrt{3}}{5n^3\sqrt{5n}}$

Key Concepts

- **Simplified Square Root** $\sqrt{a}$ is considered simplified if a has no perfect-square factors.
- **Product Property of Square Roots** If *a*, *b* are non-negative real numbers, then $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
- **Simplify a Square Root Using the Product Property** To simplify a square root using the Product Property:
    1. Find the largest perfect square factor of the radicand. Rewrite the radicand as a product using the perfect square factor.
    2. Use the product rule to rewrite the radical as the product of two radicals.
    3. Simplify the square root of the perfect square.
- **Quotient Property of Square Roots** If *a*, *b* are non-negative real numbers and $b \neq 0$, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- **Simplify a Square Root Using the Quotient Property** To simplify a square root using the Quotient Property:
    1. Simplify the fraction in the radicand, if possible.
    2. Use the Quotient Rule to rewrite the radical as the quotient of two radicals.
    3. Simplify the radicals in the numerator and the denominator.

Practice Makes Perfect

Use the Product Property to Simplify Square Roots

In the following exercises, simplify.

1. $\sqrt{27}$	2. $\sqrt{80}$
3. $\sqrt{125}$	4. $\sqrt{96}$
5. $\sqrt{200}$	6. $\sqrt{147}$
7. $\sqrt{450}$	8. $\sqrt{252}$
9. $\sqrt{800}$	10. $\sqrt{288}$
11. $\sqrt{675}$	12. $\sqrt{1250}$
13. $\sqrt{x^7}$	14. $\sqrt{y^{11}}$
15. $\sqrt{p^3}$	16. $\sqrt{q^5}$
17. $\sqrt{m^{13}}$	18. $\sqrt{n^{21}}$
19. $\sqrt{r^{25}}$	20. $\sqrt{s^{33}}$
21. $\sqrt{49n^{17}}$	22. $\sqrt{25m^9}$
23. $\sqrt{81r^{15}}$	24. $\sqrt{100s^{19}}$
25. $\sqrt{98m^5}$	26. $\sqrt{32n^{11}}$
27. $\sqrt{125r^{13}}$	28. $\sqrt{80s^{15}}$
29. $\sqrt{200p^{13}}$	30. $\sqrt{128q^3}$
31. $\sqrt{242m^{23}}$	32. $\sqrt{175n^{13}}$
33. $\sqrt{147m^7n^{11}}$	34. $\sqrt{48m^7n^5}$
35. $\sqrt{75r^{13}s^9}$	36. $\sqrt{96r^3s^3}$
37. $\sqrt{300p^9q^{11}}$	38. $\sqrt{192q^3r^7}$
39. $\sqrt{242m^{13}n^{21}}$	40. $\sqrt{150m^9n^3}$
41. $5+\sqrt{12}$	42. $8+\sqrt{96}$
43. $1+\sqrt{45}$	44. $3+\sqrt{125}$
45. $\frac{10-\sqrt{24}}{2}$	46. $\frac{8-\sqrt{80}}{4}$
47. $\frac{3+\sqrt{90}}{3}$	48. $\frac{15+\sqrt{75}}{5}$

Use the Quotient Property to Simplify Square Roots

In the following exercises, simplify. Note that all the variables are non-negative.

49. $\sqrt{\frac{49}{64}}$	50. $\sqrt{\frac{100}{36}}$
51. $\sqrt{\frac{121}{16}}$	52. $\sqrt{\frac{144}{169}}$
53. $\sqrt{\frac{72}{98}}$	54. $\sqrt{\frac{75}{12}}$
55. $\sqrt{\frac{9}{25}}$	56. $\sqrt{\frac{300}{243}}$
57. $\sqrt{\frac{x^{10}}{x^6}}$	58. $\sqrt{\frac{p^{20}}{p^{10}}}$
59. $\sqrt{\frac{y^4}{y^8}}$	60. $\sqrt{\frac{q^8}{q^{14}}}$
61. $\sqrt{\frac{200x^7}{2x^3}}$	62. $\sqrt{\frac{98y^{11}}{2y^5}}$
63. $\sqrt{\frac{96p^9}{6p}}$	64. $\sqrt{\frac{108q^{10}}{3q^2}}$
65. $\sqrt{\frac{36}{35}}$	66. $\sqrt{\frac{144}{65}}$
67. $\sqrt{\frac{20}{81}}$	68. $\sqrt{\frac{21}{196}}$
69. $\sqrt{\frac{96x^7}{121}}$	70. $\sqrt{\frac{108y^4}{49}}$
71. $\sqrt{\frac{300m^5}{64}}$	72. $\sqrt{\frac{125n^7}{169}}$
73. $\sqrt{\frac{98r^5}{100}}$	74. $\sqrt{\frac{180s^{10}}{144}}$
75. $\sqrt{\frac{28q^6}{225}}$	76. $\sqrt{\frac{150r^3}{256}}$
77. $\sqrt{\frac{75r^9}{s^8}}$	78. $\sqrt{\frac{72x^5}{y^6}}$
79. $\sqrt{\frac{28p^7}{q^2}}$	80. $\sqrt{\frac{45r^3}{s^{10}}}$
81. $\sqrt{\frac{100x^5}{36x^3}}$	82. $\sqrt{\frac{49r^{12}}{16r^6}}$
83. $\sqrt{\frac{121p^5}{81p^2}}$	84. $\sqrt{\frac{25r^8}{64r}}$
85. $\sqrt{\frac{32x^5y^3}{18x^3y}}$	86. $\sqrt{\frac{75r^6s^8}{48rs^4}}$

87. $\sqrt{\dfrac{27p^2q}{108p^5q^3}}$	88. $\sqrt{\dfrac{50r^5s^2}{128r^2s^5}}$

Everyday Math

89. a) Elliott decides to construct a square garden that will take up 288 square feet of his yard. Simplify $\sqrt{288}$ to determine the length and the width of his garden. Round to the nearest tenth of a foot. b) Suppose Elliott decides to reduce the size of his square garden so that he can create a 5-foot-wide walking path on the north and east sides of the garden. Simplify $\sqrt{288} - 5$ to determine the length and width of the new garden. Round to the nearest tenth of a foot.	90. a) Melissa accidentally drops a pair of sunglasses from the top of a roller coaster, 64 feet above the ground. Simplify $\sqrt{\dfrac{64}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground. b) Suppose the sunglasses in the previous example were dropped from a height of 144 feet. Simplify $\sqrt{\dfrac{144}{16}}$ to determine the number of seconds it takes for the sunglasses to reach the ground.

Writing Exercises

91. Explain why $\sqrt{x^4} = x^2$. Then explain why $\sqrt{x^{16}} = x^8$.	92. Explain why $7 + \sqrt{9}$ is not equal to $\sqrt{7 + 9}$.

Answers

1. $3\sqrt{3}$	3. $5\sqrt{5}$	5. $10\sqrt{2}$
7. $15\sqrt{2}$	9. $20\sqrt{2}$	11. $15\sqrt{3}$
13. $x^3\sqrt{x}$	15. $p\sqrt{p}$	17. $m^6\sqrt{m}$
19. $r^{12}\sqrt{r}$	21. $7n^8\sqrt{n}$	23. $9r^7\sqrt{r}$
25. $7m^2\sqrt{2m}$	27. $5r^6\sqrt{5r}$	29. $10p^6\sqrt{2p}$
31. $11m^{11}\sqrt{2m}$	33. $7m^3n^5\sqrt{3mn}$	35. $5r^6s^4\sqrt{3rs}$
37. $10p^4q^5\sqrt{3pq}$	39. $11m^6n^{10}\sqrt{2mn}$	41. $5+2\sqrt{3}$
43. $1+3\sqrt{5}$	45. $5-\sqrt{6}$	47. $1+\sqrt{10}$
49. $\frac{7}{8}$	51. $\frac{11}{4}$	53. $\frac{6}{7}$
55. $\frac{3}{5}$	57. x^2	59. $\frac{1}{y^2}$
61. $10x^2$	63. $4p^4$	65. $\frac{6}{\sqrt{35}}$
67. $\frac{2\sqrt{5}}{9}$	69. $\frac{4x^3\sqrt{6x}}{11}$	71. $\frac{5m^2\sqrt{3m}}{4}$
73. $\frac{7r^2\sqrt{2r}}{10}$	75. $\frac{2q^3\sqrt{7}}{15}$	77. $\frac{5r^4\sqrt{3r}}{s^4}$
79. $\frac{2p^3\sqrt{7p}}{q}$	81. $\frac{5x}{3}$	83. $\frac{11p\sqrt{p}}{9}$
85. $\frac{4xy}{3}$	87. $\frac{1}{2pq\sqrt{p}}$	89. a)17.0 feet b)12.0 feet
91. Answers will vary.		

Attributions

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5.6 Chapter Review

Review Exercises

Simplify Expressions with Exponents

In the following exercises, simplify.

1. 17^1	2. 10^4
3. $(0.5)^3$	4. $\left(\frac{2}{9}\right)^2$
5. -2^6	6. $(-2)^6$

Simplify Expressions Using the Product Property for Exponents

In the following exercises, simplify each expression.

7. $p^{15} \cdot p^{16}$	8. $x^4 \cdot x^3$
9. $8 \cdot 8^5$	10. $4^{10} \cdot 4^6$
11. $y^c \cdot y^3$	12. $n \cdot n^2 \cdot n^4$

Simplify Expressions Using the Power Property for Exponents

In the following exercises, simplify each expression.

13. $\left(5^3\right)^2$	14. $\left(m^3\right)^5$
15. $(3^r)^s$	16. $\left(y^4\right)^x$

Simplify Expressions Using the Product to a Power Property

In the following exercises, simplify each expression.

17. $(-5y)^3$	18 $(4a)^2$
19. $(10xyz)^3$	20. $(2mn)^5$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify each expression.

21. $\left(4a^3b^2\right)^3$	22. $\left(p^2\right)^5 \cdot \left(p^3\right)^6$
23. $\left(2q^3\right)^4(3q)^2$	24. $(5x)^2\,(7x)$
25. $\left(\frac{2}{5}m^2n\right)^3$	26. $\left(\frac{1}{3}x^2\right)^2\left(\frac{1}{2}x\right)^3$

Simplify Expressions Using the Quotient Property for Exponents

In the following exercises, simplify.

27. $\frac{10^{25}}{10^5}$	28. $\frac{u^{24}}{u^6}$
29. $\frac{v^{12}}{v^{48}}$	30. $\frac{3^4}{3^6}$
31. $\frac{5}{5^8}$	32. $\frac{x}{x^5}$

Simplify Expressions with Zero Exponents

In the following exercises, simplify.

33. x^0	34. 75^0
35. $\left(-12^0\right)(-12)^0$	36. -12^0
37. $(25x)^0$	38. $25x^0$
39. $(19n)^0 - (25m)^0$	40. $19n^0 - 25m^0$

Simplify Expressions Using the Quotient to a Power Property

In the following exercises, simplify.

41. $\left(\frac{m}{3}\right)^4$	42. $\left(\frac{2}{5}\right)^3$
43. $\left(\frac{x}{2y}\right)^6$	44. $\left(\frac{r}{s}\right)^8$

Simplify Expressions by Applying Several Properties

In the following exercises, simplify.

45. $\frac{n^{10}}{(n^5)^2}$	46. $\frac{\left(x^3\right)^5}{x^9}$
47. $\left(\frac{r^8}{r^3}\right)^4$	48. $\left(\frac{q^6}{q^8}\right)^3$
49. $\left(\frac{3x^4}{2y^2}\right)^5$	50. $\left(\frac{c^2}{d^5}\right)^9$
51. $\frac{\left(3n^2\right)^4\left(-5n^4\right)^3}{(-2n^5)^2}$	52. $\left(\frac{v^3v^9}{v^6}\right)^4$

Divide Monomials

In the following exercises, divide the monomials.

53. $\frac{64a^5b^9}{-16a^{10}b^3}$	54. $-65y^{14} \div 5y^2$
55. $\frac{\left(8p^6q^2\right)\left(9p^3q^5\right)}{16p^8q^7}$	56. $\frac{144x^{15}y^8z^3}{18x^{10}y^2z^{12}}$

Use the Definition of a Negative Exponent

In the following exercises, simplify.

57. $(-5)^{-3}$	58. 9^{-2}
59. $(6u)^{-3}$	60. $3 \cdot 4^{-3}$
61. $\left(\frac{3}{4}\right)^{-2}$	62. $\left(\frac{2}{5}\right)^{-1}$

Simplify Expressions with Integer Exponents

In the following exercises, simplify.

63. $q^{-6} \cdot q^{-5}$	64. $p^{-2} \cdot p^{8}$
65. $\left(y^{8}\right)^{-1}$	66. $\left(c^{-2}d\right)\left(c^{-3}d^{-2}\right)$
67. $\frac{a^{8}}{a^{12}}$	68. $\left(q^{-4}\right)^{-3}$
69. $\frac{r^{-2}}{r^{-3}}$	70. $\frac{n^{5}}{n^{-4}}$

Convert from Decimal Notation to Scientific Notation

In the following exercises, write each number in scientific notation.

71. 0.00429	72. 8,500,000
73. In 2015, the population of the world was about 7,200,000,000 people.	74. The thickness of a dime is about 0.053 inches.

Convert Scientific Notation to Decimal Form

In the following exercises, convert each number to decimal form.

75. 1.5 $\times 10^{10}$	76. 3.8 $\times 10^{5}$
77. 5.5 $\times 10^{-1}$	78. 9.1 $\times 10^{-7}$

Multiply and Divide Using Scientific Notation

In the following exercises, multiply and write your answer in decimal form.

79. $(3.5 \times 10^{-2})(6.2 \times 10^{-1})$	80. $(2 \times 10^{5})(4 \times 10^{-3})$

In the following exercises, divide and write your answer in decimal form.

81. $\frac{9 \times 10^{-5}}{3 \times 10^{2}}$	82. $\frac{8 \times 10^{5}}{4 \times 10^{-1}}$

Simplify Expressions with Square Roots

In the following exercises, simplify.

83. $\sqrt{144}$ 85. $-\sqrt{81}$ 87. $\sqrt{-36}$ 89. $\sqrt{64+225}$	84. $\sqrt{64}$ 86. $-\sqrt{25}$ 88. $\sqrt{-9}$ 90. $\sqrt{64}+\sqrt{225}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

91. $\sqrt{155}$	92. $\sqrt{28}$

Approximate Square Roots

In the following exercises, approximate each square root and round to two decimal places.

93. $\sqrt{57}$	94. $\sqrt{15}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

95. $\sqrt{64b^2}$ 97. $\sqrt{225m^2n^2}$ 99. $\sqrt{49y^2}$ 101. $\sqrt{121c^2d^2}$	96. $\sqrt{q^2}$ 98. $-\sqrt{121a^2}$ 100. $-\sqrt{100q^2}$ 102. $\sqrt{4a^2b^2}$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

Formulas:

- If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.
- If an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$.
- If the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$.

103.**Landscaping** Janet wants to plant a square flower garden in her yard. She has enough topsoil to cover an area of 30 square feet. How long can a side of the flower garden be? 105. **Accident investigation** The skid marks of a car involved in an accident were 216 feet. How fast had the car been going before applying the brakes?	104. **Art** Diego has 225 square inch tiles. He wants to use them to make a square mosaic. How long can each side of the mosaic be? 106. **Gravity** A hiker dropped a granola bar from a lookout spot 576 feet above a valley. How long did it take the granola bar to reach the valley floor?

Review Exercise Answers

1. 17	3. 0.125
5. -64	7. p^{31}
9. 8^6	11. y^{c+3}
13. 5^6	15. 3^{rs}
17. $-125y^3$	19. $1000x^3y^3z^3$
21. $64a^9b^6$	23. $144q^{14}$
25. $\frac{8}{125}m^6n^3$	27. 10^{20}
29. $\frac{1}{v^{36}}$	31. $\frac{1}{5^7}$
33. 1	35. 1
37. 1	39. 0
41. $\frac{m^4}{81}$	43. $\frac{x^6}{64y^6}$
45. 1	47. r^{20}
49. $\frac{243x^{20}}{32y^{10}}$	51. $-\frac{10{,}125n^{10}}{4}$
53. $-\frac{4b^6}{a^5}$	55. $\frac{9p}{2}$
57. $-\frac{1}{125}$	59.$\frac{1}{216u^3}$
61. $\frac{16}{9}$	63. $\frac{1}{q^{11}}$
65. $\frac{1}{y^8}$	67. $\frac{1}{a^4}$
69. r	71. 4.29×10^{-3}
73. 7.2×10^9	75. $15,000,000,000$
77. 0.55	79. 0.0217
81. 0.0000003	83. 12
85. −9	87. not a real number
89. 17	91. $12 < \sqrt{155} < 13$
93. 7.55	95. 8b
97. 15mn	99. 7y
101. 11cd	103. 5.5 feet
105. 72 mph	

Practice Test

In the following exercises, simplify each expression.

1. $\left(-\frac{2}{5}\right)^3$	2. $u \cdot u^4$
3. $\left(4a^3b^5\right)^2$	4. $\frac{n^{-2}}{n^{-10}}$
5. $\frac{3^8}{3^{10}}$	6. $\left(\frac{v^2v^6}{v^4}\right)^2$
7. $\left(87x^{15}y^3z^{22}\right)^0$	8. $\left(\frac{m^4 \cdot m}{m^3}\right)^6$
9. $\frac{80c^8d^2}{16cd^{10}}$	10. 5^{-2}
11. $q^{-4} \cdot q^{-5}$	12. $(4m)^{-3}$
13. $\frac{8.4 \times 10^{-3}}{4 \times 10^3}$	14. $\left(3.4 \times 10^9\right)\left(2.2 \times 10^{-5}\right)$
15. $\sqrt{81}$	16. $-\sqrt{49}$
17. $\sqrt{-16}$	18. $\sqrt{b^2}$
19. $-\sqrt{64a^2}$	20. $-\sqrt{144q^2}$
21. Convert 83,000,000 to scientific notation.	22. Convert 6.91×10^{-5} to decimal form.

Practice Test Answers

1. $-\frac{8}{125}$	2. u^5
3. $16a^6b^{10}$	4. n^8
5. $\frac{1}{9}$	6. v^8
7. 1	8. m^{12}
9. $\frac{5c^7}{d^8}$	10. $\frac{1}{25}$
11. $\frac{1}{q^9}$	12. $\frac{1}{64m^3}$
13. 2.1×10^{-6}	14. 7.48×10^4
15. 9	16. -7
17. not a real number	18. b
19. $-8a$	20. $-12q$
21. 8.3×10^7	22. 0.0000691

IV

CHAPTER 6 Polynomials

Architects use polynomials to design curved shapes such as this suspension bridge, the Silver Jubilee bridge in Halton, England.

We have seen that the graphs of linear equations are straight lines. Graphs of other types of equations, called polynomial equations, are curves, like the outline of this suspension bridge. Architects use polynomials to design the shape of a bridge like this and to draw the blueprints for it. Engineers use polynomials to calculate the stress on the bridge's supports to ensure they are strong enough for the intended load. In this chapter, you will explore operations with and properties of polynomials.

6.1 Add and Subtract Polynomials

Learning Objectives

By the end of this section, you will be able to:

- Identify polynomials, monomials, binomials, and trinomials
- Determine the degree of polynomials
- Add and subtract monomials
- Add and subtract polynomials
- Evaluate a polynomial for a given value

Identify Polynomials, Monomials, Binomials and Trinomials

You have learned that a *term* is a constant or the product of a constant and one or more variables. When it is of the form ax^m, where a is a constant and m is a whole number, it is called a monomial. Some examples of monomial are 8, $-2x^2$, $4y^3$, and $11z^7$.

Monomials

A monomial is a term of the form ax^m, where a is a constant and m is a positive whole number.

A monomial, or two or more monomials combined by addition or subtraction, is a polynomial. Some polynomials have special names, based on the number of terms. A monomial is a polynomial with exactly one term. A binomial has exactly two terms, and a trinomial has exactly three terms. There are no special names for polynomials with more than three terms.

Polynomials

polynomial—A monomial, or two or more monomials combined by addition or subtraction, is a polynomial.

- **monomial**—A polynomial with exactly one term is called a monomial.
- **binomial**—A polynomial with exactly two terms is called a binomial.
- **trinomial**—A polynomial with exactly three terms is called a trinomial.

Here are some examples of polynomials.

Polynomial	$b+1$	$4y^2-7y+2$	$4x^4+x^3+8x^2-9x+1$	
Monomial	14	$8y^2$	$-9x^3y^5$	-13
Binomial	$a+7$	$4b-5$	y^2-16	$3x^3-9x^2$
Trinomial	$x^2-7x+12$	$9y^2+2y-8$	$6m^4-m^3+8m$	z^4+3z^2-1

Notice that every monomial, binomial, and trinomial is also a polynomial. They are just special members of the "family" of polynomials and so they have special names. We use the words *monomial*, *binomial*, and *trinomial* when referring to these special polynomials and just call all the rest *polynomials*.

EXAMPLE 1

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial.

a. $4y^2-8y-6$

b. $-5a^4b^2$

c. $2x^5-5x^3-9x^2+3x+4$

d. $13-5m^3$

e. q

Solution

	Polynomial	**Number of terms**	**Type**
a)	$4y^2-8y-6$	3	Trinomial
b)	$-5a^4b^2$	1	Monomial
c)	$2x^5-5x^3-9x^2+3x+4$	5	Polynomial
d)	$13-5m^3$	2	Binomial
e)	q	1	Monomial

TRY IT 1.1

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial:

a) $5b$ b) $8y^3-7y^2-y-3$ c) $-3x^2-5x+9$ d) $81-4a^2$e)$-5x^6$

Show answer

a) monomial b) polynomial c) trinomial d) binomial e) monomial

TRY IT 1.2

Determine whether each polynomial is a monomial, binomial, trinomial, or other polynomial:

a) $27z^3 - 8$ b) $12m^3 - 5m^2 - 2m$ c) $\frac{5}{6}$ d) $8x^4 - 7x^2 - 6x - 5$ e) $-n^4$

Show answer

a) binomial b) trinomial c) monomial d) polynomial e) monomial

Determine the Degree of Polynomials

The degree of a polynomial and the degree of its terms are determined by the exponents of the variable.

A monomial that has no variable, just a constant, is a special case. The degree of a constant is 0—it has no variable.

Degree of a Polynomial

The degree of a term is the sum of the exponents of its variables.

The degree of a constant is 0.

The degree of a polynomial is the highest degree of all its terms.

Let's see how this works by looking at several polynomials. We'll take it step by step, starting with monomials, and then progressing to polynomials with more terms.

Monomial	14	$8y^2$	$-9x^3y^5$	$-13a$
Degree	0	2	8	1

Binomial	$a + 7$	$4b^2 - 5b$	$x^2y^2 - 16$	$3n^3 - 9n^2$
Degree of each term	0 1	2 1	4 0	3 2
Degree of polynomial	1	2	4	3

Trinomial	$x^2 - 7x + 12$	$9a^2 + 6ab + b^2$	$6m^4 - m^3n^2 + 8mn^5$	$z^4 + 3z^2 - 1$
Degree of each term	2 1 0	2 2 2	4 5 6	4 2 0
Degree of polynomial	2	2	6	4

Polynomial	$b + 1$	$4y^2 - 7y + 2$	$4x^4 + x^3 + 8x^2 - 9x + 1$
Degree of each term	1 0	2 1 0	4 3 2 1 0
Degree of polynomial	1	2	4

A polynomial is in standard form when the terms of a polynomial are written in descending order of degrees. Get in the habit of writing the term with the highest degree first.

EXAMPLE 2

Find the degree of the following polynomials.

a. $10y$

b. $4x^3 - 7x + 5$

c. -15

d. $-8b^2 + 9b - 2$

e. $8xy^2 + 2y$

Solution

a) The exponent of y is one. $y = y^1$	$10y$ The degree is 1.
b) The highest degree of all the terms is 3.	$4x^3 - 7x + 5$ The degree is 3.
c) The degree of a constant is 0.	-15 The degree is 0.
d) The highest degree of all the terms is 2.	$-8b^2 + 9b - 2$ The degree is 2.
e) The highest degree of all the terms is 3.	$8xy^2 + 2y$ The degree is 3.

EXAMPLE 2.1

Find the degree of the following polynomials:

a) $-15b$ b) $10z^4 + 4z^2 - 5$ c) $12c^5d^4 + 9c^3d^9 - 7$ d) $3x^2y - 4x$e)-9

Show answer

a) 1 b) 4 c) 12 d) 3 e) 0

TRY IT 2.2

Find the degree of the following polynomials:

a) 52 b) $a^4b - 17a^4$ c) $5x + 6y + 2z$ d) $3x^2 - 5x + 7$e)$-a^3$

Show answer

a) 0 b) 5 c) 1 d) 2 e) 3

Add and Subtract Monomials

You have learned how to simplify expressions by combining like terms. Remember, like terms must have the same variables with the same exponent. Since monomials are terms, adding and subtracting monomials is the same as combining like terms. If the monomials are like terms, we just combine them by adding or subtracting the coefficient.

EXAMPLE 3

Add: $25y^2 + 15y^2$.

Solution

	$25y^2 + 15y^2$
Combine like terms.	$40y^2$

TRY IT 3.1

Add: $12q^2 + 9q^2$.

Show answer
$21q^2$

TRY 3.2

Add: $-15c^2 + 8c^2$.

Show answer
$-7c^2$

EXAMPLE 4

Subtract: $16p - (-7p)$.

Solution

	$16p-(-7p)$
Combine like terms.	$23p$

TRY IT 4.1

Subtract: $8m-(-5m)$.

Show answer
$13m$

TRY IT 4.2

Subtract: $-15z^3-\left(-5z^3\right)$.

Show answer
$-10z^3$

Remember that like terms must have the same variables with the same exponents.

EXAMPLE 5

Simplify: $c^2+7d^2-6c^2$.

Solution

	$c^2+7d^2-6c^2$
Combine like terms.	$-5c^2+7d^2$

TRY IT 5.1

Add: $8y^2+3z^2-3y^2$.

Show answer
$5y^2+3z^2$

TRY IT 5.2

Add: $3m^2 + n^2 - 7m^2$.

Show answer
$-4m^2 + n^2$

EXAMPLE 6

Simplify: $u^2v + 5u^2 - 3v^2$.

Solution

	$u^2v + 5u^2 - 3v^2$
There are no like terms to combine.	$u^2v + 5u^2 - 3v^2$

TRY IT 6.1

Simplify: $m^2n^2 - 8m^2 + 4n^2$.

Show answer
There are no like terms to combine.

TRY IT 6.2

Simplify: $pq^2 - 6p - 5q^2$.

Show answer
There are no like terms to combine.

Add and Subtract Polynomials

We can think of adding and subtracting polynomials as just adding and subtracting a series of monomials. Look for the like terms—those with the same variables and the same exponent. The Commutative Property allows us to rearrange the terms to put like terms together.

EXAMPLE 7

Find the sum: $\left(5y^2 - 3y + 15\right) + \left(3y^2 - 4y - 11\right)$.

Solution

Identify like terms.	$\left(5y^2 - 3y + 15\right) + \left(3y^2 - 4y - 11\right)$
Rearrange to get the like terms together.	$5y^2 + 3y^2 - 3y - 4y + 15 - 11$
Combine like terms.	$8y^2 - 7y + 4$

TRY IT 7.1

Find the sum: $\left(7x^2 - 4x + 5\right) + \left(x^2 - 7x + 3\right)$.

Show answer
$8x^2 - 11x + 8$

TRY IT 7.2

Find the sum: $\left(14y^2 + 6y - 4\right) + \left(3y^2 + 8y + 5\right)$.

Show answer
$17y^2 + 14y + 1$

EXAMPLE 8

Find the difference: $\left(9w^2 - 7w + 5\right) - \left(2w^2 - 4\right)$.

Solution

	$\left(9w^2 - 7w + 5\right) - \left(2w^2 - 4\right)$
Distribute and identify like terms.	$9w^2 - 7w + 5 - 2w^2 + 4$
Rearrange the terms.	$9w^2 - 2w^2 - 7w + 5 + 4$
Combine like terms.	$7w^2 - 7w + 9$

TRY IT 8.1

Find the difference: $\left(8x^2+3x-19\right)-\left(7x^2-14\right)$.

Show answer
x^2+3x-5

TRY IT 8.2

Find the difference: $\left(9b^2-5b-4\right)-\left(3b^2-5b-7\right)$.

Show answer
$6b^2+3$

EXAMPLE 9

Subtract: $\left(c^2-4c+7\right)$ from $\left(7c^2-5c+3\right)$.

Solution

Subtract: $\left(c^2-4c+7\right)$ from $\left(7c^2-5c+3\right)$.	$\left(7c^2-5c+3\right)-\left(c^2-4c+7\right)$
Distribute and identify like terms.	$\underline{\underline{7c^2}}-\underline{5c}+\underset{\sim}{3}-\underline{\underline{c^2}}+\underline{4c}-\underset{\sim}{7}$
Rearrange the terms.	$\underline{\underline{7c^2-c^2}}-\underline{5c+4c}+\underset{\sim}{3}-\underset{\sim}{7}$
Combine like terms.	$6c^2-c-4$

TRY IT 9.1

Subtract: $\left(5z^2-6z-2\right)$ from $\left(7z^2+6z-4\right)$.

Show answer
$2z^2+12z-2$

TRY IT 9.2

Subtract: $\left(x^2-5x-8\right)$ from $\left(6x^2+9x-1\right)$.

Show answer

$5x^2 + 14x + 7$

EXAMPLE 10

Find the sum: $\left(u^2 - 6uv + 5v^2\right) + \left(3u^2 + 2uv\right)$.

Solution

	$\left(u^2 - 6uv + 5v^2\right) + \left(3u^2 + 2uv\right)$
Distribute.	$u^2 - 6uv + 5v^2 + 3u^2 + 2uv$
Rearrange the terms, to put like terms together.	$u^2 + 3u^2 - 6uv + 2uv + 5v^2$
Combine like terms.	$4u^2 - 4uv + 5v^2$

EXAMPLE 10.1

Find the sum: $\left(3x^2 - 4xy + 5y^2\right) + \left(2x^2 - xy\right)$.

Show answer
$5x^2 - 5xy + 5y^2$

EXAMPLE 10.2

Find the sum: $\left(2x^2 - 3xy - 2y^2\right) + \left(5x^2 - 3xy\right)$.

Show answer
$7x^2 - 6xy - 2y^2$

EXAMPLE 11.1

Find the difference: $\left(p^2 + q^2\right) - \left(p^2 + 10pq - 2q^2\right)$.

Solution

	$\left(p^2+q^2\right)-\left(p^2+10pq-2q^2\right)$
Distribute.	$p^2+q^2-p^2-10pq+2q^2$
Rearrange the terms, to put like terms together.	$p^2-p^2-10pq+q^2+2q^2$
Combine like terms.	$-10pq^2+3q^2$

TRY IT 11.1

Find the difference: $\left(a^2+b^2\right)-\left(a^2+5ab-6b^2\right)$.

Show answer
$-5ab+7b^2$

TRY IT 11.2

Find the difference: $\left(m^2+n^2\right)-\left(m^2-7mn-3n^2\right)$.

Show answer
$4n^2+7mn$

EXAMPLE 12

Simplify: $\left(a^3-a^2b\right)-\left(ab^2+b^3\right)+\left(a^2b+ab^2\right)$.

Solution

	$\left(a^3-a^2b\right)-\left(ab^2+b^3\right)+\left(a^2b+ab^2\right)$
Distribute.	$a^3-a^2b-ab^2-b^3+a^2b+ab^2$
Rearrange the terms, to put like terms together.	$a^3-a^2b+a^2b-ab^2+ab^2-b^3$
Combine like terms.	a^3-b^3

TRY IT 12.1

Simplify: $\left(x^3 - x^2y\right) - \left(xy^2 + y^3\right) + \left(x^2y + xy^2\right)$.

Show answer
$x^3 - y^3$

TRY IT 12.2

Simplify: $\left(p^3 - p^2q\right) + \left(pq^2 + q^3\right) - \left(p^2q + pq^2\right)$.

Show answer
$p^3 - 2p^2q + q^3$

Evaluate a Polynomial for a Given Value

We have already learned how to evaluate expressions. Since polynomials are expressions, we'll follow the same procedures to evaluate a polynomial. We will substitute the given value for the variable and then simplify using the order of operations.

EXAMPLE 13

Evaluate $5x^2 - 8x + 4$ when

a. $x = 4$
b. $x = -2$
c. $x = 0$

Solution

a) $x = 4$	
	$5x^2 - 8x + 4$
Substitute 4 for x.	$5(4)^2 - 8(4) + 4$
Simplify the exponents.	$5 \cdot 16 - 8 \cdot 4 + 4$
Multiply.	$80 - 32 + 4$
Simplify.	52

b) $x = -2$	
	$5x^2 - 8x + 4$
Substitute -2 for x .	$5(-2)^2 - 8(-2) + 4$
Simplify the exponents.	$5 \cdot 4 - 8(-2) + 4$
Multiply.	$20 + 16 + 4$
Simplify.	40

c) $x = 0$	
	$5x^2 - 8x + 4$
Substitute 0 for x .	$5(0)^2 - 8(0) + 4$
Simplify the exponents.	$5 \cdot 0 - 8 \cdot 0 + 4$
Multiply.	$0 + 0 + 4$
Simplify.	4

TRY IT 13.1

Evaluate: $3x^2 + 2x - 15$ when

a. $x = 3$
b. $x = -5$
c. $x = 0$

Show answer
a) 18 b) 50 c) -15

TRY IT 13.2

Evaluate: $5z^2 - z - 4$ when

a. $z = -2$
b. $z = 0$
c. $z = 2$

Show answer

a) 18 b) -4 c) 14

EXAMPLE 14

The polynomial $-16t^2+250$ gives the height of a ball t seconds after it is dropped from a 250 foot tall building. Find the height after $t=2$ seconds.

Solution

	$-16t^2+250$
Substitute $t=2$.	$-16(2)^2+250$
Simplify.	$-16\cdot4+250$
Simplify.	$-64+250$
Simplify.	186
	After 2 seconds the height of the ball is 186 feet.

TRY IT 14.1

The polynomial $-16t^2+250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall building. Find the height after $t=0$ seconds.

Show answer
250

TRY IT 14.2

The polynomial $-16t^2+250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall building. Find the height after $t=3$ seconds.

Show answer
106

EXAMPLE 15

The polynomial $6x^2+15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with side *x* feet and sides of height *y* feet. Find the cost of producing a box with $x=4$ feet and $y=6$ feet.

Solution

	$6x^2 + 15xy$
Substitute $x = 4, y = 6$	$6(4)^2 + 15(4)(6)$
Simplify.	$6 \cdot 16 + 15(4)(6)$
Simplify.	$96 + 360$
Simplify.	456
	The cost of producing the box i

TRY IT 15.1

The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with side *x* feet and sides of height *y* feet. Find the cost of producing a box with $x = 6$ feet and $y = 4$ feet.

Show answer
$576

TRY IT 15.2

The polynomial $6x^2 + 15xy$ gives the cost, in dollars, of producing a rectangular container whose top and bottom are squares with side *x* feet and sides of height *y* feet. Find the cost of producing a box with $x = 5$ feet and $y = 8$ feet.

Show answer
$750

Access these online resources for additional instruction and practice with adding and subtracting polynomials.

- Add and Subtract Polynomials 1
- Add and Subtract Polynomials 2
- Add and Subtract Polynomial 3
- Add and Subtract Polynomial 4

Key Concepts

- **Monomials**
    - A monomial is a term of the form ax^m, where a is a constant and m is a whole number
- **Polynomials**
    - **polynomial—**A monomial, or two or more monomials combined by addition or subtraction is a polynomial.
    - **monomial—**A polynomial with exactly one term is called a monomial.
    - **binomial—**A polynomial with exactly two terms is called a binomial.
    - **trinomial—**A polynomial with exactly three terms is called a trinomial.
- **Degree of a Polynomial**
    - The **degree of a term** is the sum of the exponents of its variables.
    - The **degree of a constant** is 0.
    - The **degree of a polynomial** is the highest degree of all its terms.

Glossary

binomial
A binomial is a polynomial with exactly two terms.

degree of a constant
The degree of any constant is 0.

degree of a polynomial
The degree of a polynomial is the highest degree of all its terms.

degree of a term
The degree of a term is the exponent of its variable.

monomial
A monomial is a term of the form ax^m, where a is a constant and m is a whole number; a monomial has exactly one term.

polynomial
A polynomial is a monomial, or two or more monomials combined by addition or subtraction.

standard form
A polynomial is in standard form when the terms of a polynomial are written in descending order of degrees.

trinomial
A trinomial is a polynomial with exactly three terms.

Type your textbox content here.

Practice Makes Perfect

Identify Polynomials, Monomials, Binomials, and Trinomials

In the following exercises, determine if each of the following polynomials is a monomial, binomial, trinomial, or other polynomial.

1. a) $81b^5 - 24b^3 + 1$ b) $5c^3 + 11c^2 - c - 8$ c) $\frac{14}{15}y + \frac{1}{7}$ d) 5 e) $4y + 17$	2. a) $x^2 - y^2$ b) $-13c^4$ c) $x^2 + 5x - 7$ d) $x^2y^2 - 2xy + 8$ e) 19
3. a) $8 - 3x$ b) $z^2 - 5z - 6$ c) $y^3 - 8y^2 + 2y - 16$ d) $81b^5 - 24b^3 + 1$ e) -18	4. a) $11y^2$ b) -73 c) $6x^2 - 3xy + 4x - 2y + y^2$ d) $4y + 17$ e) $5c^3 + 11c^2 - c - 8$

Determine the Degree of Polynomials

In the following exercises, determine the degree of each polynomial.

5. a) $6a^2 + 12a + 14$ b) $18xy^2z$ c) $5x + 2$ d) $y^3 - 8y^2 + 2y - 16$ e) -24	6. a) $9y^3 - 10y^2 + 2y - 6$ b) $-12p^4$ c) $a^2 + 9a + 18$ d) $20x^2y^2 - 10a^2b^2 + 30$ e) 17
7. a) $14 - 29x$ b) $z^2 - 5z - 6$ c) $y^3 - 8y^2 + 2y - 16$ d) $23ab^2 - 14$ e) -3	8. a) $62y^2$ b) 15 c) $6x^2 - 3xy + 4x - 2y + y^2$ d) $10 - 9x$ e) $m^4 + 4m^3 + 6m^2 + 4m + 1$

Add and Subtract Monomials

In the following exercises, add or subtract the monomials.

9. $7x^2 + 5x^2$	10. $4y^3 + 6y^3$
11. $-12w + 18w$	12. $-3m + 9m$
13. $4\text{a} - 9a$	14. $-y - 5y$
15. $28x - (-12x)$	16. $13z - (-4z)$
17. $-5b - 17b$	18. $-10x - 35x$
19. $12a + 5b - 22a$	20. $14\text{x} - 3y - 13x$
21. $2a^2 + b^2 - 6a^2$	22. $5u^2 + 4v^2 - 6u^2$
23. $xy^2 - 5x - 5y^2$	24. $pq^2 - 4p - 3q^2$
25. $a^2b - 4a - 5ab^2$	26. $x^2y - 3x + 7xy^2$
27. $12\text{a} + 8b$	28. $19\text{y} + 5z$
29. Add: $4a, -3b, -8a$	30. Add: $4x, 3y, -3x$
31. Subtract $5x^6 \text{from} - 12x^6$.	32. Subtract $2p^4 \text{from} - 7p^4$.

Add and Subtract Polynomials

In the following exercises, add or subtract the polynomials.

33. $\left(5y^2+12y+4\right)+\left(6y^2-8y+7\right)$	34. $\left(4y^2+10y+3\right)+\left(8y^2-6y+5\right)$
35. $\left(x^2+6x+8\right)+\left(-4x^2+11x-9\right)$	36. $\left(y^2+9y+4\right)+\left(-2y^2-5y-1\right)$
37. $\left(8x^2-5x+2\right)+\left(3x^2+3\right)$	38. $\left(7x^2-9x+2\right)+\left(6x^2-4\right)$
39. $\left(5a^2+8\right)+\left(a^2-4a-9\right)$	40. $\left(p^2-6p-18\right)+\left(2p^2+11\right)$
41. $\left(4m^2-6m-3\right)-\left(2m^2+m-7\right)$	42. $\left(3b^2-4b+1\right)-\left(5b^2-b-2\right)$
43. $\left(a^2+8a+5\right)-\left(a^2-3a+2\right)$	44. $\left(b^2-7b+5\right)-\left(b^2-2b+9\right)$
45. $\left(12s^2-15s\right)-(s-9)$	46. $\left(10r^2-20r\right)-(r-8)$
47. Subtract $\left(9x^2+2\right)$ from $\left(12x^2-x+6\right)$.	48. Subtract $\left(5y^2-y+12\right)$ from $\left(10y^2-8y-20\right)$.
49. Subtract $\left(7w^2-4w+2\right)$ from $\left(8w^2-w+6\right)$.	50. Subtract $\left(5x^2-x+12\right)$ from $\left(9x^2-6x-20\right)$.
51. Find the sum of $\left(2p^3-8\right)$ and $\left(p^2+9p+18\right)$.	52. Find the sum of $\left(q^2+4q+13\right)$ and $\left(7q^3-3\right)$.
53. Find the sum of $\left(8a^3-8a\right)$ and $\left(a^2+6a+12\right)$.	54. Find the sum of $\left(b^2+5b+13\right)$ and $\left(4b^3-6\right)$.
55. Find the difference of $\left(w^2+w-42\right)$ and $\left(w^2-10w+24\right)$.	56. Find the difference of $\left(z^2-3z-18\right)$ and $\left(z^2+5z-20\right)$.
57. Find the difference of $\left(c^2+4c-33\right)$ and $\left(c^2-8c+12\right)$.	58. Find the difference of $\left(t^2-5t-15\right)$ and $\left(t^2+4t-17\right)$.
59. $\left(7x^2-2xy+6y^2\right)+\left(3x^2-5xy\right)$	60. $\left(-5x^2-4xy-3y^2\right)+\left(2x^2-7xy\right)$
61. $\left(7m^2+mn-8n^2\right)+\left(3m^2+2mn\right)$	62. $\left(2r^2-3rs-2s^2\right)+\left(5r^2-3rs\right)$
63. $\left(a^2-b^2\right)-\left(a^2+3ab-4b^2\right)$	64. $\left(m^2+2n^2\right)-\left(m^2-8mn-n^2\right)$
65. $\left(u^2-v^2\right)-\left(u^2-4uv-3v^2\right)$	66. $\left(j^2-k^2\right)-\left(j^2-8jk-5k^2\right)$
67. $\left(p^3-3p^2q\right)+\left(2pq^2+4q^3\right)-\left(3p^2q+pq^2\right)$	68. $\left(a^3-2a^2b\right)+\left(ab^2+b^3\right)-\left(3a^2b+4ab^2\right)$
69. $\left(x^3-x^2y\right)-\left(4xy^2-y^3\right)+\left(3x^2y-xy^2\right)$	70. $\left(x^3-2x^2y\right)-\left(xy^2-3y^3\right)-\left(x^2y-4xy^2\right)$

Evaluate a Polynomial for a Given Value

In the following exercises, evaluate each polynomial for the given value.

71. Evaluate $8y^2 - 3y + 2$ when: a) $y = 5$ b) $y = -2$ c) $y = 0$	72. Evaluate $5y^2 - y - 7$ when: a) $y = -4$ b) $y = 1$ c) $y = 0$
73. Evaluate $4 - 36x$ when: a) $x = 3$ b) $x = 0$ c) $x = -1$	74. Evaluate $16 - 36x^2$ when: a) $x = -1$ b) $x = 0$ c) $x = 2$
75. A painter drops a brush from a platform 75 feet high. The polynomial $-16t^2 + 75$ gives the height of the brush t seconds after it was dropped. Find the height after $t = 2$ seconds.	76. A girl drops a ball off a cliff into the ocean. The polynomial $-16t^2 + 250$ gives the height of a ball t seconds after it is dropped from a 250-foot tall cliff. Find the height after $t = 2$ seconds.
77. A manufacturer of stereo sound speakers has found that the revenue received from selling the speakers at a cost of *p* dollars each is given by the polynomial $-4p^2 + 420p$. Find the revenue received when $p = 60$ dollars.	78. A manufacturer of the latest basketball shoes has found that the revenue received from selling the shoes at a cost of *p* dollars each is given by the polynomial $-4p^2 + 420p$. Find the revenue received when $p = 90$ dollars.

Everyday Math

79. **Fuel Efficiency** The fuel efficiency (in miles per gallon) of a car going at a speed of x miles per hour is given by the polynomial $-\frac{1}{150}x^2 + \frac{1}{3}x$. Find the fuel efficiency when $x = 30\text{mph}$.	80. **Stopping Distance** The number of feet it takes for a car traveling at x miles per hour to stop on dry, level concrete is given by the polynomial $0.06x^2 + 1.1x$. Find the stopping distance when $x = 40\text{mph}$.
81. **Rental Cost** The cost to rent a rug cleaner for d days is given by the polynomial $5.50d + 25$. Find the cost to rent the cleaner for 6 days.	82. **Height of Projectile** The height (in feet) of an object projected upward is given by the polynomial $-16t^2 + 60t + 90$ where t represents time in seconds. Find the height after $t = 2.5$ seconds.
83. **Temperature Conversion** The temperature in degrees Fahrenheit is given by the polynomial $\frac{9}{5}c + 32$ where c represents the temperature in degrees Celsius. Find the temperature in degrees Fahrenheit when $c = 65°$.	

Writing Exercises

84. Using your own words, explain the difference between a monomial, a binomial, and a trinomial.	85. Using your own words, explain the difference between a polynomial with five terms and a polynomial with a degree of 5.
86. Ariana thinks the sum $6y^2 + 5y^4$ is $11y^6$. What is wrong with her reasoning?	87. Jonathan thinks that $\frac{1}{3}$ and $\frac{1}{x}$ are both monomials. What is wrong with his reasoning?

Answers

1. a) trinomial b) polynomial c) binomial d) monomial e) binomial	3. a) binomial b) trinomial c) polynomial d) trinomial e) monomial
5. a) 2 b) 4 c) 1 d) 3 e) 0	7. a) 1 b) 2 c) 3 d) 3 e) 0
9. $12x^2$	11. $6w$
13. $-5a$	15. $40x$
17. $-22b$	19. $-10a+5b$
21. $-4a^2+b^2$	23. $xy^2-5x-5y^2$
25. $a^2b-4a-5ab^2$	27. $12\text{a}+8b$
29. $-4a-3b$	31. $-17x^6$
33. $11y^2+4y+11$	35. $-3x^2+17x-1$
37. $11x^2-5x+5$	39. $6a^2-4a-1$
41. $2m^2-7m+4$	43. $11a+3$
45. $12s^2-16s+9$	47. $3x^2-x+4$
49. w^2+3w+4	51. $2p^3+p^2+9p+10$
53. $8a^3+a^2-2a+12$	55. $11w-64$
57. $12c-45$	59. $10x^2-7xy+6y^2$
61. $10m^2+3mn-8n^2$	63. $-3ab+3b^2$
65. $4uv+2v^2$	67. $p^3-6p^2q+pq^2+4q^3$
69. $x^3+2x^2y-5xy^2+y^3$	71. a) 187 b) 40 c) 2
73. a) −104 b) 4 c) 40	75. 11
77. $10,800	79. 4 miles per gallon
81. $58	83. 149
85. Answers will vary.	87. Answers will vary.

Attributions

6.2 Multiply Polynomials

Learning Objectives

By the end of this section, you will be able to:

- Multiply a polynomial by a monomial
- Multiply a binomial by a binomial
- Multiply a trinomial by a binomial

Multiply a Polynomial by a Monomial

We have used the Distributive Property to simplify expressions like $2\left(x-3\right)$. You multiplied both terms in the parentheses, x and 3, by 2, to get $2x-6$. With this chapter's new vocabulary, you can say you were multiplying a binomial, $x-3$, by a monomial, 2

Multiplying a binomial by a monomial is nothing new for you! Here's an example:

EXAMPLE 1

Multiply: $4\left(x+3\right)$.

Solution

	$4(x+3)$
Distribute.	$4\cdot x+4\cdot 3$
Simplify.	$4x+12$

TRY IT 1.1

Multiply: $5\left(x+7\right)$.

Show answer

$5x+35$

TRY IT 1.2

Multiply: $3\left(y+13\right)$.

Show answer
$3y+39$

EXAMPLE 2

Multiply: $y\left(y-2\right)$.

Solution

	
Distribute.	$y\cdot y-y\cdot 2$
Simplify.	y^2-2y

TRY IT 2.1

Multiply: $x\left(x-7\right)$.

Show answer
x^2-7x

TRY IT 2.2

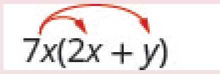
Multiply: $d\left(d-11\right)$.

Show answer
d^2-11d

EXAMPLE 3

Multiply: $7x\left(2x+y\right)$.

Solution

	
Distribute.	$7x \cdot 2x + 7x \cdot y$
Simplify.	$14x^2 + 7xy$

TRY IT 3.1

Multiply: $5x\left(x + 4y\right)$.

Show answer
$5x^2 + 20xy$

TRY IT 3.2

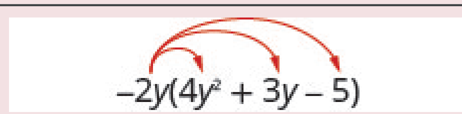
Multiply: $2p\left(6p + r\right)$.

Show answer
$12p^2 + 2pr$

EXAMPLE 4

Multiply: $-2y\left(4y^2 + 3y - 5\right)$.

Solution

	
Distribute.	$-2y \cdot 4y^2 + (-2y) \cdot 3y - (-2y) \cdot 5$
Simplify.	$-8y^3 - 6y^2 + 10y$

TRY IT 4.1

Multiply: $-3y\left(5y^2 + 8y - 7\right)$.

Show answer
$-15y^3 - 24y^2 + 21y$

TRY IT 4.2

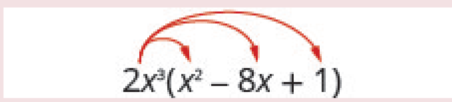
Multiply: $4x^2\left(2x^2 - 3x + 5\right)$.

Show answer
$8x^4 - 12x^3 + 20x^2$

EXAMPLE 5

Multiply: $2x^3\left(x^2 - 8x + 1\right)$.

Solution

	
Distribute.	$2x^3 \cdot x^2 + \left(2x^3\right) \cdot (-8x) + \left(2x^3\right) \cdot 1$
Simplify.	$2x^5 - 16x^4 + 2x^3$

TRY IT 5.1

Multiply: $4x\left(3x^2 - 5x + 3\right)$.

Show answer
$12x^3 - 20x^2 + 12x$

TRY IT 5.2

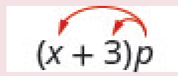
Multiply: $-6a^3\left(3a^2 - 2a + 6\right)$.

Show answer
$-18a^5 + 12a^4 - 36a^3$

EXAMPLE 6

Multiply: $(x+3)\,p$.

Solution

The monomial is the second factor.	
Distribute.	$x \cdot p + 3 \cdot p$
Simplify.	$xp + 3p$

TRY IT 6.1

Multiply: $(x+8)\,p$.

Show answer
$xp + 8p$

TRY IT 6.2

Multiply: $(a+4)\,p$.

Show answer
$ap + 4p$

Multiply a Binomial by a Binomial

Just like there are different ways to represent multiplication of numbers, there are several methods that can be used to multiply a binomial times a binomial. We will start by using the Distributive Property.

Multiply a Binomial by a Binomial Using the Distributive Property

Look at the table below, where we multiplied a binomial by a monomial.

	$(x+3)p$
We distributed the *p* to get:	$xp+3p$
What if we have $(x+7)$ instead of *p*?	$(x+3)(x+7)$
Distribute $(x+7)$.	$x(x+7)+3(x+7)$
Distribute again.	$x^2+7x+3x+21$
Combine like terms.	$x^2+10x+21$

Notice that before combining like terms, you had four terms. You multiplied the two terms of the first binomial by the two terms of the second binomial—four multiplications.

EXAMPLE 7

Multiply: $(y+5)(y+8)$.

Solution

	$(y+5)(y+8)$
Distribute $(y+8)$.	$y(y+8)+5(y+8)$
Distribute again	$y\cdot y+y\cdot 8+5\cdot y+5\cdot 8$ $y^2+8y+5y+40$
Combine like terms.	$y^2+13y+40$

TRY IT 7.1

Multiply: $(x+8)(x+9)$.

Show answer

$x^2+17x+72$

TRY IT 7.2

Multiply: $(5x+9)(4x+3)$.

Show answer
$20x^2+51x+27$

EXAMPLE 8

Multiply: $(2y+5)(3y+4)$.

Solution

	
Distribute (3y + 4).	$2y(3y+4)+5(3y+4)$
Distribute again	$2y\cdot 3y+2y\cdot 4+5\cdot 3y+5\cdot 4$ $6y^2+8y+15y+20$
Combine like terms.	$6y^2+23y+20$

TRY IT 8.1

Multiply: $(3b+5)(4b+6)$.

Show answer
$12b^2+38b+30$

TRY IT 8.2

Multiply: $(a+10)(a+7)$.

Show answer
$a^2+17a+70$

EXAMPLE 9

Multiply: $(4y+3)(2y-5)$.

Solution

	$(4y+3)(2y-5)$
Distribute.	$4y(2y-5)+3(2y-5)$
Distribute again.	$4y\cdot 2y+4y\cdot(-5)+3\cdot 2y+3\cdot(-5)$ $8y^2-20y+6y-15$
Combine like terms.	$8y^2-14y-15$

TRY IT 9.1

Multiply: $(5y+2)\,(6y-3)$.

Show answer
$30y^2-3y-6$

TRY IT 9.2

Multiply: $(3c+4)\,(5c-2)$.

Show answer
$15c^2+14c-8$

EXAMPLE 10

Multiply: $(x+2)\,(x-y)$.

Solution

	$(x-2)(x-y)$
Distribute.	$x(x-y)-2(x-y)$
Distribute again.	$x\cdot(x)+x\cdot(-y)+(-2)\cdot(x)+(-2)\cdot(-y)$ $x^2-xy-2x+2y$
There are no like terms to combine.	$x^2-xy-2x+2y$

TRY IT 10.1

Multiply: $(a+7)(a-b)$.

Show answer
$a^2-ab+7a-7b$

TRY IT 10.2

Multiply: $(x+5)(x-y)$.

Show answer
$x^2-xy+5x-5y$

Multiply a Binomial by a Binomial Using the FOIL Method

Remember that when you multiply a binomial by a binomial you get four terms. Sometimes you can combine like terms to get a trinomial, but sometimes, like in the above example, there are no like terms to combine.

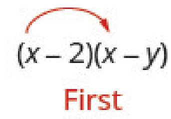
Let's look at the last example again and pay particular attention to how we got the four terms.

$$(x-2)(x-y)$$

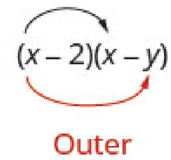
$$x^2-xy-2x+2y$$

Where did the first term, x^2, come from?

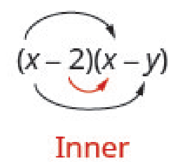
It is the product of x and x, the *first* terms in $(x - 2)$ and $(x - y)$.



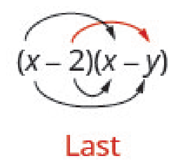
The next term, $-xy$, is the product of x and $-y$, the two *outer* terms.



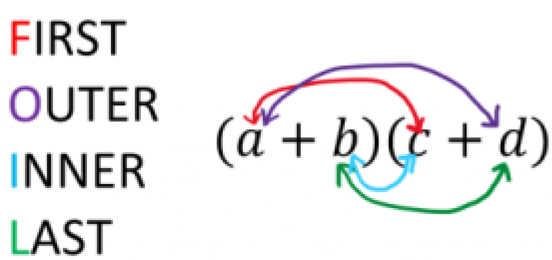
The third term, $-2x$, is the product of -2 and x, the two *inner* terms.



And the last term, $+2y$, came from multiplying the two *last* terms, -2 and $-y$.



We abbreviate "First, Outer, Inner, Last" as FOIL. The letters stand for '**F**irst, **O**uter, **I**nner, **L**ast'. The word FOIL is easy to remember and ensures we find all four products.



$(x - 2)(x - y)$
$x^2 - xy - 2x + 2y$
$F \quad O \quad I \quad L$

Let's look at $(x + 3)\,(x + 7)$.

Distibutive Property	FOIL
$(x+3)(x+7)$	$(x+3)(x+7)$
$x(x+7)+3(x+7)$	
$x^2+7x+3x+21$ F O I L	$x^2+7x+3x+21$ F O I L
$x^2+10x+21$	$x^2+10x+21$

Notice how the terms in third line fit the FOIL pattern.

Now we will do an example where we use the FOIL pattern to multiply two binomials.

EXAMPLE 11

How to Multiply a Binomial by a Binomial using the FOIL Method

Multiply using the FOIL method: $(x+5)(x+9)$.

Solution

$(x+5)(x+9)$		
Step 1. Multiply the *First* terms, $x \cdot x$	$(x+5)(x+9)$	$\underset{F}{x^2} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Step 2. Multiply the *Outer* terms, $x \cdot 9$	$(x+5)(x+9)$	$\underset{F}{x^2} + \underset{O}{9x} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Step 3. Multiply the *Inner* terms, $5 \cdot x$	$(x+5)(x+9)$	$\underset{F}{x^2} + \underset{O}{9x} + \underset{I}{5x} + \underset{L}{\_\_}$
Step 4. Multiply the *Last* terms, $5 \cdot 9$	$(x+5)(x+9)$	$\underset{F}{x^2} + \underset{O}{9x} + \underset{I}{5x} + \underset{L}{45}$
Step 5. Combine like terms.		$x^2+14x+45$

TRY IT 11.1

Multiply using the FOIL method: $(x+6)(x+8)$.

Show answer

$$x^2 + 14x + 48$$

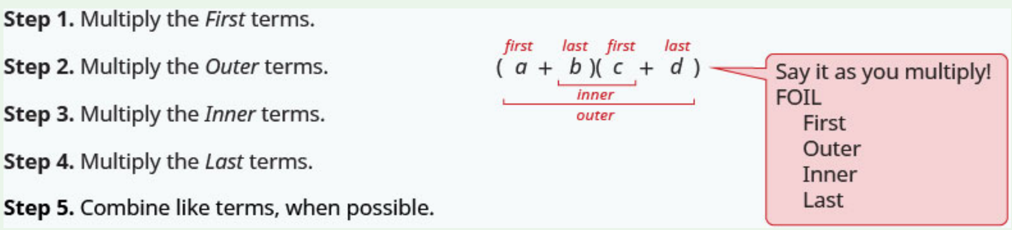
TRY IT 11.2

Multiply using the FOIL method: $(y + 17)\,(y + 3)$.

Show answer
$y^2 + 20y + 51$

We summarize the steps of the FOIL method below. The FOIL method only applies to multiplying binomials, not other polynomials!

HOW TO: Multiply two binomials using the FOIL method

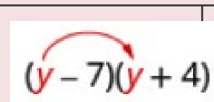
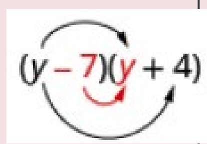
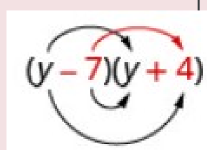


When you multiply by the FOIL method, drawing the lines will help your brain focus on the pattern and make it easier to apply.

EXAMPLE 12

Multiply: $(y - 7)\,(y + 4)$.

Solution

		$(y-7)(y+4)$
Multiply the *First* terms, $y \cdot y$		$\underset{F}{y^2} + \underset{O}{\underline{\quad}} + \underset{I}{\underline{\quad}} + \underset{L}{\underline{\quad}}$
Multiply the *Outer* terms, $y \cdot 4$		$\underset{F}{y^2} + \underset{O}{4y} + \underset{I}{\underline{\quad}} + \underset{L}{\underline{\quad}}$
Multiply the *Inner* terms, $-7 \cdot y$		$\underset{F}{y^2} + \underset{O}{4y} - \underset{I}{7y} + \underset{L}{\underline{\quad}}$
Multiply the *Last* terms, $-7 \cdot 4$		$\underset{F}{y^2} + \underset{O}{4y} - \underset{I}{7y} - \underset{L}{28}$
Combine like terms.		$y^2 - 3y - 28$

TRY IT 12.1

Multiply: $(x-7)(x+5)$.

Show answer
$x^2 - 2x - 35$

TRY IT 12.2

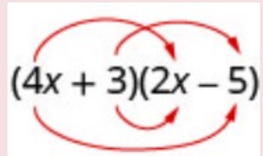
Multiply: $(b-3)(b+6)$.

Show answer
$b^2 + 3b - 18$

EXAMPLE 13

Multiply: $(4x+3)(2x-5)$.

Solution

$(4x+3)(2x-5)$	
Multiply the *First* terms, $4x \cdot 2x$	$\underset{F}{8x^2} + \underset{O}{\underline{\quad}} + \underset{I}{\underline{\quad}} + \underset{L}{\underline{\quad}}$
Multiply the *Outer* terms, $4x \cdot (-5)$	$\underset{F}{8x^2} - \underset{O}{20x} + \underset{I}{\underline{\quad}} + \underset{L}{\underline{\quad}}$
Multiply the *Inner* terms, $3 \cdot 2x$	$\underset{F}{8x^2} - \underset{O}{20x} + \underset{I}{6x} + \underset{L}{\underline{\quad}}$
Multiply the *Last* terms, $3 \cdot (-5)$	$\underset{F}{8x^2} - \underset{O}{20x} + \underset{I}{6x} - \underset{L}{15}$
Combine like terms.	$8x^2 - 14x - 15$

TRY IT 13.1

Multiply: $(3x+7)(5x-2)$.

Show answer
$15x^2 + 29x - 14$

TRY IT 13.2

Multiply: $(4y+5)(4y-10)$.

Show answer
$16y^2 - 20y - 50$

The final products in the last four examples were trinomials because we could combine the two middle terms. This is not always the case.

EXAMPLE 14

Multiply: $(3x-y)(2x-5)$.

Solution

$(3x - y)(2x - 5)$	
Multiply the *First*.	$\underset{F}{6x^2} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer*.	$\underset{F}{6x^2} - \underset{O}{15x} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner*.	$\underset{F}{6x^2} - \underset{O}{15x} - \underset{I}{2xy} + \underset{L}{\_\_}$
Multiply the *Last*.	$\underset{F}{6x^2} - \underset{O}{15x} - \underset{I}{2xy} + \underset{L}{5y}$
Combine like terms—there are none.	$6x^2 - 15x - 2xy + 5y$

TRY IT 14.1

Multiply: $(10c - d)(c - 6)$.

Show answer
$10c^2 - 60c - cd + 6d$

TRY IT 14.2

Multiply: $(7x - y)(2x - 5)$.

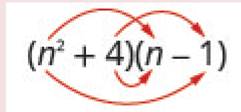
Show answer
$14x^2 - 35x - 2xy + 5y$

Be careful of the exponents in the next example.

EXAMPLE 15

Multiply: $\left(n^2 + 4\right)(n - 1)$.

Solution

$\left(n^2+4\right)(n-1)$	
Multiply the *First*.	$\underset{F}{n^3} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer*.	$\underset{F}{n^3} - \underset{O}{n^2} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner*.	$\underset{F}{n^3} - \underset{O}{n^2} + \underset{I}{4n} + \underset{L}{\_\_}$
Multiply the *Last*.	$\underset{F}{n^3} - \underset{O}{n^2} + \underset{I}{4n} - \underset{L}{4}$
Combine like terms—there are none.	n^3-n^2+4n-4

TRY IT 15.1

Multiply: $\left(x^2+6\right)(x-8)$.

Show answer
$x^3-8x^2+6x-48$

TRY IT 15.2

Multiply: $\left(y^2+7\right)(y-9)$.

Show answer
$y^3-9y^2+7y-63$

EXAMPLE 16

Multiply: $(3pq+5)(6pq-11)$.

Solution

$(3pq+5)(6pq-11)$	
Multiply the *First*.	$\underset{F}{18p^2q^2} + \underset{O}{\_\_} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Outer*.	$\underset{F}{18p^2q^2} - \underset{O}{33pq} + \underset{I}{\_\_} + \underset{L}{\_\_}$
Multiply the *Inner*.	$\underset{F}{18p^2q^2} - \underset{O}{33pq} + \underset{I}{30pq} + \underset{L}{\_\_}$
Multiply the *Last*.	$\underset{F}{18p^2q^2} - \underset{O}{33pq} + \underset{I}{30pq} - \underset{L}{55}$
Combine like terms.	$18p^2q^2 - 3pq - 55$

TRY IT 16.1

Multiply: $(2ab+5)\,(4ab-4)$.

Show answer
$8a^2b^2 + 12ab - 20$

TRY IT 16.2

Multiply: $(2xy+3)\,(4xy-5)$.

Show answer
$8x^2y^2 + 2xy - 15$

Multiply a Binomial by a Binomial Using the Vertical Method

The FOIL method is usually the quickest method for multiplying two binomials, but it *only* works for binomials. You can use the Distributive Property to find the product of any two polynomials. Another method that works for all polynomials is the Vertical Method. It is very much like the method you use to multiply whole numbers. Look carefully at this example of multiplying two-digit numbers.

23	
×46	
138 partial product	Start by multiplying 23 by 6 to get 138.
92 partial product	Next, multiply 23 by 4, lining up the partial product in the correct columns.
1058 product	Last you add the partial products.

Now we'll apply this same method to multiply two binomials.

EXAMPLE 17

Multiply using the Vertical Method: $\left(3y-1\right)\left(2y-6\right)$.

Solution

It does not matter which binomial goes on the top.

Setup for Vertical multiplication	$\begin{array}{r} 3y-1 \\ \times \quad 2y-6 \\ \hline \end{array}$	
Multiply $3y-1$ by -6 Multiple $3y-1$ by $2y$	$\begin{array}{r} -18y+6 \\ 6y^2-2y \\ \hline \end{array}$	Partial Product $-18y+6$ Partial Product $6y^2-2y$
Add like terms.	$6y^2 \quad -20y \quad +6$	Product $6y^2-20y+6$

If you use the FOIL method to multiply these binomials you will notice that the partial products are the same as the terms in the FOIL method.

TRY IT 17.1

Multiply using the Vertical Method: $\left(5m-7\right)\left(3m-6\right)$.

Show answer
$15m^2-51m+42$

TRY IT 17.2

Multiply using the Vertical Method: $\left(6b-5\right)\left(7b-3\right)$.

Show answer
$42b^2-53b+15$

We have now used three methods for multiplying binomials. Be sure to practice each method, and try to decide which one you prefer. The methods are listed here all together, to help you remember them.

HOW TO: Multiplying Two Binomials

To multiply binomials, use the:

- Distributive Property
- FOIL Method
- Vertical Method

Remember, FOIL only works when multiplying two binomials.

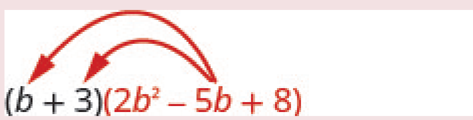
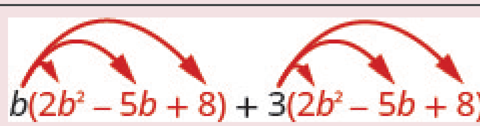
Multiply a Trinomial by a Binomial

We have multiplied monomials by monomials, monomials by polynomials, and binomials by binomials. Now we're ready to multiply a trinomial by a binomial. Remember, FOIL will not work in this case, but we can use either the Distributive Property or the Vertical Method. We first look at an example using the Distributive Property.

EXAMPLE 18

Multiply using the Distributive Property: $(b+3)\left(2b^2-5b+8\right)$.

Solution

	
Distribute.	
Multiply.	$2b^3-5b^2+8b+6b^2-15b+24$
Combine like terms.	$2b^3+b^2-7b+24$

TRY IT 18.1

Multiply using the Distributive Property: $(y-3)\left(y^2-5y+2\right)$.

Show answer

$y^3-8y^2+17y-6$

TRY IT 18.2

Multiply using the Distributive Property: $(x+4)\left(2x^2-3x+5\right)$.

Show answer
$2x^3+5x^2-7x+20$

Now let's do this same multiplication using the Vertical Method.

EXAMPLE 19

Multiply using the Vertical Method: $(b+3)\left(2b^2-5b+8\right)$.

Solution
It is easier to put the polynomial with fewer terms on the bottom because we get fewer partial products this way.

Rewrite the question in vertical setup	$\begin{array}{r} 2b^2-5b+8 \\ \times \quad b+3 \\ \hline \end{array}$
Multiply $(2b^2-5b+8)$ by 3. Multiply $(2b^2-5b+8)$ by b.	$\begin{array}{r} 6b^2-15b+24 \\ 2b^3-5b^2+8b \\ \hline \end{array}$
Add like terms.	$2b^3+b^2\quad -7b\quad +24$
There are no more like terms that can be collected.	$2b^3+b^2-7b+24$

TRY IT 19.1

Multiply using the Vertical Method: $(y-3)\left(y^2-5y+2\right)$.

Show answer
$y^3-8y^2+17y-6$

TRY IT 19.2

Multiply using the Vertical Method: $(x+4)\left(2x^2-3x+5\right)$.

Show answer
$2x^3+5x^2-7x+20$

We have now seen two methods you can use to multiply a trinomial by a binomial. After you practice each method, you'll probably find you prefer one way over the other. We list both methods are listed here, for easy reference.

HOW TO: Multiply a Trinomial by a Binomial

To multiply a trinomial by a binomial, use the:

- Distributive Property
- Vertical Method

Access these online resources for additional instruction and practice with multiplying polynomials:

- Multiplying Exponents 1
- Multiplying Exponents 2
- Multiplying Exponents 3

Key Concepts

- **FOIL Method for Multiplying Two Binomials**—To multiply two binomials:
    1. Multiply the **First** terms.
    2. Multiply the **Outer** terms.
    3. Multiply the **Inner** terms.
    4. Multiply the **Last** terms.
- **Multiplying Two Binomials**—To multiply binomials, use the:
    - Distributive Property ((Figure))
    - FOIL Method ((Figure))
- **Multiplying a Trinomial by a Binomial**—To multiply a trinomial by a binomial, use the:
    - Distributive Property ((Figure))

Practice Makes Perfect

Multiply a Polynomial by a Monomial

In the following exercises, multiply.

1. $4\left(w+10\right)$	2. $6\left(b+8\right)$
3. $-3\left(a+7\right)$	4. $-5\left(p+9\right)$
5. $2\left(x-7\right)$	6. $7\left(y-4\right)$
7. $-3\left(k-4\right)$	8. $-8\left(j-5\right)$
9. $q\left(q+5\right)$	10. $k\left(k+7\right)$
11. $-b\left(b+9\right)$	12. $-y\left(y+3\right)$
13. $-x\left(x-10\right)$	14. $-p\left(p-15\right)$
15. $6r\left(4r+s\right)$	16. $5c\left(9c+d\right)$
17. $12x\left(x-10\right)$	18. $9m\left(m-11\right)$
19. $-9a\left(3a+5\right)$	20. $-4p\left(2p+7\right)$
21. $3\left(p^2+10p+25\right)$	22. $6\left(y^2+8y+16\right)$
23. $-8x\left(x^2+2x-15\right)$	24. $-5t\left(t^2+3t-18\right)$
25. $5q^3\left(q^3-2q+6\right)$	26. $4x^3\left(x^4-3x+7\right)$
27. $-8y\left(y^2+2y-15\right)$	28. $-5m\left(m^2+3m-18\right)$
29. $5q^3\left(q^2-2q+6\right)$	30. $9r^3\left(r^2-3r+5\right)$
31. $-4z^2\left(3z^2+12z-1\right)$	32. $-3x^2\left(7x^2+10x-1\right)$
33. $\left(2m-9\right)m$	34. $\left(8j-1\right)j$
35. $\left(w-6\right)\cdot 8$	36. $\left(k-4\right)\cdot 5$
37. $4\left(x+10\right)$	38. $6\left(y+8\right)$
39. $15\left(r-24\right)$	40. $12\left(v-30\right)$
41. $-3\left(m+11\right)$	42. $-4\left(p+15\right)$
43. $-8\left(z-5\right)$	44. $-3\left(x-9\right)$
45. $u\left(u+5\right)$	46. $q\left(q+7\right)$
47. $n\left(n^2-3n\right)$	48. $s\left(s^2-6s\right)$
49. $6x\left(4x+y\right)$	50. $5a\left(9a+b\right)$
51. $5p\left(11p-5q\right)$	52. $12u\left(3u-4v\right)$
53. $3\left(v^2+10v+25\right)$	54. $6\left(x^2+8x+16\right)$

55. $2n\left(4n^2-4n+1\right)$	56. $3r\left(2r^2-6r+2\right)$
57. $-8y\left(y^2+2y-15\right)$	58. $5m\left(m^2+3m+8\right)$
59. $5q^3\left(q^2-2q+6\right)$	60. $9r^3\left(r^2-3r+5\right)$
61. $-4z^2\left(3z^2+12z-1\right)$	62. $-3x^2\left(7x^2+10x-1\right)$
63. $(2y-9)\,y$	64. $(8b-1)\,b$

Multiply a Binomial by a Binomial

In the following exercises, multiply the following binomials using: a) the Distributive Property b) the FOIL method c) the Vertical Method.

65. $(w+5)(w+7)$	66. $(y+9)(y+3)$
67. $(p+11)(p-4)$	68. $(q+4)(q-8)$

In the following exercises, multiply the binomials. Use any method.

69. $(x+8)(x+3)$	70. $(y+7)(y+4)$
71. $(y-6)(y-2)$	72. $(x-7)(x-2)$
73. $(w-4)(w+7)$	74. $(q-5)(q+8)$
75. $(p+12)(p-5)$	76. $(m+11)(m-4)$
77. $(6p+5)(p+1)$	78. $(7m+1)(m+3)$
79. $(2t-9)(10t+1)$	80. $(3r-8)(11r+1)$
81. $(5x-y)(3x-6)$	82. $(10a-b)(3a-4)$
83. $(a+b)(2a+3b)$	84. $(r+s)(3r+2s)$
85. $(4z-y)(z-6)$	86. $(5x-y)(x-4)$
87. $\left(x^2+3\right)(x+2)$	88. $\left(y^2-4\right)(y+3)$
89. $\left(x^2+8\right)\left(x^2-5\right)$	90. $\left(y^2-7\right)\left(y^2-4\right)$
91. $(5ab-1)(2ab+3)$	92. $(2xy+3)(3xy+2)$
93. $(6pq-3)(4pq-5)$	94. $(3rs-7)(3rs-4)$

Multiply a Trinomial by a Binomial

In the following exercises, multiply using a) the Distributive Property b) the Vertical Method.

95. $(x+5)\left(x^2+4x+3\right)$	96. $(u+4)\left(u^2+3u+2\right)$
97. $(y+8)\left(4y^2+y-7\right)$	98. $(a+10)\left(3a^2+a-5\right)$

In the following exercises, multiply. Use either method.

99. $(w-7)\left(w^2-9w+10\right)$	100. $(p-4)\left(p^2-6p+9\right)$
101. $(3q+1)\left(q^2-4q-5\right)$	102. $(6r+1)\left(r^2-7r-9\right)$

Mixed Practice

103. $(10y-6)+(4y-7)$	104. $(15p-4)+(3p-5)$
105. $\left(x^2-4x-34\right)-\left(x^2+7x-6\right)$	106. $\left(j^2-8j-27\right)-\left(j^2+2j-12\right)$
107. $5q\left(3q^2-6q+11\right)$	108. $8t\left(2t^2-5t+6\right)$
109. $(s-7)(s+9)$	110. $(x-5)(x+13)$
111. $\left(y^2-2y\right)(y+1)$	112. $\left(a^2-3a\right)(4a+5)$
113. $(3n-4)\left(n^2+n-7\right)$	114. $(6k-1)\left(k^2+2k-4\right)$
115. $(7p+10)(7p-10)$	116. $(3y+8)(3y-8)$
117. $\left(4m^2-3m-7\right)m^2$	118. $\left(15c^2-4c+5\right)c^4$
119. $(5a+7b)(5a+7b)$	120. $(3x-11y)(3x-11y)$
121. $(4y+12z)(4y-12z)$	

Everyday Math

122. **Mental math** You can use binomial multiplication to multiply numbers without a calculator. Say you need to multiply 13 times 15. Think of 13 as $10+3$ and 15 as $10+5$. a. Multiply $(10+3)(10+5)$ by the FOIL method. b. Multiply $13 \cdot 15$ without using a calculator. c. Which way is easier for you? Why?	123. **Mental math** You can use binomial multiplication to multiply numbers without a calculator. Say you need to multiply 18 times 17. Think of 18 as $20-2$ and 17 as $20-3$. a. Multiply $(20-2)(20-3)$ by the FOIL method. b. Multiply $18 \cdot 17$ without using a calculator. c. Which way is easier for you? Why?

Writing Exercises

124. Which method do you prefer to use when multiplying two binomials: the Distributive Property, the FOIL method, or the Vertical Method? Why?	125. Which method do you prefer to use when multiplying a trinomial by a binomial: the Distributive Property or the Vertical Method? Why?
126. Multiply the following: $(x+2)(x-2)$ $(y+7)(y-7)$ $(w+5)(w-5)$ Explain the pattern that you see in your answers.	127. Multiply the following: $(m-3)(m+3)$ $(n-10)(n+10)$ $(p-8)(p+8)$ Explain the pattern that you see in your answers.
128. Multiply the following: $(p+3)(p+3)$ $(q+6)(q+6)$ $(r+1)(r+1)$ Explain the pattern that you see in your answers.	129. Multiply the following: $(x-4)(x-4)$ $(y-1)(y-1)$ $(z-7)(z-7)$ Explain the pattern that you see in your answers.

Answers

1. $4w + 40$	3. $-3a - 21$
5. $2x - 14$	7. $-3k + 12$
9. $q^2 + 5q$	11. $-b^2 - 9b$
13. $-x^2 + 10x$	15. $24r^2 + 6rs$
17. $12x^2 - 120x$	19. $-27a^2 - 45a$
21. $3p^2 + 30p + 75$	23. $-8x^3 - 16x^2 + 120x$
25. $5q^6 - 10q^4 + 30q^3$	27. $-8y^3 - 16y^2 + 120y$
29. $5q^5 - 10q^4 + 30q^3$	31. $-12z^4 - 48z^3 + 4z^2$
33. $2m^2 - 9m$	35. $8w - 48$
37. $4x + 40$	39. $15r - 360$
41. $-3m - 33$	43. $-8z + 40$
45. $u^2 + 5u$	47. $n^3 - 3n^2$
49. $24x^2 + 6xy$	51. $55p^2 - 25pq$
53. $3v^2 + 30v + 75$	55. $8n^3 - 8n^2 + 2n$
57. $-8y^3 - 16y^2 + 120y$	59. $5q^5 - 10q^4 + 30q^3$
61. $-12z^4 - 48z^3 + 4z^2$	63. $2y^2 - 9y$
65. $w^2 + 12w + 35$	67. $p^2 + 7p - 44$
69. $x^2 + 11x + 24$	71. $y^2 - 8y + 12$
73. $w^2 + 3w - 28$	75. $p^2 + 7p - 60$
77. $6p^2 + 11p + 5$	79. $20t^2 - 88t - 9$
81. $15x^2 - 3xy - 30x + 6y$	83. $2a^2 + 5ab + 3b^2$
85. $4z^2 - 24z - zy + 6y$	87. $x^3 + 2x^2 + 3x + 6$
89. $x^4 + 3x^2 - 40$	91. $10a^2b^2 + 13ab - 3$
93. $24p^2q^2 - 42pq + 15$	95. $x^3 + 9x^2 + 23x + 15$
97. $4y^3 + 33y^2 + y - 56$	99. $w^3 - 16w^2 + 73w - 70$
101. $3q^3 - 11q^2 - 19q - 5$	103. $14y - 13$
105. $-11x - 28$	107. $15q^3 - 30q^2 + 55q$
109. $s^2 + 2s - 63$	111. $y^3 - y^2 - 2y$

113. $3n^3 - n^2 - 25n + 28$	115. $49p^2 - 100$
117. $4m^4 - 3m^3 - 7m^2$	119. $25a^2 + 70ab + 49b^2$
121. $16y^2 - 144z^2$	123. a) 306 b) 306 c) Answers will vary.
125. Answers will vary.	127. $m^2 - 9$ $n^2 - 100$ $p^2 - 64$ Answers will vary.
129. $x^2 - 8x + 16$ $y^2 - 2y + 1$ $z^2 - 14z + 49$ Answers will vary.	

Attributions

6.3 Special Products

Learning Objectives

By the end of this section, you will be able to:

- Square a binomial using the Binomial Squares Pattern
- Multiply conjugates using the Product of Conjugates Pattern
- Recognize and use the appropriate special product pattern

Square a Binomial Using the Binomial Squares Pattern

Mathematicians like to look for patterns that will make their work easier. A good example of this is squaring binomials. While you can always get the product by writing the binomial twice and using the methods of the last section, there is less work to do if you learn to use a pattern.

Let's start by looking at $(x+9)^2$.	
What does this mean?	$(x+9)^2$
It means to multiply $(x+9)$ by itself.	$(x+9)(x+9)$
Then, using FOIL, we get:	$x^2+9x+9x+81$
Combining like terms gives:	$x^2+18x+81$

Here's another one:	$(y-7)^2$
Multiply $(y-7)$ by itself.	$(y-7)(y-7)$
Using FOIL, we get:	$y^2-7y-7y+49$
And combining like terms:	$y^2-14y+49$

And one more:	$(2x+3)^2$
Multiply.	$(2x+3)(2x+3)$
Use FOIL:	$4x^2+6x+6x+9$
Combine like terms.	$4x^2+12x+9$

Look at these results. Do you see any patterns?

What about the number of terms? In each example we squared a binomial and the result was a trinomial.

$(a+b)^2 =$ ____ + ____ + ____

Now look at the ***first term*** in each result. Where did it come from?

$(x+9)^2$	$(y-7)^2$	$(2x+3)^2$
$(x+9)(x+9)$	$(y-7)(y-7)$	$(2x+3)(2x+3)$
$x^2+9x+9x+81$	$y^2-7y-7y+49$	$4x^2+6x+6x+9$
$x^2+18x+81$	$y^2-14y+49$	$4x^2+12x+9$

The first term is the product of the first terms of each binomial. Since the binomials are identical, it is just the square of the first term!

$(a+b)^2 = a^2$ + ____ + ____

To get the ***first term*** of the product, ***square the first term***.

Where did the ***last term*** come from? Look at the examples and find the pattern.

The last term is the product of the last terms, which is the square of the last term.

$(a+b)^2 =$ ____ + ____ $+ b^2$

To get the ***last term*** *of the product,* ***square the last term****.*

Finally, look at the ***middle term***. Notice it came from adding the "outer" and the "inner" terms—which are both the same! So the middle term is double the product of the two terms of the binomial.

$(a+b)^2 =$ ____ $+ 2ab +$ ____
$(a-b)^2 =$ ____ $- 2ab +$ ____

To get the ***middle term*** *of the product,* ***multiply the terms and double their product****.*

Putting it all together:

Binomial Squares Pattern

If a and b are real numbers,

$$(a+b)^2 = a^2+2ab+b^2$$
$$(a-b)^2 = a^2-2ab+b^2$$

$$\underbrace{(a+b)^2}_{(\text{binomial})^2} = \underbrace{a^2}_{(\text{first term})^2} + \underbrace{2ab}_{2(\text{product of terms})} + \underbrace{b^2}_{(\text{last term})^2}$$

HOW TO:

To square a binomial:

- square the first term
- square the last term
- double their product

A number example helps verify the pattern.

	$(10+4)^2$
Square the first term.	$10^2 +$ ____ $+$
Square the last term.	$10^2 +$ ____ $+ 4^2$
Double their product.	$10^2 + 2 \cdot 10 \cdot 4 + 4^2$
Simplify.	$100 + 80 + 16$
Simplify.	196

To multiply $(10+4)^2$ usually you'd follow the Order of Operations.

$(10+4)^2$

$(14)^2$

196

The pattern works!

EXAMPLE 1

Multiply: $(x+5)^2$.

Solution

	$\begin{pmatrix} a+b \\ x+5 \end{pmatrix}^2$
Square the first term.	$a^2+2ab+b^2$ $x^2+\_\_\_\_+\_\_\_\_$
Square the last term.	$a^2+2ab+b^2$ $x^2+\_\_\_\_+5^2$
Double the product.	$a^2+2\cdot a\cdot b+b^2$ $x^2+2\cdot x\cdot 5+25$
Simplify.	$x^2+10x+25$

TRY IT 1.1

Multiply: $(x+9)^2$.

Show answer
$x^2+18x+81$

TRY IT 1.2

Multiply: $(y+11)^2$.

Show answer
$y^2+22y+121$

EXAMPLE 2

Multiply: $(y-3)^2$.

Solution

	$\begin{pmatrix} a-b \\ y-3 \end{pmatrix}^2$
Square the first term.	$a^2 - 2ab + b^2$ $y^2 - \_\_\_\_ + \_\_\_\_$
Square the last term.	$a^2 - 2ab + b^2$ $y^2 - \_\_\_\_ + 3^2$
Double the product.	$a^2 - 2 \cdot a \cdot b + b^2$ $y^2 - 2 \cdot y \cdot 3 + 9$
Simplify.	$y^2 - 6y + 9$

TRY IT 2.1

Multiply: $(x-9)^2$.

Show answer
$x^2 - 18x + 81$

TRY IT 2.2

Multiply: $(p-13)^2$.

Show answer
$p^2 - 26p + 169$

EXAMPLE 3

Multiply: $(4x+6)^2$.

Solution

	$\begin{pmatrix} a+b \\ 4x+6 \end{pmatrix}^2$
Square the first term.	$a^2+2ab+b^2$ $(4x)^2+\_\_\_\_+\_\_\_\_$
Square the last term.	$a^2+2ab+b^2$ $16x^2+\_\_\_\_+6^2$
Double the product.	$a^2+2\cdot a\cdot b+b^2$ $16x^2+2\cdot 4x\cdot 6+36$
Simplify.	$16x^2+48x+36$

Once you become comfortable with using the pattern (or formula), you can apply it after determining the values of a and b.

	$\begin{pmatrix} a+b \\ 4x+6 \end{pmatrix}^2$
Here the value of a is $4x$ and b is 6. That means you can substitute $a=4x$ and $b=6$ in the pattern for squaring a binomial.	
Use the pattern or formula.	$a^2+2\cdot a\cdot b+b^2$ $(4x)^2+2\cdot 4x\cdot 6+(6)^2$
Simplify.	$16x^2+48x+36$

TRY IT 3.1

Multiply: $(6x+3)^2$.

Show answer
$36x^2+36x+9$

TRY IT 3.2

Multiply: $(4x+9)^2$.

Show answer

$16x^2 + 72x + 81$

EXAMPLE 4

Multiply: $(2x - 3y)^2$.

Solution

	$\begin{pmatrix} a - b \\ 2x - 3y \end{pmatrix}^2$
Here the value of a is $2x$ and b is $3y$. That means you can substitute $a = 2x$ and $b = 3y$ in the pattern for squaring a binomial.	
Use the pattern.	$a^2 - 2 \cdot a \cdot b + b^2$ $(2x)^2 - 2 \cdot 2x \cdot 3y + (3y)^2$
Simplify.	$4x^2 - 12xy + 9y^2$

TRY IT 4.1

Multiply: $(2c - d)^2$.

Show answer
$4c^2 - 4cd + d^2$

TRY IT 4.2

Multiply: $(4x - 5y)^2$.

Show answer
$16x^2 - 40xy + 25y^2$

EXAMPLE 5

Multiply: $(4u^3 + 1)^2$.

Solution

	$\left(\begin{matrix} a+b \\ 4u^3+1 \end{matrix}\right)^2$
Here the value of a is $4u^3$ and b is 1. That means you can substitute $a=4u^3$ and $b=1$ in the pattern for squaring a binomial.	
Use the pattern or formula.	$\begin{matrix} a^2 + & 2\cdot a\cdot b + & b^2 \\ (4u^3)^2 + & 2\cdot 4u^3\cdot 1 + & (1)^2 \end{matrix}$
Simplify.	$16u^6+8u^3+1$

TRY IT 5.1

Multiply: $\left(2x^2+1\right)^2$.

Show answer
$4x^4+4x^2+1$

TRY IT 5.2

Multiply: $\left(3y^3+2\right)^2$.

Show answer
$9y^6+12y^3+4$

Multiply Conjugates Using the Product of Conjugates Pattern

We just saw a pattern for squaring binomials that we can use to make multiplying some binomials easier. Similarly, there is a pattern for another product of binomials. But before we get to it, we need to introduce some vocabulary.

What do you notice about these pairs of binomials?

$(x-9)(x+9)$ $\quad$ $(y-8)(y+8)$ $\quad$ $(2x-5)(2x+5)$

Look at the first term of each binomial in each pair.

$(x-9)(x+9)$ $\quad$ $(y-8)(y+8)$ $\quad$ $(2x-5)(2x+5)$

Notice the first terms are the same in each pair.

Look at the last terms of each binomial in each pair.

$(x-9)(x+9)$ $\quad$ $(y-8)(y+8)$ $\quad$ $(2x-5)(2x+5)$

Notice the last terms are the same in each pair.

Notice how each pair has one sum and one difference.

A pair of binomials that each have the same first term and the same last term, but one is a sum and one is a difference has a special name. It is called a *conjugate pair* and is of the form $(a-b),(a+b)$.

Conjugate Pair

A conjugate pair is two binomials of the form
$(a-b),(a+b)$.

The pair of binomials each have the same first term and the same last term, but one binomial is a sum and the other is a difference.

There is a nice pattern for finding the product of conjugates. You could, of course, simply FOIL to get the product, but using the pattern makes your work easier.

Let's look for the pattern by using FOIL to multiply some conjugate pairs.

$$\begin{array}{ccc} (x-9)(x+9) & (y-8)(y+8) & (2x-5)(2x+5) \\ x^2+9x-9x-81 & y^2+8y-8y-64 & 4x^2+10x-10x-25 \\ x^2-81 & y^2-64 & 4x^2-25 \end{array}$$

$$\begin{array}{ccc} (x+9)(x-9) & (y-8)(y+8) & (2x-5)(2x+5) \\ x^2-9x+9x-81 & y^2+8y-8y-64 & 4x^2+10x-10x-25 \\ x^2-81 & y^2-64 & 4x^2-25 \end{array}$$

Each **first term** is the product of the first terms of the binomials, and since they are identical it is the square of the first term.

$$(a+b)(a-b)=a^2-\_\_\_\_$$

To get the **first term, square the first term**.

The **last term** came from multiplying the last terms, the square of the last term.

$$(a+b)(a-b)=a^2-b^2$$

To get the **last term, square the last term**.

What do you observe about the products?

The product of the two binomials is also a binomial! Most of the products resulting from FOIL have been trinomials.

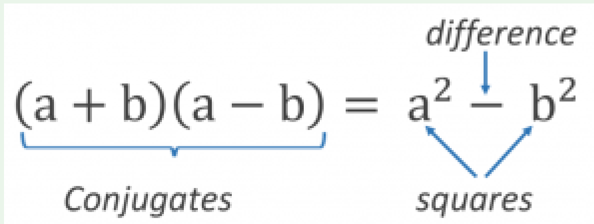
Why is there no middle term? Notice the two middle terms you get from FOIL combine to 0 in every case, the result of one addition and one subtraction.

The product of conjugates is always of the form a^2-b^2. This is called a difference of squares.

This leads to the pattern:

Product of Conjugates Pattern

If a and b are real numbers,
$(a+b)(a-b)=a^2-b^2$



The product is called a difference of squares.

To multiply conjugates, square the first term, square the last term, and write the product as a difference of squares.

Let's test this pattern with a numerical example.

	$(10-2)\,(10+2)$
It is the product of conjudgates, so the result will be the difference of two squares.	____ – ____
Square the first term.	10^2- ____
Square the last term.	10^2-2^2
Simplify.	$100-4$
Simplify.	96
What do you get using the order of operations?	
	$(10-2)\,(10+2)$ $(8)\,(12)$ 96

Notice, the result is the same!

EXAMPLE 6

Multiply: $(x-8)\,(x+8)$.

Solution

First, recognize this as a product of conjugates. The binomials have the same first terms, and the same last terms, and one binomial is a sum and the other is a difference.

It fits the pattern.	$\binom{a-b}{x-8}\binom{a+b}{x+8}$
Square the first term, *x*.	$a^2 - b^2$ $x^2 - \_\_\_$
Square the last term, 8.	$a^2 - b^2$ $x^2 - 8^2$
The product is a difference of squares.	$x^2 - 64$

TRY IT 6.1

Multiply: $(x-5)(x+5)$.

Show answer
$x^2 - 25$

TRY IT 6.2

Multiply: $(w-3)(w+3)$.

Show answer
$w^2 - 9$

EXAMPLE 7

Multiply: $(2x+5)(2x-5)$.

Solution
Are the binomials conjugates? YES

It is the product of conjugates.	$\left(\begin{array}{c} a+b \\ 2x+5 \end{array}\right)\left(\begin{array}{c} a-b \\ 2x-5 \end{array}\right)$
Square the first term, $2x$.	$a^2 - b^2$ $(2x)^2 - \_\_\_$
Square the last term, 5.	$a^2 - b^2$ $4x^2 - 5^2$
Simplify. The product is a difference of squares.	$4x^2 - 25$

TRY IT 7.1

Multiply: $(6x+5)(6x-5)$.

Show answer
$36x^2 - 25$

TRY IT 7.2

Multiply: $(2x+7)(2x-7)$.

Show answer
$4x^2 - 49$

The binomials in the next example may look backwards – the variable is in the second term. But the two binomials are still conjugates, so we use the same pattern to multiply them.

EXAMPLE 8

Find the product: $(3+5x)(3-5x)$.

Solution

It is the product of conjugates.	$\left(\begin{array}{c}a+b\\3+5x\end{array}\right)\left(\begin{array}{c}a-b\\3-5x\end{array}\right)$
Use the pattern. Here $a=3$ and $b=5x$.	a^2-b^2 $3^2-(5x)^2$
Simplify.	$9-25x^2$

TRY IT 8.1

Multiply: $(7+4x)(7-4x)$.

Show answer
$49-16x^2$

TRY IT 8.2

Multiply: $(9-2y)(9+2y)$.

Show answer
$81-4y^2$

Now we'll multiply conjugates that have two variables.

EXAMPLE 9

Find the product: $(5m-9n)(5m+9n)$.

Solution

This fits the pattern of product of conjugates.	$\left(\begin{array}{c}a-b\\5m-9n\end{array}\right)\left(\begin{array}{c}a+b\\5m+9n\end{array}\right)$
Use the pattern. Here $a=5m$ and $b=9n$.	a^2-b^2 $(5m)^2-(9n)^2$
Simplify.	$25m^2-81n^2$

TRY IT 9.1

Find the product: $(4p-7q)(4p+7q)$.

Show answer
$16p^2-49q^2$

TRY IT 9.2

Find the product: $(3x-y)(3x+y)$.

Show answer
$9x^2-y^2$

EXAMPLE 10

Find the product: $(cd-8)(cd+8)$.

Solution

This fits the pattern of product of conjugates.	$\left(\begin{matrix} a-b \\ cd-8 \end{matrix}\right)\left(\begin{matrix} a+b \\ cd+8 \end{matrix}\right)$
Use the pattern. Here $a=cd$ and $b=8$.	a^2-b^2 $(cd)^2-(8)^2$
Simplify.	c^2d^2-64

TRY IT 10.1

Find the product: $(xy-6)(xy+6)$.

Show answer
x^2y^2-36

TRY IT 10.2

Find the product: $(ab-9)(ab+9)$.

Show answer

$a^2b^2 - 81$

EXAMPLE 11

Find the product: $\left(6u^2 - 11v^5\right)\left(6u^2 + 11v^5\right)$.

Solution

This fits the pattern of product of conjugates.	$\begin{pmatrix} a & - & b \\ 6u^2 & - & 11v^5 \end{pmatrix}\begin{pmatrix} a & + & b \\ 6u^2 & + & 11v^5 \end{pmatrix}$
Use the pattern. Here $a = 6u^2$ and $b = 11v^5$.	$a^2 \quad - \quad b^2$ $(6u^2)^2 - (11v^5)^2$
Simplify.	$36^4 - 121v^{10}$

TRY IT 11.1

Find the product: $\left(3x^2 - 4y^3\right)\left(3x^2 + 4y^3\right)$.

Show answer
$9x^4 - 16y^6$

TRY IT 11.2

Find the product: $\left(2m^2 - 5n^3\right)\left(2m^2 + 5n^3\right)$.

Show answer
$4m^4 - 25n^6$

Recognize and Use the Appropriate Special Product Pattern

We just developed special product patterns for Binomial Squares and for the Product of Conjugates. The products look similar, so it is important to recognize when it is appropriate to use each of these patterns and to notice how they differ. Look at the two patterns together and note their similarities and differences.
Comparing the Special Product Patterns

Binomial Squares	**Product of Conjugates**
$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)(a+b) = a^2 - b^2$
$(a-b)^2 = a^2 - 2ab + b^2$	
– Squaring a binomial	– Multiplying conjugates
– Product is a **trinomial**	– Product is a **binomial**
– Inner and outer terms with FOIL are **the same.**	– Inner and outer terms with FOIL are **opposites.**
– Middle term is **double the product** of the terms.	– There is **no** middle term.

EXAMPLE 12

Choose the appropriate pattern and use it to find the product:

a) $(2x-3)(2x+3)$ b) $(5x-8)^2$ c) $(6m+7)^2$ d) $(5x-6)(6x+5)$

Solution

a. $(2x-3)(2x+3)$ These are conjugates. They have the same first numbers, and the same last numbers, and one binomial is a sum and the other is a difference. It fits the Product of Conjugates pattern.

	$\begin{pmatrix} a-b \\ 2x-3 \end{pmatrix}\begin{pmatrix} a+b \\ 2x+3 \end{pmatrix}$
Use the pattern.	a^2-b^2 $(2x)^2-(3)^2$
Simplify.	$4x^2-9$

b. $(8x-5)^2$ We are asked to square a binomial. It fits the **binomial squares** pattern.

	$\begin{pmatrix} a-b \\ 8x-5 \end{pmatrix}^2$
Use the pattern.	$a^2-2\cdot a\cdot b+b^2$ $(8x)^2-2\cdot 8x\cdot 5+(5)^2$
Simplify.	$64x^2-80x+25$

c. $(6m+7)^2$ Again, we will square a binomial so we use the **binomial squares** pattern.

	$\left(\begin{array}{c} a+b \\ 6m+7 \end{array} \right)^2$
Use the pattern.	$a^2+2\cdot a\cdot b+b^2$ $(6m)^2+2\cdot 6m\cdot 7+(7)^2$
Simplify.	$36m^2+84m+49$

d. $(5x-6)(6x+5)$ This product does not fit the patterns, so we will use FOIL.

	$(5x-6)(6x+5)$
Use FOIL.	$30x^2+25x-36x-30$
Simplify.	$30x^2-11x-30$

TRY IT 12.1

Choose the appropriate pattern and use it to find the product:

a) $(9b-2)(2b+9)$ b) $(9p-4)^2$ c) $(7y+1)^2$ d) $(4r-3)(4r+3)$

Show answer

a) FOIL; $18b^2+77b-18$ b) Binomial Squares; $81p^2-72p+16$ c) Binomial Squares; $49y^2+14y+1$ d) Product of Conjugates; $16r^2-9$

TRY IT 12.2

Choose the appropriate pattern and use it to find the product:

a) $(6x+7)^2$ b) $(3x-4)(3x+4)$ c) $(2x-5)(5x-2)$ d) $(6n-1)^2$

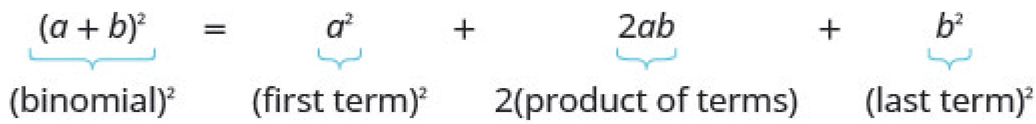
Show answer

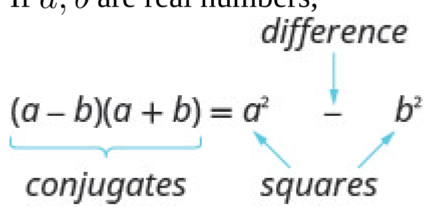
a) Binomial Squares; $36x^2+84x+49$ b) Product of Conjugates; $9x^2-16$ c) FOIL; $10x^2-29x+10$ d) Binomial Squares; $36n^2-12n+1$

Access these online resources for additional instruction and practice with special products:

- Special Products

Key Concepts

- **Binomial Squares Pattern**
  - If a, b are real numbers,
    

  - $(a+b)^2 = a^2 + 2ab + b^2$
  - $(a-b)^2 = a^2 - 2ab + b^2$
  - To square a binomial: square the first term, square the last term, double their product.
- **Product of Conjugates Pattern**
  - If a, b are real numbers,
    

  - $(a-b)(a+b) = a^2 - b^2$
  - The product is called a difference of squares.
- **To multiply conjugates:**
  - **square the first term square the last term** write it as a difference of squares

Glossary

conjugate pair
A conjugate pair is two binomials of the form $(a-b), (a+b)$; the pair of binomials each have the same first term and the same last term, but one binomial is a sum and the other is a difference.

Practice Makes Perfect

Square a Binomial Using the Binomial Squares Pattern

In the following exercises, square each binomial using the Binomial Squares Pattern.

1. $(q+12)^2$	2. $(w+4)^2$
3. $\left(x+\frac{2}{3}\right)^2$	4. $\left(y+\frac{1}{4}\right)^2$
5. $(y-6)^2$	6. $(b-7)^2$
7. $(p-13)^2$	8. $(m-15)^2$
9. $(4a+10)^2$	10. $(3d+1)^2$
11. $\left(3z+\frac{1}{5}\right)^2$	12. $\left(2q+\frac{1}{3}\right)^2$
13. $(2y-3z)^2$	14. $(3x-y)^2$
15. $\left(\frac{1}{8}x-\frac{1}{9}y\right)^2$	16. $\left(\frac{1}{5}x-\frac{1}{7}y\right)^2$
17. $\left(5u^2+9\right)^2$	18. $\left(3x^2+2\right)^2$
19. $\left(8p^3-3\right)^2$	20. $\left(4y^3-2\right)^2$

In the following exercises, multiply each pair of conjugates using the Product of Conjugates Pattern.

Multiply Conjugates Using the Product of Conjugates Pattern

21. $(c-5)(c+5)$	22. $(m-7)(m+7)$
23. $\left(b+\frac{6}{7}\right)\left(b-\frac{6}{7}\right)$	24. $\left(x+\frac{3}{4}\right)\left(x-\frac{3}{4}\right)$
25. $(8j+4)(8j-4)$	26. $(5k+6)(5k-6)$
27. $(9c+5)(9c-5)$	28. $(11k+4)(11k-4)$
29. $(13-q)(13+q)$	30. $(11-b)(11+b)$
31. $(4-6y)(4+6y)$	32. $(5-3x)(5+3x)$
33. $(7w+10x)(7w-10x)$	34. $(9c-2d)(9c+2d)$
35. $\left(p+\frac{4}{5}q\right)\left(p-\frac{4}{5}q\right)$	36. $\left(m+\frac{2}{3}n\right)\left(m-\frac{2}{3}n\right)$
37. $(xy-9)(xy+9)$	38. $(ab-4)(ab+4)$
39. $\left(rs-\frac{2}{7}\right)\left(rs+\frac{2}{7}\right)$	40. $\left(uv-\frac{3}{5}\right)\left(uv+\frac{3}{5}\right)$
41. $\left(6m^3-4n^5\right)\left(6m^3+4n^5\right)$	42. $\left(2x^2-3y^4\right)\left(2x^2+3y^4\right)$
43. $\left(15m^2-8n^4\right)\left(15m^2+8n^4\right)$	44. $\left(12p^3-11q^2\right)\left(12p^3+11q^2\right)$

In the following exercises, find each product.

Recognize and Use the Appropriate Special Product Pattern

45. a) $(2r+12)^2$ b) $(3p+8)(3p-8)$ c) $(7a+b)(a-7b)$ d) $(k-6)^2$	46. a) $(p-3)(p+3)$ b) $(t-9)^2$ c) $(m+n)^2$ d) $(2x+y)(x-2y)$
47. a) $\left(x^5+y^5\right)\left(x^5-y^5\right)$ b) $\left(m^3-8n\right)^2$ c) $(9p+8q)^2$ d) $\left(r^2-s^3\right)\left(r^3+s^2\right)$	48. a) $\left(a^5-7b\right)^2$ b) $\left(x^2+8y\right)\left(8x-y^2\right)$ c) $\left(r^6+s^6\right)\left(r^6-s^6\right)$ d) $\left(y^4+2z\right)^2$

Everyday Math

49. **Mental math** You can use the binomial squares pattern to multiply numbers without a calculator. Say you need to square 65. Think of 65 as $60+5$. a. Multiply $(60+5)^2$ by using the binomial squares pattern, $(a+b)^2=a^2+2ab+b^2$. b. Square 65 without using a calculator. c. Which way is easier for you? Why?	50. **Mental math** You can use the product of conjugates pattern to multiply numbers without a calculator. Say you need to multiply 47 times 53. Think of 47 as $50-3$ and 53 as $50+3$. a. Multiply $(50-3)(50+3)$ by using the product of conjugates pattern, $(a-b)(a+b)=a^2-b^2$. b. Multiply $47\cdot 53$ without using a calculator. c. Which way is easier for you? Why?

Writing Exercises

52. Why does $(a+b)^2$ result in a trinomial, but $(a-b)(a+b)$ result in a binomial?	51. How do you decide which pattern to use?
54. Use the order of operations to show that $(3+5)^2$ is 64, and then use that numerical example to explain why $(a+b)^2\neq a^2+b^2$.	53. Marta did the following work on her homework paper: $(3-y)^2$ 3^2-y^2 $9-y^2$ Explain what is wrong with Marta's work.

Answers

1. $q^2 + 24q + 144$	3. $x^2 + \frac{4}{3}x + \frac{4}{9}$
5. $y^2 - 12y + 36$	7. $p^2 - 26p + 169$
9. $16a^2 + 80a + 100$	11. $9z^2 + \frac{6}{5}z + \frac{1}{25}$
13. $4y^2 - 12yz + 9z^2$	15. $\frac{1}{64}x^2 - \frac{1}{36}xy + \frac{1}{81}y^2$
17. $25u^4 + 90u^2 + 81$	19. $64p^6 - 48p^3 + 9$
21. $c^2 - 25$	23. $b^2 - \frac{36}{49}$
25. $64j^2 - 16$	27. $81c^2 - 25$
29. $169 - q^2$	31. $16 - 36y^2$
33. $49w^2 - 100x^2$	35. $p^2 - \frac{16}{25}q^2$
37. $x^2y^2 - 81$	39. $r^2s^2 - \frac{4}{49}$
41. $36m^6 - 16n^{10}$	43. $225m^4 - 64n^8$
45. a) $4r^2 + 48r + 144$ b) $9p^2 - 64$ c) $7a^2 - 48ab - 7b^2$ d) $k^2 - 12k + 36$	47. a) $x^{10} - y^{10}$ b) $m^6 - 16m^3n + 64n^2$ c) $81p^2 + 144pq + 64q^2$ d) $r^5 + r^2s^2 - r^3s^3 - s^5$
49. a) 4,225 b) 4,225 c) Answers will vary.	51. Answers will vary.
53. Answers will vary.	

Attributions

6.4 Greatest Common Factor and Factor by Grouping

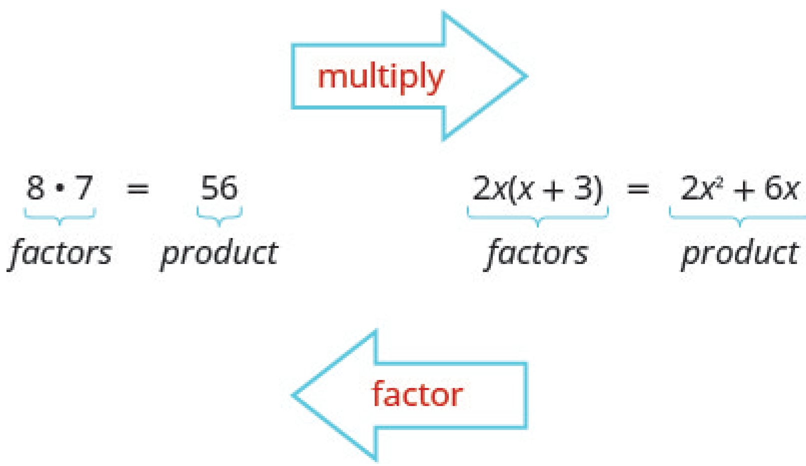
Learning Objectives

By the end of this section, you will be able to:

- Find the greatest common factor of two or more expressions
- Factor the greatest common factor from a polynomial
- Factor by grouping

Find the Greatest Common Factor of Two or More Expressions

Earlier we multiplied factors together to get a product. Now, we will be reversing this process; we will start with a product and then break it down into its factors. Splitting a product into factors is called factoring.



We have learned how to factor numbers to find the least common multiple (LCM) of two or more numbers. Now we will factor expressions and find the greatest common factor of two or more expressions. The method we use is similar to what we used to find the LCM.

Greatest Common Factor

The greatest common factor (GCF) of two or more expressions is the largest expression that is a factor of all the expressions.

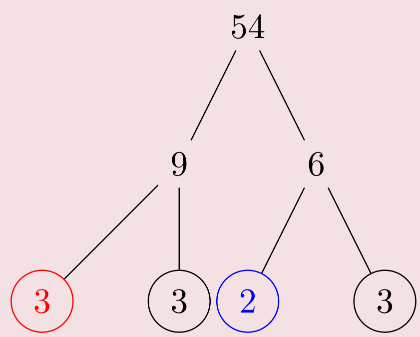
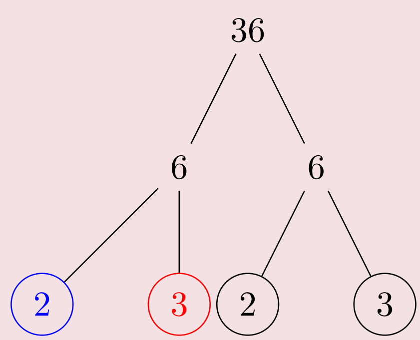
First we'll find the GCF of two numbers.

EXAMPLE 1

How to Find the Greatest Common Factor of Two or More Expressions

Find the GCF of 54 and 36

Solution

Step 1. Factor each coefficient into prime numbers. Write all the variables with exponents in expanded form	Factor 54 and 36	 
Step 2. In each column, circle the common factors.	Circle the 2,3, and 3 that are shared by both numbers.	$54 = 2 \cdot 3 \cdot 3 \cdot 3$ $36 = 2 \cdot 2 \cdot 3 \cdot 3$

Step 3. Bring down the common factors that all expressions share.	Bring down the 2,3, and 3, and then multiply.	GCF = $2 \cdot 3 \cdot 3$
Step 4. Multiply the factors.		GCF = 18 The GCF of 54 and 36 is 18

Notice that, because the GCF is a factor of both numbers, 54 and 36 can be written as multiples of 18

$54 = 18 \cdot 3$

$36 = 18 \cdot 2$

TRY IT 1.1

Find the GCF of 48 and 80.

Show answer
16

TRY IT 1.2

Find the GCF of 18 and 40.

Show answer
2

We summarize the steps we use to find the GCF below.

HOW TO:

Find the Greatest Common Factor (GCF) of two expressions

1. Factor each coefficient into primes. Write all variables with exponents in expanded form.
2. List all factors—matching common factors in a column. In each column, circle the common factors.

3. Bring down the common factors that all expressions share.
4. Multiply the factors.

In the first example, the GCF was a constant. In the next two examples, we will get variables in the greatest common factor.

EXAMPLE 2

Find the greatest common factor of $27x^3$ and $18x^4$.

Solution

Factor each coefficient into primes and write the variables with exponents in expanded form. Circle the common factors in each column.	$27x^3 = 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x$ $18x^4 = 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$
Bring down the common factors.	GCF = $3 \cdot 3 \cdot x \cdot x \cdot x$
Multiply the factors.	GCF $= 9x^3$
The GCF of $27x^3$ and $18x^4$ is $9x^3$.	

TRY IT 2.1

Find the GCF: $12x^2, 18x^3$.

Show answer
$3x^2$

TRY IT 2.2

Find the GCF: $16y^2, 24y^3$.

Show answer
$8y^2$

EXAMPLE 3

Find the GCF of $4x^2y, 6xy^3$.

Solution

Factor each coefficient into primes and write the variables with exponents in expanded form. Circle the common factors in each column.	$4x^2y = 2 \cdot 2 \cdot x \cdot x \cdot y$ $6xy^3 = 2 \cdot 3 \cdot x \cdot y \cdot y \cdot y$
Bring down the common factors.	$\text{GCF} = 2 \cdot \quad x \cdot y$
Multiply the factors.	$GCF = 2xy$
The GCF of $4x^2y$ and $6xy^3$ is $2xy$.	

TRY IT 3.1

Find the GCF: $6ab^4, 8a^2b$.

Show answer
$2ab$

TRY IT 3.2

Find the GCF: $9m^5n^2, 12m^3n$.

Show answer
$3m^3n$

EXAMPLE 4

Find the GCF of: $21x^3, 9x^2, 15x$.

Solution

Factor each coefficient into primes and write the variables with exponents in expanded form. Circle the common factors in each column.	$21x^3 = 3 \cdot 7 \cdot x \cdot x \cdot x$ $9x^2 = 3 \cdot 3 \cdot x \cdot x$ $15x = 3 \cdot 5 \cdot x$
Bring down the common factors.	$\text{GCF} = 3 \cdot x$
Multiply the factors.	$\text{GCF} = 3x$
The GCF of $21x^3, 9x^2$ and $15x$ is $3x$.	

TRY IT 4.1

Find the greatest common factor: $25m^4, 35m^3, 20m^2$.

Show answer
$5m^2$

TRY IT 4.2

Find the greatest common factor: $14x^3, 70x^2, 105x$.

Show answer
$7x$

Factor the Greatest Common Factor from a Polynomial

Just like in arithmetic, where it is sometimes useful to represent a number in factored form (for example, 12 as $2 \cdot 6$ or $3 \cdot 4$), in algebra, it can be useful to represent a polynomial in factored form. One way to do this is by finding the GCF of all the terms. Remember, we multiply a polynomial by a monomial as follows:

$$\begin{array}{ll} 2\left(x+7\right) & \text{factors} \\ 2 \cdot x + 2 \cdot 7 & \\ 2x + 14 & \text{product} \end{array}$$

Now we will start with a product, like $2x + 14$, and end with its factors, $2\left(x+7\right)$. To do this we apply the Distributive Property "in reverse."

We state the Distributive Property here just as you saw it in earlier chapters and "in reverse."

Distributive Property

If a, b, c are real numbers, then

$a\,(b+c)=ab+ac$ and $ab+ac=a\,(b+c)$

The form on the left is used to multiply. The form on the right is used to factor.

So how do you use the Distributive Property to factor a polynomial? You just find the GCF of all the terms and write the polynomial as a product!

EXAMPLE 5

How to Factor the Greatest Common Factor from a Polynomial

Factor: $4x+12$.

Solution

Step 1. Find the GCF of all the terms of the polynomial.	Find the GCF of and 12 .	$4x=2\cdot 2\cdot x$ $12=2\cdot 2\cdot 3$ $\text{GCF}=2\cdot 2$ $\text{GCF}=4$
Step 2. Rewrite each term as a product using the GCF.	Rewrite $4x$ and 12 as products of their GCF, 4 . $4x=4\cdot x$ $12=4\cdot 3$	$4x+12$ $=4\cdot x+4\cdot 3$
Step 3. Use the “reverse” Distributive Property to factor the expression.		$4(x+3)$
Step 4. Check by multiplying the factors.		$4(x+3)$ $=4\cdot x+4\cdot 3$ $=4x+12\checkmark$

TRY IT 5.1

Factor: $6a+24$.

Show answer

$6\,(a+4)$

TRY IT 5.2

Factor: $2b + 14$.

Show answer
$2\left(b+7\right)$

HOW TO:

Factor the greatest common factor from a polynomial.

1. Find the GCF of all the terms of the polynomial.
2. Rewrite each term as a product using the GCF.
3. Use the “reverse” Distributive Property to factor the expression.
4. Check by multiplying the factors.

Factor as a Noun and a Verb

We use “factor” as both a noun and a verb.

Noun	7 is a factor of 14
Verb	factor 3 from $3a + 3$

EXAMPLE 6

Factor: $5a + 5$.

Solution

Find the GCF of $5a$ and 5.	$5a = 5 \cdot a$ $5 = 5$ $\text{GCF} = 5$
Rewrite each term as a product using the GCF.	$5a + 5$ $= 5 \cdot a + 5 \cdot 1$
Use the Distributive Property “in reverse” to factor the GCF.	$5(a+1)$
Check by multiplying the factors to get the original polynomial.	$5(a+1)$ $= 5 \cdot a + 5 \cdot 1$ $= 5a + 5\checkmark$

TRY IT 6.1

Factor: $14x + 14$.

Show answer
$14\left(x+1\right)$

TRY IT 6.1

Factor: $12p + 12$.

Show answer
$12\left(p+1\right)$

The expressions in the next example have several factors in common. Remember to write the GCF as the product of all the common factors.

EXAMPLE 7

Factor: $12x - 60$.

Solution

Find the GCF of $12x$ and 60.	$12x = 2 \cdot 2 \cdot 3 \cdot x$ $60 = 2 \cdot 2 \cdot 3 \cdot 5$ $\text{GCF} = 2 \cdot 2 \cdot 3$ $\text{GCF} = 12$
Rewrite each term as a product using the GCF.	$12x - 60$ $= 12 \cdot x - 12 \cdot 5$
Factor the GCF.	$12(x-5)$
Check by multiplying the factors.	$12(x-5)$ $= 12 \cdot x - 12 \cdot 5$ $= 12x - 60\checkmark$

TRY IT 7.1

Factor: $18u - 36$.

Show answer
$18\left(u-2\right)$

TRY IT 7.2

Factor: $30y-60$.

Show answer
$30\left(y-2\right)$

Now we'll factor the greatest common factor from a trinomial. We start by finding the GCF of all three terms.

EXAMPLE 8

Factor: $4y^2+24y+28$.

Solution
We start by finding the GCF of all three terms.

Find the GCF of $4y^2$, $24y$ and 28.	$4y^2 = 2\cdot 2\cdot \quad y\cdot y$ $24y = 2\cdot 2\cdot 2\cdot 3\cdot y$ $28 = 2\cdot 2\cdot y$ $\text{GCF} = 2\cdot 2$ $\text{GCF} = 4$
Rewrite each term as a product using the GCF. Factor the GCF.	$4y^2+24y+28$ $= 4\cdot y^2+4\cdot 6y+4\cdot 7$ $= 4(y^2+6y+7)$
Factored result.	$4\left(y^2+6y+7\right)$
Check by multiplying.	$4(y^2+6y+7)$ $= 4\cdot y^2+4\cdot 6y+4\cdot 7$ $= 4y^2+24y+28\checkmark$

TRY IT 8.1

Factor: $5x^2 - 25x + 15$.

Show answer
$5\left(x^2 - 5x + 3\right)$

TRY IT 8.2

Factor: $3y^2 - 12y + 27$.

Show answer
$3\left(y^2 - 4y + 9\right)$

EXAMPLE 9

Factor: $5x^3 - 25x^2$.

Solution

Find the GCF of $5x^3$ and $25x^2$.	$5x^2 = 5 \cdot \quad x \cdot x \cdot x$ $25x^2 = 5 \cdot 5 \cdot x \cdot x$ $\text{GCF} = 5 \cdot \quad x \cdot x$ $\text{GCF} = 5x^2$
Rewrite each term.	$5x^3 - 25x^2$ $= 5x^2 \cdot x - 5x^2 \cdot 5$
Factor the GCF.	$5x^2(x - 5)$
Check.	$5x^2\,(x - 5)$ $= 5x^2 \cdot x - 5x^2 \cdot 5$ $= 5x^3 - 25x^2 \checkmark$

TRY IT 9.1

Factor: $2x^3 + 12x^2$.

Show answer
$2x^2\,(x + 6)$

TRY IT 9.2

Factor: $6y^3 - 15y^2$.

Show answer

$3y^2\left(2y - 5\right)$

EXAMPLE 10

Factor: $21x^3 - 9x^2 + 15x$.

Solution

In a previous example we found the GCF of $21x^3, 9x^2, 15x$ to be $3x$.

Rewrite each term using the GCF, 3*x*.	$21x^3 - 9x^2 + 15x$ $= 3x \cdot 7x^2 - 3x \cdot 3x + 3x \cdot 5$
Factor the GCF.	$3x\left(7x^2 - 3x + 5\right)$
Check.	$3x(7x^2 - 3x + 5)$ $= 3x \cdot 7x^2 - 3x \cdot 3x + 3x \cdot 5$ $= 21x^3 - 9x^2 + 15x$✓

TRY IT 10.1

Factor: $20x^3 - 10x^2 + 14x$.

Show answer

$2x\left(10x^2 - 5x + 7\right)$

TRY IT 10.2

Factor: $24y^3 - 12y^2 - 20y$.

Show answer

$4y\left(6y^2 - 3y - 5\right)$

EXAMPLE 11

Factor: $8m^3 - 12m^2n + 20mn^2$.

Solution

Find the GCF of $8m^3, 12m^2n, 20mn^2$.	$8m^3 = 2 \cdot 2 \cdot 2 \quad m \cdot m \cdot m$ $12m^2n = 2 \cdot 2 \cdot 3 \cdot m \cdot m \cdot n$ $20mn^2 = 2 \cdot 2 \cdot 5 \cdot m \cdot n \cdot n$ $\text{GCF} = 2 \cdot 2 \cdot \quad m$ $\text{GCF} = 4m$
Rewrite each term.	$8m^3 - 12m^2n + 20mn^2$ $= 4m \cdot 2m^2 - 4m \cdot 3mn + 4m \cdot 5n^2$
Factor the GCF.	$4m\left(2m^2 - 3mn + 5n^2\right)$
Check.	$4m\left(2m^2 - 3mn + 5n^2\right)$ $= 4m \cdot 2m^2 - 4m \cdot 3mn + 4m \cdot 5n^2$ $= 8m^3 - 12m^2n + 20mn^2 \checkmark$

TRY IT 11.1

Factor: $9xy^2 + 6x^2y^2 + 21y^3$.

Show answer
$3y^2\left(3x + 2x^2 + 7y\right)$

TRY IT 11.2

Factor: $3p^3 - 6p^2q + 9pq^3$.

Show answer
$3p\left(p^2 - 2pq + 3q^3\right)$

When the leading coefficient is negative, we factor the negative out as part of the GCF.

EXAMPLE 12

Factor: $-8y - 24$.

Solution
When the leading coefficient is negative, the GCF will be negative.

Ignoring the signs of the terms, we first find the GCF of $8y$ and 24 which is 8. Since the expression $-8y - 24$ has a negative leading coefficient, we use -8 as the GCF.	$8y = 2 \cdot 2 \cdot 2 \cdot y$ $24 = 2 \cdot 2 \cdot 2 \cdot 3$ $\text{GCF} = 2 \cdot 2 \cdot 2 = 8$ $\text{GCF} = -8$
Rewrite each term using the GCF.	$-8y - 24$ $= -8 \cdot y + -8 \cdot 3$
Factor the GCF.	$-8(y+3)$
Check.	$-8(y+3)$ $= -8 \cdot y + (-8) \cdot 3$ $= -8y - 24$✓

TRY IT 12.1

Factor: $-16z - 64$.

Show answer
$-8\left(2z + 8\right)$

TRY IT 12.2

Factor: $-9y - 27$.

Show answer
$-9\left(y + 3\right)$

EXAMPLE 13

Factor: $-6a^2 + 36a$.

Solution

The leading coefficient is negative, so what will be the sign of the GCF?

Ignoring the signs of the terms, we first find the GCF of $6a^2$ and $36a$. Since the leading coefficient is negative, the GCF is negative $6a$.	$6a^2 = 2 \cdot 3 \cdot a \cdot a$ $36a = 2 \cdot 2 \cdot 3 \cdot 3 \cdot a$ $\text{GCF} = 2 \cdot 3 \cdot a$ $\text{GCF} = 6a$ $\text{GCF} = -6a$
Rewrite each term using the GCF. *this is an important step, ask your instructor to explain it if you need help with it.	$-6a^2 + 36a$ $= -6a \cdot a - (-6a) \cdot 6$ *
Factor the GCF.	$-6a(a-6)$
Check.	$-6a(a-6)$ $= (-6a) \cdot a + (-6a) \cdot (-6)$ $= -6a^2 + 36a$✓

TRY IT 13.1

Factor: $-4b^2 + 16b$.

Show answer

$-4b\left(b-4\right)$

TRY IT 13.2

Factor: $-7a^2 + 21a$.

Show answer

$-7a\left(a-3\right)$

EXAMPLE 14

Factor: $5q\left(q+7\right) - 6\left(q+7\right)$.

Solution

Find the GCF of $5q(q+7)$ and $6(q+7)$ The GCF is the binomial $q+7$.	$5q(q+7) = 5 \cdot q \cdot (q+7)$ $6(q+7) = 2 \cdot 3 \cdot (q+7)$ $GCF = (q+7)$
Factor the GCF, $(q+7)$.	$5q(q+7) - 6(q+7)$ $= (q+7)(5q-6)$
Check by multiplying using Distributive law to distribute the binomial $q+7$.	$(q+7)(5q-6)$ $= 5q(q+7) - 6(q+7)$✓

TRY IT 14.1

Factor: $4m\,(m+3) - 7\,(m+3)$.

Show answer
$(m+3)\,(4m-7)$

TRY IT 14.2

Factor: $8n\,(n-4) + 5\,(n-4)$.

Show answer
$(n-4)\,(8n+5)$

Factor by Grouping

When there is no common factor of all the terms of a polynomial, look for a common factor in just some of the terms. When there are four terms, a good way to start is by separating the polynomial into two parts with two terms in each part. Then look for the GCF in each part. If the polynomial can be factored, you will find a common factor emerges from both parts.

(Not all polynomials can be factored. Just like some numbers are prime, some polynomials are prime.)

EXAMPLE 15

How to Factor by Grouping

Factor: $xy + 3y + 2x + 6$.

Solution

Step 1. Group terms with common factors.	Is there a greatest common factor of all four terms? No, so let's separate the first two terms from the second two.	$xy + 3y + 2x + 6$ $\underbrace{xy + 3y} + \underbrace{2x + 6}$
Step 2. Factor out the common factor in each group.	Factor the GCF from the first two terms. Factor the GCF from the second two terms.	$y(x+3) + \underbrace{2x + 6}$ $= y(x+3) + 2(x+3)$
Step 3. Factor the common factor from the expression.	Notice that each term has a common factor of $(x+3)$. Factor out the common factor.	$y(x+3) + 2(x+3)$ $= (x+3)(y+2)$
Step 4. Check.	Multiply $(x+3)(y+2)$. Is the product the original expression?	$(x+3)(y+2)$ $= xy + 2x + 3y + 6$ $= xy + 3y + 2x + 6$✓

TRY IT 15.1

Factor: $xy + 8y + 3x + 24$.

Show answer
$(x+8)(y+3)$

TRY IT 15.2

Factor: $ab + 7b + 8a + 56$.

Show answer
$(a+7)(b+8)$

HOW TO:

Factor by grouping.

1. Group terms with common factors.
2. Factor out the common factor in each group.
3. Factor the common factor from the expression.
4. Check by multiplying the factors.

EXAMPLE 16

Factor: $x^2 + 3x - 2x - 6$.

Solution

There is no GCF in all four terms.	$x^2 + 3x - 2x - 6$
Separate into two parts.	$\underbrace{x^2 + 3x}\ \underbrace{-2x - 6}$
Factor the GCF from both parts. Be careful with the signs when factoring the GCF from the last two terms.	$x(x+3) - 2(x+3)$ $(x+3)(x-2)$
Check on your own by multiplying.	

TRY IT 16.1

Factor: $x^2 + 2x - 5x - 10$.

Show answer
$(x-5)(x+2)$

TRY IT 16.2

Factor: $y^2 + 4y - 7y - 28$.

Show answer
$(y+4)(y-7)$

Access these online resources for additional instruction and practice with greatest common factors (GFCs) and factoring by grouping.

- Greatest Common Factor (GCF)

- Factoring Out the GCF of a Binomial
- Greatest Common Factor (GCF) of Polynomials

Key Concepts

- **Finding the Greatest Common Factor (GCF):** To find the GCF of two expressions:
    1. Factor each coefficient into primes. Write all variables with exponents in expanded form.
    2. List all factors—matching common factors in a column. In each column, circle the common factors.
    3. Bring down the common factors that all expressions share.
    4. Multiply the factors.
- **Factor the Greatest Common Factor from a Polynomial:** To factor a greatest common factor from a polynomial:
    1. Find the GCF of all the terms of the polynomial.
    2. Rewrite each term as a product using the GCF.
    3. Use the 'reverse' Distributive Property to factor the expression.
    4. Check by multiplying the factors.
- **Factor by Grouping:** To factor a polynomial with 4 or more terms
    1. Group terms with common factors.
    2. Factor out the common factor in each group.
    3. Factor the common factor from the expression.
    4. Check by multiplying the factors.

Glossary

factoring
Factoring is splitting a product into factors; in other words, it is the reverse process of multiplying.
greatest common factor
The greatest common factor is the largest expression that is a factor of two or more expressions is the greatest common factor (GCF).
Type your textbox content here.

Practice Makes Perfect

Find the Greatest Common Factor of Two or More Expressions

In the following exercises, find the greatest common factor.

1. 8, 18	2. 24, 40
3. 72, 162	4. 150, 275
5. 10*a*, 50	6. 5*b*, 30
7. $3x, 10x^2$	8. $21b^2, 14b$
9. $8w^2, 24w^3$	10. $30x^2, 18x^3$
11. $10p^3q, 12pq^2$	12. $8a^2b^3, 10ab^2$
13. $12m^2n^3, 30m^5n^3$	14. $28x^2y^4, 42x^4y^4$
15. $10a^3, 12a^2, 14a$	16. $20y^3, 28y^2, 40y$
17. $35x^3, 10x^4, 5x^5$	18. $27p^2, 45p^3, 9p^4$

Factor the Greatest Common Factor from a Polynomial

In the following exercises, factor the greatest common factor from each polynomial.

19. $4x + 20$	20. $8y + 16$
21. $6m + 9$	22. $14p + 35$
23. $9q + 9$	24. $7r + 7$
25. $8m - 8$	26. $4n - 4$
27. $9n - 63$	28. $45b - 18$
29. $3x^2 + 6x - 9$	30. $4y^2 + 8y - 4$
31. $8p^2 + 4p + 2$	32. $10q^2 + 14q + 20$
33. $8y^3 + 16y^2$	34. $12x^3 - 10x$
35. $5x^3 - 15x^2 + 20x$	36. $8m^2 - 40m + 16$
37. $12xy^2 + 18x^2y^2 - 30y^3$	38. $21pq^2 + 35p^2q^2 - 28q^3$
39. $-2x - 4$	40 $-3b + 12$
41. $5x(x + 1) + 3(x + 1)$	42. $2x(x - 1) + 9(x - 1)$
43. $3b(b - 2) - 13(b - 2)$	44. $6m(m - 5) - 7(m - 5)$

Factor by Grouping

In the following exercises, factor by grouping.

45. $xy + 2y + 3x + 6$	46. $mn + 4n + 6m + 24$
47. $uv - 9u + 2v - 18$	48. $pq - 10p + 8q - 80$
49. $b^2 + 5b - 4b - 20$	50. $m^2 + 6m - 12m - 72$
51. $p^2 + 4p - 9p - 36$	52. $x^2 + 5x - 3x - 15$

Mixed Practice

In the following exercises, factor.

53. $-20x - 10$	54. $5x^3 - x^2 + x$
55. $3x^3 - 7x^2 + 6x - 14$	56. $x^3 + x^2 - x - 1$
57. $x^2 + xy + 5x + 5y$	58. $5x^3 + 3x^2 - 5x - 3$

Everyday Math

59. **Area of a rectangle** The area of a rectangle with length 6 less than the width is given by the expression $w^2 - 6w$, where $w =$ width. Factor the greatest common factor from the polynomial.	60. **Height of a baseball** The height of a baseball *t* seconds after it is hit is given by the expression $-16t^2 + 80t + 4$. Factor the greatest common factor from the polynomial.

Writing Exercises

61. The greatest common factor of 36 and 60 is 12. Explain what this means.	62. What is the GCF of y^4, y^5, and y^{10}? Write a general rule that tells you how to find the GCF of y^a, y^b, and y^c.

Answers

1. 2	3. 18
5. 10	7. x
9. $8w^2$	11. $2pq$
13. $6m^2n^3$	15. $2a$
17. $5x^3$	19. $4(x+5)$
21. $3(2m+3)$	23. $9(q+1)$
25. $8(m-1)$	27. $9(n-7)$
29. $3\left(x^2+2x-3\right)$	31. $2\left(4p^2+2p+1\right)$
33. $8y^2(y+2)$	35. $5x\left(x^2-3x+4\right)$
37. $6y^2\left(2x+3x^2-5y\right)$	39. $-2(x+4)$
41. $(x+1)(5x+3)$	43. $(b-2)(3b-13)$
45. $(y+3)(x+2)$	47. $(u+2)(v-9)$
49. $(b-4)(b+5)$	51. $(p-9)(p+4)$
53. $-10(2x+1)$	55. $\left(x^2+2\right)(3x-7)$
57. $(x+y)(x+5)$	59. $w(w-6)$
61. Answers will vary.	

Attributions

6.5 Factor Quadratic Trinomials with Leading Coefficient 1

Learning Objectives

By the end of this section, you will be able to:

- Factor trinomials of the form $x^2 + bx + c$
- Factor trinomials of the form $x^2 + bxy + cy^2$

Factor Trinomials of the Form $x^2 + bx + c$

You have already learned how to multiply binomials using FOIL. Now you'll need to "undo" this multiplication—to start with the product and end up with the factors. Let's look at an example of multiplying binomials to refresh your memory.

$$(x+2)(x+3) \quad \text{factors}$$
$$F \quad O \quad I \quad L$$
$$x^2 + 3x + 2x + 6$$
$$x^2 + 5x + 6 \quad \text{product}$$

To factor the trinomial means to start with the product, $x^2 + 5x + 6$, and end with the factors, $(x+2)(x+3)$. You need to think about where each of the terms in the trinomial came from.

The *first term* came from multiplying the first term in each binomial. So to get x^2 in the product, each binomial must start with an *x*.

$x^2 + 5x + 6$

$(x)(x)$

The *last term* in the trinomial came from multiplying the last term in each binomial. So the last terms must multiply to 6

What two numbers multiply to 6?

The factors of 6 could be 1 and 6, or 2 and 3. How do you know which pair to use?

Consider the *middle term*. It came from adding the outer and inner terms.

So the numbers that must have a product of 6 will need a sum of 5. We'll test both possibilities and summarize the results in the table below—the table will be very helpful when you work with numbers that can be factored in many different ways.

Factors of 6	**Sum of factors**
$1, 6$	$1 + 6 = 7$
$2, 3$	$2 + 3 = 5$

We see that 2 and 3 are the numbers that multiply to 6 and add to 5. So we have the factors of x^2+5x+6. They are $(x+2)(x+3)$.

x^2+5x+6	product
$(x+2)(x+3)$	factors

You should check this by multiplying.

Looking back, we started with x^2+5x+6, which is of the form x^2+bx+c, where $b=5$ and $c=6$. We factored it into two binomials of the form $(x+m)$ and $(x+n)$.

x^2+5x+6	x^2+bx+c
$(x+2)(x+3)$	$(x+m)(x+n)$

To get the correct factors, we found two numbers *m* and *n* whose product is *c* and sum is *b*.

EXAMPLE 1

How to Factor Trinomials of the Form x^2+bx+c

Factor: $x^2+7x+12$.

Solution

Step 1. Write the factors as two binomials with first term x.	Write two sets of parentheses and put x as the first term.	$x^2+7x+12$ $(x\quad)(x\quad)$
Step 2. Find two numbers m and n that multiply to c and add to b: $m\cdot n=c$ $m+n=b$	In this question, $b=7$ and $c=12$. Start by writing 12 as a product of its two factors and checking if they add up to 7.	See the table below. Clearly, 3 and 4 are the required numbers.
Step 3. Use m and n as the last terms of the factors of the binomial.	Use 3 and 4 as the last terms of the factors of the given binomial.	$(x+3)(x+4)$ Note that it does not matter with factor is listed first as multiplication is commutative.
Step 4. Check by multiplying the factors.		$(x+3)(x+4)$ $=x^2+4x+3x+12$ $=x^2+7x+12$✓

Factors of 12	Sum of factors
$1,12$	$1+12=13$
$-1,-12$	$-1+(-12)=-13$
$2,6$	$2+6=8$
$-2,-6$	$-2+(-6)=-3$
$3,4$	$3+4=7$*
$-3,-4$	$-3+(-4)=-7$

TRY IT 1.1

Factor: $x^2 + 6x + 8$.

Show answer
$(x + 2)(x + 4)$

TRY IT 1.2

Factor: $y^2 + 8y + 15$.

Show answer
$(y + 3)(y + 5)$

Let's summarize the steps we used to find the factors.

HOW TO:

Factor trinomials of the form $x^2 + bx + c$.

1. Write the factors as two binomials with first terms *x*: $(x \quad)(x \quad)$.
2. Find two numbers *m* and *n* that
   Multiply to *c*, $m \cdot n = c$
   Add to *b*, $m + n = b$
3. Use *m* and *n* as the last terms of the factors: $(x + m)(x + n)$.
4. Check by multiplying the factors.

EXAMPLE 2

Factor: $u^2 + 11u + 24$.

Solution
Notice that the variable is *u*, so the factors will have first terms *u*.

Write the factors as two binomials with first terms u.	$u^2 + 11u + 24$ $(u \quad)(u \quad)$
Find two numbers that: multiply to 24 and add to 11.	see table below
Use 3 and 8 as the last terms of the binomials.	$(u+3)(u+8)$
Check.	$(u+3)(u+8)$ $u^2 + 3u + 8u + 24$ $u^2 + 11u + 24$✓

Factors of 24	**Sum of factors**
$1, 24$	$1 + 24 = 25$
$-1, -24$	$-1 - 24 = -25$
$2, 12$	$2 + 12 = 14$
$-2, -12$	$-2 - 12 = -14$
$3, 8$	$3 + 8 = 11$*
$-3, -8$	$-3 - 8 = -11$
$4, 6$	$4 + 6 = 10$
$-4, -6$	$-4 - 6 = -10$

TRY IT 2.1

Factor: $q^2 + 10q + 24$.

Show answer

$(q+4)(q+6)$

TRY IT 2.2

Factor: $t^2 + 14t + 24$.

Show answer

$(t+2)(t+12)$

EXAMPLE 3

Factor: $y^2 + 17y + 60.$

Solution

Write the factors as two binomials with first terms y.	$y^2 + 17y + 60$ $(y \quad)(y \quad)$
Find two numbers that multiply to 60 and add to 17	see table below
Use 5 and 12 as the last terms.	$(y+5)(y+12)$
Check.	$(y+5)(y+12)$ $(y^2 + 12y + 5y + 60)$ $(y^2 + 17y + 60)$✓

Factors of 60	**Sum of factors**
$1, 60$	$1 + 60 = 61$
$-1, -60$	$-1 - 60 = -61$
$2, 30$	$2 + 30 = 32$
$-2, -30$	$-2 - 30 = -32$
$3, 20$	$3 + 20 = 23$
$-3, -20$	$-3 - 20 = -23$
$4, 15$	$4 + 15 = 19$
$-4, -15$	$-4 + -15 = 19$
$5, 12$	$5 + 12 = 17^*$
$-5, -12$	$-5 - 12 = -17$
$6, 10$	$6 + 10 = 16$
$-6, -10$	$-6 - 10 = -16$

TRY IT 3.1

Factor: $x^2 + 19x + 60.$

Show answer
$(x+4)(x+15)$

TRY IT 3.2

Factor: $v^2 + 23v + 60$.

Show answer
$(v + 3)(v + 20)$

Factor Trinomials of the Form $x^2 + bx + c$ with *b* Negative, *c* Positive

In the examples so far, all terms in the trinomial were positive. What happens when there are negative terms? Well, it depends which term is negative. Let's look first at trinomials with only the middle term negative.

Remember: To get a negative sum and a positive product, the numbers must both be negative.

Again, think about FOIL and where each term in the trinomial came from. Just as before,

- the first term, x^2, comes from the product of the two first terms in each binomial factor, *x* and *y*;
- the positive last term is the product of the two last terms
- the negative middle term is the sum of the outer and inner terms.

How do you get a *positive product* and a *negative sum*? With two negative numbers.

EXAMPLE 4

Factor: $t^2 - 11t + 28$.

Solution

Write the factors as two binomials with first terms t.	$t^2 - 11t + 28$ $= (t \quad)(t \quad)$
With the positive last term, 28, and the negative middle term, $-11t$, we need two negative factors. Find two numbers that multiply 28 and add to -11.	
Find two numbers that: multiply to 28 and add to -11.	(see table below)
Use $-4, -7$ as the last terms of the binomials.	$(t-4)(t-7)$
Check.	$(t-4)(t-7)$ $t^2 - 7t - 4t + 28$ $t^2 - 11t + 28✓$

Factors of 28	Sum of factors
$-1, -28$	$-1 + (-28) = -29$
$-2, -14$	$-2 + (-14) = -16$
$-4, -7$	$-4 + (-7) = -11^*$

TRY IT 4.1

Factor: $u^2 - 9u + 18$.

Show answer

$(u-3)(u-6)$

TRY IT 4.2

Factor: $y^2 - 16y + 63$.

Show answer

$(y-7)(y-9)$

Factor Trinomials of the Form $x^2 + bx + c$ with c Negative

Now, what if the last term in the trinomial is negative? Think about FOIL. The last term is the product of the last terms in the two binomials. A negative product results from multiplying two numbers with opposite signs. You have to be very careful to choose factors to make sure you get the correct sign for the middle term, too.

Remember: To get a negative product, the numbers must have different signs.

EXAMPLE 5

Factor: $z^2 + 4z - 5$.

Solution

Factors will be two binomials with first terms z.	$z^2 + 4z - 5$ $= (z \quad)(z \quad)$
To get a negative last term, multiply one positive and one negative. We need factors of -5 that add to positive 4	<table><tr><th>Factors of -5</th><th>Sum of factors</th></tr><tr><td>$1, -5$</td><td>$1 + (-5) = -4$</td></tr><tr><td>$-1, 5$</td><td>$-1 + 5 = 4^*$</td></tr></table> Notice: We listed both $1, -5$ and $-1, 5$ to make sure we got the sign of the middle term correct.
Use $-1, 5$ as the last terms of the binomials.	$(z - 1)(z + 5)$
Check.	$(z - 1)(z + 5)$ $z^2 + 5z - 1z - 5$ $z^2 + 4z - 5$✓

TRY IT 5.1

Factor: $h^2 + 4h - 12$.

Show answer
$(h - 2)(h + 6)$

TRY IT 5.2

Factor: $k^2 + k - 20$.

Show answer
$(k - 4)(k + 5)$

Let's make a minor change to the last trinomial and see what effect it has on the factors.

EXAMPLE 6

Factor: $z^2 - 4z - 5$.

Solution

Factors will be two binomials with first terms z.	$z^2 - 4z - 5$ $= (z \quad)(z \quad)$
To get a negative last term, multiply one positive and one negative. We need factors of -5 that add to -4.	<table><tr><th>Factors of -5</th><th>Sum of factors</th></tr><tr><td>$1, -5$</td><td>$1 + (-5) = -4^*$</td></tr><tr><td>$-1, 5$</td><td>$-1 + 5 = 4^*$</td></tr></table> Notice: We listed both $1, -5$ and $-1, 5$ to make sure we got the sign of the middle term correct.
Use $1, -5$ as the last terms of the binomials.	$(z + 1)(z - 5)$
Check.	$(z + 1)(z - 5)$ $z^2 - 5z + 1z - 5$ $z^2 - 4z - 5\checkmark$

Notice that the factors of $z^2 - 4z - 5$ are very similar to the factors of $z^2 + 4z - 5$. It is very important to make sure you choose the factor pair that results in the correct sign of the middle term.

TRY IT 6.1

Factor: $x^2 - 4x - 12$.

Show answer
$(x + 2)(x - 6)$

TRY IT 6.2

Factor: $y^2 - y - 20$.

Show answer
$(y + 4)(y - 5)$

EXAMPLE 7

Factor: $q^2 - 2q - 15$.

Solution

Factors will be two binomials with first terms q.	$q^2 - 2q - 15$ $(q \quad)(q \quad)$
Find two numbers that: multiply to -15 and add to -2.	**Factors of -15** / **Sum of factors** $1, -15$: $1 + (-15) = -14$ $-1, 15$: $-1 + 15 = 14$ $3, -5$: $3 + (-5) = -2$* $-3, 5$: $-3 + 5 = 2$
Use $3, -5$ as the last terms of the binomials.	$(q + 3)(q - 5)$
Check.	$(q + 3)(q - 5)$ $q^2 - 5q + 3q - 15$ $q^2 - 2q - 15$✓

Factors of -15	Sum of factors
$1, -15$	$1 + (-15) = -14$
$-1, 15$	$-1 + 15 = 14$
$3, -5$	$3 + (-5) = -2$*
$-3, 5$	$-3 + 5 = 2$

TRY IT 7.1

Factor: $r^2 - 3r - 40$.

Show answer

$(r + 5)(r - 8)$

TRY IT 7.1

Factor: $s^2 - 3s - 10$.

Show answer

$(s + 2)(s - 5)$

Some trinomials are prime. The only way to be certain a trinomial is prime is to list all the possibilities and show that none of them work.

EXAMPLE 8

Factor: $y^2 - 6y + 15$.

Solution

Factors will be two binomials with first terms y.	$y^2 - 6y + 15$ $(y \quad)(y \quad)$
Find two numbers that: multiply to 15 and add to -6.	(see table below)

Factors of 15	Sum of factors
$-1, -15$	$-1 + (-15) = -16$
$-3, -5$	$-3 + (-5) = -8$

As shown in the table, none of the factors add to -6; therefore, **the expression is prime**.

TRY IT 8.1

Factor: $m^2 + 4m + 18$.

Show answer
prime

TRY IT 8.2

Factor: $n^2 - 10n + 12$.

Show answer
prime

EXAMPLE 9

Factor: $2x + x^2 - 48$.

Solution

First we put the terms in decreasing degree order.	$x^2 + 2x - 48$
Factors will be two binomials with first terms x.	$(x \quad)(x \quad)$
Find two numbers that: multiply to -48 and add to 2.	(see table below)
As shown in the table, you can use $-6, 8$ as the last terms of the binomials.	$(x - 6)(x + 8)$
Check.	$(x-6)(x+8)$ $= x^2 - 6q + 8q - 48$ $= x^2 + 2x - 48$✓

Factors of -48	**Sum of factors**
$-1, 48$	$-1 + 48 = 47$
$-2, 24$	$-2 + 24 = 22$
$-3, 16$	$-3 + 16 = 13$
$-4, 12$	$-4 + 12 = 8$
$-6, 8$	$6 + 8 = 2$*

TRY IT 9.1

Factor: $9m + m^2 + 18$.

Show answer
$(m+3)(m+6)$

TRY IT 9.2

Factor: $-7n + 12 + n^2$.

Show answer
$(n-3)(n-4)$

Let's summarize the method we just developed to factor trinomials of the form $x^2 + bx + c$.

HOW TO:

Factor trinomials of the form $x^2 + bx + c$.

When we factor a trinomial, we look at the signs of its terms first to determine the signs of the binomial factors.

$$x^2 + bx + c$$
$$\left(x + m\right)\left(x + n\right)$$

When *c* is positive, *m* and *n* have the same sign.

b positive	b negative
m, n positive	m, n negative
$x^2 + 5x + 6$	$x^2 - 6x + 8$
$\left(x + 2\right)\left(x + 3\right)$	$\left(x - 4\right)\left(x - 2\right)$
same signs	same signs

When *c* is negative, *m* and *n* have opposite signs.

$x^2 + x - 12$	$x^2 - 2x - 15$
$\left(x + 4\right)\left(x - 3\right)$	$\left(x - 5\right)\left(x + 3\right)$
opposite signs	opposite signs

Notice that, in the case when *m* and *n* have opposite signs, the sign of the one with the larger absolute value matches the sign of *b*.

Factor Trinomials of the Form $x^2 + bxy + cy^2$

Sometimes you'll need to factor trinomials of the form $x^2 + bxy + cy^2$ with two variables, such as $x^2 + 12xy + 36y^2$. The first term, x^2, is the product of the first terms of the binomial factors, $x \cdot x$. The y^2 in the last term means that the second terms of the binomial factors must each contain *y*. To get the coefficients *b* and *c*, you use the same process summarized in the previous objective.

EXAMPLE 10

Factor: $x^2 + 12xy + 36y^2$.

Solution

Factors will be two binomials where the first terms are x and last terms contain y.	$x^2 + 12xy + 36y^2$ $(x \quad y)(x \quad y)$
Find the numbers that multiply to 36 and add to 12.	<table><tr><th>Factors of 36</th><th>Sum of factors</th></tr><tr><td>1, 36</td><td>$1 + 36 = 37$</td></tr><tr><td>2, 18</td><td>$2 + 18 = 20$</td></tr><tr><td>3, 12</td><td>$3 + 12 = 15$</td></tr><tr><td>4, 9</td><td>$4 + 9 = 13$</td></tr><tr><td>6, 6</td><td>$6 + 6 = 12^*$</td></tr></table>
Use 6 and 6 as the coefficients of the last terms.	$(x + 6y)(x + 6y)$
Check your answer.	$(x + 6y)(x + 6y)$ $x^2 + 6xy + 6xy + 36y^2$ $x^2 + 12xy + 36y^2$ ✓

TRY IT 10.1

Factor: $u^2 + 11uv + 28v^2$.

Show answer

$(u + 4v)(u + 7v)$

TRY IT 10.2

Factor: $x^2 + 13xy + 42y^2$.

Show answer

$(x + 6y)(x + 7y)$

EXAMPLE 11

Factor: $r^2 - 8rs - 9s^2$.

Solution

Factors will be two binomials where the first terms are r and last terms contain s.	$r^2 - 8rs - 9s^2$ $(r \quad s)(r \quad s)$
Find the numbers that multiply to -9 and add to -8. The last term of the trinomial is negative, so the factors must have opposite signs.	**Factors of -9** / **Sum of factors** $1, -9$: $1 + (-9) = -8^*$ $-1, 9$: $-1 + 9 = 8$ $3, -3$: $3 + (-3) = 0$
Use 1, -9 as coefficients of the last terms.	$(r + s)(r - 9s)$
Check your answer.	$(r + s)(r - 9s)$ $r^2 - 9rs + rs - 9s^2$ $r^2 - 8rs - 9s^2 \checkmark$

Factors of -9	Sum of factors
$1, -9$	$1 + (-9) = -8^*$
$-1, 9$	$-1 + 9 = 8$
$3, -3$	$3 + (-3) = 0$

TRY IT 11.1

Factor: $a^2 - 11ab + 10b^2$.

Show answer

$(a - b)(a - 10b)$

TRY IT 11.2

Factor: $m^2 - 13mn + 12n^2$.

Show answer

$(m - n)(m - 12n)$

EXAMPLE 12

Factor: $u^2 - 9uv - 12v^2$.

Solution

We need u in the first term of each binomial and v in the second term.	$u^2 - 9uv - 12v^2$ $(u \quad v)(u \quad v)$
Find the numbers that multiply to -12 and add to -9. The last term of the trinomial is negative, so the factors must have opposite signs.	(see table below)
Note there are no factor pairs that give us -9 as a sum. **The trinomial is prime.**	

Factors of -12	Sum of factors
$1, -12$	$1 + (-12) = -11$
$1, 12$	$-1 + 12 = 11$
$2, -6$	$2 + (-6) = -4$
$-2, 6$	$-2 + 6 = 4$
$3, -4$	$3 + (-4) = -1$
$-3, 4$	$-3 + 4 = 1$

TRY IT 12.1

Factor: $x^2 - 7xy - 10y^2$.

Show answer
prime

TRY IT 12.2

Factor: $p^2 + 15pq + 20q^2$.

Show answer
prime

Key Concepts

- **Factor trinomials of the form** $x^2 + bx + c$
  1. Write the factors as two binomials with first terms *x*: $(x)\,(x)$.
  2. Find two numbers *m* and *n* that
     Multiply to *c*, $m \cdot n = c$
     Add to *b*, $m + n = b$

3. Use *m* and *n* as the last terms of the factors: $(x+m)(x+n)$.
4. Check by multiplying the factors.

Practice Makes Perfect

Factor Trinomials of the Form x^2+bx+c

In the following exercises, factor each trinomial of the form x^2+bx+c.

1. x^2+4x+3	2. y^2+8y+7
3. $m^2+12m+11$	4. $b^2+14b+13$
5. $a^2+9a+20$	6. $m^2+7m+12$
7. $p^2+11p+30$	8. $w^2+10x+21$
9. $n^2+19n+48$	10. $b^2+14b+48$
11. $a^2+25a+100$	12. $u^2+101u+100$
13. $x^2-8x+12$	14. $q^2-13q+36$
15. $y^2-18x+45$	16. $m^2-13m+30$
17. x^2-8x+7	18. y^2-5y+6
19. p^2+5p-6	20. n^2+6n-7
21. y^2-6y-7	22. v^2-2v-3
23. x^2-x-12	24. r^2-2r-8
25. $a^2-3a-28$	26. $b^2-13b-30$
27. $w^2-5w-36$	28. $t^2-3t-54$
29. x^2+x+5	30. x^2-3x-9
31. $8-6x+x^2$	32. $7x+x^2+6$
33. $x^2-12-11x$	34. $-11-10x+x^2$

Factor Trinomials of the Form $x^2+bxy+cy^2$

In the following exercises, factor each trinomial of the form $x^2+bxy+cy^2$.

35. $p^2 + 3pq + 2q^2$	36. $m^2 + 6mn + 5n^2$
37. $r^2 + 15rs + 36s^2$	38. $u^2 + 10uv + 24v^2$
39. $m^2 - 12mn + 20n^2$	40. $p^2 - 16pq + 63q^2$
41. $x^2 - 2xy - 80y^2$	42. $p^2 - 8pq - 65q^2$
43. $m^2 - 64mn - 65n^2$	44. $p^2 - 2pq - 35q^2$
45. $a^2 + 5ab - 24b^2$	46. $r^2 + 3rs - 28s^2$
47. $x^2 - 3xy - 14y^2$	48. $u^2 - 8uv - 24v^2$
49. $m^2 - 5mn + 30n^2$	50. $c^2 - 7cd + 18d^2$

Mixed Practice

In the following exercises, factor each expression.

51. $u^2 - 12u + 36$	52. $w^2 + 4w - 32$
53. $x^2 - 14x - 32$	54. $y^2 + 41y + 40$
55. $r^2 - 20rs + 64s^2$	56. $x^2 - 16xy + 64y^2$
57. $k^2 + 34k + 120$	58. $m^2 + 29m + 120$
59. $y^2 + 10y + 15$	60. $z^2 - 3z + 28$
61. $m^2 + mn - 56n^2$	62. $q^2 - 29qr - 96r^2$
63. $u^2 - 17uv + 30v^2$	64. $m^2 - 31mn + 30n^2$
65. $c^2 - 8cd + 26d^2$	66. $r^2 + 11rs + 36s^2$

Everyday Math

67. **Consecutive integers** Deirdre is thinking of two consecutive integers whose product is 56. The trinomial $x^2 + x - 56$ describes how these numbers are related. Factor the trinomial.	68. **Consecutive integers** Deshawn is thinking of two consecutive integers whose product is 182. The trinomial $x^2 + x - 182$ describes how these numbers are related. Factor the trinomial.

Writing Exercises

69. Many trinomials of the form x^2+bx+c factor into the product of two binomials $(x+m)(x+n)$. Explain how you find the values of *m* and *n*.	70. How do you determine whether to use plus or minus signs in the binomial factors of a trinomial of the form x^2+bx+c where b and c may be positive or negative numbers?
71. Will factored x^2-x-20 as $(x+5)(x-4)$. Bill factored it as $(x+4)(x-5)$. Phil factored it as $(x-5)(x-4)$. Who is correct? Explain why the other two are wrong.	72. Look at (Figure), where we factored $y^2+17y+60$. We made a table listing all pairs of factors of 60 and their sums. Do you find this kind of table helpful? Why or why not?

Answers

1. $(x+1)(x+3)$	3. $(m+1)(m+11)$
5. $(a+4)(a+5)$	7. $(p+5)(p+6)$
9. $(n+3)(n+16)$	11. $(a+5)(a+20)$
13. $(x-2)(x-6)$	15. $(y-3)(y-15)$
17. $(x-1)(x-7)$	19. $(p-1)(p+6)$
21. $(y+1)(y-7)$	23. $(x-4)(x+3)$
25. $(a-7)(a+4)$	27. $(w-9)(w+4)$
29. prime	31. $(x-4)(x-2)$
33. $(x-12)(x+1)$	35. $(p+q)(p+2q)$
37. $(r+3s)(r+12s)$	39. $(m-2n)(m-10n)$
41. $(x+8y)(x-10y)$	43. $(m+n)(m-65n)$
45. $(a+8b)(a-3b)$	47. prime
49. prime	51. $(u-6)(u-6)$
53. $(x+2)(x-16)$	55. $(r-4s)(r-16s)$
57. $(k+4)(k+30)$	59. prime
61. $(m+8n)(m-7n)$	63. $(u-15v)(u-2v)$
65. prime	67. $(x+8)(x-7)$
69. Answers may vary	71. Answers may vary

Attributions

6.6 Divide Polynomials

Learning Objectives

By the end of this section, you will be able to:

- Divide a polynomial by a monomial

Divide a Polynomial by a Monomial

In the last chapter, you learned how to divide a monomial by a monomial. As you continue to build up your knowledge of polynomials the next procedure is to divide a polynomial of two or more terms by a monomial.

The method we'll use to divide a polynomial by a monomial is based on the properties of fraction addition. So we'll start with an example to review fraction addition.

The sum,	$\frac{y}{5}+\frac{2}{5}$,
simplifies to	$\frac{y+2}{5}$.

Now we will do this in reverse to split a single fraction into separate fractions.

We'll state the fraction addition property here just as you learned it and in reverse.

Fraction Addition

If $a, b,$ and c are numbers where $c \neq 0$, then

$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$ and $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

We use the form on the left to add fractions and we use the form on the right to divide a polynomial by a monomial.

For example,	$\frac{y+2}{5}$
can be written	$\frac{y}{5}+\frac{2}{5}$.

We use this form of fraction addition to divide polynomials by monomials.

Division of a Polynomial by a Monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

EXAMPLE 1

Find the quotient: $\frac{7y^2+21}{7}$.

Solution

	$\frac{7y^2+21}{7}$
Divide each term of the numerator by the denominator.	$\frac{7y^2}{7}+\frac{21}{7}$
Simplify each fraction.	y^2+3

TRY IT 1.1

Find the quotient: $\frac{8z^2+24}{4}$.

Show answer
$2z^2+6$

TRY IT 1.2

Find the quotient: $\frac{18z^2-27}{9}$.

Show answer

$2z^2 - 3$

Remember that division can be represented as a fraction. When you are asked to divide a polynomial by a monomial and it is not already in fraction form, write a fraction with the polynomial in the numerator and the monomial in the denominator.

EXAMPLE 2

Find the quotient: $\left(18x^3 - 36x^2\right) \div 6x$.

Solution

	$\left(18x^3 - 36x^2\right) \div 6x$
Rewrite as a fraction.	$\frac{18x^3 - 36x^2}{6x}$
Divide each term of the numerator by the denominator.	$\frac{18x^3}{6x} - \frac{36x^2}{6x}$
Simplify.	$3x^2 - 6x$

TRY IT 2.1

Find the quotient: $\left(27b^3 - 33b^2\right) \div 3b$.

Show answer
$9b^2 - 11b$

TRY IT 2.2

Find the quotient: $\left(25y^3 - 55y^2\right) \div 5y$.

Show answer
$5y^2 - 11y$

When we divide by a negative, we must be extra careful with the signs.

EXAMPLE 3

Find the quotient: $\dfrac{12d^2 - 16d}{-4}$.

Solution

	$\dfrac{12d^2 - 16d}{-4}$
Divide each term of the numerator by the denominator.	$\dfrac{12d^2}{-4} - \dfrac{16d}{-4}$
Simplify. Remember, subtracting a negative is like adding a positive!	$-3d^2 + 4d$

TRY IT 3.1

Find the quotient: $\dfrac{25y^2 - 15y}{-5}$.

Show answer
$-5y^2 + 3y$

TRY IT 3.2

Find the quotient: $\dfrac{42b^2 - 18b}{-6}$.

Show answer
$-7b^2 + 3b$

EXAMPLE 4

Find the quotient: $\dfrac{105y^5 + 75y^3}{5y^2}$.

Solution

	$\dfrac{105y^5 + 75y^3}{5y^2}$
Separate the terms.	$\dfrac{105y^5}{5y^2} + \dfrac{75y^3}{5y^2}$
Simplify.	$21y^3 + 15y$

TRY IT 4.1

Find the quotient: $\dfrac{60d^7 + 24d^5}{4d^3}$.

Show answer
$15d^4 + 6d^2$

TRY IT 4.2

Find the quotient: $\dfrac{216p^7 - 48p^5}{6p^3}$.

Show answer
$36p^4 - 8p^2$

EXAMPLE 5

Find the quotient: $\left(15x^3y - 35xy^2\right) \div (-5xy)$.

Solution

	$\left(15x^3y - 35xy^2\right) \div (-5xy)$
Rewrite as a fraction.	$\dfrac{15x^3y - 35xy^2}{-5xy}$
Separate the terms.	$\dfrac{15x^3y}{-5xy} - \dfrac{35xy^2}{-5xy}$
Simplify.	$-3x^2 + 7y$

TRY IT 5.1

Find the quotient: $\left(32a^2b - 16ab^2\right) \div (-8ab)$.

Show answer
$-4a + 2b$

TRY IT 5.2

Find the quotient: $\left(-48a^8b^4 - 36a^6b^5\right) \div \left(-6a^3b^3\right)$.

Show answer
$8a^5b + 6a^3b^2$

EXAMPLE 6

Find the quotient: $\dfrac{36x^3y^2 + 27x^2y^2 - 9x^2y^3}{9x^2y}$.

Solution

	$\dfrac{36x^3y^2 + 27x^2y^2 - 9x^2y^3}{9x^2y}$
Separate the terms.	$\dfrac{36x^3y^2}{9x^2y} + \dfrac{27x^2y^2}{9x^2y} - \dfrac{9x^2y^3}{9x^2y}$
Simplify.	$4xy + 3y - y^2$

TRY IT 6.1

Find the quotient: $\dfrac{40x^3y^2 + 24x^2y^2 - 16x^2y^3}{8x^2y}$.

Show answer
$5xy + 3y - 2y^2$

TRY IT 6.2

Find the quotient: $\dfrac{35a^4b^2 + 14a^4b^3 - 42a^2b^4}{7a^2b^2}$.

Show answer
$5a^2 + 2a^2b - 6b^2$

EXAMPLE 7

Find the quotient: $\dfrac{10x^2 + 5x - 20}{5x}$.

Solution

	$\dfrac{10x^2 + 5x - 20}{5x}$
Separate the terms.	$\dfrac{10x^2}{5x} + \dfrac{5x}{5x} - \dfrac{20}{5x}$
Simplify.	$2x + 1 + \dfrac{4}{x}$

TRY IT 7.1

Find the quotient: $\dfrac{18c^2 + 6c - 9}{6c}$.

Show answer

$$3c + 1 - \frac{3}{2c}$$

TRY IT 7.2

Find the quotient: $\dfrac{10d^2 - 5d - 2}{5d}$.

Show answer

$$2d - 1 - \frac{2}{5d}$$

Access these online resources for additional instruction and practice with dividing polynomials:

- Divide a Polynomial by a Monomial
- Divide a Polynomial by a Monomial 2

Key Concepts

- **Fraction Addition**
  - If a, b, and c are numbers where $c \neq 0$, then $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$
- **Division of a Polynomial by a Monomial**
  - To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Practice Makes Perfect

Dividing Polynomial by Monomial

In the following exercises, divide each polynomial by the monomial.

1. $\frac{30b+75}{5}$	2. $\frac{45y+36}{9}$
3. $\frac{42x^2-14x}{7}$	4. $\frac{8d^2-4d}{2}$
5. $\left(55w^2-10w\right)\div 5w$	6. $\left(16y^2-20y\right)\div 4y$
7. $\left(8x^3+6x^2\right)\div 2x$	8. $\left(9n^4+6n^3\right)\div 3n$
9. $\frac{20b^2-12b}{-4}$	10. $\frac{18y^2-12y}{-6}$
11. $\frac{51m^4+72m^3}{-3}$	12. $\frac{35a^4+65a^2}{-5}$
13. $\frac{412z^8-48z^5}{4z^3}$	14. $\frac{310y^4-200y^3}{5y^2}$
15. $\frac{51y^4+42y^2}{3y^2}$	16. $\frac{46x^3+38x^2}{2x^2}$
17. $\left(35x^4-21x\right)\div(-7x)$	18. $\left(24p^2-33p\right)\div(-3p)$
19. $\left(48y^4-24y^3\right)\div\left(-8y^2\right)$	20. $\left(63m^4-42m^3\right)\div\left(-7m^2\right)$
21. $\left(45x^3y^4+60xy^2\right)\div(5xy)$	22. $\left(63a^2b^3+72ab^4\right)\div(9ab)$
23. $\frac{49c^2d^2-70c^3d^3-35c^2d^4}{7cd^2}$	24. $\frac{52p^5q^4+36p^4q^3-64p^3q^2}{4p^2q}$
25. $\frac{72r^5s^2+132r^4s^3-96r^3s^5}{12r^2s^2}$	26. $\frac{66x^3y^2-110x^2y^3-44x^4y^3}{11x^2y^2}$
27. $\frac{12q^2+3q-1}{3q}$	28. $\frac{4w^2+2w-5}{2w}$
29. $\frac{20y^2+12y-1}{-4y}$	30. $\frac{10x^2+5x-4}{-5x}$
31. $\frac{63a^3-108a^2+99a}{9a^2}$	32. $\frac{36p^3+18p^2-12p}{6p^2}$

Everyday Math

33. Handshakes At a company meeting, every employee shakes hands with every other employee. The number of handshakes is given by the expression $\frac{n^2-n}{2}$, where n represents the number of employees. How many handshakes will there be if there are 10 employees at the meeting?	34. Average cost Pictures Plus produces digital albums. The company's average cost (in dollars) to make x albums is given by the expression $\frac{7x+500}{x}$. a. Find the quotient by dividing the numerator by the denominator. b. What will the average cost (in dollars) be to produce 20 albums?

Writing Exercises

35. Divide $\frac{10x^2+x-12}{2x}$ and explain with words how you get each term of the quotient.	36. James divides $48y+6$ by 6 this way: $\frac{48y+\cancel{6}}{\cancel{6}}=48y$. What is wrong with his reasoning?

Answers

1. $6b+15$	3. $6x^2-2x$	5. $11w-2$
7. $4x^2+3x$	9. $-5b^2+3b$	11. $-17m^4-24m^3$
13. $103z^5-12z^2$	15. $17y^2+14$	17. $-5x^3+3$
19. $-6y^2+3y$	21. $9x^2y^3+12y$	23. $7c-10c^2d-5cd^2$
25. $6r^3+11r^2s-8rs^3$	27. $4q+1-\frac{1}{3q}$	29. $-5y-3+\frac{1}{4y}$
31. $7a-12+\frac{11}{a}$	33. 45	35. Answers will vary.

Attributions

6.7 Factor Binomials - Difference of Squares

Learning Objectives

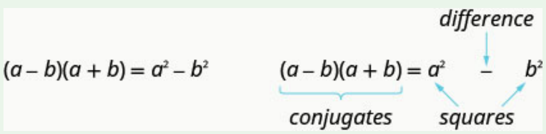
By the end of this section, you will be able to:

- Factor Binomials of the Form $a^2x^2 - b^2y^2$

Let us first review briefly what we learned in Chapter 6.3:

Product of Conjugates Pattern

If a and b are real numbers,



The product is called a difference of squares.

Factor the Difference of Squares

To be more specific, we are going to see how we can factorize the difference of **two** squares. From above, it is very clear that factors of a binomial of the type $a^2 - b^2$ are always $(a + b)(a - b)$. Once we know what a and b are, the factorization becomes very easy.

EXAMPLE 1

Factor the difference of square: $x^2 - 64$

Solution
First, we rewrite each term of $x^2 - 64$ as a perfect square of an expression.

Rewrite each term as a perfect square	$x^2 - 64$ $(x)^2 - (8)^2$
Treating x as a and 8 as b	$a^2 - b^2$ $(x)^2 - (8)^2$
Apply the difference of squares formula	$(a + b)(a - b)$ $(x + 8)(x - 8)$
Hence,	$x^2 - 64 = (x + 8)(x - 8)$

Difference of Squares

If a and b are real numbers, then

$$a^2 - b^2 = (a+b)(a-b)$$

TRY IT 1.1

Factor: $n^2 - 49$

Show answer
$(n + 7)(n - 7)$

TRY IT 1.2

Factor: $w^2 - 9$.

Show answer
$(w + 3)\,(w - 3)$

EXAMPLE 2

Factor completely: $4m^2 - 81$.

Solution
We know that $4m^2 = (2m)^2$ and $81 = 9^2$

Rewrite each term as a perfect square	$(2m)^2 - 9^2$
Treating $2m$ as a and 9 as b	$a = 2m, b = 9$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$(2m+9)(2m-9)$
Hence,	$4m^2 - 81 = (2m+9)(2m-9)$

TRY IT 2.1

Factor completely: $36x^2 - 49$.

Show answer
$(6x+7)\,(6x-7)$

TRY IT 2.2

Factor completely: $49x^2 - \frac{25}{121}$.

Show answer
$\left(7x + \frac{5}{11}\right)\left(7x - \frac{5}{11}\right)$

Now we'll factor a similar binomial that has two variables.

EXAMPLE 3

Factor completely: $25x^2 - y^2$.

Solution
We know that $25 = 5^2$, $x^2 = (x)^2$ and $y^2 = (y)^2$

Rewrite each term as a perfect square	$(5x)^2 - (y)^2$
Assigning values to a and b	$a = 5x, b = y$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$(5x+y)(5x-y)$
Hence,	$25x^2 - y^2 = (5x+y)(5x-y)$

TRY IT 3.1

Factor: $49y^2 - 16x^2$.

Show answer
$(7y+4x)(7y-4x)$

TRY IT 3.2

Factor: $81m^2 - 4n^2$.

Show answer
$(9m+2n)(9m-2n)$

EXAMPLE 4

Factor completely: $25x^2y^2 - 36$.

Solution
We know that $25 = 5^2, x^2 = (x)^2, y^2 = (y)^2$ and $36 = 6^2$

Rewrite each term as a perfect square	$(5xy)^2 - (6)^2$
Assigning values to a and b	$a = 5xy, b = 6$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$(5xy+6)(5xy-6)$
Hence,	$25x^2y^2 - 36 = (5xy+6)(5xy-6)$

TRY IT 4.1

Factor: $x^2y^2 - 1$.

Show answer
$(xy-1)(xy+1)$

TRY IT 4.2

Factor: $a^2b^2 - 81$.

Show answer
$(ab-9)(ab+9)$

Let us now try to factor a binomial where it is not apparent that we can use the Difference of Squares formula right away.

EXAMPLE 5

Factor: $3a^2 - 27b^2$

Solution

We know that $a^2 = (a)^2$ and $b^2 = (b)^2$ but neither 3 nor 27 is a perfect square. In a question like this check whether the numbers have a GCF other than 1. In this case, the GCF of 3 and 27 is 3. Now apply your knowledge of factoring out the GCF and check if you can factor the binomial any further:

Factor out the GCF	$3(a^2 - 9b^2)$
Rewrite each term as a perfect square	$3((a)^2 - (3b)^2)$
Assigning values to a and b	$a = a, b = 3b$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$3(a+3b)(a-3b)$
Hence,	$3a^2 - 27b^2 = 3(a+3b)(a-3b)$

TRY IT 5.1

Factor completely: $36x^2 - 9y^2$

Show answer
$9(2x+y)(2x-y)$

TRY IT 5.2

Factor completely: $3.6x^2 - 0.9$

Show answer
$0.9(2x+1)(2x-1)$

EXAMPLE 6

Factor: $x^4 - y^6$

Solution
We know that $x^4 = (x^2)^2$ and $y^6 = (y^3)^2$

Factor out the GCF (if any)	
Rewrite each term as a perfect square	$((x^2)^2 - (y^3)^2)$
Assigning values to a and b	$a = x^2, b = y^3$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$(x^2+y^3)(x^2-y^3)$
Hence,	$x^4 - y^6 = (x^2+y^3)(x^2-y^3)$

TRY IT 6.1

Factor completely: $x^4 - 9$

Show answer
$(x^2+3)(x^2-3)$

TRY IT 6.2

Factor completely: $2m^4n^2 - 50$

Show answer
$2(m^2n+5)(m^2n-5)$

Can you factor Sum of Squares?

Note that, except for a common factor, a sum of squares or a binomial of the form $a^2 + b^2$ is not factorable over the set of real numbers.

EXAMPLE 7

Factor completely: $x^4 - 16$

Solution
We know that $x^4 = (x^2)^2$ and $16 = (4)^2$

Factor out the GCF (if any)	
Rewrite each term as a perfect square	$((x^2)^2 - (4)^2)$
Assigning values to a and b	$a = x^2, b = 4$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$(x^2 + 4)(x^2 - 4)$
So far, this question is not much different from the previous examples but note that this factorization is incomplete. Why? Because the second factor $(x^2 - 4)$ can also be rewritten as a difference of squares. The first factor $(x^2 + 4)$ is a sum of squares and cannot be factored further over the set of real numbers.	
Rewrite each term of the second factor as a perfect square	$(x^2 + 4)((x)^2 - (2)^2)$
Assigning values to a and b	$a = x, b = 2$
Apply the difference of squares formula $a^2 - b^2 = (a+b)(a-b)$	$(x^2 + 4)(x + 2)(x - 2)$
Hence,	$x^4 - 16 = (x^2 + 4)(x + 2)(x - 2)$

TRY IT 7.1

Factor completely: $y^4 - 81$

Show answer
$(y^2 + 9)(y + 3)(y - 3)$

TRY IT 7.2

Factor completely: $5p^4 - 80$

Show answer
$5(p^2+4)(p+2)(p-2)$

EXAMPLE 8

Factor completely: $36-(x-5)^2$

Solution
We can apply the difference of squares formula not only on monomials that are perfect squares but also on any polynomial that is a perfect square. In this example, we know that $36=(6)^2$ and the second term is already in the form of a perfect square.

Factor out the GCF (if any)	
Rewrite each term as a perfect square	$((6)^2-(x-5)^2)$
Assigning values to a and b	$a=6, b=x-5$
Apply the difference of squares formula $a^2-b^2=(a+b)(a-b)$	$(6+(x-5))(6-(x-5))$
Note the brackets around the $(x-5)$. This is especially important in the second factor where we have a negative sign to take care of.	
Rewrite to remove the inner brackets	$(6+x-5)(6-x+5)$
Combine like terms	$(1+x)(11-x)$
Hence,	$36-(x-5)^2=(1+x)(11-x)$

TRY IT 8.1

Factor completely: $(x-7)^2-49$

Show answer
$x(x-14)$

TRY IT 8.2

Factor completely: $16-(k+3)^2$

Show answer

$(k+7)(1-k)$

Practice Makes Perfect

In the following exercises, factorize using the difference of squares

1. c^2-25	2. m^2-49
3. $b^2-\frac{36}{49}$	4. $x^2-\frac{9}{16}$
5. $64j^2-16$	6. $25k^2-36$
7. $81c^2-25$	8. $121k^2-16$
9. $169-q^2$	10. $121-b^2$
11. $16-36y^2$	12. $25-9x^2$
13. $49w^2-100x^2$	14. $81c^2-4d^2$
15. $p^2-\frac{16}{25}q^2$	16. $m^2-\frac{4}{9}n^2$
17. x^2y^2-81	18. a^2b^2-16
19. $r^2s^2-\frac{4}{49}$	20. $u^2v^2-\frac{9}{25}$
21. $7x^2-28y^2$	22. $4x^2m-36y^2m$
23. $2x^4r-72y^4r$	24. $7x^4-343y^4$
25. $25x^6-y^6$	26. $16m^4-n^4$
27. $16-(x-2)^2$	28. $9y^2-(y+9)^2$
29. $0.25a^2-0.16b^2$	30. $25-(n+3)^2$

Answers

1. $(c-5)(c+5)$	2. $(m-7)(m+7)$
3. $\left(b+\frac{6}{7}\right)\left(b-\frac{6}{7}\right)$	4. $\left(x+\frac{3}{4}\right)\left(x-\frac{3}{4}\right)$
5. $(8j+4)(8j-4)$	6. $(5k+6)(5k-6)$
7. $(9c+5)(9c-5)$	8. $(11k+4)(11k-4)$
9. $(13-q)(13+q)$	10. $(11-b)(11+b)$
11. $(4-6y)(4+6y)$	12. $(5-3x)(5+3x)$
13. $(7w+10x)(7w-10x)$	14. $(9c-2d)(9c+2d)$
15. $\left(p+\frac{4}{5}q\right)\left(p-\frac{4}{5}q\right)$	16. $\left(m+\frac{2}{3}n\right)\left(m-\frac{2}{3}n\right)$
17. $(xy-9)(xy+9)$	18. $(ab-4)(ab+4)$
19. $\left(rs-\frac{2}{7}\right)\left(rs+\frac{2}{7}\right)$	20. $\left(uv-\frac{3}{5}\right)\left(uv+\frac{3}{5}\right)$
21. $7(x+2y)(x-2y)$	22. $4m(x+3y)(x-3y)$
23. $2r(x^2+6y^2)(x^2-6y^2)$	24. $7(x^2+7y^2)(x^2-7y^2)$
25. $(5x^3+y^3)(5x^3-y^3)$	26. $(4m^2+n^2)(2m+n)(2m-n)$
27. $(x+2)(6-x)$	28. $(4y+9)(2y-9)$
29. $0.01(5a+4b)(5a-4b)$ OR $(0.5a+0.4b)(0.5a-0.4b)$	30. $(n+8)(2-n)$

6.8 Chapter Review

Review Exercises

Identify Polynomials, Monomials, Binomials and Trinomials

In the following exercises, determine if each of the following polynomials is a monomial, binomial, trinomial, or other polynomial.

1. a) $a^2 - b^2$ b) $24d^3$ c) $x^2 + 8x - 10$ d) $m^2n^2 - 2mn + 6$ e) $7y^3 + y^2 - 2y - 4$	2. a) $11c^4 - 23c^2 + 1$ b) $9p^3 + 6p^2 - p - 5$ c) $\frac{3}{7}x + \frac{5}{14}$ d) 10 e) $2y - 12$

Determine the Degree of Polynomials

In the following exercises, determine the degree of each polynomial.

3. a) $5p^3 - 8p^2 + 10p - 4$ b) $-20q^4$ c) $x^2 + 6x + 12$ d) $23r^2s^2 - 4rs + 5$ e) 100	4. a) $3x^2 + 9x + 10$ b) $14a^2bc$ c) $6y + 1$ d) $n^3 - 4n^2 + 2n - 8$ e) -19

Add and Subtract Monomials

In the following exercises, add or subtract the monomials.

5. $-14k + 19k$	6. $5\text{y}^3 + 8y^3$
7. $-9c - 18c$	8. $12q - (-6q)$
9. $3m^2 + 7n^2 - 3m^2$	10. $12\text{x} - 4y - 9x$
11. $13\text{a} + b$	12. $6x^2y - 4x + 8xy^2$

Add and Subtract Polynomials

In the following exercises, add or subtract the polynomials.

13. $\left(9p^2 - 5p + 3\right) + \left(4p^2 - 4\right)$	14. $\left(5x^2 + 12x + 1\right) + \left(6x^2 - 8x + 3\right)$
15. $\left(7y^2 - 8y\right) - (y - 4)$	16. $\left(10m^2 - 8m - 1\right) - \left(5m^2 + m - 2\right)$
17. Find the sum of $\left(a^2 + 6a + 9\right)$ and $\left(5a^3 - 7\right)$	18. Subtract $\left(3s^2 + 10\right)$ from $\left(15s^2 - 2s + 8\right)$

Evaluate a Polynomial for a Given Value of the Variable

In the following exercises, evaluate each polynomial for the given value.

19. Evaluate $10 - 12x$ when: a) $x = 3$ b) $x = 0$ c) $x = -1$	20. Evaluate $3y^2 - y + 1$ when: a) $y = 5$ b) $y = -1$ c) $y = 0$
21. A manufacturer of stereo sound speakers has found that the revenue received from selling the speakers at a cost of *p* dollars each is given by the polynomial $-4p^2 + 460p$. Find the revenue received when $p = 75$ dollars.	22. Randee drops a stone off the 200 foot high cliff into the ocean. The polynomial $-16t^2 + 200$ gives the height of a stone t seconds after it is dropped from the cliff. Find the height after $t = 3$ seconds.

Multiply Monomials

In the following exercises, multiply the monomials.

23. $\left(-9n^7\right)(-16n)$	24. $\left(-15x^2\right)\left(6x^4\right)$
25. $\left(\frac{5}{9}ab^2\right)\left(27ab^3\right)$	26. $\left(7p^5q^3\right)\left(8pq^9\right)$

Multiply a Polynomial by a Monomial

In the following exercises, multiply.

27. $-4(y + 13)$	28. $7(a + 9)$
29. $p(p + 3)$	30. $-5(r - 2)$
31. $-6u(2u + 7)$	32. $-m(m + 15)$
33. $3q^2\left(q^2 - 7q + 6\right)$	34. $9\left(b^2 + 6b + 8\right)$
35. $(b - 4) \cdot 11$	36. $(5z - 1)z$

Multiply a Binomial by a Binomial

In the following exercises, multiply the binomials using: a) the Distributive Property, b) the FOIL method, c) the Vertical Method.

37. $(6y-7)(2y-5)$	38. $(x-4)(x+10)$

In the following exercises, multiply the binomials. Use any method.

39. $(y-4)(y-8)$	40. $(x+3)(x+9)$
41. $(q+16)(q-3)$	42. $(p-7)(p+4)$
43. $(u^2+6)(u^2-5)$	44. $(5m-8)(12m+1)$
45. $(8mn+3)(2mn-1)$	46. $(9x-y)(6x-5)$

Multiply a Trinomial by a Binomial

In the following exercises, multiply using a) the Distributive Property, b) the Vertical Method.

47. $(3x-4)(6x^2+x-10)$	48. $(n+1)(n^2+5n-2)$

In the following exercises, multiply. Use either method.

49. $(7m+1)(m^2-10m-3)$	50. $(y-2)(y^2-8y+9)$

Square a Binomial Using the Binomial Squares Pattern

In the following exercises, square each binomial using the Binomial Squares Pattern.

51. $(q-15)^2$	52. $(c+11)^2$
53. $(8u+1)^2$	54. $\left(x+\frac{1}{3}\right)^2$
55. $(4a-3b)^2$	56. $(3n^3-2)^2$

Multiply Conjugates Using the Product of Conjugates Pattern

In the following exercises, multiply each pair of conjugates using the Product of Conjugates Pattern.

57. $\left(y+\frac{2}{5}\right)\left(y-\frac{2}{5}\right)$	58. $(s-7)(s+7)$
59. $(6-r)(6+r)$	60. $(12c+13)(12c-13)$
61. $\left(5p^4-4q^3\right)\left(5p^4+4q^3\right)$	62. $\left(u+\frac{3}{4}v\right)\left(u-\frac{3}{4}v\right)$

Recognize and Use the Appropriate Special Product Pattern

In the following exercises, find each product.

63. $(6a+11)(6a-11)$	64. $(3m+10)^2$
65. $\left(c^4+9d\right)^2$	66. $(5x+y)(x-5y)$
67. $\left(a^2+4b\right)\left(4a-b^2\right)$	68. $\left(p^5+q^5\right)\left(p^5-q^5\right)$

Divide a Polynomial by a Monomial

In the following exercises, divide each polynomial by the monomial.

69. $\left(35x^2-75x\right)\div 5x$	70. $\frac{42z^2-18z}{6}$
71. $\frac{550p^6-300p^4}{10p^3}$	72. $\frac{81n^4+105n^2}{-3}$
73. $\frac{96a^5b^2-48a^4b^3-56a^2b^4}{8ab^2}$	74. $\left(63xy^3+56x^2y^4\right)\div(7xy)$
75. $\frac{105y^5+50y^3-5y}{5y^3}$	76. $\frac{57m^2-12m+1}{-3m}$

Greatest Common Factor of two or more expressions

In the following exercises, find the greatest common factor.

77. $56, 24$	78. $3, 18$
79. $58r, 38r^2$	80. $54r^5m^5, 14rm^4$
81. $32m^7y^9, 31m^5y^2, 34m^8y^7$	82. $14t^3p^7, 27t^5p, 3t^2p^2$

Factor the Greatest Common Factor from a polynomial

In the following exercises, factor the greatest common factor from each polynomial.

83. $3z+9$	84. $5m+5$
85. $-26t-44$	86. $-12t-33$
87. $6t^3-12t^2+15t$	88. $5p^2-15p-5$
89. $6m(m+5)-7(m+5)$	90. $z(z-9)-8(z-9)$

Factor by Grouping

In the following exercises, factor by grouping.

91. $mn-4n+6m-24$	92. $at+10t-7a-70$
93. $xy-10x+8y-80$	94. $a^2-7a+5a-35$

Factor Trinomials of the Form x^2+bx+c

In the following exercises, factor each trinomial.

95. $-11-10y+y^2$	96. t^2+3t+8
97. $x^2-9x+42$	98. $w^2+4w+16$
99. $7s+s^2+6$	100. $5x+x^2-14$

Factor Trinomials of the Form $x^2+bxy+cy^2$

In the following exercises, factor each trinomial.

101. $m^2+6mn+5n^2$	102. $b^2+12by+27y^2$
103. $v^2-10vx-24x^2$	104. $x^2+10xb+16b^2$
105. $p^2-8pq-65q^2$	106. $t^2-3ts+2s^2$

Divide a Polynomial by a Monomial

In the following exercises, divide each polynomial by the monomial.

107. $\frac{6x^2-4}{2}$	108. $\frac{21y^2-7}{7}$
109. $\frac{-30x^4-6}{-6}$	110. $\frac{6c^5-6c^4}{6c^3}$
111. $(4y^3-10y^5)\div 2y^2$	112. $(14r^4y^3-28r^4y^4)\div 7r^3y^2$
113. $\frac{66x^3y^2-110x^2y^3-44x^4y^3}{11x^2y^2}$	114. $\frac{4w^2+2w-5}{2w}$

Factor Binomials of the Form $a^2x^2-b^2y^2$

In the following exercises, factorize using the difference of squares.

115. $x^2-\dfrac{9}{16}$	116. $25k^2-36$
117. $81c^2-4d^2$	118. $u^2v^2-\dfrac{9}{25}$
119. $9y^2-(y+9)^2$	120. $25-(n+3)^2$

Review Exercise Answers

1. a) binomial b) monomial c) trinomial d) trinomial e) other polynomial	3. a) 3 b) 4 c) 2 d) 4 e) 0	5. $5k$
7. $-27c$	9. $7n^2$	11. $13\text{a}+b$
13. $13p^2-5p-1$	15. $7y^2-9y+4$	17. $5a^3+a^2+6a+2$
19. a) -26 b) 10 c) 22	21. 12,000	23. $144n^8$
25. $15a^2b^5$	27. $-4y-52$	29. p^2+3p
31. $-12u^2-42u$	33. $3q^4-21q^3+18q^2$	35. $11b-44$
37. a) $12y^2-44y+35$ b) $12y^2-44y+35$ c) $12y^2-44y+35$	39. $y^2-12y+32$	41. $q^2+13q-48$
43. u^4+u^2-30	45. $16m^2n^2-2mn-3$	47. a) $18x^3-21x^2-34x+40$ b) $18x^3-21x^2-34x+40$
49. $7m^3-69m^2-31m-3$	51. $q^2-30q+225$	53. $64u^2+16u+1$
55. $16a^2-24ab+9b^2$	57. $y^2-\frac{4}{25}$	59. $36-r^2$
61. $25p^8-16q^6$	63. $36a^2-121$	65. $c^8+18c^4d+81d^2$
67. $4a^3+3a^2b-4b^3$	69. $7x-15$	71. $55p^3-30p$
73. $12a^4-6a^3b-7ab^2$	75. $21y^2+10-\frac{1}{y^2}$	77. 8
79. $2r$	81. m^5y^2	83. $3(z+3)$
85. $-2(13t+22)$	87. $3t(2t^2-4t+5)$	89. $(6m-7)(m+5)$
91. $(n+6)(m-4)$	93. $(x+8)(y-10)$	95. $(y-11)(y+1)$
97. Prime or not factorable.	99. $(s+6)(s+1)$	101. $(m+5n)(m+n)$
103. $(v+2x)(v-12x)$	105. $(p-13q)(p+5q)$	107. $3x^2-2$
109. $5x^4+1$	111. $2y-5y^3$	113. $6x-10y-4x^2y$
115. $(x+\frac{3}{4})(x-\frac{3}{4})$	117. $(9c+2d)(9c-2d)$	119. $(4y+9)(2y-9)$

Chapter Practice Test

In the following exercises, simplify each expression. 1. $\left(12a^2-7a+4\right)+\left(3a^2+8a-10\right)$	2. For the polynomial $10x^4+9y^2-1$ a) Is it a monomial, binomial, or trinomial? b) What is its degree?
3. $\left(9p^2-5p+1\right)-\left(2p^2-6\right)$	4. $\left(-9r^4s^5\right)\left(4rs^7\right)$
5. $(v-9)(9v-5)$	6. $(m+6)(m+12)$
7. $(n-6)\left(n^2-5n+4\right)$	8. $(4c-11)(3c-8)$
9. $(7p-5)(7p+5)$	10. $(2x-15y)(5x+7y)$
11. $(9v-2)^2$	12. $\frac{12x^3+42x^2-6x}{2x}$
13. $\frac{64x^3-x}{4x}$	14. $\frac{70xy^4+95x^3y}{5xy}$
15. $\frac{y^2-5y-18}{y}$	16. A helicopter flying at an altitude of 1000 feet drops a rescue package. The polynomial $-16t^2+1000$ gives the height of the package t seconds after it was dropped. Find the height when $t=6$ seconds.
17. Divide: $\frac{51z^4+42z^2}{-3z^2}$	18. Divide: $\left(42a^4b^3-28a^4b^4\right)\div 7a^3b^3$
19. Multiply: $(2x+y)^2$	20. Factor: $8y^2(3y-8)-7(3y-8)$
21. Factor: $25x^3+5x^2+30x+6$	22. Factor: $m^2-29mn-96n^2$
23. Factor: $x^2-9x+18$	24. Factor: $0.36x^2-0.49y^2$

Practice Test Answers

1. $15a^2 + a - 6$	2. a) Trinomial, b) 4	3. $7p^2 - 5p + 7$
4. $-36r^5s^{12}$	5. $9v^2 - 86v + 45$	6. $m^2 + 18m + 72$
7. $n^3 - 11n^2 + 34n - 24$	8. $12c^2 - 65c + 88$	9. $49p^2 - 25$
10. $10x^2 - 61xy - 105y^2$	11. $81v^2 - 36v + 4$	12. $6x^2 + 21x - 3$
13. $16x^2 - \frac{1}{4}$	14. $14y^3 + 19x^2$	15. $y - 5 - \frac{18}{y}$
16. 424 feet	17. $-17z^2 - 14$	18. $6a - 4ab$
19. $4x^2 + 4xy + y^2$	20. $(8y^2 - 7)(3y - 8)$	21. $(5x^2 + 6)(5x + 1)$
22. $(m - 32n)(m + 3n)$	23. $(x - 3)(x - 6)$	24. $0.01(6x + 7y)(6x - 7y)\text{OR}(0.6x + 0.7y)(0.6x - 0.7y)$