physics0312chooge

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Contents

Part XVII. Chapter 24 Electromagnetic Waves

Preface to College Physics

About OpenStax

OpenStax is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of modern college courses. Unlike traditional textbooks, OpenStax resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax is working to improve access to higher education for all. OpenStax is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations.

About This Book

Welcome to *College Physics*, an OpenStax resource created with several goals in mind: accessibility, affordability, customization, and student engagement—all while encouraging learners toward high levels of learning. Instructors and students alike will find that this textbook offers a strong foundation in introductory physics, with algebra as a prerequisite. It is available for free online and in low-cost print and e-book editions.

To broaden access and encourage community curation, College Physics is "open source" licensed under a Creative Commons Attribution (CC-BY) license. Everyone is invited to submit examples, emerging research, and other feedback to enhance and strengthen the material and keep it current and relevant for today's students. You can make suggestions by contacting us at info@openstaxcollege.org.

To the Student

This book is written for you. It is based on the teaching and research experience of numerous physicists and influenced by a strong recollection of their own struggles as students. After reading this book, we hope you see that physics is visible everywhere. Applications range from driving a car to launching a rocket, from a skater whirling on ice to a neutron star spinning in space, and from taking your temperature to taking a chest X-ray.

To the Instructor

This text is intended for one-year introductory courses requiring algebra and some trigonometry, but no calculus. OpenStax provides the essential supplemental resources at http://openstaxcollege.org ; however, we have pared down the number of supplements to keep costs low. College Physics can be easily customized for your course using Connexions (http://cnx.org/content/col11406). Simply select the content most relevant to your curriculum and create a textbook that speaks directly to the needs of your class.

General Approach

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Organization, Level, and Content

There is considerable latitude on the part of the instructor regarding the use, organization, level, and content of this book. By choosing the types of problems assigned, the instructor can determine the level of sophistication required of the student.

Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

Modern Perspective

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

Supplements

Accompanying the main text are a Student Solutions Manual and an Instructor Solutions Manual. The Student Solutions Manual provides worked-out solutions to select end-of-module Problems and Exercises. The Instructor Solutions Manual provides worked-out solutions to all Exercises.

Features of OpenStax *College Physics*

The following briefly describes the special features of this text.

Modularity

This textbook is organized on Connexions (http://cnx.org) as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado (http://phet.colorado.edu). There they can further explore the physics concepts they have learned about in the module.

Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Glossary

At the end of every module or chapter is a glossary containing definitions of all of the key terms in the module or chapter.

End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online, every other problem includes an answer that students can reveal immediately by clicking on a "Show Solution" button. Fully worked solutions to select problems are available in the Student Solutions Manual and the Teacher Solutions Manual.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

Integrated Concept Problems

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer. Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

Appendices

Appendix A: Atomic Masses

Appendix B: Selected Radioactive Isotopes

Appendix C: Useful Information

Appendix D: Glossary of Key Symbols and Notation

Acknowledgements

This text is based on the work completed by Dr. Paul Peter Urone in collaboration with Roger Hinrichs, Kim Dirks, and Manjula Sharma. We would like to thank the authors as well as the numerous professors (a partial list follows) who have contributed their time and energy to review and provide feedback on the manuscript. Their input has been critical in maintaining the pedagogical integrity and accuracy of the text.

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Webassign is an independent online homework and assessment system that has been available commercially since 1998. WebAssign has recently begun to support the Open Education Resource community by creating a high quality online homework solution for selected open-source textbooks, available at an affordable price to students. These question collections include randomized values and variables, immediate feedback, links to the open-source textbook, and a variety of text-specific resources and tools; as well as the same level of rigorous coding and accuracy-checking as any commercially available online homework solution supporting traditionally available textbooks.

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PART 1

Chapter 1 Introduction: The Nature of Science and Phyiscs

Figure 1. Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics)

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

1.1 Physics: An Introduction

Summary

- Explain the difference between a principle and a law.
- Explain the difference between a model and theory.

Figure 1. The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett).

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone (Figure 2). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

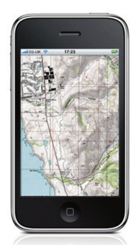


Figure 2. The Apple "iPhone" is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images).

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See Figure 3 and Figure 4.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (Figure 5 and Figure 6). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

Figure 3. The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz).

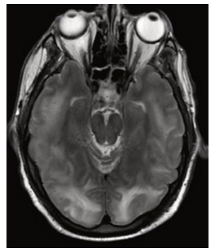


Figure 4. These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik).

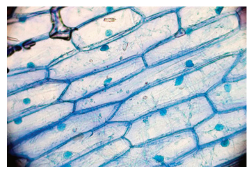


Figure 5. Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin).

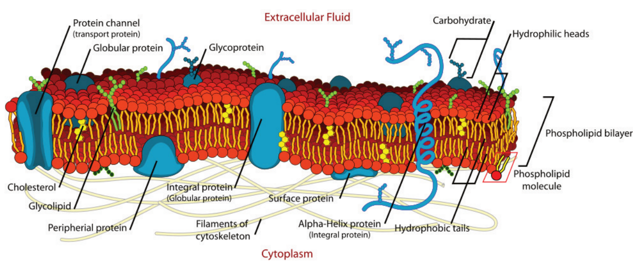


Figure 6. An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz).

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See Figure 7 and Figure 8.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

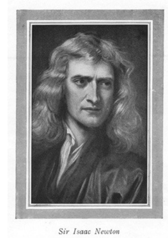


Figure 7. Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: Britain's Heritage of Science. London, 1917.).

Figure 8. Marie Curie (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons).

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See Figure 9.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $F = ma$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

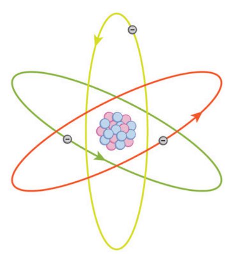


Figure 9. What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

MODELS, THEORIES, AND LAWS

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unob-*

served. These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

THE SCIENTIFIC METHOD

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See Figure 10, Figure 11, and Figure 12.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

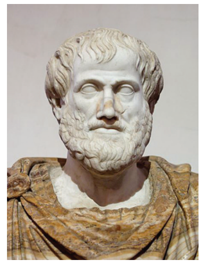


Figure 10. Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher Aristotle (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection).

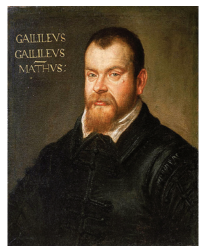


Figure 11. Galileo Galilei (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto).

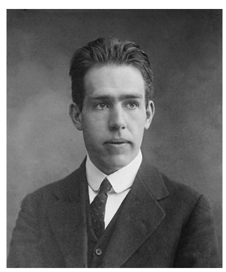


Figure 12. Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division).

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

LIMITS ON THE LAWS OF CLASSICAL PHYSICS

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

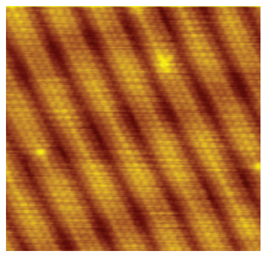


Figure 13. Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen).

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

Check Your Understanding

1: A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

PHET EXPLORATIONS: EQUATION GRAPHER

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

Figure 14. Equation Grapher.

Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

Conceptual Questions

1: Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

2: How does a model differ from a theory?

3: If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

4: What determines the validity of a theory?

5: Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

6: Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

7: Classical physics is a good approximation to modern physics under certain circumstances. What are they?

8: When is it *necessary* to use relativistic quantum mechanics?

9: Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

Glossary

classical physics
physics that was developed from the Renaissance to the end of the 19th century
physics
the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon
model
representation of something that is often to difficult (or impossible) to display directly
theory
an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers
law
a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated examples
scientific method
a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion
modern physics
the study of relativity, quantum mechanics, or both
relativity
the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field
quantum mechanics
the study of objects smaller than can be seen with a microscope

Solutions

Check Your Understanding

1: Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

1.2 Physical Quantities and Units

Summary

- Perform unit conversions both in the SI and English units
- Explain the most common prefixes in the SI units and be able to write them in scientific notation

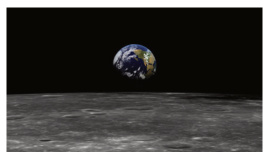


Figure 1. The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA).

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See Figure 2.)

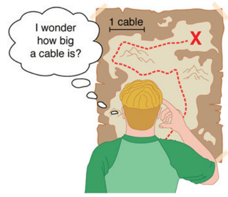


Figure 2. Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym "SI" is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

Table 1. Fundamental SI Units.

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the second(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See Figure 3.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

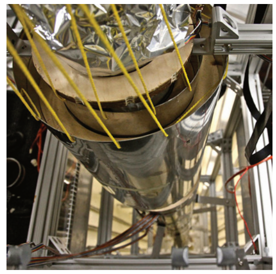


Figure 3. An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr).

The Meter

The SI unit for length is the meter (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See Figure 4.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

The Kilogram

The SI unit for mass is the kilogram (abbreviated kg); it is defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass.

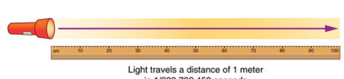


Figure 4. The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

Electric current and its accompanying unit, the ampere, will be introduced in Chapter 20 Introduction to Electric Current, Resistance, and Ohm's Law when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. Table 2 gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, $10^1, 10^2, 10^3$, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9} m, while the diameter of the Sun is on the order of 10^9 m.

THE QUEST FOR MICROSCOPIC STANDARDS FOR BASIC UNITS

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Prefix	Symbol	Value[1]	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	10^{18} m	distar
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 m
tera	T	10^{12}	terawatt	TW	10^{12} W	powe
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a mic
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high
kilo	k	10^{3}	kilometer	km	10^{3} m	about
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 ga
deka	da	10^{1}	dekagram	dag	10^{1} g	teasp
—	—	10^{0} (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less t
centi	c	10^{-2}	centimeter	cm	10^{-2} m	finge
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea a
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size c
atto	a	10^{-18}	attosecond	as	10^{-18} s	time

Table 2. Metric Prefixes for Powers of 10 and their Symbols.

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in Table 3. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See Figure 5 and Figure 6.)

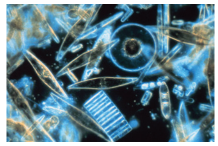


Figure 5. Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections).

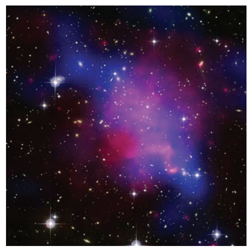


Figure 6. Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.).

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\ \cancel{\text{m}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} = 0.080\ \text{km}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click Appendix C Useful Information for a more complete list of conversion factors.

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron (9.11×10^{-31} kg)	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom (1.67×10^{-27} kg)	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day (8.64×10^{4} s)
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) (3.16×10^{7} s)
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon (7.35×10^{22} kg)	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth (5.97×10^{24} kg)	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun (1.99×10^{30} kg)		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Table 3. Approximate Values of Length, Mass, and Time.

Example 1: Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\,\frac{\text{km}}{\text{min}}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,

$$\text{average speed} = 0.500\,\frac{\text{km}}{\text{min}} \times \frac{60\text{ min}}{1\text{ h}} = 30.0\,\frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\,\frac{\text{km}\cdot\text{hr}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer $30.0\,\frac{\text{km}}{\text{hr}}$ does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\,\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3{,}600\text{ s}} \times \frac{1{,}000\text{ m}}{1\text{ km}},$$

$$\text{Average speed} = 8.33\,\frac{\text{m}}{\text{s}}.$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Chapter 1.3 Accuracy, Precision, and Significant Figures will help you answer these questions.

NONSTANDARD UNITS

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different "weights and measures." Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding 1

1: Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Check Your Understanding 2

1: One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Summary

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the

second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.

- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

1: Identify some advantages of metric units.

Problems & Exercises

1: The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

2: A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

3: Show that $1.0 \text{ m/s} = 3.6 \text{ km/h}$. Hint: Show the explicit steps involved in converting $1.0 \text{ m/s} = 3.6 \text{ km/h}$.

4: American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

5: Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

6: What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

7: Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

8: The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

9: Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

10: (a) Refer to Table 3 to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

Glossary

physical quantity
a characteristic or property of an object that can be measure or calculated from other measurements

units
a standard used for expressing and comparing measurements
SI units
the international system of units that scientist in most countries have agreed to use; includes units such as meters, liters, and grams
English units
system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds
fundamental units
units that can only be expressed relative to the procedure used to measure them
derived units
units that can be calculated using algebraic combinations of the fundamental units
second
the SI unit for time, abbreviated (s)
meter
the SI unit for length, abbreviated (m)
kilogram
the SI unit for mass, abbreviated (kg)
metric system
a system in which values can be calculated in factors of 10
order of magnitude
refers to the size of a quantity as it related to a power of 10
conversion factor
a ratio expression how many of one unit are equal to another unit

Solutions

Check Your Understanding 1

1: The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding 2

1: The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

Problems & Exercises

1:

1. 27.8 m/s
2. 62.1 mph

3:

$$\frac{1.0\text{ m}}{\text{s}} = \frac{1.0\text{ m}}{\text{s}} \times \frac{3600\text{ s}}{1\text{ hr}} \times \frac{1\text{ km}}{1000\text{ m}}$$

$= 3.6\ \mathrm{km/h}.$

5:

length: $377\ \mathrm{ft};\ 4.53 \times 10^3\ \mathrm{in}.$ width: $280\ \mathrm{ft};\ 3.3 \times 10^3\ \mathrm{in}.$

7:

$8.847\ \mathrm{km}$

9:

(a) $1.3 \times 10^{-9}\ \mathrm{m}$

(b) $40\ \mathrm{km/My}$

1.3 Accuracy, Precision, and Significant Figures

Summary

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

Figure 1. A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki).

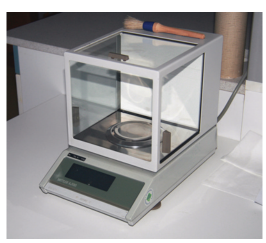


Figure 2. Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec).

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system is refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 3, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 4, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

Figure 3. A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil).

Figure 4. In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil).

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA (“delta A”), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{ in.} \pm 0.2$.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

MAKING CONNECTIONS: REAL-WORLD CONNECTIONS – FEVER OR CHILLS?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check his or her temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C? If the child’s temperature reading was $37.0°C$ (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic $34.0°C$ to a dangerously high $40.0°C$. A thermometer with an uncertainty of $3.0°C$ would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the **percent uncertainty** (%unc) is defined to be:

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%$$

Example 1: Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.8\ \text{lb}$
- Week 2 weight: $5.3\ \text{lb}$
- Week 3 weight: $4.9\ \text{lb}$
- Week 4 weight: $5.4\ \text{lb}$

You determine that the weight of the 5-lb bag has an uncertainty of $\pm 0.4\ \text{lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%$$

Solution

Plug the known values into the equation:

$$\%\ \text{unc} = \frac{0.4\ \text{lb}}{5\ \text{lb}} \times 100\% = 8\%$$

Discussion

We can conclude that the weight of the apple bag is $5\ \text{lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3%. (Expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)

Check Your Understanding 1

1: A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of $\pm 0.05\text{ s}$. Runners on the track coach’s team regularly clock 100-m sprints of 11.49 s to 15.01 s. At the school’s last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s. Will the coach’s new stopwatch be helpful in timing the sprint team? Why or why not?

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be **36.7 cm**. You could not express this value as **36.71 cm** because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between **36.6 cm** and **36.7 cm**, and he or she must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value **36.7 cm** has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

Check Your Understanding 2

2: Determine the number of significant figures in the following measurements:

a: 0.0009

b: 15,450.0

c: 6×10^3

d: 87.990

e: 30.42

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\ \text{m}$. Then,

$$A = \pi r^2 = (3.1415927\ldots) \times (1.2\ \text{m})^2 = 4.5238934\ \text{m}^2$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A = 4.5\ \text{m}^2,$$

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement*. Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision

0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{r} 7.56 \text{ kg} \\ -6.052 \text{ kg} \\ +\ \ 13.7 \text{ kg} \\ \hline 15.208 \text{ kg} = 15.2 \text{ kg} \end{array}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Check Your Understanding 3

1: Perform the following calculations and express your answer using the correct number of significant digits.

(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?

(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of 0.0255 m/s^2, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

PHET EXPLORATION: ESTIMATION

Explore size estimation in one, two, and three dimensions! Multiple levels of difficulty allow for progressive skill improvement.

Figure 5. Estimation.

Summary

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

Conceptual Questions

1: What is the relationship between the accuracy and uncertainty of a measurement?

2: Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

Express your answer to problems in this section to the correct number of significant figures and proper units.

1: Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

2: A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

3: (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads 90 km/h? (b) Convert this range to miles per hour. $(1\text{ km} = 0.6214\text{ mi})$

4: An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

5: (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

6: A can contains 375 mL of soda. How much is left after 308 mL is removed?

7: State how many significant figures are proper in the results of the following calculations: (a)$(106.7)(98.2)\backslash(46.210)(1.01)$ (b)$(18.7)^2$(c)$(1.60 \times 10^{-19})(3712)$.

8: (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

9: (a) If your speedometer has an uncertainty of$2.0\ \text{km/h}$at a speed of$90\ \text{km/h}$, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads$60\ \text{km/h}$, what is the range of speeds you could be going?

10: (a) A person's blood pressure is measured to be$120 \pm 2\ \text{mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of$80\ \text{mm Hg}$?

11: A person measures his or her heart rate by counting the number of beats in$30\ \text{s}$. If40 ± 1beats are counted in $30.0 \pm 0.5\ \text{s}$, what is the heart rate and its uncertainty in beats per minute?

12: What is the area of a circle$3.102\ \text{cm}$in diameter?

13: If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

14: A marathon runner completes a42.188-kmcourse in$2\ \text{h}$, 30 min, and$12\ \text{s}$. There is an uncertainty of$25\ \text{m}$ in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

15: The sides of a small rectangular box are measured to be$1.80 \pm 0.01\ \text{cm}$,$2.05 \pm 0.0\ 2\text{cm}$, and$3.1 \pm 0.1\ \text{cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

16: When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where$1\ \text{lbm} = 0.4539\ \text{kg}$. (a) If there is an uncertainty of$0.0001\ \text{kg}$in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

17: The length and width of a rectangular room are measured to be$3.955 \pm 0.005\ \text{m}and3.050 \pm 0.005\ \text{m}$. Calculate the area of the room and its uncertainty in square meters.

18: A car engine moves a piston with a circular cross section of$7.500 \pm 0.002\ \text{cm}$ diameter a distance of$3.250 \pm 0.001\ \text{cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

Glossary

accuracy
the degree to which a measure value agrees with the correct value for that measurement
method of adding percents
the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation
percent uncertainty
the ratio of the uncertainty of a measurement to the measure value, express as a percentage
precision
the degree to which repeated measurements agree with each other

significant figures

express the precision of a measuring tool used to measure a value

uncertainty

a quantitative measure of how much your measured values deviate from a standard or expected value

Solutions

Check Your Understanding 1

1: No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Check Your Understanding 2

1: (a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

Check Your Understanding 3

1: (a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

Problems & Exercises

1:

2 kg

3:

1. $85.5 \text{ to } 94.5 \text{ km/h}$
2. $53.1 \text{ to } 58.7 \text{ mi/h}$

5:

(a) $7.6 \times 10^6 \text{ beats}$

(b) $7.57 \times 10^7 \text{ beats}$

(c) $7.57 \times 10^7 \text{ beats}$

7:

1. 3
2. 3
3. 3

9:

(a) 2.2%

(b) 59 to 61 km/h

11:

80 ± 3 beats/min

13:

2.8 h

15:

$11.1 \pm 1\ m^3$

17:

$12.06 \pm 0.04\ m^2$

1.4 Approximation

Summary

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or “guesstimates” for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

Example 1: Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

$$\frac{2\text{ m}}{1\text{ person}} \times \frac{2\text{ person}}{1\text{ story}} \times 39\text{ stories} = 156\text{ m}$$

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

Example 2: Approximating Vast Numbers: a Trillion Dollars

Figure 1. A bank stack contains one-hundred \$100 bills, and is worth \$10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill).

The U.S. federal deficit in the 2008 fiscal year was a little greater than \$10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in \$100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped \$100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\begin{aligned}\text{volume of stack} &= \text{length} \times \text{width} \times \text{height},\\ \text{volume of stack} &= 6\text{ in.} \times 3\text{ in.} \times 0.5\text{ in.},\\ \text{volume of stack} &= 9\text{ in.}^3.\end{aligned}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $\$100$ bills is equal to $\$10,000$, or $\$1 \times 10^4$. The number of stacks you will have is:

$$\$1 \times 10^{12}(\text{a trillion dollars})/\$1 \times 10^4\text{ per stack} = 1 \times 10^8\text{ stacks.}$$

(3) Calculate the area of a football field in square inches. The area of a football field is $100\text{ yd} \times 50\text{ yd},$ which gives 5000 yd^2 Because we are working in inches, we need to convert square yards to square inches:

$$\text{Area} = 5000\text{ yd}^2 \times \frac{3\text{ ft}}{1\text{ yd}} \times \frac{3\text{ ft}}{1\text{ yd}} \times \frac{12\text{ in.}}{1\text{ ft.}} \times \frac{12\text{ in.}}{1\text{ ft.}} = 6,480,000\text{ in.}^2$$

$$\text{Area} \approx 6 \times 10^6\text{ in.}^2$$

This conversion gives us $6 \times 10^6\text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $\$100\text{-bill}$ stacks is $9\text{ in.}^3/\text{stack} \times 10^8\text{ stacks} = 9 \times 10^8\text{ in.}^3$.

(5) Calculate the height. To determine the height of the bills, use the equation:

$$\begin{aligned}
\text{volume of bills} &= \text{area of field} \times \text{height of money:} \\
\text{Height of money} &= \frac{\text{volume of bills}}{\text{area of field}}, \\
\text{Height of money} &= \frac{9\times 10^{8}\ \text{in.}^3}{6\times 10^{6}\ \text{in.}^2} = 1.33 \times 10^{2}\ \text{in.}, \\
\text{Height of money} &\approx 1 \times 10^{2}\ \text{in.} = 100\ \text{in.}
\end{aligned}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100\ \text{in.} \times \frac{1\ \text{ft}}{12\ \text{in.}} = 8.33\ \text{ft} \approx 8\ \text{ft}.$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?

Check Your Understanding

1: Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Summary

Scientists often approximate the values of quantities to perform calculations and analyze system.

Problems & Exercises

1: How many heartbeats are there in a lifetime?

2: A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

3: How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of $10^{-22}\ \text{s}$.)

4: Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of $10^{-27}\ \text{kg}$ and the mass of a bacterium is on the order of $10^{-15}\ \text{kg}$.)

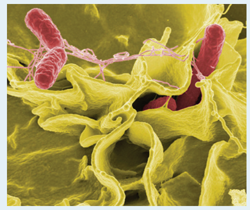


Figure 2. This color-enhanced photo shows Salmonella typhimurium (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH).

5: Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

6: (a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

7: (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

8: Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

Glossary

approximation

an estimated value based on prior experience and reasoning

Solutions

Check Your Understanding

1: An average male is about two meters tall. It would take approximately 15 men laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of $420\ m^2$.

Problems & Exercises

1:

Sample answer: 2×10^9 heartbeats

3:

Sample answer: 2×10^{31} if an average human lifetime is taken to be about 70 years.

5:

Sample answer: 50 atoms

7:

Sample answers:

(a) 10^{12} cells/hummingbird

(b) 10^{16} cells/human

PART 2

Chapter 2 One-Dimensional Kinematics

Figure 1. The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons).

Introduction to One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering* *its causes*. The word "kinematics" comes from a Greek term meaning motion and is related to other English words such as "cinema" (movies) and "kinesiology" (the study of human motion). In one-dimensional kinematics and Chapter 3 Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Chapter 3 Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

2.1 Displacement

Summary

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position and the path between the two.

Figure 1. These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia).

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See Figure 2.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See Figure 3.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

DISPLACEMENT

Displacement is the *change in position* of an object:

$$\Delta x = x_f - x_0$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final x_f.

Note that the SI unit for displacement is the meter (m) (see Chapter 1.2 Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

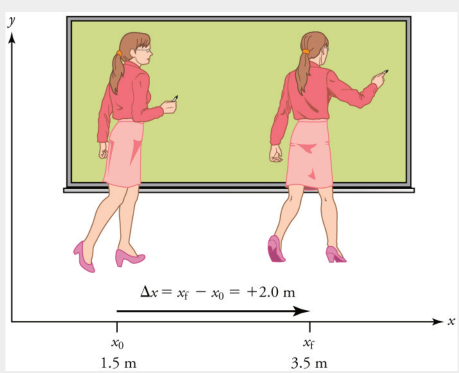


Figure 2. A professor paces left and right while lecturing. Her position relative to Earth is given by ***x***. The **+2.0 m** displacement of the professor relative to Earth is represented by an arrow pointing to the right.

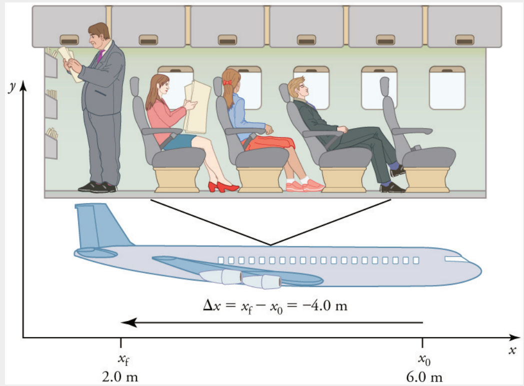


Figure 3. A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by ***x***. The **-4.0-m** displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in Figure 2.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5 \text{ m}$ and her final position is $x_f = 3.5 \text{ m}$. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5 \text{ m} - 1.5 \text{ m} = +2.0 \text{ m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0 \text{ m}$ and his final position is $x_f = 2.0 \text{ m}$, so his displacement is

$$\Delta x = x_f - x_0 = 2.0 \text{ m} - 6.0 \text{ m} = -4.0 \text{ m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

MISCONCEPTION ALERT: DISTANCE TRAVELED VS. MAGNITUDE OF DISPLACEMENT

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding 1

1: A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be

$$\Delta x = x_f - x_0$$

  where x_0 is the initial position and x_f is the final position. In this text, the Greek letter Δ (delta) always means "change in" what ever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

Conceptual Questions

1: Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2: Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3: Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \text{um/s}\ (50 \times 10^{-6}\ \text{m/s})$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

Problems & Exercises

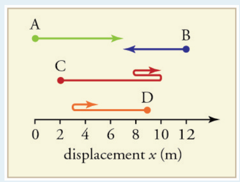


Figure 4.

1: Find the following for path A in Figure 4: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2: Find the following for path B in Figure 4: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3: Find the following for path C in Figure 4: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4: Find the following for path D in Figure 4: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Glossary

kinematics
the study of motion without considering its causes
position
the location of an object at a particular time
displacement
the change in position of an object
distance
the magnitude of displacement between two positions
distance traveled
the total length of the path traveled between two positions

Solutions

Check Your Understanding 1

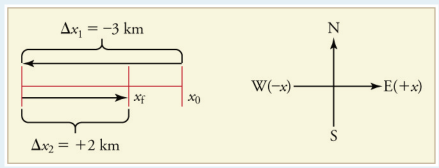


Figure 5.

1: (a) The rider's displacement is $\Delta x = x_f - x_0 = -1 \text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3 \text{ km} + 2 \text{ km} = 5 \text{ km}$.

(c) The magnitude of the displacement is 1 km.

Problems & Exercises

1:

(a) 7 m

(b) 7 m

(c) $+7 \text{ m}$

3:

(a) 13 m

(b) 9 m

(c) $+9 \text{ m}$

2.2 Vectors, Scalars, and Coordinate Systems

Summary

- Define and distinguished between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

Figure 1. The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the ***x***-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr).

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus (+) or minus (−) sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a $20^{\circ}C$ temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a $-20^{\circ}C$ temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure 1, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

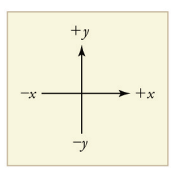


Figure 2. It is usually convenient to consider motion upward or to the right as positive **(+)** and motion downward or to the left as negative **(−)**.

Check Your Understanding

1: A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Conceptual Questions

1: A student writes, "*A bird that is diving for prey has a speed of* $-10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

2: What is the speed of the bird in Question 1?

3: Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

4: A weather forecast states that the temperature is predicted to be $-5°C$ the following day. Is this temperature a vector or a scalar quantity? Explain.

Glossary

scalar
a quantity that is described by magnitude, but not direction
vector
a quantity that is described by both magnitude and direction

Solutions

Check Your Understanding

1: Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

2.3 Time, Velocity, and Speed

Summary

- Explain the relationship between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial position, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

Figure 1. The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr).

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in Chapter 1.2 Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

$$\Delta t = t_f - t_0,$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_f \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol t is used for elapsed time unless otherwise specified $(\Delta x = t_f \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

AVERAGE VELOCITY

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}.$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\text{ m}}{5\text{ s}} = -0.8\ m/s$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

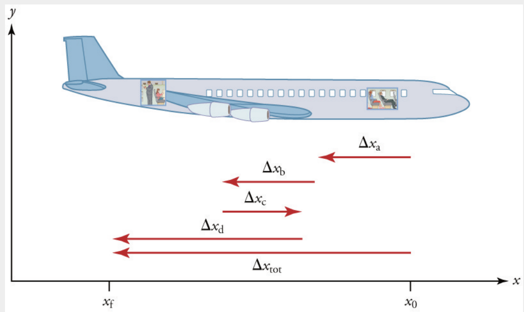


Figure 2. A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity or the velocity at a specific instant.* A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

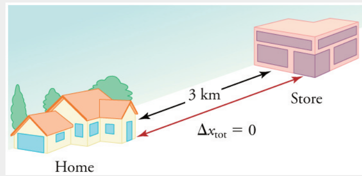


Figure 3. During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 4. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

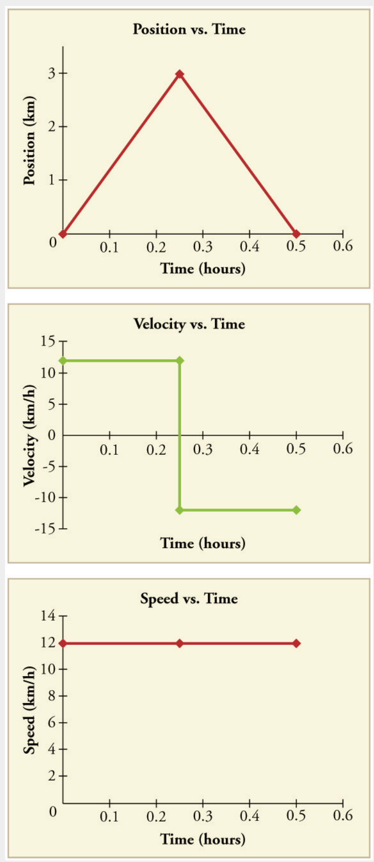


Figure 4. Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION — GETTING A SENSE OF SPEED

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

1: A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

  $$\Delta t = t_f - t_0$$

  where t_f is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Conceptual Questions

1: Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

2: There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

3: Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

4: If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

5: How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

Problems & Exercises

1: (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

2: A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

3: The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

4: Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

5: On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

6: Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by 3.84×10^6 m (1%)?

7: A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

8: The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

9: Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00 \times 10^8 \text{ m/s})$.

10: A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

11: The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06 \times 10^{-10} \text{ m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6 \text{ m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

Glossary

average speed
distance traveled divided by time during which motion occurs
average velocity
displacement divided by time over which displacement occurs
instantaneous velocity
velocity at a specific instant, or the average velocity over an infinitesimal time interval
instantaneous speed
magnitude of the instantaneous velocity
time
change, or the interval over which change occurs
model
simplified description that contains only those elements necessary to describe the physics of a physical situation
elapsed time
the difference between the ending time and beginning time

Solutions

Check Your Understanding

1: (a) The average velocity of the train is zero because $x_f = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}}$$

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

Problems & Exercises

1:

(a) $3.0 \times 10^4 \text{ m/s}$

(b) 0 m/s

3:

$2 \times 10^7 \text{ years}$

5:

$34.689 \text{ m/s} = 124.88 \text{ km/h}$

7:

(a) 40.0 km/h

(b) 34.3 km/h, 25^o S of E.

(c) $\text{average speed} = 3.20 \text{ km/h}, \bar{v} = 0$.

9:

384,000 km

11:

(a) $6.61 \times 10^{15} \text{ rev/s}$

(b) 0 m/s

2.4 Acceleration

Summary

- Define and distinguish instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, and final velocity.

Figure 1. A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr).

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

AVERAGE ACCELERATION

Average Acceleration is *the rate at which velocity changes,*

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0},$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

ACCELERATION AS A VECTOR

Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

Figure 2. A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)Misconception Alert: Deceleration vs. Negative Acceleration.

MISCONCEPTION ALERT: DECELERATION VS. NEGATIVE ACCELERATION

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider Figure 3.

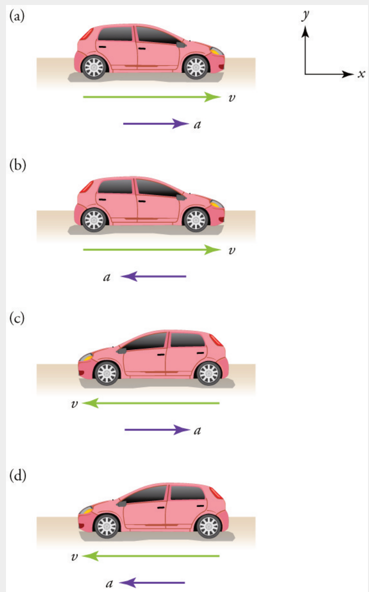


Figure 3. (a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (not decelerating).

Example 1: Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 4. (credit: Jon Sullivan, PD Photo.org).

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

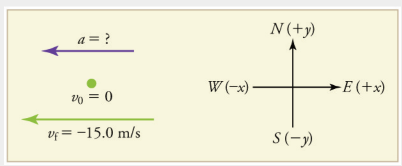


Figure 5.

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2.$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in Chapter 2.3 Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 6 shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 6(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In Figure 6(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0 \text{ m/s}^2$ and -2.0 m/s^2, respectively.

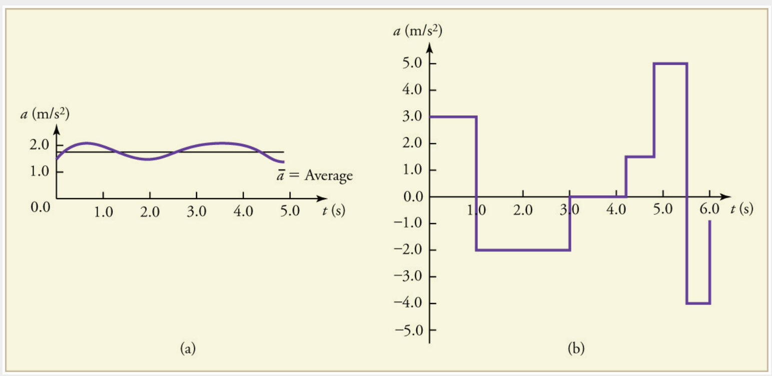


Figure 6. Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in Figure 7. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

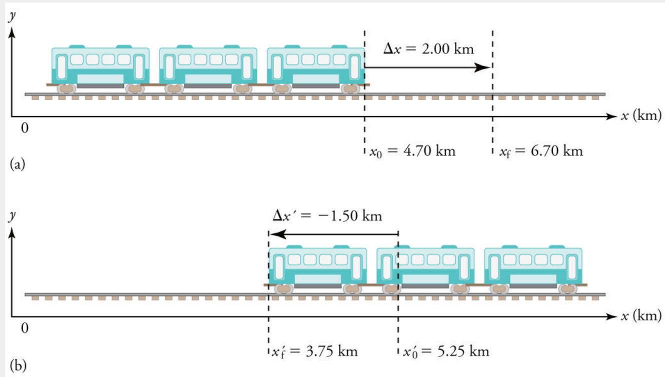


Figure 7. One-dimensional motion of a subway train considered in Example 2, Example 3, Example 4, Example 5, Example 6, and Example 7. Here we have chosen the x-axis so that + means to the right and − means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_f. Its displacement $\Delta x'$ is **-1.5 km**. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.).

Example 2: Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 7?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70\text{ km}$ and $x_0 = 4.70\text{ km}$ for part (a), and $x'_f = 3.75\text{ km}$ and $x'_0 = 5.25\text{ km}$ for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_f - x_0 = 6.70\text{ km} - 4.70\text{ km} = +2.00\text{ km}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_f - x'_0 = 3.75\text{ km} - 5.25\text{ km} = -1.50\text{ km}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 3: Calculating Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure 7?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2. Distance traveled is the total length of the path traveled between the two positions. (See Chapter 2.1 Displacement.) In the case of the subway train shown in Figure 7, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was -1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 4: Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure 7(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

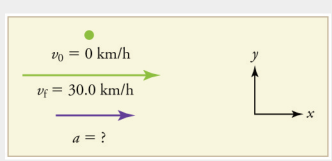


Figure 8.

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_f = 30.0 \text{ km/h}$, and $\Delta t = 20.0 \text{ s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0 \text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 \text{ km/h}}{20.0 \text{ s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Chapter 1.2 Physical Quantities and Units for more guidance.)

$$\bar{a} = \left(\frac{+30.0 \text{ km/h}}{20.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 0.417 \text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example 5: Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in Figure 7(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

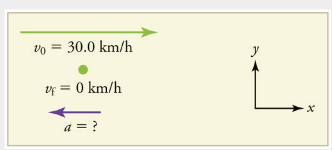


Figure 9.

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0 \text{ km/h}$, $v_f = 0 \text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00 \text{ s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0 \text{ km/h} = -30.0 \text{ km/h}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0 \text{ km/h}}{8.00 \text{ s}}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0 \text{ km/h}}{8.00 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = -1.04 \text{ m/s}^2.$$

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in Example 4 and Example 5 are displayed in Figure 10**.** (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

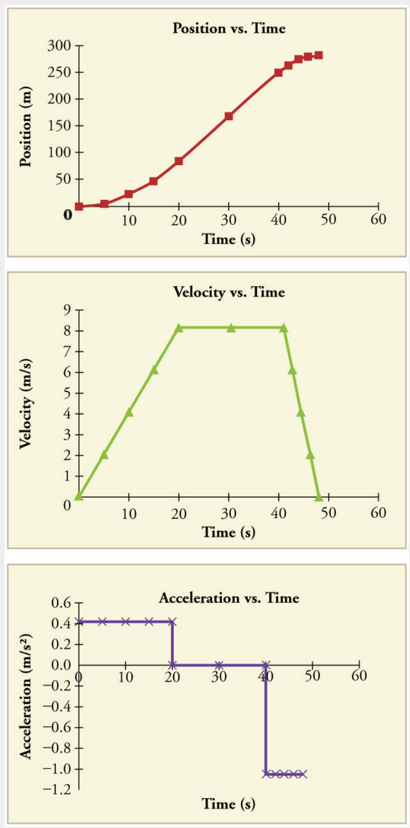


Figure 10. (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 6: Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 2, and shown again below, if it takes 5.00 min to make its trip?

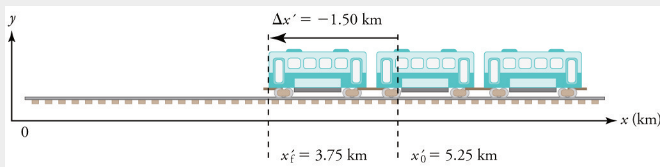


Figure 11.

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_f = 3.75 \text{ km}$, $x'_0 = 5.25 \text{ km}$, $\Delta t = 5.00 \text{ min}$.
2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5 km in Example 2.
3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50 \text{ km}}{5.00 \text{ min}}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50 \text{ km}}{5.00 \text{ min}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = -18.0 \text{ km/h}$$

Discussion

The negative velocity indicates motion to the left.

Example 7: Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 11 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

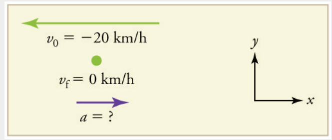


Figure 12.

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20 \text{ km/h}$, $v_f = 0 \text{ km/h}$, $\Delta t = 10.0 \text{ s}$.
2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20 \text{ km/h}) = +20 \text{ km/h}.$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0 \text{ km/h}}{10.0 \text{ s}}$$

4. Convert units

$$\bar{a} = \left(\frac{+20.0 \text{ km/h}}{10.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = +0.556 \text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in Example 5, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in Example 7, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in Figure 11 is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

Check Your Understanding

1: An airplane lands on a runway traveling east. Describe its acceleration.

PHET EXPLORATIONS: MOVING MAN SIMULATION

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

PhET Interactive Simulation

Figure 13. Moving Man.

Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.
- The SI unit for acceleration is m/s^2.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

Conceptual Questions

1: Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

2: Is it possible for velocity to be constant while acceleration is not zero? Explain.

3: Give an example in which velocity is zero yet acceleration is not.

4: If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

5: Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

Problems & Exercises

1: A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

2: Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of $g\ (9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

3: A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

4: Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of $g\ (9.80\ \text{m/s}^2)$?

Glossary

acceleration
the rate of change in velocity; the change in velocity over time
average acceleration
the change in velocity divided by the time over which it changes
instantaneous acceleration
acceleration at a specific point in time
deceleration
acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

Solutions

Check Your Understanding

1: If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

Problems & Exercises

1:

$4.29 m/s^2$

3:

(a) $1.43 s$

(b) $-2.50 m/s^2$

2.5 Motion Equations for Constant Acceleration in One Dimension

Summary

- Calculate displacement of an object that is not acceleration, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

Figure 1. Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr).

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{array}{rcl}\Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0\end{array}\right\}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

SOLVING FOR DISPLACEMENT (Δx) AND FINAL POSITION (x) FROM AVERAGE VELOCITY WHEN ACCELERATION (a) IS CONSTANT

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for x yields

$$x = x_0 + \bar{v}t,$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{)}.$$

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30 \text{ km/h} + 60 \text{ km/h}}{2} = 45 \text{ km/h},$$

which seems logical.

Example 1: Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

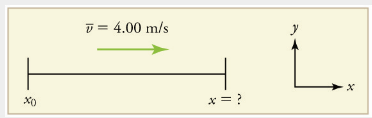


Figure 2.

The final position x is given by the equation

$$x = x_0 + \bar{v}t.$$

To find x, we identify the values of x_0, $\bar{v}$, and t *from the statement of the problem and substitute them into the equation.*

Solution

1. Identify the knowns. $\bar{v} = 4.00\text{ m/s}$, $\Delta t = 2.00\text{ min}$, and $x_0 = 0\text{ m}$.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00\text{ m/s})(120\text{ s}) = 480\text{ m}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

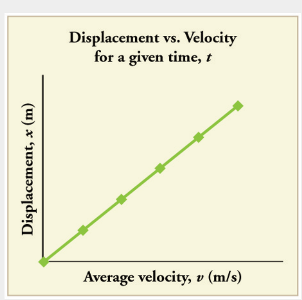


Figure 3. There is a linear relationship between displacement and average velocity. For a given time ***t***, an object moving twice as fast as another object will move twice as far as the other object.

SOLVING FOR FINAL VELOCITY

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{)}.$$

Solving for v yields

$$v = v_0 + at \text{ (constant } a\text{)}.$$

Example 2: Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

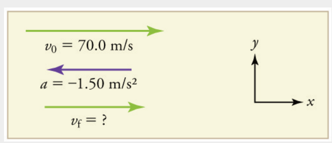


Figure 4.

Solution

1. Identify the knowns. $v_0 = 70.0 \text{ m/s}$, $a = -1.50 \text{ m/s}^2$, $t = 40.0 \text{ s}$.

2. Identify the unknown. In this case, it is final velocity, v_f.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0 \text{ m/s} + (-1.50 \text{ m/s}^2)(40.0 \text{ s}) = 10.0 \text{ m/s}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

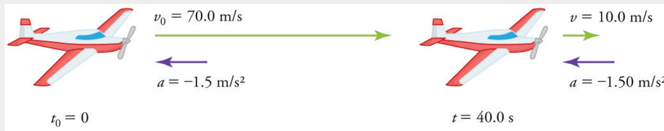


Figure 5. The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

MAKING CONNECTIONS: REAL WORLD CONNECTION

Figure 6. The Space Shuttle Endeavor blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr).

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

SOLVING FOR FINAL POSITION WHEN VELOCITY IS NOT CONSTANT (a ≠ 0)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at.$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at$$

Since $\frac{v_0 + v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 (\text{constant } a).$$

Example 3: Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 7. U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.).

Strategy

Draw a sketch.

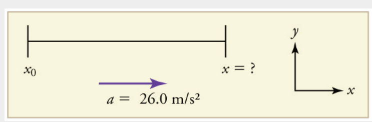


Figure 8.

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0 m/s^2 and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}at^2.$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}(26.0\ \text{m/s}^2)(5.56\ \text{s})^2,$$

yielding

$$x = 402\ \text{m}.$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example 3, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity $(v_0 = \bar{v})$ and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

SOLVING FOR FINAL VELOCITY WHEN VELOCITY IS NOT CONSTANT (a ≠ 0)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t, we get

$$t = \frac{v - v_0}{a}.$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0)(\text{constant } a).$$ Example: Calculating Final Velocity: Dragsters

Example 4: Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 3 without using information about time.

Strategy

Draw a sketch.

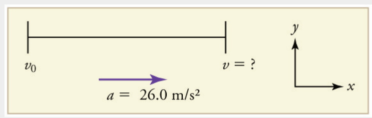


Figure 9.

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\ \text{m}$ (this was the answer in Example 3). Finally, the average acceleration was given to be $a = 26.0\ \text{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2(26.0\ \text{m/s}^2)(402\ \text{m}).$$

Thus

$$v^2 = 2.09 \times 10^4\ \text{m}^2/\ \text{s}^2$$

To get v, we take the square root:

$$v = \sqrt{2.09 \times 10^4\ \text{m}^2/\ \text{s}^2} = 145\ \text{m/s}.$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts

- For a fixed deceleration, a car that is going twice as fast doesn’t simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

SUMMARY OF KINEMATIC EQUATIONS(CONSTANT *a*)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example 5: Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of $7.00\ \mathrm{m/s^2}$, whereas on wet concrete it can decelerate at only $5.00\ \mathrm{m/s^2}$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

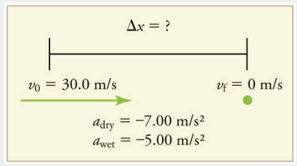


Figure 10.

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\ \text{m/s}$; $v = 0$; $a = -7.00\ \text{m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0\ \text{m/s})^2}{2(-7.00\ \text{m/s}^2)}$$

Thus,

$$x = 64.3\ \text{m on dry concrete.}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is $-5.00\ \text{m/s}^2$. The result is

$$x_{\text{wet}} = 90.0\ \text{m on wet concrete.}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0 \text{ m/s}$; $t_{\text{reaction}} = 0.500 \text{ s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m}.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

(a) 64.3 m + 15.0 m = 79.3 m when dry
(b) 90.0 m + 15.0 m = 105 m when wet

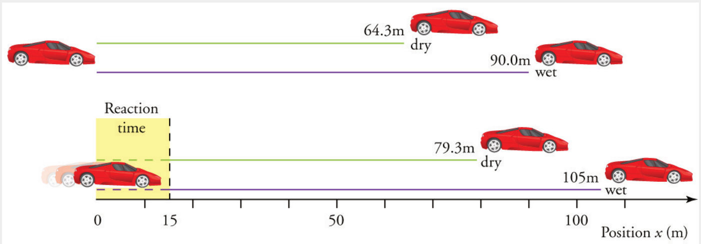


Figure 11. The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

Example 6: Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

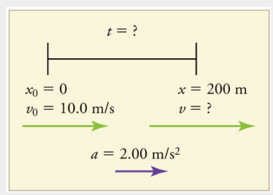


Figure 12.

We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10 \text{ m/s}$; $a = 2.00 \text{ m/s}^2$; and $x = 200 \text{ m}$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200 \text{ m} = 0 \text{ m} + (10.0 \text{ m/s})t + \frac{1}{2}(2.00 \text{ m/s}^2)t^2$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t \text{ s}$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2.$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0,$$

where the constants are $a = 1.00, \ b = 10.0, \ \text{and} \ c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This yields two solutions for t, which are

$$t = 10.0 \ \text{and} \ -20.0.$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0 \ \text{s and} \ -20.0 \ \text{s}.$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0 \ \text{s}.$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Chapter 2.6 Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—BREAKING NEWS

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

1: A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\left.\begin{array}{lll} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}\right\}$$

- The following kinematic equations for motion with constant a are useful:

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

Problems & Exercises

1: An Olympic-class sprinter starts a race with an acceleration of $4.50\ \text{m/s}^2$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

2: A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10 \times 10^4\ \text{m/s}^2$, and 1.85 ms $(1\ \text{ms} = 10^{-3}\ \text{s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

3: A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20 \times 10^5\ \text{m/s}^2$ for $8.10 \times 10^{-4}\ \text{s}$. What is its muzzle velocity (that is, its final velocity)?

4: (a) A light-rail commuter train accelerates at a rate of $1.35\ \text{m/s}^2$. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of $1.65\ \text{m/s}^2$. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in m/s^2?

5: While entering a freeway, a car accelerates from rest at a rate of $2.40\ \text{m/s}^2$ for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

6: At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of $2.00\ \text{m/s}^2$. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

7: Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

8: In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33 \times 10^{-2}\ \mathrm{s}$, calculate the distance over which the puck accelerates.

9: A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

10: Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of $0.0500\ \mathrm{m/s^2}$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \mathrm{m/s^2}$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

11: A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

12: A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \mathrm{m/s^2}$, how far will it travel before becoming airborne? (b) How long does this take?

13: Professional Application:

A woodpecker’s brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker’s head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in $\mathrm{m/s^2}$ and in multiples of $g\ (g = 9.80\ \mathrm{m/s^2})$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain’s deceleration, expressed in multiples of g?

14: An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

15: In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

16: Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

17: An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of $0.150\ \text{m/s}^2$ as it goes through. The station is 210 m long. (a) How long is the nose of the train in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

18: Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example 3 and Example 4. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

19: A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500\ \text{m/s}^2$ for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

20: In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

21: (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

Solutions

Check Your Understanding

1: To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + at$$

Rearrange to solve for t.

$$t = \frac{v - v_0}{a} = \frac{400\ \text{m/s} - 0\ \text{m/s}}{20\ \text{m/s}^2} = 20\ \text{s}$$

Problems & Exercises

1:

(a) $10.8\ \text{m/s}$

(b)

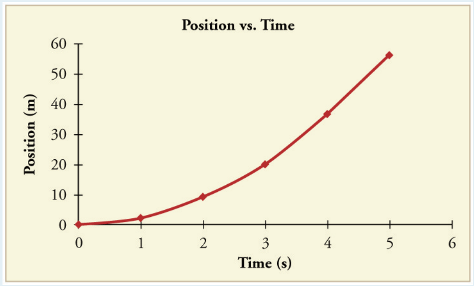


Figure 13.

2:
38.9 m/s (about 87 miles per hour)

4:
(a) 16.5 s

(b) 13.5 s

(c) -2.68 m/s^2

6:
(a) 20.0 m

(b) -1.00 m/s

(c) This result does not really make sense. If the runner starts at 9.00 m/s and decelerates at 2.00 m/s^2, then she will have stopped after 4.50 s. If she continues to decelerate, she will be running backwards.

8:
0.799 m

10:
(a) 28.0 m/s

(b) 50.9 s

(c) 7.68 km to accelerate and 713 m to decelerate

12:
(a) 51.4 m

(b) 17.1 s

14:

(a) -80.4 m/s^2

(b) $9.33 \times 10^{-2} \text{ s}$

16:
(a) 7.7 m/s

(b) $-15 \times 10^2 \text{ m/s}^2$. This is about 3 times the deceleration of the pilots, who were falling from thousands of meters high!

18:
(a) 32.6 m/s^2

(b) 162 m/s

(c) $v > v_{max}$, because the assumption of constant acceleration is not valid for a dragster. A dragster changes gears, and would have a greater acceleration in first gear than second gear than third gear, etc. The acceleration would be greatest at the beginning, so it would not be accelerating at 32.6 m/s^2 during the last few meters, but substantially less, and the final velocity would be less than 162 m/s.

20:
104 s
21:
(a) $v = 12.2 \text{ m/s}$; $a = 4.07 \text{ m/s}^2$

(b) $v = 11.2 \text{ m/s}$

2.6 Problem-Solving Basics for One-Dimensional Kinematics

Summary

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

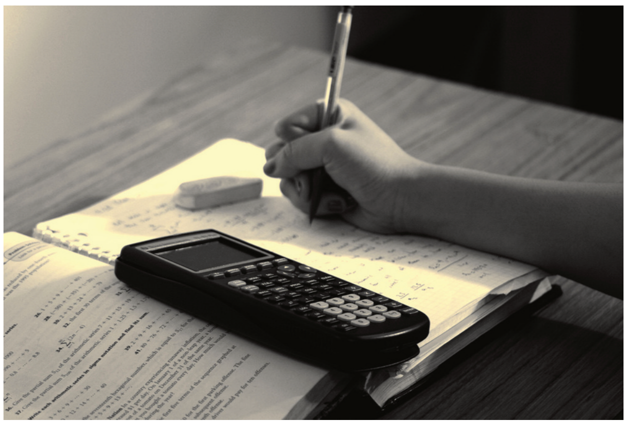


Figure 1. Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr).

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text's examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v_0 + at = 0 + (0.40 \text{ m/s}^2)(100 \text{ s}) = 40 \text{ m/s}.$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, …? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\text{ m}}{\text{s}}\right)\left(\frac{3.28\text{ ft}}{\text{m}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)=89\text{ mph}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s^2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s^2 for 100 s (almost two minutes).

Section Summary

- *The six basic problem solving steps for physics are:*

  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

  Step 6. Check the answer to see if it is reasonable: Does it make sense?

Conceptual Questions

1: What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

2: What is the last thing you should do when solving a problem? Explain.

2.7 Falling Objects

Summary

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

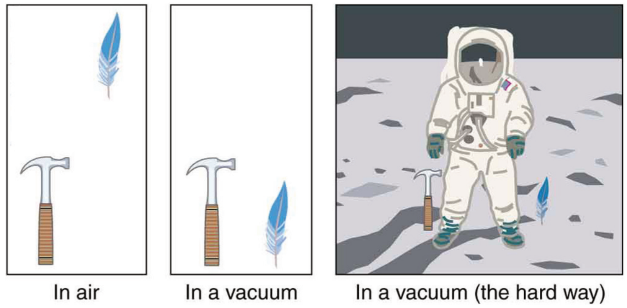


Figure 1. A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only **1.67 m/s²**.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, gg size 12{g} {}. It is constant at any given location on Earth and has the average value

$$g = 9.80\ \text{m/s}^2.$$

Although g varies from $9.78\ \text{m/s}^2$ to $9.83\ \text{m/s}^2$, depending on latitude, altitude, underlying geological formations, and local topography, the average value of $9.80\ \text{m/s}^2$ will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\ \text{m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80\ \text{m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

KINEMATIC EQUATIONS FOR OBJECTS IN FREE FALL WHERE ACCELERATION = -G

$$v = v_0 - gt$$

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Example 1: Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

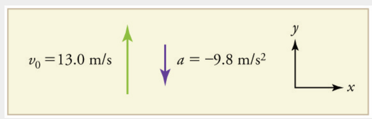


Figure 2.

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2} a t^2$ because it includes only one unknown, y (or y_1, here), *which is the value we want to find.*

3. Plug in the known values and solve for y_1.

$$y_1 = 0 + (13.0\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = 8.10\ \text{m}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$. We also know from the solution above that $y_1 = 8.10\ \text{m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0\ \text{m/s} - (9.80\ \text{m/s}^2)(1.00\ \text{s}) = 3.20\ \text{m/s}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\ \text{s}$. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\ \text{s}$ and $3.00\ \text{s}$ are the same as those above. The results are summarized in Table 1 and illustrated in Figure 3.

Time, *t*	**Position, *y***	**Velocity, *v***	**Acceleration, *a***
1.00 s	8.10 m	3.20 m/s	−9.80 m/s2
2.00 s	6.40 m	−6.60 m/s	−9.80 m/s2
3.00 s	−5.10 m	−16.4 m/s	−9.80 m/s2

Table 1. Results.

Graphing the data helps us understand it more clearly.

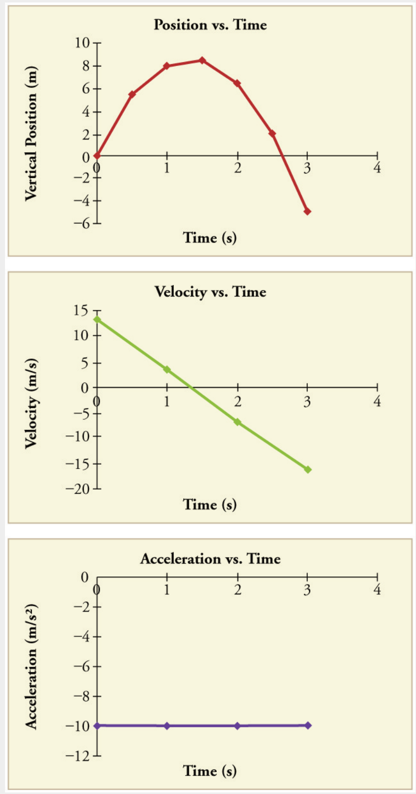


Figure 3. Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is time, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

MAKING CONNECTIONS: TAKE HOME EXPERIMENT—REACTION TIME

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 2: Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

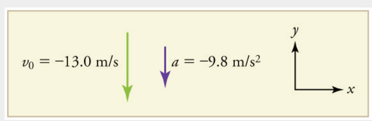


Figure 4.

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4\text{ m/s}.$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4\text{ m/s}.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See Example 1 and Figure 5(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example 1) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20\ \text{m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

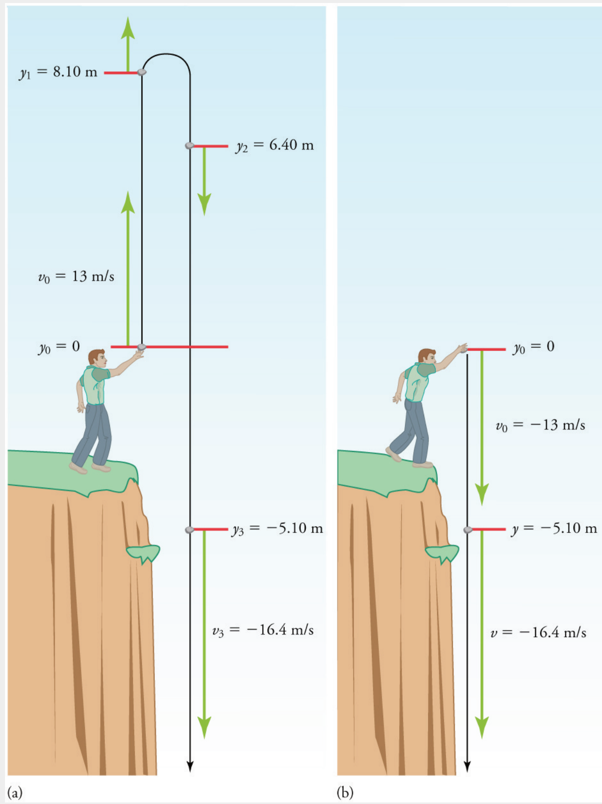
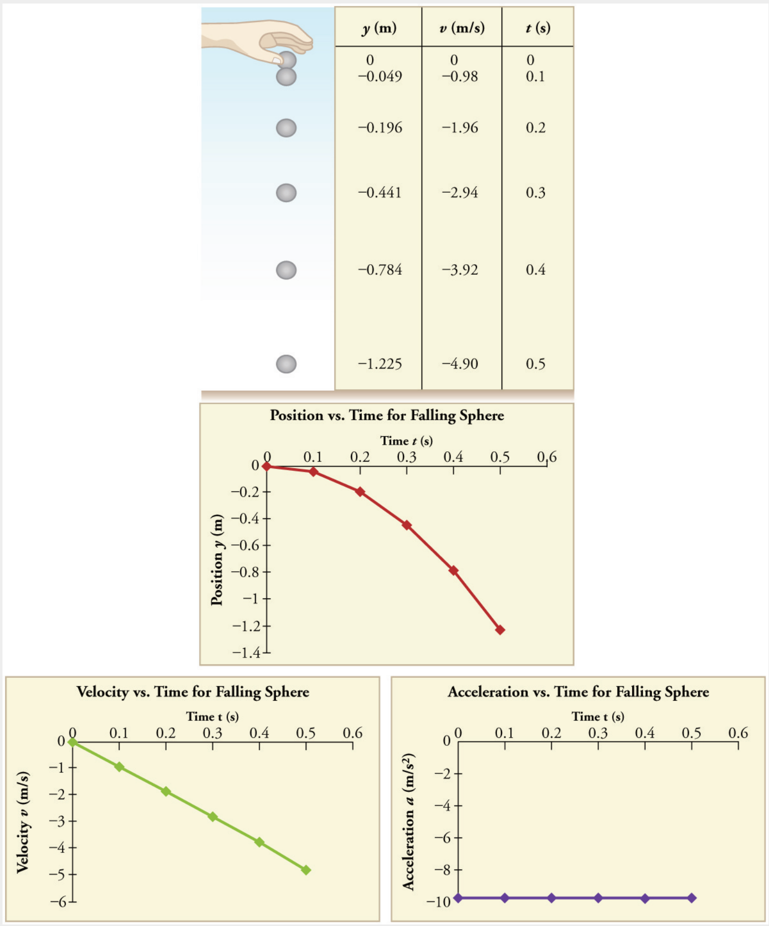


Figure 5. (a) A person throws a rock straight up, as explored in Example 1. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in Example 2. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In Example 1, the rock is thrown up with an initial velocity of $13.0\ \text{m/s}$. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is $-13.0\ \text{m/s}$. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10\ \text{m}$ to be the same whether we have thrown it upwards at $+13.0\ \text{m/s}$ or thrown it downwards at $-13.0\ \text{m/s}$. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 3: Find g from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure 6. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Figure 6. Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

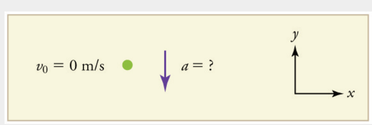


Figure 7.

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = -1.0000\ \text{m}$; $t = 0.45173\ \text{s}$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2.$$

Solving for a gives

$$a = \frac{2(y - y_0)}{t^2}.$$

4. Substitute known values yields

$$a = \frac{2(-1.0000\ \text{m} - 0)}{(0.45173\ \text{s})^2} = -9.8010\ \text{m/s}^2,$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\ \text{m/s}^2.$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of $9.80\ \text{m/s}^2$, so $9.8010\ \text{m/s}^2$ makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of $9.80\ \text{m/s}^2$; it represents the local value for the acceleration due to gravity.

Check Your Understanding

1: A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

PHET EXPLORATIONS: EQUATION GRAPHER

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

Figure 8. Equation Grapher.

Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages

$$g = 9.80 \text{ m/s}^2.$$

- Whether the acceleration *a* should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80 \text{ m/s}^2$ is negative. In the opposite case, $a = +g = 9.80 \text{ m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Conceptual Questions

1: What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

2: An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

3: Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

4: If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

5: The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

6: How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

Problems & Exercises

Assume air resistance is negligible unless otherwise stated.

1: Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

2: Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

3: A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

4: A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

5: A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

6: A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

7: (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

8: A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

9: You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

10: A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

11: Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can't see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

12: An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

13: There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

14: A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball's initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

15: Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

16: A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5}\ s)$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

17: A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

18: A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3}\ s)$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

Glossary

free-fall
the state of movement that results from gravitational force only
acceleration due to gravity
acceleration of an object as a result of gravity

Solutions

Check Your Understanding

1: We know that initial position $y_0 = 0$, final position $y = -30.0\ m$, and $a = -g = -9.80\ m/s^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$y = 0 + 0 - \frac{1}{2}gt^2$

$t^2 = \frac{2y}{-g}$

$t = \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0\ \text{m}}{-9.80\ \text{m/s}^2}} = \pm\sqrt{6.12\ \text{s}^2} = 2.47\ \text{s} \approx 2.5\ \text{s}$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

Problems & Exercises

1:

(a)$y_1 = 6.28\ \text{m};\ v_1 = 10.1\ \text{m/s}$

(b)$y_2 = 10.1\ \text{m};\ v_2 = 5.20\ \text{m/s}$

(c)$y_3 = 11.5\ \text{m};\ v_3 = 0.300\ \text{m/s}$

(d)$y_4 = 10.4\ \text{m};\ v_4 = -4.60\ \text{m/s}$

3:

$v_0 = 4.95\ \text{m/s}$

5:

(a)$a = -9.80\ \text{m/s}^2;\ v_0 = 13.0\ \text{m/s};\ y_0 = 0\ \text{m}$

(b)$v = 0\ \text{m/s}$. Unknown is distance y to top of trajectory, where velocity is zero. Use equation $v^2 = v_0^2 + 2a(y - y_0)$ because it contains all known values except for y, so we can solve for y. Solving for y gives

$$v^2 - v_0^2 = 2a(y - y_0)$$
$$\frac{v^2 - v_0}{2a} = y - y_0$$
$$y = y_0 + \frac{v^2 - v_0^2}{2a} = 0\ \text{m} + \frac{(0\ \text{m/s})^2 - (13.0\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 8.62\ \text{m}$$

Dolphins measure about 2 meters long and can jump several times their length out of the water, so this is a reasonable result.

(c)$2.65\ \text{s}$

7:

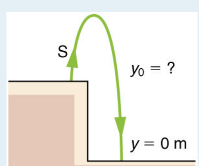


Figure 9.

(a) 8.26 m

(b) 0.717 s

9:

1.91 s

11:

(a) 94.0 m

(b) 3.13 s

13:

(a) -70.0 m/s (downward)

(b) 6.10 s

15:

(a) $19.6\ \mathrm{m}$

(b) $18.5\ \mathrm{m}$

17:

(a) 305 m

(b) 262 m, -29.2 m/s

(c) 8.91 s

2.8 Graphical Analysis of One-Dimensional Motion

Summary

- Describe a straight-line graph in terms of its slope and y-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of displacement, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure 1, a straight-line graph has the general form

$$y = mx + b.$$

Here m is the **slope,** defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.

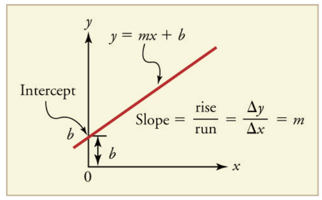


Figure 1. A straight-line graph. The equation for a straight line is ***y* = *mx* + *b*.**

Graph of Displacement vs. Time (*a* = 0, so *v* is constant)

Time is usually an independent variable that other quantities, such as displacement, depend upon. A graph of displacement versus time would, thus, have x on the vertical axis and t on the horizontal axis. Figure 2 is just such a straight-line graph. It shows a graph of displacement versus time for a jet-powered car on a very flat dry lake bed in Nevada**.**

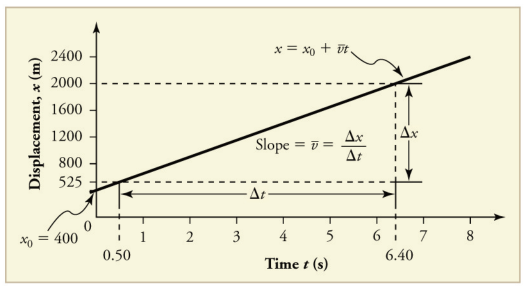


Figure 2. Graph of displacement versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is displacement at time zero—that is, x_0. Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0$$

or

$$x = x_0 + \bar{v}t.$$

Thus a graph of displacement versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation.

THE SLOPE OF X VS. T

The slope of the graph of displacement x vs. time t is velocity v.

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}$$

Notice that this equation is the same as that derived algebraically from other motion equations in Chapter 2.5 Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a displacement of 25 m at 0.50 s and 2000 m at 6.40 s. Its displacement at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

Example 1: Determining Average Velocity from a Graph of Displacement versus Time: Jet Car

Find the average velocity of the car whose position is graphed in Figure 2.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in displacement and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}.$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\ \text{m} - 525\ \text{m}}{6.4\ \text{s} - 0.50\ \text{s}},$$

yielding

$$\bar{v} = 250\ \text{m/s}.$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but a≠0

The graphs in Figure 3 below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

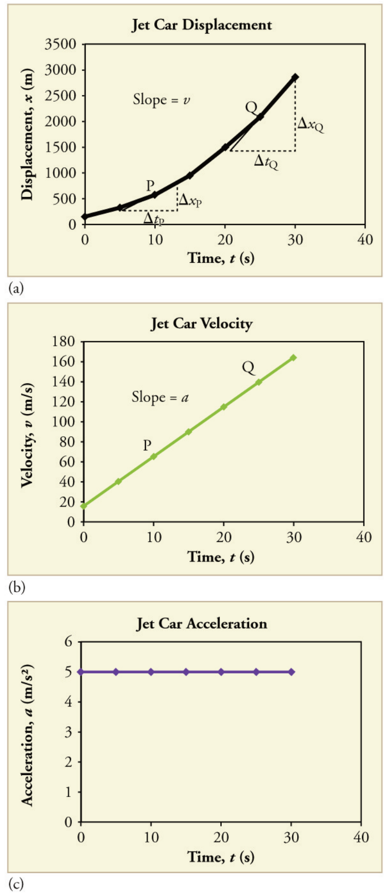


Figure 3. Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an$\boldsymbol{x}$ vs. $\boldsymbol{t}$ graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the $\boldsymbol{v}$ vs. $\boldsymbol{t}$ graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of $\mathbf{5.0\ m/s^2}$ over the time interval plotted.

Figure 4. A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr).

The graph of displacement versus time in Figure 3(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a displacement-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 3(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure 3(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 3(c).

Example 2: Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.

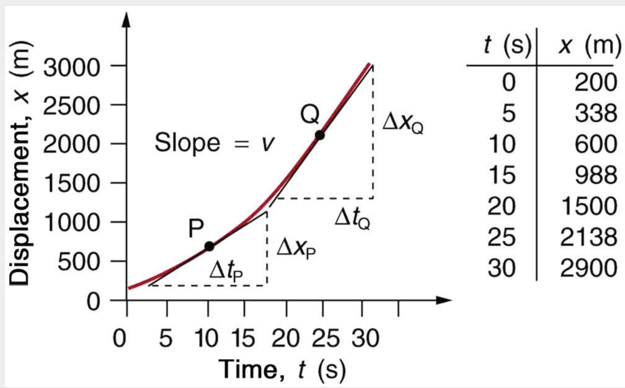


Figure 5. The slope of an ***x*** vs.***t*** graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure 5, where Q is the point at $t = 25\text{ s}$.

Solution

1. Find the tangent line to the curve at $t = 25\text{ s}$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_Q = \frac{\Delta x_Q}{\Delta t_Q} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})}$$

Thus,

$$v_Q = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}.$$

Discussion

This is the value given in this figure's table for v at $t = 25\text{ s}$. The value of 140 m/s for v_Q is plotted in Figure 5. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

THE SLOPE OF V VS. T

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a$$

Since the velocity versus time graph in Figure 3(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure 3(c).

Additional general information can be obtained from Figure 5 and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is v, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

$$v = v_0 + at.$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Chapter 2.5 Motion Equations for Constant Acceleration in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure 6. Time again starts at zero, and the initial displacement and velocity are 2900 m and 165 m/s, respectively. (These were the final displacement and velocity of the car in the motion graphed in Figure 3.) Acceleration gradually decreases from $5.0\ \text{m/s}^2$ to zero when the car hits 250 m/s. The slope of the x vs. t graph increases until $t = 55\ \text{s}$, after which time the slope is constant. Similarly, velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.

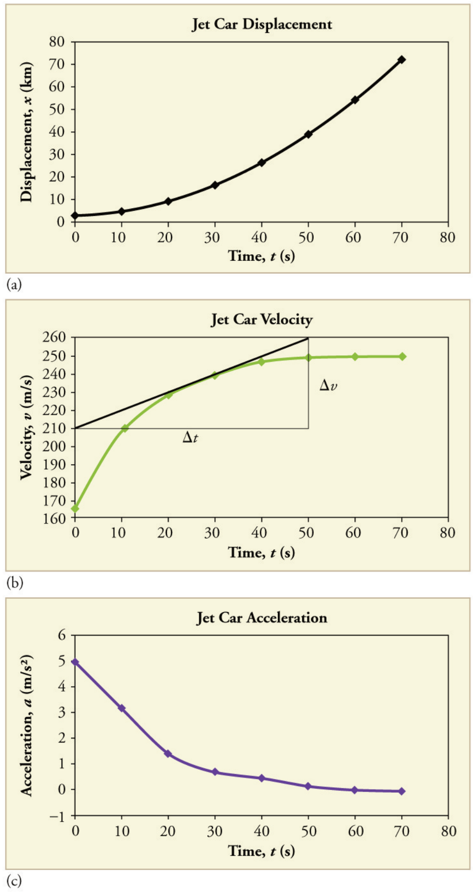


Figure 6. Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in Figure 3 ends. (a) The slope of this graph is velocity; it is plotted in the next graph. (b) The velocity gradually approaches its top value. The slope of this graph is acceleration; it is plotted in the final graph. (c) Acceleration gradually declines to zero when velocity becomes constant.

Example 3: Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in Figure 6(b).

Strategy

The slope of the curve at $t = 25\text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in Figure 6(b).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\text{ m/s} - 210\text{ m/s})}{(51\text{ s} - 1.0\text{ s})}$$

$$a = \frac{50\text{ m/s}}{50\text{ s}} = 1.0\text{ m/s}^2.$$

Discussion

Note that this value for a is consistent with the value plotted in Figure 6(c) at $t = 25\text{ s}$.

A graph of displacement versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

Check Your Understanding

1: A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?

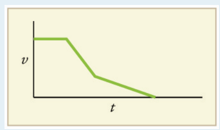


Figure 7.

Section Summary

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing graphs.

Conceptual Questions

1: (a) Explain how you can use the graph of position versus time in Figure 8 to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

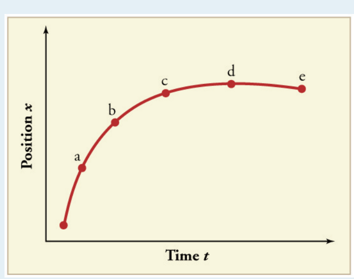


Figure 8.

2: (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in Figure 9. (b) Identify the time or times (t_a, t_b, t_c etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

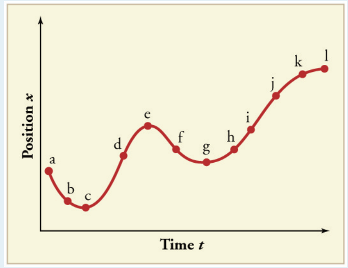


Figure 9.

3: (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 10. (b) Based on the graph, how does acceleration change over time?

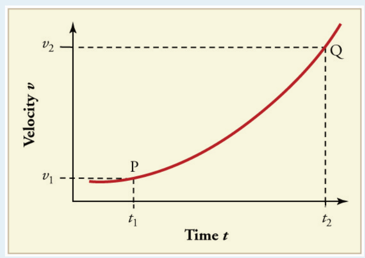


Figure 10.

4: (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 11. (b) Identify the time or times (t_a, t_b, t_c etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

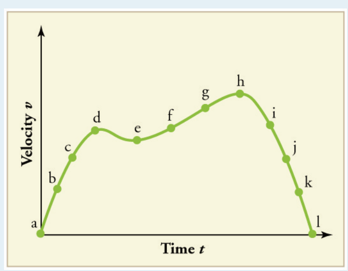


Figure 11.

5: Consider the velocity vs. time graph of a person in an elevator shown in Figure 12. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Chapter 2.5 Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

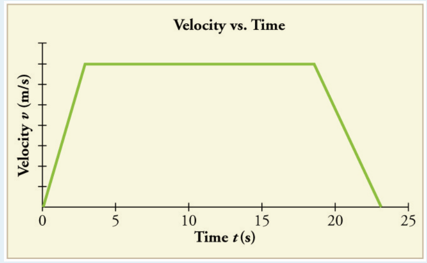


Figure 12.

6: A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

1: (a) By taking the slope of the curve in Figure 13, verify that the velocity of the jet car is 115 m/s at $t = 20$ s. (b) By taking the slope of the curve at any point in Figure 14, verify that the jet car's acceleration is 5.0 m/s^2.

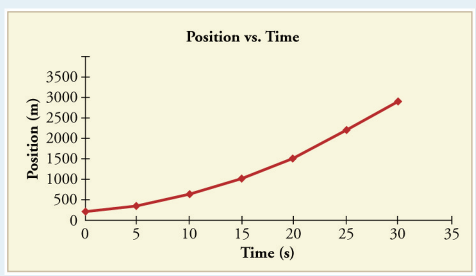


Figure 13.

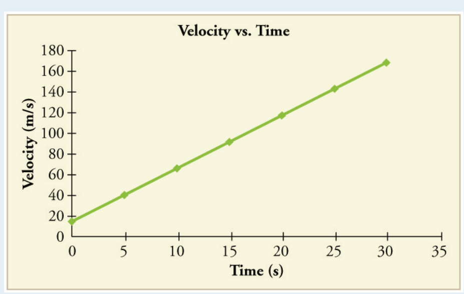


Figure 14.

2: Using approximate values, calculate the slope of the curve in Figure 15 to verify that the velocity at $t = 10.0$ s is 0.208 m/s. Assume all values are known to 3 significant figures.

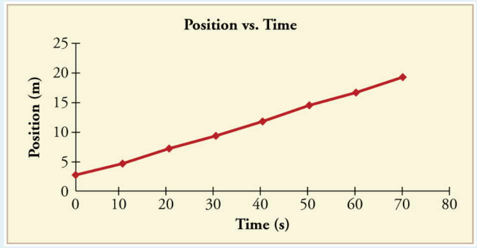


Figure 15.

3: Using approximate values, calculate the slope of the curve in Figure 15 to verify that the velocity at $t = 30.0\ \text{s}$ is 0.238 m/s. Assume all values are known to 3 significant figures.

4: By taking the slope of the curve in Figure 16, verify that the acceleration is $3.2\ \text{m/s}^2$ at $t = 10\ \text{s}$.

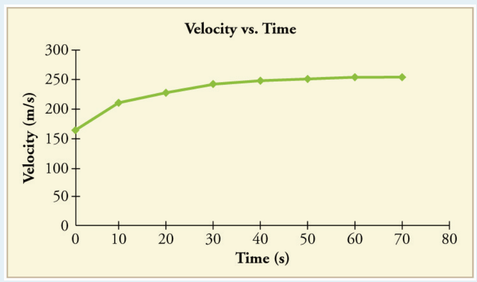


Figure 16.

5: Construct the displacement graph for the subway shuttle train as shown in Chapter 2.4 Figure 7(a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

6: (a) Take the slope of the curve in Figure 17 to find the jogger's velocity at $t = 2.5\ \text{s}$. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 18.

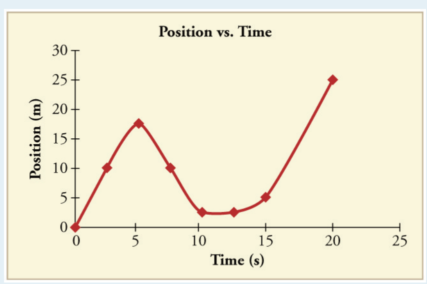


Figure 17.

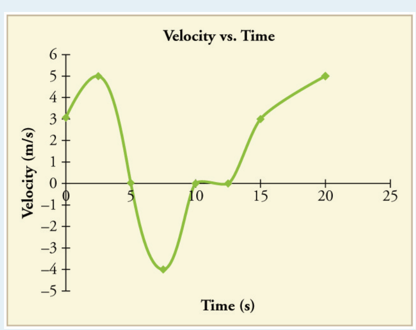


Figure 18.

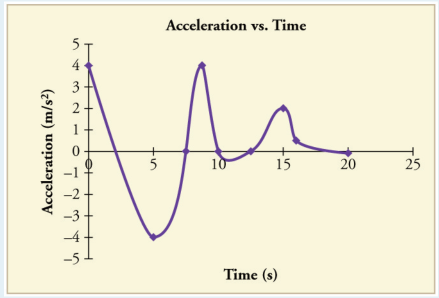


Figure 19.

7: A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure 20). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5\text{ s}$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

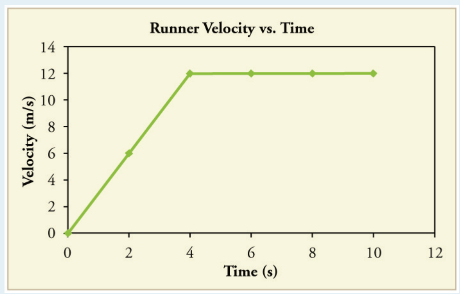


Figure 20.

8: Figure 21 shows the displacement graph for a particle for 5 s. Draw the corresponding velocity and acceleration graphs.

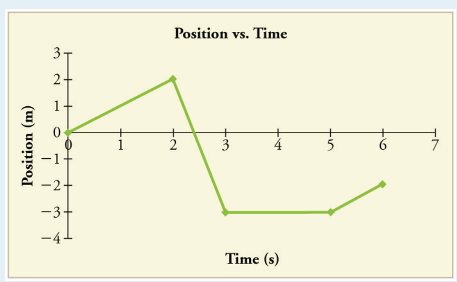


Figure 21.

Glossary

independent variable

the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

dependent variable

the variable that is being measured; usually plotted along the y-axis

slope

the difference in y-value *(the rise) divided by the difference in* x-value *(the run) of two points on a straight line*

y-intercept

the y-value *when* $x = 0$, *or when the graph crosses the* y-axis

Solutions

Check Your Understanding

1: (a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

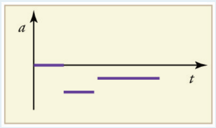


Figure 22.

Problems & Exercises

1:

(a) $115\ \text{m/s}$

(b) $5.0\ \text{m/s}^2$

3:

$$v = \frac{(11.7 - 6.95) \times 10^3\ \text{m}}{(40.0 - 20.0)\ \text{s}} = 238\ \text{m/s}$$

5:

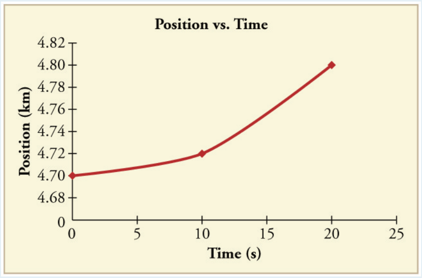


Figure 23.

7:

(a) 6 m/s

(b) 12 m/s

(c) $3\ \text{m/s}^2$

(d) 10 s

PART 3

Chapter 3 Two-Dimensional Kinematics

Figure 1. Everyday motion that we experience is, thankfully, rarely as tortuous as a rollercoaster ride like this—the Dragon Khan in Spain's Universal Port Aventura Amusement Park. However, most motion is in curved, rather than straight-line, paths. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Boris23/Wikimedia Commons).

The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

3.1 Kinematics in Two Dimensions: An Introduction

Summary

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

Figure 1. Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers).

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in Figure 2.

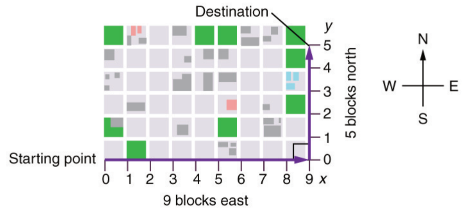


Figure 2. A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

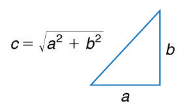


Figure 3. The Pythagorean theorem relates the length of the legs of a right triangle, labeled ***a*** and ***b***, with the hypotenuse, labeled ***c***. The relationship is given by: $a^2+b^2=c^2$. This can be rewritten, solving for ***c***: $c = \sqrt{(a^2+b^2)}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9\text{ blocks})^2 + (5\text{ blocks})^2} = 10.3\text{ blocks},$ considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that "9" and "5" have only one significant digit, they are discrete numbers. In this case "9 blocks" is the same as "9.0 or 9.00 blocks." We have decided to use three significant figures in the answer in order to show the result more precisely.)

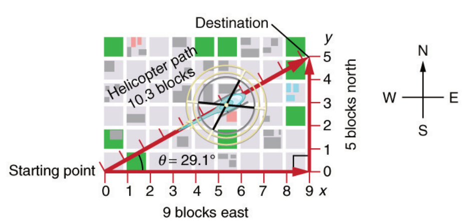


Figure 4. The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in Figure 4 is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector's magnitude. The arrow's length is indicated by hash marks in Figure 2 and Figure 4. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure 4. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Chapter 3.2 Vector Addition and Subtraction: Graphical Methods and Chapter 3.3 Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

The person taking the path shown in Figure 4 walks east and then north (two perpendicular directions). How far he or she walks east is only affected by his or her motion eastward. Similarly, how far he or she walks north is only affected by his or her motion northward.

INDEPENDENCE OF MOTION

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let's compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

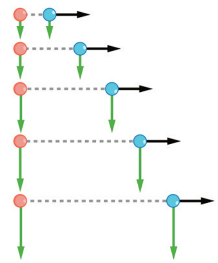


Figure 5. This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Chapter 3.2 Vector Addition and Subtraction: Graphical Methods and Chapter 3.3 Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

PHET EXPLORATIONS: LADYBUG MOTION IN 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Figure 6. Ladybug Motion 2D

Summary

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Glossary

vector
a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

3.2 Vector Addition and Subtraction: Graphical Methods

Summary

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

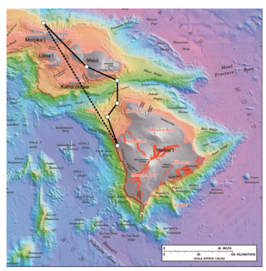


Figure 1. Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey).

Vectors in Two Dimensions

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure 2 shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Chapter 3.1 Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

VECTORS IN THIS TEXT

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

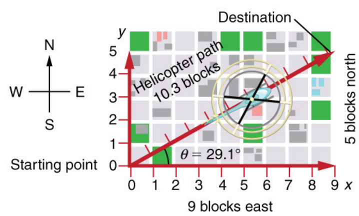


Figure 2. A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle $\mathbf{29.1^{o}}$ north of east.

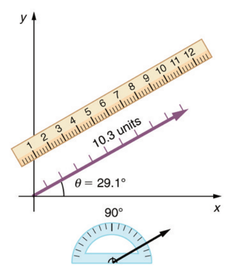


Figure 3. To describe the resultant vector for the person walking in a city considered in Figure 2 graphically, draw an arrow to represent the total displacement vector **D**. Using a protractor, draw a line at an angle **θ** relative to the east-west axis. The length ***D*** of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude ***D*** of the vector is 10.3 units, and the direction **θ** is $\mathbf{29.1^{o}}$ north of east.

Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in Figure 4 below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

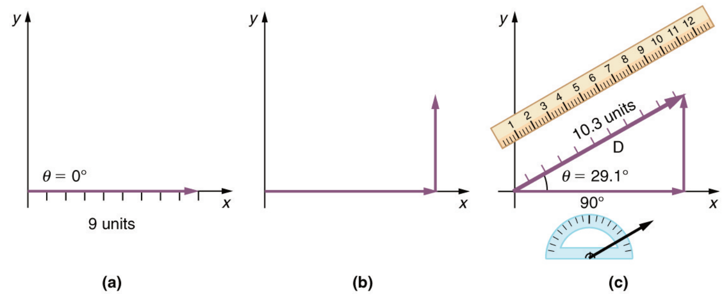


Figure 4. Head-to-Tail Method: The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in Figure 2. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector D**. The length of the arrow **D** is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) **θ** is measured with a protractor to be **29.1$^{\mathbf{0}}$**.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor*.

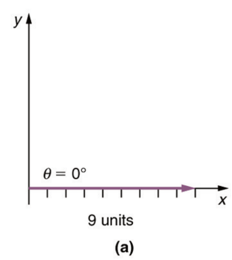


Figure 5.

Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector*.

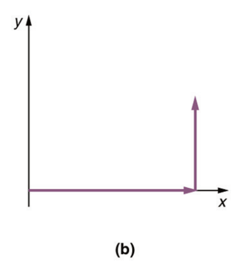


Figure 6.

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail*.

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.

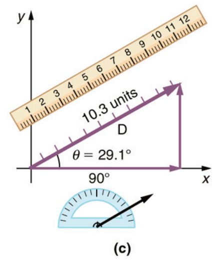


Figure 7.

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

Example 1: Adding Vectors Graphically Using the Head-to-Tail Method: A Women Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0° north of east. Then, she walks 23.0 m heading 15.0° north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first $\mathbf{A}$, the second $\mathbf{B}$, and the third $\mathbf{C}$, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted $\mathbf{R}$.

Solution

(1) Draw the three displacement vectors.

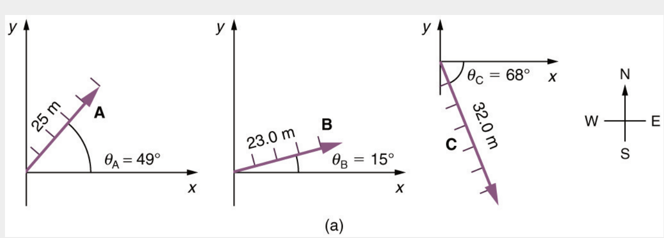


Figure 8.

(2) Place the vectors head to tail retaining both their initial magnitude and direction.

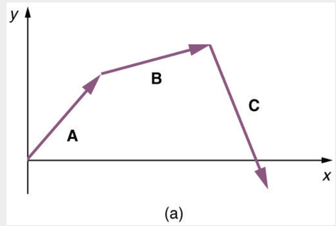


Figure 9.

(3) Draw the resultant vector, $\mathbf{R}$.

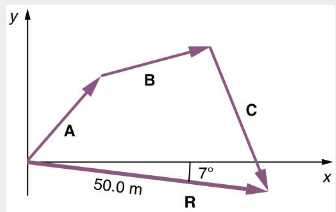


Figure 10.

(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of $\mathbf{R}$. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

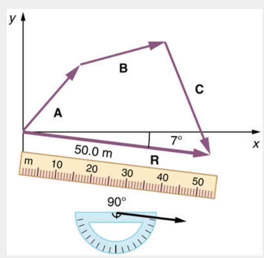


Figure 11.

In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.0 m and to lie in a direction 7.0° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.0\text{ m}$ and $\theta = 7.0^\circ$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in Figure 12 and we will still get the same solution.

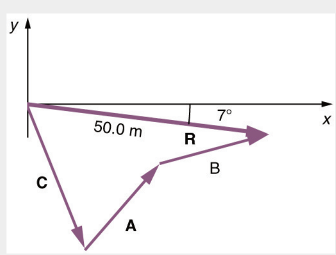


Figure 12.

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $2 + 3$ or $3 + 2$, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A} - \mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $-\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in Figure 13. In other words, $\mathbf{B}$ has the same length as $-\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

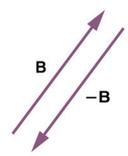


Figure 13. The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So **B** is the negative of **-B**; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $-\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}).$$

This is analogous to the subtraction of scalars (where, for example, $5 - 2 = 5 + (-2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

Example 2: Subtracting Vectors Graphically: A Women Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

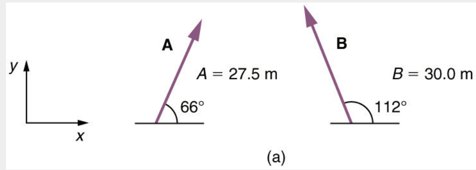


Figure 14.

Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A} + \mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180° - 112° = 68°$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A} + (-\mathbf{B})$, or $\mathbf{A} - \mathbf{B}$.

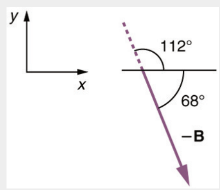


Figure 15.

We will perform vector addition to compare the location of the dock, $\mathbf{A} + \mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathbf{A} + (-\mathbf{B})$.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.

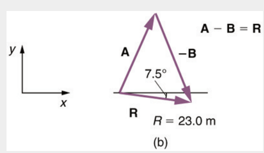


Figure 16.

In this case, $R = 23.0\text{ m}$ and $\theta = 7.5°$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R'}$:

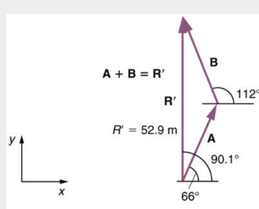


Figure 17.

In this case $R = 52.9\text{ m}$ and $\theta = 90.1°$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5 \text{ m}$, or 82.5 m, in a direction $66.0°$ north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5 \text{ m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x– and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction $29.0°$ north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Chapter 3.4 Projectile Motion, and much more when we cover **forces** in Chapter 4 Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Chapter 3.3 Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

PHET EXPLORATIONS: MAZE GAME

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

PhET Interactive Simulation

Figure 18. Maze Game

Summary

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by $c\mathbf{A}$. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

Conceptual Questions

1: Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?

2: Give a specific example of a vector, stating its magnitude, units, and direction.

3: What do vectors and scalars have in common? How do they differ?

4: Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?

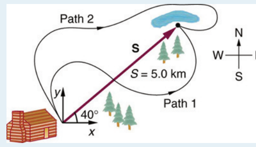


Figure 19.

5: If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure 20. What other information would he need to get to Sacramento?

Figure 20.

6: Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A}+\mathbf{B}$ the sum of the lengths of the two steps?

7: Explain why it is not possible to add a scalar to a vector.

8: If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

Problems & Exercises

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

1: Find the following for path A in Figure 21: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

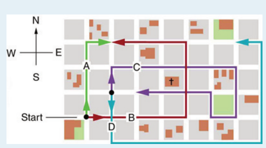


Figure 21. The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

2: Find the following for path B in Figure 21: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

3: Find the north and east components of the displacement for the hikers shown in Figure 19.

4: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 22, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

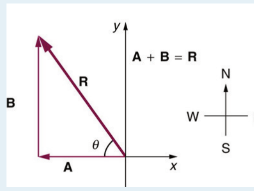


Figure 22. The two displacements **A** and **B** add to give a total displacement **R** having magnitude ***R*** and direction **θ**.

5: Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 23, then this problem finds their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

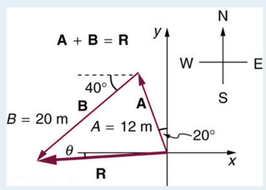


Figure 23.

6: Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)

7: (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0° north of east (which is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, to finding $\mathbf{R'} = \mathbf{A} - \mathbf{B}$). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° south of west and then 12.0 m in a direction 20.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$—that is, to finding $\mathbf{R''} = \mathbf{B} - \mathbf{A} = -\mathbf{R'}$). Show that this is the case.

8: Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathbf{A} + \mathbf{B} + \mathbf{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)

9: Show that the sum of the vectors discussed in Example 2 gives the result shown in Figure 17.

10: Find the magnitudes of velocities v_A and v_B in Figure 24

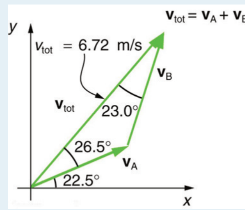


Figure 24. The two velocities $\mathbf{v_A}$ and $\mathbf{v_B}$ add to give a total $\mathbf{v_{tot}}$.

11: Find the components of v_{tot} along the *x*– and *y*-axes in Figure 24.

12: Find the components of v_{tot} along a set of perpendicular axes rotated $30°$ counterclockwise relative to those in Figure 24.

Glossary

component (of a 2-d vector)
a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components
commutative
refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum
direction (of a vector)
the orientation of a vector in space
head (of a vector)
the end point of a vector; the location of the tip of the vector's arrowhead; also referred to as the "tip"
head-to-tail method
a method of adding vectors in which the tail of each vector is placed at the head of the previous vector
magnitude (of a vector)
the length or size of a vector; magnitude is a scalar quantity
resultant
the sum of two or more vectors
resultant vector
the vector sum of two or more vectors
scalar
a quantity with magnitude but no direction
tail
the start point of a vector; opposite to the head or tip of the arrow

Solutions

Problems & Exercises

1:

(a) 480 m

(b) $379\ \text{m}, 18.4^{\circ}$ east of north

3:

north component 3.21 km, east component 3.83 km

5:

$19.5\ \text{m}, 4.65^{\circ}$ south of west

7:

(a) $26.6\ \text{m}, 65.1^{\circ}$ north of east

(b) $26.6\ \text{m}, 65.1^{\circ}$ south of west

9:

$52.9\ \text{m}, 90.1^{\circ}$ with respect to the *x*-axis.

11:

x-component 4.41 m/s

y-component 5.07 m/s

3.3 Vector Addition and Subtraction: Analytical Methods

Summary

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in Figure 1, we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.

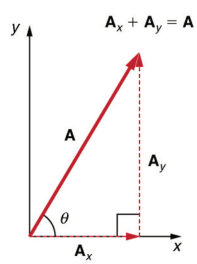


Figure 1. The vector **A**, with its tail at the origin of an x, y-coordinate system, is shown together with its x- and y-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

A_x and A_y are defined to be the components of A the x- and y-axes. The three vectors A, A_x, and A_y form a right triangle:

$$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}.$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $A_x = 3\text{ m}$ east, $A_y = 4\text{ m}$ north, and $A = 5\text{ m}$ north-east, then it is true that the vectors $A_x + A_y = A$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$3\text{ m} + 4\text{ m} \neq 5\text{ m}$$

Thus,

$$\mathbf{A_x} + \mathbf{A_y} \neq \mathbf{A}$$

If the vector A is known, then its magnitude A and its angle θ (its direction) are known. To find A_x and A_y, its x- and y-components, we use the following relationships for a right triangle.

$$A_x = A \cos \theta$$

and

$$A_y = A \sin \theta.$$

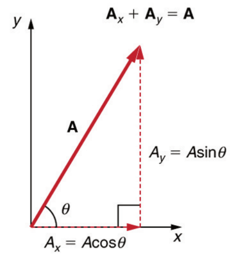


Figure 2. The magnitudes of the vector components $\mathbf{A_x}$ and $\mathbf{A_y}$ can be related to the resultant vector $\mathbf{A}$ and the angle $\boldsymbol{\theta}$ with trigonometric identities. Here we see that $A_x = A \cos \theta$ and $A_y = A \sin\theta$.

Suppose, for example, that A is the vector representing the total displacement of the person walking in a city considered in Chapter 3.1 Kinematics in Two Dimensions: An Introduction and Chapter 3.2 Vector Addition and Subtraction: Graphical Methods.

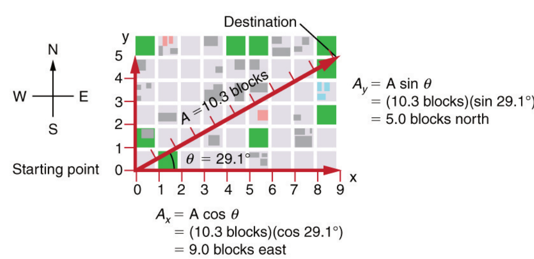


Figure 3. We can use the relationships $A_x = A \cos \theta$ and $A_y = A \sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A = 10.3 \text{ blocks}$ and $\theta = 29.1^{o}$, so that

$$A_x = A \cos\theta = (10.3 \text{ blocks})(\cos 29.1^{o}) = 9.0 \text{ blocks}$$

$$A_y = A \sin\theta = (10.3 \text{ blocks})(\sin 29.1^{o}) = 5.0 \text{ blocks}.$$

Calculating a Resultant Vector

If the perpendicular components A_x and A_y of a vector A are known, then A can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components A_x and A_y, we use the following relationships:

$$A = \sqrt{A_x{}^2 + A_y{}^2}$$

$$\theta = \tan^{-1}(A_y / A_x).$$

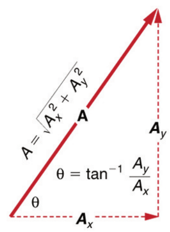


Figure 4. The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x{}^2 + A_y{}^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2 + 5^2} = 10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9) = 29.1^{o}$, as before.

DETERMINING VECTORS AND VECTOR COMPONENTS WITH ANALYTICAL METHODS

Equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x{}^2 + A_y{}^2}$ and $\theta = \tan^{-1}(A_y / A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider Figure 5, in which the vectors A and B are added to produce the resultant R.

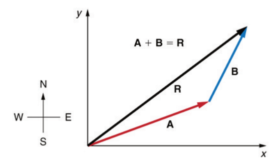


Figure 5. Vectors **A** and **B** are two legs of a walk, and **R** is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of **R**.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*– and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find $\mathbf{R}$ and θ using the equations $A = \sqrt{A_x{}^2 + A_y{}^2}$ and $\theta = \tan^{-1}(A_y / A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x = A \cos \theta$ and $A_y = A \sin \theta$ to find the components. In Figure 6, these components are A_x, A_y, B_x, and B_y. The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_A and θ_B, respectively.

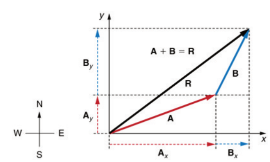


Figure 6. To add vectors **A** and **B**, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in Figure 7,

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x$$

and

$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y.$$

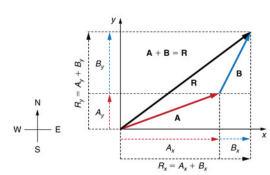


Figure 7. The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block east-ward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude* R *of the resultant, use the Pythagorean theorem:*

Formula does not parse

Step 4. *To get the direction of the resultant:*

$$\theta = \tan^{-1}(R_y / R_x).$$

The following example illustrates this technique for adding vectors using perpendicular components.

Example 1: Adding Vectors Using Analytical Methods

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in Figure 8, using perpendicular components along the *x*– and *y*-axes. The *x*– and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks $53.0\ \text{m}$ in a direction 20.0^o north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of $34.0\ \text{m}$ in a direction 63.0^o north of east.

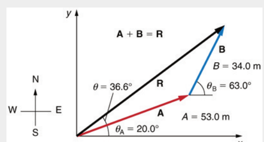


Figure 8. Vector **A** has magnitude **53.0 m** and direction $\mathbf{20.0^0}$ north of the x-axis. Vector **B** has magnitude **34.0 m** and direction $\mathbf{63.0^0}$ north of the x-axis. You can use analytical methods to determine the magnitude and direction of **R**.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*– and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*– and *y*-axes. Note that $A = 53.0\ \text{m}, \theta_A = 20.0^o, B = 34.0\ \text{m},$ and $\theta_B = 63.0^o$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$A_x = A\cos\theta_A = (53.0\ \text{m})(\cos 20.0^o)$$

$$= (53.0\ \text{m})(0.940) = 49.8\ \text{m}$$

and

$$B_x = B\cos\theta_B = (34.0\ \text{m})(\cos 63.0^o)$$

$$= (34.0\ \text{m})(0.454) = 15.4\ \text{m}.$$

Similarly, the *y*-components are found using $A_y = A\sin\theta$:

$$A_y = A\sin\theta_A = (53.0\ \text{m})(\sin 20.0^o)$$

$$= (53.0\ \text{m})(0.342) = 18.1\ \text{m}$$

and

$$B_y = B\sin\theta_B = (34.0\ \text{m})(\sin 63.0^o)$$

$$= (34.0\ \text{m})(0.891) = 30.3\ \text{m}.$$

The *x*– and *y*-components of the resultant are thus

$$\mathbf{R}_x = \mathbf{A}_x + \mathbf{B}_x = 49.8 \text{ m} + 15.4 \text{ m} = 65.2 \text{ m}$$

and

$$\mathbf{R}_y = \mathbf{A}_y + \mathbf{B}_y = 18.1 \text{ m} + 30.3 \text{ m} = 48.4 \text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$\mathbf{R} = \sqrt{\mathbf{R}_x^2 + \mathbf{R}_y^2} = \sqrt{(65.2)^2 + (48.4)^2} \text{ m}$$

so that

$$\mathbf{R} = 81.2 \text{ m}.$$

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(\mathbf{R}_y / \mathbf{R}_x) = +\tan^{-1}(48.4/65.2).$$

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6^{\circ}.$$

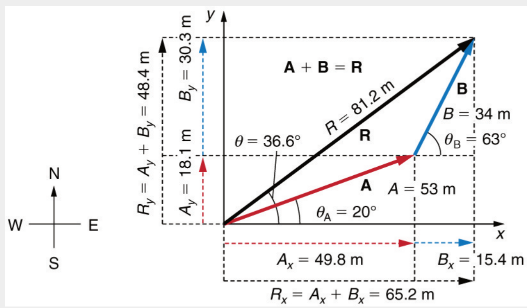


Figure 9. Using analytical methods, we see that the magnitude of **R** is **81.2 m** and its direction is **36.6⁰** north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*– and *y*-components of the resultant $\mathbf{A} - \mathbf{B} = \mathbf{R}$ are thus

$$\mathbf{R}_x = \mathbf{A}_x + (-\mathbf{B}_x)$$

and

$$\mathbf{R}_y = \mathbf{A}_y + (-\mathbf{B}_y)$$

and the rest of the method outlined above is identical to that for addition. (See Figure 10.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Chapter 3.4 Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

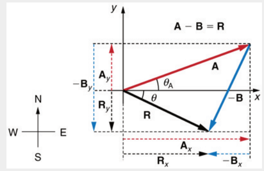


Figure 10. The subtraction of the two vectors shown in Figure 5. The components of **-B** are the negatives of the components of **B**. The method of subtraction is the same as that for addition.

PHET EXPLORATIONS: VECTOR ADDITION

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

PhET Interactive Simulation

Figure 11. Vector Addition

Summary

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:
  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

  $$A_x = A \cos \theta$$

  $$B_x = B \cos \theta$$

  and

  $$A_y = A \sin \theta$$

  $$B_y = B \sin \theta.$$

  Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:

  $$R_x = A_x + B_x$$

  and

  $$R_y = A_y + B_y.$$

  Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:

  $$R = \sqrt{R_x{}^2 + R_y{}^2}.$$

  Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$:

  $$\theta = \tan^{-1}(R_y / R_x).$$

Conceptual Questions

1: Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?

2: Give an example of a nonzero vector that has a component of zero.

3: Explain why a vector cannot have a component greater than its own magnitude.

4: If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

Problems & Exercises

1: Find the following for path C in Figure 12: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

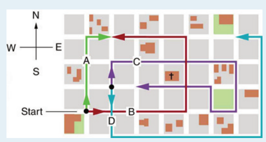


Figure 12. The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

2: Find the following for path D in Figure 12: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

3: Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure 13.

Figure 13.

4: Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 14, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

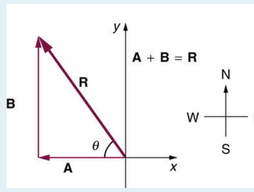


Figure 14. The two displacements **A** and **B** add to give a total displacement **R** having magnitude ***R*** and direction ***θ***.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

5: Repeat Exercise 4 using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking you other path.

6: You drive $7.50\ \text{km}$ in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

7: Do Exercise 4 again using analytical techniques and change the second leg of the walk to $25.0\ \text{m}$ straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R'} = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk $25.0\ \text{m}$ north and then $18.0\ \text{m}$ east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

8: A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in Figure 15. She then correctly calculates the length and orientation of the third side $\mathbf{C}$. What is her result?

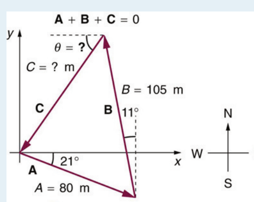


Figure 15.

9: You fly $32.0\ \text{km}$ in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.

10: A farmer wants to fence off his four-sided plot of flat land. He measures the first three sides,

shown as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in Figure 16, and then correctly calculates the length and orientation of the fourth side $\mathbf{D}$.
What is his result?

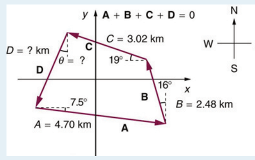


Figure 16.

11: In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: $2.50\text{ km }45.0^\circ$ north of west; then $4.70\text{ km }60.0^\circ$ south of east; then $1.30\text{ km }25.0^\circ$ south of west; then 5.10 km straight east; then $1.70\text{ km }5.00^\circ$ east of north; then $7.20\text{ km }55.0^\circ$ south of west; and finally $2.80\text{ km }10.0^\circ$ north of east. What is his final position relative to the island?

12: Suppose a pilot flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east as shown in Figure 17. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.

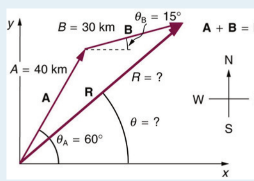


Figure 17.

Glossary

analytical method
the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

Solutions

Problems & Exercises

1:

(a) 1.56 km

(b) 120 m east

3:

North-component 87.0 km, east-component 87.0 km

5:

30.8 m, 35.8 west of north

7:

(a) $30.8\text{ m}, 54.2^\circ$ south of west

(b) $30.8\text{ m}, 54.2^\circ$ north of east

9:

(a) 18.4 km south, then 26.2 km west

(b) 31.5 km at 45.0° south of west, then 5.56 km at 45.0° west of north

11:

$7.34\text{ km}, 63.5^\circ$ south of east

3.4 Projectile Motion

Summary

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in Chapter 2.6 Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in Chapter 3.1 Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. Figure 1 illustrates the notation for displacement, where $\mathbf{s}$ is defined to be the total displacement and $\mathbf{x}$ and $\mathbf{y}$ are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are *s*, *x*, and *y*. (Note that in the last section we used the notation $\mathbf{A}$ to represent a vector with components $\mathbf{A_x}$ and $\mathbf{A_y}$. If we continued this format, we would call displacement $\mathbf{s}$ with components $\mathbf{s_x}$ and $\mathbf{s_y}$. However, to simplify the notation, we will simply represent the component vectors as $\mathbf{x}$ and $\mathbf{y}$.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the *x*– and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.80 \text{ m/s}^2$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

REVIEW OF KINEMATIC EQUATIONS (CONSTANT α)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v_2 = v_0^2 + 2a(x - x_0).$$

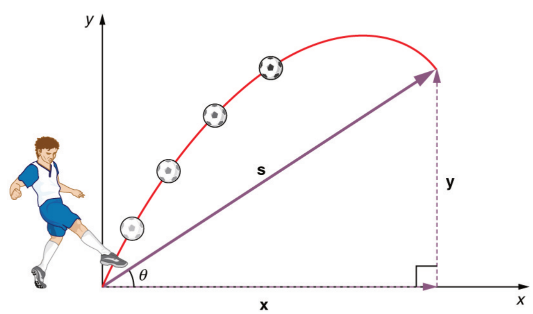


Figure 1. The total displacement **s** of a soccer ball at a point along its path. The vector ***s*** has components **x** and **y** along the horizontal and vertical axes. Its magnitude is ***s***, and it makes an angle ***θ*** with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = A \cos\theta$ and $A_y = A \sin\theta$ are used. The magnitude of the components of displacement s along these axes are x and y. The magnitudes of the components of the velocity v are $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where v is the magnitude of the velocity and θ is its direction, as shown in Figure 2. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion}(a_x = 0)$$

$$x = x_0 + v_x t$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant.}$$

$$\text{Vertical Motion}(\text{assuming positive is up } a_y = -g = -9.80 \text{ m/s}^2)$$

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* s *and velocity* v. Because the x – and y -motions are perpendicular, we determine these vectors by using the techniques outlined in the Chapter 3.3 Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y / A_x)$ in the following form, where θ is the direction of the displacement s and θ_v is the direction of the velocity v:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_v = \tan^{-1}(v_y / v_x).$$

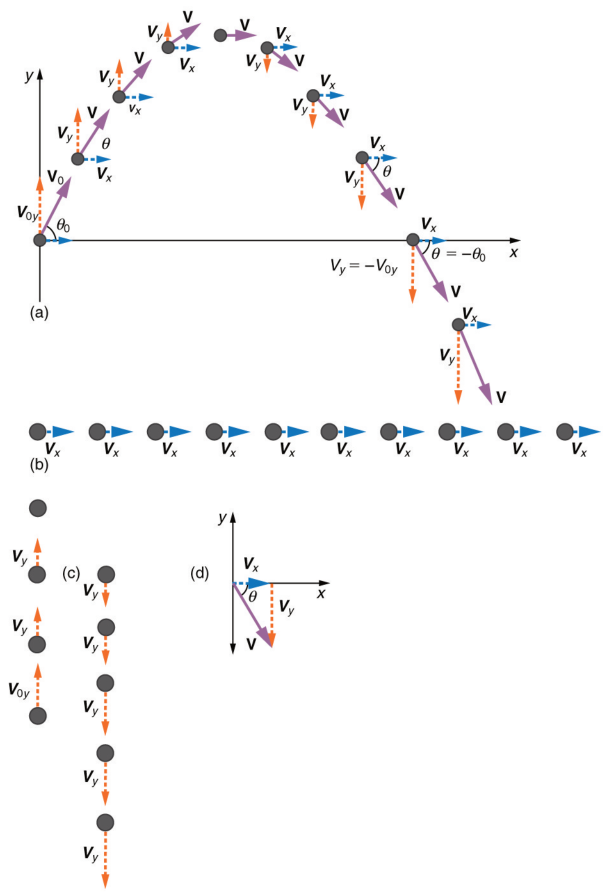


Figure 2. (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $\boldsymbol{a_x = 0}$ and $\boldsymbol{v_x}$ is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x – and y -motions are recombined to give the total velocity at any given point on the trajectory.

Example 1: A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0^{o} above the horizontal, as illustrated in Figure 3. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By "height" we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

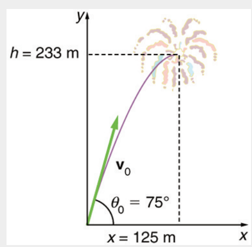


Figure 3. The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy.$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}.$$

Now we must find v_{0y}, the component of the initial velocity in the *y*-direction. It is given by $v_{0y} = v_0 \sin\theta$, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0^{o}$ is the initial angle. Thus,

$$v_{0y} = v_0 \sin\theta_0 = (70.0 \text{ m/s})(\sin 75^{o}) = 67.6 \text{ m/s}.$$

and y is

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)},$$

so that

$$y = 233 \text{ m}.$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y_0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t.$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$t = \frac{2y}{(v_{0y} + v_y)} = \frac{2(233\text{ m})}{(67.6\text{ m/s})}$$

$$= 6.90\text{ s}.$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t,$$

where v_x is the *x*-component of the velocity, which is given by $v_x = v_0 \cos\theta_0$. Now,

$$v_x = v_0 \cos\theta_0 = (70.0\text{ m/s})(\cos 75.0^\circ) = 18.1\text{ m/s}.$$

The time t for both motions is the same, and so x is

$$x = (18.1\text{ m/s})(6.90\text{ s}) = 125\text{ m}.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}.$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

DEFINING A COORDINATE SYSTEM

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

Example 2: Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in Figure 4. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

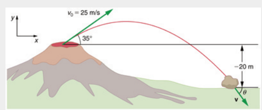


Figure 4. The trajectory of a rock ejected from the Kilauea volcano.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2.$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0\text{ m}.$ Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin\theta_0 = (25.0\text{ m/s})(\sin 35.0^{\circ}) = 14.3\text{ m/s}.$ Substituting known values yields

$$-20.0\text{ m} = (14.3\text{ m/s})t - 4.90\text{ m/s}^2 t^2.$$

Rearranging terms gives a quadratic equation in t:

$$4.90\text{ m/s}^2 t^2 - 14.3\text{ m/s}t - 20.0\text{ m} = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0,$ where the constants are $a = 4.90, b = -14.3,$ and $c = -20.0.$ Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This equation yields two solutions: $t = 3.96$ and $t = -1.03.$ (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96\ \text{s}$ or $-1.03\ \text{s}$. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96\ \text{s}.$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 \cos\theta_0 = (25.0\ \text{m/s})(\cos 35^\circ) = 20.5\ \text{m/s}.$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - gt,$$

where v_{0y} was found in part (a) to be $14.3\ \text{m/s}$. Thus,

$$v_y = 14.3\ \text{m/s} - (9.80\ \text{m/s}^2)(3.96\ \text{s})$$

so that

$$v_y = -24.5\ \text{m/s}.$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5\ \text{m/s})^2 + (-24.5\ \text{m/s})^2},$$

which gives

$$v = 31.9\ \text{m/s}.$$

The direction θ_v is found from the equation:

$$\theta_v = \tan^{-1}(v_y\ /\ v_x)$$

so that

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19).$$

Thus,

$$\theta_v = -50.1^\circ.$$

Discussion for (b)

The negative angle means that the velocity is 50.1° below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure 4.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance R traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

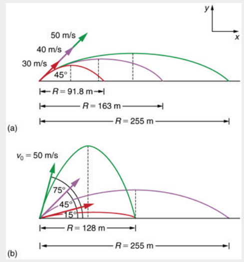


Figure 5. Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for 15^o and 75^o, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in Figure 5(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in Figure 5(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45^o$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38^o. Interestingly, for every initial angle except 45^o there are two angles that give the same range—the sum of those angles is 90^o. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure 6.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Chapter 3.5 Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

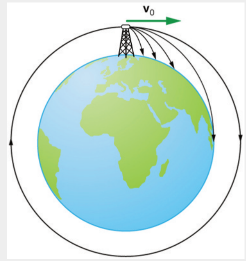


Figure 6. Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

PHET EXPLORATIONS: PROJECTILE MOTION

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

PhET Interactive Simulation

Figure 7. Projectile Motion

Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:

    1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position s are given by the quantities x and y, and the components of the velocity $\mathbf{v}$ are given by $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where v is the magnitude of the velocity and θ is its direction.
    2. Analyze the motion of the projectile in the horizontal direction using the following equations:

    $$\text{Horizontal motion}(a_x = 0)$$

    $$x = x_0 + v_x t$$

    $$v_x = v_{0x} = \mathrm{v}_x = \text{velocity is a constant.}$$

    3. Analyze the motion of the projectile in the vertical direction using the following equations:

    $$\text{Vertical motion (Assuming positive direction is up; } a_y = -g = -9.80\ \text{m/s}^2\text{)}$$

    $$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:

$$s = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta_v = \tan^{-1}(v_y / v_x).$$

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by
$$h = \frac{v_{0y}^2}{2g}.$$
- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by
$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

Conceptual Questions

1: Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

2: Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

3: For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

4: During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

Problems & Exercises

1: A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

2: A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

3: A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

4: (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32° ramp at a speed of $40.0 \text{ m/s } (144 \text{ km/h})$. How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

5: An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

6: A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

7: Verify the ranges for the projectiles in Figure 5(a) for $\theta = 45^\circ$ and the given initial velocities.

8: Verify the ranges shown for the projectiles in Figure 5(b) for an initial velocity of 50 m/s at the given initial angles.

9: The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is 6.37×10^3 km. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

10: An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60° above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

11: In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g. How far can they jump?

State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

12: The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

13: Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The service line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose out line is 6.40 m from the net?

14: A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25° relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?

15: Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

16: An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

17: An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle 30.0° below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.

18: Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be 40° above the horizontal.

19: Can a goalkeeper at her/ his goal kick a soccer ball into the opponent’s goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

20: The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 7.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.

21: In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0° above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at 45° when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus, 38° will give a longer range than 45° in the shot put.)

22: A basketball player is running at $5.00\ \text{m/s}$ directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

23: A football player punts the ball at a 45.0° angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

24: Prove that the trajectory of a projectile is parabolic, having the form $y = ax + bx^2$. To obtain this expression, solve the equation $x = v_{0x}t$ for t and substitute it into the expression for $y = v_{0y}t - (1/2)gt^2$ (These equations describe the x and y positions of a projectile that starts at the origin.) You should obtain an equation of the form $y = ax + bx^2$ where a and b are constants.

25: Derive $R = \frac{v_0^2 \sin 2\theta_0}{g}$ for the range of a projectile on level ground by finding the time t at which y becomes zero and substituting this value of t into the expression for $x - x_0$, noting that $R = x - x_0$

26: Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

27: Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

Glossary

air resistance
a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

kinematics
the study of motion without regard to mass or force

motion
displacement of an object as a function of time

projectile
an object that travels through the air and experiences only acceleration due to gravity

projectile motion
the motion of an object that is subject only to the acceleration of gravity

range
the maximum horizontal distance that a projectile travels
trajectory
the path of a projectile through the air

Solutions

Problems & Exercises

1:

$x = 1.30\ \text{m} \times 10^2$

$y = 30.9\ \text{m}.$

3:

(a) 3.50 s

(b) 28.6 m/s (c) 34.3 m/s

(d) 44.7 m/s, 50.2^0 below horizontal

5:

(a) 18.4^0

(b) The arrow will go over the branch.

7:

$R = \frac{v_0^2}{\sin 2\theta_0 g}$

For $\theta = 45^0$, $R = \frac{v_0^2}{g}$

$R = 91.8\ \text{m}$ for $v_0 = 30\ \text{m/s}$; $R = 163\ \text{m}$ for $v_0 = 40\ \text{m/s}$; $R = 255\ \text{m}$ for $v_0 = 50\ \text{m/s}.$

9:

(a) 560 m/s

(b) $8.00 \times 10^3\ \text{m}$

(c) 80.0 m. This error is not significant because it is only 1% of the answer in part (b).

11:

1.50 m, assuming launch angle of 45^0

13:

$\theta = 6.1^0$

yes, the ball lands at 5.3 m from the net

15:

(a) −0.486 m

(b) The larger the muzzle velocity, the smaller the deviation in the vertical direction, because the time of flight would be smaller. Air resistance would have the effect of decreasing the time of flight, therefore increasing the vertical deviation.

17:

4.23 m. No, the owl is not lucky; he misses the nest.

19:

No, the maximum range (neglecting air resistance) is about 92 m.

21:

15.0 m/s

23:

(a) 24.2 m/s

(b) The ball travels a total of 57.4 m with the brief gust of wind.

25:

$y - y_0 = 0 = v_{0_y}t - \frac{1}{2}gt^2 = (v_0 \sin \theta)t - \frac{1}{2}gt^2,$

so that $t = \frac{2(v_0 \sin \theta)}{g}$

$x - x_0 = v_{0x}t = (v_0 \cos \theta)t = R,$ and substituting for t gives:

$R = v_0 \cos \theta (\frac{2v_0 \sin \theta}{g}) = (\frac{2v_0^2 \sin \theta \cos \theta}{g})$

since $2\sin\theta\cos\theta = \sin 2\theta,$ the range is:

$\mathrm{R} = \frac{v_0^2 \sin 2\theta}{g}.$

3.5 Addition of Velocities

Summary

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in Figure 1. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in Figure 2. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

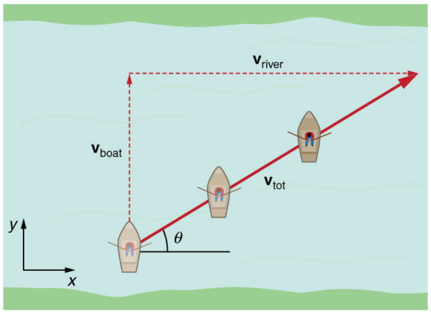


Figure 1. A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

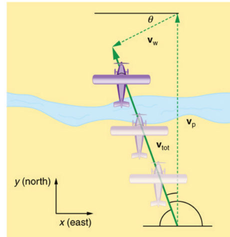


Figure 2. An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in Figure 1 and Figure 2. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in Chapter 3.2 Vector Addition and Subtraction: Graphical Methods and Chapter 3.3 Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*– and *y*-axes of an appropriately chosen coordinate system:

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}(v_y/v_x).$$

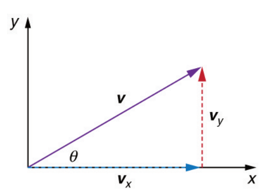


Figure 3. The velocity, ***v***, of an object traveling at an angle ***θ*** to the horizontal axis is the sum of component vectors $\mathbf{v_x}$ and $\mathbf{v_y}$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

TAKE-HOME EXPERIMENT: RELATIVE VELOCITY OF A BOAT

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

Example 1: Adding Velocities: A Boat on a River

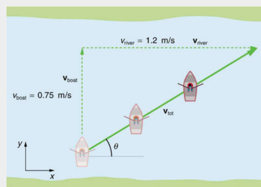


Figure 4. A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?

Refer to Figure 4, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, v_{tot}. The velocity of the boat, v_{boat}, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, v_{river}, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its xx-axis parallel to the velocity of the river, as shown in Figure 4. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{tot} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{tot} = \sqrt{v_x^2 + v_y^2},$$

where

$$v_x = v_{river} = 1.20 \text{ m/s}$$

and

$$v_y = v_{boat} = 0.750 \text{ m/s}.$$

Thus,

$$v_{tot} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2}$$

yielding

$$v_{tot} = 1.42 \text{ m/s}.$$

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(0.750/1.20).$$

This equation gives

$$\theta = 32.0^{\circ}.$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with Figure 4. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0°) the total velocity has relative to the riverbank.

Example 2: Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in Figure 5. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0° west of north.

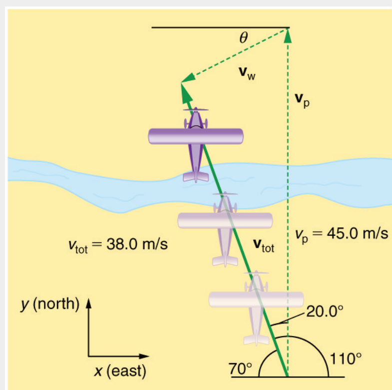


Figure 5. An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{\text{tot}}$ and that it is the sum of two other velocities, $\mathbf{v}_{\text{w}}$ (the wind) and $\mathbf{v}_{\text{p}}$ (the plane relative to the air mass). The quantity $\mathbf{v}_{\text{p}}$ is known, and we are asked to find $\mathbf{v}_{\text{w}}$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_{\text{w}}$, then we can combine them to solve for its magnitude and direction. As shown in Figure 5, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_{\text{p}}$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Chapter 3.3 Vector Addition and Subtraction: Analytical Methods.)

Solution

Because $\mathbf{v}_{\text{tot}}$ is the vector sum of the $\mathbf{v}_{\text{w}}$ and $\mathbf{v}_{\text{p}}$, its *x*– and *y*-components are the sums of the *x*– and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{px} = 0$ and $v_{py} = v_p$. That is,

$$v_{\text{tot}x} = v_{wx}$$

and

$$v_{toty} = v_{wy} + v_p.$$

We can use the first of these two equations to find v_{wx}:

$$v_{wy} = v_{totx} = v_{tot} \cos\ 110^0.$$

Because $v_{tot} = 38.0\ \mathrm{m/s}$ and $\cos\ 110^0 = -0.342$ we have

$$v_{wy} = (38.0\ \mathrm{m/s})(-0.342) = -13\ \mathrm{m/s}.$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find v_{wy} we note that

$$v_{toty} = v_{wy} + v_p$$

Here $v_{toty} = v_{tot} \sin\ 110^0$; thus,

$$v_{wy} = (38.0\ \mathrm{m/s})(0.940) - 45.0\ \mathrm{m/s} = -9.29\ \mathrm{m/s}.$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity v_{wx} and v_{wy} are known, we can find the magnitude and direction of v_w. First, the magnitude is

$$v_w = \sqrt{v_{wx}^2 + v_{wy}^2}$$

$$= (-13.0\ \mathrm{m/s})^2 + (-9.29\ \mathrm{m/s})^2$$

so that

$$v_w = 16.0\ \mathrm{m/s}.$$

The direction is:

$$\theta = \tan^{-1}(v_{wy}/v_{wx}) = \tan^{-1}(-9.29/-13.0)$$

giving

$$\theta = 35.6^0.$$

Discussion

The wind's speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in Figure 5. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3,000 \text{ km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops his binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See Figure 6.) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in Figure 6. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

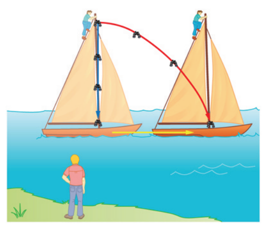


Figure 6. Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

Example 3: Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

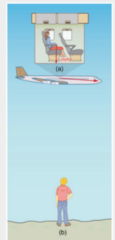


Figure 7. The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

Substituting known values into the equation, we get

$$v_y^2 = 0^2 - 2(9.80\ \text{m/s}^2)(-1.50\ \text{m} - 0\ \text{m}) = 29.4\ \text{m}^2/\ \text{s}^2$$

yielding

$$v_y = -5.42\ \text{m/s}.$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42\ \text{m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260\ \text{m/s}$. The *x*– and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$\sqrt{v = v_x{}^2 + v_y{}^2}.$$

Thus,

$$v = \sqrt{(260\ \text{m/s})^2 + (-5.42\ \text{m/s})^2}$$

yielding

$$v = 260.06\ \text{m/s}.$$

The direction is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-5.42/260)$$

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^0.$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 - 5.42)\ \text{m/s}$; rather, it is $260.06\ \text{m/s}$. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See Figure 7.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

MAKING CONNECTIONS: RELATIVITY AND EINSTEIN

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PHET EXPLORATIONS: MOTION IN 2D

Try the new "Ladybug Motion 2D" simulation for the latest updated version. Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

PhET Interactive Simulation

Figure 8. Motion in 2D

Summary

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

$$v_x = v \cos\theta$$

$$v_y = v \sin\theta$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}(v_y/v_x).$$

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

1: What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?

2: A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?

3: If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?

4: The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.

5: A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

1: Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction 45°south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?

2: A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.

3: Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?

4: Verify that the coin dropped by the airline passenger in the Example 3 travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.

5: A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of 25.0° relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback* ?

6: A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to the Earth?

7: (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction 5.0° south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction 15° south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.

8: (a) In what direction would the ship in Exercise 6 have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains $7.00\ \text{m/s}$? (b) What would its speed be relative to the Earth?

9: (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20° south of east (as in Exercise 7). Its direction of motion relative to the Earth is 45.0° south of west, while its direction of travel relative to the air is 5.00° south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?

10: A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.

11: The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction 30.0° east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of 50.0° south of west relative to the Earth. What is the velocity of the wind relative to the water?

12: The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. Figure 9 illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.

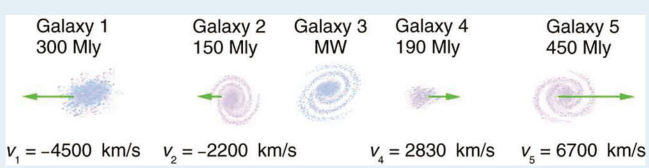


Figure 9. Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

13: (a) Use the distance and velocity data in Figure 9 to find the rate of expansion as a function of distance.

(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.

14: An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. He reaches the opposite side at a distance 40 m downstream from his starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?

15: A ship sailing in the Gulf Stream is heading 25.0° west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s 5.00° west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)

16: An ice hockey player is moving at 8.00 m/s when he hits the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a 90.0° angle relative to his path as shown in Figure 10. What angle must the puck's velocity make relative to the player (in his frame of reference) to hit the center of the goal?

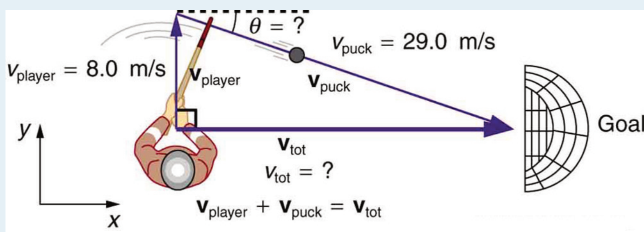


Figure 10. An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

15: Unreasonable Results Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.

16: Unreasonable Results A commercial airplane has an air speed of 280 m/s due east and flies with a strong tailwind. It travels 3000 km in a direction 5° south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?

17: Construct Your Own Problem Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

Glossary

classical relativity
the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s
relative velocity
the velocity of an object as observed from a particular reference frame

relativity
the study of how different observers moving relative to each other measure the same phenomenon
velocity
speed in a given direction
vector addition
the rules that apply to adding vectors together

Solution

Problems & Exercises:

1:

(a) 35.8 km south of east

(b) 5.53 m/s south of east

(c) 56.1 km south of east

3:

(a) 0.70 m/s faster

(b) Second runner wins

(c) 4.17 m

5:

$17.0\text{ m/s},\ 22.1^{0}$

7:

(a) $230\text{ m/s},\ 8.0^{0}$ south of west

(b) The wind should make the plane travel slower and more to the south, which is what was calculated

9:

(a) 63.5 m/s

(b) 29.6 m/s

11:

$6.68\text{ m/s},\ 53.3^{0}$ south of west

13:

(a) $H_{\text{average}} = 14.9\,\frac{\text{km/s}}{\text{Mly}}$

(b) 20.2 billion years

15:

$1.72\text{ m/s},\ 42.3^{0}$ north of east

PART 4

Chapter 4 Dynamics: Force and Newton's Laws of Motion

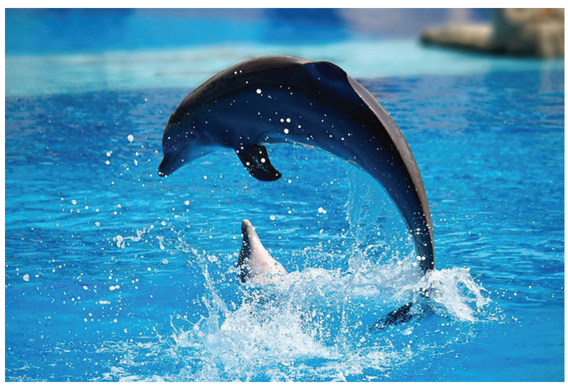
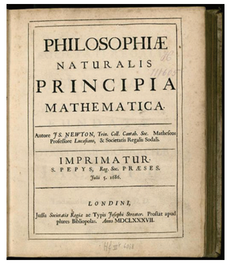


Figure 1. Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore JS. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Figure 2. Isaac Newton's monumental work, Philosophiae Naturalis Principia Mathematica, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about $\boldsymbol{10^{-9}}\textbf{m}$ in diameter). These constraints define the realm of classical mechanics, as discussed in Chapter 1 Introduction to the Nature of Science and Physics. At the beginning of the 20^{th} century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in Chapter 28 Special Relativity, are in the realm of classical physics.

MAKING CONNECTIONS: PAST AND PRESENT PHILOSOPHY

The importance of observation and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

4.1 Development of Force Concept

Summary

- Understand the definition of force.

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in Figure 1, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in Figure 2(a) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in Chapter 3 Two-Dimensional Kinematics.

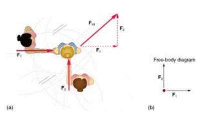


Figure 1. Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

Figure 1(b) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure 2, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in Chapter 22 Magnetism is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.

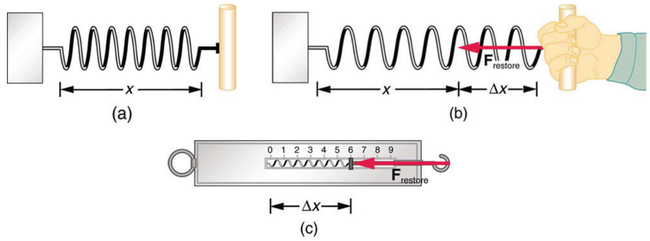


Figure 2. The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\textbf{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\textbf{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\textbf{restore}}$ has a magnitude of 6 units in the force standard being employed.

TAKE-HOME EXPERIMENT: FORCE STANDARDS

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

Section Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

Conceptual Questions

1: Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

2: What properties do forces have that allow us to classify them as vectors?

Glossary

dynamics
the study of how forces affect the motion of objects and systems

external force
a force acting on an object or system that originates outside of the object or system

free-body diagram
a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

force
a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

4.2 Newton's First Law of Motion: Inertia

Summary

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

NEWTON'S FIRST LAW OF MOTION

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton's first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Check Your Understanding

1: Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Section Summary

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

Conceptual Questions

1: How are inertia and mass related?

2: What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

Glossary

inertia
the tendency of an object to remain at rest or remain in motion

law of inertia
see Newton's first law of motion

mass
the quantity of matter in a substance; measured in kilograms

Newton's first law of motion
a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Solutions

Check Your Understanding

1: They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

4.3 Newton's Second Law of Motion: Concept of a System

Summary

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in Figure 1(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at Figure 1(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external*. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

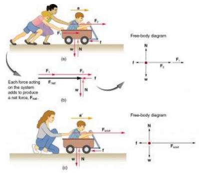


Figure 1. Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight **w** of the system and the support of the ground **N** are also shown for completeness and are assumed to cancel. The vector **f** represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force, $\mathbf{F_{net}}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration (**a'>a**) when an adult pushes the child.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in Figure 1. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\mathbf{w}$ and the support of the ground $\mathbf{N}$, and the horizontal force $\mathbf{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define friction as a force that opposes the motion past each other of objects that are touching. Figure 1(b) shows how vectors representing the external forces add together to produce a net force, $\mathbf{F_{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a} \propto \mathbf{F_{net}},$$

where the symbol $\propto$ means "proportional to," and $\mathbf{F_{net}}$ is the net external force. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in Chapter 3 Two-Dimensional Kinematics.) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in Figure 2, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{1}{m}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

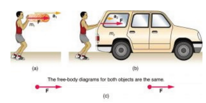


Figure 2. The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

NEWTON'S SECOND LAW OF MOTION

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}.$$

This is often written in the more familiar form

$$\mathbf{F}_{\text{net}} = m\mathbf{a}.$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\text{net}} = ma.$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of $1\ \text{m/s}^2$. That is, $\mathbf{F}_{\text{net}} = m\mathbf{a}$,

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** $\vec{w}$. Weight can be denoted as a vector $\vec{w}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is $F_{net} = ma$.

Since the object experiences only the downward force of gravity, $F_{net} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

WEIGHT

This is the equation for *weight*—the gravitational force on a mass m:

$$w = mg.$$

Since $g = 9.80\ \text{m/s}^2$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = mg = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = 9.8\ \text{N}.$$

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only $1.67\ \text{m/s}^2$. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

COMMON MISCONCEPTIONS: MASS VS. WEIGHT

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is 1.67 m/s^2 (which is much less than the acceleration due to gravity on Earth, 9.80 m/s^2). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

TAKE-HOME EXPERIMENT: MASS AND WEIGHT

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

Example 1: What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration**?**

Figure 3. The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy

Since F_{net} and m are given, the acceleration can be calculated directly from Newton's second law as stated in $F_{net} = ma$.

Solution

The magnitude of the acceleration a is $a = \frac{F_{net}}{m}$. Entering known values gives

$$a = \frac{51 \text{ N}}{24 \text{ kg}}$$

Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$a = \frac{51 \text{ kg} \cdot \text{m/s}^2}{24 \text{ kg}} = 2.1 \text{ m/s}^2.$$

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

Example 2: What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust T, for the four-rocket propulsion system shown in Figure 4. The sled's initial acceleration is 49 m/s^2, the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

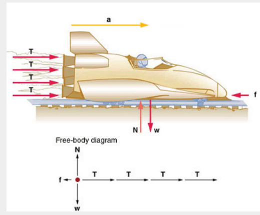


Figure 4. A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust **T**. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force **N** on the system that is equal in magnitude and opposite in direction to its weight, **w**. The system here is the sled, its rockets, and rider, so none of the forces between these objects are considered. The arrow representing friction (**f**) is drawn larger than scale.

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{net} = ma,$$

where F_{net} net is the net force along the horizontal direction. We can see from Figure 4 that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{net} = 4T - f.$$

Substituting this into Newton's second law gives

$$F_{net} = ma = 4T - f.$$

Using a little algebra, we solve for the total thrust $4T$:

$$4T = ma + f.$$

Substituting known values yields

$$4T = ma + f = (2100\ \text{kg})(49\ \text{m/s}^2) + 650\ \text{N}.$$

So the total thrust is

$$4T = 1.0 \times 10^5\ \text{N},$$

and the individual thrusts are

$$T = \frac{1.0\times10^5\ \text{N}}{4} = 2.6 \times 10^4\ \text{N}.$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 ***g***'s. (Recall that g, the acceleration due to gravity, is $9.80\ \text{m/s}^2$. When we say that an acceleration is 45 ***g***'s, it is $45 \times 9.80\ \text{m/s}^2$, which is approximately $440\ \text{m/s}^2$.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

Section Summary

- Acceleration, $\mathbf{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$.
- This is often written in the more familiar form: $\mathbf{F}_{\text{net}} = m\mathbf{a}$.
- The weight $\mathbf{w}$ of an object is defined as the force of gravity acting on an object of mass m. The object experiences an acceleration due to gravity $\mathbf{g}$:

$$\mathbf{w} = m\mathbf{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

Conceptual Questions

1: Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

2: Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

3: Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

4: Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

5: A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

6: A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

7: (a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

8: If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

9: If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

10: The gravitational force on the basketball in Figure 2 is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

Problems & Exercises

You may assume data taken from illustrations is accurate to three digits.

1: A 63.0-kg sprinter starts a race with an acceleration of $4.20\ \text{m/s}^2$. What is the net external force on him?

2: If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

3: A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

4: Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be $0.893\ \text{m/s}^2$. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

5: In Figure 3, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

6: The same rocket sled drawn in Figure 5 is decelerated at a rate of $196\ m/s^2$. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

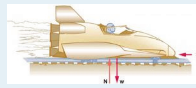


Figure 5.

7: (a) If the rocket sled shown in Figure 6 starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is $2.4 \times 10^4\ N$ and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

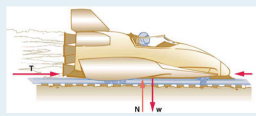


Figure 6.

8: What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

9: Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

10: A powerful motorcycle can produce an acceleration of $3.50\ m/s^2$ while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

11: The rocket sled shown in Figure 7 accelerates at a rate of $49.0\ m/s^2$. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

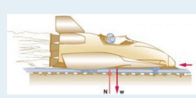


Figure 7.

12: Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of $201\ m/s^2$. In this problem, the forces are exerted by the seat and restraining belts.

13: The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

14: Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

Glossary

acceleration
the rate at which an object's velocity changes over a period of time
free-fall
a situation in which the only force acting on an object is the force due to gravity
friction
a force past each other of objects that are touching; examples include rough surfaces and air resistance
net external force
the vector sum of all external forces acting on an object or system; causes a mass to accelerate
Newton's second law of motion
the net external force $\mathbf{F}_{\text{net}}$ *on an object with mass* m *is proportional to and in the same direction as the acceleration of the object,* $\mathbf{a}$, *and inversely proportional to the mass; defined mathematically as* $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$
system
defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces
weight
the force $\mathbf{w}$ *due to gravity acting on an object of mass* m; *defined mathematically as:* $\mathbf{w} = m\mathbf{g}$, *where* $\mathbf{g}$ *is the magnitude and direction of the acceleration due to gravity*

Solutions

Problems & Exercises

1:

265 N

3:

13.3 m/s

7:

(a)

12 m/s^2.

(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

9:

(a) The system is the child in the wagon plus the wagon.

(b)

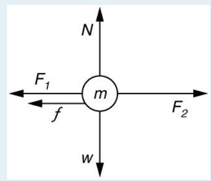


Figure 8.

(c) $a = 0.130\ \text{m/s}^2$ in the direction of the second child's push.

(d) $a = 0.00\ \text{m/s}^2$

11:

(a) $3.68 \times 10^3\ \text{N}$. This force is 5.00 times greater than his weight.

(b) $3750\ \text{N}$; 11.3° above horizontal

13:

$1.5 \times 10^3\ \text{N}$, 150 kg, 150 kg

4.4 Newton’s Third Law of Motion: Symmetry in Forces

Summary

- Understand Newton’s third law of motion.
- Apply Newton’s third law to define systems and solve problems of motion.

There is a passage in the musical *Man of la Mancha* that relates to Newton’s third law of motion. Sancho, in describing a fight with his wife to Don Quixote, says, “Of course I hit her back, Your Grace, but she’s a lot harder than me and you know what they say, ‘Whether the stone hits the pitcher or the pitcher hits the stone, it’s going to be bad for the pitcher.’” This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton’s third law of motion**.

NEWTON’S THIRD LAW OF MOTION

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as “action-reaction,” where the force exerted is the action and the force experienced as a consequence is the reaction. Newton’s third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton’s third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 1. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $F_{\text{wall on feet}}$. In contrast, the force $F_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $F_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $F_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

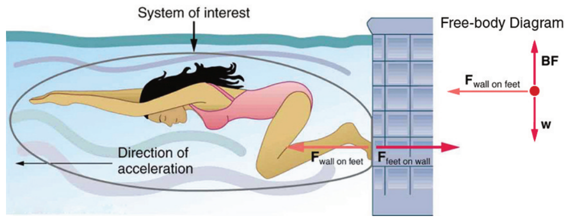


Figure 1. When the swimmer exerts a force $\mathbf{F}_{\textbf{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $\mathbf{F}_{\textbf{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force $\mathbf{F}_{\textbf{wall on feet}}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $\mathbf{F}_{\textbf{feet on wall}}$ does not act on this system (the swimmer) and, thus, does not cancel $\mathbf{F}_{\textbf{wall on feet}}$. Thus the free-body diagram shows only $F_{\textbf{wall on feet}}$, **w**, the gravitational force, and **BF**, the buoyant force of the water supporting the swimmer's weight. The vertical forces **w** and **BF** cancel since there is no vertical motion.

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Example 1: Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in Figure 2. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

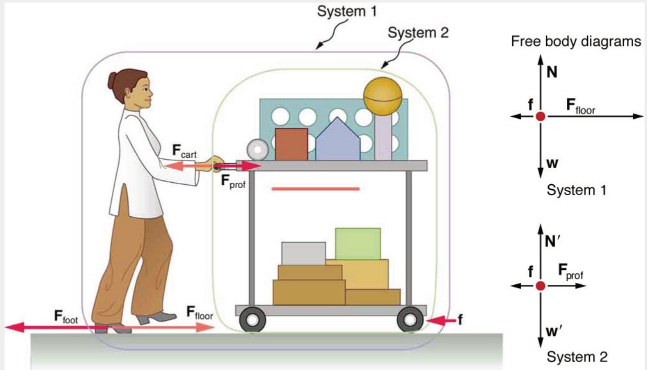


Figure 2. A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for **f**, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for Example 2, since it asks for the acceleration of the entire group of objects. Only $\mathbf{F_{floor}}$ and **f** are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for this example so that $\mathbf{F_{prof}}$ will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure 2. The professor pushes backward with a force $\mathbf{F_{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F_{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, $\mathbf{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F_{floor}}$. Note that we do not include the forces $\mathbf{F_{prof}}$ or $\mathbf{F_{cart}}$ because these are internal forces, and we do not include $\mathbf{F_{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{\text{net}}}{m}.$$

The net external force on System 1 is deduced from Figure 2 and the discussion above to be

$$\mathbf{F_{net}} = \mathbf{F_{floor}} - \mathbf{f} = 150 \text{ N} - 24.0 \text{ N} = 126 \text{ N}.$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0) \text{ kg} = 84 \text{ kg}.$$

These values of F_{net} and m produce an acceleration of

$$a = \frac{F_{\text{net}}}{m}$$
$$a = \frac{126 \text{ N}}{84 \text{ kg}} = 1.5 \text{ m/ s}^2$$

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 2: Force of the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in Figure 2 using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in Figure 2), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, F_{prof} is an external force acting on System 2. F_{prof} was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find F_{prof}. Starting with

$$a = \frac{F_{net}}{m}$$

and noting that the magnitude of the net external force on System 2 is

$$F_{net} = F_{prof} - f,$$

we solve for F_{prof}, the desired quantity:

$$F_{prof} = F_{net} + f.$$

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{net} = ma,$$

where the mass of System 2 is 19.0 kg (m= 12.0 kg + 7.0 kg) and its acceleration was found to be $a = 1.5\ \text{m/s}^2$ in the previous example. Thus,

$$F_{net} = ma,$$

$$F_{net} = (19.0\ \text{kg})(1.5\ \text{m/s}^2) = 29\ \text{N}.$$

Now we can find the desired force:

$$F_{prof} = F_{net} + f,$$

$$F_{prof} = 29\ \text{N} + 24.0\ \text{N} = 53\ \text{N}.$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

PHET EXPLORATIONS: GRAVITY FORCE LAB

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

Figure 3. Gravity Force Lab

Section Summary

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

Conceptual Questions

1: When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

2: A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

3: Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

4: Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?

5: An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

6: Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the “system of interest” affects whether one such pair of forces cancels.

Problems & Exercises

1: What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40 \times 10^4 \text{ m/s}^2$? What is the magnitude of the force exerted on the ship by the artillery shell?

2: A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at 1.20 m/s^2 backward. (a) What is the force of friction between the losing player’s feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

Glossary

Newton’s third law of motion
whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts
thrust
a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

Solutions

Problems & Exercises

1:

Force on shell: $2.64 \times 10^7 \text{ N}$

Force exerted on ship = $-2.64 \times 10^7 \text{ N}$, by Newton’s third law

4.5 Normal, Tension, and Other Examples of Forces

Summary

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in Figure 1(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in Figure 1(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

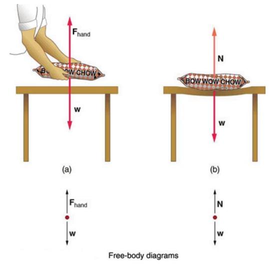


Figure 1. (a) The person holding the bag of dog food must supply an upward force $\mathbf{F_{hand}}$ equal in magnitude and opposite in direction to the weight of the food **w**. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force **N** equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol $\mathbf{N}$. (This is not the unit for force N.) The word normal means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

COMMON MISCONCEPTIONS: NORMAL FORCE (N) VS. NEWTON (N)

In this section we have introduced the quantity normal force, which is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force $\mathbf{N}$ that the floor exerts on a chair might be $\mathbf{N} = 100\ \text{N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

Example 1: Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in Figure 2. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be 45.0 N?

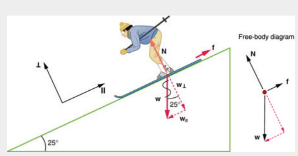


Figure 2. Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** is perpendicular to the slope and **f** is parallel to the slope, but **w** has components along both axes, namely **w?** and $\mathbf{w}_{||}$. **N** is equal in magnitude to **w?**, so that there is no motion perpendicular to the slope, but **f** is less than $\mathbf{w}_{||}$, so that there is a downslope acceleration (along the parallel axis).

Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols $\perp$ and $\parallel$ to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled $\mathbf{w}$, $\mathbf{f}$, and $\mathbf{N}$ in Figure 2. $\mathbf{N}$ is always perpendicular to the slope, and $\mathbf{f}$ is parallel to it. But $\mathbf{w}$ is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining $w_\parallel$ to be the component of weight parallel to the slope and $w_\perp$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $w_\parallel = w\sin(25^\circ) = mg\sin(25^\circ)$, and the magnitude of the component of the weight perpendicular to the slope is $w_\perp = w\cos(25^\circ) = mg\cos(25^\circ)$.

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope $w_\parallel$ and friction f. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_\parallel = \frac{F_{\text{net}\parallel}}{m}$$

where $F_{\text{net}\parallel} = w_\parallel = mg\sin(25^\circ)$, assuming no friction for this part, so that

$$a_\parallel = \frac{F_{\text{net}\parallel}}{m} = \frac{mg\sin(25^\circ)}{m} = g\sin(25^\circ)$$

$$(9.80\ \text{m/s}^2)(0.4226) = 4.14\ \text{m/s}^2$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{\text{net}\parallel} = w_\parallel - f,$$

and substituting this into Newton's second law, $a_\parallel = \frac{F_{\text{net}\parallel}}{m}$, gives

$$a_\parallel = \frac{F_{\text{net}\parallel}}{m} = \frac{w_\parallel - f}{m} = \frac{mg\sin(25^\circ) - f}{m}.$$

We substitute known values to obtain

$$a_\parallel = \frac{(60.0\ \text{kg})(9.80\ \text{m/s}^2)(0.4226) - 45.0\ \text{N}}{60.0\ \text{kg}},$$

which yields

$$a_\parallel = 3.39\ \text{m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g\sin\theta$, *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

RESOLVING WEIGHT INTO COMPONENTS

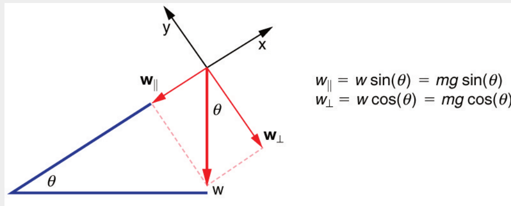


Figure 3. An object rests on an incline that makes an angle θ with the horizontal.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, $w_{\perp}$, and a force acting parallel to the plane, $w_{\parallel}$. The perpendicular force of weight, $w_{\perp}$, is typically equal in magnitude and opposite in direction to the normal force, N. The force acting parallel to the plane, $w_{\parallel}$, causes the object to accelerate down the incline. The force of friction, f, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_{\parallel} = w\sin(\theta) = mg\sin(\theta)$$

and

$$w_{\perp} = w\cos(\theta) = mg\cos(\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle θ of the incline is the same as the angle formed between w and w_{perp}. Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$\cos(\theta) = \frac{w_{\perp}}{w}$$
$$w_{\perp} = w\cos(\theta) = mg\cos(\theta)$$

$$\sin(\theta) = \frac{w_{\parallel}}{w}$$
$$w_{\parallel} = w\sin(\theta) = mg\sin(\theta)$$

TAKE-HOME EXPERIMENT: FORCE PARALLEL

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure 4.

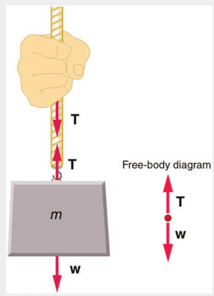


Figure 4. When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force **T**, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $F_{net} = 0$. The only external forces acting on the mass are its weight w and the tension T supplied by the rope. Thus,

$$F_{net} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T = w = mg$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T = mg = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in Figure 5 (a) and (b).

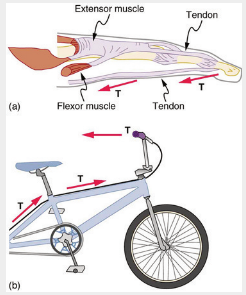


Figure 5. (a) Tendons in the finger carry force **T** from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension **T** from the handlebars to the brake mechanism. Again, the direction but not the magnitude of **T** is changed.

Example 2: What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in Figure 6.

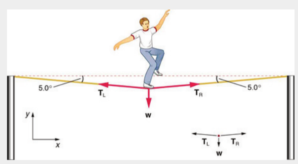


Figure 6. The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight w and the two tensions T_L (left tension) and T_R (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions T_L and T_R must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are T_L and T_R. Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the x-axis and the vertical the y-axis.

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.

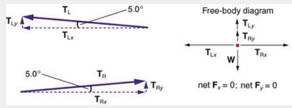


Figure 7. When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in **T** being much greater than **w**.

Consider the horizontal components of the forces (denoted with a subscript x):

$$F_{\text{net}x} = T_{\text{L}x} - T_{\text{R}x}.$$

The net external horizontal force $F_{\text{net}x} = 0$ since the person is stationary. Thus,

$$\begin{aligned} F_{\text{net}x} &= 0 = T_{\text{L}x} - T_{\text{R}x} \\ T_{\text{L}x} &= T_{\text{R}x} \end{aligned}$$

Now, observe Figure 7. You can use trigonometry to determine the magnitude of T_{L} and T_{R}. Notice that:

$$\begin{aligned} \cos(5.0^\circ) &= \frac{T_{\text{L}x}}{T_{\text{L}}} \\ T_{\text{L}x} &= T_{\text{L}}\cos(5.0^\circ) \\ \cos(5.0^\circ) &= \frac{T_{\text{R}x}}{T_{\text{R}}} \\ T_{\text{R}x} &= T_{\text{R}}\cos(5.0^\circ) \end{aligned}$$

Equating $T_{\text{L}x}$ and $T_{\text{R}x}$:

$$T_{\text{L}}\cos(5.0^\circ) = T_{\text{R}}\cos(5.0^\circ).$$

Thus,

$$T_{\text{L}} = T_{\text{R}} = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript y), we can solve for T. Again, since the person is stationary, Newton’s second law implies that net $F_y = 0$. Thus, as illustrated in the free-body diagram in Figure 7,

$$F_{\text{net}y} = T_{\text{L}y} + T_{\text{R}y} - w = 0.$$

Observing Figure 7, we can use trigonometry to determine the relationship between $T_{\text{L}y}$, $T_{\text{R}y}$, and T. As we determined from the analysis in the horizontal direction, $T_{\text{L}} = T_{\text{R}} = T$:

$$\begin{aligned} \sin(5.0^\circ) &= \frac{T_{\text{L}y}}{T_{\text{L}}} \\ T_{\text{L}y} = T_{\text{L}}\sin(5.0^\circ) &= T\sin(5.0^\circ) \\ \sin(5.0^\circ) &= \frac{T_{\text{R}y}}{T_{\text{R}}} \\ T_{\text{R}y} = T_{\text{R}}\sin(5.0^\circ) &= T\sin(5.0^\circ) \end{aligned}$$

Now, we can substitute the values for $T_{\text{L}y}$ and $T_{\text{R}y}$ into the net force equation in the vertical direction:

$$\begin{aligned} F_{\text{net}y} &= T_{\text{L}y} + T_{\text{L}y} - w = 0 \\ F_{\text{net}y} &= T\sin(5.0^\circ) + T\sin(5.0^\circ) - w = 0 \\ 2T\sin(5.0^\circ) - w &= 0 \\ 2T\sin(5.0^\circ) &= w \end{aligned}$$

and

$$T = \frac{w}{2\sin(5.0^\circ)} = \frac{mg}{2\sin(5.0^\circ)},$$

so that

$$T = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)}{2(0.0872)},$$

and the tension is

$$T = 3900\ \text{N}.$$

Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in Figure 8. As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2\sin(\theta)}.$$

We can extend this expression to describe the tension T created when a perpendicular force ($F_{\perp}$) is exerted at the middle of a flexible connector:

$$T = \frac{F_{\perp}}{2\sin(\theta)}.$$

Note that θ is the angle between the horizontal and the bent connector. In this case, T becomes very large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta = 0$ and $\sin\theta = 0$). (See Figure 8.)

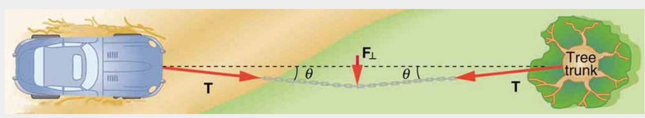


Figure 8. We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by **T = F? / 2 sin(θ)**; since ***θ*** is small, ***T*** is very large. This situation is analogous to the tightrope walker shown in Figure 6, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where **F?** is applied.

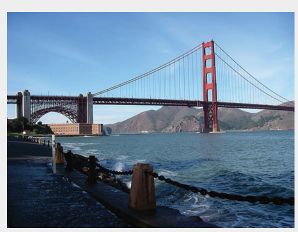


Figure 9. Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.

PHET EXPLORATIONS: FORCES IN 1 DIMENSION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

Figure 10. Forces in 1 Dimension

Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, N.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:

$$N = mg.$$

- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular ($w_{\perp}$) and parallel ($w_{\parallel}$) to the surface of the plane. These components can be calculated using:

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta)$$

$$w_{\perp} = w \cos(\theta) = mg \cos(\theta).$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, T. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

$$T = mg.$$

- In any inertial frame of reference (one that is not accelerated or rotated), Newton’s laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

Conceptual Questions

1: If a leg is suspended by a traction setup as shown in Figure 11, what is the tension in the rope?

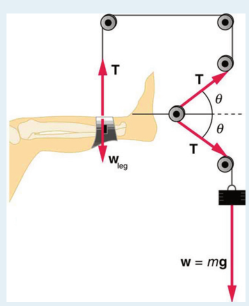


Figure 11. A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force T without changing its magnitude.

2: In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See Figure 11.) (Note that the tibia is the shin bone shown in this image.)

Problems & Exercises

1: Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

2: What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at 7.50 m/s^2? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

3: (a) Calculate the tension in a vertical strand of spider web if a spider of mass $8.00 \times 10^{-5} \text{ kg}$ hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure 6. The strand sags at an angle of $12°$ below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

4: Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of 1.50 m/s^2?

5: Show that, as stated in the text, a force $F_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure 6) gives rise to a tension of magnitude $T = \frac{F_\perp}{2\sin(\theta)}$.

6: Consider the baby being weighed in Figure 12. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

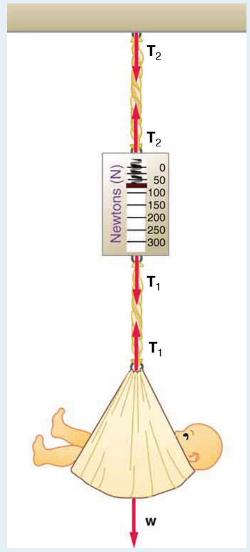


Figure 12. A baby is weighed using a spring scale.

Glossary

inertial frame of reference
a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

normal force
the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

tension
the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

Solutions

Problems & Exercises

1:

(a)$0.11\ \text{m/s}^2$

(b)$1.2 \times 10^4\ \text{N}$

3:

(a)$7.84 \times 10^{-4}\ \text{N}$

(b)$1.89 \times 10^{-3}\ \text{N}$. This is 2.41 times the tension in the vertical strand.

5:

Newton’s second law applied in vertical direction gives

$$F_y = F - 2T \sin\theta = 0$$

$$F = 2T \sin\theta$$

$$T = \frac{F}{2\sin\theta}.$$

4.6 Problem-Solving Strategies

Summary

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation*. Such a sketch is shown in Figure 1(a). Then, as in Figure 1(b), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).

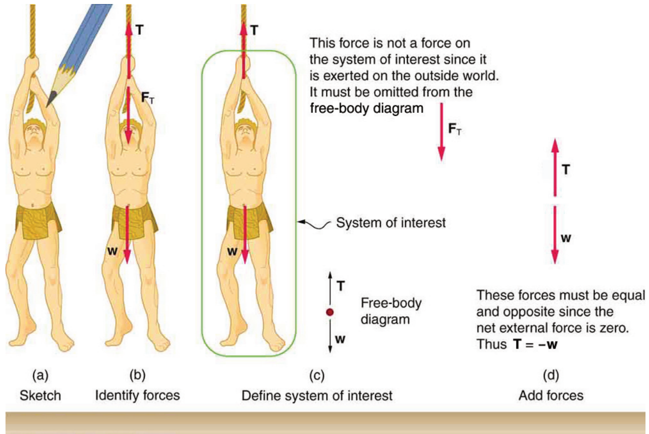


Figure 1. (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces. **T** is the tension in the vine above Tarzan, $\mathbf{F_T}$ is the force he exerts on the vine, and **w** is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram. $\mathbf{F_T}$ is no longer shown, because it is not a force acting on the system of interest; rather, $\mathbf{F_T}$ acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that **T = -w**, if Tarzan is stationary.

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest*. This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See Figure 1(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. Figure 1(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem*. This is done in Figure 1(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation: $F_{net} = ma$.

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{net\ x} = ma,$$

$$F_{net\ y} = 0.$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

Section Summary

- To solve problems involving Newton's laws of motion, follow the procedure described:

  1. Draw a sketch of the problem.

  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.

  3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x-direction) then $F_{net\ x} = 0$. If the object does accelerate in that direction, $F_{net\ x} = ma$.

  4. Check your answer. Is the answer reasonable? Are the units correct?

Problems & Exercises

1: A $5.00 \times 10^5\text{-kg}$ rocket is accelerating straight up. Its engines produce $1.250 \times 10^7\ \text{N}$ of thrust, and air resistance is $4.50 \times 10^6\ \text{N}$. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

2: The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is $1.80\ \text{m/s}^2$, what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.

3: Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

4: When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.

5: A freight train consists of two $8.00 \times 10^4\text{-kg}$ engines and 45 cars with average masses of $5.50 \times 10^4\ \text{kg}$. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2}\ \text{m/s}^2$ if the force of friction is $7.50 \times 10^5\ \text{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

6: Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of $1.75 \times 10^4\ \text{N}$ backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is $0.150\ \text{m/s}^2$, what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.

7: A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of $0.550\ \text{m/s}^2$? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?

8: (a) Find the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ that add to give the total force $\mathbf{F}_{\text{tot}}$ shown in Figure 2. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of $\mathbf{F}_1$ and $\mathbf{F}_2$. (c) Find the direction and magnitude of some other pair of vectors that add to give $\mathbf{F}_{\text{tot}}$. Draw these to scale on the same drawing used in part (b) or a similar picture.

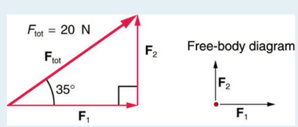


Figure 2.

9: Two children pull a third child on a snow saucer sled exerting forces $\mathbf{F}_1$ and $\mathbf{F}_2$ as shown from above in Figure 3. Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of $\mathbf{F}_1$ and $\mathbf{F}_2$.

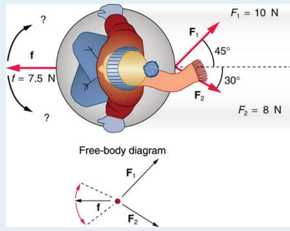


Figure 3. An overhead view of the horizontal forces acting on a child's snow saucer sled.

10: Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in Figure 4 to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is 2.00°? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to 7.00° and you still apply the force found in part (a) to its center?

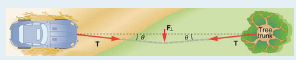


Figure 4.

11: What force is exerted on the tooth in Figure 5 if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

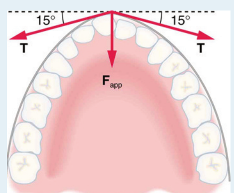


Figure 5. Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire, $\mathbf{F}_{\mathbf{app}}$, points straight toward the back of the mouth.

12: Figure 6 shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.

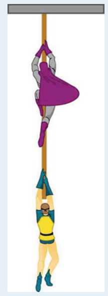


Figure 6. Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?

13: A nurse pushes a cart by exerting a force on the handle at a downward angle 35.0° below the horizontal. The loaded cart has a mass of 28.0 kg, and the force of friction is 60.0 N. (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?

14: Construct Your Own Problem Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.

15: Construct Your Own Problem Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.

16: Unreasonable Results (a) Repeat Exercise 7, but assume an acceleration of 1.20 m/s^2 is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?

17: Unreasonable Results (a) What is the initial acceleration of a rocket that has a mass of $1.50 \times 10^6\text{ kg}$ at takeoff, the engines of which produce a thrust of $2.00 \times 10^6\text{ N}$? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

Solutions

Problems & Exercises

1:

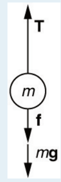


Figure 7.

Using the free-body diagram:

$F_{\text{net}} = T - f - mg = ma,$

so that

$a = \frac{T - f - mg}{m} = \frac{1.250 \times 10^7\ \text{N} - 4.50 \times 10^6\ N - (5.00 \times 10^5\ \text{kg})(9.80\ \text{m/s}^2)}{5.00 \times 10^5\ \text{kg}} = 6.20\ \text{m/s}^2$

3:

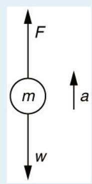


Figure 8.

1. Use Newton's laws of motion
2. Given : $a = 4.00g = (4.00)(9.80\ \text{m/s}^2) = 39.2\ \text{m/s}^2$; $m = 70.0\ \text{kg},$
   Find: $F.$
3. $\sum F = +F - w = ma,$ so that $F = ma + w = ma + mg = m(a + g).$
   $F = (70.0\ \text{kg})[(39.2\ \text{m/s}^2) + (9.80\ \text{m/s}^2)] = 3.43 \times 10^3\ \text{N}.$ The force exerted by the high-jumper is actually down on the ground, but F is up from the ground and makes him jump.
4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of $10^3\ \text{N}.$

5:

(a) $4.41 \times 10^5\ \text{N}$

(b) $1.50 \times 10^5\ \text{N}$

7:

(a) $910\ \text{N}$

(b) $1.11 \times 10^3\ \text{N}$

9:

$a = 0.139\ \text{m/s}, \theta = 12.4^{\circ}$ north of east

11:

1. Use Newton's laws since we are looking for forces.
2. Draw a free-body diagram:

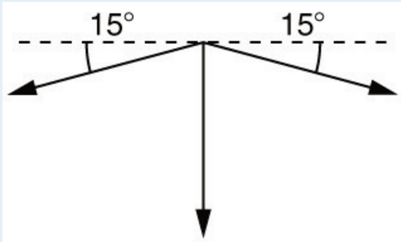


Figure 9.

The tension is given as $T = 25.0\ \text{N}$. Find F_{app}. Using Newton's laws gives: $\Sigma F_y = 0$, so that applied force is due to the *y*-components of the two tensions: $F_{\text{app}} = 2T\sin\theta = 2(25.0\ \text{N})\sin(15^\circ) = 12.9\ \text{N}$

The *x*-components of the tension cancel. $\Sigma F_x = 0$.

This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.

4.7 Further Applications of Newton's Laws of Motion

Summary

- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

Example 1: Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in Figure 1. The first tugboat exerts a force of $2.7 \times 10^5\ \text{N}$ in the *x*-direction, and the second tugboat exerts a force of $3.6 \times 10^5\ \text{N}$ in the *y*-direction.

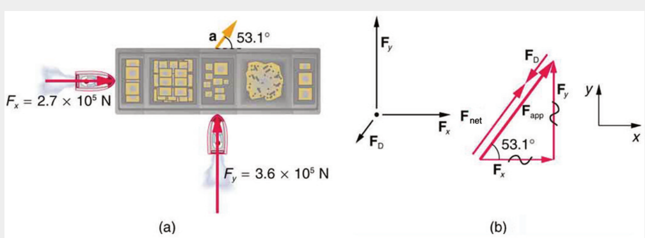


Figure 1. (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the x- and y-axes are in the same direction as $\mathbf{F}_x$ and $\mathbf{F}_y$. The problem quickly becomes a one-dimensional problem along the direction of $\mathbf{F}_{\text{app}}$, since friction is in the direction opposite to $\mathbf{F}_{\text{app}}$.

If the mass of the barge is $5.0 \times 10^6\ \text{kg}$ and its acceleration is observed to be $7.5 \times 10^{-2}\ \text{m/s}^2$ in the direction shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

Strategy

The directions and magnitudes of acceleration and the applied forces are given in Figure 1**(a)**. We will define the total force of the tugboats on the barge as $\mathbf{F}_{\text{app}}$ so that:

$$\mathbf{F}_{\text{app}} = \mathbf{F}_x + \mathbf{F}_y$$

Since the barge is flat bottomed, the drag of the water $\mathbf{F}_{\text{D}}$ will be in the direction opposite to $\mathbf{F}_{\text{app}}$, as shown in the free-body diagram in Figure 1(b). The system of interest here is the barge, since the forces on *it* are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force $\mathbf{F}_{\text{app}}$, and then apply Newton's second law to solve for the drag force $\mathbf{F}_{\text{D}}$.

Solution

Since $\mathbf{F}_x$ and $\mathbf{F}_y$ are perpendicular, the magnitude and direction of $\mathbf{F}_{\text{app}}$ are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$F_{\text{app}} = \sqrt{\mathbf{F}_x^2 + \mathbf{F}_y^2}$$

$$F_{\text{app}} = \sqrt{(2.7 \times 10^5\ \text{N})^2 + (3.6 \times 10^5\ \text{N})^2} = 4.5 \times 10^5\ \text{N}.$$

The angle is given by

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{3.6 \times 10^5\ \text{N}}{2.7 \times 10^5\ \text{N}}\right) = 53^o,$$

which we know, because of Newton's first law, is the same direction as the acceleration. $\mathbf{F}_{\text{D}}$ is in the opposite direction of $\mathbf{F}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\mathbf{F}_{\text{app}}$, but its magnitude is slightly less than $\mathbf{F}_{\text{app}}$. The problem is now one-dimensional. From Figure 1**(b)**, we can see that

$$F_{\text{net}} = F_{\text{app}} - F_{\text{D}}.$$

But Newton's second law states that

$$F_{\text{net}} = ma.$$

Thus,

$$F_{\text{app}} - F_{\text{D}} = ma.$$

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

$$F_{\text{D}} = F_{\text{app}} - ma.$$

Substituting known values gives

$$F_{\text{D}} = (4.5 \times 10^5\ \text{N}) - (5.0 \times 10^6\ \text{kg})(7.5 \times 10^{-2}\ \text{m/s}^2) = 7.5 \times 10^4\ \text{N}.$$

The direction of $\mathbf{F}_{\text{D}}$ has already been determined to be in the direction opposite to $\mathbf{F}_{\text{app}}$, or at an angle of 53^o south of west.

Discussion

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

Example 2: Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in Figure 2. Find the tension in each wire, neglecting the masses of the wires.

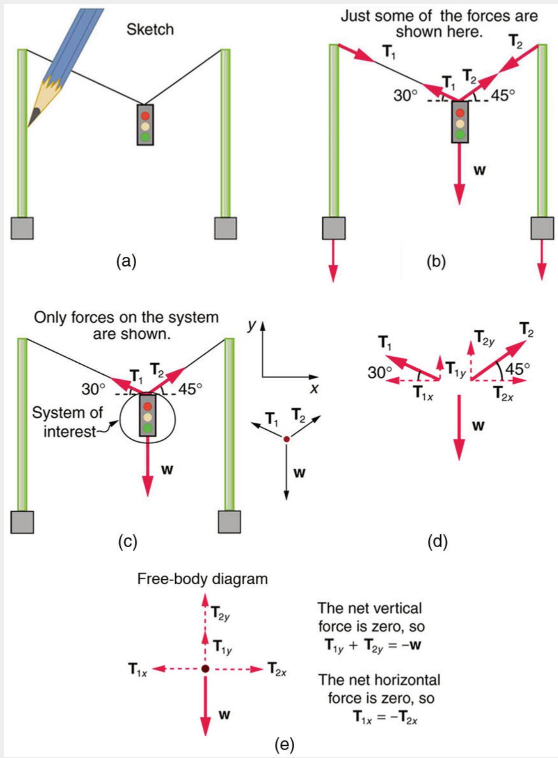


Figure 2. A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in Figure 2(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

$$T_{1x} = T_{2x}.$$

This gives us the following relationship between T_1 and T_2:

$$T_1 \cos(30^\circ) = T_2 \cos(45^\circ).$$

Thus,

$$T_2 = (1.225)T_1.$$

Note that T_1 and T_2 are not equal in this case, because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1 because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

$$F_{\text{net}\,y} = T_{1y} + T_{2y} - w = 0.$$

This implies

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

$$T_1 \sin(30^\circ) + T_2 \sin(45^\circ) = w.$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg,$$

which yields

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2).$$

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108 \text{ N}.$$

Finally, the magnitude of T_2 is determined using the relationship between them, T_2 = 1.225 T_1 found above. Thus we obtain

$$T_2 = 132 \text{ N}.$$

Discussion

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

Example 3: What Does the Bathroom Scale Read in an Elevator?

Figure 3 shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of $1.20\ \mathrm{m/s^2}$, and (b) if the elevator moves upward at a constant speed of 1 m/s.

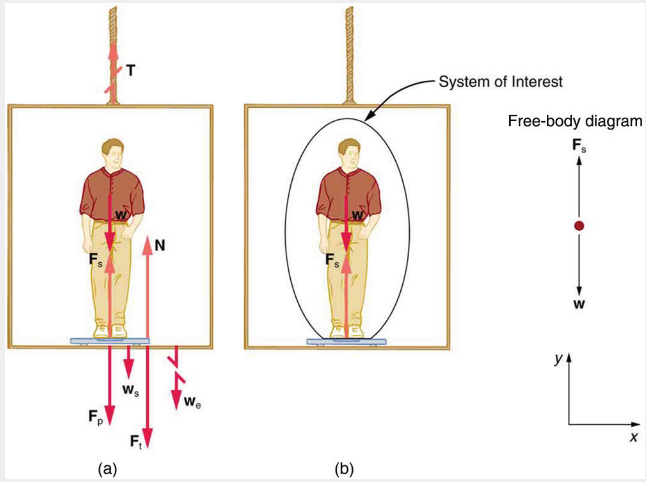


Figure 3. (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. **T** is the tension in the supporting cable, **w** is the weight of the person, $\mathbf{w_s}$ is the weight of the scale, $\mathbf{w_e}$is the weight of the elevator, $\mathbf{F_s}$ is the force of the scale on the person, $\mathbf{F_p}$ is the force of the person on the scale, $\mathbf{F_t}$ is the force of the scale on the floor of the elevator, and **N** is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

Strategy

If the scale is accurate, its reading will equal F_p the magnitude of the force the person exerts downward on it. Figure 3(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in Figure 3(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight w and the upward force of the scale F_s. According to Newton's third law F_p and F_s are equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$F_{net} = ma.$$

From the free-body diagram we see that $F_{net} = F_s - w$, so that

$$F_s - w = ma.$$

Solving for F_s gives an equation with only one unknown:

$$F_s = ma + w,$$

or, because $w = mg$, simply

$$F_s = ma + mg.$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

Solution for (a)

In this part of the problem, $a = 1.20 \text{ m/s}^2$, so that

$$F_s = (75.0 \text{ kg})(1.20 \text{ m/s}^2) + (75.0 \text{ kg})(9.80 \text{ m/s}^2),$$

yielding

$$F_s = 825 \text{ N}.$$

Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$F_{net} = ma = 0 = F_s - w$$

$$F_s = w = mg$$

$$F_s = (75.0 \text{ kg})(9.80 \text{ m/s}^2)$$

$$F_s = 735 \text{ N}.$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$, and $\Delta v = 0$.

Thus,

$$F_s = ma + mg = 0 + mg.$$

Now

$$F_s = (75.0 \text{ kg})(9.80 \text{ m/s}^2),$$

which gives

$$F_s = 735 \text{ N}.$$

Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, a is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at g, then the scale reading will be zero and the person will *appear* to be weightless.

Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

Problem-Solving Strategy

Step 1. *Identify which physical principles are involved*. Listing the givens and the quantities to be calculated will allow you to identify the principles involved.

Step 2. *Solve the problem using strategies outlined in the text*. If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.

Example 4: What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

Strategy

1. *To solve an integrated concept problem,* we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics.* Part (b) deals with force, a topic of *dynamics* found in this chapter.

2. The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\ \text{m/s}$. We are given the elapsed time, and so $\Delta t = 2.50\ \text{s}$. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields

$$a = \frac{8.00\ \text{m/s}}{2.50\ \text{s}}$$

$$= 3.20\ \text{m/s}^2.$$

Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = ma.$$

Substituting the known values of m and a gives

$$F_{\text{net}} = (70.0\ \text{kg})(3.20\ \text{m/s}^2)$$

$$= 224\ \text{N}.$$

Discussion for (b)

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Summary

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.

- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{net} = ma$ or $F_{net} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.
- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

Conceptual Questions

1: To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

2: A cartoon shows the toupee coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

Problems & Exercises

1: A flea jumps by exerting a force of $1.20 \times 10^{-5}\ \text{N}$ straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of $0.500 \times 10^{-6}\ \text{N}$ on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is $6.00 \times 10^{-7}\ \text{kg}$. Do not neglect the gravitational force.

2: Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in Figure 4. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

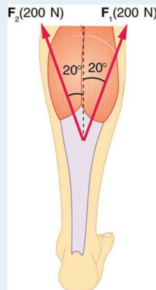


Figure 4. Achilles tendon

3: A 76.0-kg person is being pulled away from a burning building as shown in Figure 5. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.

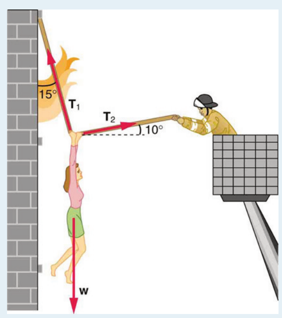


Figure 5. The force $\mathbf{T_2}$ needed to hold steady the person being rescued from the fire is less than her weight and less than the force $\mathbf{T_1}$ in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force)

4: Integrated Concepts A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

5: Integrated Concepts When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

6: Integrated Concepts A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of 3.50×10^7 N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.

7: Integrated Concepts A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.

8: Integrated Concepts A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

9: Integrated Concepts Repeat Exercise 8 for a shell fired at an angle 10.0° from the vertical.

10: Integrated Concepts An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of 1.20 m/s^2 for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of 0.600 m/s^2 for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

11: Unreasonable Results (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of 0.400 m/s^2 for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

12: Unreasonable Results A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

Solutions

Problems & Exercises

1:

$10.2\ \text{m/s}^2, 4.67^0$ from vertical

3:

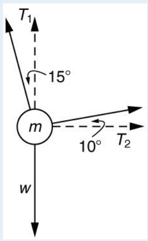


Figure 6.

$T_1 = 736\ \text{N}$

$T_2 = 194\ \text{N}$

5:

(a) $7.43\ \text{m/s}$

(b) 2.97 m

7:

(a) $4.20\ \text{m/s}$

(b) $29.4\ \text{m/s}^2$

(c) $4.31 \times 10^3\ \text{N}$

9:

(a) 47.1 m/s

(b) $2.47 \times 10^3\ \text{m/s}^2$

(c) $6.18 \times 10^3\ \text{N}$. The average force is 252 times the shell's weight.

4.8 Extended Topic: The Four Basic Forces—An Introduction

Summary

- Understand the four basic forces that underlie the processes in nature.

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a **force field** rather than by "physical contact."

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in Table 1. Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

CONCEPT CONNECTIONS: THE FOUR BASIC FORCES

The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in Chapter 6 Uniform Circular Motion and Gravitation, electric force in Chapter 18 Electric

Charge and Electric Field, magnetic force in Chapter 22 Magnetism, and nuclear forces in Chapter 31 Radioactivity and Nuclear Physics. On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Force	Approximate Relative Strengths	Range	Attraction/Repulsion	Carrier Particle
Gravitational	10^{-38}	∞	attractive only	Graviton
Electromagnetic	10^{-2}	∞	attractive and repulsive	Photon
Weak nuclear	10^{-13}	$< 10^{-18} m$	attractive and repulsive	W^+, W^-, Z^0
Strong nuclear	1	$< 10^{-15} m$	attractive and repulsive	gluons

Table 1. Properties of the Four Basic Forces[1].

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

CONCEPT CONNECTIONS: UNIFYING FORCES

Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By "unify" we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

Action at a Distance: Concept of a Field

All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a **force field** surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the “thing” that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth’s gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields $w = mg$ at Earth’s surface), and motions can be calculated from these equations. (See Figure 1.)

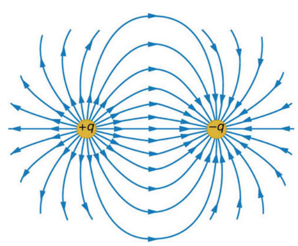


Figure 1. The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines.

CONCEPT CONNECTIONS: FORCE FIELDS

The concept of a *force field* is also used in connection with electric charge and is presented in Chapter 18 Electric Charge and Electric Field. It is also a useful idea for all the basic forces, as will be seen in Chapter

33 Particle Physics. Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa's (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See Figure 2.)

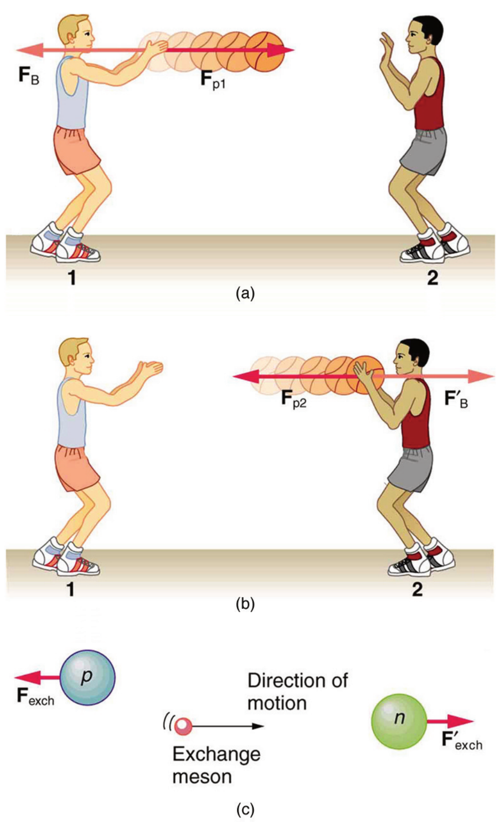


Figure 2. The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force $\mathbf{F_{p1}}$ on it toward the other person and feels a reaction force $\mathbf{F_B}$ away from the second person. (b) The person catching the basketball exerts a force $\mathbf{F_{p2}}$ on it to stop the ball and feels a reaction force $\mathbf{F'_B}$away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces $\mathbf{F_{exch}}$ and $\mathbf{F'_{exch}}$ between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive.

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. Table 1 lists the exchange or **carrier particles**, both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See Figure 3.) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.

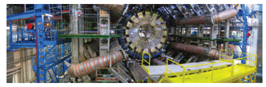


Figure 3. The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. (credit: Frank Hommes)

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Almost 100 years ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) (Figure 4). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project might be as early as 2018.

"I'm sure LIGO will tell us something about the universe that we didn't know before. The history of science tells us that any time you go where you haven't been before, you usually find something that really shakes the scientific paradigms of the day. Whether gravitational wave astrophysics will do that, only time will tell." —David Reitze, LIGO Input Optics Manager, University of Florida

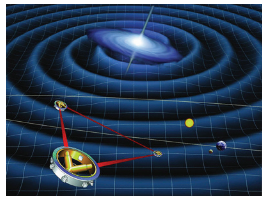


Figure 4. Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves. (credit: NASA)

The ideas presented in this section are but a glimpse into topics of modern physics that will be covered in much greater depth in later chapters.

Summary

- The various types of forces that are categorized for use in many applications are all manifestations of the *four basic forces* in nature.
- The properties of these forces are summarized in Table 1.
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

Conceptual Questions

1: Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.

2: What is the dominant force between astronomical objects? Why are the other three basic forces less significant over these very large distances?

3: Give a detailed example of how the exchange of a particle can result in an *attractive* force. (For example, consider one child pulling a toy out of the hands of another.)

Problems & Exercises

1: (a) What is the strength of the weak nuclear force relative to the strong nuclear force? (b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a type of nuclear decay not explained by other forces.

2: (a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force? (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force? (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?

3: What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.

Footnotes

1. 1 The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles W^+, W^-, and Z^0 are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms.

Glossary

carrier particle
a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

force field
a region in which a test particle will experience a force

Solutions

Problems & Exercises

1:

(a) 1×10^{-13}

(b) 1×10^{-11}

3:

10^2

PART 5

Chapter 5 Further Applications of Newton's Laws: Friction, Drag and Elasticity

Figure 1. Total hip replacement surgery has become a common procedure. The head (or ball) of the patient's femur fits into a cup that has a hard plastic-like inner lining. (credit: National Institutes of Health, via Wikimedia Commons)

Describe the forces on the hip joint. What means are taken to ensure that this will be a good movable joint? From the photograph (for an adult) in Figure 1, estimate the dimensions of the artificial device.

It is difficult to categorize forces into various types (aside from the four basic forces discussed in previous chapter). We know that a net force affects the motion, position, and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton's laws of motion. We have in mind the forces of friction, air or liquid drag, and deformation.

5.1 Friction

Summary

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

FRICTION

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

KINETIC FRICTION

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 1 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

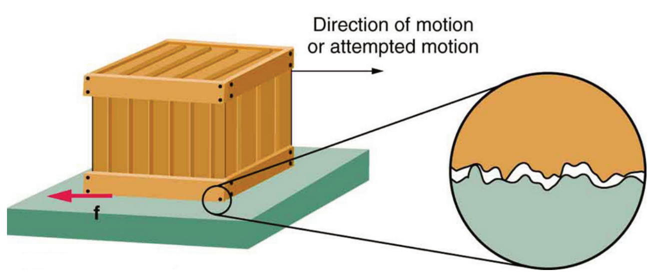


Figure 1. Frictional forces, such as **f**, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction** is

$$f_s \leq \mu_s N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

MAGNITUDE OF STATIC FRICTION

Magnitude of static friction f_s is

$$f_s \leq \mu_s N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_s \mathrm{N}$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{s(\max)}$, the object will move. Thus

$$f_{s(\max)} = \mu_s \mathrm{N}.$$

Once an object is moving, the **magnitude of kinetic friction** f_k is given by

$$f_k = \mu_k \mathrm{N},$$

where mu_k is the coefficient of kinetic friction. A system in which $f_k = \mu_k \mathrm{N}$ is described as a system in which *friction behaves simply*.

MAGNITUDE OF KINETIC FRICTION

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k \mathrm{N},$$

where μ_k is the coefficient of kinetic friction.

As seen in Table 1, the coefficients of kinetic friction are less than their static counterparts. That values of μ in Table 1 are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	Static friction, μ_s	Kinetic friction, μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Table 1. Coefficients of Static and Kinetic Friction.

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 980\ \mathrm{N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{s(\mathrm{max})} = \mu N = (0.45)\ (980\ \mathrm{N}) = 440\ \mathrm{N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_k = \mu_k N = (0.30)(980\ \mathrm{N}) = 290\ \mathrm{N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

TAKE-HOME EXPERIMENT

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 2). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

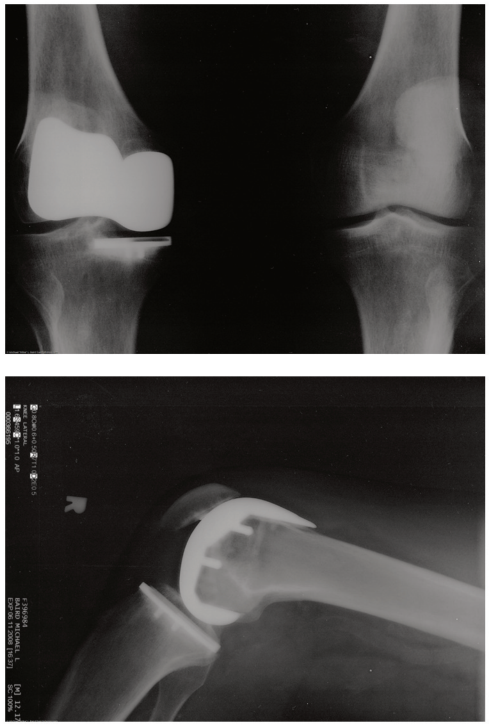


Figure 2. Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

Example 1: Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force$_N$

as $f_k = \mu_k N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in Figure 3.)

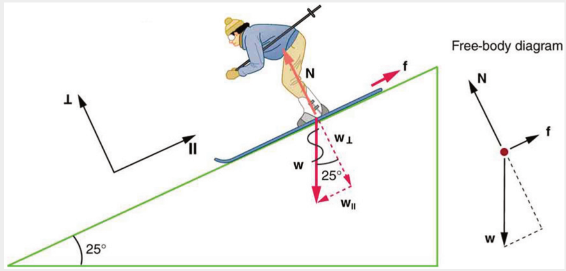


Figure 3. The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). **N** (the normal force) is perpendicular to the slope, and **f** (the friction) is parallel to the slope, but **w** (the skier's weight) has components along both axes, namely **w?** and **w//**. **N** is equal in magnitude to **w?**, so there is no motion perpendicular to the slope. However, **f** is less than **w//** in magnitude, so there is acceleration down the slope (along the x-axis).

That is,

$$N = w_\perp = w\cos 25^\circ = mg\cos 25^\circ.$$

Substituting this into our expression for kinetic friction, we get

$$f_k = \mu_k mg\cos 25^\circ,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w\cos 25^\circ} = \frac{f_k}{mg\cos 25^\circ}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\ \text{N}}{(62\ \text{kg})(9.80\ \text{m/s}^2)(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table 1 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k mg\cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

TAKE-HOME EXPERIMENT

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in Example 1, the kinetic friction on a slope $f_k = \mu_k mg\cos\theta$. The component of the weight down the slope is equal to $mg\sin\theta$ (see the free-body diagram in Figure 3). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_k = Fg_x$$

$$\mu_k mg \cos\theta = mg \sin\theta.$$

Solving for μ_k, we find that

$$\mu_k = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta.$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_k. Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_k and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

MAKING CONNECTIONS: SUBMICROSCOPIC EXPLANATIONS OF FRICTION

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 4 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

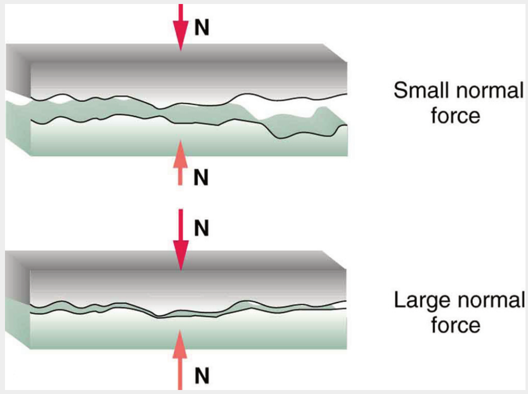


Figure 4. Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure 5 shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

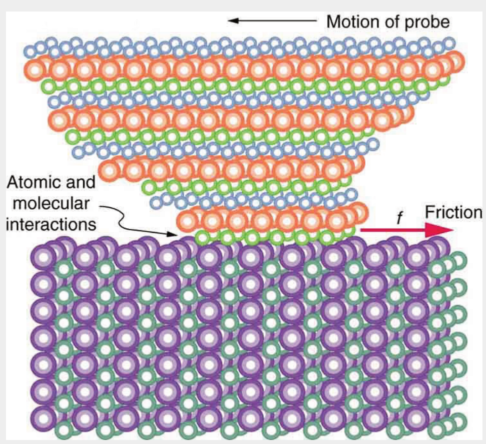


Figure 5. The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

PHET EXPLORATIONS: FORCES AND MOTION

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Figure 6. Forces and Motion

Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_s between systems stationary relative to one another is given by

  $$f_s \leq \mu_s N,$$

  where μ_s is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_k between systems moving relative to one another is given by

  $$f_k = \mu_k N,$$

  where μ_k is the coefficient of kinetic friction, which also depends on both materials.

Conceptual Questions

1: Define normal force. What is its relationship to friction when friction behaves simply?

2: The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

3: When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

4: When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

Problems & Exercises

1: A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

2: (a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

3: (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

4: Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

5: (a) If half of the weight of a small $1.00 \times 10^3\ \text{kg}$ utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

6: A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

7: Consider the 65.0-kg ice skater being pushed by two others shown in Figure 7. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

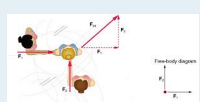


Figure 7.

8: Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin\theta$. (Note that this acceleration is independent of mass.)

9: Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_k = \mu_k N$) is $a = g(\sin\theta - \mu_k \cos\theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small $(\mu_k = 0)$.

10: Calculate the deceleration of a snow boarder going up a 5.0° slope assuming the coefficient of friction for waxed wood on wet snow. The result of Exercise 9 may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in Chapter 4.6 Problem-Solving Strategies.

11: (a) Calculate the acceleration of a skier heading down a 10.0° slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of Exercise 9 to be useful. Explicitly show how you follow the steps in the Chapter 4.6 Problem-Solving Strategies.

12: If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1}\mu_s$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction has reached its maximum value.

13: Calculate the maximum deceleration of a car that is heading down a 6° slope (one that makes an angle of 6° with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

14: Calculate the maximum acceleration of a car that is heading up a 4° slope (one that makes an angle of 4° with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

15: Repeat Exercise 14 for a car with four-wheel drive.

16: A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of 5.00×10^{-2} m/s^2 if the force of friction is 7.50×10^5 N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

17: Consider the 52.0-kg mountain climber in Figure 8. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?

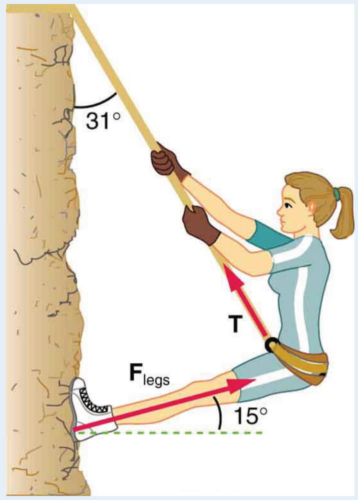


Figure 8. Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

18: A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in Figure 9(a). (a) Calculate the minimum force he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

19: Repeat Exercise 18 with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in Figure 9(b).

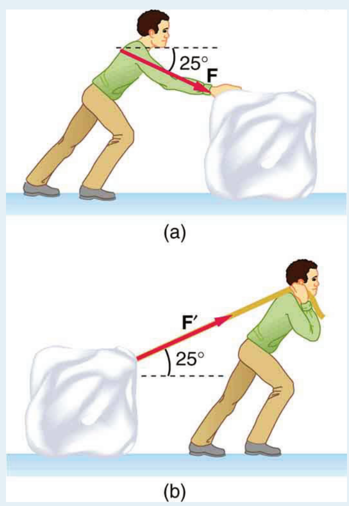


Figure 9. Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

Glossary

friction
a force that opposes relative motion or attempts at motion between systems in contact
kinetic friction
a force that opposes the motion of two systems that are in contact and moving relative to one another
static friction
a force that opposes the motion of two systems that are in contact and are not moving relative to one another
magnitude of static friction
$f_s \leq \mu_s N$, *where* μ_s *is the coefficient of static friction and* N *is the magnitude of the normal force*
magnitude of kinetic friction
$f_k = \mu_k N$, *where* μ_k *is the coefficient of kinetic friction*

Solutions

Problems & Exercises

1:

$5.00\ \text{N}$

4:

(a) 588 N

(b) $1.96\ \text{m/s}^2$

6:

(a) 3.29 m/s^2

(b) 3.52 m/s^2

(c) 980 N; 945 N

10:

$1.83\ \text{m/s}^2$

14:

(a) $4.20\ \text{m/s}^2$

(b) $2.74\ \text{m/s}^2$

(c) $-0.195\ \text{m/s}^2$

16:

(a) $1.03 \times 10^6\ \mathrm{N}$

(b) $3.48 \times 10^5\ \mathrm{N}$

18:

(a) $51.0\ \mathrm{N}$

(b) $0.720\ \mathrm{m/s^2}$

5.2 Drag Forces

Summary

- Express mathematically the drag force.
- Discuss the applications of drag force.
- Define terminal velocity.
- Determine the terminal velocity given mass.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_D is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_D = \frac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_D = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

DRAG FORCE

Drag force F_D is found to be proportional to the square of the speed of the object. Mathematically

$$F_D \propto v^2$$

$$F_D = \frac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See Figure 1). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.

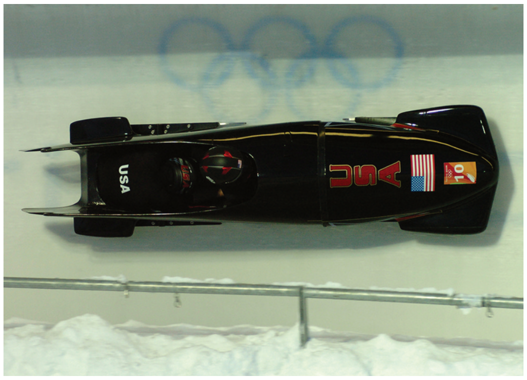


Figure 1. From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient, C, is determined empirically, usually with the use of a wind tunnel. (See Figure 2).

Figure 2. NASA researchers test a model plane in a wind tunnel. (credit: NASA/ Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. Table 2 lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Table 2. Drag Coefficient Values Typical values of drag coefficient C.

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full body-suits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See Figure 3). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Figure 3. Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother "skin" and more compression forces on a swimmer's body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* (v_t). Since F_D is proportional to the speed, a heavier skydiver must go faster for F_D to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{\text{net}} = mg - F_D = ma = 0.$$

Thus,

$$mg = F_D.$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}\rho C A v^2.$$

Solving for the velocity, we obtain

$$v = \sqrt{\frac{2mg}{\rho C A}}.$$

Assume the density of air is $\rho = 1.21\ \text{kg/m}^3$. A 75-kg skydiver descending head first will have an area approximately $A = 0.18\ \text{m}^2$ and a drag coefficient of approximately $C = 0.70$. We find that

$$v = \sqrt{\frac{2(75\ \text{kg})(9.80\ \text{m/s}^2)}{(1.21\ \text{kg/m}^3)(0.70)(0.18\ \text{m}^2)}}$$

$$= 98\ \text{m/s}$$

$$= 350\ \text{km/h}.$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

TAKE-HOME EXPERIMENT

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity v versus mass. Also plot v^2 versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

Example 1: A Terminal Velocity

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_{\text{net}} = 0$. Thus the drag force on the skydiver must equal the force of gravity (the person’s weight). Using the equation of drag force, we find $mg = \frac{1}{2}\rho CAv^2$.

Thus the terminal velocity v_t can be written as

$$v_t = \sqrt{\frac{2mg}{\rho CA}}.$$

Solution

All quantities are known except the person’s projected area. This is an adult (82 kg) falling spread eagle. We can estimate the frontal area as

$$A = (2\ \text{m})(0.35\ \text{m}) = 0.70\ \text{m}^2.$$

Using our equation for v_t, we find that

$$v_t = \sqrt{\frac{2(85\ \text{kg})(9.80\ \text{m/s}^2)}{(1.21\ \text{kg/m}^3)(1.0)(0.70\ \text{m}^2)}}$$

$$= 44\ \text{m/s}.$$

Discussion

This result is consistent with the value for v_t mentioned earlier. The 75-kg skydiver going feet first had a $v = 98\ \text{m/s}$. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by **Stokes' law**, which states that

$$F_s = 6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

STOKE'S LAW

$$F_s = 6\pi r\eta v,$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about $1\ \mu$) can be about $2\ \mu/s.$ To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about $5\ \mu/s$), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see Figure 4). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Figure 4. Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

GALILEO'S EXPERIMENT

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren't readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

PHET EXPLORATIONS: MASSES & SPRINGS

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

PhET Interactive Simulation

Figure 5. Masses & Springs

Section Summary

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity v in air, the drag force is given by

$$F_D = \frac{1}{2}C\rho A v^2,$$

where C is the drag coefficient (typical values are given in Table 2), A is the area of the object facing the fluid, and ρ is the fluid density.

- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law,

$$F_s = 6\pi\eta r v,$$

where r is the radius of the object, η is the fluid viscosity, and v is the object's velocity.

Conceptual Questions

1: Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

2: Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

3: As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

4: Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Problems & Exercises

1: The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of $0.140\ \text{m}^2$.

2: A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

3: A 560-g squirrel with a surface area of $930\ \text{cm}^2$ falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

4: To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is $0.70\ \text{m}^2$) (b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is $2.44\ \text{m}^2$) Assume all values are accurate to three significant digits.

5: By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

6: Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be $1.00 \times 10^3\ \text{kg/m}^3$, and the surface area to be πr^2.

7: Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.

8: Find the terminal velocity of a spherical bacterium (diameter $2.00\ \mu\text{m}$) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be $1.10 \times 10^3\ \text{kg/m}^3$.

9: Stokes' law describes seimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density $7.8 \times 10^3\ \text{kg/m}^3$, diameter $3.0\ \text{mm}$) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

Glossary

drag force

F_D, found to be proportional to the square of the speed of the object; mathematically

$$F_D \propto v^2$$

$$F_D = \frac{1}{2} C \rho A v^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid

Stokes' law

$F_s = 6\pi r \eta v$, where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity

Solutions

Problems & Exercises

1:

$115\ \text{m/s};\ 414\ \text{km/hr}$

3:

$25\ \text{m/s};\ 9.9\ \text{m/s}$

5:

2.9

7:

$$[\eta] = \frac{[F_s]}{[r][v]} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}\cdot\text{m/s}} = \frac{\text{kg}}{\text{m}\cdot\text{s}}$$

9:

$0.76\ \text{kg/m}\cdot\text{s}$

5.3 Elasticity: Stress and Strain

Summary

- State Hooke's law.
- Explain Hooke's law using graphical representation between deformation and applied force.
- Discuss the three types of deformations such as changes in length, sideways shear and changes in volume.
- Describe with examples the young's modulus, shear modulus and bulk modulus.
- Determine the change in length given mass, length and radius.

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, **Hooke's law** is obeyed. In equation form, Hooke's law is given by

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation ΔL—it is not constant as a kinetic friction force is. Rearranging this to

$$\Delta L = \frac{F}{k}$$

makes it clear that the deformation is proportional to the applied force. Figure 1 shows the Hooke's law relationship between the extension ΔL of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

HOOKE'S LAW

$$F = k\Delta L,$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\Delta L = \frac{F}{k}$$

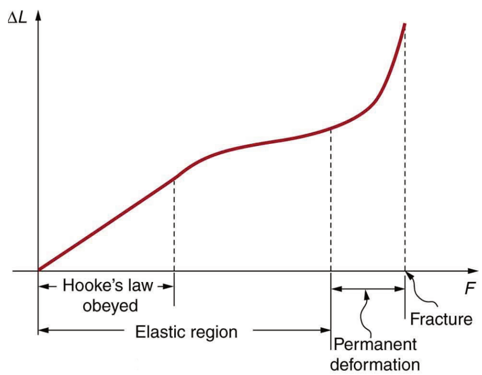


Figure 1. A graph of deformation **Δ*L*** versus applied force ***F***. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is **1 / k**. For larger forces, the graph is curved but the deformation is still elastic—**Δ*L*** will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force ***F*** is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in ***F*** is producing a large increase in ***L*** near the fracture.

The proportionality constant k depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation ΔL is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger k (see Figure 2). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in 10^3.

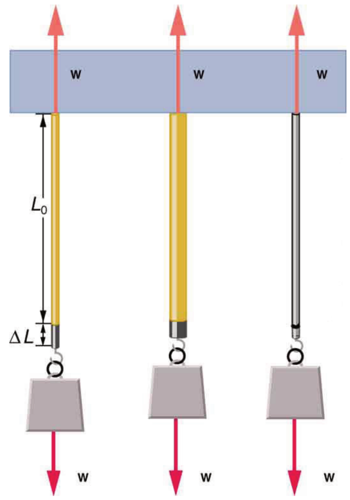


Figure 2. The same force, in this case a weight (***w***), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.

STRETCH YOURSELF A LITTLE

How would you go about measuring the proportionality constant k of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

We now consider three specific types of deformations: changes in length (tension and compression), sideways shear (stress), and changes in volume. All deformations are assumed to be small unless otherwise stated.

Changes in Length—Tension and Compression: Elastic Modulus

A change in length ΔL is produced when a force is applied to a wire or rod parallel to its length L_0, either stretching it (a tension) or compressing it. (See Figure 3.)

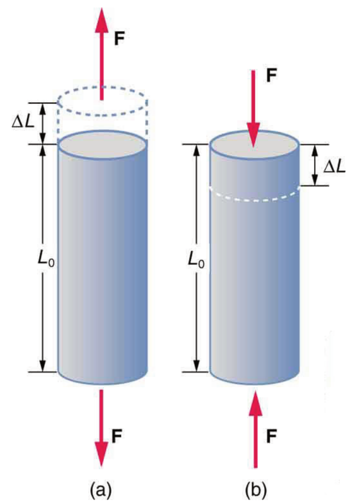


Figure 3. (a) Tension. The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L_0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for ΔL:

$$\Delta L = \frac{1}{Y}\frac{F}{A}L_0,$$

where ΔL is the change in length, F the applied force, Y is a factor, called the elastic modulus or Young's modulus, that depends on the substance, A is the cross-sectional area, and L_0 is the original length. Table 3 lists values of Y for several materials—those with a large Y are said to have a large tensile stiffness because they deform less for a given tension or compression.

Material	Young's modulus (tension–compression) Y $(10^9 N/m^2)$	Shear modulus S $(10^9 N/m^2)$	Bulk modulus B $(10^9 N/m^2)$
Aluminum	70	25	75
Bone – tension	16	80	8
Bone – compression	9		
Brass	90	35	75
Brick	15		
Concrete	20		
Glass	70	20	30
Granite	45	20	45
Hair (human)	10		
Hardwood	15	10	
Iron, cast	100	40	90
Lead	16	5	50
Marble	60	20	70
Nylon	5		
Polystyrene	3		
Silk	6		
Spider thread	3		
Steel	210	80	130
Tendon	1		
Acetone			0.7
Ethanol			0.9
Glycerin			4.5
Mercury			25
Water			2.2

Table 3. Elastic Moduli[1].

Young's moduli are not listed for liquids and gases in Table 3 because they cannot be stretched or compressed in only one direction. Note that there is an assumption that the object does not accelerate, so that there are actually two applied forces of magnitude F acting in opposite directions. For example, the strings in Figure 3 are being pulled down by a force of magnitude w and held up by the ceiling, which also exerts a force of magnitude w.

Example 1: The Stretch of a Long Cable

Suspension cables are used to carry gondolas at ski resorts. (See Figure 4) Consider a suspension cable that includes an unsupported span of 3 km. Calculate the amount of stretch in the steel cable. Assume that the cable has a diameter of 5.6 cm and the maximum tension it can withstand is $3.0 \times 10^6\ \mathrm{N}$.

Figure 4. Gondolas travel along suspension cables at the Gala Yuzawa ski resort in Japan. (credit: Rudy Herman, Flickr)

Strategy

The force is equal to the maximum tension, or $F = 3.0 \times 10^6\ \mathrm{N}$. The cross-sectional area is $\pi r^2 = 2.46 \times 10^{-3}\ \mathrm{m}^2$. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities are known. Thus,

$$\Delta L = \left(\frac{1}{210 \times 10^9\ \mathrm{N/m^2}}\right)\left(\frac{3.0 \times 10^6\ \mathrm{N}}{2.46 \times 10^{-3}\ \mathrm{m^2}}\right)(3020\ \mathrm{m})$$

$$= 18\ \mathrm{m}.$$

Discussion

This is quite a stretch, but only about 0.6% of the unsupported length. Effects of temperature upon length might be important in these environments.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Another biological example of Hooke's law occurs in tendons. Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. Figure 5 shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.

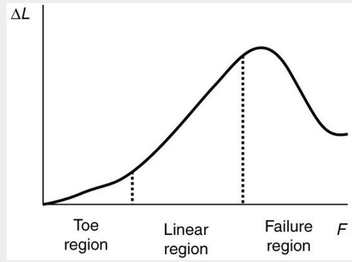


Figure 5. Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling exactly this—the elastic behavior of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out. Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

Example 2: Calculating Deformation: How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

Strategy

The force is equal to the weight supported, or

$$F = mg = (62.0 \text{ kg})(9.80 \text{ m/s}^2) = 607.6 \text{ N},$$

and the cross-sectional area is $\pi r^2 = 1.257 \times 10^{-3} \text{ m}^2$. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities except ΔL are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\Delta L = \left(\frac{1}{9 \times 10^9 \text{ N/m}^2}\right)\left(\frac{607.6 \text{ N}}{1.257 \times 10^{-3} \text{ m}^2}\right)(0.400 \text{ m})$$

$$= 2 \times 10^{-5} \text{ m}.$$

Discussion

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in Table 3 have larger values of Young's modulus Y. In other words, they are more rigid.

The equation for change in length is traditionally rearranged and written in the following form:

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}.$$

The ratio of force to area, $\frac{F}{A}$, is defined as **stress** (measured in N/m^2), and the ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as **strain** (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form

$$F = YA\frac{\Delta L}{L_0},$$

we see that it is the same as Hooke's law with a proportionality constant

$$k = \frac{YA}{L_0}.$$

This general idea—that force and the deformation it causes are proportional for small deformations—applies to changes in length, sideways bending, and changes in volume.

STRESS

The ratio of force to area, $\frac{F}{A}$, is defined as stress measured in N/m^2.

STRAIN

The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as strain (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.$$

Sideways Stress: Shear Modulus

Figure 6 illustrates what is meant by a sideways stress or a *shearing force*. Here the deformation is called Δx and it is perpendicular to L_0, rather than parallel as with tension and compression. Shear deformation behaves similarly to tension and compression and can be described with similar equations. The expression for shear deformation is

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

where S is the shear modulus (see Table 3) and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A. Again, to keep the object from accelerating, there are actually two equal and opposite forces F applied across opposite faces, as illustrated in Figure 6. The equation is logical—for example, it is easier to bend a long thin pencil (small A) than a short thick one, and both are more easily bent than similar steel rods (large S).

SHEAR DEFORMATION

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.

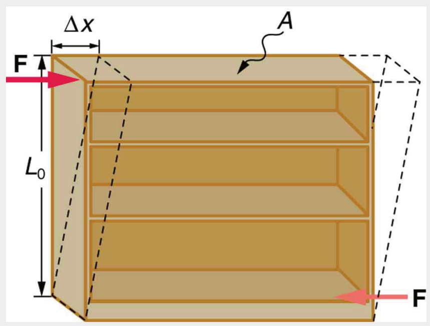


Figure 6. Shearing forces are applied perpendicular to the length $\boldsymbol{L_0}$ and parallel to the area **A**, producing a deformation **Δx**. Vertical forces are not shown, but it should be kept in mind that in addition to the two shearing forces, **F**, there must be supporting forces to keep the object from rotating. The distorting effects of these supporting forces are ignored in this treatment. The weight of the object also is not shown, since it is usually negligible compared with forces large enough to cause significant deformations.

Examination of the shear moduli in Table 3 reveals some telling patterns. For example, shear moduli are less than Young's moduli for most materials. Bone is a remarkable exception. Its shear modulus is not only greater than its Young's modulus, but it is as large as that of steel. This is why bones are so rigid.

The spinal column (consisting of 26 vertebral segments separated by discs) provides the main support for the head and upper part of the body. The spinal column has normal curvature for stability, but this curvature can be increased, leading to increased shearing forces on the lower vertebrae. Discs are better at withstanding compressional forces than shear forces. Because the spine is not vertical, the weight of the upper body exerts some of both. Pregnant women and people that are overweight (with large abdomens) need to move their shoulders back to maintain balance, thereby increasing the curvature in their spine and so increasing the shear component of the stress. An increased angle due to more curvature increases the shear forces along the plane. These higher shear forces increase the risk of back injury through ruptured discs. The lumbosacral disc (the wedge shaped disc below the last vertebrae) is particularly at risk because of its location.

The shear moduli for concrete and brick are very small; they are too highly variable to be listed. Concrete used in buildings can withstand compression, as in pillars and arches, but is very poor against shear, as might be encountered in heavily loaded floors or during earthquakes. Modern structures were made possible by the use of steel and steel-reinforced concrete. Almost by definition, liquids and gases have shear moduli near zero, because they flow in response to shearing forces.

Example 3: Calculating Force Required to Deform: That Nail Does Not Bend Much Under a Load

Find the mass of the picture hanging from a steel nail as shown in Figure 7, given that the nail bends only $1.80\ \mu m$. (Assume the shear modulus is known to two significant figures.)

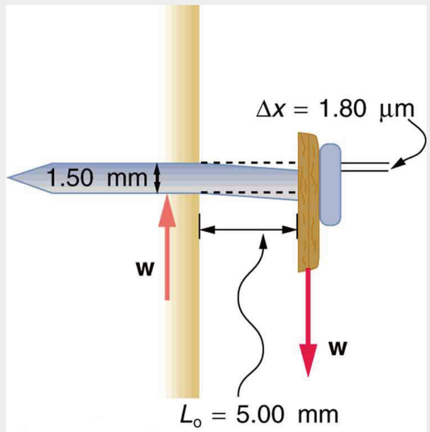


Figure 7. Side view of a nail with a picture hung from it. The nail flexes very slightly (shown much larger than actual) because of the shearing effect of the supported weight. Also shown is the upward force of the wall on the nail, illustrating that there are equal and opposite forces applied across opposite cross sections of the nail. See Example 3 for a calculation of the mass of the picture.

Strategy

The force F on the nail (neglecting the nail's own weight) is the weight of the picture w. If we can find w, then the mass of the picture is just $\frac{w}{g}$. The equation $\Delta x = \frac{1}{S}\frac{F}{A}L_0$ can be solved for F.

Solution

Solving the equation $\Delta x = \frac{1}{S}\frac{F}{A}L_0$ for F, we see that all other quantities can be found:

$$F = \frac{SA}{L_0}\Delta x.$$

S is found in Table 3 and is $S = 80 \times 10^9\ \text{N/m}^2$. The radius r is 0.750 mm (as seen in the figure), so the cross-sectional area is

$$A = \pi r^2 = 1.77 \times 10^{-6}\ \text{m}^2.$$

The value for L_0 is also shown in the figure. Thus,

$$F = \frac{(80 \times 10^9\ \text{N/m}^2)(1.77 \times 10^{-6}\ \text{m}^2)}{(5.00 \times 10^{-3}\ \text{m})}(1.80 \times 10^{-6}\ \text{m}) = 51\ \text{N}.$$

This 51 N force is the weight w of the picture, so the picture's mass is

$$m = \frac{w}{g} = \frac{F}{g} = 5.2\ \text{kg}.$$

Discussion

This is a fairly massive picture, and it is impressive that the nail flexes only $1.80\ \mu m$—an amount undetectable to the unaided eye.

Changes in Volume: Bulk Modulus

An object will be compressed in all directions if inward forces are applied evenly on all its surfaces as in Figure 8. It is relatively easy to compress gases and extremely difficult to compress liquids and solids. For example, air in a wine bottle is compressed when it is corked. But if you try corking a brim-full bottle, you cannot compress the wine—some must be removed if the cork is to be inserted. The reason for these different compressibilities is that atoms and molecules are separated by large empty spaces in gases but packed close together in liquids and solids. To compress a gas, you must force its atoms and molecules closer together. To compress liquids and solids, you must actually compress their atoms and molecules, and very strong electromagnetic forces in them oppose this compression**.**

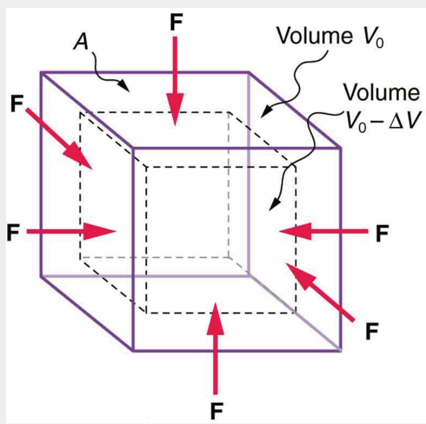


Figure 8. An inward force on all surfaces compresses this cube. Its change in volume is proportional to the force per unit area and its original volume, and is related to the compressibility of the substance.

We can describe the compression or volume deformation of an object with an equation. First, we note that a force "applied evenly" is defined to have the same stress, or ratio of force to area F/A on all surfaces. The deformation produced is a change in volume ΔV, which is found to behave very similarly to the shear, tension, and compression previously discussed. (This is not surprising, since a compression of the entire object is equivalent to compressing each of its three dimensions.) The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

where B is the bulk modulus (see Table 3), V_0 is the original volume, and F/A is the force per unit area applied uniformly inward on all surfaces. Note that no bulk moduli are given for gases.

What are some examples of bulk compression of solids and liquids? One practical example is the manufacture of industrial-grade diamonds by compressing carbon with an extremely large force per unit area. The carbon atoms rearrange their crystalline structure into the more tightly packed pattern of diamonds.

In nature, a similar process occurs deep underground, where extremely large forces result from the weight of overlying material. Another natural source of large compressive forces is the pressure created by the weight of water, especially in deep parts of the oceans. Water exerts an inward force on all surfaces of a submerged object, and even on the water itself. At great depths, water is measurably compressed, as the following example illustrates.

Example 4: Calculating Change in Volume with Deformation: How Much Is Water Compressed at Great Ocean Depths?

Calculate the fractional decrease in volume ($\frac{\Delta V}{V_0}$) for seawater at 5.00 km depth, where the force per unit area is $5.00 \times 10^7\ \mathrm{N/m^2}$.

Strategy

Equation $\Delta V = \frac{1}{B}\frac{F}{A}V_0$ is the correct physical relationship. All quantities in the equation except $\frac{\Delta V}{V_0}$ are known.

Solution

Solving for the unknown $\frac{\Delta V}{V_0}$ gives

$$\frac{\Delta V}{V_0} = \frac{1}{B}\frac{F}{A}.$$

Substituting known values with the value for the bulk modulus B from Table 3,

$$\frac{\Delta V}{V_0} = \frac{5.00 \times 10^7\ \mathrm{N/m^2}}{2.2 \times 10^9\ \mathrm{N/m^2}}$$

$$= 0.023 = 2.3\%.$$

Discussion

Although measurable, this is not a significant decrease in volume considering that the force per unit area is about 500 atmospheres (1 million pounds per square foot). Liquids and solids are extraordinarily difficult to compress.

Conversely, very large forces are created by liquids and solids when they try to expand but are constrained from doing so—which is equivalent to compressing them to less than their normal volume. This often occurs when a contained material warms up, since most materials expand when their temperature increases. If the materials are tightly constrained, they deform or break their container. Another very common example occurs when water freezes. Water, unlike most materials, expands when it freezes, and it can easily fracture a boulder, rupture a biological cell, or crack an engine block that gets in its way.

Other types of deformations, such as torsion or twisting, behave analogously to the tension, shear, and bulk deformations considered here.

Section Summary

- Hooke's law is given by

  $$F = k\Delta L,$$

  where ΔL is the amount of deformation (the change in length), F is the applied force, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

$$\Delta L = \frac{1}{Y}\frac{F}{A}L_0,$$

where Y is *Young's modulus*, which depends on the substance, A is the cross-sectional area, and L_0 is the original length.

- The ratio of force to area, $\frac{F}{A}$, is defined as *stress*, measured in N/m^2.
- The ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as *strain* (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.$$

- The expression for shear deformation is

$$\Delta x = \frac{1}{S}\frac{F}{A}L_0,$$

where S is the shear modulus and F is the force applied perpendicular to L_0 and parallel to the cross-sectional area A.

- The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B}\frac{F}{A}V_0,$$

where B is the bulk modulus, V_0 is the original volume, and $\frac{F}{A}$ is the force per unit area applied uniformly inward on all surfaces.

Conceptual Questions

1: The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).

2: What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?

3: Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?

4: Would you expect your height to be different depending upon the time of day? Why or why not?

5: Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

6: Explain why pregnant women often suffer from back strain late in their pregnancy.

7: An old carpenter's trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?

8: When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)

Problems & Exercises

1: During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.

2: During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex his already moribund adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

3: (a) The "lead" in pencils is a graphite composition with a Young's modulus of about $1 \times 10^9 \text{ N/m}^2$. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?

4: TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?

5: (a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when she hangs 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?

6: A 20.0-m tall hollow aluminum flagpole is equivalent in stiffness to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?

7: As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in stiffness to a solid cylinder 5.00 cm in diameter.

8: Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.

9: A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.

10: A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of $1 \times 10^9 \text{ N/m}^2$. The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.

11: When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of 20.0° to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?

12: To consider the effect of wires hung on poles, we take data from Chapter 4.7 Example 2, in which tensions in wires supporting a traffic light were calculated. The left wire made an angle 30.0° below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in stiffness to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?

13: A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2 \times 10^{-3}$) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is $1.8 \times 10^{9}\ \text{N/m}^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?

14: (a) When water freezes, its volume increases by 9.05% (that is, $\Delta V/V_0 = 9.05 \times 10^{-2}$). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?

15: This problem returns to the tightrope walker studied in Chapter 4.5 Example 2, who created a tension of $3.94 \times 10^{3}\ \text{N}$ in a wire making an angle 5.0° below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

16: The pole in Figure 9 is at a 90.0° bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is $4.00 \times 10^{4}\ \text{N}$, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the stiffness of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of 30.0° with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)

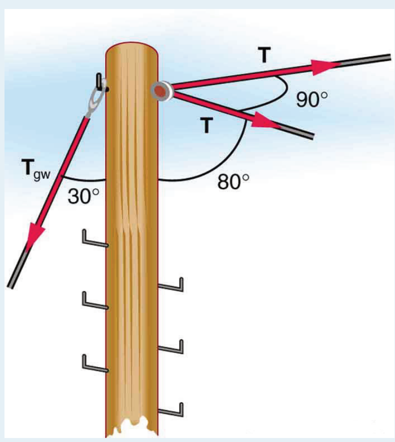


Figure 9. This telephone pole is at a $\mathbf{90^0}$ bend in a power line. A guy wire is attached to the top of the pole at an angle of $\mathbf{30^0}$ with the vertical.

Footnotes

1. 1 Approximate and average values. Young's moduli Y for tension and compression sometimes differ but are averaged here. Bone has significantly different Young's moduli for tension and compression.

Glossary

deformation
change in shape due to the application of force
Hooke's law
proportional relationship between the force F on a material and the deformation ΔL it causes, $F = k\Delta L$
tensile strength
the breaking stress that will cause permanent deformation or fraction of a material
stress
ratio of force to area
strain
ratio of change in length to original length
shear deformation
deformation perpendicular to the original length of an object

Solutions

Problems & Exercises

1:

1.90×10^{-3} cm

3:

(a)1 mm

(b) This does seem reasonable, since the lead does seem to shrink a little when you push on it.

5:

(a)9 cm

(b)This seems reasonable for nylon climbing rope, since it is not supposed to stretch that much.

7:

8.59 mm

9:

1.49×10^{-7} m

11:

(a)3.99×10^{-7} m

(b) 9.67×10^{-8} m

13:

$4 \times 10^{6}\ \mathrm{N/m^2}$. This is about 36 atm, greater than a typical jar can withstand.

15:

1.4 cm

PART 6

Chapter 6 Uniform Circular Motion and Gravitation

Figure 1. This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly—the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles are involved in each. (credit: Richard Munckton)

Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of Chapter 4 Dynamics: Newton's Laws of Motion as we study more applications of Newton's laws of motion.

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

Glossary

uniform circular motion

the motion of an object in a circular path at constant speed

6.1 Rotation Angle and Angular Velocity

Summary

- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In Chapter 2 Kinematics, we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. Chapter 3 Two-Dimensional Kinematics dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) in Figure 1 rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each pit used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the **rotation angle** $\Delta\theta$ to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r}.$$

Figure 1. All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle $\Delta\theta$ in a time Δt.

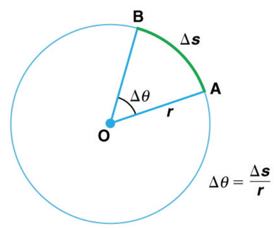


Figure 2. The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

The **arc length** Δs is the distance traveled along a circular path as shown in Figure 2 Note that r is the **radius of curvature** of the circular path.

We know that for one complete revolution, the arc length is the circumference of a circle of radius r. The circumference of a circle is $2\pi r$. Thus for one complete revolution the rotation angle is

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi .$$

This result is the basis for defining the units used to measure rotation angles, $\Delta\theta$ to be **radians** (rad), defined so that

$$2\pi \text{ rad} = 1 \text{ revolution}.$$

A comparison of some useful angles expressed in both degrees and radians is shown in Table 1.

Degree Measures	Radian Measure
30^{o}	$\frac{\pi}{6}$
60^{o}	$\frac{\pi}{3}$
90^{o}	$\frac{\pi}{2}$
120^{o}	$\frac{2\pi}{3}$
135^{o}	$\frac{3\pi}{4}$
180^{o}	π

Table 1. Comparison of Angular Units.

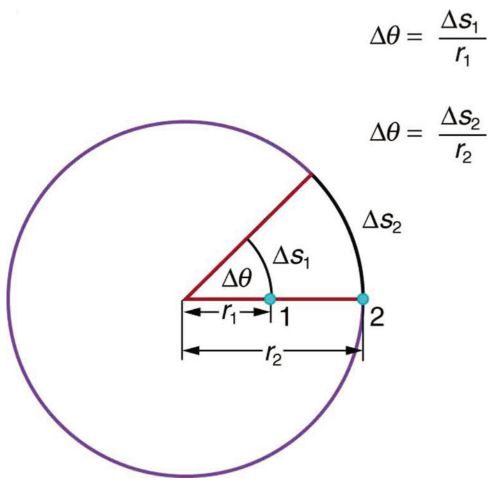


Figure 3. Points 1 and 2 rotate through the same angle **($\Delta\theta$)**, but point 2 moves through a greater arc length **(Δs)** because it is at a greater distance from the center of rotation **(r)**.

If $\Delta\theta = 2\pi$ rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are 360^{o} in a circle or one revolution, the relationship between radians and degrees is thus

$$2\pi \text{rad} = 360^{o}$$

so that

$$1 \text{ rad} = \frac{360^{o}}{2\pi} \approx 57.3^{o}.$$

Angular Velocity

How fast is an object rotating? We define **angular velocity** ω as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t},$$

where an angular rotation $\Delta\theta$ takes place in a time Δt. The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity ω is analogous to linear velocity v. To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length Δs in a time Δt, and so it has a linear velocity

$$v = \frac{\Delta s}{\Delta t}.$$

From $\Delta\theta = \frac{\Delta s}{r}$ we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for v gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega.$$

We write this relationship in two different ways and gain two different insights:

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

The first relationship in $v = r\omega \text{ or } \omega = \frac{v}{r}$ states that the linear velocity v is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest r), as you might expect. We can also call this linear speed v of a point on the rim the *tangential speed*. The second relationship in $v = r\omega \text{ or } \omega = \frac{v}{r}$ can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed v of the car. See Figure 4. So the faster the car moves, the faster the tire spins—large v means a large ω because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity (ω) will produce a greater linear speed (v) for the car.

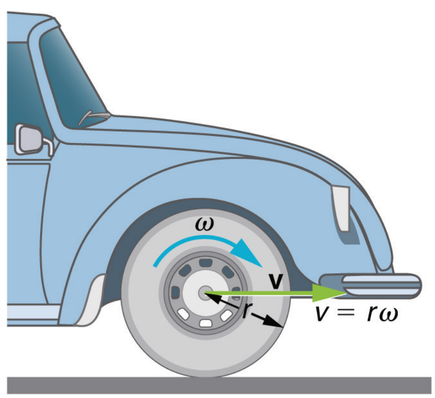


Figure 4. A car moving at a velocity ***v*** to the right has a tire rotating with an angular velocity **ω**. The speed of the tread of the tire relative to the axle is ***v***, the same as if the car were jacked up. Thus the car moves forward at linear velocity ***v=rω***, where ***r*** is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Example 1: How Fast Does a Car Tire Spin?

Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See Figure 4.

Strategy

Because the linear speed of the tire rim is the same as the speed of the car, we have $v = 15.0 \text{ m/s}$. The radius of the tire is given to be $r = 0.300$ m. Knowing v and r, we can use the second relationship in $v = r\omega, \omega = \frac{v}{r}$ to calculate the angular velocity.

Solution

To calculate the angular velocity, we will use the following relationship:

$$\omega = \frac{v}{r}.$$

Substituting the knowns,

$$\omega = \frac{15.0\text{ m/s}}{0.300\text{ m}} = 50.0\text{ rad/s}.$$

Discussion

When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

$$\omega = (15.0\text{ m/s})/(1.20\text{ m}) = 12.5\text{ rad/s}.$$

Both ω and v have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in Figure 5.

TAKE-HOME EXPERIMENT

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.

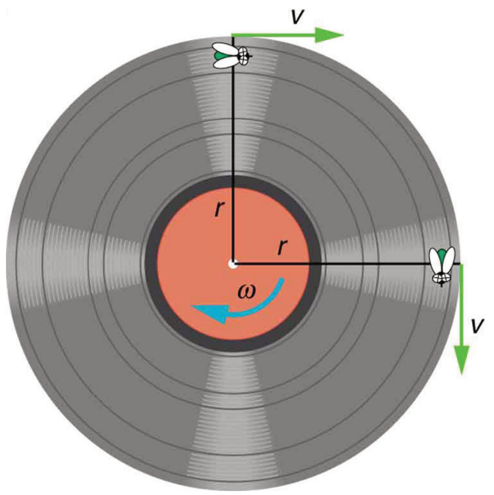


Figure 5. As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case.

PHET EXPLORATIONS: LADYBUG REVOLUTION

PhET Interactive Simulation

Figure 6. Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

Section Summary

- Uniform circular motion is motion in a circle at constant speed. The rotation angle $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature:

  $$\Delta\theta = \frac{\Delta s}{r},$$

  where arc length Δs is distance traveled along a circular path and r is the radius of curvature of the circular path. The quantity $\Delta\theta$ is measured in units of radians (rad), for which

  $$2\pi \text{ rad} = 360^\circ = 1 \text{ revolution}.$$

- The conversion between radians and degrees is $1 \text{ rad} = 57.3^\circ$.
- Angular velocity ω is the rate of change of an angle,

$$\omega = \frac{\Delta\theta}{\Delta t},$$

where a rotation $\Delta\theta$ takes place in a time Δt. The units of angular velocity are radians per second (rad/s). Linear velocity v and angular velocity ω are related by

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

Conceptual Questions

1: There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

Problems & Exercises

1: Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?

2: Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?

3: An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?

4: (a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of 6.4×10^6 m at its equator, what is the linear velocity at Earth's surface?

5: A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?

6: In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?

7: A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?

8: Integrated Concepts

When kicking a football, the kicker rotates his leg about the hip joint.

(a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?

(b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?

(c) Find the maximum range of the football, neglecting air resistance.

9: Construct Your Own Problem

Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

Glossary

arc length

Δs, *the distance traveled by an object along a circular path*

pit

a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

rotation angle

the ratio of the arc length to the radius of curvature on a circular path:

$$\Delta\theta = \frac{\Delta s}{r}$$

radius of curvature

radius of a circular path

radians

a unit of angle measurement

angular velocity

ω, *the rate of change of the angle with which an object moves on a circular path*

Solutions

Problems & Exercises

1:

723 km

3:

5×10^7 rotations

5:

117 rad/s

7:

76.2 rad/s

728 rpm

8:

(a) 33.3 rad/s

(b) 500 N

(c) 40.8 m

6.2 Centripetal Acceleration

Summary

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

Figure 1 shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration**(a_c); centripetal means “toward the center” or “center seeking.”

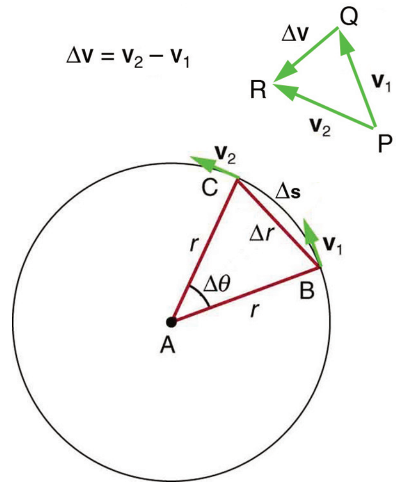


Figure 1. The directions of the velocity of an object at two different points are shown, and the change in velocity **Δv** is seen to point directly toward the center of curvature. (See small inset.) Because **ac = Δ*v*/Δ*t***, the acceleration is also toward the center; ac is called centripetal acceleration. (Because **Δ*θ*** is very small, the arc length **Δ*s*** is equal to the chord length **Δ*r*** for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii r and Δs are similar. Both the triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds $v_1 = v_2 = v$. Using the properties of two similar triangles, we obtain

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}.$$

Acceleration is $\Delta v / \Delta t$ and so we first solve this expression for Δv:

$$\Delta v = \frac{v}{r}\Delta s.$$

Then we divide this by Δt, yielding

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}.$$

Finally, noting that $\Delta v / \Delta t = a_c$ and that $\Delta s / \Delta t = v$, the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r},$$

which is the acceleration of an object in a circle of radius r at a speed v. So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_c is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_c is greater for tighter turns, as you have probably noticed.

It is also useful to express a_c in terms of angular velocity. Substituting $v = r\omega$ into the above expression, we find $a_c = (r\omega)^2/r = r\omega^2$. We can express the magnitude of centripetal acceleration using either of two equations:

$$a_c = \frac{v^2}{r};\ a_c = r\omega^2.$$

Recall that the direction of a_c is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A centrifuge (see Figure 2b) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity (g); maximum centripetal acceleration of several hundred thousand g is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.

Example 1: How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See Figure 2(a).

Strategy

Because v and r are given, the first expression in $a_c = \frac{v^2}{r};\ a_c = r\omega^2$ is the most convenient to use.

Solution

Entering the given values of $v = 25.0\ \text{m/s}$ and $r = 500\ \text{m}$ into the first expression for a_c gives

$$a_c = \frac{v^2}{r} = \frac{(25.0\ \text{m/s})^2}{500\ \text{m}} = 1.25\ \text{m/s}^2.$$

Discussion

To compare this with the acceleration due to gravity $(g = 9.80\ \text{m/s}^2)$, we take the ratio of $a_c/g = (1.25\ \text{m/s}^2)/(9.80\ \text{m/s}^2) = 0.128$. Thus, $a_c = 0.128\ \text{g}$ and is noticeable especially if you were not wearing a seat belt.

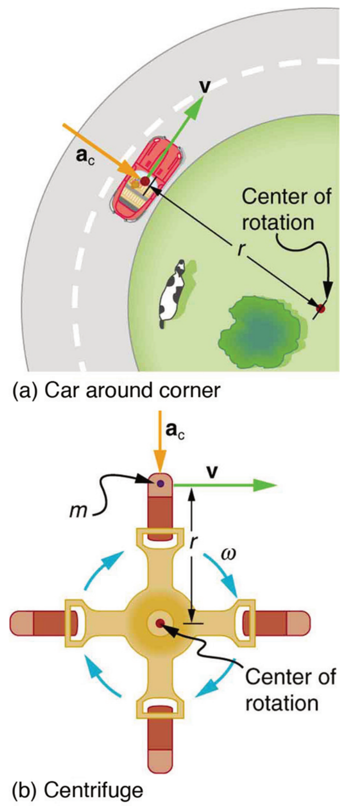


Figure 2. (a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in Example 1. (b) A particle of mass in a centrifuge is rotating at constant angular velocity . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in Example 2.

Example 2: How Big Is The Centripetal Acceleration in an Ultracentrifuge?

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at 7.5×10^4 rev/min. Determine the ratio of this acceleration to that due to gravity. See Figure 2(b).

Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity ω. Because r is given, we can use the second expression in the equation $a_c = \frac{v^2}{r};\ a_c = r\omega^2$ to calculate the centripetal acceleration.

Solution

To convert 7.5×10^4 rev/min to radians per second, we use the facts that one revolution is $2\pi \text{rad}$ and one minute is 60.0 s. Thus,

$$\omega = 7.50 \times 10^4 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60.0 \text{ s}} = 7854 \text{ rad/s}.$$

Now the centripetal acceleration is given by the second expression in $a_c = \frac{v^2}{r};\ a_c = r\omega^2$ as

$$a_c = r\omega^2.$$

Converting 7.50 cm to meters and substituting known values gives

$$a_c = (0.0750\ \text{m})(7854\ \text{rad/s})^2 = 4.63 \times 10^6\ \text{m/s}^2.$$

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of a_c to g yields

$$\frac{a_c}{g} = \frac{4.63 \times 10^6}{9.80} = 4.72 \times 10^5.$$

Discussion

This last result means that the centripetal acceleration is 472,000 times as strong as g. It is no wonder that such high ω centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In Chapter 6.3 Centripetal Force, we will consider the forces involved in circular motion.

PHET EXPLORATIONS: LADYBUG MOTION 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Figure 3. Ladybug Motion in 2D

Section Summary

- Centripetal acceleration a_c is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude

$$a_c = \frac{v^2}{r};\ a_c = r\omega^2.$$

- The unit of centripetal acceleration is m/s^2.

Conceptual Questions

1: Can centripetal acceleration change the speed of circular motion? Explain.

Problems & Exercises

1: A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?

2: A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

3: Taking the age of Earth to be about 4×10^9 years and assuming its orbital radius of 1.5×10^{11} has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).

4: The propeller of a World War II fighter plane is 2.30 m in diameter.

(a) What is its angular velocity in radians per second if it spins at 1200 rev/min?

(b) What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac?

(c) What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of g.

5: An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.

(a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g.

(b) What is the linear speed of a point on its edge?

6: Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

(a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.

(b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).

7: Olympic ice skaters are able to spin at about 5 rev/s.

(a) What is their angular velocity in radians per second?

(b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?

(c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?

(d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

8: What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?

9: Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating:

(a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.

(b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).

10: A rotating space station is said to create "artificial gravity"—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an "artificial gravity" of 9.80 m/s^2 at the rim?

11: At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m.

(a) At how many rev/min are the tires rotating?

(b) What is the centripetal acceleration at the edge of the tire?

(c) With what force must a determined $1.00 \times 10^{-15} \text{ kg}$ bacterium cling to the rim?

(d) Take the ratio of this force to the bacterium's weight.

12: Integrated Concepts

Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity.

(a) Assuming negligible friction, find the speed of the riders at the bottom of its arc, given the system's center of mass travels in an arc having a radius of 14.0 m and the riders are near the center of mass.

(b) What is the centripetal acceleration at the bottom of the arc?

(c) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.

(d) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.

(e) Discuss whether the answer seems reasonable.

13: Unreasonable Results

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

Glossary

centripetal acceleration
the acceleration of an object moving in a circle, directed toward the center
ultracentrifuge
a centrifuge optimized for spinning a rotor at very high speeds

Solutions

Problems & Exercises

1:

12.9 rev/min

3:

$4 \times 10^{21}\ \text{m}$

5:

a)$3.47 \times 10^{4}\ \text{m/s}^{2}$, $3.55 \times 10^{3}\ g$

b)$51.1\ \text{m/s}$

7:

a)$31.4\ \text{rad/s}$

b)$118\ \text{m/s}$

c)$384\ \text{m/s}$

d)The centripetal acceleration felt by Olympic skaters is 12 times larger than the acceleration due to gravity. That's quite a lot of acceleration in itself. The centripetal acceleration felt by Button's nose was 39.2 times larger than the acceleration due to gravity. It is no wonder that he ruptured small blood vessels in his spins.

9:

a) 0.524 km/s

b) 29.7 km/s

11:

(a)$1.35 \times 10^{3}\ \text{rpm}$

(b)$8.47 \times 10^{3}\ \text{m/s}^{2}$

(c)$8.47 \times 10^{-12}\ \text{N}$

(d)865

12:

(a)$16.6\ \text{m/s}$

(b)$19.6\ \text{m/s}^{2}$

(c)

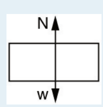


Figure 4.

(d)$1.76 \times 10^{3}\ \text{N}$ or $3.00\ w$,that is, the normal force (upward) is three times her weight.

(e) This answer seems reasonable, since she feels like she's being forced into the chair MUCH stronger than just by gravity.

13:

a) $40.5\ m/s^2$

b) 905 N

c) The force in part (b) is very large. The acceleration in part (a) is too much, about 4 g.

d) The speed of the swing is too large. At the given velocity at the bottom of the swing, there is enough kinetic energy to send the child all the way over the top, ignoring friction.

6.3 Centripetal Force

Summary

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net $F = ma$. For uniform circular motion, the acceleration is the centripetal acceleration— $a = a_c$. Thus, the magnitude of centripetal force F_c is

$$F_c = ma_c.$$

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r}; a_c = r\omega^2$, we get two expressions for the centripetal force F_c in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r}; F_c = mr\omega^2.$$

You may use whichever expression for centripetal force is more convenient. Centripetal force F_c is always perpendicular to the path and pointing to the center of curvature, because a_c is perpendicular to the velocity and pointing to the center of curvature.

Note that if you solve the first expression for r, you get

$$r = \frac{mv^2}{F_c}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

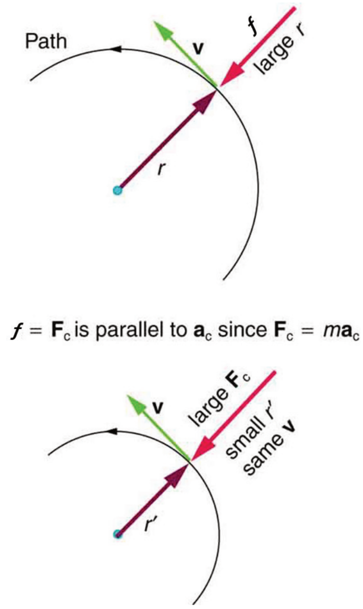


Figure 1. The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the $\mathbf{F_c}$, the smaller the radius of curvature ***r*** and the sharper the curve. The second curve has the same ***v***, but a larger $\mathbf{F_c}$ produces a smaller ***r'***.

Example 1: What Coefficient of Friction Do Care Tires Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

(b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see Figure 2).

Strategy and Solution for (a)

We know that $F_c = \frac{mv^2}{r}$. Thus,

$$F_c = \frac{mv^2}{r} = \frac{(900\ \text{kg})(25.0\ \text{m/s})^2}{(500\ \text{m})} = 1125\ \text{N}.$$

Strategy for (b)

Figure 2 shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_s \text{N}$, where μ_s is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so that $\text{N} = mg$. Thus the centripetal force in this situation is

$$\mathbf{F_c} = f = \mu_s \text{N} = \mu_s mg.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for F_c from the equation

$$\left.\begin{aligned} F_c &= m\frac{v^2}{r} \\ F_c &= mr\omega^2 \end{aligned}\right\},$$

$$m\frac{v^2}{r} = \mu_s mg.$$

We solve this for μ_s, noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}.$$

Solution for (b)

Substituting the knowns,

$$\mu_s = \frac{(25.0 \text{ m/s})^2}{(500 \text{ m})(9.80 \text{ m/s}^2)} = 0.13.$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Discussion

We could also solve part (a) using the first expression in $F_c = m\frac{v^2}{r}$; $F_c = mr\omega^2$, because m, v, and r are given. The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

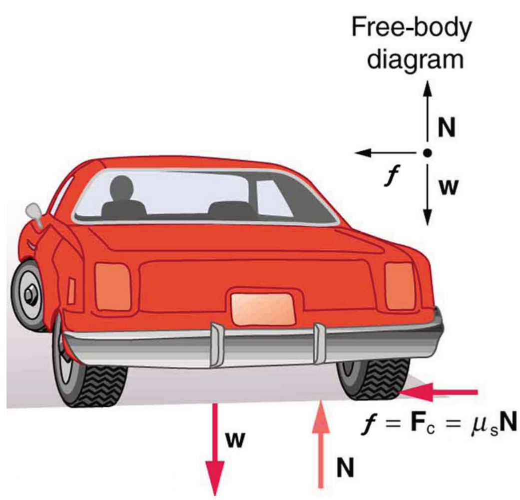


Figure 2. This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve. See Figure 3. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

Figure 3 shows a free body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight w and the normal force of the road N. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is,

$$N \sin\theta = \frac{mv^2}{r}.$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is $N\cos\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$N \cos\theta = mg.$$

Now we can combine the last two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for $N = mg/(\cos\theta)$, and substituting this into the first yields

$$mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r}$$

$$mg\tan(\theta) = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{rg}.$$

Taking the inverse tangent gives

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \text{(ideally banked curve, no friction).}$$

This expression can be understood by considering how θ depends on v and r. A large θ will be obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that θ does not depend on the mass of the vehicle.

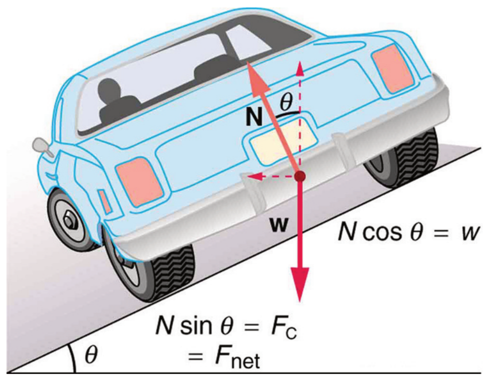


Figure 3. The car on this banked curve is moving away and turning to the left.

Example 2: What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 65.0° should be driven if the road is frictionless.

Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution

Starting with

$$\tan\theta = \frac{v^2}{rg}$$

we get

$$v = (rg\tan\theta)^{1/2}.$$

Noting that tan 65.0º = 2.14, we obtain

$$\begin{aligned} v &= [(100\text{ m})(9.80\text{ m/s}^2)(2.14)]^{1/2} \\ &= 45.8\text{ m/s}. \end{aligned}$$

Discussion

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter’s Problems and Exercises.

TAKE-HOME EXPERIMENT

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

PHET EXPLORATIONS: GRAVITY AND ORBITS

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!

Figure 4. Gravity and Orbits

Section Summary

- Centripetal force F_c is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude

$$F_c = ma_c,$$

which can also be expressed as

$$F_c = m\frac{v^2}{r}$$

or

$$F_c = mr\omega^2$$

Conceptual Questions

Conceptual Questions

1: If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

2: Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

3: If centripetal force is directed toward the center, why do you feel that you are ‘thrown’ away from the center as a car goes around a curve? Explain.

4: Race car drivers routinely cut corners as shown in Figure 5. Explain how this allows the curve to be taken at the greatest speed.

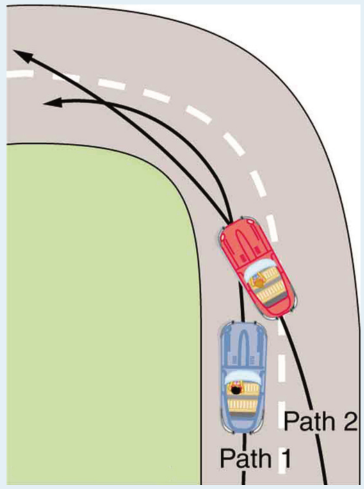


Figure 5. Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

5: A number of amusement parks have rides that make vertical loops like the one shown in Figure 6. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b)The car goes over the top at slower than this speed?

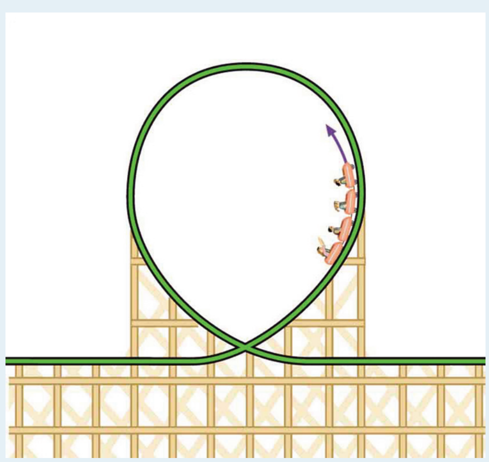


Figure 6. Amusement rides with a vertical loop are an example of a form of curved motion.

7: What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in Figure 6 under the following circumstances:

(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

8: As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.

9: Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in Figure 7 will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.

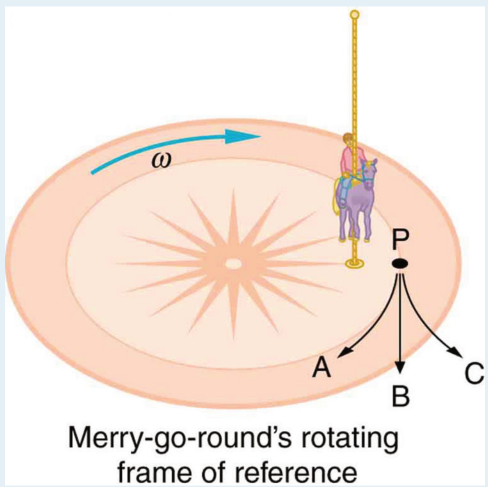


Figure 7. A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?

10: Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

11: Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.

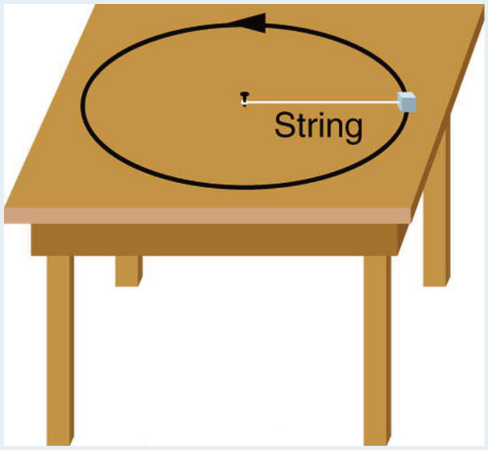


Figure 8. A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

Problems & Exercises

1: (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?

(b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?

(c) Compare each force with her weight.

2: Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

3: What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

4: What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?

5: (a) What is the radius of a bobsled turn banked at 75.0° and taken at 30.0 m/s, assuming it is ideally banked?

(b) Calculate the centripetal acceleration.

(c) Does this acceleration seem large to you?

6: Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in Figure 9. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).

(a) Show that θ (as defined in the figure) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1} v^2 / rg$

(b) Calculate θ for a 12.0 m/s turn of radius 30.0 m (as in a race).

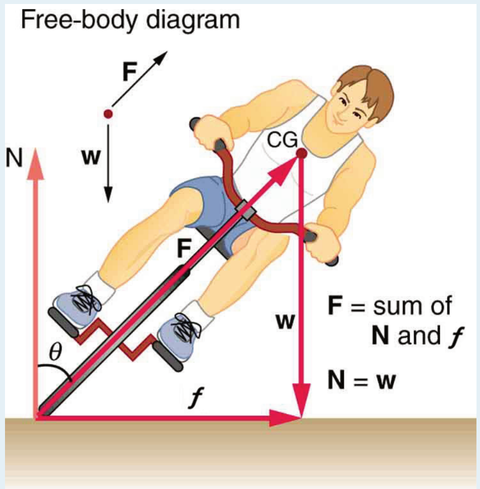


Figure 9. A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle $\boldsymbol{\theta}$, the speed $\boldsymbol{v}$, and the radius of curvature $\boldsymbol{r}$ of the turn similar to that for the ideal banking of roadways.

7: A large centrifuge, like the one shown in Figure 10(a), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries.

(a) At what angular velocity is the centripetal acceleration $10\ g$ if the rider is 15.0 m from the center of rotation?

(b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in Figure 10(b). At what angle θ below the horizontal will the cage hang when the centripetal acceleration is $10\ g$? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free body diagram of the forces to see what the angle θ should be.)

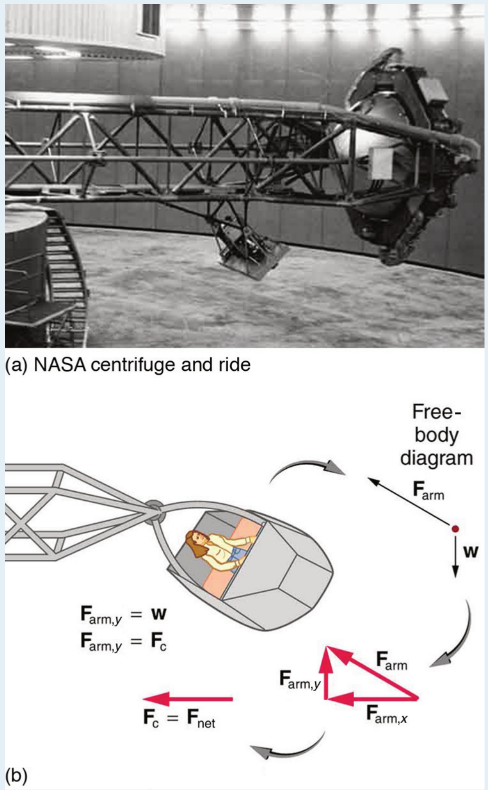


Figure 10. (a) NASA centrifuge used to subject trainees to accelerations similar to those experienced in rocket launches and reentries. (credit: NASA) (b) Rider in cage showing how the cage pivots outward during rotation. This allows the total force exerted on the rider by the cage to be along its axis at all times.

8: Integrated Concepts

If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a 100 m radius curve banked at 15.0º. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

9: Modern roller coasters have vertical loops like the one shown in Figure 11. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?

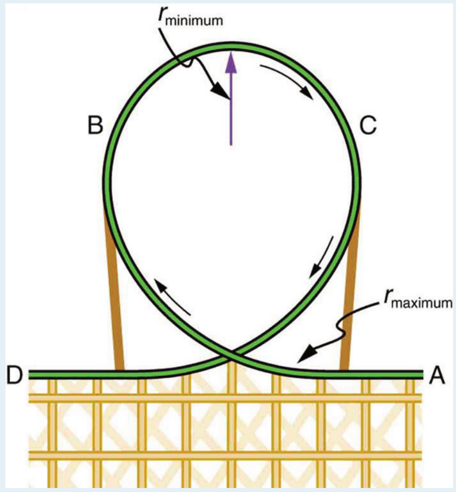


Figure 11. Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than g so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place.

10: Unreasonable Results

(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s.

(b) What is unreasonable about the result?

(c) Which premises are unreasonable or inconsistent?

Glossary

centripetal force
any net force causing uniform circular motion
ideal banking
the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction
ideal speed
the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road
ideal angle
the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

banked curve
the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

Solutions

Problems & Exercises

1:

a) 483 N

b) 17.4 N

c) 2.24 times her weight, 0.0807 times her weight

3:

4.14^0

5:

a) 24.6 m

b) $36.6\ \text{m/s}^2$

c) $a_c = 3.73\ \text{g}$. This does not seem too large, but it is clear that bobsledders feel a lot of force on them going through sharply banked turns.

7:

a) 2.56 rad/s

b) 5.71^0

8:

a) 16.2 m/s

b) 0.234

10:

a) 1.84

b) A coefficient of friction this much greater than 1 is unreasonable .

c) The assumed speed is too great for the tight curve.

6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

Summary

- Discuss the inertial frame of reference.
- Discuss the non-inertial frame of reference.
- Describe the effects of the Coriolis force.

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits fictitious forces—unreal forces that arise from motion and may *seem* real, because the observer's frame of reference is accelerating or rotating.

When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you, and there is no real force backward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right. You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line but the *car* moves to the right, and there is no real force on you to the left. Recall Newton's first law.

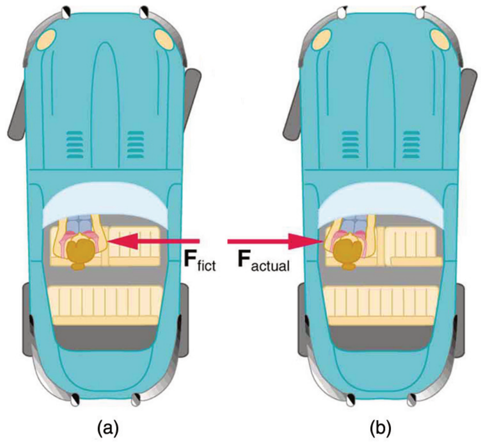


Figure 1. (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is a fictitious force arising from the use of the car as a frame of reference. (b) In the Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no real force to the left on the driver relative to Earth. There is a real force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, while a physicist uses Earth. The physicist chooses Earth because it is very nearly an inertial frame of reference—one in which all forces are real (that is, in which all forces have an identifiable physical origin). In such a frame of reference, Newton's laws of motion take the form given in Chapter 4 Dynamics: Newton's Laws of Motion The car is a **non-inertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is a **fictitious force** having no physical origin. There is nothing real pushing them left—the car, as well as the driver, is actually accelerating to the right.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round. You take the merry-go-round to be your frame of reference because you rotate together. In that non-inertial frame, you feel a fictitious force, named **centrifugal force (**not to be confused with centripetal force**)**, trying to throw you off. You must hang on tightly to counteract the centrifugal force. In Earth's frame of reference, there is no force trying to throw you off. Rather you must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round.

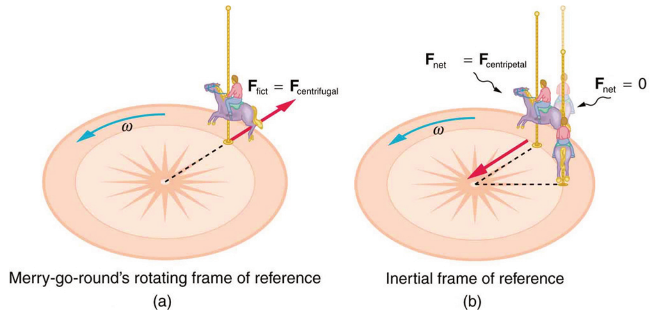


Figure 2. (a) A rider on a merry-go-round feels as if he is being thrown off. This fictitious force is called the centrifugal force—it explains the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off and not a real force (the unshaded rider has $\boldsymbol{F}_{\mathbf{net}}\mathbf{=0}$ and heads in a straight line). A real force, $\boldsymbol{F}_{\mathbf{centripetal}}$, is needed to cause a circular path.

This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (see Figure 3). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the fictitious centrifugal force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.

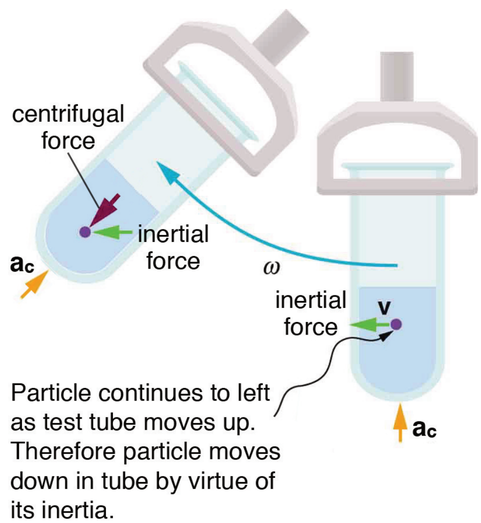


Figure 3. Centrifuges use inertia to perform their task. Particles in the fluid sediment come out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles will come into contact with the test tube walls, which will then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a frame of reference that rotates. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in Figure 4? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using a fictitious force, called the **Coriolis force**, that causes the ball to curve to the right. The fictitious Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's Laws in non-inertial frames of reference.

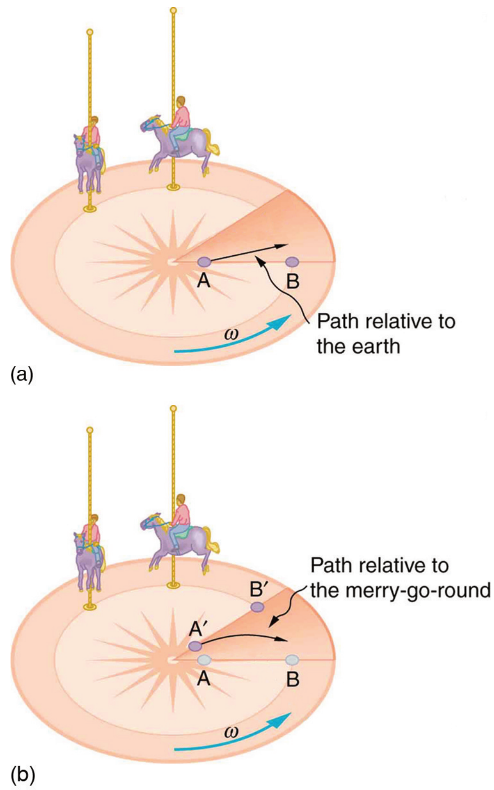


Figure 4. Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point B, starting at point A. Both points rotate to the shaded positions (A' and B') shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in Figure 4. As on the merry-go-round, any motion in Earth's northern hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the southern hemisphere; there, the force is to the left. Because Earth's angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the northern hemisphere to rotate in the counterclockwise direction, while the tropical cyclones (what hurricanes are called below the equator) in the southern hemisphere rotate in the clockwise direction. The terms hurricane, typhoon, and tropical storm are regionally-specific names for tropical cyclones, storm systems characterized by low pressure centers, strong winds, and heavy rains. Figure 5 helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the northern hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the northern hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When non-inertial frames are used, fictitious forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these fictitious forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest and truest, in the sense that all forces have real origins and explanations.

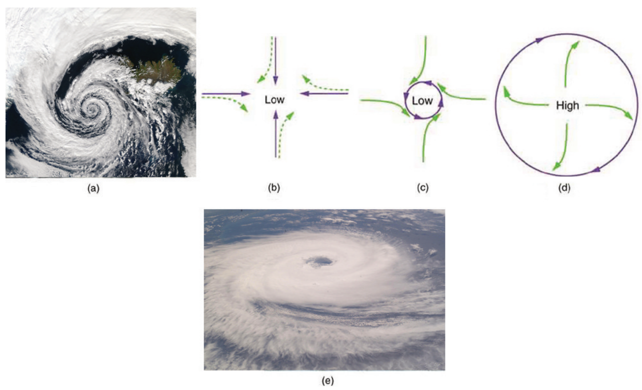


Figure 5. (a) The counterclockwise rotation of this northern hemisphere hurricane is a major consequence of the Coriolis force. (credit: NASA) (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the southern hemisphere, leading to tropical cyclones. (credit: NASA)

Section Summary

- Rotating and accelerated frames of reference are non-inertial.
- Fictitious forces, such as the Coriolis force, are needed to explain motion in such frames.

Conceptual Questions

1: When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the northern hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?

2: Is there a real force that throws water from clothes during the spin cycle of a washing machine? Explain how the water is removed.

3: In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is a fictitious force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all of the real forces acting on them.

4: Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

5: Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80\,\mathrm{m/s^2}$. Who do you agree with and why?

6: A non-rotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

Glossary

fictitious force
a force having no physical origin
centrifugal force
a fictitious force that tends to throw an object off when the object is rotating in a non-inertial frame of reference
Coriolis force
the fictitious force causing the apparent deflection of moving objects when viewed in a rotating frame of reference
non-inertial frame of reference
an accelerated frame of reference

6.5 Newton’s Universal Law of Gravitation

Summary

- Explain Earth’s gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth’s gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth’s surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 1. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton’s contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.

Figure 1. According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

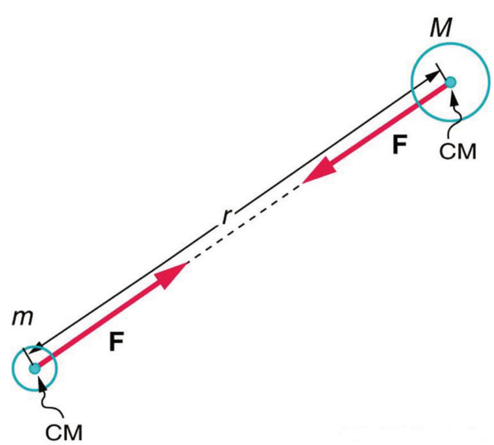


Figure 2. Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

MISCONCEPTION ALERT

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM), which will be further explored in Chapter 8 Linear Momentum and Collisions. For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$F = G\frac{mM}{r^2},$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G = 6.674 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

in SI units. Note that the units of G are such that a force in newtons is obtained from $F = G\frac{mM}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of $6.674 \times 10^{-11}\ \text{N}$. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of $6 \times 10^{24}\ \text{kg}$.

Recall that the acceleration due to gravity g is about $9.80\ \text{m/s}^2$ on Earth. We can now determine why this is so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

$$mg = G\frac{mM}{r^2},$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure 3. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}.$$

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = \left(6.67 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \times \frac{5.98 \times 10^{24}\ \text{kg}}{(6.38 \times 10^6\ \text{m})^2},$$

and we obtain a value for the acceleration of a falling body:

$$g = 9.80\ \text{m/s}^2.$$

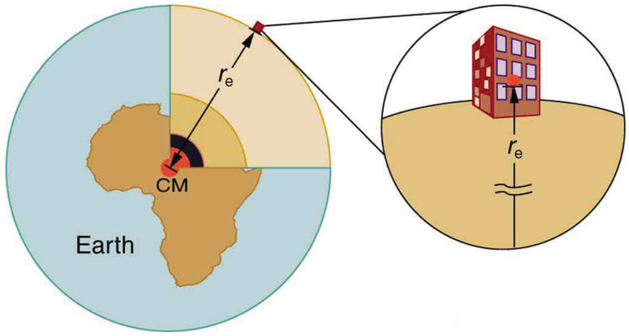


Figure 3. The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

TAKE-HOME EXPERIMENT

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

MAKING CONNECTIONS

Attempts are still being made to understand the gravitational force. As we shall see in Chapter 33 Particle Physics, modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed "pretty nearly."

Example 1: Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path

(a) Find the acceleration due to Earth's gravity at the distance of the Moon.

(b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.

Strategy for (a)

This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that r is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is $3.84 \times 10^8\ \text{m}$.

Solution for (a)

Substituting known values into the expression for g found above, remembering that M is the mass of Earth not the Moon, yields

$$g = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \times \frac{5.98 \times 10^{24}\ \text{kg}}{(3.84 \times 10^8\ \text{m})^2}$$

$$= 2.70 \times 10^{-3}\ \text{m/s}^2 .$$

Strategy for (b)

Centripetal acceleration can be calculated using either form of

$$\left.\begin{array}{rcl} a_c & = & \frac{v^2}{r} \\ a_c & = & r\omega^2 \end{array}\right\}$$

We choose to use the second form:

$$a_c = r\omega^2,$$

where ω is the angular velocity of the Moon about Earth.

Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

$$1\ \text{d} \times 24\frac{\text{hr}}{\text{d}} \times 60\frac{\text{min}}{\text{hr}} \times 60\frac{\text{s}}{\text{min}} = 86,400\ \text{s}$$

we see that

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\ \text{rad}}{(27.3\ \text{d})(86{,}400\ \text{s/d})} = 2.66 \times 10^{-6}\ \frac{\text{rad}}{\text{s}}.$$

The centripetal acceleration is

$$\begin{array}{rcl} a_c & = & r\omega^2 = (3.84 \times 10^8\ \text{m})(2.66 \times 10^{-6}\ \text{rad/s})^2 \\ & = & 2.72 \times 10^{-3}\ \text{m/s}^2 \end{array}$$

The direction of the acceleration is toward the center of the Earth.

Discussion

The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see Figure 4). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force as discussed in Chapter 6.6 Satellites and Kepler's Laws: An Argument for Simplicity.

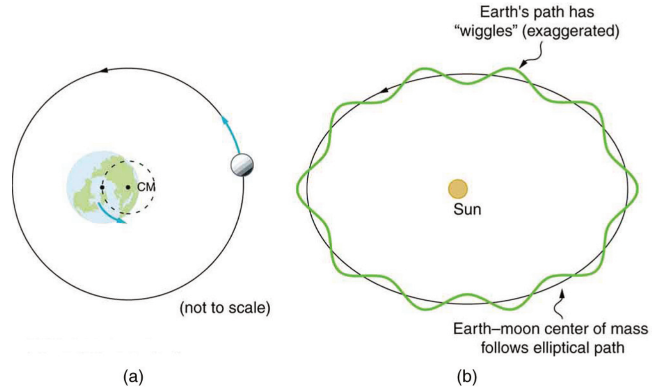


Figure 4. (a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.

Tides

Ocean tides are one very observable result of the Moon's gravity acting on Earth. Figure 5 is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).

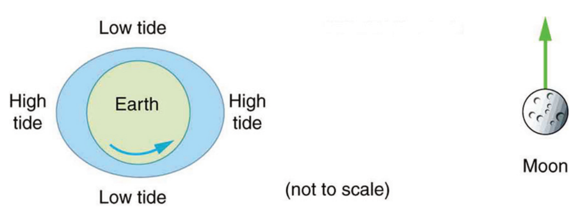


Figure 5. The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90° angle to the Earth-Moon alignment.

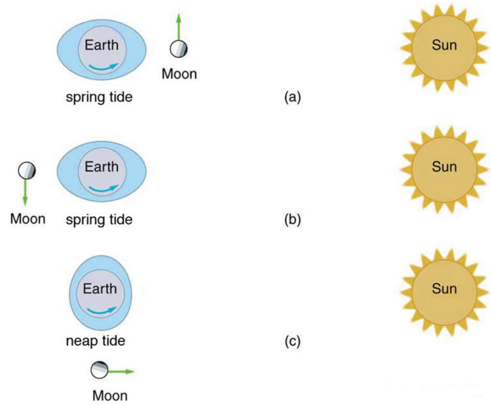


Figure 6. (a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at $\mathbf{90^0}$ to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see Figure 7). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

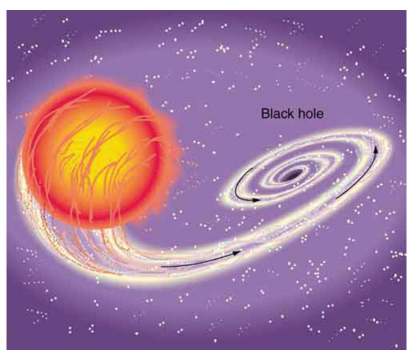


Figure 7. A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

"Weightlessness" and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of "weightlessness" upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn't mean that an astronaut is not being acted upon by the gravitational force. There is no "zero gravity" in an astronaut's orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.

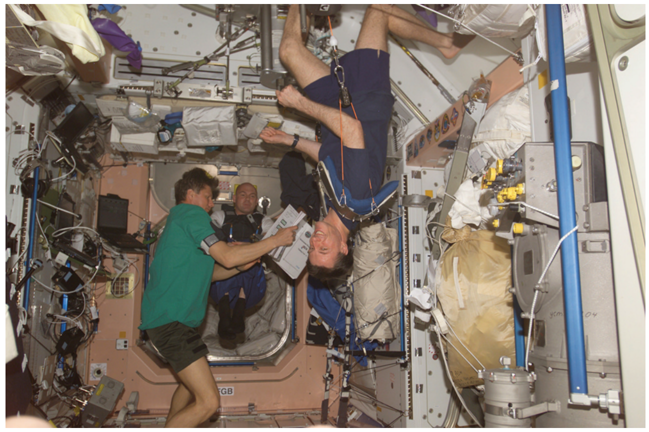


Figure 8. Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

Microgravity refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant G is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in Figure 9. Remarkably, his value for G differs by less than 1% from the best modern value.

One important consequence of knowing G was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth M from the relationship Newton's universal law of gravitation gives

$$mg = G\frac{mM}{r^2},$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See Figure 2. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}.$$

Rearranging to solve for M yields

$$M = \frac{gr^2}{G}.$$

So M can be calculated because all quantities on the right, including the radius of Earth r, are known from direct measurements. We shall see in Chapter 6.6 Satellites and Kepler's Laws: An Argument for Simplicity that knowing G also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics, G is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.

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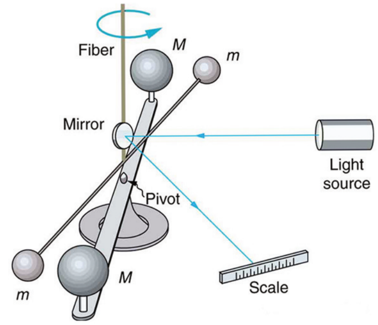

Figure 9. Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres **(*m*)** and the two on the stand **(*M*)** by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Section Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

  $$F = G\frac{mM}{r^2},$$

  where F is the magnitude of the gravitational force. G is the gravitational constant, given by

  $$G = 6.674 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$$

- Newton's law of gravitation applies universally.

Conceptual Questions

1: Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

2: Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80\ \text{m/s}^2$. Who do you agree with and why?

3: Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.

4: Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

Problems & Exercises

1: (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is 9.830 m/s^2 and the radius of the Earth is 6371 km from center to pole.

(b) Compare this with the accepted value of $5.979 \times 10^{24} \text{ kg}$.

2: (a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.

(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.

(c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this number.

3: (a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is $6.418 \times 10^{23} \text{ kg}$ and its radius is $3.38 \times 10^6 \text{ m}$.

4: (a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

5: The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)

(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.

(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.

6: Solve part (b) of Example 1 using $a_c = v^2/r$.

7: Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29 \times 10^{11} \text{ m}$ away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)

8: The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:

(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are $4.50 \times 10^{12} \text{ m}$ apart, as they are at present. The mass of Pluto is $1.4 \times 10^{22} \text{ kg}$.

(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about $2.50 \times 10^{12} \text{ m}$ apart, and compare it with that due to Pluto. The mass of Uranus is $8.62 \times 10^{25} \text{ kg}$.

9: (a) The Sun orbits the Milky Way galaxy once each $2.60 \times 10^8 \text{ y}$, with a roughly circular orbit averaging 3.00×10^4 light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?

(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?

10: Unreasonable Result

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

Glossary

gravitational constant, *G*
a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

center of mass
the point where the entire mass of an object can be thought to be concentrated

microgravity
an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

Newton's universal law of gravitation
every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

Solutions

Problems & Exercises

1:

a) $5.979 \times 10^{24} \text{ kg}$

b) This is identical to the best value to three significant figures.

3:

a) 1.62 m/s^2

b) $3.75\ \mathrm{m/s^2}$

5:

a) $3.42 \times 10^{-5}\ \mathrm{m/s^2}$

b) $3.34 \times 10^{-5}\ \mathrm{m/s^2}$

The values are nearly identical. One would expect the gravitational force to be the same as the centripetal force at the core of the system.

7:

a) $7.01 \times 10^{-7}\ \mathrm{N}$

b) $1.35 \times 10^{-6}\ \mathrm{N},\ 0.521$

9:

a) $1.66 \times 10^{-10}\ \mathrm{m/s^2}$

b) $2.17 \times 10^{5}\ \mathrm{m/s}$

10:

a) $2.937 \times 10^{17}\ \mathrm{kg}$

b) 4.91×10^{-8}

of the Earth's mass.

c) The mass of the mountain and its fraction of the Earth's mass are too great.

d) The gravitational force assumed to be exerted by the mountain is too great.

6.6 Satellites and Kepler's Laws: An Argument for Simplicity

Summary

- State Kepler's laws of planetary motion.
- Derive the third Kepler's law for circular orbits.
- Discuss the Ptolemaic model of the universe.

Examples of gravitational orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The Moon's orbit about Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets about the Sun are no less interesting. If we look further, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force, and it is possible to describe them to various degrees of precision. Precise descriptions of complex systems must be made with large computers. However, we can describe an important class of orbits without the use of computers, and we shall find it instructive to study them. These orbits have the following characteristics:

1. *A small mass m orbits a much larger mass M.* This allows us to view the motion as if M were stationary—in fact, as if from an inertial frame of reference placed on M—without significant error. Mass m is the satellite of M, if the orbit is gravitationally bound.
2. *The system is isolated from other masses*. This allows us to neglect any small effects due to outside masses.

The conditions are satisfied, to good approximation, by Earth's satellites (including the Moon), by objects orbiting the Sun, and by the satellites of other planets. Historically, planets were studied first, and there is a classical set of three laws, called Kepler's laws of planetary motion, that describe the orbits of all bodies satisfying the two previous conditions (not just planets in our solar system). These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630), who devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed.

Kepler's Laws of Planetary Motion

Kepler's First Law

The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

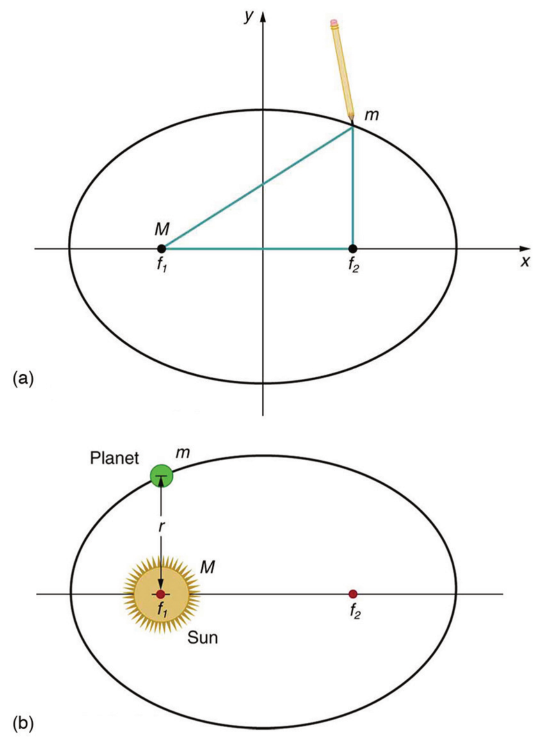

Figure 1. (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Kepler's Second Law

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times (see Figure 2).

Kepler's Third Law

The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun. In equation form, this is

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3},$$

where T is the period (time for one orbit) and r is the average radius. This equation is valid only for comparing two small masses orbiting the same large one. Most importantly, this is a descriptive equation only, giving no information as to the cause of the equality.

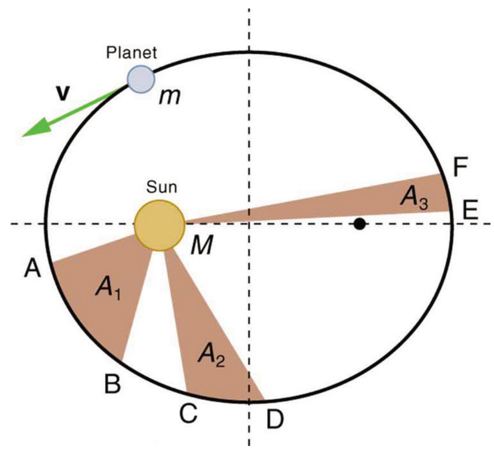

Figure 2. The shaded regions have equal areas. It takes equal times for ***m*** to go from A to B, from C to D, and from E to F. The mass ***m*** moves fastest when it is closest to ***M***. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Note again that while, for historical reasons, Kepler's laws are stated for planets orbiting the Sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Example 1: Find the Time for One Orbit of an Earth Satellite

Given that the Moon orbits Earth each 27.3 d and that it is an average distance of $3.84\times10^8\text{ m}$ from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.

Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form in $\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the Moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the Moon is $r_1=3.84\times10^8\text{ m}$, and that the period of the Moon is $T_1=27.3\text{ d}$. The height of the artificial satellite above Earth's surface is given, and so we must add the radius of Earth (6380 km) to get $r_2=(1500+6380)\text{ km}=7880\text{ km}$. Now all quantities are known, and so T_2 can be found.

Solution

Kepler's third law is

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3}.$$

To solve for T_2, we cross-multiply and take the square root, yielding

$$T_2^2=T_1^2\left(\frac{r_2}{r_1}\right)^3$$

$$T_2=T_1\left(\frac{r_2}{r_1}\right)^{3/2}.$$

Substituting known values yields

$$\begin{aligned} T_2 &= 27.3\ \text{d} \times \frac{24.0\ \text{h}}{\text{d}} \times \left(\frac{7380\ \text{km}}{3.84 \times 10^5\ \text{km}}\right)^{3/2} \\ &= 1.93\ \text{h} \end{aligned}$$

Discussion This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will orbit in the same amount of time. This fact is related to the condition that the satellite's mass is small compared with that of Earth.

People immediately search for deeper meaning when broadly applicable laws, like Kepler's, are discovered. It was Newton who took the next giant step when he proposed the law of universal gravitation. While Kepler was able to discover *what* was happening, Newton discovered that gravitational force was the cause.

Derivation of Kepler's Third Law for Circular Orbits

We shall derive Kepler's third law, starting with Newton's laws of motion and his universal law of gravitation. The point is to demonstrate that the force of gravity is the cause for Kepler's laws (although we will only derive the third one).

Let us consider a circular orbit of a small mass m around a large mass M, satisfying the two conditions stated at the beginning of this section. Gravity supplies the centripetal force to mass m. Starting with Newton's second law applied to circular motion,

$$F_{\text{net}} = ma_c = m\frac{v^2}{r}.$$

The net external force on mass m is gravity, and so we substitute the force of gravity for F_{net}:

$$G\frac{mM}{r^2} = m\frac{v^2}{r}.$$

The mass m cancels, yielding

$$G\frac{M}{r} = v^2.$$

The fact that m cancels out is another aspect of the oft-noted fact that at a given location all masses fall with the same acceleration. Here we see that at a given orbital radius r, all masses orbit at the same speed. (This was implied by the result of the preceding worked example.) Now, to get at Kepler's third law, we must get the period T into the equation. By definition, period T is the time for one complete orbit. Now the average speed v is the circumference divided by the period—that is,

$$v = \frac{2\pi r}{T}.$$

Substituting this into the previous equation gives

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}.$$

Solving for T^2 yields

$$T^2 = \frac{4\pi^2}{GM}r^3.$$

Using subscripts 1 and 2 to denote two different satellites, and taking the ratio of the last equation for satellite 1 to satellite 2 yields

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}.$$

This is Kepler's third law. Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body M cancel.

Now consider what we get if we solve $T^2 = \frac{4\pi^2}{GM} r^3$. We obtain a relationship that can be used to determine the mass M of a parent body from the orbits of its satellites:

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M.$$

If r and T are known for a satellite, then the mass M of the parent can be calculated. This principle has been used extensively to find the masses of heavenly bodies that have satellites. Furthermore, the ratio r^3/T^2 should be a constant for all satellites of the same parent body (because $r^3/T^2 = GM/4\pi^2$). (See Table 2).

It is clear from Table 2 that the ratio of r^3/T^2 is constant, at least to the third digit, for all listed satellites of the Sun, and for those of Jupiter. Small variations in that ratio have two causes—uncertainties in the r and T data, and perturbations of the orbits due to other bodies. Interestingly, those perturbations can be—and have been—used to predict the location of new planets and moons. This is another verification of Newton's universal law of gravitation.

MAKING CONNECTIONS

Newton's universal law of gravitation is modified by Einstein's general theory of relativity, as we shall see in Chapter 33 Particle Physics. Newton's gravity is not seriously in error—it was and still is an extremely good approximation for most situations. Einstein's modification is most noticeable in extremely large gravitational fields, such as near black holes. However, general relativity also explains such phenomena as small but long-known deviations of the orbit of the planet Mercury from classical predictions.

The Case for Simplicity

The development of the universal law of gravitation by Newton played a pivotal role in the history of ideas. While it is beyond the scope of this text to cover that history in any detail, we note some important points. The definition of planet set in 2006 by the International Astronomical Union (IAU) states that in the solar system, a planet is a celestial body that:

1. is in orbit around the Sun,
2. has sufficient mass to assume hydrostatic equilibrium and
3. has cleared the neighborhood around its orbit.

A non-satellite body fulfilling only the first two of the above criteria is classified as "dwarf planet."

In 2006, Pluto was demoted to a 'dwarf planet' after scientists revised their definition of what constitutes a "true" planet.

Parent	Satellite	Average orbital radius r(km)	Period $T(y)$	r^3 / T^2 (km^3 / y^2)
Earth	Moon	3.84×10^5	0.07481	1.01×10^{19}
Sun	Mercury	5.79×10^7	0.2409	3.34×10^{24}
	Venus	1.082×10^8	0.6150	3.35×10^{24}
	Earth	1.496×10^8	1.000	3.35×10^{24}
	Mars	2.279×10^8	1.881	3.35×10^{24}
	Jupiter	7.783×10^8	11.86	3.35×10^{24}
	Saturn	1.427×10^9	29.46	3.35×10^{24}
	Neptune	4.497×10^9	164.8	3.35×10^{24}
	Pluto	5.90×10^9	248.3	3.33×10^{24}
Jupiter	Io	4.22×10^5	0.00485 (1.77 d)	3.19×10^{21}
	Europa	6.71×10^5	0.00972 (3.55 d)	3.20×10^{21}
	Ganymede	1.07×10^6	0.0196 (7.16 d)	3.19×10^{21}
	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

Table 2. Orbital Data and Kepler's Third Law.

The universal law of gravitation is a good example of a physical principle that is very broadly applicable. That single equation for the gravitational force describes all situations in which gravity acts. It gives a cause for a vast number of effects, such as the orbits of the planets and moons in the solar system. It epitomizes the underlying unity and simplicity of physics.

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in Figure 3(a). This is called the Ptolemaic view, for the Greek philosopher who lived in the second century AD. This model is characterized by a list of facts for the motions of planets with no cause and effect explanation. There tended to be a different rule for each heavenly body and a general lack of simplicity.

Figure 3(b) represents the modern or Copernican model. In this model, a small set of rules and a single underlying force explain not only all motions in the solar system, but all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling. As our knowledge of nature has grown, the basic simplicity of its laws has become ever more evident.

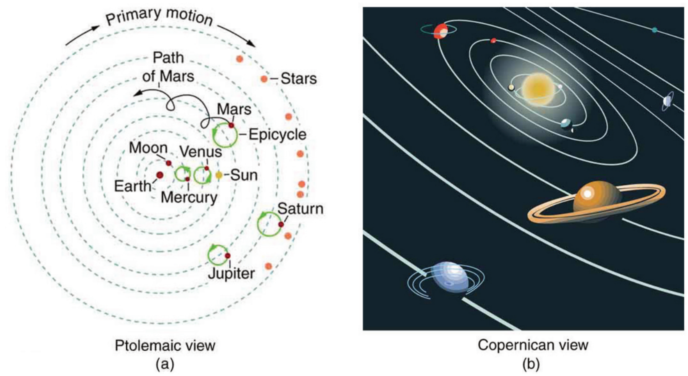

Figure 3. (a) The Ptolemaic model of the universe has Earth at the center with the Moon, the planets, the Sun, and the stars revolving about it in complex superpositions of circular paths. This geocentric model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints as to what are the causes of these motions. (b) The Copernican model has the Sun at the center of the solar system. It is fully explained by a small number of laws of physics, including Newton's universal law of gravitation.

Section Summary

- Kepler's laws are stated for a small mass m orbiting a larger mass M in near-isolation. Kepler's laws of planetary motion are then as follows:

  Kepler's first law

  The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

  Kepler's second law

  Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

  Kepler's third law

  The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun:

  $$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

  where T is the period (time for one orbit) and rr size 12{m} {} is the average radius of the orbit.

- The period and radius of a satellite's orbit about a larger body M are related by

  $$T^2 = \frac{4\pi^2}{GM} r^3$$

  or

  $$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M.$$

Conceptual Questions 1: In what frame(s) of reference are Kepler's laws valid? Are Kepler's laws purely descriptive, or do they contain causal information?

Problems & Exercises

1: A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit based on the data for the moon in Table 2.

2: Calculate the mass of the Sun based on data for Earth's orbit and compare the value obtained with the Sun's actual mass.

3: Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.

4: Find the ratio of the mass of Jupiter to that of Earth based on data in Table 2.

5: Astronomical observations of our Milky Way galaxy indicate that it has a mass of about 8.0×10^{11} solar masses. A star orbiting on the galaxy's periphery is about 6.0×10^{4} light years from its center. (a) What should the orbital period of that star be? (b) If its period is 6.0×10^{7} instead, what is the mass of the galaxy? Such calculations are used to imply the existence of "dark matter" in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.

6: Integrated Concepts

Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same radius that intersects the satellite's orbit at an angle of 90° relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)

7: Unreasonable Results

(a) Based on Kepler's laws and information on the orbital characteristics of the Moon, calculate the orbital radius for an Earth satellite having a period of 1.00 h. (b) What is unreasonable about this result? (c) What is unreasonable or inconsistent about the premise of a 1.00 h orbit?

8: Construct Your Own Problem

On February 14, 2000, the NEAR spacecraft was successfully inserted into orbit around Eros, becoming the first artificial satellite of an asteroid. Construct a problem in which you determine the orbital speed for a satellite near Eros. You will need to find the mass of the asteroid and consider such things as a safe distance for the orbit. Although Eros is not spherical, calculate the acceleration due to gravity on its surface at a point an average distance from its center of mass. Your instructor may also wish to have you calculate the escape velocity from this point on Eros.

Solutions

Problems & Exercises

2:

$1.98 \times 10^{30}\ \text{kg}$

4:

$\frac{M_J}{M_E} = 316$

6:

a) $7.4 \times 10^{3}\ \text{m/s}$

b) $1.05 \times 10^{3}\ \text{m/s}$

c) $2.86 \times 10^{-7}\ \text{s}$

d) $1.84 \times 10^{7}\ \text{N}$

e) $2.76 \times 10^{4}\ \text{J}$

7:

a) $5.08 \times 10^{3}\ \text{km}$

b) This radius is unreasonable because it is less than the radius of earth.

c) The premise of a one-hour orbit is inconsistent with the known radius of the earth.

PART 7

Chapter 7 Work, Energy, and Energy Resources

Figure 1. How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $\boldsymbol{E} = mc^2$).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

7.1 Work: The Scientific Definition

Summary

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = |\mathbf{F}|(\cos\theta)|\mathbf{d}|,$$

where W is work, $\mathbf{d}$ is the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$, as in Figure 1. We can also write this as

$$W = Fd\cos\theta.$$

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

WHAT IS WORK?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd\cos\theta,$$

where W is work, F is the magnitude of the force on the system, d is the magnitude of the displacement of the system, and θ is the angle between the force vector $\mathbf{F}$ and the displacement vector $\mathbf{d}$.

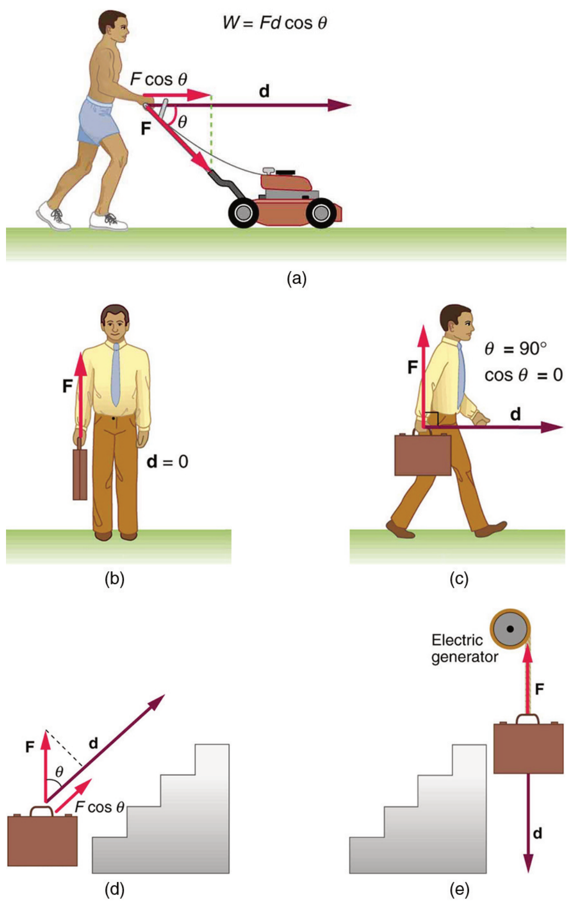

Figure 1. Examples of work. (a) The work done by the force **F** on this lawn mower is $Fd\cos\theta$. Note that $F\cos\theta$ is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force **F** in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because **F** and **d** are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in Figure 1. The person holding the briefcase in Figure 1(b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”—see Chapter 7.3 Gravitational Potential Energy for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure 1(c) does no work on it, because the force is perpendicular to the motion. That is, $\cos 90^\circ = 0$, and so $W = 0$.

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure 1(d), work *is* done—energy is transferred to the briefcase. Finally, in Figure 1(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase’s weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes $\theta = 180^\circ$, and $\cos 180^\circ = -1$; therefore, W is negative.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\ \text{J} = 1\ \text{N} \cdot \text{m} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 1: Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in Figure 1(a) if he exerts a constant force of $75.0\ \text{N}$ at an angle 35° below the horizontal and pushes the mower $25.0\ \text{m}$ on level ground? Convert the amount of work from joules to kilocalories and compare it with this person’s average daily intake of $10,000\ \text{kJ}$ (about $2400\ \text{kcal}$) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by $1^\circ C$, and is equivalent to $4.184\ \text{J}$, while one *food calorie* (1 kcal) is equivalent to $4184\ \text{J}$.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd \cos\theta$. The force, angle, and displacement are given, so that only the work W is unknown.

Solution

The equation for the work is

$$W = Fd \cos\theta.$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0\ \text{N})(25.0\ \text{m}) \cos(35.0^\circ) \\ &= 1536\ \text{J} = 1.54 \times 10^3\ \text{J} \end{aligned}.$$

Converting the work in joules to kilocalories yields $W = (1536\ \text{J})(1\ \text{kcal}/4184\ \text{J}) = 0.367\ \text{kcal}$. The ratio of the work done to the daily consumption is

$$\frac{W}{2400\ \text{kcal}} = 1.53 \times 10^{-4}.$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force F does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols,

$$W = Fd\cos\theta.$$

- The SI unit for work and energy is the joule (J), where $1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

1: Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

2: Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

3: Describe a situation in which a force is exerted for a long time but does no work. Explain.

Problems & Exercises

1: How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.

2: A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

3: (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

4: Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See Chapter 7.6 Table 1 for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?

5: Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See Figure 2.) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.

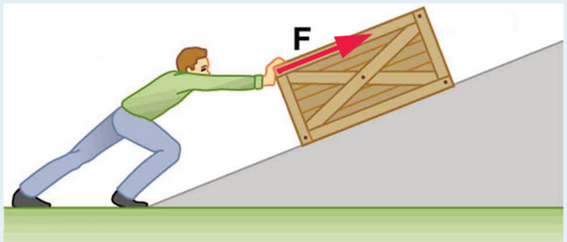

Figure 2. A man pushes a crate up a ramp.

6: How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in Figure 3? Assume no friction acts on the wagon.

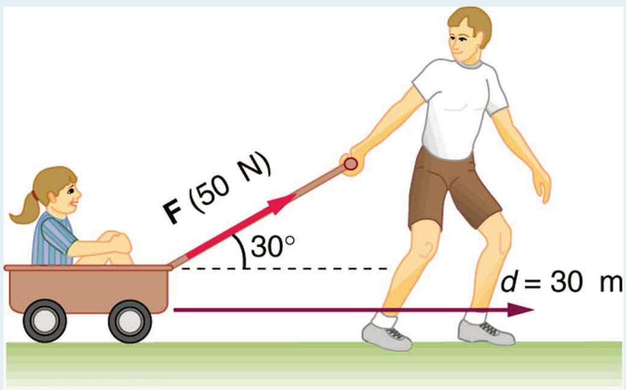

Figure 3. The boy does work on the system of the wagon and the child when he pulls them as shown.

7: A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

8: Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in Figure 4. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

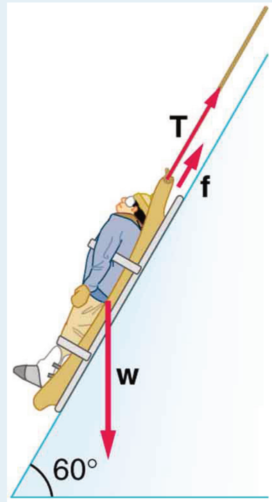

Figure 4. A rescue sled and victim are lowered down a steep slope.

Glossary

energy
the ability to do work
work
the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement
joule
SI unit of work and energy, equal to one newton-meter

Solutions

Problems & Exercises

1:

$3.00 \text{ J} = 7.17 \times 10^{-4} \text{ kcal}$

3:

(a) $5.92 \times 10^{5} \text{ J}$

(b) $-5.88 \times 10^{5} \text{ J}$

(c) The net force is zero.

5:

$3.14 \times 10^{3} \text{ J}$

7:

(a) -700 J

(b) 0

(c) 700 J

(d) 38.6 N

(e) 0

7.2 Kinetic Energy and the Work-Energy Theorem

Summary

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in Chapter 7.1 Figure 1(a) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in Chapter 7.1 Figure 1(d) is stored in the briefcase-Earth system and can be recovered at any time, as shown in Chapter 7.1 Figure 1(e). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in Chapter 4 Dynamics: Force and Newton's Laws of Motion that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force F_{net}. In equation form, this is $W_{\text{net}} = F_{\text{net}} d \cos \theta$ where θ is the angle between the force vector and the displacement vector.

Figure 1(a) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an $F \cos \theta$ vs. d graph. In this case, $F \cos \theta$ is constant. You can see that the area under the graph is $Fd \cos\theta$, or the work done. Figure 1(b) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force $(F \cos \theta)_{i(\text{ave})}$. The work done is $(F \cos\theta)_{i(\text{ave})} d_i$ for each strip, and the total work done is the sum of the W_i. Thus the total work done is the total area under the curve, a useful property to which we shall refer later.

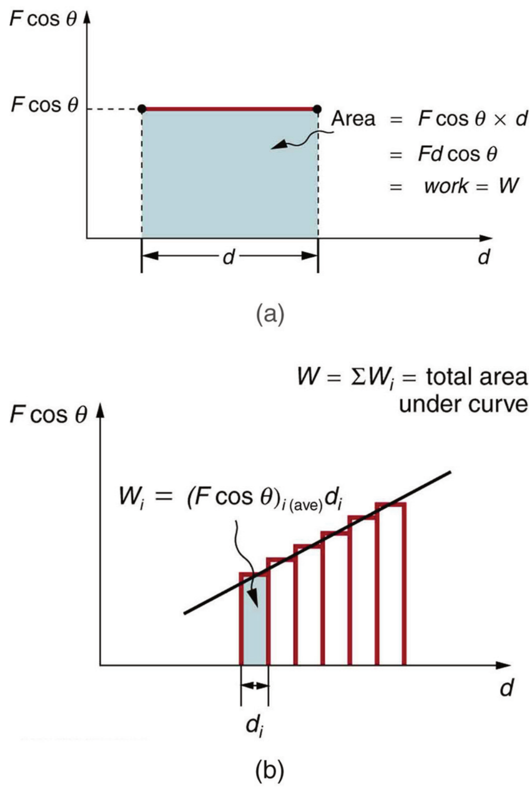

Figure 1. (a) A graph of ***F* cos *θ*** vs.***d***, when ***F* cos *θ*** is constant. The area under the curve represents the work done by the force. (b) A graph of ***F* cos *θ*** vs. ***d*** in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 2.

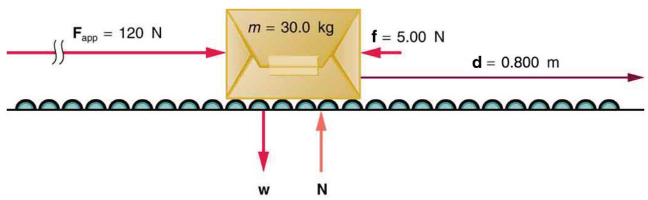

Figure 2. A package on a roller belt is pushed horizontally through a distance **d**.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force F_{app} and the horizontal friction force f. Thus, as expected, the net force is parallel to the displacement, so that $\theta = 0^{\circ}$ and $\cos \theta = 1$, and the net work is given by

$$W_{net} = F_{net} d.$$

The effect of the net force F_{net} is to accelerate the package from v_0 to v. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See Example 1.) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{net} = ma$ from Newton's second law gives

$$W_{net} = mad.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in Chapter 2.5 Motion Equations for Constant Acceleration in One Dimension for the change in speed over a distance d if the acceleration has the constant value a; namely, $v^2 = v_0^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0^2}{2d}$. When a is substituted into the preceding expression for W_{net}, we obtain

$$W_{net} = m\left(\frac{v^2 - v_0^2}{2d}\right)d.$$

The d cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

THE WORK-ENERGY THEOREM

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational kinetic energy (KE) of a mass m moving at a speed v. (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\mathrm{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in Figure 2, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example 1: Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in Figure 2 is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\mathrm{KE} = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$\mathrm{KE} = \frac{1}{2}mv^2,$$

Entering known values gives

$$\mathrm{KE} = 0.5(30.0\ \mathrm{kg})(0.500\ \mathrm{m/s})^2,$$

which yields

$$\mathrm{KE} = 3.75\ \mathrm{k\cdot m^2/s^2} = 3.75\ \mathrm{J}.$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 2: Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in Figure 2 with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See Figure 2.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{\text{net}} = 120\text{ N} - 5.00\text{ N} = 115\text{ N}$. Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}} d = (115\text{ N})(0.800\text{ m}) \\ &= 92.0 \text{N} \cdot \text{m} = 92.0\text{ J}. \end{aligned}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}} d \cos(0^\circ) = F_{\text{app}} d \\ &= (120\text{ N})(0.800\text{ m}) \\ &= 96.0\text{ J} \end{aligned}$$

The friction force and displacement are in opposite directions, so that $\theta = 180^\circ$, and the work done by friction is

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}} d \cos 180^\circ = -F_{\text{fr}} d \\ &= (-5.00\text{ N})(0.800\text{ m}) \\ &= -4.00\text{ J}. \end{aligned}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_{\text{N}} &= 0, \\ W_{\text{app}} &= 96.0\text{ J}, \\ W_{\text{fr}} &= -4.00\text{ J}. \end{aligned}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0\text{ J}.$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 3: Determining Speed from Work and Energy

Find the speed of the package in Figure 2 at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}mv_0^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{net} + \frac{1}{2}mv_0^2.$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\ \text{J} + 3.75\ \text{J} = 95.75\ \text{J}.$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\ \text{J})}{m}} = \sqrt{\frac{191.5\ \text{kg}\cdot\text{m}^2/\text{s}^2}{30.0\ \text{kg}}} \\ &= 2.53\ \text{m/s}. \end{aligned}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 4: Work and Energy Can Reveal Distance, Too

How far does the package in Figure 2 coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so $\theta = 180^\circ$. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus $W_{fr} = -95.75\ \text{J}$. Furthermore, $W_{fr} = fd'\cos\theta = -fd'$, where d' is the distance it takes to stop. Thus,

$$d' = -\frac{W_{fr}}{f} = -\frac{-95.75\ \text{J}}{5.00\ \text{N}},$$

and so

$$d' = 19.2\ \text{m}.$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Section Summary

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $KE = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

Conceptual Questions

1: The person in Figure 3 does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

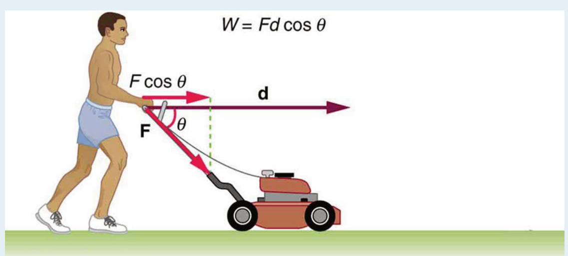

Figure 3.

2: Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

3: When solving for speed in Example 3, we kept only the positive root. Why?

Problems & Exercises 1: Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.

2: (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

3: Confirm the value given for the kinetic energy of an aircraft carrier in Chapter 7.6 Table 1. You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).

4: (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

5: A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

6: Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

7: Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

Glossary

net work
work done by the net force, or vector sum of all the forces, acting on an object
work-energy theorem
the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy
kinetic energy
the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ *for the translational (i.e., non-rotational) motion of an object of mass* m *moving at speed* v

Solutions

Problems & Exercises

1:

$1/250$

3:

1.1×10^{10} J

5:

2.8×10^3 N

7:

102 N

7.3 Gravitational Potential Energy

Summary

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $PE_g = mgh$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in Figure 1. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of $\mathrm{PE_g}$ to KE without explicitly considering the intermediate step of work. (See Example 2.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

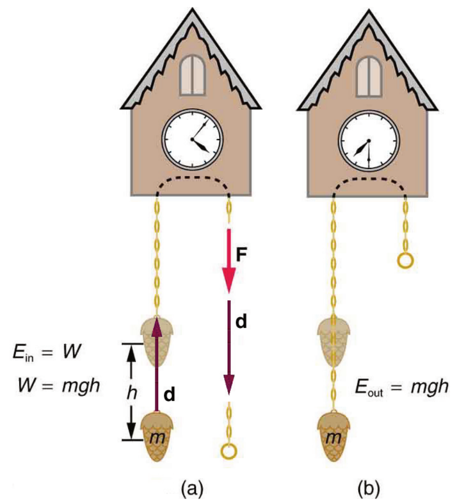

Figure 1. (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy $\Delta\mathrm{PE_g}$ to be

$$\Delta\mathrm{PE_g} = mgh,$$

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned} mgh &= (0.500\ \mathrm{kg})(9.80\ \mathrm{m/s})(21.00\ \mathrm{m}) \\ &= 4.90\ \mathrm{kg{\cdot}m^2/s^2} = 4.90\ \mathrm{J.} \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta\mathrm{PE_g} = mgh$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See Figure 2.) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

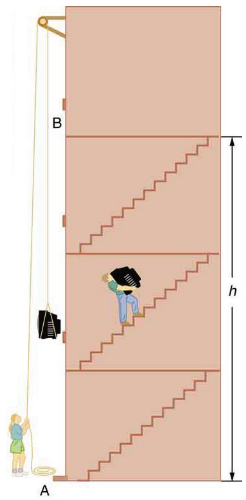

Figure 2. The change in gravitational potential energy **(ΔPE_g)** between points A and B is independent of the path. **$\Delta PE_g = mgh$** for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example 1: The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W = Fd\cos\theta = -Fd,$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(\cos\theta = \cos 180^\circ = 1)$. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:

$$KE = -\Delta PE_g = -mgh,$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\text{KE} = mgh.$$

Combining this equation with the expression for W gives

$$-Fd = mgh.$$

Recalling that hh size 12{h} {} is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mgh}{d} = -\frac{(60.0\ \text{kg})(9.80\ \text{m/s}^2)(-3.00\ \text{m})}{5.00\times10^{-3}\ \text{m}} = 3.53 \times 10^5\ \text{N}.$$

Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See Figure 3.)

Figure 3. The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example 2: Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in Figure 4 if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

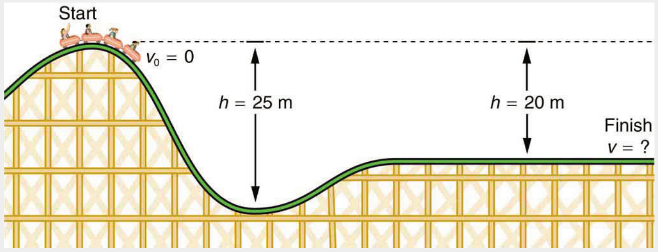

Figure 4. The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all $\mathbf{\Delta PE_g}$ is converted to **KE**.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta PE_g = \Delta KE$. Using the equations for PE_g and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

Here the initial kinetic energy is zero, so that $\Delta KE = \frac{1}{2}mv^2$. The equation for change in potential energy states that $\Delta PE_g = mgh$. Since h is negative in this case, we will rewrite this as $\Delta PE_g = -mg|h|$ to show the minus sign clearly. Thus,

$$-\Delta PE_g = \Delta KE$$

becomes

$$mg|h| = \frac{1}{2}mv^2.$$

Solving for v, we find that mass cancels and that

$$v = \sqrt{2g|h|}.$$

Substituting known values,

$$\begin{aligned} v &= \sqrt{2(9.80\ \text{m/s})(220.0\ \text{m})} \\ &= 19.8\ \text{m/s}. \end{aligned}$$

Solution for (b)

Again $-\Delta PE_g = \Delta KE$. In this case there is initial kinetic energy, so $\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$. Thus,

$$mg|h| = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg|h| + \frac{1}{2}mv_0^2.$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g|h| + v_0^2}.$$

This equation is very similar to the kinematics equation $v = \sqrt{v_0^2 + 2ad}$, but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m}) + (5.00\ \text{m/s})^2} \\ &= 20.4\ \text{m/s}. \end{aligned}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in Chapter 2.7 Falling Objects that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth.

When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION— CONVERTING POTENTIAL TO KINETIC ENERGY

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see Figure 5). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.

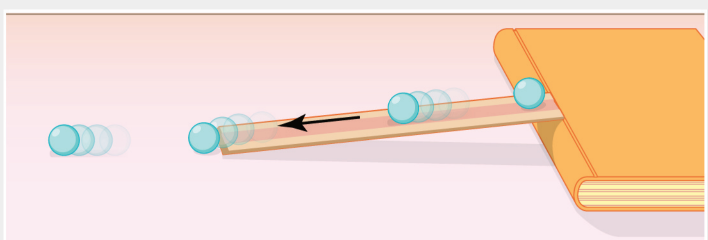

Figure 5. A marble rolls down a ruler, and its speed on the level surface is measured.

Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g, is $\Delta PE_g = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta KE = -\Delta PE_g$.

Conceptual Questions

1: In Example 2, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that it had the same final speed. Explain in terms of conservation of energy.

2: Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

Problems & Exercises

1: A hydroelectric power facility (see Figure 6) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume $50.0\ \text{km}^3 \text{mass} = 5.00 \times 10^{13}\ \text{kg},$ given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.

Figure 6. Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

2: (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^{9}\ \text{kg}$ and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?

3: Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

4: In Example 2, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta PE >> KE_i$. Confirm this statement by taking the ratio of ΔPE to KE_i. (Note that mass cancels.)

5: A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in Figure 7. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

Figure 7. A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

6: In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

Glossary

gravitational potential energy
the energy an object has due to its position in a gravitational field

Solutions

Problems & Exercises

1:

(a) $1.96 \times 10^{16}\ \text{J}$

(b) The ratio of gravitational potential energy in the lake to the energy stored in the bomb is 0.52. That is, the energy stored in the lake is approximately half that in a 9-megaton fusion bomb.

3:

(a) 1.8 J

(b) 8.6 J

5:

$$v_f = \sqrt{2gh + {v_0}^2} = \sqrt{2(9.80\ \text{m/s}^2)(-0.180\ \text{m}) + (2.00\ \text{m/s})^2} = 0.687\ \text{m/s}$$

7.4 Conservative Forces and Potential Energy

Summary

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

POTENTIAL ENERGY AND CONSERVATIVE FORCES

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring (PE_s). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in Chapter 5.3 Elasticity: Stress and Strain, and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See Figure 1.) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $F = kx$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is $W_s = Fd = \left(\frac{kx}{2}\right)x = \frac{1}{2}kx^2$. Alternatively, we noted in Chapter 7.2 Kinetic Energy and the Work-Energy Theorem that the area under a graph of F vs. x is the work done by the force. In Figure 1(c) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, PE_s, to be

$$PE_s = \frac{1}{2}kx^2,$$

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x. The potential energy of the spring PE_s does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.

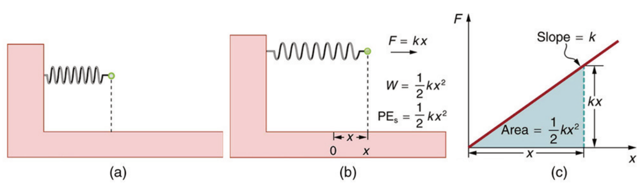

Figure 1. (a) An undeformed spring has no PE_s stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude $F=kx$, and the work done to stretch (or compress) it is $1/2\ kx^2$. Because the force is conservative, this work is stored as potential energy (PE_s) in the spring, and it can be fully recovered. (c) A graph of F vs. x has a slope of k, and the area under the graph is $1/2\ kx^2$. Thus the work done or potential energy stored is $1/2\ kx^2$.

The equation $PE_s = \frac{1}{2}kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $PE_s = \frac{1}{2}kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in Figure 2 for a guitar string.

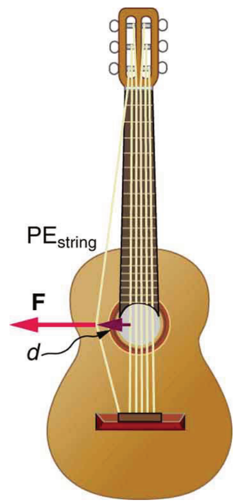

Figure 2. Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta\text{KE}.$$

If only conservative forces act, then

$$W_{net} = W_c,$$

where W_c is the total work done by all conservative forces. Thus,

$$W_c = \Delta\text{KE}.$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is, $W_c = -\Delta\text{PE}$. Therefore,

$$-\Delta\text{PE} = \Delta\text{KE}$$

or

$$\Delta\text{KE} + \Delta\text{PE} = 0.$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\left.\begin{array}{c}\mathrm{KE}+\mathrm{PE}=\text{constant}\\ \text{or}\\ \mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f}\end{array}\right\}\text{(conservative forces only)},$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(\mathrm{KE}+\mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

Example 1: Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in Figure 3. The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

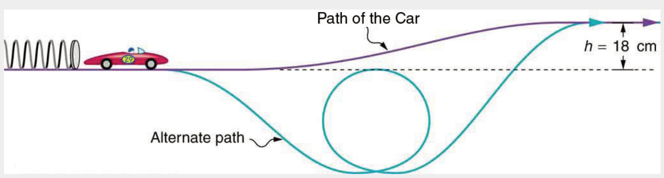

Figure 3. A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathrm{KE_i}+\mathrm{PE_i}=\mathrm{KE_f}+\mathrm{PE_f}$$

or

$$\frac{1}{2}mv_{\mathrm{i}}^2+mgh_{\mathrm{i}}+\frac{1}{2}kx_{\mathrm{i}}^2=\frac{1}{2}mv_{\mathrm{f}}^2+mgh_{\mathrm{f}}+\frac{1}{2}kx_{\mathrm{f}}^2,$$

where h is the height (vertical position) and x is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_{i} and h_{f} are zero. Furthermore, the initial speed v_{i} is zero and the final compression of the spring x_{f} is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_{\mathrm{i}}^2=\frac{1}{2}mv_{\mathrm{f}}^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}}x_i \\ &= \sqrt{\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}}(0.0400 \text{ m}) \\ &= 2.00 \text{ m/s}. \end{aligned}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f.$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$\begin{aligned} v_f &= \sqrt{\frac{kx_i^2}{m} - 2gh_f} \\ &= \sqrt{\left(\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}\right)(0.0400 \text{ m})^2 - 2(9.80 \text{ m/s}^2)(0.180 \text{ m})} \\ &= 0.687 \text{ m/s}. \end{aligned}$$

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in Example 1. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

PHET EXPLORATIONS: ENERGY SKATE PARK

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

Figure 4. Energy Skate Park

Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $PE_s = \frac{1}{2}kx^2$, where k is the spring's force constant and x is the displacement

from its undeformed position.

- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\left.\begin{array}{c}\text{KE} + \text{PE} = \text{constant} \\ \text{or} \\ \text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f}\end{array}\right\}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

Conceptual Questions

1: What is a conservative force?

2: The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

3: Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

4: What is the relationship of potential energy to conservative force?

Problems & Exercises

1: A $5.00 \times 10^5\text{-kg}$ subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?

2: A pogo stick has a spring with a force constant of $2.50 \times 10^4 \text{ N/m,}$ which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the Chapter 7.6 Problem-Solving Strategies for Energy.

Glossary

conservative force
a force that does the same work for any given initial and final configuration, regardless of the path followed

potential energy
energy due to position, shape, or configuration

potential energy of a spring
the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ *where* x *is the distance the spring is compressed or extended and* k *is the spring constant*

conservation of mechanical energy
the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system
mechanical energy
the sum of kinetic energy and potential energy

Solutions

Problems & Exercises

1:

$7.81 \times 10^5\ \mathrm{N/m}$

7.5 Nonconservative Forces

Summary

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

Nonconservative Forces and Friction

Forces are either conservative or nonconservative. Conservative forces were discussed in Chapter 7.4 Conservative Forces and Potential Energy. A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in Figure 1, work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

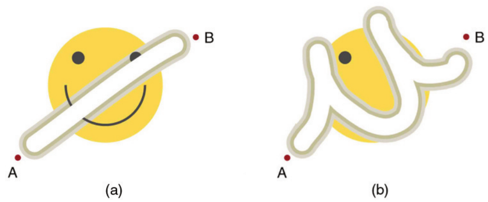

Figure 1. The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

How Nonconservative Forces Affect Mechanical Energy

Mechanical energy *may* not be conserved when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. Figure 2 compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in Figure 2(a) first before studying more complicated systems as in Figure 2(b).

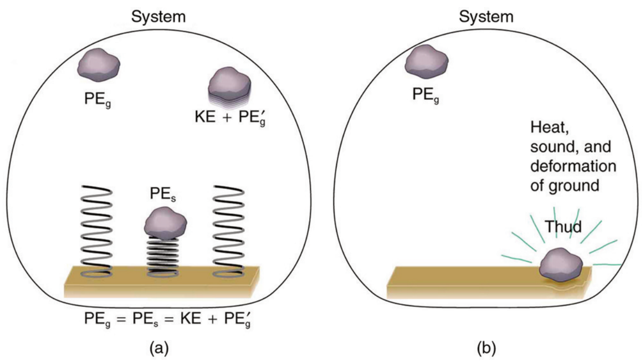

Figure 2. Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

How the Work-Energy Theorem Applies

Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in Chapter 7.2 Kinetic Energy and the Work-Energy Theorem, the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or $W_{net} = \Delta KE$. The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{net} = W_{nc} + W_c,$$

so that

$$W_{nc} + W_c = \Delta KE,$$

where W_{nc} is the total work done by all nonconservative forces and W_c is the total work done by all conservative forces.

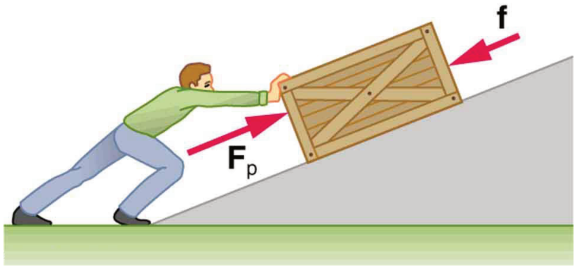

Figure 3. A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider Figure 3, in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that $W_c = -\Delta PE$. Substituting this equation into the previous one and solving for W_{nc} gives

$$W_{nc} = \Delta KE + \Delta PE.$$

This equation means that the total mechanical energy $(KE + PE)$ changes by exactly the amount of work done by nonconservative forces. In Figure 3, this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange $W_{nc} = \Delta KE + \Delta PE$ to obtain

$$KE_i + PE_i + W_{nc} = KE_f + PE_f.$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If W_{nc} is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in Figure 3. If W_{nc} is negative, then mechanical energy is decreased, such as when the rock hits the ground in Figure 2(b). If W_{nc} is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying $KE_i + PE_i + W_{nc} = KE_f + PE_f$ amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation $KE_i + PE_i + W_{nc} = KE_f + PE_f$ says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

Example 1: Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in Figure 4, where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.

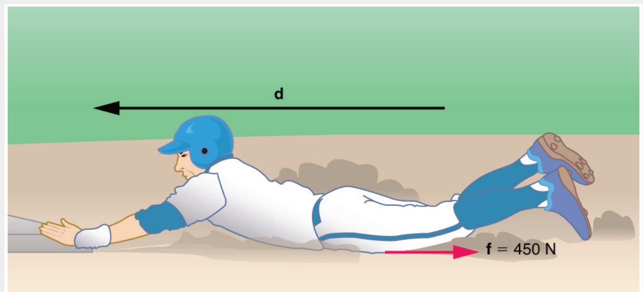

Figure 4. The baseball player slides to a stop in a distance ***d***. In the process, friction removes the player's kinetic energy by doing an amount of work ***fd*** equal to the initial kinetic energy.

Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because f is in the opposite direction of the motion (that is, $\theta = 180^\circ$, and so $\cos\theta = -1$). Thus $W_{nc} = -fd$. The equation simplifies to

$$\frac{1}{2}mv_i^2 - fd = 0$$

or

$$fd = \frac{1}{2}mv_i^2.$$

This equation can now be solved for the distance d.

Solution

Solving the previous equation for d and substituting known values yields

$$\begin{aligned} d &= \frac{mv_i^2}{2f} \\ &= \frac{(65.0\text{ kg})(6.00\text{ m/s})^2}{(2)(450\text{ N})} \\ &= 2.60\text{ m}. \end{aligned}$$

Discussion

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

Example 2: Calculating Distance Traveled: Sliding Up an Incline

Suppose that the player from Example 1 is running up a hill having a 5.00° incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed. Determine how far he slides.

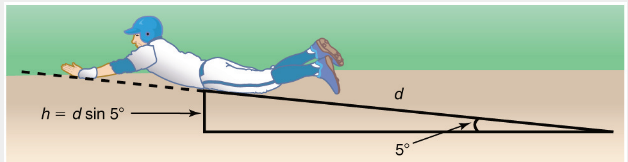

Figure 5. The same baseball player slides to a stop on a $\mathbf{5.00^0}$ slope.

Strategy

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance d to reach height h along the hill, with $h = d \sin 5.00°$. This is expressed by the equation

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} = \mathrm{KE_f} + \mathrm{PE_f}.$$

Solution

The work done by friction is again $W_{\mathrm{nc}} = -fd$; initially the potential energy is $\mathrm{PE_i} = mg \cdot 0 = 0$ and the kinetic energy is $\mathrm{KE_i} = \frac{1}{2}mv_i^2$; the final energy contributions are $\mathrm{KE_f} = 0$ for the kinetic energy and $\mathrm{PE_f} = mgh = mgd \sin\theta$ for the potential energy.

Substituting these values gives

$$\frac{1}{2}mv_i^2 + 0 + (-fd) = 0 + mgd \sin\theta .$$

Solve this for d to obtain

$$\begin{aligned} d &= \frac{\left(\frac{1}{2}\right)mv_i^2}{f + mg \sin\theta} \\ &= \frac{(0.5)(65.0\ \mathrm{kg})(6.00\ \mathrm{m/s})^2}{450\ \mathrm{N} + (65.0\ \mathrm{kg})(9.80\ \mathrm{m/s^2})\sin(5.00^0)} \\ &= 2.31\ \mathrm{m}. \end{aligned}$$

Discussion

As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance d that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy mgh, without combining and resolving force vectors. This simplifies the solution considerably.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—DETERMINING FRICTION FROM STOPPING DISTANCE

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from Chapter 7.3 Take-Home Investigation—Converting Potential to Kinetic Energy. In addition, you will need a foam cup with a small hole in the side, as shown in Figure 6. From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance d the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these data to find the coefficient of kinetic friction μ_k of the cup on the table. The force of friction f on the cup is $\mu_k N$, where the normal force N is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is fd. You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?

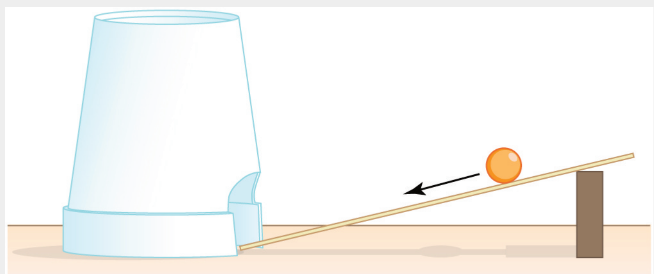

Figure 6. Rolling a marble down a ruler into a foam cup.

PHET EXPLORATIONS: THE RAMP

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

Figure 7. The Ramp

Section Summary

- A nonconservative force is one for which work depends on the path.
- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation

form, $W_{nc} = \Delta\text{KE} + \Delta\text{PE}$ or, equivalently, $\text{KE}_i + \text{PE}_i + W_{nc} = \text{KE}_f + \text{PE}_f$.

- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

Problems & Exercises

1: A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in Figure 8. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)

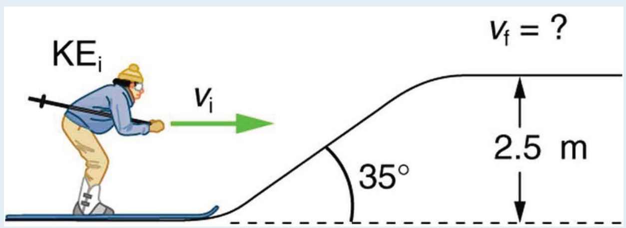

Figure 8. The skier's initial kinetic energy is partially used in coasting to the top of a rise.

2: (a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope 2.5° above the horizontal?

Glossary

nonconservative force
a force whose work depends on the path followed between the given initial and final configurations
friction
the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

Solutions

Problems & Exercises

1:

9.46 m/s

7.6 Conservation of Energy

Summary

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy $(\mathbf{KE} + \mathbf{PE})$ and energy transferred via work done by nonconservative forces $(\boldsymbol{W}_{\mathbf{nc}})$. But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called **other energy** $(\mathbf{OE})$. Then we can state the conservation of energy in equation form as

$$\mathbf{KE_i} + \mathbf{PE_i} + \boldsymbol{W}_{\mathbf{nc}} + \mathbf{OE_i} = \mathbf{KE_f} + \mathbf{PE_f} + \mathbf{OE_f}.$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is $\mathbf{KE}$, work done by a conservative force is represented by $\mathbf{PE}$, work done by nonconservative forces is $\boldsymbol{W}_{\mathbf{nc}}$, and all other energies are included as OE. This equation applies to all previous examples; in those situations OE was constant, and so it subtracted out and was not directly considered.

MAKING CONNECTIONS: USEFULNESS OF THE ENERGY CONSERVATION PRINCIPLE

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does OE play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of OE).

Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Table 1 gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

PROBLEM SOLVING STRATEGIES FOR ENERGY

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$$

Step 4. If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}.$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate W_c, the work done by conservative forces; it is already incorporated in the PE terms.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that $\mathrm{PE_g}$ is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see Figure 1) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure 1. Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	10^{68}
Energy released in a supernova	10^{44}
Fusion of all the hydrogen in Earth's oceans	10^{34}
Annual world energy use	4×10^{20}
Large fusion bomb (9 megaton)	3.8×10^{16}
1 kg hydrogen (fusion to helium)	6.4×10^{14}
1 kg uranium (nuclear fission)	8.0×10^{13}
Hiroshima-size fission bomb (10 kiloton)	4.2×10^{13}
90,000-ton aircraft carrier at 30 knots	1.1×10^{10}
1 barrel crude oil	5.9×10^{9}
1 ton TNT	4.2×10^{9}
1 gallon of gasoline	1.2×10^{8}
Daily home electricity use (developed countries)	7×10^{7}
Daily adult food intake (recommended)	1.2×10^{7}
1000-kg car at 90 km/h	3.1×10^{5}
1 g fat (9.3 kcal)	3.9×10^{4}
ATP hydrolysis reaction	3.2×10^{4}
1 g carbohydrate (4.1 kcal)	1.7×10^{4}
1 g protein (4.1 kcal)	1.7×10^{4}
Tennis ball at 100 km/h	22
Mosquito $(10^{-2}\ \text{g at } 0.5\ \text{m/s})$	1.3×10^{-6}
Single electron in a TV tube beam	4.0×10^{-15}
Energy to break one DNA strand	10^{-19}

Table 1. Energy of Various Objects and Phenomena.

Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency** Eff of an energy conversion process is defined as

$$\text{Efficiency}(Eff) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}.$$

Table 2 lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%)[1]
Cycling and climbing	20
Swimming, surface	2
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

Table 2. Efficiency of the Human Body and Mechanical Devices.

PHET EXPLOTATIONS: MASSES AND SPRINGS

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

Figure 2. Masses and Springs

Section Summary

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.

- When all forms of energy are considered, conservation of energy is written in equation form as $\mathrm{KE_i} + \mathrm{PE_i} + W_{\mathrm{nc}} + \mathrm{OE_i} = \mathrm{KE_f} + \mathrm{PE_f} + \mathrm{OE_f}$, where OE is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be $Eff = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

Conceptual Questions 1: Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See Figure 3.)

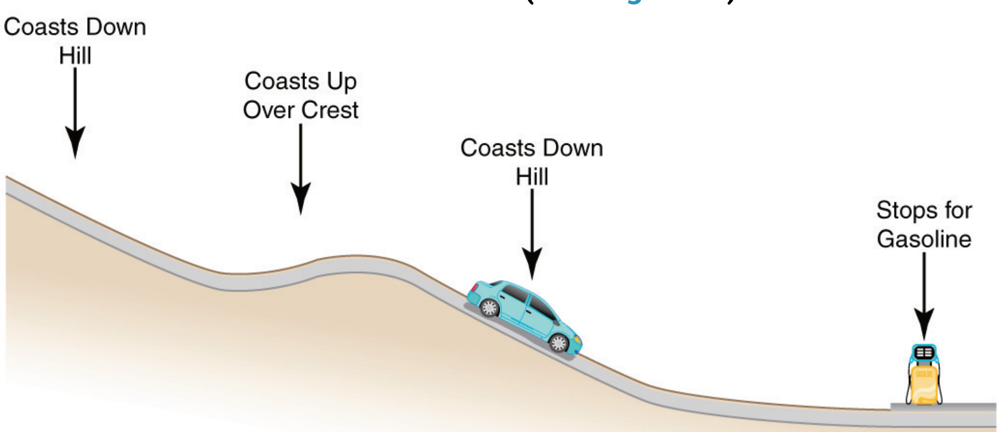

Figure 3. A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

2: Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

3: Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.

4: List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

5: List the energy conversions that occur when riding a bicycle.

Problems & Exercises 1: Using values from Table 1, how many DNA molecules could be

broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

2: Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

3: If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from Table 1)? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.

4: (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from Table 1. To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

Footnotes

1. 1 Representative values

Glossary

law of conservation of energy
the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same
electrical energy
the energy carried by a flow of charge
chemical energy
the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction
radiant energy
the energy carried by electromagnetic waves
nuclear energy
energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy

the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

efficiency

a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

Solutions

Problems & Exercises

1:

$4 \times 10^4 \text{ molecules}$

2:

Equating ΔPE_g and ΔKE, we obtain $v = \sqrt{2gh + v_0^2} = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (15.0 \text{ m/s})^2} = 24.8 \text{ m/s}$

4:

(a) $25 \times 10^6 \text{ years}$

(b) This is much, much longer than human time scales.

7.7 Power

Summary

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in Figure 1.

Figure 1. This powerful rocket on the Space Shuttle Endeavor did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

POWER

Power is the rate at which work is done.

$$P = \frac{W}{t}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $(1\ \text{W} = 1\ \text{J/s})$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example 1: Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See Figure 2.)

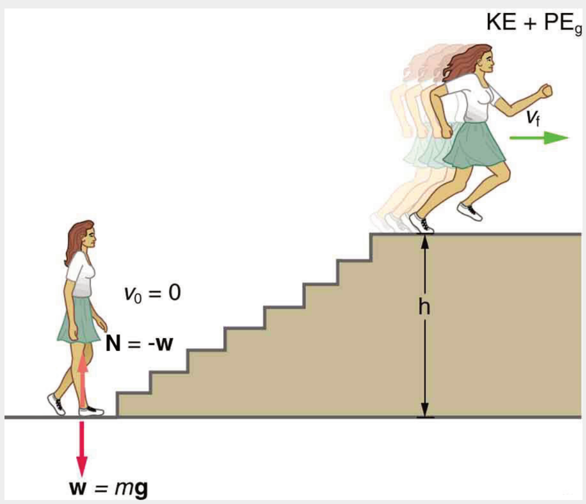

Figure 2. When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = \text{KE} + \text{PE}$. At the bottom of the stairs, we take both KE and PE_g initially zero; thus, $W = \text{KE}_f + \text{PE}_g = \frac{1}{2}mv_f^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t}.$$

Entering known values yields

$$\begin{aligned} P &= \frac{0.5(60.0\ \text{kg})(2.00\ \text{m/s})^2 + (60.0\ \text{kg})(9.80\ \text{m/s}^2)(3.00\ \text{m})}{3.50\ \text{s}} \\ &= \frac{120\ \text{J} + 1764\ \text{J}}{3.50\ \text{s}} \\ &= 538\ \text{W}. \end{aligned}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1\ \text{hp} = 746\ \text{W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—MEASURE YOUR POWER RATING

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See Table 3 for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is $10^6\ \text{W}$ of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See Figure 3.)

Figure 3. Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Table 3. Power Output or Consumption.

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P=W/t=E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E=Pt.$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\text{kW}\cdot\text{h})$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example 2: Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per $\text{kW}\cdot\text{h}$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E=Pt$ and then calculate the cost. Because electrical energy is expressed in $\text{kW}\cdot\text{h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\text{kW}\cdot\text{h}$ is

$$\begin{aligned} E &= Pt=(0.200\ \text{kW})(6.00\ \text{h/d})(30.0\ \text{d}) \\ &= 36.0\ \text{kW}\cdot\text{h}, \end{aligned}$$

and the cost is simply given by

$$\text{cost}=(36.0\ \text{kW}\cdot\text{h})(\$0.120\ \text{per kW}\cdot\text{h})=\$4.32\ \text{per month}.$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in Chapter 15 Thermodynamics, the potential for energy to produce useful work has been "degraded" in the energy transformation.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\text{ W} = 1\text{ J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\text{ hp} = 746\text{ W}$.

Conceptual Questions

1: Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

2: Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

3: A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Problems & Exercises

1: The Crab Nebula (see Figure 4) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table 3, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.

Figure 4. Crab Nebula (credit: ESO, via Wikimedia Commons)

2: Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from Table 3: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

3: A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

4: What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

5: A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per $\text{kW} \cdot \text{h}$?

6: (a) What is the average power consumption in watts of an appliance that uses $5.00 \text{ kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

7: (a) What is the average useful power output of a person who does $6.00 \times 10^6 \text{ J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

8: A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

9: (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

10: (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

11: (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00 \times 10^4 \text{ J}$ run a pocket calculator that consumes energy at the rate of $1.00 \times 10^{-3} \text{ W}$?

12: (a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

13: Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the Chapter 7.7 Problem-Solving Strategies for Energy.

14: (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26} \text{ W}$.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of 1.30 kW/m^2 reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05 \times 10^{20} \text{ J})$? Australia's energy needs $(5.4 \times 10^{18} \text{ J})$? China's energy needs $(6.3 \times 10^{19} \text{ J})$? (These energy consumption values are from 2006.)

Glossary

power
the rate at which work is done
watt
(W) SI unit of power, with $1\text{ W} = 1\text{ J/s}$
horsepower
an older non-SI unit of power, with $1\text{ hp} = 746\text{ W}$
kilowatt-hour
$(\text{kW}\cdot\text{h})$ *unit used primarily for electrical energy provided by electric utility companies*

Solutions

Problems & Exercises

1:

2×10^{-10}

3:

(a) 40

(b) 8 million

5:

$149

7:

(a) 208 W

(b) 141 s

9:

(a) 3.20 s

(b) 4.04 s

11:

(a) $9.46 \times 10^{7}\text{ J}$

(b) 2.54 y

13:

Identify knowns: $m = 950\text{ kg},\ \text{slope angle } \theta = 2.00^{0},\ v = 3.00\text{ m/s},\ f = 600\text{ N}$

Identify unknowns: power P of the car, force F that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv,$$

where F is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$F = f + w = 600\ \text{N} + mg \sin\theta$$

Insert this into the expression for power and solve:

$$\begin{aligned} P &= (f + mg \sin\theta)v \\ &= [600\ \text{N} + (950\ \text{kg})(9.80\ \text{m/s}^2) \sin 2^0](30.0 m/s) \\ &= 2.77 \times 10^4\ \text{W} \end{aligned}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

7.8 Work, Energy, and Power in Humans

Summary

- Explain the human body's consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See Figure 1.) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.

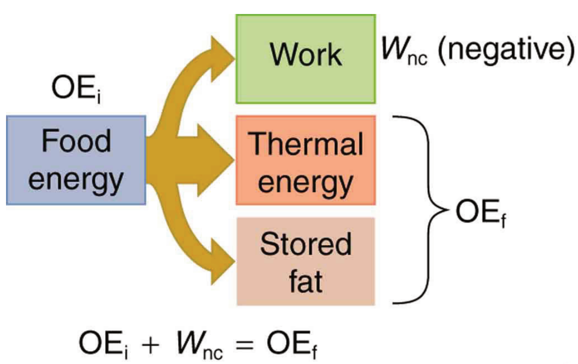

Figure 1. Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate** (BMR) and is divided among various systems in the body, as shown in Table 4. The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

Table 4. Basal Metabolic Rates (BMR).

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See Figure 2.) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. Table 5 shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy $(\mathrm{KE} + \mathrm{PE})$ of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as Example 1 illustrates.

Example 1: Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, he will steadily gain weight. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

Table 5 states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000\ \text{kJ}}{400\ \text{W}} = 2500\ \text{s} = 42\ \text{min.}$$

Discussion

If this person uses more energy than he or she consumes, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$\text{Fat loss} = (1000\ \text{kJ})\left(\frac{1.0\ \text{g fat}}{39\ \text{kJ}}\right) = 26\ \text{g},$$

assuming the energy content of fat to be 39 kJ/g.

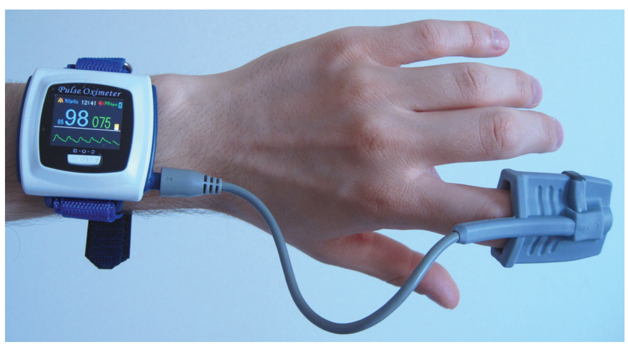

Figure 2. A pulse oxymeter is an apparatus that measures the amount of oxygen in blood. Oxymeters can be used to determine a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

Activity	Energy consumption in watts	Oxygen consumption in liters O_2/min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

Table 5. Energy and Oxygen Consumption Rates[1] (Power).

All bodily functions, from thinking to lifting weights, require energy. (See Figure 3.) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all it that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

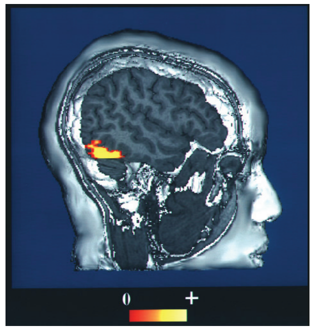

Figure 3. This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being asked to recognize faces. (credit: NIH via Wikimedia Commons)

Section Summary

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate when at rest is called the basal metabolic rate (BMR)
- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

Conceptual Questions

1: Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?

2: Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?

3: Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?

4: Discuss the relative effectiveness of dieting and exercise in losing weight, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal). Specifically, is it likely that exercise alone will be sufficient to lose weight? You may wish to consider that regular exercise may increase the metabolic rate, whereas protracted dieting may reduce it.

Problems & Exercises

1: (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?

2: (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?

3: Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)

Figure 4. Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

4: (a) What is the efficiency of an out-of-condition professor who does $2.10 \times 10^5\ \text{J}$ of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?

5: Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from Table 5 for the energy consumption rates of these activities.

6: Using data from Table 5, calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)

7: What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See Table 5.)

8: Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?

9: Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b)

In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.

10: Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.

11: (a) Calculate the energy in kJ used by a 55.0-kg woman who does 50 deep knee bends in which her center of mass is lowered and raised 0.400 m. (She does work in both directions.) You may assume her efficiency is 20%. (b) What is the average power consumption rate in watts if she does this in 3.00 min?

12: Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see Figure 5). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from Table 2, calculate the food energy in kilojoules he metabolized during the flight.

Figure 5. The Daedalus 88 in flight. (credit: NASA photo by Beasley)

13: The swimmer shown in Figure 6 exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

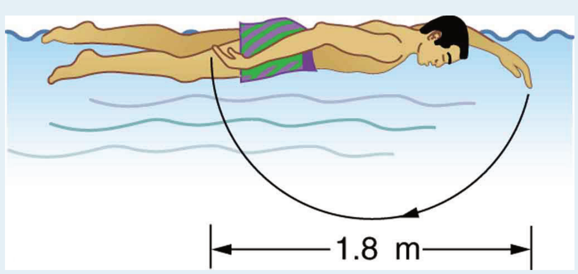

Figure 6.

14: Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

15: The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about 7×10^9 kg. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 7), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

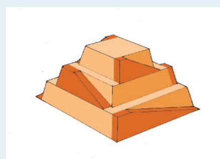

Figure 7. Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

16: (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

Footnotes

1. 1 for an average 76-kg male

Glossary

metabolic rate
the rate at which the body uses food energy to sustain life and to do different activities
basal metabolic rate
the total energy conversion rate of a person at rest
useful work
work done on an external system

Solutions

Problems & Exercises

1:

(a) 9.5 min

(b) 69 flights of stairs

3:

641 W, 0.860 hp

5:

31 g

7:

14.3%

9:

(a) $3.21 \times 10^4 \text{ N}$

(b) $2.35 \times 10^3 \text{ N}$

(c) Ratio of net force to weight of person is 41.0 in part (a); 3.00 in part (b)

11:

(a) 108 kJ

(b) 599 W

13:

(a) 144 J

(b) 288 W

15:

(a) $2.50 \times 10^{12} \text{ J}$

(b) 2.52%

(c) $1.4 \times 10^4 \text{ kg}$ (14 metric tons)

7.9 World Energy Use

Summary

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. About 40% of the world's energy comes from oil, and much of that goes to transportation uses. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.5% of the world's population, consumes 24% of the world's oil production per year; 66% of that oil is imported!

Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in Figure 1. The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

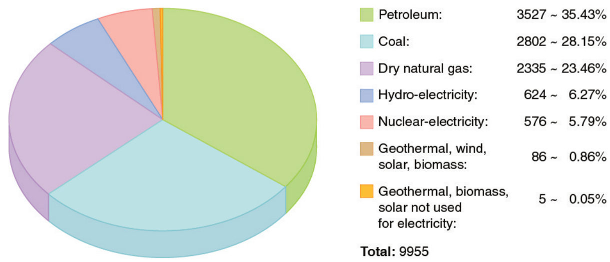

Figure 1. World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

The World's Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See Figure 2.) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 20% of its electricity and 10% of its overall energy needs with renewable resources by the year 2020. (See Figure 3.) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world's second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO_2. In India, the main energy resources are biomass (wood and dung) and coal. Half of India's oil is imported. About 70% of India's electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world.

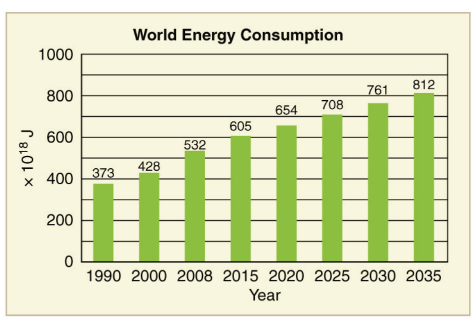

Figure 2. Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2011)

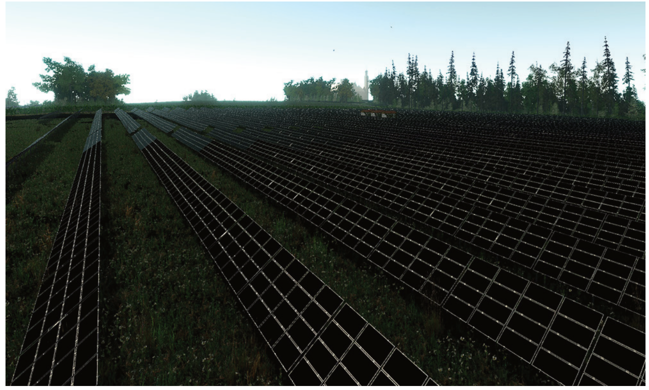

Figure 3. Solar cell arrays at a power plant in Steindorf, Germany (credit: Michael Betke, Flickr)

Table 6 displays the 2006 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about 67% of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total contributions of renewable energy in some countries with a large rural population, so these percentages in this table are left blank.

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables	Electricity Use per capita (kWh/yr)	Energy Use per capita (GJ/yr)
Australia	5.4	34%	17%	44%	0%	3%	1%	10000	260
Brazil	9.6	48%	7%	5%	1%	35%	2%	2000	50
China	63	22%	3%	69%	1%	6%		1500	35
Egypt	2.4	50%	41%	1%	0%	6%		990	32
Germany	16	37%	24%	24%	11%	1%	3%	6400	173
India	15	34%	7%	52%	1%	5%		470	13
Indonesia	4.9	51%	26%	16%	0%	2%	3%	420	22
Japan	24	48%	14%	21%	12%	4%	1%	7100	176
New Zealand	0.44	32%	26%	6%	0%	11%	19%	8500	102
Russia	31	19%	53%	16%	5%	6%		5700	202
U.S.	105	40%	23%	22%	8%	3%	1%	12500	340
World	**432**	**39%**	**23%**	**24%**	**6%**	**6%**	**2%**	**2600**	**71**

Table 6. Energy Consumption—Selected Countries (2006).

Energy and Economic Well-being

The last two columns in this table examine the energy and electricity use per capita. Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in Figure 4. Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.

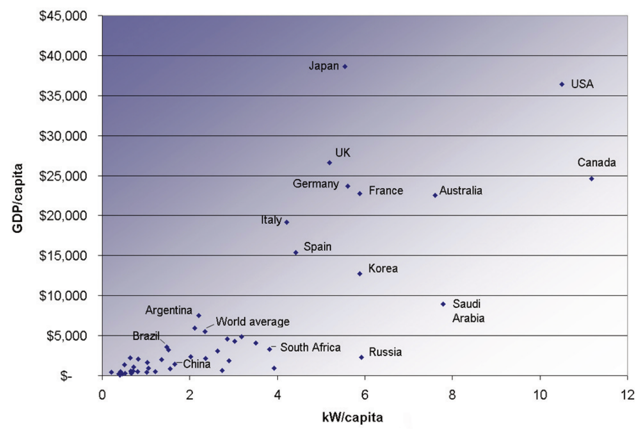

Figure 4. Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the "law of the conservation of energy" is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been "degraded" in the energy transformation. (This will be discussed in more detail in Chapter 15 Thermodynamics.)

Section Summary

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas and solar contributions are increasing.
- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Conceptual Questions 1: What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.

2: If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?

Problems & Exercises

1: Integrated Concepts

(a) Calculate the force the woman in Figure 5 exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in Chapter 7.8 Work, Energy, and Power in Humans.

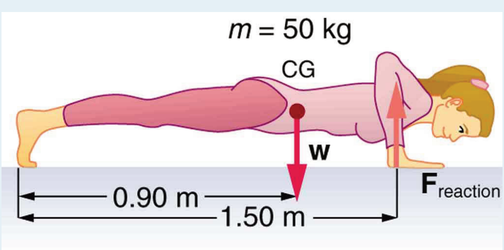

Figure 5. Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

2: Integrated Concepts

A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of

25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

3: Integrated Concepts

The 70.0-kg swimmer in Chapter 7.8 Work, Energy, and Power in Humans starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

4: Integrated Concepts

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

5: Integrated Concepts

(a) What force must be supplied by an elevator cable to produce an acceleration of 0.800 m/s^2 against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

6: Unreasonable Results

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

7: Unreasonable Results

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can lose 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

8: Construct Your Own Problem

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

9: Construct Your Own Problem

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

10: Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, his feet leave the floor and his center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate his velocity when he leaves the floor. (b) What average force did he exert on the floor? (Do not neglect the force to support his weight as well as that to accelerate him.) (c) What was his power output during the acceleration phase?

Glossary

renewable forms of energy
those sources that cannot be used up, such as water, wind, solar, and biomass
fossil fuels
oil, natural gas, and coal

Solutions

Problems & Exercises

1:

(a) 294 N

(b) 118 J

(c) 49.0 W

3:

(a) $0.500\ \text{m}/\text{s}^2$

(b) $62.5\ \text{N}$

(c) Assuming the acceleration of the swimmer decreases linearly with time over the 5.00 s interval, the frictional force must therefore be increasing linearly with time, since $f = F - ma$. If the acceleration decreases linearly with time, the velocity will contain a term dependent on time squared (t^2). Therefore, the water resistance will not depend linearly on the velocity.

5:

(a) $16.1 \times 10^3\ \text{N}$

(b) $3.22 \times 10^5\ \text{J}$

(c) 5.66 m/s

(d) 4.00 kJ

7:

(a) $4.65 \times 10^3\ \text{kcal}$

(b) 38.8 kcal/min

(c) This power output is higher than the highest value on Table 5, which is about 35 kcal/min (corresponding to 2415 watts) for sprinting.

(d) It would be impossible to maintain this power output for 2 hours (imagine sprinting for 2 hours!).

10:

(a) 4.32 m/s

(b) $3.47 \times 10^3\ \text{N}$

(c) 8.93 kW

PART 8

Chapter 8 Linear Momentum and Collisions

Figure 1. Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

8.1 Linear Momentum and Force

Summary

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\mathbf{p} = m\mathbf{v}.$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\mathbf{p}$ is a vector having the same direction as the velocity $\mathbf{v}$. The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.

LINEAR MOMENTUM

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\mathbf{p} = m\mathbf{v}.$$

Example 1: Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p.(As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$p_{\text{player}} = (110 \text{ kg})(8.00 \text{ m/s}) = 880 \text{ kg·m/s}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$p_{\text{ball}} = (0.410 \text{ kg})(25.0 \text{ m/s}) = 10.3 \text{ kg·m/s}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9 .$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

MAKING CONNECTIONS: FORCE AND MOMENTUM

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta\mathbf{v}.$$

So that for constant mass, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta\mathbf{v}}{\Delta t}.$$

Because $\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example 2: Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by

Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\mathbf{F}_{\text{net}}=\frac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p=m\Delta v=m(v_{\text{f}}-v_{\text{i}}).$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}}=\frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned}\Delta p &= m(v_{\text{f}}-v_{\text{i}})\\ &= (0.057\text{ kg})(58\text{ m/s}-0\text{ m/s})\\ &= 3.306\text{ kg}\cdot\text{m/s}\approx 3.3\text{ kg}\cdot\text{m/s}\end{aligned}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}}=\frac{\Delta p}{\Delta t}$:

$$\begin{aligned}F_{\text{net}} &= \frac{\Delta p}{\Delta t}=\frac{3.306\text{ kg}\cdot\text{m/s}}{5.0\times10^{-3}\text{ s}}\\ &= 661\text{ N}\approx 660\text{ N},\end{aligned}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}}=ma$, but one additional step would be required compared with the strategy used in this example.

Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\mathbf{p}$ is defined to be

  $$\mathbf{p}=m\mathbf{v},$$

  where m is the mass of the system and $\mathbf{v}$ is its velocity.
- The SI unit for momentum is $\text{kg}\cdot\text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

  $$\mathbf{F}_{\text{net}}=\frac{\Delta \mathbf{p}}{\Delta t},$$

  $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change time.

Conceptual Questions

1: An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2: An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3: Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

4: How can a small force impart the same momentum to an object as a large force?

Problems & Exercises

1: (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of $7.50\ \text{m/s}$. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of $600\ \text{m/s}$. (c) What is the momentum of the 90.0-kg hunter running at $7.40\ \text{m/s}$ after missing the elephant?

2: (a) What is the mass of a large ship that has a momentum of $1.60 \times 10^9\ \text{kg} \cdot \text{m/s}$, when the ship is moving at a speed of $48.0\ \text{km/h}$? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of $1200\ \text{m/s}$.

3: (a) At what speed would a $2.00 \times 10^4\text{-kg}$ airplane have to fly to have a momentum of $1.60 \times 10^9\ \text{kg} \cdot \text{m/s}$ (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of $60.0\ \text{m/s}$? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

4: (a) What is the momentum of a garbage truck that is $1.20 \times 10^4\ \text{kg}$ and is moving at $10.0\ \text{m/s}$? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

5: A runaway train car that has a mass of 15,000 kg travels at a speed of $5.4\ \text{m/s}$ down a track. Compute the time required for a force of 1500 N to bring the car to rest.

6: The mass of Earth is $5.972 \times 10^{24}\ \text{kg}$ and its orbital radius is an average of $1.496 \times 10^{11}\ \text{m}$. Calculate its linear momentum.

Glossary

linear momentum
the product of mass and velocity

second law of motion
physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Solutions

Problems & Exercises

1:

(a) $1.50 \times 10^4 \text{ kg} \cdot \text{m/s}$

(b) 625 to 1

(c) $6.66 \times 10^2 \text{ kg} \cdot \text{m/s}$

3:

(a) $8.00 \times 10^4 \text{ m/s}$

(b) $1.20 \times 10^6 \text{ kg} \cdot \text{m/s}$

(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be -0.0100 m/s, which is probably not noticeable.

5:

54 s

8.2 Impulse

Summary

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In [link], a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta \mathbf{p}$.

By rearranging the equation $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$ to be

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t,$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\mathbf{F}_{\text{net}} \Delta t$ is given the name **impulse**. Impulse is the same as the change in momentum.

IMPULSE: CHANGE IN MOMENTUM

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$$

The quantity $\mathbf{F}_{\text{net}} \Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Example 1: Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30° from the perpendicular, and bounces off at an angle of 30° from perpendicular to the wall.

(a) Determine the direction of the force on the wall due to each ball.

(b) Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the x-axis to be normal to the wall and to be positive in the initial direction of motion. Choose the y-axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the $+x$ direction. Therefore the wall exerts a force on the ball in the $-x$ direction. The second ball continues with the same momentum component in the y direction, but reverses its x-component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the $-x$ direction, so the force of the wall on each ball is along the $-x$ direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x-axis and y-axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$p_{xi} = mu;\ p_{yi} = 0$$

$$p_{xf} = -mu;\ p_{yf} = 0$$

Impulse is the change in momentum vector. Therefore the x-component of impulse is equal to $-2mu$ and the y -component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{xi} = mu\cos 30^0;\ p_{yi} = -mu\sin 30^0$$

$$p_{xf} = -mu\cos 30^0;\ p_{yf} = -mu\sin 30^0$$

It should be noted here that while p_x changes sign after the collision, p_y does not. Therefore the x-component of impulse is equal to $-2mu\cos 30^0$ and the y-component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2mu}{2mu\cos 30^0} = \frac{2}{\sqrt{3}} = 1.155\ .$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative x–direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive x-direction.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt. *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. Figure 1 shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2. That area is equal to the area inside the rectangle bounded by F_{eff}, t_1, and t_2. Thus the impulses and their effects are the same for both the actual and effective forces.

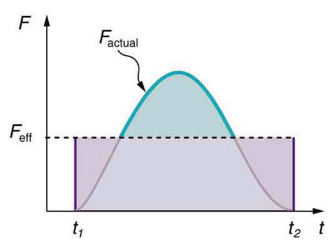

Figure 1. A graph of force versus time with time along the ***x***-axis and force along the ***y***-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—HAND MOVEMENT AND IMPULSE

Try catching a ball while "giving" with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

MAKING CONNECTIONS: CONSTANT FORCE AND CONSTANT ACCELERATION

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

Section Summary

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}}\Delta t.$$

- Forces are usually not constant over a period of time.

Conceptual Questions

1: Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

2: While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

3: Professional Application

Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

Problems & Exercises

1: A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

2: Professional Application

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

3: A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

4: Professional Application

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

5: Professional Application

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

6: Professional Application

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00 \times 10^3 \text{ m/s}$, given the collision lasts $6.00 \times 10^{-8} \text{ s}$.

7: Professional Application

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

8: Professional Application

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

9: A cruise ship with a mass of $1.00 \times 10^7 \text{ kg}$ strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

10: Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76 \times 10^4 \text{ N}$.

11: Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

12: A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

13: Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

14: A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$-direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$-direction with the same speed. What is the impulse delivered by the wall?

15: When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

16: A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

Glossary

change in momentum

the difference between the final and initial momentum; the mass times the change in velocity

impulse

the average net external force times the time it acts; equal to the change in momentum

Solutions

Problems & Exercises

1:

$9.00 \times 10^{3}\ \mathrm{N}$

3:

a) $2.40 \times 10^{3}\ \mathrm{N}$ toward the leg

b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the change in momentum and the time interval are the same.

5:

a) $800\ \mathrm{kg \cdot m/s}$ away from the wall

b) $1.20\ \mathrm{m/s}$ away from the wall

7:

(a) $1.50 \times 10^{6}\ \mathrm{N}$ away from the dashboard

(b) $1.00 \times 10^{5}\ \mathrm{N}$ away from the dashboard

9:

$4.69 \times 10^{5}\ \mathrm{N}$ in the boat's original direction of motion

11:

$2.10 \times 10^{3}\ \mathrm{N}$ away from the wall

13:

$$\mathrm{p} = mv \Rightarrow p^2 = m^2v^2 \Rightarrow \frac{p^2}{m} = mv^2$$
$$\Rightarrow \frac{p^2}{2m} = \frac{1}{2}mv^2 = \mathrm{KE}$$
$$\mathrm{KE} = \frac{p^2}{2m}$$

15:

60.0 g

8.3 Conservation of Momentum

Summary

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the principle of conservation of momentum as it relates to atomic and subatomic particles.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in Chapter 8.2 Impulse and Chapter 8.1 Linear Momentum and Force, where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in Figure 1. Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

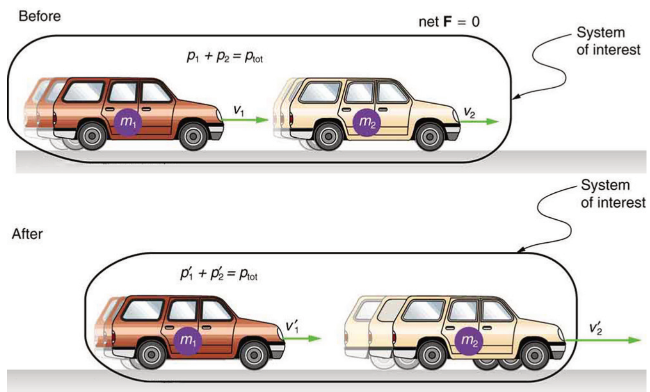

Figure 1. A car of mass $\boldsymbol{m_1}$ moving with a velocity of $\boldsymbol{v_1}$ bumps into another car of mass $\boldsymbol{m_2}$and velocity $\boldsymbol{v_2}$ that it is following. As a result, the first car slows down to a velocity of $\boldsymbol{v'_1}$and the second speeds up to a velocity of $\boldsymbol{v'_2}$. The momentum of each car is changed, but the total momentum $\mathbf{p_{tot}}$of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t,$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t,$$

where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1.$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta p_1 + \Delta p_2 = 0.$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = \text{constant},$$
$$p_1 + p_2 = p'_1 + p'_2,$$

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\mathbf{p}_{\text{tot}} = \text{constant},$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}},$$

where $\mathbf{p}_{\text{tot}}$ is the total momentum (the sum of the momenta of the individual objects in the system) and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

CONSERVATION OF MOMENTUM PRINCIPLE

$$\begin{aligned}\mathbf{p}_{\text{tot}} &= \text{constant}\\ \mathbf{p}_{\text{tot}} &= \mathbf{p}'_{\text{tot}}\ \text{(isolated system)}\end{aligned}$$

ISOLATED SYSTEM

An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}_{\text{tot}}}{\Delta t}$. For an isolated system, $\mathbf{F}_{\text{net}} = 0$; thus, $\Delta \mathbf{p}_{\text{tot}} = 0$, and $\mathbf{p}_{\text{tot}}$ is constant.

We have noted that the three length dimensions in nature—x, y, and z—are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See Figure 2.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

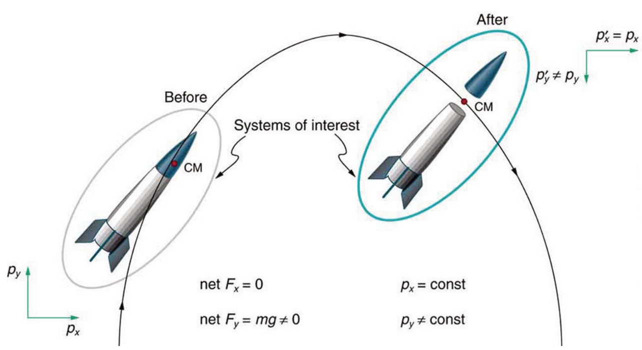

Figure 2. The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $\boldsymbol{F}_{x\text{-net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $\boldsymbol{F}_{y\text{-net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—DROP OF TENNIS BALL AND A BASEBALL

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—TWO TENNIS BALLS IN A BALLISTIC TRAJECTORY

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

MAKING CONNECTIONS: CONSERVATION OF MOMENTUM AND COLLISION

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. Figure 3 below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that **quarks** make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

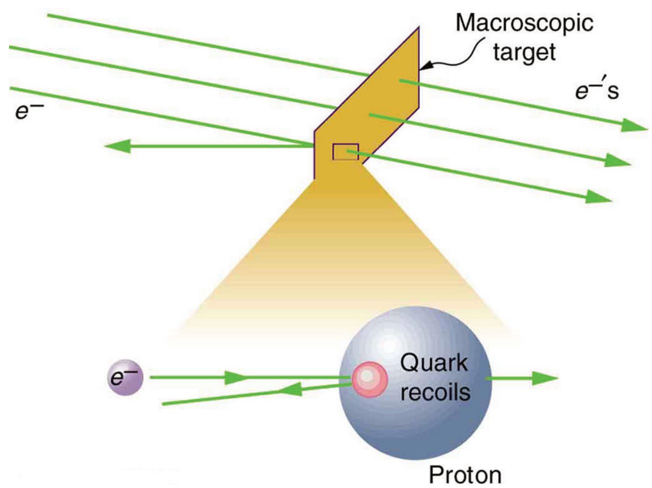

Figure 3. A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.

Section Summary

- The conservation of momentum principle is written

$$\mathbf{p}_{\text{tot}} = \text{constant}$$

  or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \text{ (isolated system),}$$

$\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.

- An isolated system is defined to be one for which the net external force is zero $(\mathbf{F}_{\text{net}} = 0)$.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

Conceptual Questions

1: Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

2: Under what circumstances is momentum conserved?

3: Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

4: Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

5: Professional Application

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

6: Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

7: Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

Problems & Exercises

1: Professional Application

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s. (The minus indicates direction of motion.) What is their final velocity?

2: Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

3: Professional Application

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

4: What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

5: A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

Glossary

conservation of momentum principle

when the net external force is zero, the total momentum of the system is conserved or constant

isolated system

a system in which the net external force is zero

quark

fundamental constituent of matter and an elementary particle

Exercises

Problems & Exercises

1:

0.122 m/s

3:

In a collision with an identical car, momentum is conserved. Afterwards $v_f = 0$ for both cars. The change in momentum will be the same as in the crash with the tree. However, the force on the body is not determined since the time is not known. A padded stop will reduce injurious force on body.

5:

22.4 m/s in the same direction as the original motion

8.4 Elastic Collisions in One Dimension

Summary

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. Figure 1 illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

ELASTIC COLLISION

An **elastic collision** is one that conserves internal kinetic energy.

INTERNAL KINETIC ENERGY

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.

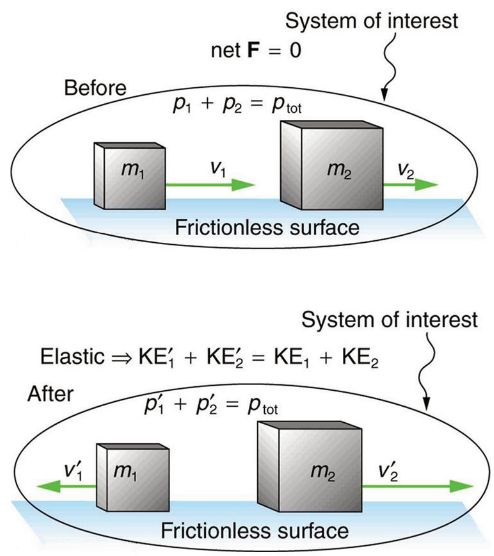

Figure 1. An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \ \ (F_{\text{net}} = 0)$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \ \ (F_{\text{net}} = 0),$$

where the primes (‘) indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2} m_1 {v_1}^2 + \frac{1}{2} m_2 {v_2}^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \ \ \text{(two-object elastic collision)}$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

Example 1: Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500 \text{ kg}, \ m_2 = 3.50 \text{ kg}, \ v_1 = 4.00 \text{ m/s}, \text{ and } v_2 = 0.$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in Figure 1 where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2.$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2.$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1).$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2, leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00 \text{ m/s}$$

and

$$v'_1 = -3.00 \text{ m/s}.$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v'_1 = -3.00 \text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}}[4.00 - (-3.00 \text{ m/s})]$$

or

$$v'_2 = 1.00 \text{ m/s}.$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

MAKING CONNECTIONS: TAKE-HOME INVESTIGATION—ICE CUBES AND ELASTIC COLLISION

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

PHET EXPLORATIONS: COLLISION LAB

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

PhET Interactive Simulation

Figure 2. Collision Lab

Section Summary

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

Conceptual Questions

1: What is an elastic collision?

Problems & Exercises

1: Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

2: Professional Application

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00 \times 10^3\ \text{kg}$, and the second a mass of $7.50 \times 10^3\ \text{kg}$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

3: A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

Glossary

elastic collision
a collision that also conserves internal kinetic energy
internal kinetic energy
the sum of the kinetic energies of the objects in a system

Solutions

Problems & Exercises

2:

0.250 m/s

8.5 Inelastic Collisions in One Dimension

Summary

- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collisio**n is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

INELASTIC COLLISION

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

Figure 1 shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

PERFECTLY INELASTIC COLLISION

A collision in which the objects stick together is sometimes called "perfectly inelastic."

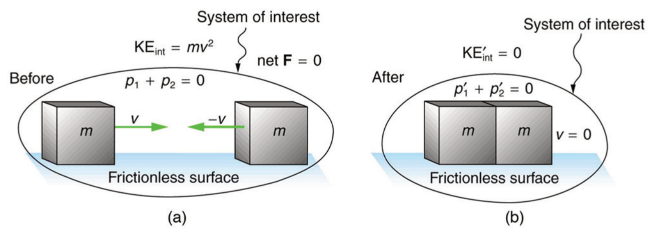

Figure 1. An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

Example 1: Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See Figure 2)

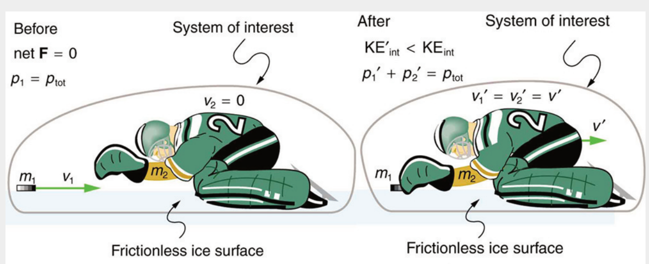

Figure 2. An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2$$

or

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'.$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v_1' = v_2' = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1v_1 = (m_1 + m_2)v'.$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2}v_1.$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg} + 0.150 \text{ kg}}\right)(35.0 \text{ m/s}) = 7.48 \times 10^{-2} \text{ m/s}.$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$\begin{aligned} \text{KE}_{\text{int}} &= \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J.} \end{aligned}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \tfrac{1}{2}(m+M)v^2 = \tfrac{1}{2}(70.15 \text{ kg})(7.48 \times 10^{-2} \text{ m/s})^2 \\ &= 0.196 \text{ J.} \end{aligned}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 0.196 \text{ J} - 91.9 \text{ J} \\ &= -91.7 \text{ J} \end{aligned}$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. KE_{int} is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. Figure 3 shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. Example 2 deals with data from such a collision.

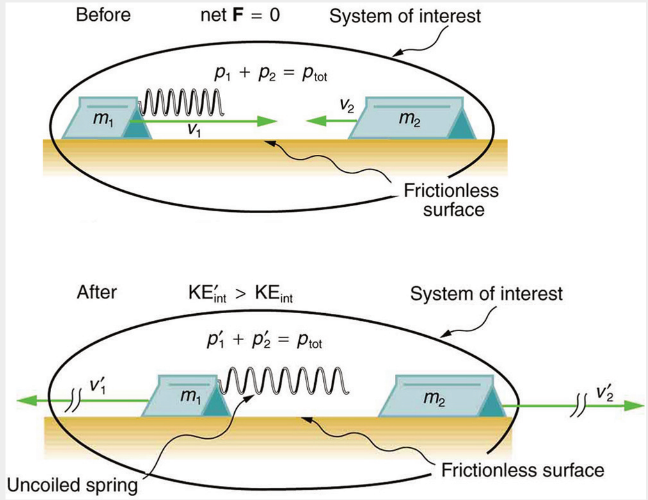

Figure 3. An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in Example 2, the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the "sweet spot" on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

TAKE-HOME EXPERIMENT—BOUNCING OF TENNIS BALL

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend's hand during the collision. Explain your observations and measurements.

2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).

Example 2: Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in Figure 3, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in Figure 3) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2.$$

The only unknown in this equation is v'_2. Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350\text{ kg})(2.00\text{ m/s}) + (0.500\text{ kg})(-0.500\text{ m/s})}{0.500\text{ kg}} - \frac{(0.350\text{ kg})(-4.00\text{ m/s})}{0.500\text{ kg}} \\ &= 3.70\text{ m/s}. \end{aligned}$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned} \text{KE}_{\text{int}} &= \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 \\ &= \tfrac{1}{2}(0.350\text{ kg})(2.00\text{ m/s})^2 + \tfrac{1}{2}(0.500\text{ kg})(-0.500\text{ m/s})^2 \\ &= 0.763\text{ J}. \end{aligned}$$

After the collision, the internal kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2 \\ &= \tfrac{1}{2}(0.350\text{ kg})(-4.00\text{ m/s})^2 + \tfrac{1}{2}(0.500\text{ kg})(3.70\text{ m/s})^2 \\ &= 6.22\text{ J}. \end{aligned}$$

The change in internal kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22\text{ J} - 0.763\text{ J} \\ &= 5.46\text{ J}. \end{aligned}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

Section Summary

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it

reduces internal kinetic energy more than does any other type of inelastic collision.

- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Conceptual Questions

1: What is an inelastic collision? What is a perfectly inelastic collision?

2: Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

3: A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

Problems & Exercises

1: A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

2: During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

3: Professional Application

Using mass and speed data from Chapter 8.1 Example 1 and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

4: A battleship that is $6.00 \times 10^7 \text{ kg}$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

5: Professional Application

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of 4.00×10^3 kg, and the second a mass of 7.50×10^3 kg. (a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

6: Professional Application

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

7: Professional Application

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

8: A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

9: Professional Application

One of the waste products of a nuclear reactor is plutonium-239 (^{239}Pu). This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus $(^{4}He + ^{235}U)$, the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is 8.40×10^{-13} J and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is 6.68×10^{-27} kg, while that of the uranium is 3.92×10^{-25} kg (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

10: Professional Application

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of 5.00×10^{12} kg (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is 7.36×10^{22} kg) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

11: Professional Application

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

12: What is the speed of a garbage truck that is 1.20×10^4 kg and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

13: During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

14: (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

Glossary

inelastic collision
a collision in which internal kinetic energy is not conserved
perfectly inelastic collision
a collision in which the colliding objects stick together

Exercises

Problems & Exercises

1:

(a) 86.4 N perpendicularly away from the bumper

(b) 0.389 J

(c) 64.0%

3:

(a) 8.06 m/s

(b) -56.0 J

(c)(i) 7.88 m/s; (ii) -223 J

5:

(a) 0.163 m/s in the direction of motion of the more massive satellite

(b) 81.6 J

(c) $8.70 \times 10^{-2}\ \text{m/s}$ in the direction of motion of the less massive satellite, 81.5 J. Because there are no external forces, the velocity of the center of mass of the two-satellite system is unchanged by the collision. The two velocities calculated above are the velocity of the center of mass in each of the two different individual reference frames. The loss in KE is the same in both reference frames because the KE lost to internal forces (heat, friction, etc.) is the same regardless of the coordinate system chosen.

7:

0.704 m/s

–2.25 m/s

8:

(a) 4.58 m/s away from the bullet

(b) 31.5 J

(c) –0.491 m/s

(d) 3.38 J

10:

(a) $1.02 \times 10^{-6}\ \text{m/s}$

(b) $5.63 \times 10^{20}\ \text{J}$ (almost all KE lost)

(c) Recoil speed is $6.79 \times 10^{-17}\ \text{m/s}$, energy lost is $6.25 \times 10^{9}\ \text{J}$. The plume will not affect the momentum result because the plume is still part of the Moon system. The plume may affect the kinetic energy result because a significant part of the initial kinetic energy may be transferred to the kinetic energy of the plume particles.

12:

24.8 m/s

14:

(a) 4.00 kg

(b) 210 J

(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

8.6 Collisions of Point Masses in Two Dimensions

Summary

- Discuss two dimensional collisions as an extension of one dimensional analysis.
- Define point masses.
- Derive an expression for conservation of momentum along x-axis and y-axis.
- Describe elastic collisions of two objects with equal mass.
- Determine the magnitude and direction of the final velocity given initial velocity, and scattering angle.

In the previous two sections, we considered only one-dimensional collisions; during such collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and we shall see that their study is an extension of the one-dimensional analysis already presented. The approach taken (similar to the approach in discussing two-dimensional kinematics and dynamics) is to choose a convenient coordinate system and resolve the motion into components along perpendicular axes. Resolving the motion yields a pair of one-dimensional problems to be solved simultaneously.

One complication arising in two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass by one another, they will spin in circles. We will not consider such rotation until later, and so for now we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. (See Figure 1.) The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in Figure 1. Because momentum is conserved, the components of momentum along the x– and y-axes $(p_x \text{ and } p_y)$ will also be conserved, but with the chosen coordinate system, p_y is initially zero and p_x is the momentum of the incoming particle. Both facts simplify the analysis. (Even with the simplifying assumptions of point masses, one particle initially at rest, and a convenient coordinate system, we still gain new insights into nature from the analysis of two-dimensional collisions.)

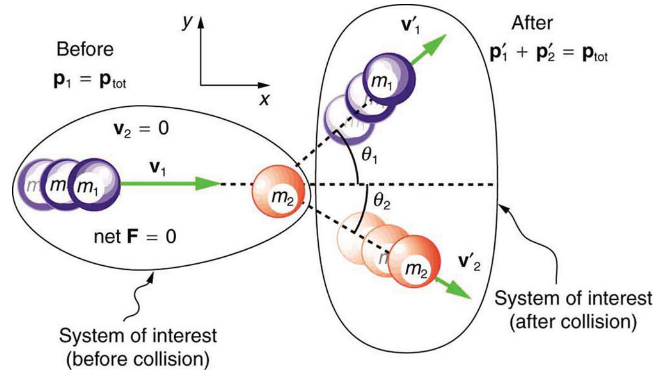

Figure 1. A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and v_1 is parallel to the x-axis. This coordinate system is sometimes called the laboratory coordinate system, because many scattering experiments have a target that is stationary in the laboratory, while particles are scattered from it to determine the particles that make-up the target and how they are bound together. The particles may not be observed directly, but their initial and final velocities are.

Along the x-axis, the equation for conservation of momentum is

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}.$$

Where the subscripts denote the particles and axes and the primes denote the situation after the collision. In terms of masses and velocities, this equation is

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}.$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}.$$

The components of the velocities along the x-axis have the form $v \cos \theta$. Because particle 1 initially moves along the x-axis, we find $v_{1x} = v_1$.

Conservation of momentum along the x-axis gives the following equation:

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2,$$

where θ_1 and θ_2 are as shown in Figure 1.

CONSERVATION OF MOMENTUM ALONG THE *x*-AXIS

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2,$$

Along the y-axis, the equation for conservation of momentum is

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$$

or

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}.$$

But v_{1y} is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, v_{2y} is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 v'_{1y} + m_2 v'_{2y}.$$

The components of the velocities along the y-axis have the form $v \sin \theta$.

Thus, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2.$$

CONSERVATION OF MOMENTUM ALONG THE *y*-AXIS

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2$$

The equations of conservation of momentum along the x-axis and y-axis are very useful in analyzing two-dimensional collisions of particles, where one is originally stationary (a common laboratory situation). But two equations can only be used to find two unknowns, and so other data may be necessary when collision experiments are used to explore nature at the subatomic level.

Example 1: Determining the Final Velocity of an Unseen Object from the Scattering of Another Object

Suppose the following experiment is performed. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45.0° with its incoming direction.

The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity $(v'_2$ and $\theta_2)$ of the 0.400-kg object after the collision.

Strategy

Momentum is conserved because the surface is frictionless. The coordinate system shown in Figure 2 is one in which m_2 is originally at rest and the initial velocity is parallel to the x-axis, so that conservation of momentum along the x– and y-axes is applicable.

Everything is known in these equations except v'_2 and θ_2, which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the x– and y– directions.

Solution

Solving $m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$ for $v'_2 \cos \theta_2$ and $0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2$ for $v'_2 \sin \theta_2$ and taking the ratio yields an equation (in which θ_2 is the only unknown quantity. Applying the identity $\left(\tan \theta = \frac{\sin \theta}{\cos \theta}\right)$, we obtain:

$$\tan \theta_2 = \frac{v'_1 \sin \theta_1}{v'_1 \cos \theta_1 - v_1}.$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50\text{ m/s})(0.7071)}{(1.50\text{ m/s})(0.7071) - 2.00\text{ m/s}} = -1.129 .$$

Thus,

$$\theta_2 = \tan^{-1} -1.129 = 311.5^0 \approx 312^0.$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in Figure 2, as expected (this angle is in the fourth quadrant). Either equation for the x– or y-axis can now be used to solve for v_2', but the latter equation is easiest because it has fewer terms.

$$v_2' = -\frac{m_1}{m_2} v_1' \frac{\sin\theta_1}{\sin\theta_2}$$

Entering known values into this equation gives

$$v_2' = -\left(\frac{0.250\text{ kg}}{0.400\text{ kg}}\right) 1.50\text{ m/s} - \left(\frac{0.7071}{-0.7485}\right).$$

Thus,

$$v_2' = 0.886\text{ m/s}.$$

Discussion

It is instructive to calculate the internal kinetic energy of this two-object system before and after the collision. (This calculation is left as an end-of-chapter problem.) If you do this calculation, you will find that the internal kinetic energy is less after the collision, and so the collision is inelastic. This type of result makes a physicist want to explore the system further.

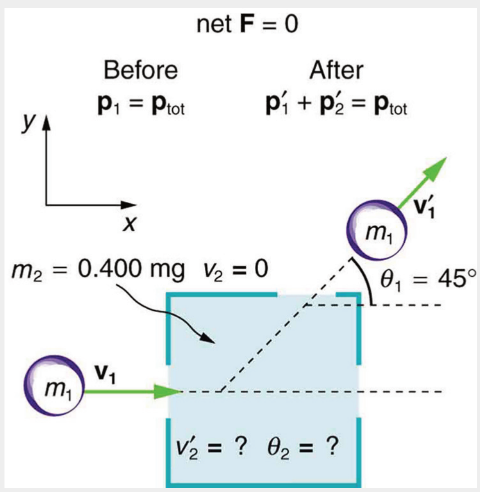

Figure 2. A collision taking place in a dark room is explored in Example 1. The incoming object m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Elastic Collisions of Two Objects with Equal Mass

Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic.

This situation is nearly the case with colliding billiard balls, and precisely the case with some subatomic particle collisions. We can thus get a mental image of a collision of subatomic particles by thinking about billiards (or pool). (Refer to Figure 1 for masses and angles.) First, an elastic collision conserves internal kinetic energy. Again, let us assume object 2 m_2 is initially at rest. Then, the internal kinetic energy before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2.$$

Because the masses are equal, $m_1 = m_2 = m$. Algebraic manipulation (left to the reader) of conservation of momentum in the x– and y-directions can show that

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2'(\cos\theta_1 - \theta_2).$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv_1'v_2'\cos(\theta_1 - \theta_2) = 0.$$

There are three ways that this term can be zero. They are

- $v_1' = 0$: head-on collision; incoming ball stops
- $v_2' = 0$: no collision; incoming ball continues unaffected
- $\cos(\theta_1 - \theta_2) = 0$: angle of separation $(\theta_1 - \theta_2)$ is 90° after the collision

All three of these ways are familiar occurrences in billiards and pool, although most of us try to avoid the second. If you play enough pool, you will notice that the angle between the balls is very close to 90° after the collision, although it will vary from this value if a great deal of spin is placed on the ball. (Large spin carries in extra energy and a quantity called *angular momentum*, which must also be conserved.) The assumption that the scattering of billiard balls is elastic is reasonable based on the correctness of the three results it produces. This assumption also implies that, to a good approximation, momentum is conserved for the two-ball system in billiards and pool. The problems below explore these and other characteristics of two-dimensional collisions.

CONNECTIONS TO NUCLEAR AND PARTICLE PHYSICS

Two-dimensional collision experiments have revealed much of what we know about subatomic particles, as we shall see in Chapter 32 Medical Applications of Nuclear Physics and Chapter 33 Particle Physics. Ernest Rutherford, for example, discovered the nature of the atomic nucleus from such experiments.

Section Summary

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x-axis), stated by $m_1v_1 = m_1v_1'\cos\theta_1 + m_2v_2'\cos\theta_2$ and along the direction perpendicular to the initial direction (the y-axis) stated by $0 = m_1v_{1y}' + m_2v_{2y}'$.
- The internal kinetic before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2' \cos(\theta_1 - \theta_2).$$

- Point masses are structureless particles that cannot spin.

Conceptual Questions

1: Figure 3 shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b, the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.

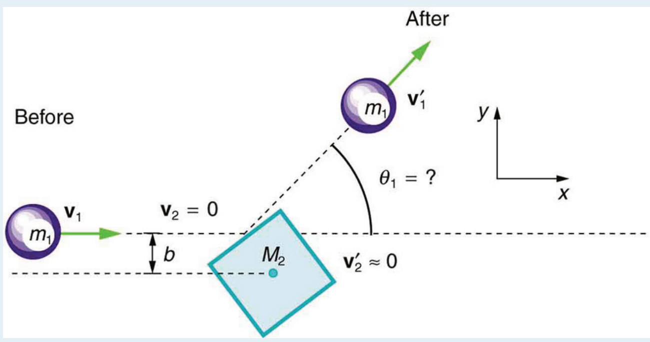

Figure 3. A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1. The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b.

Problems & Exercises

1: Two identical pucks collide on an air hockey table. One puck was originally at rest. (a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0°, what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^\circ$ for elastic collisions of objects that have identical masses.) (b) Confirm that the collision is elastic.

2: Confirm that the results of the example Example 1 do conserve momentum in both the x– and y -directions.

3: A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction. (a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0° above the horizontal. (b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil. (c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

4: Professional Application

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0° to the initial direction of the bowling ball and with a speed of 15.0 m/s. (a) Calculate the final velocity (magnitude and direction) of the bowling ball. (b) Is the collision elastic? (c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

5: Professional Application

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei (^{4}He) from gold-197 nuclei (^{197}Au). The energy of the incoming helium nucleus was $8.00 \times 10^{-13}\ \text{J}$, and the masses of the helium and gold nuclei were $6.68 \times 10^{-27}\ \text{kg}$ and $3.29 \times 10^{-25}\ \text{kg}$, respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of 120° during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?

6: Professional Application

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at $8.00\ \text{m/s}$ due south. The second car has a mass of 850 kg and is approaching at $17.0\ \text{m/s}$ due west. (a) Calculate the final velocity (magnitude and direction) of the cars. (b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x-axis and y-axis; instead, you must look for other simplifying aspects.

7: Starting with equations $m_1 v_1 = m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2$ and $0 = m_1 v_1' \sin\theta_1 + m_2 v_2' \sin\theta_2$ for conservation of momentum in the x– and y-directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2'\cos(\theta_1 - \theta_2)$$

as discussed in the text.

8: Integrated Concepts

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

Glossary

point masses
structureless particles with no rotation or spin

Solutions

Problems & Exercises

1:

(a) 3.00 m/s, 60^0 below x-axis

(b) Find speed of first puck after collision: $0 = mv_1' \sin 30^0 - mv_2' \sin 60^0 \Rightarrow v_1' = v_2' \frac{\sin 60^0}{\sin 30^0} = 5.196 \text{ m/s}$

Verify that ratio of initial to final KE equals one:

$$\left.\begin{array}{l}\text{KE} = \frac{1}{2}mv_1^2 = 18m \text{ J} \\ \text{KE} = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 = 18m \text{ J}\end{array}\right\}\frac{\text{KE}}{\text{KE}'} = 1.00$$

3:

(a) -2.26 m/s

(b) $7.63 \times 10^3 \text{ J}$

(c) The ground will exert a normal force to oppose recoil of the cannon in the vertical direction. The momentum in the vertical direction is transferred to the earth. The energy is transferred into the ground, making a dent where the cannon is. After long barrages, cannon have erratic aim because the ground is full of divots.

5:

(a) $5.36 \times 10^5 \text{ m/s}$ at -29.5^0

(b) $7.52 \times 10^{-13} \text{ J}$

7:

We are given that $m_1 = m_2 \equiv m.$ The given equations then become:

$$v_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2$$

and

$$0 = v_1' \sin\theta_1 + v_2' \sin\theta_2.$$

Square each equation to get

$$\begin{array}{lll} v_1^2 & = & v_1'^2 \cos^2\theta_1 + v_2'^2 \cos^2\theta_2 + 2v_1'v_2' \cos\theta_1 \cos\theta_2 \\ 0 & = & v_1'^2 \sin^2\theta_1 + v_2'^2 \sin^2\theta_2 + 2v_1'v_2' \sin\theta_1 \sin\theta_2. \end{array}$$

Add these two equations and simplify:

$$\begin{array}{lll} v_1^2 & = & v_1'^2 + v_2'^2 + 2v_1'v_2'(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2) \\ & = & v_1'^2 + v_2'^2 + 2v_1'v_2'[\frac{1}{2}\cos(\theta_1 - \theta_2) + \frac{1}{2}\cos(\theta_1 + \theta_2) + \frac{1}{2}\cos(\theta_1 - \theta_2) - \frac{1}{2}\cos(\theta_1 + \theta_2)] \\ & = & v_1'^2 + v_2'^2 + 2v_1'v_2' \cos(\theta_1 - \theta_2). \end{array}$$

Multiply the entire equation by $\frac{1}{2}m$ to recover the kinetic energy:

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2' \cos(\theta_1 - \theta_2)$$

8.7 Introduction to Rocket Propulsion

Summary

- State Newton's third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket and discuss the factors that affect the acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT—PROPULSION OF A BALLOON

Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

Figure 1 shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv. In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass $(m - \Delta m)$ now has a greater velocity $(v + \Delta v)$. The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt, producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over Δt and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$$

"The rocket" is that part of the system remaining after the gas is ejected, and g is the acceleration due to gravity.

ACCELERATION OF A ROCKET

Acceleration of a rocket is

$$a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g,$$

where a is the acceleration of the rocket, v_e is the escape velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

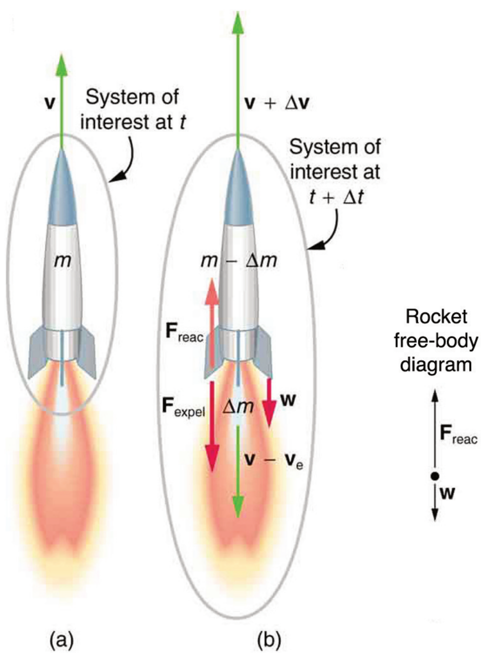

Figure 1. (a) This rocket has a mass $\boldsymbol{m}$ and an upward velocity $\boldsymbol{v}$. The net external force on the system is $-\boldsymbol{mg}$, if air resistance is neglected. (b) A time $\boldsymbol{\Delta t}$ later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket. First, the greater the exhaust velocity of the gases relative to the rocket, v_e, the greater the acceleration is. The practical limit for v_e is about 2.5×10^3 m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\Delta m/\Delta t$ in the equation. The quantity $(\Delta m/\Delta t)v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

FACTORS AFFECTING A ROCKETS ACCELERATION

- The greater the exhaust velocity v_e of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

Example 1: Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was $2.80 \times 10^6\ \text{kg}$, its fuel-burn rate was $1.40 \times 10^4\ \text{kg/s}$, and the exhaust velocity was $2.40 \times 10^3\ \text{m/s}$. Calculate its initial acceleration.

Strategy

This problem is a straightforward application of the expression for acceleration because a is the unknown and all of the terms on the right side of the equation are given.

Solution

Substituting the given values into the equation for acceleration yields

$$\begin{aligned} a &= \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g \\ &= \frac{2.40 \times 10^3\ \text{m/s}}{2.80 \times 10^6\ \text{kg}}(1.40 \times 10^4\ \text{kg/s}) - 9.80\ \text{m/s}^2 \\ &= 2.20\ \text{m/s}^2. \end{aligned}$$

Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while v_e and $\frac{\Delta m}{\Delta t}$ remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was $3.36 \times 10^7\ \text{N}$.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln \frac{m_0}{m_r},$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about $11.2 \times 10^3\ \text{m/s}$, and assuming an exhaust velocity $v_e = 2.5 \times 10^3\ \text{m/s}$.

$$\ln\frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2 \times 10^3 \text{ m/s}}{2.5 \times 10^3 \text{ m/s}} = 4.48$$

Solving for m_0/m_r gives

$$\frac{m_0}{m_r} = e^{4.48} = 88\,.$$

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}.$$

This result means that only $1/88$ of the mass is left when the fuel is burnt, and $87/88$ of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass m_r remaining can only be about $m_0/180$. It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See Figure 2) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unmanned rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.

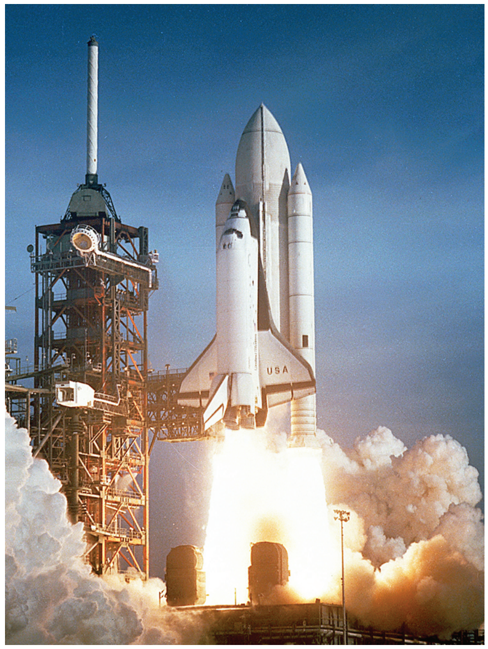

Figure 2. The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)

PHET EXPLORATIONS: LUNAR LANDER

Can you avoid the boulder field and land safely, just before your fuel runs out, as Neil Armstrong did in 1969? Our version of this classic video game accurately simulates the real motion of the lunar lander with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is very hard to control.

PhET Interactive Simulation

Figure 3. Lunar Lander

Section Summary

- Newton’s third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is $a = \frac{v_e}{m}\frac{\Delta m}{\Delta t} - g$.
- A rocket’s acceleration depends on three main factors. They are

1. The greater the exhaust velocity of the gases, the greater the acceleration.
2. The faster the rocket burns its fuel, the greater its acceleration.
3. The smaller the rocket's mass, the greater the acceleration.

Conceptual Questions

1: Professional Application

Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?

2: Professional Application

During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.

3: Professional Application

It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

Problems & Exercises

1: Professional Application

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of $2.50 \times 10^3 \text{ m/s}$?

2: Professional Application

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only 1.6 m/s^2, if the rocket expels 8.00 kg of gas per second at an exhaust velocity of $2.20 \times 10^3 \text{ m/s}$?

3: Professional Application

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of $2.00 \times 10^3 \text{ m/s}$. You may assume the gravitational force is negligible at the probe's location.

4: Professional Application

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as $8.00 \times 10^6 \text{ m/s}$. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact: (a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity. (b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels $4.50 \times 10^{-6} \text{ kg/s}$ at the given velocity, assuming the acceleration due to gravity is negligible.

5: Derive the equation for the vertical acceleration of a rocket.

6: Professional Application

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is $2.40 \times 10^3 \text{ m/s}$. Assume that the acceleration of gravity is the same as on Earth's surface (9.80 m/s^2). (b) Why might it be necessary to limit the acceleration of a rocket?

7: Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.

8: How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of 8.00 km/s, given that it expels gases at an exhaust velocity of $2.20 \times 10^3 \text{ m/s}$?

9: Professional Application

(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?

10: Unreasonable Results

Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0°, assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

11: Construct Your Own Problem

Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.

12: Construct Your Own Problem

Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

Solutions

Problems & Exercises

1:

39.2 m/s^2

3:

$4.16 \times 10^3 \text{ m/s}$

5:

The force needed to give a small mass Δm an acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed v_e requires $v_e = a_{\Delta m} \Delta t$, so $F = v_e \frac{\Delta m}{\Delta t}$. By Newton’s third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_{\text{thrust}} = v_e \frac{\Delta m}{\Delta t}$, where all quantities are positive. Applying Newton’s second law to the rocket gives $F_{\text{thrust}} - mg = ma \Rightarrow a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

8:

$2.63 \times 10^3 \text{ kg}$

9:

(a) 0.421 m/s away from the ejected fluid.

(b) 0.237 J.

PART 9

Chapter 9 Statics and Torque

Figure 1. On a short time scale, rocks like these in Australia's Kings Canyon are static, or motionless relative to the Earth. (credit: freeaussiestock.com)

What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton's second law states that net $\boldsymbol{F} = ma$, and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

STATICS

Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton's second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.

9.1 The First Condition for Equilibrium

Summary

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\text{net } \mathbf{F} = 0$$

Note that if net F is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical *x*– and *y*-axes are zero. This is written as

$$\text{net } F_x = 0 \text{ and } F_y = 0$$

Figure 1 and Figure 2 illustrate situations where $\text{net } F = 0$ for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

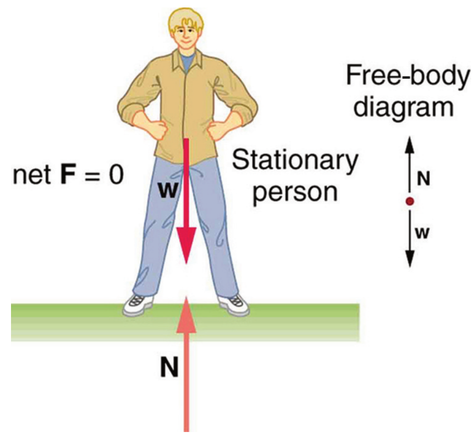

Figure 1. This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.

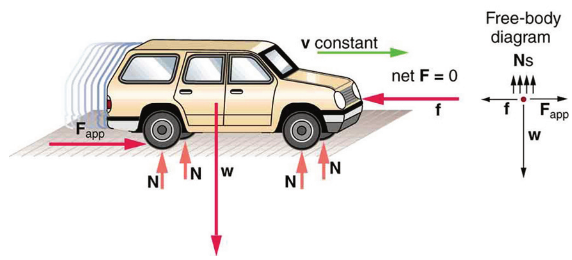

Figure 2. This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force$\boldsymbol{F}_{\mathbf{app}}$ between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in Figure 3 and Figure 4 where forces are applied to an ice hockey stick lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In Figure 3, the ice hockey stick remains motionless. But in Figure 4, with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

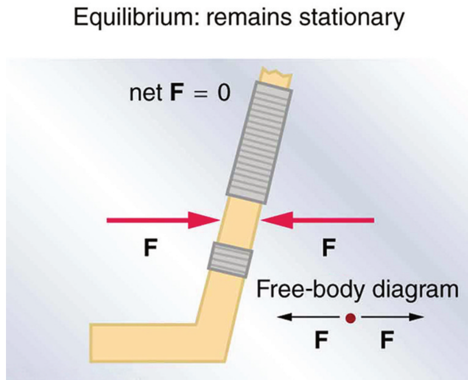

Figure 3. An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, **net *F* = 0**. Equilibrium is achieved, which is static equilibrium in this case.

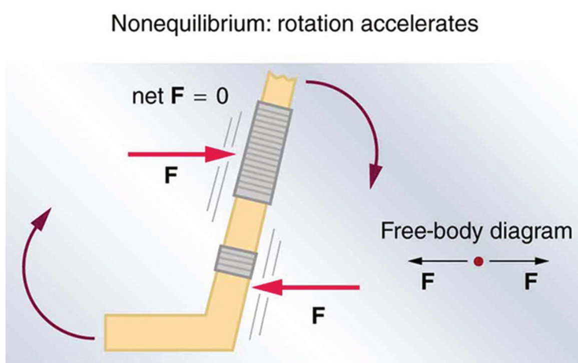

Figure 4. The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here **net *F* = 0** but the system is not at equilibrium. Hence, the **net *F* = 0** is a necessary—but not sufficient—condition for achieving equilibrium.

PHET EXPLORATIONS: TORQUE

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

PhET Interactive Simulation

Figure 5. Torque

Section Summary

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that $\text{net } \mathbf{F} = \mathbf{0}$.

Conceptual Questions

1: What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.

2: Under what conditions can a rotating body be in equilibrium? Give an example.

Glossary

static equilibrium
a state of equilibrium in which the net external force and torque acting on a system is zero

dynamic equilibrium
a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

9.2 The Second Condition for Equilibrium

Summary

- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics

TORQUE

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a constant angular velocity. A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See Figure 1. First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

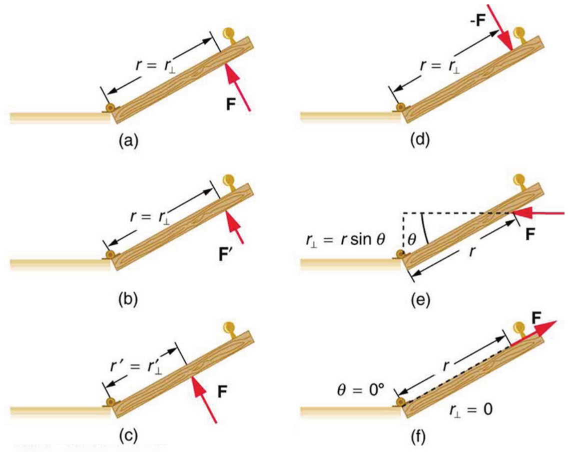

Figure 1. Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to **F**. Note that **r?** is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force **F′** acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here, ***θ*** is less than **90º**. (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case, ***θ* = 0º**.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\tau = rF \sin \theta$$

where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in Figure 1 and Figure 2. An alternative expression for torque is given in terms of the **perpendicular lever arm** $r_\perp$ as shown in Figure 1 and Figure 2, which is defined as

$$r_\perp = r \sin \theta$$

so that

$$\tau = r_\perp F.$$

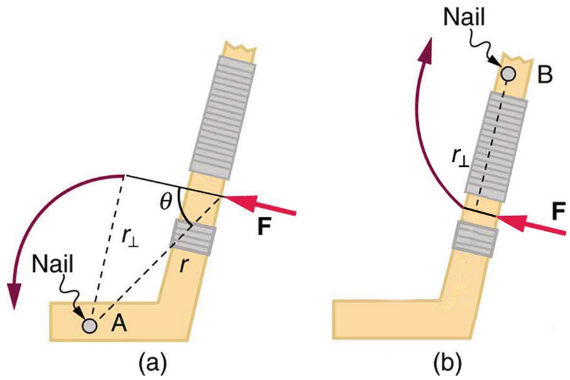

Figure 2. A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors ***r***, ***F***, and ***θ*** for pivot point A on a body are shown here—***r*** is the distance from the chosen pivot point to the point where the force ***F*** is applied, and ***θ*** is the angle between **F** and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.

The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which F acts; it is shown as a dashed line in Figure 1 and Figure 2. Note that the line segment that defines the distance $r_\perp$ is perpendicular to F, as its name implies. It is sometimes easier to find or visualize $r_\perp$ than to find both r and θ. In such cases, it may be more convenient to use $\tau = r_\perp F$ rather than $tau = rF \sin \theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as $N \cdot m$. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of $32\ N \cdot m (0.800\ m \times 40\ N \times \sin 90^\circ)$ relative to the hinges. If you reduce the force to 20 N, the torque is reduced to $16\ N \cdot m$, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both r and θ depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen "pivot point."

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in Figure 2. If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\text{net } \tau = 0$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in Figure 3, they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.

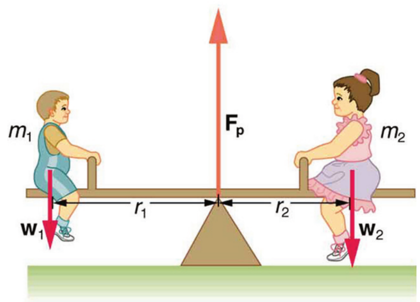

Figure 3. Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

Example 1: She Saw Torques On A Seesaw

The two children shown in Figure 3 are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot.(a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is F_p, the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\tau = rF \sin \theta.$$

Here $\theta = 90^\circ$, so that $\sin \theta = 1$ for all three forces. That means $r_\perp = r$ for all three. The torques exerted by the three forces are first,

$$\tau_1 = r_1 w_1$$

second,

$$\tau_2 = -r_2 w_2$$

and third,

$$\begin{aligned} \tau_p &= r_p F_p \\ &= 0 \cdot F_p \\ &= 0. \end{aligned}$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since F_p acts directly on the pivot point, the distance r_p is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\tau_2 = -\tau_1,$$

or

$$r_2 w_2 = r_1 w_1.$$

Weight is mass times the acceleration due to gravity. Entering mg for w, we get

$$r_2 m_2 g = r_1 m_1 g.$$

Solve this for the unknown r_2:

$$r_2 = r_1 \frac{m_1}{m_2}.$$

The quantities on the right side of the equation are known; thus, r_2 is

$$r_2 = (1.60\text{ m})\frac{26.0\text{ kg}}{32.0\text{ kg}} = 1.30\text{ m}.$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_p. The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net } F = 0.$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net } F_y = 0$$

where we again call the vertical axis the *y*-axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_p - w_1 - w_2 = 0.$$

This equation yields what might have been guessed at the beginning:

$$F_p = w_1 + w_2.$$

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_p = m_1 g + m_2 g.$$

Entering known values gives

$$\begin{aligned} F_p &= (26.0\text{ kg})(9.80\text{ m/s}^2) + (32.0\text{ kg})(9.80\text{ m/s}^2) \\ &= 568\text{ N}. \end{aligned}$$

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since F_p is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force F_p is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case*. Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force were exerted at a single point. This is not an approximation—the distances r_1 and r_2 are the distances to points directly below the **center of gravity** of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

TAKE-HOME EXPERIMENT

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

Section Summary

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be

  $$\tau = rF \sin \theta$$

  where τ is torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between F and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm $r_\perp$ is defined to be

  $$r_\perp = r \sin \theta$$

  so that

  $$\tau = r_\perp F.$$

- The perpendicular lever arm $r_\perp$ is the shortest distance from the pivot point to the line along which F acts. The SI unit for torque is newton-meter $(\mathrm{N \cdot m})$. The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:

  $$\text{net } \tau = 0$$

  By convention, counterclockwise torques are positive, and clockwise torques are negative.

Conceptual Questions

1: What three factors affect the torque created by a force relative to a specific pivot point?

2: A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.

3: Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

Problems & Exercises

1: (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

2: When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton × meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.

3: Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.

4: Use the second condition for equilibrium $(\text{net } \tau = 0)$ to calculate F_p in Example 1, employing any data given or solved for in part (a) of the example.

5: Repeat the seesaw problem in Example 1 with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

Glossary

torque

turning or twisting effectiveness of a force

perpendicular lever arm

the shortest distance from the pivot point to the line along which FF lies

SI units of torque

newton times meters, usually written as N·m

center of gravity

the point where the total weight of the body is assumed to be concentrated

Solutions

Problems & Exercises

1:

a) $46.8\ \mathrm{N\cdot m}$

b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

3:

23.3 N

5:

Given:

$$\begin{aligned} m_1 &= 26.0\ \text{kg},\ m_2 = 32.0\ \text{kg},\ m_s = 12.0\ \text{kg}, \\ r_1 &= 1.60\ \text{m},\ r_s = 0.160\ \text{m},\ \text{find (a)} r_2,\ \text{(b)} F_\text{p} \end{aligned}$$

a) Since children are balancing:

$$\text{net } \tau_\text{cw} = -\text{net } \tau_\text{ccw}$$

$$\Rightarrow w_1 r_1 + m_s g r_s = w_2 r_2$$

So, solving for r_2 gives:

$$\begin{aligned} r_2 &= \frac{w_1 r_1 + m_s g r_s}{w_2} = \frac{m_1 g r_1 + m_s g r_s}{m_2 g} = \frac{m_1 r_1 + m_s r_s}{m_2} \\ &= \frac{(26.0\ \text{kg})(1.60\ \text{m}) + (12.0\ \text{kg})(0.160\ \text{m})}{32.0\ \text{kg}} \\ &= 1.36\ \text{m} \end{aligned}$$

b) Since the children are not moving:

$$\text{net F} = 0 = F_\text{p} - w_1 - w_2 - w_s$$

$$\Rightarrow F_\text{p} = w_1 + w_2 + w_s$$

So that

$$\begin{aligned} F_\text{p} &= (26.0\ \text{kg} + 32.0\ \text{kg} + 12.0\ \text{kg})(9.80\ \text{m/s}^2) \\ &= 686\ \text{N} \end{aligned}$$

9.3 Stability

Summary

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in Figure 1, for example, is not in stable equilibrium. There are *three types of equilibrium*: *stable*, *unstable*, and *neutral*. Figures throughout this module illustrate various examples.

Figure 1 presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

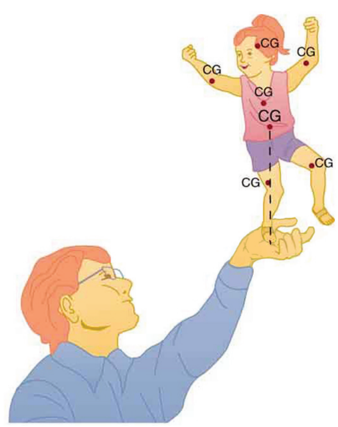

Figure 1. A man balances a toy doll on one hand.

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in Figure 2.

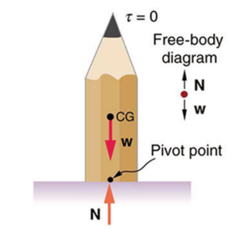

Figure 2. This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

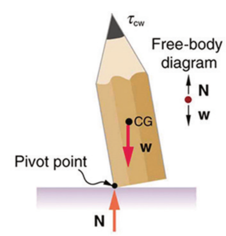

Figure 3. If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.

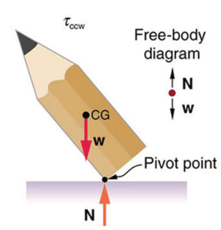

Figure 4. If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase.

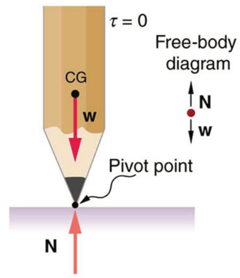

Figure 5. This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied.

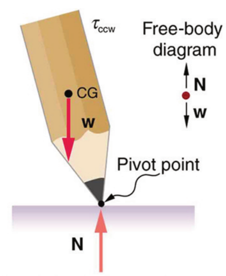

Figure 6. If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.

A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. Figure 7 shows another example of neutral equilibrium.

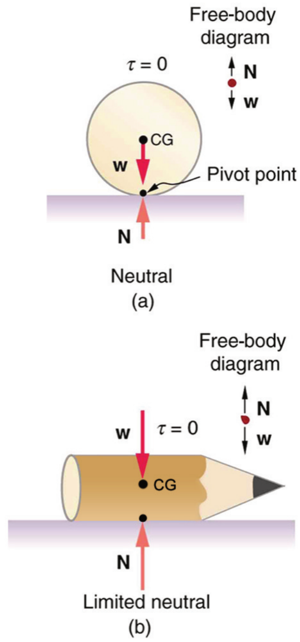

Figure 7. (a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil is in neutral equilibrium for displacements perpendicular to its length.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The pencil in Figure 2 and the person in Figure 8(a) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

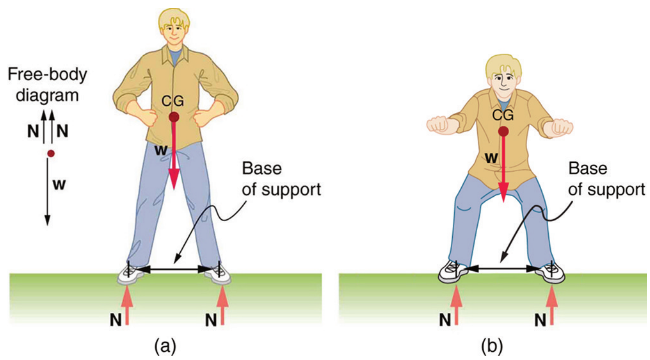

Figure 8. (a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.

Animals such as chickens have easier systems to control. Figure 9 shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

Figure 9 shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

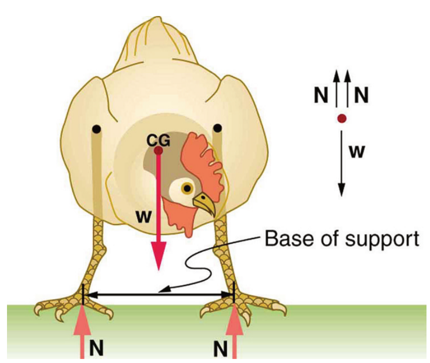

Figure 9. The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

TAKE-HOME EXPERIMENT

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

Section Summary

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

Conceptual Questions

1: A round pencil lying on its side as in Figure 4 is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?

2: Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

Problems & Exercises

1: Suppose a horse leans against a wall as in Figure 10. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

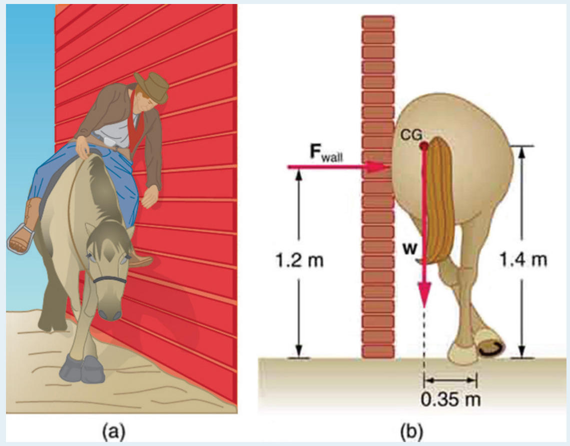

Figure 10.

2: Two children of mass 20.0 kg and 30.0 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3.00 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?

3: (a) Calculate the magnitude and direction of the force on each foot of the horse in Figure 10 (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.

4: A person carries a plank of wood 2.00 m long with one hand pushing down on it at one end with a force F_1 and the other hand holding it up at .500 m from the end of the plank with force F_2. If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces F_1 and F_2?

5: A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in Figure 11. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

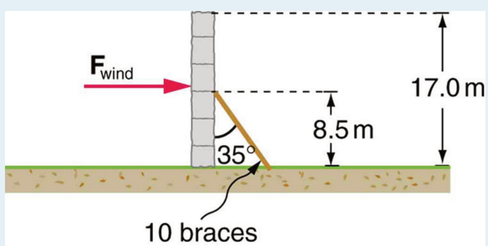

Figure 11.

6: (a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in Figure 12? (b) What is the ratio of this force to the chicken’s weight? (c) Does this support the contention that the chicken has a relatively stable construction?

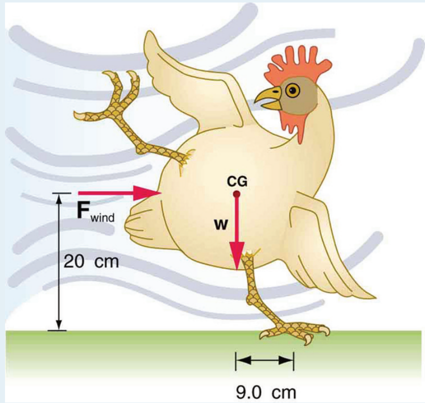

Figure 12.

7: Suppose the weight of the drawbridge in Figure 13 is supported entirely by its hinges and the opposite shore, so that its cables are slack. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances? The mass of the bridge is 2500 kg.

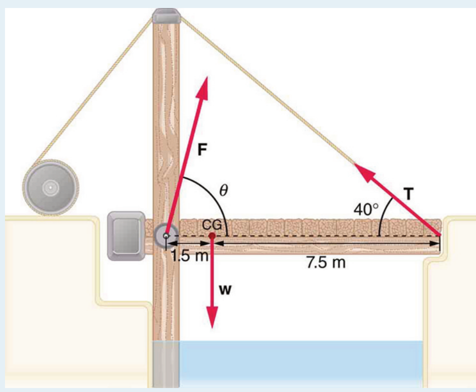

Figure 13. A small drawbridge, showing the forces on the hinges (**F**), its weight (**w**), and the tension in its wires (**T**).

8: Suppose a 900-kg car is on the bridge in Figure 13 with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.

9: A sandwich board advertising sign is constructed as shown in Figure 14. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?

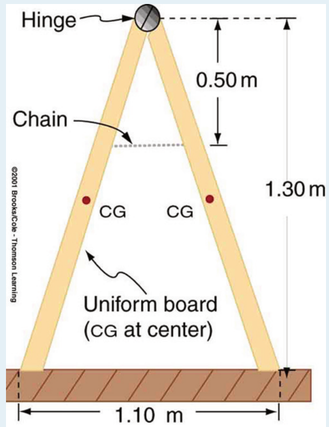

Figure 14. A sandwich board advertising sign demonstrates tension.

10: (a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in Figure 14 in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?

11: A gymnast is attempting to perform splits. From the information given in Figure 15, calculate the magnitude and direction of the force exerted on each foot by the floor.

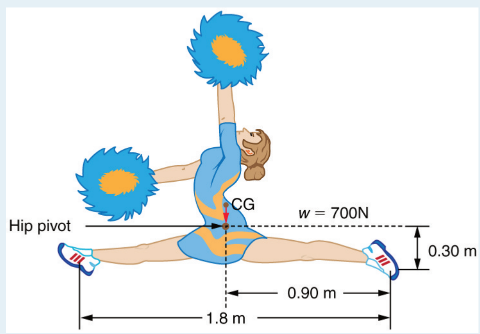

Figure 15. A gymnast performs full split. The center of gravity and the various distances from it are shown.

Glossary

neutral equilibrium
a state of equilibrium that is independent of a system's displacements from its original position

stable equilibrium
a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

unstable equilibrium
a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

Solutions

Problems & Exercises

1:

$F_{\text{wall}} = 1.43 \times 10^3\ \text{N}$

3:

a) $2.55 \times 10^3\ \text{N},\ 16.3^{\circ}$ to the left of vertical (i.e., toward the wall)

b) 0.292

5:

$F_{\text{B}} = 2.12 \times 10^4\ \text{N}$

7:

a) 0.167, or about one-sixth of the weight is supported by the opposite shore.

b) $F = 2.0 \times 10^4\ \text{N},$ straight up.

9:

a) 21.6 N

b) 21.6 N

11:

350 N directly upwards

9.4 Applications of Statics, Including Problem-Solving Strategies

Summary

- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in Chapter 4.6 Problem-Solving Strategies, still apply.

PROBLEM-SOLVING STRATEGY: STATIC EQUILIBRIUM SITUATIONS

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.

2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.

3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations $\text{net } F = 0$ and $\text{net } \tau = 0$, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$). Always choose a convenient coordinate system for projecting forces.

4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with

experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In Figure 1, the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium$(\text{net } F = 0)$. The second condition$(\text{net } \tau = 0)$is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In Figure 1, a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See Figure 1. If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

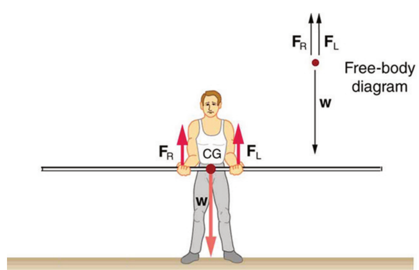

Figure 1. A pole vaulter holds a pole horizontally with both hands.

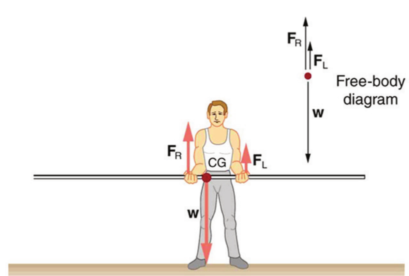

Figure 2. A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.

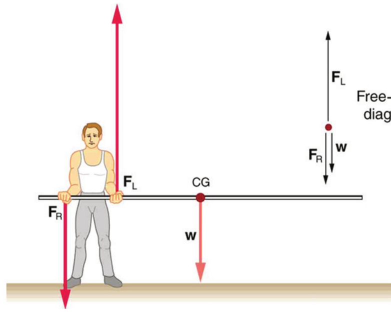

Figure 3. A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in Figure 2, the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $F_L = F_R$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces F_L and F_R is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole (Figure 3), the direction of the force applied by the right hand of the vaulter reverses its direction.

Example 1: What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in Figure 2, calculate: (a) F_R, the force exerted by the right hand, and (b) F_L, the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

Figure 2 includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium $(\text{net } F = 0)$, since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium $(\text{net } \tau = 0)$ if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force (τ_w) and from the push or pull of the right hand (τ_R). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\text{net } \tau_{cw} = -\text{net } \tau_{ccw}.$$

or the algebraic sum of the torques is zero.

Here this is

$$\tau_R = -\tau_w$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque, $\tau = rF \sin \theta$, noting that $\theta = 90^\circ$, and substituting known values, we obtain

$$(0.900 \text{ m})(F_R) = (0.600 \text{ m})(mg)$$

Thus,

$$\begin{aligned} F_{\mathrm{R}} &= (0.667)(5.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) \\ &= 32.7\ \mathrm{N}. \end{aligned}$$

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$F_{\mathrm{L}} + F_{\mathrm{R}} - mg = 0$$

From this we can conclude:

$$F_{\mathrm{L}} + F_{\mathrm{R}} = w = mg$$

Solving for F_{L}, we obtain

$$\begin{aligned} F_{\mathrm{L}} &= mg - F_{\mathrm{R}} \\ &= mg - 32.7\ \mathrm{N} \\ &= (5.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) - 32.7\ \mathrm{N} \\ &= 16.3\ \mathrm{N} \end{aligned}$$

Discussion

F_{L} is seen to be exactly half of F_{R}, as we might have guessed, since F_{L} is applied twice as far from the cg as F_{R}.

If the pole vaulter holds the pole as he might at the start of a run, shown in Figure 3, the forces change again. Both are considerably greater, and one force reverses direction.

TAKE-HOME EXPERIMENT

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

PHET EXPLORATIONS: BALANCING ACT

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

Figure 4. Balancing Act

Summary

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in Chapter 4.6 Problem-Solving Strategies, still apply.

Conceptual Questions

1: When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

Problems & Exercises

1: To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

2: In Figure 3, the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in Figure 1, show that the second condition for equilibrium $(\text{net } \tau = 0)$ is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

Glossary

static equilibrium
equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

9.5 Simple Machines

Summary

- Describe different simple machines.
- Calculate the mechanical advantage.

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for “machine” comes from the Greek word meaning “to help make things easier.” Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$\mathbf{MA} = \frac{F_o}{F_i}$$

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

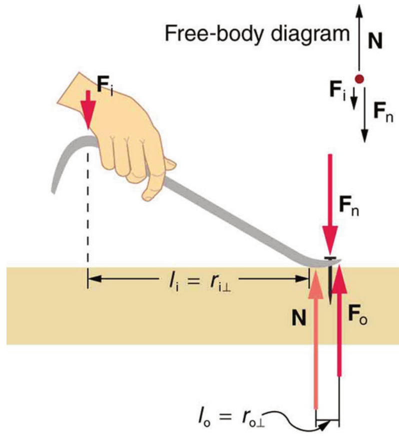

Figure 1. A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail ($\mathbf{F_o}$) is not a force on the nail puller. The reaction force the nail exerts back on the puller ($\mathbf{F_n}$) is an external force and is equal and opposite to $\mathbf{F_o}$. The perpendicular lever arms of the input and output forces are l_i and l_o.

Figure 1 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. F_i is the input force and F_o is the output force. There are three vertical forces acting on the nail puller (the system of interest) – these are F_i, F_o and N. F_n is the reaction force back on the system, equal and opposite to F_o. (Note that F_o is not a force on the system.) N is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to F_i and F_n must be equal to each other if the nail is not moving, to satisfy the second condition for equilibrium $(\text{net } \tau = 0)$. (In order for the nail to actually move, the torque due to F_i must be ever-so-slightly greater than torque due to F_n.) Hence,

$$l_i F_i = l_o F_o$$

where l_i and l_o are the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_o}{F_i} = \frac{l_i}{l_o}.$$

What interests us most here is that the magnitude of the force exerted by the nail puller, F_o, is much greater than the magnitude of the input force applied to the puller at the other end, F_i. For the nail puller,

$$\text{MA} = \frac{F_o}{F_i} = \frac{l_i}{l_o}.$$

This equation is true for levers in general. For the nail puller, the MA is certainly greater than one. The longer the handle on the nail puller, the greater the force you can exert with it.

Two other types of levers that differ slightly from the nail puller are a wheelbarrow and a shovel, shown in Figure 2. All these lever types are similar in that only three forces are involved – the input force, the output force, and the force on the pivot – and thus their MAs are given by $\text{MA} = \frac{F_o}{F_i}$ and $\text{MA} = \frac{d_i}{d_o}$, with distances being measured relative to the physical pivot. The wheelbarrow and shovel differ from the nail puller because both the input and output forces are on the same side of the pivot.

In the case of the wheelbarrow, the output force or load is between the pivot (the wheel's axle) and the input or applied force. In the case of the shovel, the input force is between the pivot (at the end of the handle) and the load, but the input lever arm is shorter than the output lever arm. In this case, the MA is less than one.

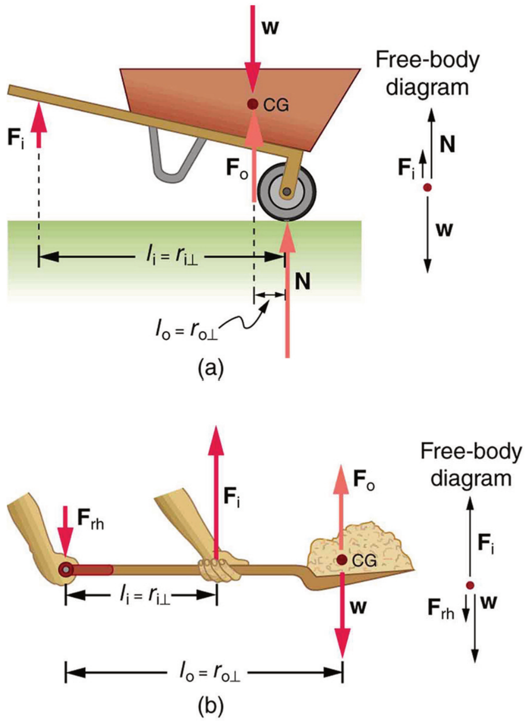

Figure 2. (a) In the case of the wheelbarrow, the output force or load is between the pivot and the input force. The pivot is the wheel's axle. Here, the output force is greater than the input force. Thus, a wheelbarrow enables you to lift much heavier loads than you could with your body alone. (b) In the case of the shovel, the input force is between the pivot and the load, but the input lever arm is shorter than the output lever arm. The pivot is at the handle held by the right hand. Here, the output force (supporting the shovel's load) is less than the input force (from the hand nearest the load), because the input is exerted closer to the pivot than is the output.

Example 1: What is the Advantage for the Wheelbarrow?

In the wheelbarrow of Figure 2, the load has a perpendicular lever arm of 7.50 cm, while the hands have a perpendicular lever arm of 1.02 m. (a) What upward force must you exert to support the wheelbarrow and its load if their combined mass is 45.0 kg? (b) What force does the wheelbarrow exert on the ground?

Strategy

Here, we use the concept of mechanical advantage.

Solution

(a) In this case, $\frac{F_o}{F_i} = \frac{l_i}{l_o}$ becomes

$$F_i = F_o \frac{l_o}{l_i}.$$

Adding values into this equation yields

$$F_i = (45.0\ \text{kg})(9.80\ \text{m/s}^2)\frac{0.075\ \text{m}}{1.02\ \text{m}} = 32.4\ \text{N}.$$

The free-body diagram (see Figure 2) gives the following normal force: $F_i + N = w$. Therefore, $N = (45.0\ \text{kg})(9.80\ \text{m/s}^2) - 32.4\ \text{N} = 409\ \text{N}$. N is the normal force acting on the wheel; by Newton's third law, the force the wheel exerts on the ground is $409\ \text{N}$.

Discussion

An even longer handle would reduce the force needed to lift the load. The MA here is $MA = 1.02/0.0750 = 13.6$.

Another very simple machine is the inclined plane. Pushing a cart up a plane is easier than lifting the same cart straight up to the top using a ladder, because the applied force is less. However, the work done in both cases (assuming the work done by friction is negligible) is the same. Inclined lanes or ramps were probably used during the construction of the Egyptian pyramids to move large blocks of stone to the top.

A crank is a lever that can be rotated 360° about its pivot, as shown in Figure 3. Such a machine may not look like a lever, but the physics of its actions remain the same. The MA for a crank is simply the ratio of the radii r_i/r_o. Wheels and gears have this simple expression for their MAs too. The MA can be greater than 1, as it is for the crank, or less than 1, as it is for the simplified car axle driving the wheels, as shown. If the axle's radius is $2.0\ \text{cm}$ and the wheel's radius is $24.0\ \text{cm}$, then $MA = 2.0/24.0 = 0.083$ and the axle would have to exert a force of $12,000\ \text{N}$ on the wheel to enable it to exert a force of $1000\ \text{N}$ on the ground.

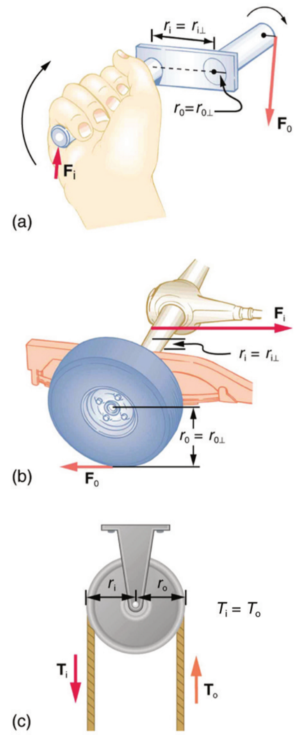

Figure 3. (a) A crank is a type of lever that can be rotated **360º** about its pivot. Cranks are usually designed to have a large MA. (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. The MA is less than 1. (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force ***T*** exerted by the cord without changing its magnitude. Hence, this machine has an MA of 1.

An ordinary pulley has an MA of 1; it only changes the direction of the force and not its magnitude. Combinations of pulleys, such as those illustrated in Figure 4, are used to multiply force. If the pulleys are friction-free, then the force output is approximately an integral multiple of the tension in the cable. The number of cables pulling directly upward on the system of interest, as illustrated in the figures given below, is approximately the MA of the pulley system. Since each attachment applies an external force in approximately the same direction as the others, they add, producing a total force that is nearly an integral multiple of the input force T.

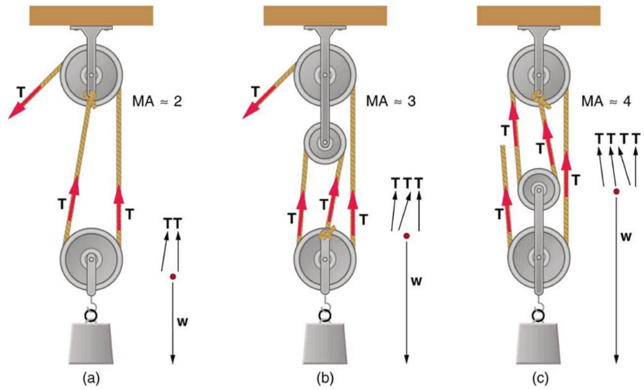

Figure 4. (a) The combination of pulleys is used to multiply force. The force is an integral multiple of tension if the pulleys are frictionless. This pulley system has two cables attached to its load, thus applying a force of approximately $\mathbf{2}\boldsymbol{T}$. This machine has **MA≈2**. (b) Three pulleys are used to lift a load in such a way that the mechanical advantage is about 3. Effectively, there are three cables attached to the load. (c) This pulley system applies a force of $\mathbf{4}\boldsymbol{T}$, so that it has **MA≈4**. Effectively, four cables are pulling on the system of interest.

Section Summary

- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

Conceptual Questions

1: Scissors are like a double-lever system. Which of the simple machines in Figure 1 and Figure 2 is most analogous to scissors?

2: Suppose you pull a nail at a constant rate using a nail puller as shown in Figure 1. Is the nail puller in equilibrium? What if you pull the nail with some acceleration – is the nail puller in equilibrium then? In which case is the force applied to the nail puller larger and why?

3: Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

4: Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces (see previous Question)?

Problems & Exercises

1: What is the mechanical advantage of a nail puller—similar to the one shown in Figure 1 —where you exert a force 45 cm from the pivot and the nail is 1.8 cm on the other side? What minimum force must you exert to apply a force of 1250 N to the nail?

2: Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?

3: a) What is the mechanical advantage of a wheelbarrow, such as the one in Figure 2, if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?

4: A typical car has an axle with 1.10 cm radius driving a tire with a radius of 27.5 cm. What is its mechanical advantage assuming the very simplified model in Figure 3(b)?

5: What force does the nail puller in Exercise 1 exert on the supporting surface? The nail puller has a mass of 2.10 kg.

6: If you used an ideal pulley of the type shown in Figure 4(a) to support a car engine of mass 115 kg, (a) What would be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system's mass.

7: Repeat Exercise 6 for the pulley shown in Figure 4(c), assuming you pull straight up on the rope. The pulley system's mass is 7.00 kg.

Glossary

mechanical advantage
the ratio of output to input forces for any simple machine

Solutions

Problems & Exercises

1:

25

50 N

3:

a) $MA = 18.5$

b) $F_{\text{i}} = 29.1 \text{ N}$

c) 510 N downward

5:

$1.3 \times 10^3 \text{ N}$

7:

a) $T = 299 \text{ N}$

b) 897 N upward

9.6 Forces and Torques in Muscles and Joints

Summary

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. Figure 1 shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in Figure 1.

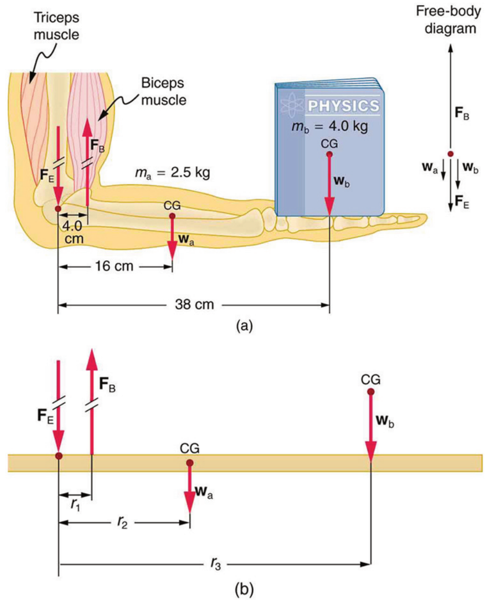

Figure 1. (a) The figure shows the forearm of a person holding a book. The biceps exert a force $\mathbf{F_B}$ to support the weight of the forearm and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in Example 1.

Example 1: Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in Figure 1, and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is F_B; that of the elbow joint is F_E; that of the weights of the forearm is w_a, and its load is w_b. Two of these are unknown (F_B and F_E), so that the first condition for equilibrium cannot by itself yield F_B. But if we use the second condition and choose the pivot to be at the elbow, then the torque due to F_E is zero, and the only unknown becomes F_B.

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium $(\text{net } \tau = 0)$ becomes

$$r_2 w_a + r_3 w_b = r_1 F_B.$$

Note that $\sin \theta = 1$ for all forces, since $\theta = 90^\circ$ for all forces. This equation can easily be solved for F_B in terms of known quantities, yielding

$$F_B = \frac{r_2 w_a + r_3 w_b}{r_1}.$$

Entering the known values gives

$$F_B = \frac{(0.160 \text{ m})(2.50 \text{ kg})(9.80 \text{ m/s}^2) + (0.380 \text{ m})(4.00 \text{ kg})(9.80 \text{ m/s}^2)}{0.0400 \text{ m}}$$

which yields

$$F_B = 470 \text{ N}.$$

Now, the combined weight of the arm and its load is $(6.50 \text{ kg})(9.80 \text{ m/s}^2) = 63.7 \text{ N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_{\text{B}}}{w_{\text{a}}+w_{\text{b}}} = \frac{470}{63.7} = 7.38.$$

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90°. If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force F_{E} exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of F_{E} is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is, $470 \text{ N} - 407 \text{ N} = 63 \text{ N}$, approximately equal to the weight supported.) Forces in muscles and joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as "tennis elbow," can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the "sweet spot" on the racquet will result in little vibration or impact force being felt in the racquet and the body—less torque as explained in Chapter 10.6 Collisions of Extended Bodies in Two Dimensions. Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

Figure 2 shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body's cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body's weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body's cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body's CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of Figure 2. This is compulsory; otherwise the person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in Figure 3.

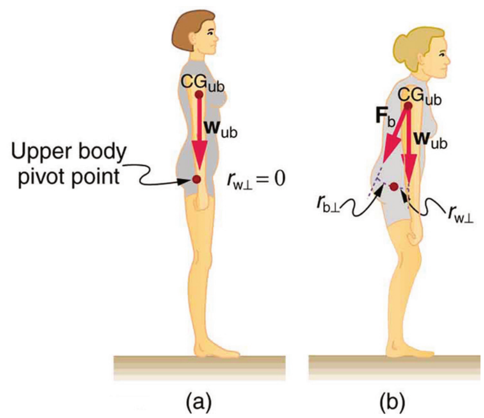

Figure 2. (a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm, $\boldsymbol{r}_{\mathbf{b?}}$, and must therefore exert a large force $\mathbf{F_b}$. Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet.

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

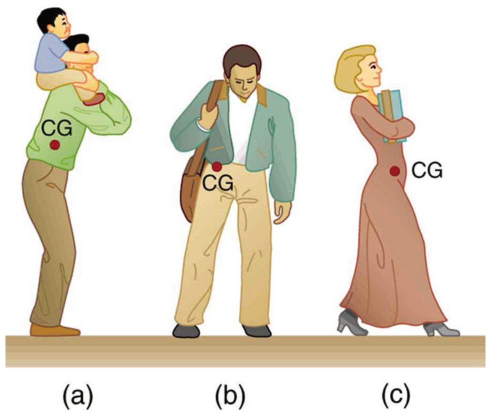

Figure 3. People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over his feet. (c) Another student carrying a load of books in her arms leans backward for the same reason.

Example 2: Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in Figure 4. (a) Calculate the magnitude of the force F_B in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude and direction of the force F_V exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

Strategy

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for F_B if the pivot is chosen to be at the hips. The torques created by w_{ub} and w_{box} are clockwise, while that created by F_B is counterclockwise.

Solution for (a)

Using the perpendicular lever arms given in the figure, the second condition for equilibrium $(\text{net } \tau = 0)$ becomes

$$(0.350\ \text{m})(55.0\ \text{kg})(9.80\ \text{m/s}^2) + (0.500\ \text{m})(30.0\ \text{kg})(9.80\ \text{m/s}^2) = (0.0800\ \text{m})F_B.$$

Solving for F_B yields

$$F_B = 4.20 \times 10^3\ \text{N}.$$

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{F_B}{w_{ub} + w_{box}} = \frac{4200\ \text{N}}{833\ \text{N}} = 5.04.$$

This force is considerably larger than it would be if the load were not present.

Solution for (b)

More important in terms of its damage potential is the force on the vertebrae F_V. The first condition for equilibrium $(\text{net } F = 0)$ can be used to find its magnitude and direction. Using y for vertical and x for horizontal, the condition for the net external forces along those axes to be zero

$$\text{net } F_y = 0 \text{ and net } F_x = 0.$$

Starting with the vertical (y) components, this yields

$$F_{Vy} - w_{ub} - w_{box} - F_B \sin 29.0^\circ = 0.$$

Thus,

$$\begin{aligned} F_{Vy} &= w_{ub} + w_{box} + F_B \sin 29.0^\circ \\ &= 833\ \text{N} + (4200\ \text{N}) \sin 29.0^\circ \end{aligned}$$

yielding

$$F_{Vy} = 2.87 \times 10^3\ \text{N}.$$

Similarly, for the horizontal (x) components,

$$F_{Vx} - F_B \cos 29.0^\circ = 0$$

yielding

$$F_{Vx} = 3.67 \times 10^3\ \text{N}.$$

The magnitude of F_V is given by the Pythagorean theorem:

$$F_V = \sqrt{F_{Vx}^2 + F_{Vy}^2} = 4.66 \times 10^3\ \text{N}.$$

The direction of F_V is

$$\theta = \tan^{-1}\left(\frac{F_{Vy}}{F_{Vx}}\right) = 38.0^\circ .$$

Note that the ratio of F_V to the weight supported is

$$\frac{F_V}{w_{ub} + w_{box}} = \frac{4660\ \text{N}}{833\ \text{N}} = 5.59.$$

Discussion

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and load, creates much smaller forces in the back—in this case, about 5.6 times smaller.

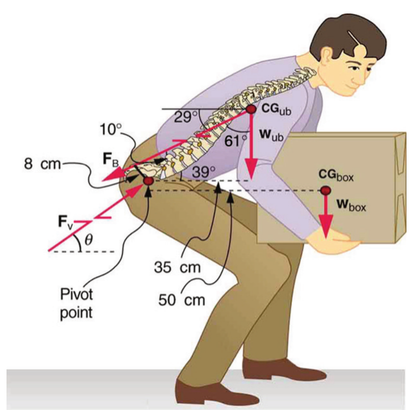

Figure 4. This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, Example 2.

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

Section Summary

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many of our muscles are attached close to joints.
- Someone with good posture stands or sits in such as way that their center of gravity lies directly above the pivot point in their hips, thereby avoiding back strain and damage to disks.

Conceptual Questions

1: Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?

2: Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces?

3: Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?

4: Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.

5: If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.

6: Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?

7: Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

Problems & Exercises

1: Verify that the force in the elbow joint in Example 1 is 407 N, as stated in the text.

2: Two muscles in the back of the leg pull on the Achilles tendon as shown in Figure 5. What total force do they exert?

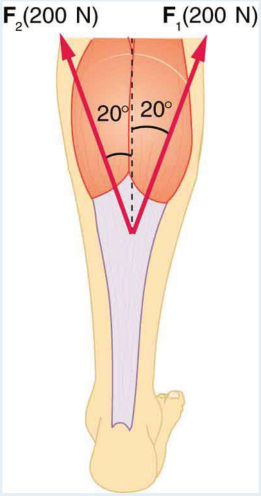

Figure 5. The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone.

3: The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in Figure 6. Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

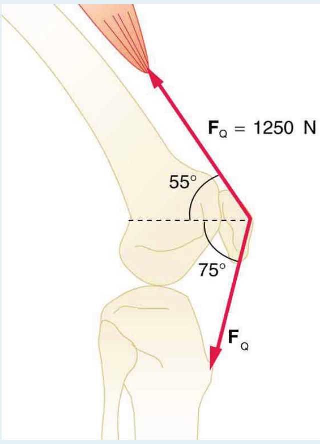

Figure 6. The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot.

4: A device for exercising the upper leg muscle is shown in Figure 7, together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in Chapter 9.4 Applications of Statistics, Including Problem-Solving Strategies.

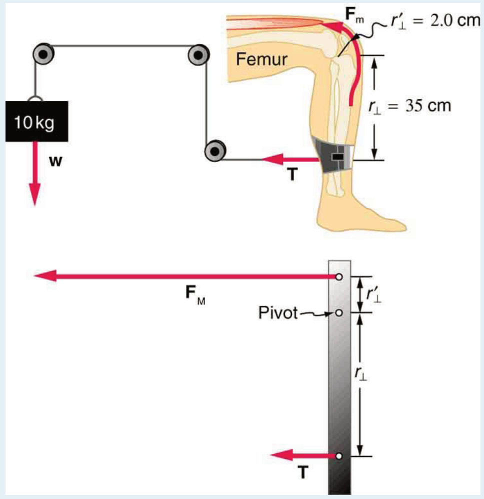

Figure 7. A mass is connected by pulleys and wires to the ankle in this exercise device.

5: A person working at a drafting board may hold her head as shown in Figure 8, requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae F_V to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

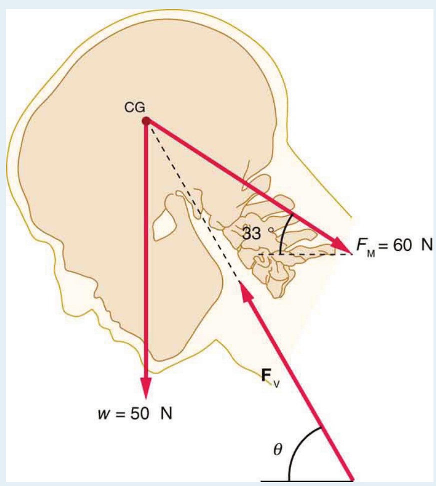

Figure 8.

6: We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90°. Using the same numbers as in Example 1, find the force exerted by the biceps muscle when the angle is 120° and the forearm is in a downward position.

7: Even when the head is held erect, as in Figure 9, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

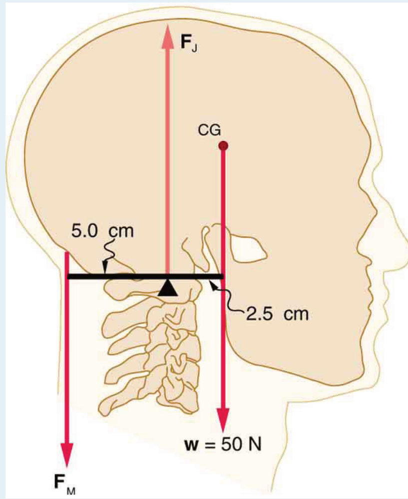

Figure 9. The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown.

8: A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in Figure 10. (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

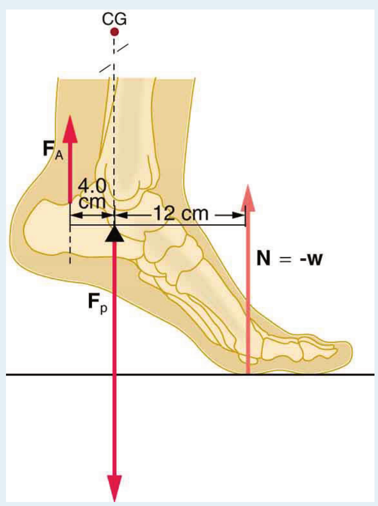

Figure 10. The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown.

9: A father lifts his child as shown in Figure 11. What force should the upper leg muscle exert to lift the child at a constant speed?

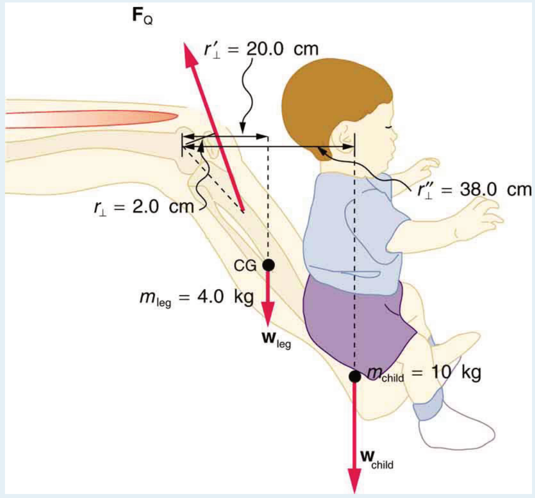

Figure 11. A child being lifted by a father's lower leg.

10: Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in Figure 12, is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the lower teeth on the bullet. (b) Calculate the force on the joint.

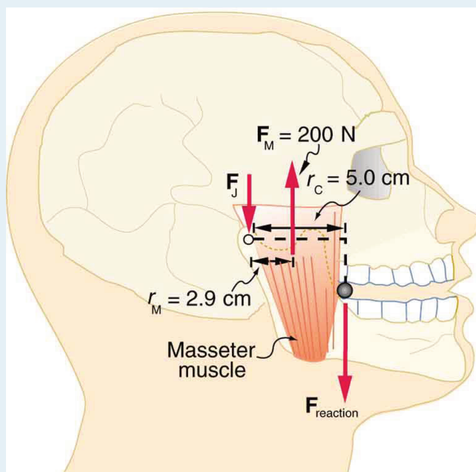

Figure 12. A person clenching a bullet between his teeth.

11: Integrated Concepts

Suppose we replace the 4.0-kg book in Exercise 6 of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant $k = 600 \text{ N/m}$. (a) How much is the rope stretched (past equilibrium) to provide the same force F_B as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25° with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

12: (a) What force should the woman in Figure 13 exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight.

(c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

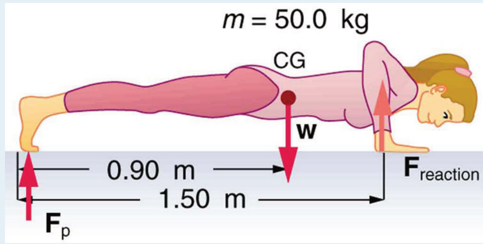

Figure 13. A woman doing pushups.

13: You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother's birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?

14: Unreasonable Results

Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

15: Construct Your Own Problem

Consider a method for measuring the mass of a person's arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its center of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.

Solutions

Problems & Exercises

1:

$$\begin{aligned} F_{\mathrm{B}} &= 470\text{ N}; r_1 = 4.00\text{ cm}; w_{\mathrm{a}} = 2.50\text{ kg}; r_2 = 16.0\text{ cm}; w_{\mathrm{b}} = 4.00\text{ kg}; r_3 = 38.0\text{ cm} \\ F_{\mathrm{E}} &= w_{\mathrm{a}}\left(\frac{r_2}{r_1} - 1\right) + w_{\mathrm{b}}\left(\frac{r_3}{r_1} - 1\right) \\ &= (2.50\text{ kg})(9.80\text{ m/s}^2)\left(\frac{16.0\text{ cm}}{4.0\text{ cm}} - 1\right) \\ &\quad + (4.00\text{ kg})(9.80\text{ m/s}^2)\left(\frac{38.0\text{ cm}}{4.00\text{ cm}} - 1\right) \\ &= 407\text{ N} \end{aligned}$$

3:

$1.1 \times 10^3\text{ N}$

$\theta = 190^0$ ccw from positive x axis

5:

$F_{\mathrm{V}} = 97\ \mathrm{N}, \theta = 59^{0}$

7:

(a) 25 N downward

(b) 75 N upward

8:

(a)$F_{\mathrm{A}} = 2.21 \times 10^{3}\ \mathrm{N}$upward

(b)$F_{\mathrm{B}} = 2.94 \times 10^{3}\ \mathrm{N}$downward

10:

(a)$F_{\text{teeth on bullet}} = 1.2 \times 10^{2}\ \mathrm{N}$upward

(b)$F_{\mathrm{J}} = 84\ \mathrm{N}$downward

12:

(a) 147 N downward

(b) 1680 N, 3.4 times her weight

(c) 118 J

(d) 49.0 W

14:

a)$\bar{x}_2 = 2.33\ \mathrm{m}$

b) The seesaw is 3.0 m long, and hence, there is only 1.50 m of board on the other side of the pivot. The second child is off the board.

c) The position of the first child must be shortened, i.e. brought closer to the pivot.

PART 10

Chapter 10 Rotational Motion and Angular Momentum

Figure 1. The mention of a tornado conjures up images of raw destructive power. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. They descend from clouds in funnel-like shapes that spin violently, particularly at the bottom where they are most narrow, producing winds as high as 500 km/h. (credit: Daphne Zaras, U.S. National Oceanic and Atmospheric Administration).

Why do tornadoes spin at all? And why do tornados spin so rapidly? The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner as seen in Figure 2. The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion. These and other aspects of rotational motion are covered in this chapter. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogs in linear motion. First, we look at angular acceleration—the rotational analog of linear acceleration.

Figure 2. This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. (credit: Luu, Wikimedia Commons)

10.1 Angular Acceleration

Summary

- Describe uniform circular motion.
- Explain non-uniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.

Chapter 6 Uniform Circular Motion and Gravitation discussed only uniform circular motion, which is motion in a circle at constant speed and, hence, constant angular velocity. Recall that angular velocity ω was defined as the time rate of change of angle θ:

$$\omega = \frac{\Delta\theta}{\Delta t},$$

where θ is the angle of rotation as seen in Figure 1. The relationship between angular velocity ω and linear velocity v was also defined in Chapter 6.1 Rotation Angle and Angular Velocity as

$$v = r\omega$$

or

$$\omega = \frac{v}{r},$$

where r is the radius of curvature, also seen in Figure 1. According to the sign convention, the counter clockwise direction is considered as positive direction and clockwise direction as negative

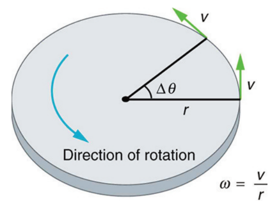

Figure 1. This figure shows uniform circular motion and some of its defined quantities.

Angular velocity is not constant when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an **angular acceleration**, in which ω changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration α is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

$$\alpha = \frac{\Delta\omega}{\Delta t},$$

where $\Delta\omega$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are $(\text{rad/s})/\text{s}$, or rad/s^2. If ω increases, then α is positive. If ω decreases, then α is negative.

Example 1: Calculating the Angular Acceleration and Deceleration of a Bike Wheel

Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in rad/s^2. (b) If she now slams on the brakes, causing an angular acceleration of $-87.3\ \text{rad/s}^2$, how long does it take the wheel to stop?

Strategy for (a)

The angular acceleration can be found directly from its definition in $\alpha = \frac{\Delta\omega}{\Delta t}$ because the final angular velocity and time are given. We see that $\Delta\omega$ is 250 rpm and Δt is 5.00 s.

Solution for (a)

Entering known information into the definition of angular acceleration, we get

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t}\\ &= \frac{250\ \text{rpm}}{5.00\ \text{s}}.\end{aligned}$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert $\Delta\omega$ from rpm to rad/s:

$$\begin{aligned}\Delta\omega &= 250\,\frac{\text{rev}}{\text{min}}\cdot\frac{2\pi\ \text{rad}}{\text{rev}}\cdot\frac{1\ \text{min}}{60\ \text{sec}}\\ &= 26.2\ \text{rads}.\end{aligned}$$

Entering this quantity into the expression for α, we get

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t}\\ &= \frac{26.2\ \text{rad/s}}{5.00\ \text{s}}\\ &= 5.24\ \text{rad/s}.^2\end{aligned}$$

Strategy for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for Δt, yielding

$$\Delta t = \frac{\Delta\omega}{\alpha}.$$

Solution for (b)

Here the angular velocity decreases from $26.2\ \text{rad/s}$ (250 rpm) to zero, so that $\Delta\omega$ is $-26.2\ \text{rad/s}$, and α is given to be $-87.3\ \text{rad/s}^2$. Thus,

$$\begin{aligned}\Delta t &= \frac{-26.2\ \text{rad/s}}{-87.3\ \text{rad/s}^2}\\ &= 0.300\ \text{s}.\end{aligned}$$

Discussion

Note that the angular acceleration as the girl spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero. In both cases, the relationships are analogous to what happens with linear motion. For example, there is a large deceleration when you crash into a brick wall—the velocity change is large in a short time interval.

If the bicycle in the preceding example had been on its wheels instead of upside-down, it would first have accelerated along the ground and then come to a stop. This connection between circular motion and linear motion needs to be explored. For example, it would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in Figure 2. Thus, linear acceleration is called tangential acceleration a_t.

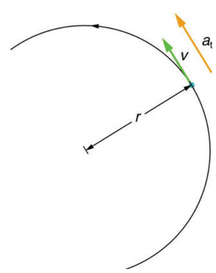

Figure 2. In circular motion, linear acceleration ***a***, occurs as the magnitude of the velocity changes: ***a*** is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration $\boldsymbol{a_t}$.

Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction. We know from Chapter 6 Uniform Circular Motion and Gravitation that in circular motion centripetal acceleration, a_c, refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration, as seen in Figure 3. Thus, a_t and a_c are perpendicular and independent of one another. Tangential acceleration a_t is directly related to the angular acceleration α and is linked to an increase or decrease in the velocity, but not its direction.

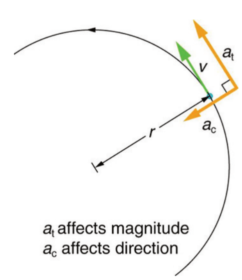

Figure 3. Centripetal acceleration $\boldsymbol{a_c}$ occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.

Now we can find the exact relationship between linear acceleration a_t and angular acceleration α. Because linear acceleration is proportional to a change in the magnitude of the velocity, it is defined (as it was in Chapter 2 One-Dimensional Kinematics) to be

$$a_t = \frac{\Delta v}{\Delta t}.$$

For circular motion, note that $v = r\omega$, so that

$$a_t = \frac{\Delta(r\omega)}{\Delta t}.$$

The radius r is constant for circular motion, and so $\Delta(r\omega) = r(\Delta\omega)$. Thus,

$$a_t = r\frac{\Delta\omega}{\Delta t}.$$

By definition, $\alpha = \frac{\Delta\omega}{\Delta t}$. Thus,

$$a_t = r\alpha,$$

or

$$\alpha = \frac{a_t}{r}.$$

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a car's drive wheels, the greater the acceleration of the car. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration α.

Example 2: Calculating the Angular Acceleration of a Motorcycle Wheel

A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels? (See Figure 4.)

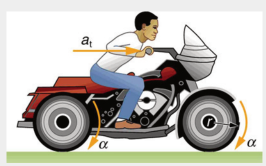

Figure 4. The linear acceleration of a motorcycle is accompanied by an angular acceleration of its wheels.

Strategy

We are given information about the linear velocities of the motorcycle. Thus, we can find its linear acceleration a_t. Then, the expression $\alpha = \frac{a_t}{r}$ can be used to find the angular acceleration.

Solution

The linear acceleration is

$$\begin{aligned} a_t &= \frac{\Delta v}{\Delta t} \\ &= \frac{30.0\ \text{m/s}}{4.20\ \text{s}} \\ &= 7.14\ \text{m/s}^2. \end{aligned}$$

We also know the radius of the wheels. Entering the values for a_t and r into $\alpha = \frac{a_t}{r}$, we get

$$\begin{aligned} \alpha &= \frac{a_t}{r} \\ &= \frac{7.14\ \text{m/s}^2}{0.320\ \text{m}} \\ &= 22.3\ \text{rad/s}^2. \end{aligned}$$

Discussion

Units of radians are dimensionless and appear in any relationship between angular and linear quantities.

So far, we have defined three rotational quantities— θ, ω, and α. These quantities are analogous to the translational quantities x, v, and a. Table 1 displays rotational quantities, the analogous translational quantities, and the relationships between them.

Rotational	Translational	Relationship
θ	x	$\theta = \frac{x}{r}$
ω	v	$\omega = \frac{v}{r}$
α	a	$\alpha = \frac{a_t}{r}$

Table 1. Rotational and Translational Quantities.

MAKING CONNECTIONS: TAKE-HOME EXPERIMENT

Sit down with your feet on the ground on a chair that rotates. Lift one of your legs such that it is unbent (straightened out). Using the other leg, begin to rotate yourself by pushing on the ground. Stop using your leg to push the ground but allow the chair to rotate. From the origin where you began, sketch the angle, angular velocity, and angular acceleration of your leg as a function of time in the form of three separate graphs. Estimate the magnitudes of these quantities.

Check Your Understanding

1: Angular acceleration is a vector, having both magnitude and direction. How do we denote its magnitude and direction? Illustrate with an example.

PHET EXPLORATIONS: LADYBUG REVOLUTION

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

Figure 5. Ladybug Revolution

Section Summary

- Uniform circular motion is the motion with a constant angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\alpha = \frac{\Delta\omega}{\Delta t}$.
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $a_t = \frac{\Delta v}{\Delta t}$.
- For circular motion, note that $v = r\omega$, so that

  $$a_t = \frac{\Delta(r\omega)}{\Delta t}.$$

- The radius r is constant for circular motion, and so $\Delta(r\omega) = r\Delta\omega$. Thus,

  $$a_t = r\frac{\Delta\omega}{\Delta t}.$$

- By definition, $\Delta\omega/\Delta t = \alpha$. Thus,

  $$a_t = r\alpha$$

  or

  $$\alpha = \frac{a_t}{r}.$$

Conceptual Questions

1: Analogies exist between rotational and translational physical quantities. Identify the rotational term analogous to each of the following: acceleration, force, mass, work, translational kinetic energy, linear momentum, impulse.

2: Explain why centripetal acceleration changes the direction of velocity in circular motion but not its magnitude.

3: In circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction. Explain your answer.

4: Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience nonzero tangential acceleration, centripetal acceleration, or both when: (a) The plate starts to spin? (b) The plate rotates at constant angular velocity? (c) The plate slows to a halt?

Problems & Exercises

1: At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?

2: Integrated Concepts

An ultracentrifuge accelerates from rest to 100,000 rpm in 2.00 min. (a) What is its angular acceleration in rad/s^2? (b) What is the tangential acceleration of a point 9.50 cm from the axis of rotation? (c) What is the radial acceleration in m/s^2 and multiples of g of this point at full rpm?

3: Integrated Concepts

You have a grindstone (a disk) that is 90.0 kg, has a 0.340-m radius, and is turning at 90.0 rpm, and you press a steel axe against it with a radial force of 20.0 N. (a) Assuming the kinetic coefficient of friction between steel and stone is 0.20, calculate the angular acceleration of the grindstone. (b) How many turns will the stone make before coming to rest?

4: Unreasonable Results

You are told that a basketball player spins the ball with an angular acceleration of $100\ \text{rad/s}^2$. (a) What is the ball's final angular velocity if the ball starts from rest and the acceleration lasts 2.00 s? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

Glossary

angular acceleration
the rate of change of angular velocity with time

change in angular velocity
the difference between final and initial values of angular velocity

tangential acceleration
the acceleration in a direction tangent to the circle at the point of interest in circular motion

Solutions

Check Your Understanding

1: The magnitude of angular acceleration is α and its most common units are rad/s^2. The direction of angular acceleration along a fixed axis is denoted by a + or a – sign, just as the direction of linear acceleration in one dimension is denoted by a + or a – sign. For example, consider a gymnast doing a forward flip. Her angular momentum would be parallel to the mat and to her left. The magnitude of her angular acceleration would be proportional to her angular velocity (spin rate) and her moment of inertia about her spin axis.

Problems & Exercises

1:

$\omega = 0.737\ \text{rev/s}$

3:

(a) $-0.26\ \mathrm{rad/s^2}$

(b) $27\ \mathrm{rev}$

10.2 Kinematics of Rotational Motion

Summary

- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.
- Evaluate problem solving strategies for rotational kinematics.

Just by using our intuition, we can begin to see how rotational quantities like θ, ω, and α are related to one another. For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. In more technical terms, if the wheel's angular acceleration α is large for a long period of time t, then the final angular velocity ω and angle of rotation θ are large. The wheel's rotational motion is exactly analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large.

Kinematics is the description of motion. The **kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω, α, and t. To determine this equation, we recall a familiar kinematic equation for translational, or straight-line, motion:

$$v = v_0 + at \text{ (constant a)}$$

Note that in rotational motion $a = a_t$, and we shall use the symbol a for tangential or linear acceleration from now on. As in linear kinematics, we assume a is constant, which means that angular acceleration α is also a constant, because $a = r\alpha$. Now, let us substitute $v = r\omega$ and $a = r\alpha$ into the linear equation above:

$$r\omega = r\omega_0 + r\alpha t.$$

The radius r cancels in the equation, yielding

$$\omega = \omega_0 + \alpha t \text{ (constant a)},$$

where ω_0 is the initial angular velocity. This last equation is a *kinematic relationship* among ω, α, and t—that is, it describes their relationship without reference to forces or masses that may affect rotation. It is also precisely analogous in form to its translational counterpart.

MAKING CONNECTIONS

Kinematics for rotational motion is completely analogous to translational kinematics, first presented in Chapter 2 One-Dimensional Kinematics. Kinematics is concerned with the description of motion without regard to force or mass. We will find that translational kinematic quantities, such as displacement, velocity, and acceleration have direct analogs in rotational motion.

Starting with the four kinematic equations we developed in Chapter 2 One-Dimensional Kinematics, we can derive the following four rotational kinematic equations (presented together with their translational counterparts):

Rotational	Translational	
$\theta = \bar{\omega} t$	$x = \bar{v} t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α, a)
$\omega = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	(constant α, a)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	(constant α, a)

Table 2. Rotational Kinematic Equations.

In these equations, the subscript 0 denotes initial values (θ_0, x_0, and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}.$$

The equations given above in Table 2 can be used to solve any rotational or translational kinematics problem in which a and α are constant.

PROBLEM-SOLVING STRATEGY FOR ROTATIONAL KINEMATICS

1. *Examine the situation to determine that rotational kinematics (rotational motion) is involved*. Rotation must be involved, but without the need to consider forces or masses that affect the motion.
2. *Identify exactly what needs to be determined in the problem (identify the unknowns)*. A sketch of the situation is useful.
3. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns)*.
4. *Solve the appropriate equation or equations for the quantity to be determined (the unknown)*. It can be useful to think in terms of a translational analog because by now you are familiar with such motion.

5. *Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units*. Be sure to use units of radians for angles.

6. *Check your answer to see if it is reasonable: Does your answer make sense*?

Example 1: Calculating the Acceleration of a Fishing Reel

A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of $110\ \text{rad/s}^2$ for 2.00 s as seen in Figure 1.

(a) What is the final angular velocity of the reel?

(b) At what speed is fishing line leaving the reel after 2.00 s elapses?

(c) How many revolutions does the reel make?

(d) How many meters of fishing line come off the reel in this time?

Strategy

In each part of this example, the strategy is the same as it was for solving problems in linear kinematics. In particular, known values are identified and a relationship is then sought that can be used to solve for the unknown.

Solution for (a)

Here α and t are given and ω needs to be determined. The most straightforward equation to use is $\omega = \omega_0 + \alpha t$ because the unknown is already on one side and all other terms are known. That equation states that

$$\omega = \omega_0 + \alpha t.$$

We are also given that $\omega_0 = 0$ (it starts from rest), so that

$$\omega = 0 + (110\ \text{rad/s}^2)(2.00\ \text{s}) = 220\ \text{rad/s}.$$

Solution for (b)

Now that ω is known, the speed v can most easily be found using the relationship

$$v = r\omega,$$

where the radius r of the reel is given to be 4.50 cm; thus,

$$v = (0.0450\ \text{m})(220\ \text{rad/s}) = 9.90\ \text{m/s}.$$

Note again that radians must always be used in any calculation relating linear and angular quantities. Also, because radians are dimensionless, we have $\text{m} \times \text{rad} = \text{m}$.

Solution for (c)

Here, we are asked to find the number of revolutions. Because $1\ \text{rev} = 2\pi\ \text{rad}$, we can find the number of revolutions by finding θ in radians. We are given α and t, and we know ω_0 is zero, so that θ can be obtained using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

$$\begin{aligned} \theta &= \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ &= 0 + (0.500)(110\ \text{rad/s}^2)(2.00\ \text{s})^2 = 220\ \text{rad}. \end{aligned}$$

Converting radians to revolutions gives

$$\theta = (220\ \text{rad})\frac{1\ \text{rev}}{2\pi\ \text{rad}} = 35.0\ \text{rev}.$$

Solution for (d)

The number of meters of fishing line is x, which can be obtained through its relationship with θ:

$$x = r\theta = (0.0450\ \text{m})(220\ \text{rad}) = 9.90\ \text{m}.$$

Discussion

This example illustrates that relationships among rotational quantities are highly analogous to those among linear quantities. We also see in this example how linear and rotational quantities are connected. The answers to the questions are realistic. After unwinding for two seconds, the reel is found to spin at 220 rad/s, which is 2100 rpm. (No wonder reels sometimes make high-pitched sounds.) The amount of fishing line played out is 9.90 m, about right for when the big fish bites.

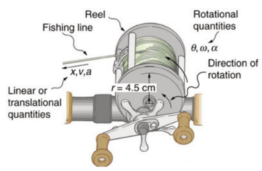

Figure 1. Fishing line coming off a rotating reel moves linearly. Example 1 and Example 2 consider relationships between rotational and linear quantities associated with a fishing reel.

Example 2: Calculating the Duration When the Fishing Reel Slows Down and Stops

Now let us consider what happens if the fisherman applies a brake to the spinning reel, achieving an angular acceleration of $-300\ \text{rad/s}^2$. How long does it take the reel to come to a stop?

Strategy

We are asked to find the time t for the reel to come to a stop. The initial and final conditions are different from those in the previous problem, which involved the same fishing reel. Now we see that the initial angular velocity is $\omega_0 = 220\ \text{rad/s}$ and the final angular velocity ω is zero. The angular acceleration is given to be $\alpha = -300\ \text{rad/s}^2$. Examining the available equations, we see all quantities but *t* are known in $\omega = \omega_0 + \alpha t$, making it easiest to use this equation.

Solution

The equation states

$$\omega = \omega_0 + \alpha t.$$

We solve the equation algebraically for *t*, and then substitute the known values as usual, yielding

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 220\ \text{rad/s}}{-300\ \text{rad/s}^2} = 0.733\ \text{s}.$$

Discussion

Note that care must be taken with the signs that indicate the directions of various quantities. Also, note that the time to stop the reel is fairly small because the acceleration is rather large. Fishing lines sometimes snap because of the accelerations involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration.

Example 3: Calculating the Slow Acceleration of Trains and Their Wheels

Large freight trains accelerate very slowly. Suppose one such train accelerates from rest, giving its 0.350-m-radius wheels an angular acceleration of $0.250\ \text{rad/s}^2$. After the wheels have made 200 revolutions (assume no slippage): (a) How far has the train moved down the track? (b) What are the final angular velocity of the wheels and the linear velocity of the train?

Strategy

In part (a), we are asked to find x, and in (b) we are asked to find ω and v. We are given the number of revolutions θ, the radius of the wheels r, and the angular acceleration α.

Solution for (a)

The distance x is very easily found from the relationship between distance and rotation angle:

$$\theta = \frac{x}{r}.$$

Solving this equation for x yields

$$x = r\theta.$$

Before using this equation, we must convert the number of revolutions into radians, because we are dealing with a relationship between linear and rotational quantities:

$$\theta = (200\ \text{rev})\frac{2\pi\text{rad}}{1\ \text{rev}} = 1257\ \text{rad}.$$

Now we can substitute the known values into $x = r\theta$ to find the distance the train moved down the track:

$$x = r\theta = (0.350\ \text{m})(1257\ \text{rad}) = 440\ \text{m}.$$

Solution for (b)

We cannot use any equation that incorporates t to find ω, because the equation would have at least two unknown values. The equation $\omega^2 = \omega_0^2 + 2\alpha\theta$ will work, because we know the values for all variables except ω:

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Taking the square root of this equation and entering the known values gives

$$\begin{aligned}\omega &= [0 + 2(0.250\ \text{rad/s}^2)(1257\ \text{rad})]^{1/2}\\ &= 25.1\ \text{rad/s}.\end{aligned}$$

We can find the linear velocity of the train, v, through its relationship to ω:

$$v = r\omega = (0.350\ \text{m})(25.1\ \text{rad/s}) = 8.77\ \text{m/s}.$$

Discussion

The distance traveled is fairly large and the final velocity is fairly slow (just under 32 km/h).

There is translational motion even for something spinning in place, as the following example illustrates. Figure 2 shows a fly on the edge of a rotating microwave oven plate. The example below calculates the total distance it travels.

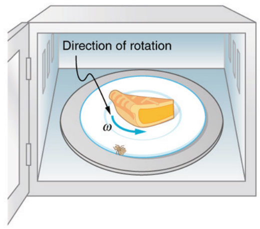

Figure 2. The image shows a microwave plate. The fly makes revolutions while the food is heated (along with the fly).

Example 4: Calculating the Distance Traveled by a Fly on the Edge of a Microwave Oven Plate

A person decides to use a microwave oven to reheat some lunch. In the process, a fly accidentally flies into the microwave and lands on the outer edge of the rotating plate and remains there. If the plate has a radius of 0.15 m and rotates at 6.0 rpm, calculate the total distance traveled by the fly during a 2.0-min cooking period. (Ignore the start-up and slow-down times.)

Strategy

First, find the total number of revolutions θ, and then the linear distance x traveled. $\theta = \bar{\omega}t$ can be used to find θ because $\bar{\omega}$ is given to be 6.0 rpm.

Solution

Entering known values into $\theta = \bar{\omega}t$ gives

$$\theta = \bar{\omega}t = (6.0\ \text{rpm})(2.0\ \text{min}) = 12\ \text{rev}.$$

As always, it is necessary to convert revolutions to radians before calculating a linear quantity like x from an angular quantity like θ:

$$\theta = (12\ \text{rev})\frac{2\pi\ \text{rad}}{1\ \text{rev}} = 75.4\ \text{rad}.$$

Now, using the relationship between x and θ, we can determine the distance traveled:

$$x = r\omega = (0.15\ \text{m})(75.4\ \text{rad}) = 11\ \text{m}.$$

Discussion

Quite a trip (if it survives)! Note that this distance is the total distance traveled by the fly. Displacement is actually zero for complete revolutions because they bring the fly back to its original position. The distinction between total distance traveled and displacement was first noted in Chapter 2 One-Dimensional Kinematics.

Check Your Understanding

1: Rotational kinematics has many useful relationships, often expressed in equation form. Are these relationships laws of physics or are they simply descriptive? (Hint: the same question applies to linear kinematics.)

Section Summary

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time.
- Starting with the four kinematic equations we developed in the Chapter 2 One-Dimensional Kinematics, we can derive the four rotational kinematic equations (presented together with their translational counterparts) seen in Table 2.
- In these equations, the subscript 0 denotes initial values (x_0 and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:

$$\bar{\omega} = \frac{\omega_0+\omega}{2} \text{ and } \bar{v} = \frac{v_0+v}{2}.$$

Problems & Exercises

1: With the aid of a string, a gyroscope is accelerated from rest to 32 rad/s in 0.40 s.

(a) What is its angular acceleration in rad/s^2?

(b) How many revolutions does it go through in the process?

2: Suppose a piece of dust finds itself on a CD. If the spin rate of the CD is 500 rpm, and the piece of dust is 4.3 cm from the center, what is the total distance traveled by the dust in 3 minutes? (Ignore accelerations due to getting the CD rotating.)

3: A gyroscope slows from an initial rate of 32.0 rad/s at a rate of 0.700 rad/s^2.

(a) How long does it take to come to rest?

(b) How many revolutions does it make before stopping?

4: During a very quick stop, a car decelerates at 7.00 m/s^2.

(a) What is the angular acceleration of its 0.280-m-radius tires, assuming they do not slip on the pavement?

(b) How many revolutions do the tires make before coming to rest, given their initial angular velocity is 95.0 rad/s?

(c) How long does the car take to stop completely?

(d) What distance does the car travel in this time?

(e) What was the car’s initial velocity?

(f) Do the values obtained seem reasonable, considering that this stop happens very quickly?

Figure 3. Yo-yos are amusing toys that display significant physics and are engineered to enhance performance based on physical laws. (credit: Beyond Neon, Flickr)

5: Everyday application: Suppose a yo-yo has a center shaft that has a 0.250 cm radius and that its string is being pulled.

(a) If the string is stationary and the yo-yo accelerates away from it at a rate of $1.50\ \text{m/s}^2$, what is the angular acceleration of the yo-yo?

(b) What is the angular velocity after 0.750 s if it starts from rest?

(c) The outside radius of the yo-yo is 3.50 cm. What is the tangential acceleration of a point on its edge?

Glossary

kinematics of rotational motion
describes the relationships among rotation angle, angular velocity, angular acceleration, and time

Solutions

Check Your Understanding

1: Rotational kinematics (just like linear kinematics) is descriptive and does not represent laws of nature. With kinematics, we can describe many things to great precision but kinematics does not consider causes. For example, a large angular acceleration describes a very rapid change in angular velocity without any consideration of its cause.

Problems & Exercises

1:

(a) $80\ \text{rad/s}^2$

(b) 1.0 rev

3:

(a) 45.7 s

(b) 116 rev

5:

a) $600\ \text{rad/s}^2$

b) 450 rad/s

c) 21.0 m/s

10.3 Dynamics of Rotational Motion: Rotational Inertia

Summary

- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in Figure 1. In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

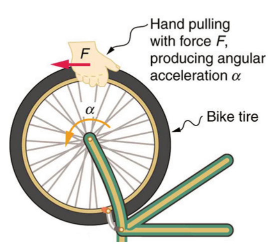

Figure 1. Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in Figure 2. Because the force is perpendicular to r, an acceleration $a = \frac{F}{m}$ is obtained in the direction of F. We can rearrange this equation such that $F = ma$ and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

$$F = mr\alpha.$$

Recall that **torque** is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$rF = mr^2\alpha$$

or

$$\tau = mr^2\alpha.$$

This last equation is the rotational analog of Newton's second law ($F = ma$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the **rotational inertia** or **moment of inertia** of a point mass m a distance r from the center of rotation.

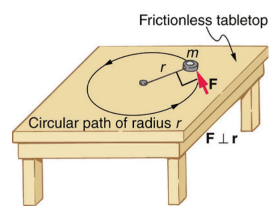

Figure 2. An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force ***F*** is applied to the object perpendicular to the radius ***r***, causing it to accelerate about the pivot point. The force is kept perpendicular to ***r***.

MAKING CONNECTIONS: ROTATIONAL MOTION DYNAMICS

Dynamics for rotational motion is completely analogous to linear or translational dynamics. Dynamics is concerned with force and mass and their effects on motion. For rotational motion, we will find direct analogs to force and mass that behave just as we would expect from our earlier experiences.

Rotational Inertia and Moment of Inertia

Before we can consider the rotation of anything other than a point mass like the one in Figure 2, we must extend the idea of rotational inertia to all types of objects. To expand our concept of rotational inertia, we define the **moment of inertia** I of an object to be the sum of mr^2 for all the point masses of which it is composed. That is, $I = \Sigma mr^2$. Here I is analogous to m in translational motion. Because of the distance r, the moment of inertia for any object depends on the chosen axis. Actually, calculating I is beyond the scope of this text except for one simple case—that of a hoop, which has all its mass at the same distance from its axis. A hoop's moment of inertia around its axis is therefore MR^2, where M is its total mass and R its radius. (We use M and R for an entire object to distinguish them from m and r for point masses.) In all other cases, we must consult Figure 3 (note that the table is piece of artwork that has shapes as well as formulae) for formulas for I that have been derived from integration over the continuous body. Note that I has units of mass multiplied by distance squared ($\text{kg} \cdot \text{m}^2$), as we might expect from its definition.

The general relationship among torque, moment of inertia, and angular acceleration is

$$\text{net } \tau = I\alpha$$

or

$$\alpha = \frac{\text{net } \tau}{I},$$

where net τ is the total torque from all forces relative to a chosen axis. For simplicity, we will only consider torques exerted by forces in the plane of the rotation. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = I\alpha,\ \alpha = \frac{\text{net } \tau}{I}$ is the rotational analog to Newton's second law and is very generally applicable. This equation is actually valid for *any* torque, applied to *any* object, relative to *any* axis.

As we might expect, the larger the torque is, the larger the angular acceleration is. For example, the harder a child pushes on a merry-go-round, the faster it accelerates. Furthermore, the more massive a merry-go-round, the slower it accelerates for the same torque. The basic relationship between moment of inertia and angular acceleration is that the larger the moment of inertia, the smaller is the angular acceleration. But there is an additional twist. The moment of inertia depends not only on the mass of an object, but also on its *distribution* of mass relative to the axis around which it rotates. For example, it will be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge. The mass is the same in both cases; but the moment of inertia is much larger when the children are at the edge.

TAKE-HOME EXPERIMENT

Cut out a circle that has about a 10 cm radius from stiff cardboard. Near the edge of the circle, write numbers 1 to 12 like hours on a clock face. Position the circle so that it can rotate freely about a horizontal axis through its center, like a wheel. (You could loosely nail the circle to a wall.) Hold the circle stationary and with the number 12 positioned at the top, attach a lump of blue putty (sticky material used for fixing posters to walls) at the number 3. How large does the lump need to be to just rotate the circle? Describe how you can change the moment of inertia of the circle. How does this change affect the amount of blue putty needed at the number 3 to just rotate the circle? Change the circle's moment of inertia and then try rotating the circle by using different amounts of blue putty. Repeat this process several times.

PROBLEM-SOLVING STRATEGY FOR ROTATIONAL DYNAMICS

1. *Examine the situation to determine that torque and mass are involved in the rotation*. Draw a careful sketch of the situation.
2. *Determine the system of interest*.
3. *Draw a free body diagram*. That is, draw and label all external forces acting on the system of interest.
4. *Apply* $\text{net}\ \tau = I\alpha,\ \alpha = \frac{\text{net}\ \tau}{I}$*, the rotational equivalent of Newton's second law, to solve the problem*. Care must be taken to use the correct moment of inertia and to consider the torque about the point of rotation.
5. *As always, check the solution to see if it is reasonable*.

MAKING CONNECTIONS

In statics, the net torque is zero, and there is no angular acceleration. In rotational motion, net torque is the cause of angular acceleration, exactly as in Newton's second law of motion for rotation.

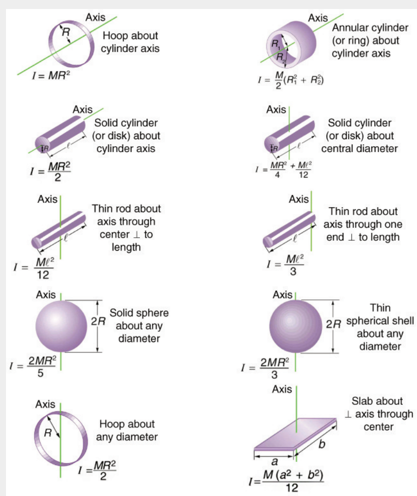

Figure 3. Some rotational inertias.

Example 1: Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in Figure 4. He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible retarding friction.

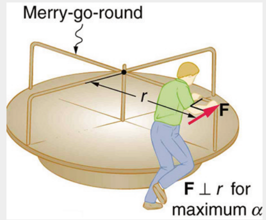

Figure 4. A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Strategy

Angular acceleration is given directly by the expression $\alpha = \frac{\text{net }\tau}{I}$:

$$\alpha = \frac{\tau}{I}.$$

To solve for α, we must first calculate the torque τ (which is the same in both cases) and moment of inertia I (which is greater in the second case). To find the torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF\sin\theta = (1.50\text{ m})(250\text{ N}) = 375\text{ N}\cdot\text{m}.$$

Solution for (a)

The moment of inertia of a solid disk about this axis is given in Figure 3 to be

$$\frac{1}{2}MR^2,$$

where $M = 50.0\text{ kg}$ and $R = 1.50\text{ m}$, so that

$$I = (0.500)(50.0\text{ kg})(1.50\text{ m})^2 = 56.25\text{ kg}\cdot\text{m}^2.$$

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375\text{ N}\cdot\text{m}}{56.25\text{ kg}\cdot\text{m}^2} = 6.67\frac{\text{rad}}{\text{s}^2}.$$

Solution for (b)

We expect the angular acceleration for the system to be less in this part, because the moment of inertia is greater when the child is on the merry-go-round. To find the total moment of inertia I, we first find the child's moment of inertia I_c by considering the child to be equivalent to a point mass at a distance of 1.25 m from the axis. Then,

$$I_c = MR^2 = (18.0\text{ kg})(1.25\text{ m})^2 = 28.13\text{ kg}\cdot\text{m}^2.$$

The total moment of inertia is the sum of moments of inertia of the merry-go-round and the child (about the same axis). To justify this sum to yourself, examine the definition of I:

$$I = 28.13\text{ kg}\cdot\text{m}^2 + 56.25\text{ kg}\cdot\text{m}^2 = 84.38\text{ kg}\cdot\text{m}^2.$$

Substituting known values into the equation for α gives

$$\alpha = \frac{\tau}{I} = \frac{375\text{ N}\cdot\text{m}}{84.38\text{ kg}\cdot\text{m}^2} = 4.44\frac{rad}{s^2}.$$

Discussion

The angular acceleration is less when the child is on the merry-go-round than when the merry-go-round is empty, as expected. The angular accelerations found are quite large, partly due to the fact that friction was considered to be negligible. If, for example, the father kept pushing perpendicularly for 2.00 s, he would give the merry-go-round an angular velocity of 13.3 rad/s when it is empty but only 8.89 rad/s when the child is on it. In terms of revolutions per second, these angular velocities are 2.12 rev/s and 1.41 rev/s, respectively. The father would end up running at about 50 km/h in the first case. Summer Olympics, here he comes! Confirmation of these numbers is left as an exercise for the reader.

Check Your Understanding

1: Torque is the analog of force and moment of inertia is the analog of mass. Force and mass are physical quantities that depend on only one factor. For example, mass is related solely to the numbers of atoms of various types in an object. Are torque and moment of inertia similarly simple?

Section Summary

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r, an acceleration $a = F/m$ is obtained in the direction of F. We can rearrange this equation such that

  $$F = ma,$$

  and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

  $$F = mr\alpha$$

- Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = rF$. If we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

  $$rF = mr^2\alpha$$

  or

  $$\tau = mr^2\alpha.$$

- The moment of inertia I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,

  $$I = \sum mr^2.$$

- The general relationship among torque, moment of inertia, and angular acceleration is

  $$\tau = I\alpha$$

  or

  $$\alpha = \frac{\text{net } \tau}{I}$$

Conceptual Questions

1: The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $ML^2/3$. Why is this moment of inertia greater than it would be if you spun a point mass M at the location of the center of mass of the rod (at $L/2$)? (That would be $ML^2/4$.)

2: Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius? Why is the moment of inertia of a spherical shell that has a mass M and a radius R greater than that of a solid sphere that has the same mass and radius?

3: Give an example in which a small force exerts a large torque. Give another example in which a large force exerts a small torque.

4: While reducing the mass of a racing bike, the greatest benefit is realized from reducing the mass of the tires and wheel rims. Why does this allow a racer to achieve greater accelerations than would an identical reduction in the mass of the bicycle's frame?

Figure 5. The image shows a side view of a racing bicycle. Can you see evidence in the design of the wheels on this racing bicycle that their moment of inertia has been purposely reduced? (credit: Jesús Rodriguez)

5: A ball slides up a frictionless ramp. It is then rolled without slipping and with the same initial velocity up another frictionless ramp (with the same slope angle). In which case does it reach a greater height, and why?

Problems & Exercises

1: This problem considers additional aspects of Example 1. (a) How long does it take the father to give the merry-go-round an angular velocity of 1.50 rad/s? (b) How many revolutions must he go through to generate this velocity? (c) If he exerts a slowing force of 300 N at a radius of 1.35 m, how long would it take him to stop them?

2: Calculate the moment of inertia of a skater given the following information. (a) The 60.0-kg skater is approximated as a cylinder that has a 0.110-m radius. (b) The skater with arms extended is approximately a cylinder that is 52.5 kg, has a 0.110-m radius, and has two 0.900-m-long arms which are 3.75 kg each and extend straight out from the cylinder like rods rotated about their ends.

3: The triceps muscle in the back of the upper arm extends the forearm. This muscle in a professional boxer exerts a force of $2.00 \times 10^3\ \text{N}$ with an effective perpendicular lever arm of 3.00 cm, producing an angular acceleration of the forearm of $120\ \text{rad/s}^2$. What is the moment of inertia of the boxer's forearm?

4: A soccer player extends her lower leg in a kicking motion by exerting a force with the muscle above the knee in the front of her leg. She produces an angular acceleration of $30.00\ \text{rad/s}^2$ and her lower leg has a moment of inertia of $0.750\ \text{kg}\cdot\text{m}^2$. What is the force exerted by the muscle if its effective perpendicular lever arm is 1.90 cm?

5: Suppose you exert a force of 180 N tangential to a 0.280-m-radius 75.0-kg grindstone (a solid disk).

(a)What torque is exerted? (b) What is the angular acceleration assuming negligible opposing friction? (c) What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

6: Consider the 12.0 kg motorcycle wheel shown in Figure 6. Assume it to be approximately an annular ring with an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. (a) If the drive chain exerts a force of 2200 N at a radius of 5.00 cm, what is the angular acceleration of the wheel? (b) What is the tangential acceleration of a point on the outer edge of the tire? (c) How long, starting from rest, does it take to reach an angular velocity of 80.0 rad/s?

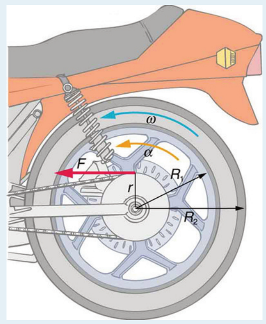

Figure 6. A motorcycle wheel has a moment of inertia approximately that of an annular ring.

7: Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of 4.00×10^7 N (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Explicitly show how you follow the steps found in Problem-Solving Strategy for Rotational Dynamics.

8: An automobile engine can produce 200 N · m of torque. Calculate the angular acceleration produced if 95.0% of this torque is applied to the drive shaft, axle, and rear wheels of a car, given the following information. The car is suspended so that the wheels can turn freely. Each wheel acts like a 15.0 kg disk that has a 0.180 m radius. The walls of each tire act like a 2.00-kg annular ring that has inside radius of 0.180 m and outside radius of 0.320 m. The tread of each tire acts like a 10.0-kg hoop of radius 0.330 m. The 14.0-kg axle acts like a rod that has a 2.00-cm radius. The 30.0-kg drive shaft acts like a rod that has a 3.20-cm radius.

9: Starting with the formula for the moment of inertia of a rod rotated around an axis through one end perpendicular to its length $(I = M\ell^2/3)$, prove that the moment of inertia of a rod rotated about an axis through its center perpendicular to its length is $I = M\ell^2/12$. You will find the graphics in Figure 3 useful in visualizing these rotations.

10: Unreasonable Results

A gymnast doing a forward flip lands on the mat and exerts a 500-N · m torque to slow and then reverse her angular velocity. Her initial angular velocity is 10.0 rad/s, and her moment of inertia is $0.050\ \text{kg}\cdot\text{m}^2$. (a) What time is required for her to exactly reverse her spin? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

11: Unreasonable Results

An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel, which can accelerate the car from rest to a speed of 30.0 m/s. The flywheel is a disk with a 0.150-m radius. (a) Calculate the angular velocity the flywheel must have if 95.0% of its rotational energy is used to get the car up to speed. (b) What is unreasonable about the result? (c) Which premise is unreasonable or which premises are inconsistent?

Glossary

torque
the turning effectiveness of a force
rotational inertia
resistance to change of rotation. The more rotational inertia an object has, the harder it is to rotate
moment of inertia
mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = mr^2$ *and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia*

Exercises

Check Your Understanding

1: No. Torque depends on three factors: force magnitude, force direction, and point of application. Moment of inertia depends on both mass and its distribution relative to the axis of rotation. So, while the analogies are precise, these rotational quantities depend on more factors.

Problems & Exercises

1:

(a) 0.338 s

(b) 0.0403 rev

(c) 0.313 s

3:

$0.50\ \text{kg}\cdot\text{m}^2$

5:

(a) $50.4\ \text{N}\cdot\text{m}$

(b) $17.1\ \text{rad/s}^2$

(c) $17.0\ \text{rad/s}^2$

7:

$3.96 \times 10^{18}\ \text{s}$

or $1.26 \times 10^{11}\ \text{y}$

9:

$$I_{\text{end}} = I_{\text{center}} + m\left(\frac{l}{2}\right)^2$$
$$\text{Thus, } I_{\text{center}} = I_{\text{end}} - \frac{1}{4}ml^2 = \frac{1}{3}ml^2 - \frac{1}{4}ml^2 = \frac{1}{12}ml^2$$

10:

(a) 2.0 ms

(b) The time interval is too short.

(c) The moment of inertia is much too small, by one to two orders of magnitude. A torque of $500\ \text{N}\cdot\text{m}$ is reasonable.

11:

(a) 17,500 rpm

(b) This angular velocity is very high for a disk of this size and mass. The radial acceleration at the edge of the disk is > 50,000 gs.

(c) Flywheel mass and radius should both be much greater, allowing for a lower spin rate (angular velocity).

10.4 Rotational Kinetic Energy: Work and Energy Revisited

Summary

- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the Law of Conservation of Energy.

In this module, we will learn about work and energy associated with rotational motion. Figure 1 shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable **rotational kinetic energy**.

Figure 1. The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell)

Work must be done to rotate objects such as grindstones or merry-go-rounds. Work was defined in Chapter 6 Uniform Circular Motion and Gravitation for translational motion, and we can build on that knowledge when considering work done in rotational motion. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disk (as shown in Figure 2) and remains perpendicular as the disk starts to rotate. The force is parallel to the displacement, and so the net work done is the product of the force times the arc length traveled:

$$\text{net } W = (\text{net } F)\Delta s.$$

To get torque and other rotational quantities into the equation, we multiply and divide the right-hand side of the equation by r, and gather terms:

$$\text{net } W = (r \text{ net } F)\frac{\Delta s}{r}.$$

We recognize that $r \text{ net } F = \text{net } \tau$ and $\Delta s / r = \theta$, so that

$$\text{net } W = (\text{net } \tau)\theta.$$

This equation is the expression for rotational work. It is very similar to the familiar definition of translational work as force multiplied by distance. Here, torque is analogous to force, and angle is analogous to distance. The equation $\text{net } W = (\text{net } \tau)\theta$ is valid in general, even though it was derived for a special case.

To get an expression for rotational kinetic energy, we must again perform some algebraic manipulations. The first step is to note that $\text{net } \tau = I\alpha$, so that

$$\text{net } W = I\alpha\theta.$$

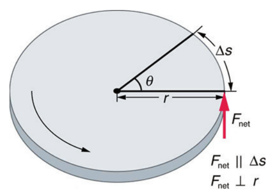

Figure 2. The net force on this disk is kept perpendicular to its radius as the force causes the disk to rotate. The net work done is thus **(net *F*)Δ*s***. The net work goes into rotational kinetic energy.

MAKING CONNECTIONS

Work and energy in rotational motion are completely analogous to work and energy in translational motion, first presented in Chapter 6 Uniform Circular Motion and Gravitation.

Now, we solve one of the rotational kinematics equations for $\alpha\theta$. We start with the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta.$$

Next, we solve for $\alpha\theta$:

$$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}.$$

Substituting this into the equation for net W and gathering terms yields

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2.$$

This equation is the **work-energy theorem** for rotational motion only. As you may recall, net work changes the kinetic energy of a system. Through an analogy with translational motion, we define the term $\left(\frac{1}{2}\right)I\omega^2$ to be **rotational kinetic energy** $\mathrm{KE}_{\mathrm{rot}}$ for an object with a moment of inertia I and an angular velocity ω:

$$\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}I\omega^2.$$

The expression for rotational kinetic energy is exactly analogous to translational kinetic energy, with I being analogous to m and ω to v. Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in Figure 3.

Figure 3. Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into **KE$_{\mathbf{rot}}$**. It can also convert translational kinetic energy, when the bus stops, into **KE$_{\mathbf{rot}}$**. The flywheel's energy can then be used to accelerate, to go up another hill, or to keep the bus from going against friction.

Example 1: Calculating the Work and Energy for Spinning a Grindstone

Consider a person who spins a large grindstone by placing her hand on its edge and exerting a force through part of a revolution as shown in Figure 4. In this example, we verify that the work done by the torque she exerts equals the change in rotational energy. (a) How much work is done if she exerts a force of 200 N through a rotation of $1.00\text{ rad}(57.3^\circ)$? The force is kept perpendicular to the grindstone's 0.320-m radius at the point of application, and the effects of friction are negligible. (b) What is the final angular velocity if the grindstone has a mass of 85.0 kg? (c) What is the final rotational kinetic energy? (It should equal the work.)

Strategy

To find the work, we can use the equation $\text{net } W = (\text{net } \tau)\theta$. We have enough information to calculate the torque and are given the rotation angle. In the second part, we can find the final angular velocity using one of the kinematic relationships. In the last part, we can calculate the rotational kinetic energy from its expression in $\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}I\omega^2$.

Solution for (a)

The net work is expressed in the equation

$$\text{net } W = (\text{net } \tau)\theta,$$

where net τ is the applied force multiplied by the radius (rF) because there is no retarding friction, and the force is perpendicular to r. The angle θ is given. Substituting the given values in the equation above yields

$$\begin{aligned}\text{net } W &= rF\theta = (0.320\text{ m})(200\text{ N})(1.00\text{ rad})\\ &= 64.0\text{ N}\cdot\text{m}.\end{aligned}$$

Noting that $1\text{ N}\cdot\text{m} = 1\text{ J}$,

$$\text{net } W = 64.0\text{ J}.$$

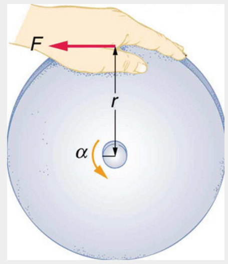

Figure 4. A large grindstone is given a spin by a person grasping its outer edge.

Solution for (b)

To find ω from the given information requires more than one step. We start with the kinematic relationship in the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta.$$

Note that $\omega_0 = 0$ because we start from rest. Taking the square root of the resulting equation gives

$$\omega = (2\alpha\theta)^{1/2}.$$

Now we need to find α. One possibility is

$$\alpha = \frac{\text{net }\tau}{I},$$

where the torque is

$$\text{net }\tau = rF = (0.320\text{ m})(200\text{ N}) = 64.0\text{ N}\cdot\text{m}.$$

The formula for the moment of inertia for a disk is found in Making Connections:

$$I = \frac{1}{2}MR^2 = 0.5(85.0\text{ kg})(0.320\text{ m})^2 = 4.352\text{ kg}\cdot\text{m}^2.$$

Substituting the values of torque and moment of inertia into the expression for α, we obtain

$$\alpha = \frac{64.0\text{ N}\cdot\text{m}}{4.352\text{ kg}\cdot\text{m}^2} = 14.7\,\frac{rad}{s^2}.$$

Now, substitute this value and the given value for θ into the above expression for ω:

$$\omega = (2\alpha\theta)^{1/2} = \left[2\left(14.7\,\frac{\text{rad}}{s^2}\right)(1.00\text{ rad})\right]^{1/2} = 5.42\,\frac{\text{rad}}{s}.$$

Solution for (c)

The final rotational kinetic energy is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

Both I and ω were found above. Thus,

$$\text{KE}_{\text{rot}} = (0.5)(4.352\text{ kg}\cdot\text{m}^2)(5.42\text{ rad/s})^2 = 64.0\text{ J}.$$

Discussion

The final rotational kinetic energy equals the work done by the torque, which confirms that the work done went into rotational kinetic energy. We could, in fact, have used an expression for energy instead of a kinematic relation to solve part (b). We will do this in later examples.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

PROBLEM-SOLVING STRATEGY FOR ROTATIONAL ENERGY

1. *Determine that energy or work is involved in the rotation*.
2. *Determine the system of interest*. A sketch usually helps.
3. *Analyze the situation to determine the types of work and energy involved*.
4. *For closed systems, mechanical energy is conserved*. That is, $\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$. Note that $\mathrm{KE_i}$ and $\mathrm{KE_f}$ may each include translational and rotational contributions.
5. *For open systems*, mechanical energy may not be conserved, and other forms of energy (referred to previously as OE), such as heat transfer, may enter or leave the system. Determine what they are, and calculate them as necessary.
6. *Eliminate terms wherever possible to simplify the algebra*.
7. *Check the answer to see if it is reasonable*.

Example 2: Calculating Helicopter Energies

A typical small rescue helicopter, similar to the one in Figure 5, has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

$$\mathrm{KE_{rot}} = \frac{1}{2}I\omega^2.$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find $\mathrm{KE_{rot}}$. The angular velocity ω is

$$\omega = \frac{300\ \mathrm{rev}}{1.00\ \mathrm{min}} \cdot \frac{2\pi\ \mathrm{rad}}{1\ \mathrm{rev}} \cdot \frac{1.00\ \mathrm{min}}{60.0\ \mathrm{s}} = 31.4\ \frac{\mathrm{rad}}{\mathrm{s}}.$$

The moment of inertia of one blade will be that of a thin rod rotated about its end, found in Making Connections. The total I is four times this moment of inertia, because there are four blades. Thus,

$$I = 4\frac{M\ell^2}{3} = 4\times\frac{(50.0\ \text{kg})(4.00\ \text{m})^2}{3} = 1067\ \text{kg}\cdot\text{m}^2.$$

Entering ω and I into the expression for rotational kinetic energy gives

$$\begin{aligned}\text{KE}_{\text{rot}} &= 0.5(1067\ \text{kg}\cdot\text{m}^2)(31.4\ \text{rad/s})^2\\ &= 5.26\times10^5\ \text{J}\end{aligned}$$

Solution for (b)

Translational kinetic energy was defined in Chapter 6 Uniform Circular Motion and Gravitation. Entering the given values of mass and velocity, we obtain

$$\text{KE}_{\text{trans}} = \frac{1}{2}mv^2 = 0.5(1000\ \text{kg})(20.0\ \text{m/s})^2 = 2.00\times10^5\ \text{J}.$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00\times10^5\ \text{J}}{5.26\times10^5\ \text{J}} = 0.380.$$

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

$$\text{KE}_{\text{rot}} = \text{PE}_{\text{grav}}$$

or

$$\frac{1}{2}I\omega^2 = mgh.$$

We now solve for h and substitute known values into the resulting equation

$$h = \frac{\frac{1}{2}I\omega^2}{mg} = \frac{5.26\times10^5\ \text{J}}{(1000\ \text{kg})(9.80\ \text{m/s}^2)} = 53.7\ \text{m}.$$

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.

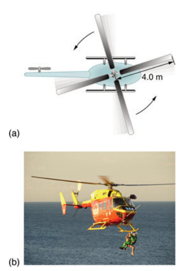

Figure 5. The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. (credit: 111 Emergency, Flickr)

MAKING CONNECTIONS

Conservation of energy includes rotational motion, because rotational kinetic energy is another form of KE. Chapter 6 Uniform Circular Motion and Gravitation has a detailed treatment of conservation of energy.

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy PE_{grav}, which is converted entirely to KE, provided each rolls without slipping. KE, however, can take the form of KE_{trans} or KE_{rot}, and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in Figure 6.

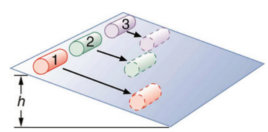

Figure 6. Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

$$PE_i = KE_f.$$

More specifically,

$$PE_{grav} = KE_{trans} + KE_{rot}$$

or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

TAKE-HOME EXPERIMENT

Locate several cans each containing different types of food. First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. You could also do this experiment by collecting several empty cylindrical containers of the same size and filling them with different materials such as wet or dry sand.

Example 3: Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with vv as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Before we can solve for v, we must get an expression for I from Chapter 10.3 Making Connections. Because v and ω are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v/R$ into the expression. These substitutions yield

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right).$$

Interestingly, the cylinder's radius R and mass m cancel, yielding

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2.$$

Solving algebraically, the equation for the final velocity v gives

$$v = \left(\frac{4gh}{3}\right)^{1/2}.$$

Substituting known values into the resulting expression yields

$$v = \left[\frac{4(9.80\text{ m/s}^2)(2.00\text{ m})}{3}\right]^{1/2} = 5.11\text{ m/s}.$$

Discussion

Because m and R cancel, the result $v = \left(\frac{4}{3}gh\right)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}mv^2 = mgh$ and $v = (2gh)^{1/2}$, which is 22% greater than $(4gh/3)^{1/2}$. That is, the cylinder would go faster at the bottom.

Check Your Understanding

Analogy of Rotational and Translational Kinetic Energy

1: Is rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

PHET EXPLORATIONS: MY SOLAR SYSTEM

Build your own system of heavenly bodies and watch the gravitational ballet. With this orbit simulator, you can set initial positions, velocities, and masses of 2, 3, or 4 bodies, and then see them orbit each other.

PhET Interactive Simulation

Figure 7. My Solar System

Section Summary

- The rotational kinetic energy KE_{rot} for an object with a moment of inertia I and an angular velocity ω is given by

$$KE_{rot} = \frac{1}{2}I\omega^2.$$

- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.
- The equation for the **work-energy theorem** for rotational motion is,

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2.$$

Conceptual Questions

1: Describe the energy transformations involved when a yo-yo is thrown downward and then climbs back up its string to be caught in the user’s hand.

2: What energy transformations are involved when a dragster engine is revved, its clutch let out rapidly, its tires spun, and it starts to accelerate forward? Describe the source and transformation of energy at each step.

3: The Earth has more rotational kinetic energy now than did the cloud of gas and dust from which it formed. Where did this energy come from?

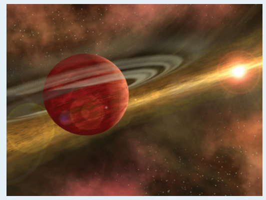

Figure 8. An immense cloud of rotating gas and dust contracted under the influence of gravity to form the Earth and in the process rotational kinetic energy increased. (credit: NASA)

Problems & Exercises

1: This problem considers energy and work aspects of Chapter 10.3 Example 1—use data from that example as needed. (a) Calculate the rotational kinetic energy in the merry-go-round plus child when they have an angular velocity of 20.0 rpm. (b) Using energy considerations, find the number of revolutions the father will have to push to achieve this angular velocity starting from rest. (c) Again, using energy considerations, calculate the force the father must exert to stop the merry-go-round in two revolutions

2: What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?

3: (a) Calculate the rotational kinetic energy of Earth on its axis. (b) What is the rotational kinetic energy of Earth in its orbit around the Sun?

4: Calculate the rotational kinetic energy in the motorcycle wheel (Figure 6) if its angular velocity is 120 rad/s. Assume M = 12.0 kg, R_1 = 0.280 m, and R_2 = 0.330 m.

5: A baseball pitcher throws the ball in a motion where there is rotation of the forearm about the elbow joint as well as other movements. If the linear velocity of the ball relative to the elbow joint is 20.0 m/s at a distance of 0.480 m from the joint and the moment of inertia of the forearm is $0.500\ \text{kg}\cdot\text{m}^2$, what is the rotational kinetic energy of the forearm?

6: While punting a football, a kicker rotates his leg about the hip joint. The moment of inertia of the leg is $3.75\ \text{kg}\cdot\text{m}^2$ and its rotational kinetic energy is 175 J. (a) What is the angular velocity of the leg? (b) What is the velocity of tip of the punter's shoe if it is 1.05 m from the hip joint? (c) Explain how the football can be given a velocity greater than the tip of the shoe (necessary for a decent kick distance).

7: A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg. (a) Calculate the angular velocity the flywheel must have to contain enough energy to take the bus from rest to a speed of 20.0 m/s, assuming 90.0% of the rotational kinetic energy can be transformed into translational energy. (b) How high a hill can the bus climb with this stored energy and still have a speed of 3.00 m/s at the top of the hill? Explicitly show how you follow the steps in the Problem-Solving Strategy for Rotational Energy.

8: A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches. (b) Repeat the calculation for the same ball if it slides up the hill without rolling.

9: While exercising in a fitness center, a man lies face down on a bench and lifts a weight with one lower leg by contacting the muscles in the back of the upper leg. (a) Find the angular acceleration produced given the mass lifted is 10.0 kg at a distance of 28.0 cm from the knee joint, the moment of inertia of the lower leg is $0.900\ \text{kg}\cdot\text{m}^2$, the muscle force is 1500 N, and its effective perpendicular lever arm is 3.00 cm. (b) How much work is done if the leg rotates through an angle of 20.0° with a constant force exerted by the muscle?

10: To develop muscle tone, a woman lifts a 2.00-kg weight held in her hand. She uses her biceps muscle to flex the lower arm through an angle of 60.0°. (a) What is the angular acceleration if the weight is 24.0 cm from the elbow joint, her forearm has a moment of inertia of $0.250\ \text{kg}\cdot\text{m}^2$, and the net force she exerts is 750 N at an effective perpendicular lever arm of 2.00 cm? (b) How much work does she do?

11: Consider two cylinders that start down identical inclines from rest except that one is frictionless. Thus one cylinder rolls without slipping, while the other slides frictionlessly without rolling. They both travel a short distance at the bottom and then start up another incline. (a) Show that they both reach the same height on the other incline, and that this height is equal to their original height. (b) Find the ratio of the time the rolling cylinder takes to reach the height on the second incline to the time the sliding cylinder takes to reach the height on the second incline. (c) Explain why the time for the rolling motion is greater than that for the sliding motion.

12: What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest, and has a final velocity of 6.00 m/s? Express the moment of inertia as a multiple of MR^2, where M is the mass of the object and R is its radius.

13: Suppose a 200-kg motorcycle has two wheels like, the one described in Problem 10.15 and is heading toward a hill at a speed of 30.0 m/s. (a) How high can it coast up the hill, if you neglect friction? (b) How much energy is lost to friction if the motorcycle only gains an altitude of 35.0 m before coming to rest?

14: In softball, the pitcher throws with the arm fully extended (straight at the elbow). In a fast pitch the ball leaves the hand with a speed of 139 km/h. (a) Find the rotational kinetic energy of the pitcher's arm given its moment of inertia is $0.720\ \text{kg}\cdot\text{m}^2$ and the ball leaves the hand at a distance of 0.600 m from the pivot at the shoulder. (b) What force did the muscles exert to cause the arm to rotate if their effective perpendicular lever arm is 4.00 cm and the ball is 0.156 kg?

15: Construct Your Own Problem

Consider the work done by a spinning skater pulling her arms in to increase her rate of spin. Construct a problem in which you calculate the work done with a "force multiplied by distance" calculation and compare it to the skater's increase in kinetic energy.

Glossary

work-energy theorem

if one or more external forces act upon a rigid object, causing its kinetic energy to change from KE_1 *to* KE_2, *then the work* W *done by the net force is equal to the change in kinetic energy*

rotational kinetic energy

the kinetic energy due to the rotation of an object. This is part of its total kinetic energy

Solutions

Check Your Understanding

1: Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

Problems & Exercises

1:

(a) 185 J

(b) 0.0785 rev

(c) $W = 9.81\ \text{N}$

3:

(a) $2.57 \times 10^{29}\ \text{J}$

(b) $\text{KE}_{\text{rot}} = 2.65 \times 10^{33}\ \text{J}$

5:

$\text{KE}_{\text{rot}} = 434\ \text{J}$

7:

(a) $128\ \text{rad/s}$

(b) $19.9\ \text{m}$

9:

(a) $10.4\ \text{rad/s}^2$

(b) $\text{net}\ W = 6.11\ \text{J}$

14:

(a) 1.49 kJ

(b) $2.52 \times 10^{4}\ \text{N}$

10.5 Angular Momentum and Its Conservation

Summary

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

Why does Earth keep on spinning? What started it spinning to begin with? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster? Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

By now the pattern is clear—every rotational phenomenon has a direct translational analog. It seems quite reasonable, then, to define **angular momentum** L as

$$L = I\omega.$$

This equation is an analog to the definition of linear momentum as $p = mv$. Units for linear momentum are $\text{kg} \cdot \text{m/s}$ while units for angular momentum are $\text{kg} \cdot \text{m}^2/\text{s}$. As we would expect, an object that has a large moment of inertia I, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

MAKING CONNECTIONS

Angular momentum is completely analogous to linear momentum, first presented in Chapter 6 Uniform Circular Motion and Gravitation. It has the same implications in terms of carrying rotation forward, and it is conserved when the net external torque is zero. Angular momentum, like linear momentum, is also a property of the atoms and subatomic particles.

Example 1: Calculating Angular Momentum of the Earth

Strategy

No information is given in the statement of the problem; so we must look up pertinent data before we can calculate $L = I\omega$. First, according to Chapter 10.3 Making Connections, the formula for the moment of inertia of a sphere is

$$I = \frac{2MR^2}{5}$$

so that

$$L = I\omega = \frac{2MR^2\omega}{5}.$$

Earth's mass M is $5.979 \times 10^{24}\ \text{kg}$ and its radius R is $6.376 \times 10^6\ \text{m}$. The Earth's angular velocity ω is, of course, exactly one revolution per day, but we must covert ω to radians per second to do the calculation in SI units.

Solution

Substituting known information into the expression for L and converting ω to radians per second gives

$$\begin{aligned} L &= 0.4(5.979 \times 10^{24}\ \text{kg})(6.376 \times 10^6\ \text{m})^2\left(\frac{1\ \text{rev}}{\text{d}}\right) \\ &= 9.72 \times 10^{37}\ \text{kg} \cdot \text{m}^2 \cdot \text{rev/d}. \end{aligned}$$

Substituting 2π rad for 1 rev and $8.64 \times 10^4\ \text{s}$ for 1 day gives

$$\begin{aligned} L &= (9.72 \times 10^{37}\ \text{kg} \cdot \text{m}^2)\left(\frac{2\pi\ \text{rad/rev}}{8.64 \times 10^4\ \text{s/d}}\right)(1\ \text{rev/d}) \\ &= 7.07 \times 10^{33}\ \text{kg} \cdot \text{m}^2/\text{s}. \end{aligned}$$

Discussion

This number is large, demonstrating that Earth, as expected, has a tremendous angular momentum. The answer is approximate, because we have assumed a constant density for Earth in order to estimate its moment of inertia.

When you push a merry-go-round, spin a bike wheel, or open a door, you exert a torque. If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases. The greater the net torque, the more rapid the increase in L. The relationship between torque and angular momentum is

$$\text{net } \tau = \frac{\Delta L}{\Delta t}.$$

This expression is exactly analogous to the relationship between force and linear momentum, $F = \Delta p / \Delta t$. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ is very fundamental and broadly applicable. It is, in fact, the rotational form of Newton's second law.

Example 2: Calculating the Torque Putting Angular Momentum Into a Lazy Susan

Figure 1 shows a Lazy Susan food tray being rotated by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?

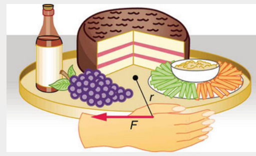

Figure 1. A partygoer exerts a torque on a lazy Susan to make it rotate. The equation **net τ=Δ*L*/Δ*t*** gives the relationship between torque and the angular momentum produced.

Strategy

We can find the angular momentum by solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL, and using the given information to calculate the torque. The final angular momentum equals the change in angular momentum, because the lazy Susan starts from rest. That is, $\Delta L = L$. To find the final velocity, we must calculate ω from the definition of L in $L = I\omega$.

Solution for (a)

Solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL gives

$$\Delta L = \text{net } \tau \Delta t.$$

Because the force is perpendicular to r, we see that $\text{net } \tau = rF$, so that

$$\begin{aligned} L &= rF\Delta t = (0.260\text{ m})(2.50\text{ N})(0.150\text{ s}) \\ &= 9.75 \times 10^{-2}\text{ kg}\cdot\text{m}^2/\text{s}. \end{aligned}$$

Solution for (b)

The final angular velocity can be calculated from the definition of angular momentum,

$$L = I\omega.$$

Solving for ω and substituting the formula for the moment of inertia of a disk into the resulting equation gives

$$\omega = \frac{L}{I} = \frac{L}{\frac{1}{2}MR^2}.$$

And substituting known values into the preceding equation yields

$$\omega = \frac{9.75 \times 10^{-2}\text{ kg}\cdot\text{m}^2/\text{s}}{(0.500)(4.00\text{ kg})(0.260\text{ m})} = 0.721\text{ rad/s}.$$

Discussion

Note that the imparted angular momentum does not depend on any property of the object but only on torque and time. The final angular velocity is equivalent to one revolution in 8.71 s (determination of the time period is left as an exercise for the reader), which is about right for a lazy Susan.

Example 3: Calculating the Torque in a Kick

The person whose leg is shown in Figure 2 kicks his leg by exerting a 2000-N force with his upper leg muscle. The effective perpendicular lever arm is 2.20 cm. Given the moment of inertia of the lower leg is $1.25\text{ kg}\cdot\text{m}^2$, (a) find the angular acceleration of the leg. (b) Neglecting the gravitational force, what is the rotational kinetic energy of the leg after it has rotated through 57.3° (1.00 rad)?

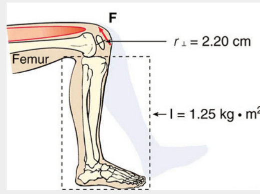

Figure 2. The muscle in the upper leg gives the lower leg an angular acceleration and imparts rotational kinetic energy to it by exerting a torque about the knee. **F** is a vector that is perpendicular to ***r***. This example examines the situation.

Strategy

The angular acceleration can be found using the rotational analog to Newton's second law, or $\alpha = \text{net } \tau / I$. The moment of inertia I is given and the torque can be found easily from the given force and perpendicular lever arm. Once the angular acceleration α is known, the final angular velocity and rotational kinetic energy can be calculated.

Solution to (a)

From the rotational analog to Newton's second law, the angular acceleration α is

$$\alpha = \frac{\text{net } \tau}{I}.$$

Because the force and the perpendicular lever arm are given and the leg is vertical so that its weight does not create a torque, the net torque is thus

$$\begin{aligned} \text{net } \tau &= r_\perp F \\ &= (0.0220 \text{ m})(2000 \text{ N}) \\ &= 44.0 \text{ N} \cdot \text{m}. \end{aligned}$$

Substituting this value for the torque and the given value for the moment of inertia into the expression for α gives

$$\alpha = \frac{44.0 \text{ N}\cdot\text{m}}{1.25 \text{ kg}\cdot\text{m}^2} = 35.2 \text{ rad/s}^2.$$

Solution to (b)

The final angular velocity can be calculated from the kinematic expression

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

or

$$\omega^2 = 2\alpha\theta$$

because the initial angular velocity is zero. The kinetic energy of rotation is

$$\text{KE}_{\text{rot}} = \frac{1}{2} I \omega^2$$

so it is most convenient to use the value of ω^2 just found and the given value for the moment of inertia. The kinetic energy is then

$$\begin{aligned} \text{KE}_{\text{rot}} &= 0.5(1.25 \text{ kg}\cdot\text{m}^2)(70.4 \text{ rad}^2/\text{s}^2) \\ &= 44.0 \text{ J}. \end{aligned}$$

Discussion

These values are reasonable for a person kicking his leg starting from the position shown. The weight of the leg can be neglected in part (a) because it exerts no torque when the center of gravity of the lower leg is directly beneath the pivot in the knee. In part (b), the force exerted by the upper leg is so large that its torque is much greater than that created by the weight of the lower leg as it rotates. The rotational kinetic energy given to the lower leg is enough that it could give a ball a significant velocity by transferring some of this energy in a kick.

MAKING CONNECTIONS: CONSERVATION LAWS

Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conservation of Angular Momentum

We can now understand why Earth keeps on spinning. As we saw in the previous example, $\Delta L = (\text{net } \tau)\Delta t$. This equation means that, to change angular momentum, a torque must act over some period of time. Because Earth has a large angular momentum, a large torque acting over a long time is needed to change its rate of spin. So what external torques are there? Tidal friction exerts torque that is slowing Earth's rotation, but tens of millions of years must pass before the change is very significant. Recent research indicates the length of the day was 18 h some 900 million years ago. Only the tides exert significant retarding torques on Earth, and so it will continue to spin, although ever more slowly, for many billions of years.

What we have here is, in fact, another conservation law. If the net torque is *zero*, then angular momentum is constant or *conserved*. We can see this rigorously by considering $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for the situation in which the net torque is zero. In that case,

$$\text{net} \tau = 0$$

implying that

$$\frac{\Delta L}{\Delta t} = 0 .$$

If the change in angular momentum ΔL is zero, then the angular momentum is constant; thus,

$$L = \text{constant} \;\; (\text{net } \tau = 0)$$

or

$$L = L'(\text{net } \tau = 0).$$

These expressions are the **law of conservation of angular momentum**. Conservation laws are as scarce as they are important.

An example of conservation of angular momentum is seen in Figure 3, in which an ice skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice and because the friction is exerted very close to the pivot point. (Both F and r are small, and so τ is negligibly small.) Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that

$$L = L'.$$

Expressing this equation in terms of the moment of inertia,

$$I\omega = I'\omega',$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity ω' must increase to keep the angular momentum constant. The change can be dramatic, as the following example shows.

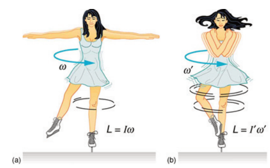

Figure 3. (a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

Example 4: Calculating the Angular Momentum of a Spinning Skater

Suppose an ice skater, such as the one in Figure 3, is spinning at 0.800 rev/ s with her arms extended. She has a moment of inertia of $2.34\ \text{kg}\cdot\text{m}^2$ with her arms extended and of $0.363\ \text{kg}\cdot\text{m}^2$ with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

Strategy

In the first part of the problem, we are looking for the skater's angular velocity ω' after she has pulled in her arms. To find this quantity, we use the conservation of angular momentum and note that the moments of inertia and initial angular velocity are given. To find the initial and final kinetic energies, we use the definition of rotational kinetic energy given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

Solution for (a)

Because torque is negligible (as discussed above), the conservation of angular momentum given in $I\omega = I'\omega'$ is applicable. Thus,

$$L = L'$$

or

$$I\omega = I'\omega'$$

Solving for ω' and substituting known values into the resulting equation gives

$$\begin{aligned} \omega' &= \frac{I}{I'}\omega = \left(\frac{2.34\ \text{kg}\cdot\text{m}^2}{0.363\ \text{kg}\cdot\text{m}^2}\right)(0.800\ \text{rev/s}) \\ &= 5.16\ \text{rev/s}. \end{aligned}$$

Solution for (b)

Rotational kinetic energy is given by

$$\mathrm{KE_{rot}} = \frac{1}{2} I \omega^2.$$

The initial value is found by substituting known values into the equation and converting the angular velocity to rad/s:

$$\begin{aligned} \mathrm{KE_{rot}} &= (0.5)(2.34\ \mathrm{kg \cdot m^2})((0.800\ \mathrm{rev/s})(2\pi\ \mathrm{rad/rev}))^2 \\ &= 29.6\ \mathrm{J}. \end{aligned}$$

The final rotational kinetic energy is

$$\mathrm{KE'_{rot}} = \frac{1}{2} I' \omega'^2$$

Substituting known values into this equation gives

$$\begin{aligned} \mathrm{KE'_{rot}} &= (0.5)(0.363\ \mathrm{kg \cdot m^2})[(5.16\ \mathrm{rev/s})(2\pi\ \mathrm{rad/rev})]^2 \\ &= 191\ \mathrm{J}. \end{aligned}$$

Discussion

In both parts, there is an impressive increase. First, the final angular velocity is large, although most world-class skaters can achieve spin rates about this great. Second, the final kinetic energy is much greater than the initial kinetic energy. The increase in rotational kinetic energy comes from work done by the skater in pulling in her arms. This work is internal work that depletes some of the skater's food energy.

There are several other examples of objects that increase their rate of spin because something reduced their moment of inertia. Tornadoes are one example. Storm systems that create tornadoes are slowly rotating. When the radius of rotation narrows, even in a local region, angular velocity increases, sometimes to the furious level of a tornado. Earth is another example. Our planet was born from a huge cloud of gas and dust, the rotation of which came from turbulence in an even larger cloud. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result. (See Figure 4.)

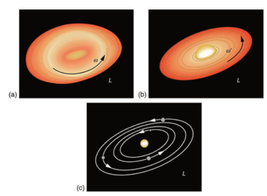

Figure 4. The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

Check Your Understanding

1: Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

Section Summary

- Every rotational phenomenon has a direct translational analog , likewise angular momentum L can be defined as $L = I\omega$.
- This equation is an analog to the definition of linear momentum as $p = mv$. The relationship between torque and angular momentum is $\text{net } \tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conceptual Questions

1: When you start the engine of your car with the transmission in neutral, you notice that the car rocks in the opposite sense of the engine's rotation. Explain in terms of conservation of angular momentum. Is the angular momentum of the car conserved for long (for more than a few seconds)?

2: Suppose a child walks from the outer edge of a rotating merry-go round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.

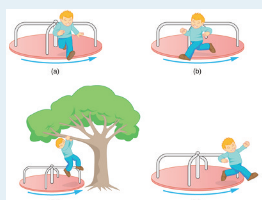

Figure 5. A child may jump off a merry-go-round in a variety of directions.

3: Suppose a child gets off a rotating merry-go-round. Does the angular velocity of the merry-go-round increase, decrease, or remain the same if: (a) He jumps off radially? (b) He jumps backward to land motionless? (c) He jumps straight up and hangs onto an overhead tree branch? (d) He jumps off forward, tangential to the edge? Explain your answers. (Refer to Figure 5).

4: Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton's third law why the helicopter body rotates in the opposite direction to the blades.

5: Whenever a helicopter has two sets of lifting blades, they rotate in opposite directions (and there will be no tail propeller). Explain why it is best to have the blades rotate in opposite directions.

6: Describe how work is done by a skater pulling in her arms during a spin. In particular, identify the force she exerts on each arm to pull it in and the distance each moves, noting that a component of the force is in the direction moved. Why is angular momentum not increased by this action?

7: When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.

8: Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?

9: Jet turbines spin rapidly. They are designed to fly apart if something makes them seize suddenly, rather than transfer angular momentum to the plane's wing, possibly tearing it off. Explain how flying apart conserves angular momentum without transferring it to the wing.

10: An astronaut tightens a bolt on a satellite in orbit. He rotates in a direction opposite to that of the bolt, and the satellite rotates in the same direction as the bolt. Explain why. If a handhold is available on the satellite, can this counter-rotation be prevented? Explain your answer.

11: Competitive divers pull their limbs in and curl up their bodies when they do flips. Just before entering the water, they fully extend their limbs to enter straight down. Explain the effect of both actions on their angular velocities. Also explain the effect on their angular momenta.

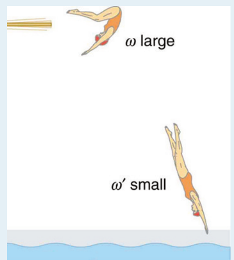

Figure 6. The diver spins rapidly when curled up and slows when she extends her limbs before entering the water.

12: Draw a free body diagram to show how a diver gains angular momentum when leaving the diving board.

13: In terms of angular momentum, what is the advantage of giving a football or a rifle bullet a spin when throwing or releasing it?

Figure 7. The image shows a view down the barrel of a cannon, emphasizing its rifling. Rifling in the barrel of a canon causes the projectile to spin just as is the case for rifles (hence the name for the grooves in the barrel). (credit: Elsie esq., Flickr)

Problems & Exercises

1: (a) Calculate the angular momentum of the Earth in its orbit around the Sun.

(b) Compare this angular momentum with the angular momentum of Earth on its axis.

2: (a) What is the angular momentum of the Moon in its orbit around Earth?

(b) How does this angular momentum compare with the angular momentum of the Moon on its axis? Remember that the Moon keeps one side toward Earth at all times.

(c) Discuss whether the values found in parts (a) and (b) seem consistent with the fact that tidal effects with Earth have caused the Moon to rotate with one side always facing Earth.

3: Suppose you start an antique car by exerting a force of 300 N on its crank for 0.250 s. What angular momentum is given to the engine if the handle of the crank is 0.300 m from the pivot and the force is exerted to create maximum torque the entire time?

4: A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.

5: Three children are riding on the edge of a merry-go-round that is 100 kg, has a 1.60-m radius, and is spinning at 20.0 rpm. The children have masses of 22.0, 28.0, and 33.0 kg. If the child who has a mass of 28.0 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm?

6: (a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is $0.400 \text{ kg} \cdot \text{m}^2$. (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?

7: Construct Your Own Problem
Consider the Earth-Moon system. Construct a problem in which you calculate the total angular momentum of the system including the spins of the Earth and the Moon on their axes and the orbital angular momentum of the Earth-Moon system in its nearly monthly rotation. Calculate what happens to the Moon's orbital radius if the Earth's rotation decreases due to tidal drag. Among the things to be considered are the amount by which the Earth's rotation slows and the fact that the Moon will continue to have one side always facing the Earth.

Glossary

angular momentum
the product of moment of inertia and angular velocity
law of conservation of angular momentum
angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system

Solutions

Check Your Understanding

1: Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

Problems & Exercises

1:

(a) $2.66 \times 10^{40} \text{ kg} \cdot \text{m}^2/\text{s}$

(b) $7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}$

The angular momentum of the Earth in its orbit around the Sun is 3.77×10^6 times larger than the angular momentum of the Earth around its axis.

3:

$22.5 \text{ kg} \cdot \text{m}^2/\text{s}$

5:

25.3 rpm

10.6 Collisions of Extended Bodies in Two Dimensions

Summary

- Observe collisions of extended bodies in two dimensions.
- Examine collision at the point of percussion.

Bowling pins are sent flying and spinning when hit by a bowling ball—angular momentum as well as linear momentum and energy have been imparted to the pins. (See Figure 1). Many collisions involve angular momentum. Cars, for example, may spin and collide on ice or a wet surface. Baseball pitchers throw curves by putting spin on the baseball. A tennis player can put a lot of top spin on the tennis ball which causes it to dive down onto the court once it crosses the net. We now take a brief look at what happens when objects that can rotate collide.

Consider the relatively simple collision shown in Figure 2, in which a disk strikes and adheres to an initially motionless stick nailed at one end to a frictionless surface. After the collision, the two rotate about the nail. There is an unbalanced external force on the system at the nail. This force exerts no torque because its lever arm is zero. Angular momentum is therefore conserved in the collision. Kinetic energy is not conserved, because the collision is inelastic. It is possible that momentum is not conserved either because the force at the nail may have a component in the direction of the disk's initial velocity. Let us examine a case of rotation in a collision in Example 1.

Figure 1. The bowling ball causes the pins to fly, some of them spinning violently. (credit: Tinou Bao, Flickr)

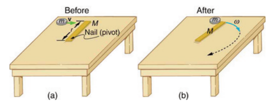

Figure 2. (a) A disk slides toward a motionless stick on a frictionless surface. (b) The disk hits the stick at one end and adheres to it, and they rotate together, pivoting around the nail. Angular momentum is conserved for this inelastic collision because the surface is frictionless and the unbalanced external force at the nail exerts no torque.

Example 1: Rotation in a Collision

Suppose the disk in Figure 2 has a mass of 50.0 g and an initial velocity of 30.0 m/s when it strikes the stick that is 1.20 m long and 2.00 kg.

(a) What is the angular velocity of the two after the collision?

(b) What is the kinetic energy before and after the collision?

(c) What is the total linear momentum before and after the collision?

Strategy for (a)

We can answer the first question using conservation of angular momentum as noted. Because angular momentum is $I\omega$, we can solve for angular velocity.

Solution for (a)

Conservation of angular momentum states

$$L = L',$$

where primed quantities stand for conditions after the collision and both momenta are calculated relative to the pivot point. The initial angular momentum of the system of stick-disk is that of the disk just before it strikes the stick. That is,

$$L = I\omega,$$

where I is the moment of inertia of the disk and ω is its angular velocity around the pivot point. Now, $I = mr^2$ (taking the disk to be approximately a point mass) and $\omega = v/r$, so that

$$L = mr^2 \frac{v}{r} = mvr.$$

After the collision,

$$L' = I'\omega'.$$

It is ω' that we wish to find. Conservation of angular momentum gives

$$I'\omega' = mvr.$$

Rearranging the equation yields

$$\omega' = \frac{mvr}{I'},$$

where I' is the moment of inertia of the stick and disk stuck together, which is the sum of their individual moments of inertia about the nail. Chapter 10.3 Making Connections gives the formula for a rod rotating around one end to be $I = Mr^2/3$. Thus,

$$I' = mr^2 + \frac{Mr^2}{3} = \left(m + \frac{M}{3}\right)r^2.$$

Entering known values in this equation yields,

$$I' = (0.0500\ \text{kg} + 0.667\ \text{kg})(1.20\ \text{m})^2 = 1.032\ \text{kg}\cdot\text{m}^2.$$

The value of I' is now entered into the expression for ω', which yields

$$\begin{aligned}\omega' &= \frac{mvr}{I'} = \frac{(0.0500\ \text{kg})(30.0\ \text{m/s})(1.20\ \text{m})}{1.032\ \text{kg}\cdot\text{m}^2} \\ &= 1.744\ \text{rad/s} \approx 1.74\ \text{rad/s}.\end{aligned}$$

Strategy for (b)

The kinetic energy before the collision is the incoming disk's translational kinetic energy, and after the collision, it is the rotational kinetic energy of the two stuck together.

Solution for (b)

First, we calculate the translational kinetic energy by entering given values for the mass and speed of the incoming disk.

$$\text{KE} = \frac{1}{2}mv^2 = (0.500)(0.0500\ \text{kg})(30.0\ \text{m/s})^2 = 22.5\ \text{J}$$

After the collision, the rotational kinetic energy can be found because we now know the final angular velocity and the final moment of inertia. Thus, entering the values into the rotational kinetic energy equation gives

$$\begin{aligned}\text{KE}' &= \frac{1}{2}I'\omega'^2 = (0.5)(1.032\ \text{kg}\cdot\text{m}^2)\left(1.744\ \frac{\text{rad}}{\text{s}}\right)^2 \\ &= 1.57\ \text{J}.\end{aligned}$$

Strategy for (c)

The linear momentum before the collision is that of the disk. After the collision, it is the sum of the disk's momentum and that of the center of mass of the stick.

Solution of (c)

Before the collision, then, linear momentum is

$$p = mv = (0.0500\ \text{kg})(30.0\ \text{m/s}) = 1.50\ \text{kg}\cdot\text{m/s}.$$

After the collision, the disk and the stick's center of mass move in the same direction. The total linear momentum is that of the disk moving at a new velocity $v' = r\omega'$ plus that of the stick's center of mass,

which moves at half this speed because $v_{\text{CM}} = \left(\frac{r}{2}\right)\omega' = \frac{v'}{2}$. Thus,

$$p' = mv' + Mv_{\text{CM}} = mv' + \frac{Mv'}{2}.$$

Gathering similar terms in the equation yields,

$$p' = \left(m + \frac{M}{2}\right)v'$$

so that

$$p' = \left(m + \frac{M}{2}\right)r\omega'.$$

Substituting known values into the equation,

$$p' = (1.050\ \text{kg})(1.20\ \text{m})(1.744\ \text{rad/s}) = 2.20\ \text{kg}\cdot\text{m/s}.$$

Discussion

First note that the kinetic energy is less after the collision, as predicted, because the collision is inelastic. More surprising is that the momentum after the collision is actually greater than before the collision. This result can be understood if you consider how the nail affects the stick and vice versa. Apparently, the stick pushes backward on the nail when first struck by the disk. The nail's reaction (consistent with Newton's third law) is to push forward on the stick, imparting momentum to it in the same direction in which the disk was initially moving, thereby increasing the momentum of the system.

The above example has other implications. For example, what would happen if the disk hit very close to the nail? Obviously, a force would be exerted on the nail in the forward direction. So, when the stick is struck at the end farthest from the nail, a backward force is exerted on the nail, and when it is hit at the end nearest the nail, a forward force is exerted on the nail. Thus, striking it at a certain point in between produces no force on the nail. This intermediate point is known as the *percussion point*.

An analogous situation occurs in tennis as seen in Figure 3. If you hit a ball with the end of your racquet, the handle is pulled away from your hand. If you hit a ball much farther down, for example, on the shaft of the racquet, the handle is pushed into your palm. And if you hit the ball at the racquet's percussion point (what some people call the "sweet spot"), then little or *no* force is exerted on your hand, and there is less vibration, reducing chances of a tennis elbow. The same effect occurs for a baseball bat.

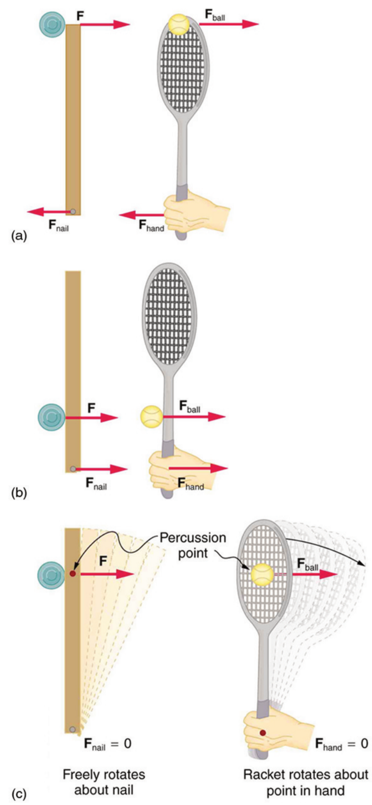

Figure 3. A disk hitting a stick is compared to a tennis ball being hit by a racquet. (a) When the ball strikes the racquet near the end, a backward force is exerted on the hand. (b) When the racquet is struck much farther down, a forward force is exerted on the hand. (c) When the racquet is struck at the percussion point, no force is delivered to the hand.

Check Your Understanding

1: Is rotational kinetic energy a vector? Justify your answer.

Section Summary

- Angular momentum L is analogous to linear momentum and is given by $L = I\omega$.

- Angular momentum is changed by torque, following the relationship $\text{net }\tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum is conserved if the net torque is zero $L = \text{constant}\ (\text{net }\tau = 0)$ or $L = L'(\text{net }\tau = 0)$. This equation is known as the law of conservation of angular momentum, which may be conserved in collisions.

Conceptual Questions

1: Describe two different collisions—one in which angular momentum is conserved, and the other in which it is not. Which condition determines whether or not angular momentum is conserved in a collision?

2: Suppose an ice hockey puck strikes a hockey stick that lies flat on the ice and is free to move in any direction. Which quantities are likely to be conserved: angular momentum, linear momentum, or kinetic energy (assuming the puck and stick are very resilient)?

3: While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

Problems & Exercises

Problems & Exercises

1: Repeat Example 1 in which the disk strikes and adheres to the stick 0.100 m from the nail.

2: Repeat Example 1 in which the disk originally spins clockwise at 1000 rpm and has a radius of 1.50 cm.

3: Twin skaters approach one another as shown in Figure 4 and lock hands. (a) Calculate their final angular velocity, given each had an initial speed of 2.50 m/s relative to the ice. Each has a mass of 70.0 kg, and each has a center of mass located 0.800 m from their locked hands. You may approximate their moments of inertia to be that of point masses at this radius. (b) Compare the initial kinetic energy and final kinetic energy.

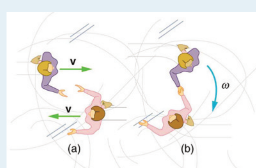

Figure 4. Twin skaters approach each other with identical speeds. Then, the skaters lock hands and spin.

4: Suppose a 0.250-kg ball is thrown at 15.0 m/s to a motionless person standing on ice who catches it with an outstretched arm as shown in Figure 5.

(a) Calculate the final linear velocity of the person, given his mass is 70.0 kg.

(b) What is his angular velocity if each arm is 5.00 kg? You may treat the ball as a point mass and treat the person's arms as uniform rods (each has a length of 0.900 m) and the rest of his body as a uniform cylinder of radius 0.180 m. Neglect the effect of the ball on his center of mass so that his center of mass remains in his geometrical center.

(c) Compare the initial and final total kinetic energies.

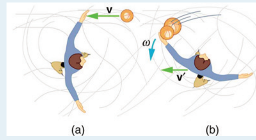

Figure 5. The figure shows the overhead view of a person standing motionless on ice about to catch a ball. Both arms are outstretched. After catching the ball, the skater recoils and rotates.

5: Repeat Example 1 in which the stick is free to have translational motion as well as rotational motion.

Solutions

Check Your Understanding

1: No, energy is always scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

Problems & Exercises

1:

(a) 0.156 rad/s

(b) $1.17 \times 10^{-2} \text{ J}$

(c) $0.188 \text{ kg}\cdot\text{m/s}$

3:

(a) 3.13 rad/s

(b) Initial KE = 438 J, final KE = 438 J

5:

(a) 1.70 rad/s

(b) Initial KE = 22.5 J, final KE = 2.04 J

(c) $1.50 \text{ kg}\cdot\text{m/s}$

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

Summary

- Describe the right-hand rule to find the direction of angular velocity, momentum, and torque.
- Explain the gyroscopic effect.
- Study how Earth acts like a gigantic gyroscope.

Angular momentum is a vector and, therefore, *has direction as well as magnitude*. Torque affects both the direction and the magnitude of angular momentum. What is the direction of the angular momentum of a rotating object like the disk in Figure 1? The figure shows the **right-hand rule** used to find the direction of both angular momentum and angular velocity. Both $\mathbf{L}$ and $\boldsymbol{\omega}$ are vectors—each has direction and magnitude. Both can be represented by arrows. The right-hand rule defines both to be perpendicular to the plane of rotation in the direction shown. Because angular momentum is related to angular velocity by $\mathbf{L} = I\boldsymbol{\omega}$, the direction of $\mathbf{L}$ is the same as the direction of $\boldsymbol{\omega}$. Notice in the figure that both point along the axis of rotation.

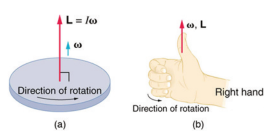

Figure 1. Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity **ω** size and angular momentum **L** are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation as shown.

Now, recall that torque changes angular momentum as expressed by

$$\text{net } \boldsymbol{\tau} = \frac{\Delta \mathbf{L}}{\Delta t}.$$

This equation means that the direction of $\Delta \mathbf{L}$ is the same as the direction of the torque $\boldsymbol{\tau}$ that creates it. This result is illustrated in Figure 2, which shows the direction of torque and the angular momentum it creates.

Let us now consider a bicycle wheel with a couple of handles attached to it, as shown in Figure 3. (This device is popular in demonstrations among physicists, because it does unexpected things.) With the wheel rotating as shown, its angular momentum is to the woman's left. Suppose the person holding the wheel tries to rotate it as in the figure. Her natural expectation is that the wheel will rotate in the direction she pushes it—but what happens is quite different. The forces exerted create a torque that is horizontal toward the person, as shown in Figure 3(a). This torque creates a change in angular momentum $\mathbf{L}$ in the same direction, perpendicular to the original angular momentum $\mathbf{L}$, thus changing the direction of $\mathbf{L}$ but not the magnitude of $\mathbf{L}$. Figure 3 shows how $\Delta \mathbf{L}$ and $\mathbf{L}$ add, giving a new angular momentum with direction that is inclined more toward the person than before. The axis of the wheel has thus moved *perpendicular to the forces exerted on it*, instead of in the expected direction.

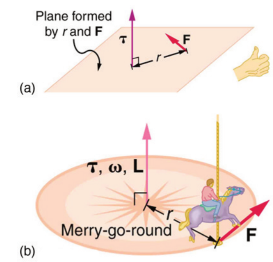

Figure 2. In figure (a), the torque is perpendicular to the plane formed by ***r*** and **F** and is the direction your right thumb would point to if you curled your fingers in the direction of **F**. Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.

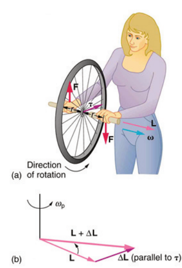

Figure 3. In figure (a), a person holding the spinning bike wheel lifts it with her right hand and pushes down with her left hand in an attempt to rotate the wheel. This action creates a torque directly toward her. This torque causes a change in angular momentum **ΔL** in exactly the same direction. Figure (b) shows a vector diagram depicting how **ΔL** and **L** add, producing a new angular momentum pointing more toward the person. The wheel moves toward the person, perpendicular to the forces she exerts on it.

This same logic explains the behavior of gyroscopes. Figure 4 shows the two forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the torque is changed, but not its magnitude. The gyroscope *precesses* around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is *not* spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta \mathbf{L}$), and it rotates around a horizontal axis, falling over just as we would expect.

Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star. But Earth is slowly precessing (once in about 26,000 years) due to the torque of the Sun and the Moon on its nonspherical shape.

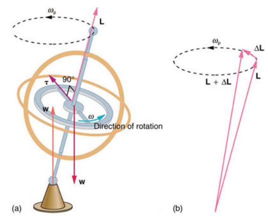

Figure 4. As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum **ΔL** that is also horizontal. In figure (b), **ΔL** and **L** add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

Check Your Understanding

1: Rotational kinetic energy is associated with angular momentum? Does that mean that rotational kinetic energy is a vector?

Section Summary

- Torque is perpendicular to the plane formed by $\mathbf{r}$ and $\mathbf{F}$ and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of $\mathbf{F}$. The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$), and it rotates about a horizontal axis, falling over just as we would expect.
- Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star.

Conceptual Questions

1: While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

2: Gyroscopes used in guidance systems to indicate directions in space must have an angular momentum that does not change in direction. Yet they are often subjected to large forces and accelerations. How can the direction of their angular momentum be constant when they are accelerated?

Problems & Exercises

1: Integrated Concepts

The axis of Earth makes a 23.5° angle with a direction perpendicular to the plane of Earth's orbit. As shown in Figure 5, this axis precesses, making one complete rotation in 25,780 y.

(a) Calculate the change in angular momentum in half this time.

(b) What is the average torque producing this change in angular momentum?

(c) If this torque were created by a single force (it is not) acting at the most effective point on the equator, what would its magnitude be?

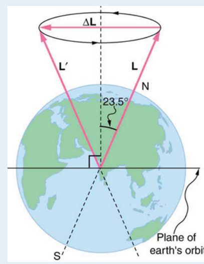

Figure 5. The Earth's axis slowly precesses, always making an angle of 23.5° with the direction perpendicular to the plane of Earth's orbit. The change in angular momentum for the two shown positions is quite large, although the magnitude **L** is unchanged.

Glossary

right-hand rule

direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation

Solutions

Check Your Understanding

1: No, energy is always a scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

Problems & Exercises

1:

(a) $5.64 \times 10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}$

(b) $1.39 \times 10^{22}\ \text{N}\cdot\text{m}$

(c) $2.17 \times 10^{15}\ \text{N}$

PART 11

Chapter 18 Electric Charge and Electric Field

Figure 1. Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See Figure 2.) In this experiment, Franklin demonstrated a connection between lightning and static electricity. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.

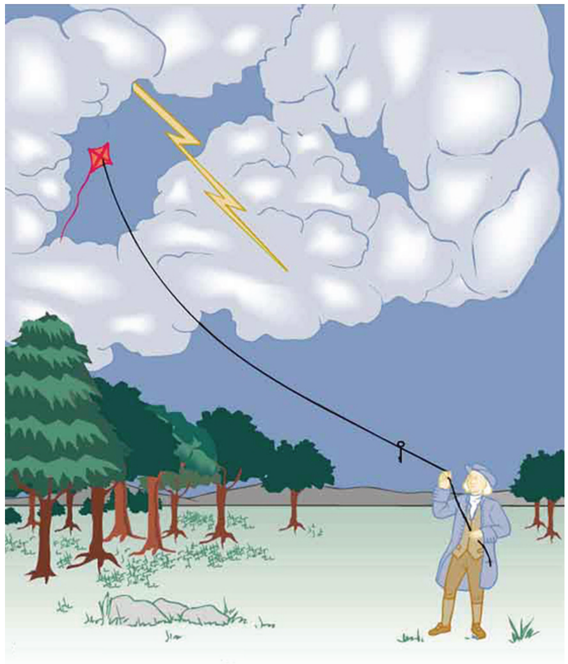

Figure 2. When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the electromagnetic force. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

Glossary

static electricity
a buildup of electric charge on the surface of an object
electromagnetic force
one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

18.1 Static Electricity and Charge: Conservation of Charge

Summary

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.

Figure 1. Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see Figure 1). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge "positive", and the other type "negative." For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. Figure 2 shows how these simple materials can be used to explore the nature of the force between charges.

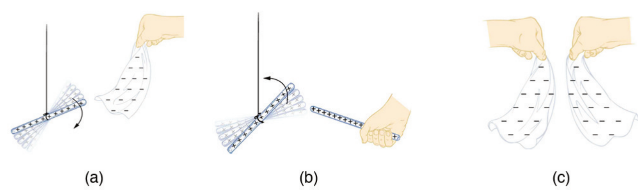

Figure 2. A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

Figure 3 shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.

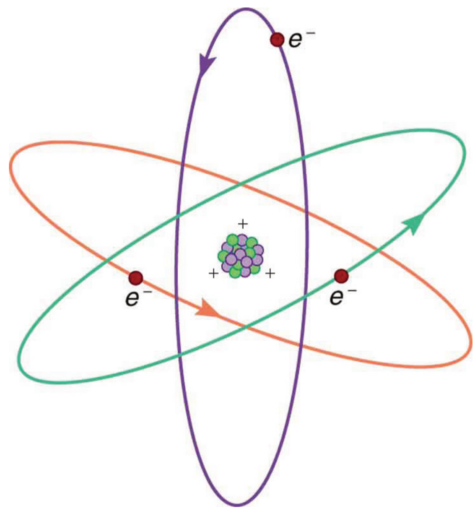

Figure 3. This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

$$|q_e| = 1.60 \times 10^{-19}\ \mathrm{C}.$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

$$1.00\ \mathrm{C} \times \frac{1\ \text{proton}}{1.60 \times 10^{-19}\ \mathrm{C}} = 6.25 \times 10^{18}\ \text{protons}.$$

Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see Things Great and Small: The Submicroscopic Origin of Charge), and all observed charges are integral multiples of $|q_e|$.

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See Figure 4.) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

Figure 4 shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.

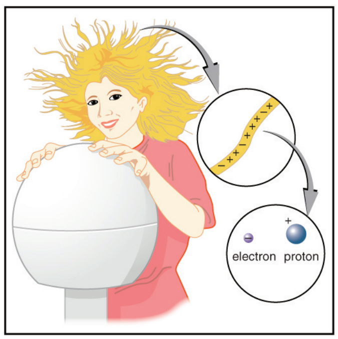

Figure 4. When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in Figure 5. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

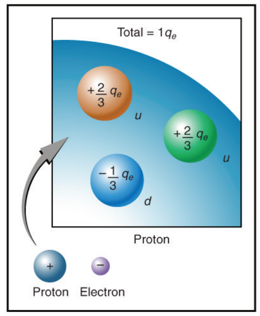

Figure 5. Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton: $-1/3q_e + 2/3q_e + 2/3q_e = +1q_e$.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See Figure 6.) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.

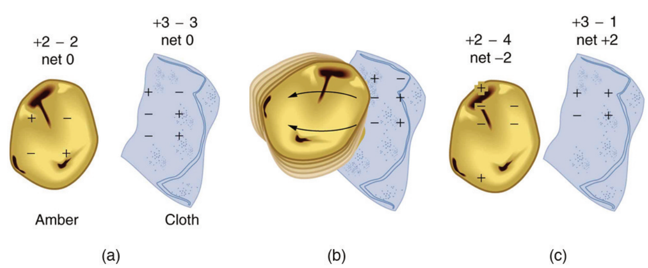

Figure 6. When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm, can be created from energy in the amount

$\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are "matter-antimatter" counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See Figure 7.) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy *E*, again obeying the relationship $\Delta m = fracEc^2$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.

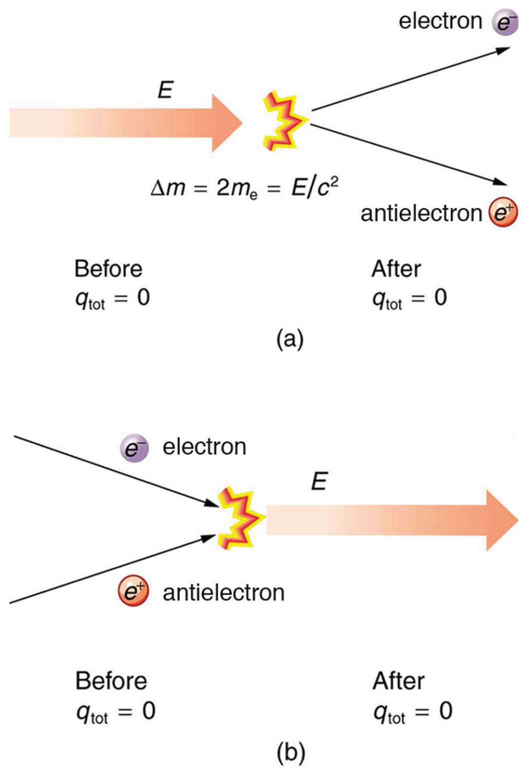

Figure 7. (a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

PhET Interactive Simulation

Figure 8. Balloons and Static Electricity

Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is
  $latex \boldsymbol{|q_e| = 1.60 \times 10^{-19} \;\textbf{C.} }
- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

Conceptual Questions

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

Problems & Exercises

1: Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\;\mu\text{C}$?

2: If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

3: To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

4: A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

Glossary

electric charge
a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

law of conservation of charge
states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

electron
a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

proton
a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

Solutions

1:

(a) 1.25×10^{10}

(b) 3.13×10^{12}

2:

-600 C

18.2 Conductors and Insulators

Summary

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.

Figure 1. This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as 10^{23} times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.

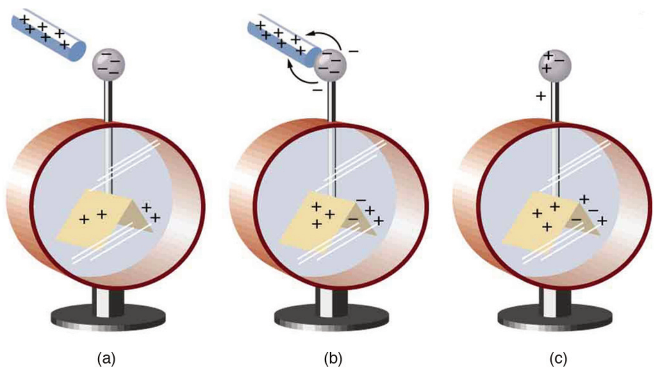

Figure 2. An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

Charging by Contact

Figure 2 shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

Electrostatic repulsion in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. Figure 3 shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world. A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in Figure 4. The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.

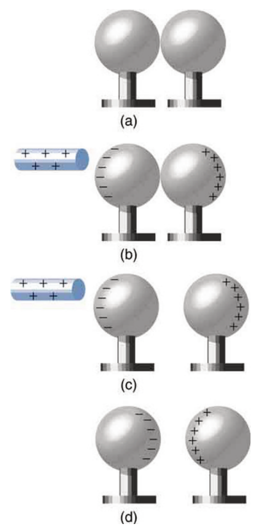

Figure 3. Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

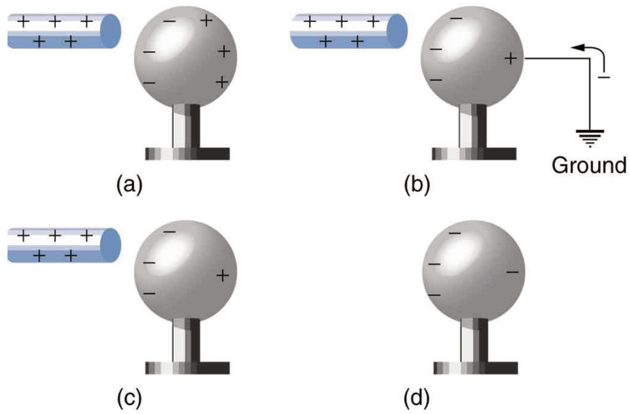

Figure 4. Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is removed, leaving the sphere with an induced negative charge.

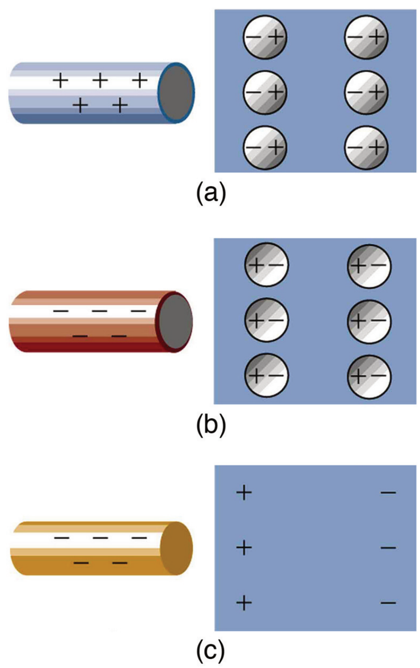

Figure 5. Both positive and negative objects attract a neutral object by polarizing its molecules. (a) A positive object brought near a neutral insulator polarizes its molecules. There is a slight shift in the distribution of the electrons orbiting the molecule, with unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. Figure 5 shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

Check Your Understanding

Can you explain the attraction of water to the charged rod in the figure below?

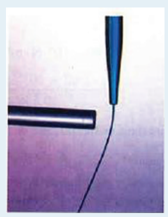

Figure 6.

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

John Travoltage

PhET Interactive Simulation

Figure 7.

Section Summary

- Polarization is the separation of positive and negative charges in a neutral object.

- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

Conceptual Questions

1: An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

2: If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

3: When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

4: Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

5: Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

6: What is grounding? What effect does it have on a charged conductor? On a charged insulator?

Problems & Exercises

1: Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of –5.00 nC (a very large charge for a small speck). How many electrons does it have?

2: An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

3: A 50.0 g ball of copper has a net charge of $2.00\ \mu C$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

4: What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in 10^{12} of its atoms? (Sulfur has an atomic mass of 32.1.)

5: How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

Glossary

free electron
an electron that is free to move away from its atomic orbit

conductor
a material that allows electrons to move separately from their atomic orbits

insulator
a material that holds electrons securely within their atomic orbits

grounded
when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

induction
the process by which an electrically charged object brought near a neutral object creates a charge in that object

polarization
slight shifting of positive and negative charges to opposite sides of an atom or molecule

electrostatic repulsion
the phenomenon of two objects with like charges repelling each other

Solutions

Check Your Understanding

Answer
Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

Problems & Exercises

1.03×10^{12}

9.09×10^{-13}

$1.48 \times 10^{8}\ \text{C}$

18.3 Coulomb's Law

Summary

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.

Figure 1. This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the electrostatic force—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called Coulomb's law after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r2}.$$

Coulomb's law calculates the magnitude of the force FF between two point charges, q_1 and q_2, separated by a distance r. In SI units, the constant k is equal to

$$k = 8.988 \times 10^9 \ \frac{N \cdot m^2}{C^2} \approx 8.99 \times 10^9 \ \frac{N \cdot m^2}{C^2}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See Figure 2.)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $(F \propto 1/r^2)$ to an accuracy of 1 part in 10^{16}. No exceptions have ever been found, even at the small distances within the atom.

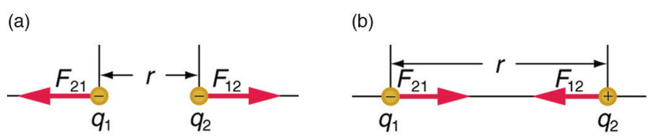

Figure 2. The magnitude of the electrostatic force ***F*** between point charges q_1 and q_2 separated by a distance ***r*** is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges. (b) Unlike charges.

How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by 0.530×10^{-10} m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k \frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

$$F = k \frac{|q_1 q_2|}{r2}$$
$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(0.530 \times 10^{-10} \text{ m})^2}$$

Thus the Coulomb force is

$$F = 8.19 \times 10^{-8} \text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99 \times 10^{22} \text{ m/s}^2$ (verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

$$F_G = G \frac{mM}{r^2},$$

where $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

$$F_G = (6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(0.530 \times 10^{-10}\ \text{m})^2} = 3.61 \times 10^{-47}\ \text{N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive Coulomb forces nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

Section Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is

  $$F = k\frac{|q_1 q_2|}{r2},$$

  where q_1 and q_2 are two point charges separated by a distance r, and

  Formula does not parse

- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

Conceptual Questions

1: Figure 3 shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

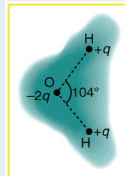

Figure 3. Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

2: Using Figure 3, explain, in terms of Coulomb's law, why a polar molecule (such as in Figure 3) is attracted by both positive and negative charges.

3: Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

Problems & Exercises

1: What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of –30.0 nC?

2: (a) How strong is the attractive force between a glass rod with a 0.700μC0.700μC charge and a silk cloth with a –0.600μC–0.600μC charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

3: Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

4: Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

5: How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

6: If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

7: A test charge of $+2\,\mu C$ is placed halfway between a charge of $+6\,\mu C$ and another of $+4\,\mu C$ separated by 10 cm. (a)

What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\ \mu C$ charge)?

8: Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

9: (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

10: Suppose you have a total charge $\boldsymbol{q_{tot}}$ that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

11: (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

12: (a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

13: At what distance is the electrostatic force between two protons equal to the weight of one proton?

14: A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

15: (a) Two point charges totaling $8.00\ \mu C$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

16: Point charges of $5.00\ \mu C$ and $-3.00\ \mu C$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

17: Two point charges q_1 and q_2 are 3.00 m apart, and their total charge is $20\ \mu C$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

Glossary

Coulomb's law
the mathematical equation calculating the electrostatic force vector between two charged particles

Coulomb force
another term for the electrostatic force

electrostatic force
the amount and direction of attraction or repulsion between two charged bodies

Solutions

2:

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

4:

The separation decreased by a factor of 5.

$$
\begin{aligned}
F &= k\frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2} \\
&= \frac{(9.00\times 10^9\ \mathrm{N\cdot m^2/C^2})(1.60\times 10^{-19}\ \mathrm{m})^2}{(1.67\times 10^{-27}\ \mathrm{kg})(2.00\times 10^{-9}\ \mathrm{m})^2} \\
&= 3.45 \times 10^{16}\ \mathrm{m/s^2}
\end{aligned}
$$

9:

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

11:

(a) $1.04 \times 10^{-9}\ \mathrm{C}$

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

14:

1.02×10^{-11}

16:

(a). 0.859 m beyond negative charge on line connecting two charges
(b). 0.109 m from lesser charge on line connecting two charges

18.4 Electric Field: Concept of a Field Revisited

Summary

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to “touch.” That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb’s law, $F = k|q_1 q_2|/r^2$, its magnitude is given by the equation $F = k|qQ|/r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see [link]). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q.

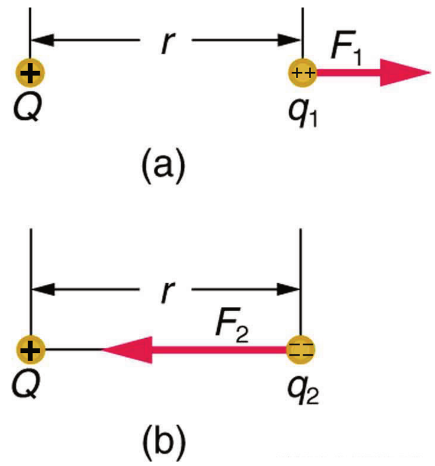

Figure 1. The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force $F1$ acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1, and so the force F_2 acting on it is attractive and stronger than F_1. The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

$$E=\frac{F}{q},$$

where F is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that E is in the same direction as F. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $F=qE$. Consider the electric field due to a point charge Q. According to Coulomb's law, the force it exerts on a test charge q is $F=k|qQ|/r^2$. Thus the magnitude of the electric field, E, for a point charge is

$$E=\left|\frac{F}{q}\right|=k\left|\frac{qQ}{qr^2}\right|=k\frac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

$$E=k\frac{|Q|}{r^2}$$

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ/r^2$.

Solution

Here $Q = 2.00 \times 10^{-9}\ \mathrm{C}$ and $r = 5.00 \times 10^{-3}\ \mathrm{m}$. Entering those values into the above equation gives

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \times \frac{(2.00 \times 10^{-9}\ \mathrm{C})}{(5.00 \times 10^{-3}\ \mathrm{m})^2} \\ &= 7.19 \times 10^5\ \mathrm{N/C}. \end{aligned}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250\ \mu\mathrm{C}$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250\ \mu\mathrm{C}$ exerted by a field of strength $E = 7.20 \times 10^5$ N/C is thus,

$$\begin{aligned} F &= -qE \\ &= (0.250 \times 10^{-6}\ \mathrm{C})(7.20 \times 10^5\ \mathrm{N/C}) \\ &= 0.180\ \mathrm{N}. \end{aligned}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

PhET Explorations: Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

PhET Interactive Simulation

Figure 2. Electric Field of Dreams

Section Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

  $$E = \frac{\mathbf{F}}{q},$$

  where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.
- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is

  $$\mathbf{E} = k\frac{|Q|}{r^2}.$$

  where r is the distance from Q. The electric field E is a vector and fields due to multiple charges add like vectors.

Conceptual Questions

1: Why must the test charge q in the definition of the electric field be vanishingly small?

2: Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

Problem Exercises

1: What is the magnitude and direction of an electric field that exerts a $2.00 \times 10^{-5}\ \text{N}$ upward force on a $-1.75\ \mu\text{C}$ charge?

2: What is the magnitude and direction of the force exerted on a $3.50\ \mu\text{C}$ charge by a 250 N/C electric field that points due east?

3: Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

4: (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

5: Calculate the initial (from rest) acceleration of a proton in a $5.00 \times 10^{6}\ \text{N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

6: (a) Find the direction and magnitude of an electric field that exerts a $4.80 \times 10^{-17}\ \text{N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

Glossary

field
a map of the amount and direction of a force acting on other objects, extending out into space

point charge
A charged particle, designated Q, generating an electric field

test charge
A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Problem Exercises

2: $8.75 \times 10{-}4\ \text{N}$

4:

(a) $6.94 \times 10^{-8}\ \text{C}$

(b) $6.25\ \text{N/C}$

6:

(a) 300 N/C (east)

(b) $4.80 \times 10^{-17}\ \text{N (east)}$

18.5 Electric Field Lines: Multiple Charges

Summary

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent electric fields around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all vectors, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 1 shows two pictorial representations of the same electric field created by a positive point charge Q. Figure 1 (b) shows the standard representation using continuous lines. Figure 1 (b) shows numerous individual arrows with each arrow representing the force on a test charge q. Field lines are essentially a map of infinitesimal force vectors.

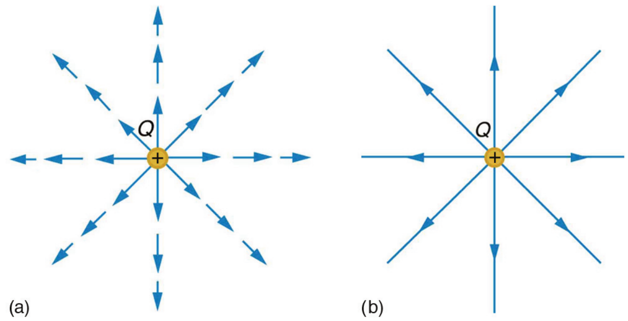

Figure 1. Two equivalent representations of the electric field due to a positive charge ***Q***. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See Figure 2.) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2. This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.

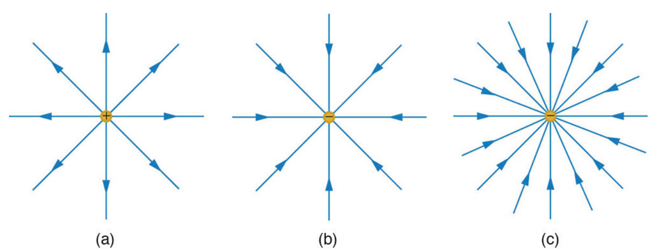

Figure 2. The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, q_1 and q_2, at the origin of the coordinate system as shown in Figure 3.

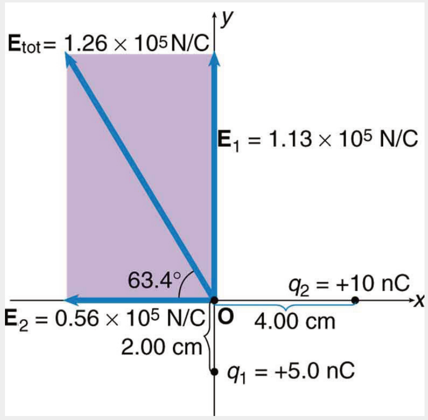

Figure 3. The electric fields $\mathbf{E_1}$ and $\mathbf{E_2}$ at the origin O add to $\mathbf{E_{tot}}$.

Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q, at point O, which allows us to determine the direction of the fields E_1 and E_2. Once those fields are found, the total field can be determined using vector addition.

Solution

The electric field strength at the origin due to q_1 is labeled E_1 and is calculated:

$$E_1 = k\frac{q_1}{r_1^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{5.00 \times 10^{-9}\ \mathrm{C}}{2.00 \times 10^{-2}\ \mathrm{m^2}}$$
$$E_1 = 1.124 \times 10^5\ \mathrm{N/C}.$$

Similarly, E_2 is

$$E_2 = k\frac{q_2}{r_2^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{10.0 \times 10^{-9}\ \mathrm{C}}{4.00 \times 10^{-2}\ \mathrm{m^2}}$$
$$E_1 = 0.5619 \times 10^5\ \mathrm{N/C}.$$

Four digits have been retained in this solution to illustrate that E_1 is exactly twice the magnitude of E_2. Now arrows are drawn to represent the magnitudes and directions of E_1 and E_2. (See Figure 3.) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for E_1 is exactly twice the length of that for E_2. The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field E_{tot} is

$$\begin{aligned} E_{tot} &= (E_1^2 + E_2^2)^{1/2} \\ &= \{(1.124 \times 10^5\ \mathrm{N/C})^2 + (0.5619 \times 10^5\ \mathrm{N/C})^2\}^{1/2} \\ &= 1.26 \times 10^5\ \mathrm{N/C}. \end{aligned}$$

The direction is

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{E_1}{E_2}\right) \\ &= \tan^{-1}\left(\frac{1.124 \times 10^5\ \mathrm{N/C}}{0.5619 \times 10^5\ \mathrm{N/C}}\right) \\ &= 63.4^\circ \end{aligned}$$

or 63.4° above the *x*-axis.

Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

Figure 4 shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See Figure 4 and Figure 5(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

Figure 5(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.

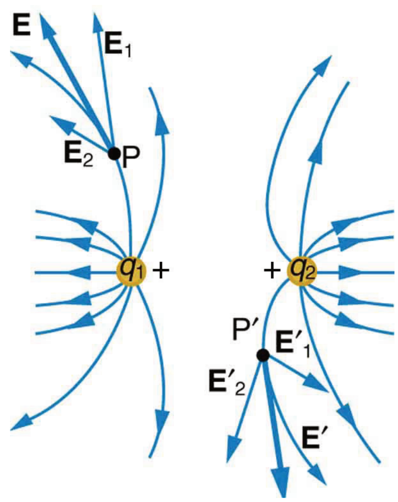

Figure 4. Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.

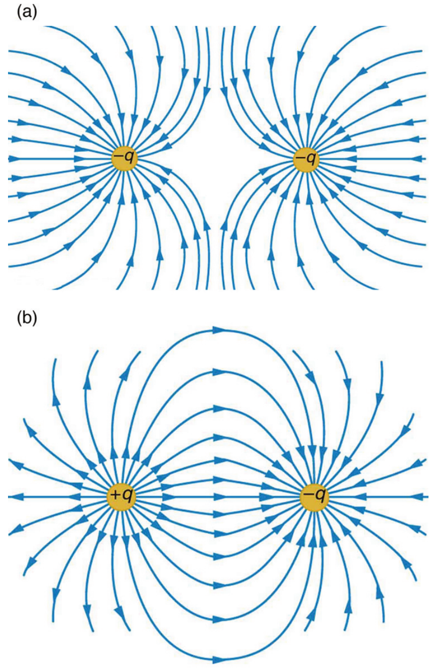

Figure 5. (a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

PhET Interactive Simulation

Figure 6. Charges and Fields

Section Summary

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

Conceptual Questions

1: Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

2: Figure 7 shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?

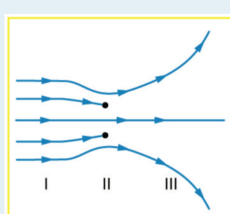

Figure 7.

Problem Exercises

1: (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$

2: Sketch the electric field lines a long distance from the charge distributions shown in Figure 5 (a) and (b)

3: Figure 8 shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.

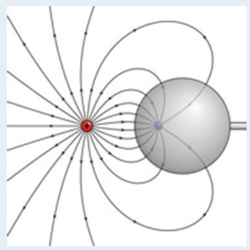

Figure 8. The electric field near two charges.

4: Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See Figure 8 for a similar situation).

Glossary

electric field

a three-dimensional map of the electric force extended out into space from a point charge

electric field lines

a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

vector

a quantity with both magnitude and direction

vector addition

mathematical combination of two or more vectors, including their magnitudes, directions, and positions

18.6 Electric Forces in Biology

Summary

- Describe how a water molecule is polar.
- Explain electrostatic screening by a water molecule within a living cell.

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. Figure 1 is a schematic of the DNA double helix.

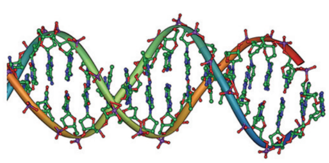

Figure 1. DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which "code" the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. (credit: Jerome Walker)

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance $(F \propto 1/r^2)$, the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about $2q_e$ (fundamental charge) per $0.3 \times 10^{-9}\ \text{m}$. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is “diluted” due to **screening** between molecules. This is due to the presence of other charges in the cell.

Polarity of Water Molecules

The best example of this charge screening is the water molecule, represented as H2O. Water is a strongly **polar molecule**. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a **dipole**, as illustrated in Figure 2. The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the **Coulomb interaction**. One might say that screening makes the Coulomb force a short range force rather than long range.

Other ions of importance in biology that can reduce or screen Coulomb interactions are Na^+, and K^+, and Cl^-. These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by “microtubules” within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).

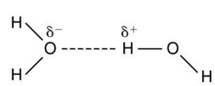

Figure 2. This schematic shows water (H_2O) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole. The symbols δ^- and δ^+ indicate that the oxygen side of the H_2O molecule tends to be more negative, while the hydrogen ends tend to be more positive. This leads to an attraction of opposite charges between molecules.

Section Summary

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.

- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

Conceptual Question

1: A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of $-2.5 \times 10^{-6}\ \text{C/m}^2$ on its inner surface and $+2.5 \times 10^{-6}\ \text{C/m}^2$ on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

Glossary

dipole
a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative
polar molecule
a molecule with an asymmetrical distribution of positive and negative charge
screening
the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby
Coulomb interaction
the interaction between two charged particles generated by the Coulomb forces they exert on one another

18.7 Conductors and Electric Fields in Static Equilibrium

Summary

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

Figure 1 shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.

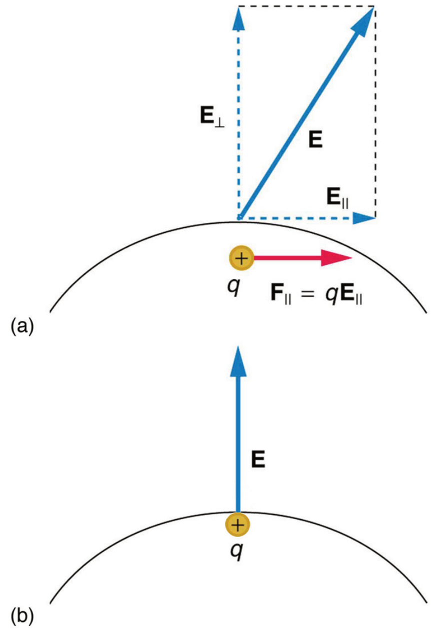

Figure 1. When an electric field **E** is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component ($\mathbf{E}_{||}$) exerts a force ($\mathbf{F}_{||}$) on the free charge *q*, which moves the charge until $\mathbf{F}_{||} = \mathbf{0}$. (b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium.

A conductor placed in an **electric field** will be **polarized**. Figure 2 shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.

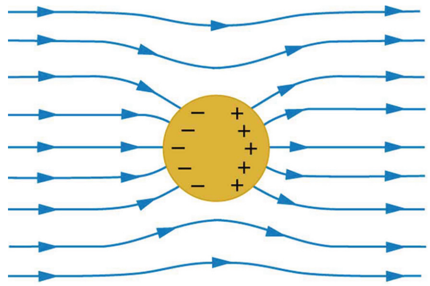

Figure 2. This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in Figure 3. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.

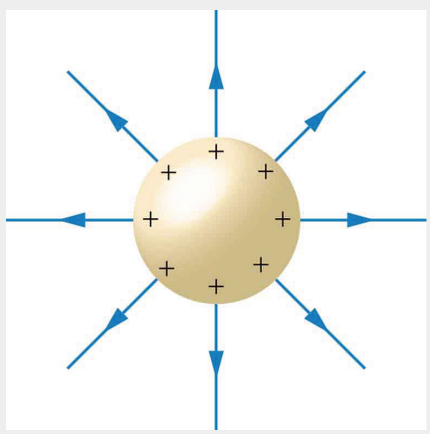

Figure 3. The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in Figure 4. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.

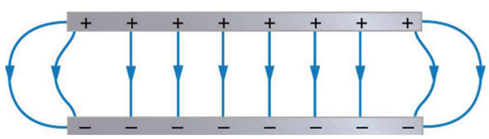

Figure 4. Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube.

Earth's Electric Field

A near uniform electric field of approximately 150 N/C, directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field (Figure 5(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction (Figure 5(b)). The exact charge distributions depend on the local conditions, and variations of Figure 5(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3×10^6 N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.

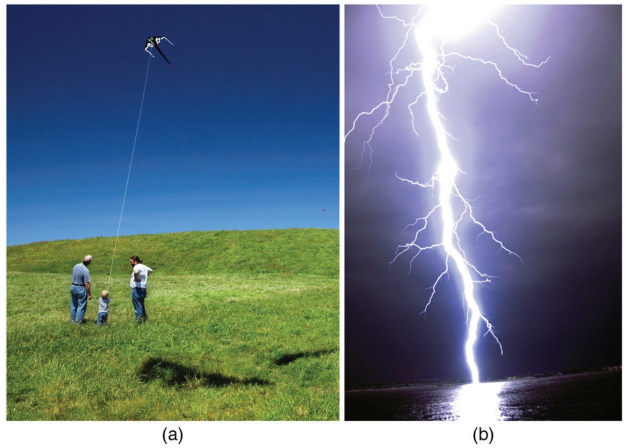

Figure 5. Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in Figure 6. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in Figure 6 (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.

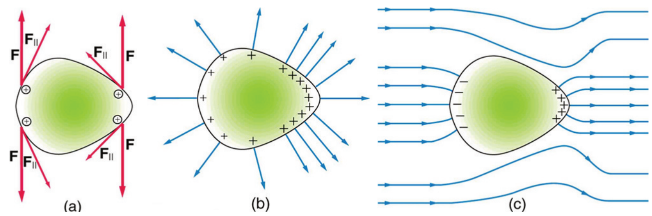

Figure 6. Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{||}$ that moves the charges apart once they have reached the surface. (b) $\mathbf{F}_{||}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

On a very sharply curved surface, such as shown in Figure 7, the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See Figure 8.) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.

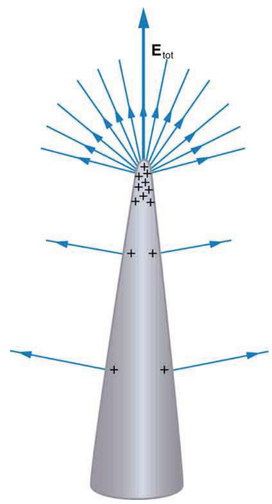

Figure 7. A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.

Figure 8. (a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

Section Summary

- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.

- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

Conceptual Questions

1: Is the object in Figure 9 a conductor or an insulator? Justify your answer.

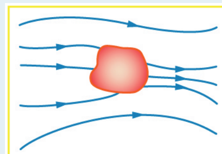

Figure 9.

2: If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

3: The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

4: Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

5: Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

6: Can the belt of a Van de Graaff accelerator be a conductor? Explain.

7: Are you relatively safe from lightning inside an automobile? Give two reasons.

8: Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

9: Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below (Figure 10) is zero if the charges on the four corners are exactly equal.

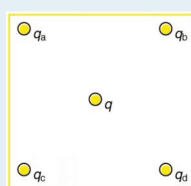

Figure 10. Four point charges q_a, q_b, q_c, and q_d lie on the corners of a square and q is located at its center.

10: (a) Using the symmetry of the arrangement, show that the electric field at the center of the square in Figure 10 is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$

11: (a) What is the direction of the total Coulomb force on q in Figure 10 if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

12: Considering Figure 10, suppose that $q_a = q_d$ and $q_b = q_c$. First show that *qq size 12{q} {}* is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

13: If $q_a = 0$ in Figure 10, under what conditions will there be no net Coulomb force on q?

14: In regions of low humidity, one develops a special "grip" when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one's fingers. Discuss the induced charge and explain why this is done.

15: Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

16: Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

Problems & Exercises

1: Sketch the electric field lines in the vicinity of the conductor in Figure 11 given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

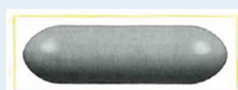

Figure 11.

2: Sketch the electric field lines in the vicinity of the conductor in Figure 12 given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

Figure 12.

3: Sketch the electric field between the two conducting plates shown in Figure 13, given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.

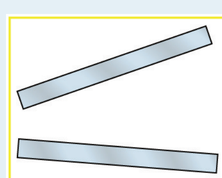

Figure 13.

4: Sketch the electric field lines in the vicinity of the charged insulator in Figure 14 noting its nonuniform charge distribution.

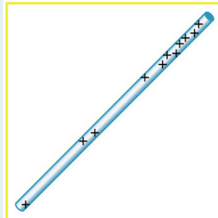

Figure 14. A charged insulating rod such as might be used in a classroom demonstration.

5: What is the force on the charge located at x=8.00 cm in Figure 15(a) given that $latex \boldsymbol{q = 1.00 \;\mu \textbf{C}}?

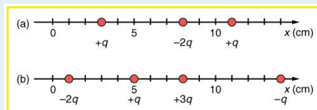

Figure 15. (a) Point charges located at 3.00, 8.00, and 11.0 cm along the *x*-axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the *x*-axis.

6: (a) Find the total electric field at x=1.00 cmx=1.00 cm in Figure 15(b) given that $q = 5.00\text{ nC}$. (b) Find the total electric field at $latex \boldsymbol{x = 11.00 \;\textbf{cm}} in Figure 15(b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

7: (a) Find the electric field at $x = 5.00\text{ cm}$ in Figure 15(a), given that $q = 1.00\;\mu\text{C}$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm? (d) At very large positive or negative values of *x*, the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

8: (a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00\text{ cm}$ in Figure 15 (b), given that $q = 1.00\;\mu\text{C}$. (b) Find the *x*-position at which the electric field is zero in Figure 15 (b).

9: Using the symmetry of the arrangement, determine the direction of the force on *q* in the figure below, given that $q_a = q_b = +7.50\;\mu\text{C}$ and $q_c = q_d = -7.50\;\mu\text{C}$. (b) Calculate the magnitude of the force on the charge *q*, given that the square is 10.0 cm on a side and $q = 2.00\;\mu\text{C}$.

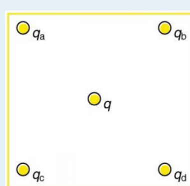

Figure 16.

10: (a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in Figure 16, given that $q_a = q_b = -1.00\;\mu\text{C}$ and $q_c = q_d = +1.00\;\mu\text{C}$. (b) Calculate the magnitude of the electric field at the location of *q*, given that the square is 5.00 cm on a side.

11: Find the electric field at the location of q_a in Figure 16 given that $q_b = q_c = q_d = +2.00\text{ nC}$, $q = -1.00\text{ nC}$, and the square is 20.0 cfm on a side.

12: Find the total Coulomb force on the charge q in Figure 16, given that $q = 1.00\ \mu\text{C}$, $q_a = 2.00\ \mu\text{C}$, $q_b = -3.00\ \mu\text{C}$, $q_c = -4.00\ \mu\text{C}$, and $q_d = +1.00\ \mu\text{C}$. The square is 50.0 cm on a side.

13: (a) Find the electric field at the location of qaqa in Figure 17, given that $q_b = +10.00\ \mu\text{C}$ and $q_c = -5.00\ \mu\text{C}$. (b) What is the force on q_a, given that $q_a = +1.50\ \text{nC}$?

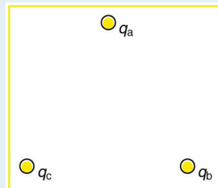

Figure 17. Point charges located at the corners of an equilateral triangle 25.0 cm on a side.

14: (a) Find the electric field at the center of the triangular configuration of charges in Figure 17, given that $q_a = +2.50\ \text{nC}$, $q_b = -8.00\ \text{nC}$, and $q_c = +1.50\ \text{nC}$. (b) Is there any combination of charges, other than $q_a = q_b = q_c$, that will produce a zero strength electric field at the center of the triangular configuration?

Glossary

conductor
an object with properties that allow charges to move about freely within it
free charge
an electrical charge (either positive or negative) which can move about separately from its base molecule
electrostatic equilibrium
an electrostatically balanced state in which all free electrical charges have stopped moving about
polarized
a state in which the positive and negative charges within an object have collected in separate locations
ionosphere
a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth
Faraday cage
a metal shield which prevents electric charge from penetrating its surface

Solutions

Problems & Exercises

6:

(a) $E_{x=1.00\ \text{cm}} = -\infty$

(b) $2.12 \times 10^5\ \text{N/C}$

(c) one charge of $+q$

8:

(a) 0.252 N to the left

(b) $x = 6.07\ \text{cm}$

10:

(a)The electric field at the center of the square will be straight up, since q_a and q_b are positive and q_c and q_d are negative and all have the same magnitude.

(b) $2.04 \times 10^7\ \text{N/C (upward)}$

12: $0.102\ \text{N}$,0.102N, in the $-y$ direction

14:

(a) $\vec{E} = 4.36 \times 10^3\ \text{N/C}, 35.0°$, below the horizontal.

(b) No

18.8 Applications of Electrostatics

Summary

- Name several real-world applications of the study of electrostatics.

The study of electrostatics has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. Figure 1 shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

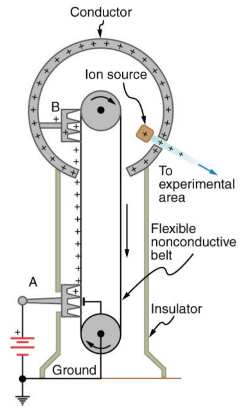

Figure 1. Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

Xerography

Most copy machines use an electrostatic process called xerography—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in Figure 2.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a photoconductor. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is grounded so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.

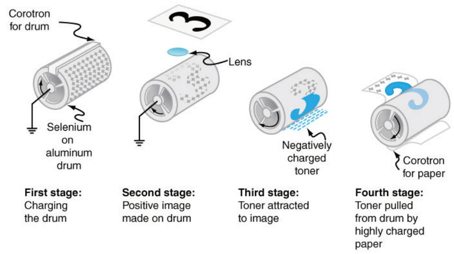

Figure 2. Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in Figure 3. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

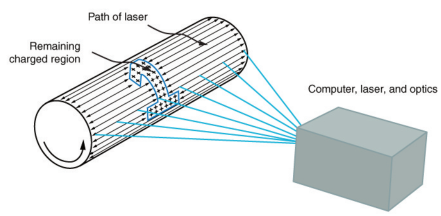

Figure 3. In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The ink jet printer, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See Figure 4.)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

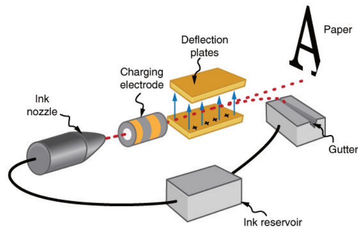

Figure 4. The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See Figure 5.)

Large electrostatic precipitators are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

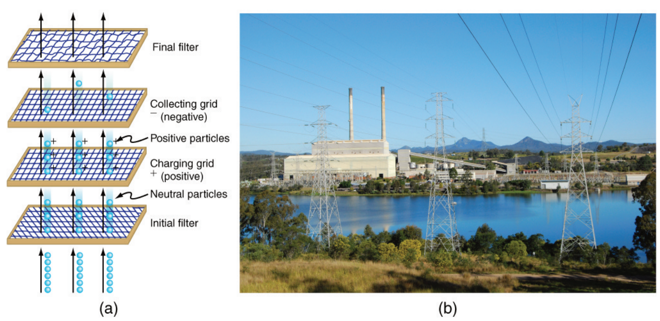

Figure 5. (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

Problem-Solving Strategies for Electrostatics

1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E, for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

Integrated Concepts

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, and hence the relevance of Chapter 4 Dynamics: Force and Newton's Laws of Motion. The following topics are involved in some or all of the problems labeled "Integrated Concepts":

Chapter 2 Kinematics
Chapter 3 Two-Dimensional Kinematics
Chapter 4 Dynamics: Force and Newton's Laws of Motion
Chapter 6 Uniform Circular Motion and Gravitation
Chapter 9 Statics and Torque
Chapter 11 Fluid Statics

The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of 4.00×10^{-15} kg and is given a positive charge of 3.20×10^{-19} C. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength $3.00 \times 10^5\ \text{N/C}$ due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

Strategy

To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in Chapter 4 Dynamics: Force and Newton's Laws of Motion. Part (b) deals with electric force on a charge, a topic of Chapter 18 Electric Charge and Electric Field. Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in Chapter 4 Dynamics: Force and Newton's Laws of Motion.

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

Weight is mass times the acceleration due to gravity, as first expressed in

$$w = mg.$$

Entering the given mass and the average acceleration due to gravity yields

$$w = (4.00 \times 10^{-15}\ \text{kg})(9.80\ \text{m/s}^2) = 3.92 \times 10^{-14}\ \text{N}.$$

Discussion for (a)

This is a small weight, consistent with the small mass of the drop.

Solution for (b)

The force an electric field exerts on a charge is given by rearranging the following equation:

$$F = qE.$$

Here we are given the charge ($3.20 \times 10^{-19}\ \text{C}$ is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

$$F = (3.20 \times 10^{-19}\ \text{C})(3.00 \times 10^5\ \text{N/C}) = 9.60 \times 10^{-14}\ \text{N}.$$

Discussion for (b)

While this is a small force, it is greater than the weight of the drop.

Solution for (c)

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

$$a = \frac{F_{net}}{m}.$$ $

where $F_{net} = F - w$. Entering this and the known values into the expression for Newton's second law yields

$$\begin{aligned} a &= \frac{F - w}{m} \\ &= \frac{9.60 \times 10^{-14}\ \text{N} - 3.92 \times 10^{-14}\ \text{N}}{4.00 \times 10^{-15}\ \text{kg}} \\ &= 14.2\ \text{m/s}^2. \end{aligned}$$

Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Problem-Solving Strategy
To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

Section Summary

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Problems & Exercises

1: (a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\ \mu C$ charge on the Van de Graaff's belt?

2: (a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

3: A simple and common technique for accelerating electrons is shown in Figure 6, where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $2.50 \times 10^4\ \text{N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.

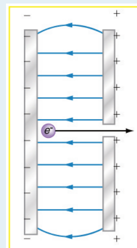

Figure 6. Parallel conducting plates with opposite charges on them create a relatively uniform electric field used to accelerate electrons to the right. Those that go through the hole can be used to make a TV or computer screen glow or to produce X-rays.

4: Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

5: Point charges of $25.0\ \mu\text{C}$ and $45.0\ \mu\text{C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

6: What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?

Problems & Exercises

Integrated Concepts

1: Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is $0.530 \times 10^{-10}\ \text{m}$. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

2: An electron has an initial velocity of $5.00 \times 10^6\ \text{m/s}$ in a uniform $2.00 \times 10^5\ \text{N/C}$ strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

3: The practical limit to an electric field in air is about $3.00 \times 10^6 \text{ N/C}$. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

4: A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in Figure 7. Given the charge on the ball is 1.00μC1.00μC, find the strength of the field.

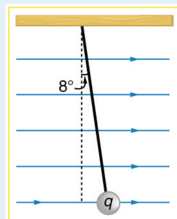

Figure 7. A horizontal electric field causes the charged ball to hang at an angle of 8.00º.

5: Figure 8 shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is $3.00 \times 10^6 \text{ m/s}$, and the horizontal distance it travels in the uniform field is 4.00 cm. (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.

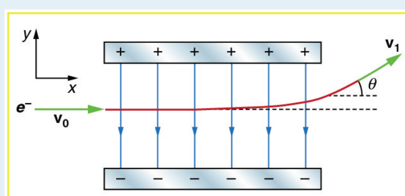

Figure 8.

6: The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See Figure 9.) Given the oil drop to be $1.00 \ \mu\text{m}$ in radius and have a density of 920 kg/m^3: (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.

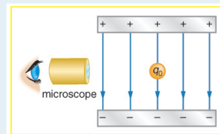

Figure 9. In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge $\mathbf{q_e}$ by measuring the electric field and mass of the drop.

7: (a) In Figure 10, four equal charges q lie on the corners of a square. A fifth charge Q is on a mass mm directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q, m, and d, if the Coulomb force is to equal the weight of m. (b) Is this equilibrium stable or unstable? Discuss.

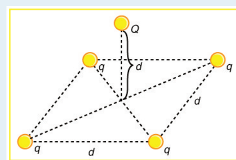

Figure 10. Four equal charges on the corners of a horizontal square support the weight of a fifth charge located directly above the center of the square.

Unreasonable Results

1: (a) Calculate the electric field strength near a 10.0 cm diameter conducting sphere that has 1.00 C of excess charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

2: (a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

3: A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

Construct Your Own Problems

1: Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric field off axis or for a more complex array of charges, such as those in a water molecule.

2: Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.

Glossary

Van de Graaff generator
a machine that produces a large amount of excess charge, used for experiments with high voltage
electrostatics
the study of electric forces that are static or slow-moving
photoconductor
a substance that is an insulator until it is exposed to light, when it becomes a conductor
xerography
a dry copying process based on electrostatics
grounded
connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object
laser printer
uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image
ink-jet printer
small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper
electrostatic precipitators
filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

Solutions

Problems & Exercises

2: (a) $5.58 \times 10^{-11}\ \mathrm{N/C}$

(b)the coulomb force is extraordinarily stronger than gravity

4: (a) $-6.76 \times 10^{5}\ \mathrm{C}$

(b) $2.63 \times 10^{13}\ \mathrm{m/s^2}$ (upward)

(c) $2.45 \times 10^{-18}\ \mathrm{kg}$

6: The charge q_2 is 9 times greater than q_1.

PART 12

Chapter 19 Electric Potential and Electric Field

Figure 1. Automated external defibrillator unit (AED) (credit: U.S. Defense Department photo/Tech. Sgt. Suzanne M. Day)

In Chapter 18 Electric Charge and Electric Field, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

19.1 Electric Potential Energy: Potential Difference

Summary

- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

When a free positive charge q is accelerated by an electric field, such as shown in Figure 1, it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

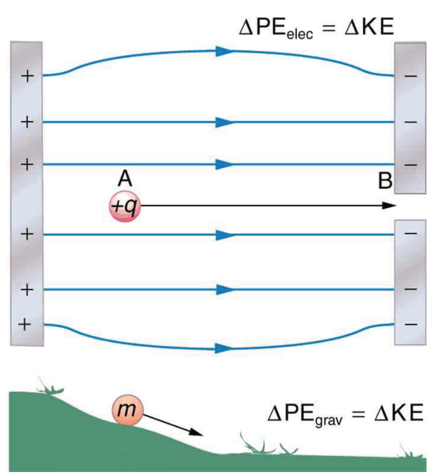

Figure 1. A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force is conservative, we can write $\boldsymbol{W = -\Delta PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE, is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE. There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Potential Energy

$W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE. There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = Fd\cos\theta$ and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence ΔPE, is proportional to the test charge q. To have a physical quantity that is independent of test charge, we define electric potential V (or simply potential, since electric is understood) to be the potential energy per unit charge:

$$V = \frac{\text{PE}}{q}.$$

Electric Potential

This is the electric potential energy per unit charge.

$$V = \frac{\text{PE}}{q}$$

Since PE is proportional to q , the dependence on q cancels. Thus V does not depend on q. The change in potential energy ΔPE is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_B - V_A = \frac{\Delta\text{PE}}{q}.$$

The potential difference between points A and B, $V_B - V_A$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\text{ V} = 1\ \frac{\text{J}}{\text{C}}$$

Potential Difference

The potential difference between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\text{ V} = 1\ \frac{\text{J}}{\text{C}}$$

The familiar term voltage is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta\text{V}.$$

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta\text{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta PE = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000 \text{ C}$ and $\Delta V = 12.0 \text{ V}$. The total energy delivered by the motorcycle battery is

$$\begin{aligned} \Delta PE_{\text{cycle}} &= (5000 \text{ C})(12.0 \text{ V}) \\ &= (5000 \text{ C})(12.0 \text{ J/C}) \\ &= 6.00 \times 10^4 \text{ J}. \end{aligned}$$

Similarly, for the car battery, $q = 60,000 \text{ C}$ and

$$\begin{aligned} \Delta PE_{\text{cycle}} &= (60,000 \text{ C})(12.0 \text{ V}) \\ &= 7.20 \times 10^5 \text{ J} \end{aligned}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in Figure 2. The change in potential is $\Delta V = V_B - V_A = +12 \text{ V}$ and the charge q is negative, so that $\Delta PE = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

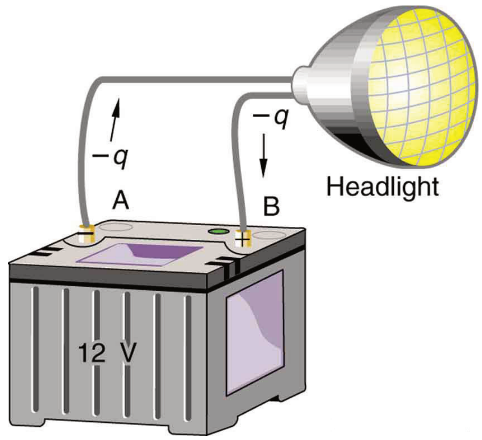

Figure 2. A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta \text{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta \text{PE} = -30.0 \text{ J}$ and, since the electrons are going from the negative terminal to the positive, we see that

$\Delta V = +12.0 \text{ V}$.

Solution

To find the charge q moved, we solve the equation $\Delta \text{PE} = q\Delta V$:

$$q = \frac{\Delta \text{PE}}{\Delta V}.$$

Entering the values for ΔPE and ΔV, we get

$$q = \frac{-30.0 \text{ J}}{+12.0 \text{ V}} = \frac{-30.0 \text{ J}}{+12.0 \text{ J/C}} = -2.50 \text{ C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

$$n_e = \frac{-2.50 \text{ C}}{-1.60 \times 10^{-19} \text{ C/e}^-} = 1.56 \times 10^{19} \text{ electrons.}$$

Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. Figure 3 shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta \mathrm{PE} = q\Delta V$, we can think of the joule as a coulomb-volt.

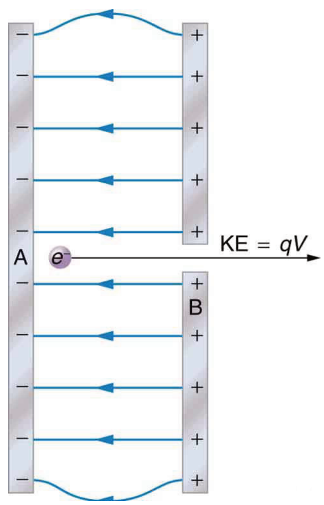

Figure 3. A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1 \text{eV} &= (1.60 \times 10^{-19}\ \text{C})\ (1\ \text{V}) = (1.60 \times 10^{-19}\ \text{C})\ (1\ \text{J/C}) \\ &= 1.60 \times 10^{-19}\ \text{J}. \end{aligned}$$

Electron Volt

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An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules $(30,000\ \text{eV} \div 5\ \text{eV per molecule} = 6000\ \text{molecules})$. Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\text{KE} + \text{PE} = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

$$\text{KE} + \text{PE} = \text{constant}$$

or

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f},$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\mathrm{KE_i} = 0$, $\mathrm{KE_f} = \frac{1}{2}mv^2$, $\mathrm{PE_i} = qV$, and $\mathrm{PE_f} = 0$.

Solution

Conservation of energy states that

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}.$$

We solve this for v:

$$v = \sqrt{\frac{2qV}{m}}.$$

Entering values for q, V, and m gives

$$\begin{aligned} v &= \sqrt{\frac{2(-1.60 \times 10^{-19}\ \mathrm{C})(-100\ \mathrm{J/C})}{9.11 \times 10^{-31}\ \mathrm{kg}}} \\ &= 5.93 \times 10^{6}\ \mathrm{m/s} \end{aligned}.$$

Discussion

Note that both the charge and the initial voltage are negative, as in Figure 3. From the discussions in Chapter 18 Electric Charge and Electric Field, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol

  ΔV.

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1\text{eV} &= (1.60 \times 10^{-19}\ \text{C})(1\ \text{V}) = 1.60 \times 10^{-19}\ \text{C})(1\ \text{J/C}) \\ &= 1.60 \times 10^{-19}\ \text{J}. \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\text{KE} + \text{PE}$. This sum is a constant.

Conceptual Questions

1: Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

2: If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

3: What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

4: Voltages are always measured between two points. Why?

5: How are units of volts and electron volts related? How do they differ?

Problems & Exercises

1: Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be $1.67 \times 10^{-27}\ \text{kg}$.

2: An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

3: A bare helium nucleus has two positive charges and a mass of $6.64 \times 10^{-27}\ \text{kg}$. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

4: Integrated Concepts
Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

5: Integrated Concepts
The temperature near the center of the Sun is thought to be 15 million degrees Celsius $(1.5 \times 10^{7}\ ^\circ\text{C})$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

6: Integrated Concepts
(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

7: Integrated Concepts

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of 1.00×10^{2} MV. (a) What energy was dissipated? (b) What mass of water could be raised from $15\ °C$ to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

8: Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^{2} g of baby formula, and 2.00×10^{2} g of aluminum from $20.0°C$ to $90.0°C$. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

9: Integrated Concepts

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a 2.00×10^{2} m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a 5.00×10^{2} N force for an hour.

10: Integrated Concepts

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by 1.00×10^{-12} m by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

11: Unreasonable Results

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

12: Construct Your Own Problem

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

Glossary

electric potential

potential energy per unit charge

potential difference (or voltage)

change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

electron volt

the energy given to a fundamental charge accelerated through a potential difference of one volt

mechanical energy

sum of the kinetic energy and potential energy of a system; this sum is a constant

Solutions

Problems & Exercises

1: 42.8

4: $1.00 \times 10^{5}\ \text{K}$

6: (a) $4 \times 10^{4}\ \text{W}$

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

8: (a) $7.40 \times 10^{3}\ \text{C}$

(b) 1.54×10^{20} electrons per second

9: $3.89 \times 10^{6}\ \text{C}$

11:
(a) $1.44 \times 10^{12}\ \text{V}$

(b) This voltage is very high. A 10.0 cm diameter sphere could never maintain this voltage; it would discharge.

(c) An 8.00 C charge is more charge than can reasonably be accumulated on a sphere of that size.

19.2 Electric Potential in a Uniform Electric Field

Learning Objectives

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See Figure 1.) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either ΔV or $\mathbf{E}$ can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. ΔV is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in Chapter 19.1 Electric Potential Energy: Potential Difference, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.

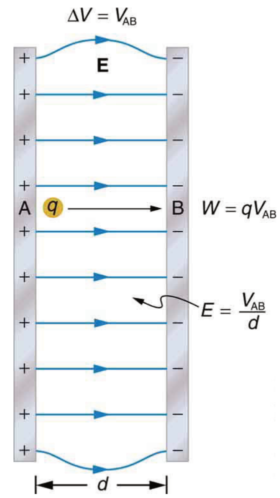


Figure 1. The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{AB}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_A - V_B = V_{AB}$. See the text for details.)

The work done by the electric field in Figure 1 to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta \text{PE} = -q\Delta V.$$

The potential difference between points A and B is

$$-\Delta V = -(V_B - V_A) = V_{A-V_B} = V_{AB}.$$

Entering this into the expression for work yields

$$W = qV_{AB}.$$

Work is $W = Fd\cos\theta$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

$$qEd = qV_{AB}.$$

The charge cancels, and so the voltage between points A and B is seen to be

$$\left.\begin{array}{l} V_{AB} = Ed \\ E = \frac{V_{AB}}{d} \end{array}\right\} \text{(uniform } E \text{ - field only)},$$

where d is the distance from A to B, or the distance between the plates in Figure 1. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1\text{ N/C} = 1\text{ V/m}.$$

Voltage between Points A and B

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \frac{V_{\text{AB}}}{d} \end{array}\right\} \text{(uniform } E \text{ - field only)},$$

where d is the distance from A to B, or the distance between the plates.

Example 1: What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about $3.0 \times 10^6\text{ V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{\text{AB}} = Ed$ can thus be used to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$V_{\text{AB}} = Ed.$$

Entering the given values for E and d gives

$$V_{\text{AB}} = (3.0 \times 10^6\text{ V/m})(0.025\text{ m}) = 7.5 \times 10^4\text{ V}$$

or

$$V_{\text{AB}} = 75\text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.

Figure 2. A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

Example 2: Field and Force inside an Electron Gun

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{\text{AB}}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q\mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q\,E$.

Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{\text{AB}}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\text{ kV}}{0.0400\text{ m}} = 6.25 \times 10^5\text{ V/m}.$$

Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE.$$

Substituting known values gives

$$F = (0.500 \times 10^{-6}\ \text{C})(6.25 \times 10^{5}\ \text{V/m}) = 0.313\ \text{N}.$$

Discussion

Note that the units are newtons, since $1\ \text{V/m} = 1\ \text{N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

Section Summary

- The voltage between points A and B is

  $$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \frac{V_{\text{AB}}}{d} \end{array}\right\} \text{(uniform } E\text{ - field only)},$$

  where d is the distance from A to B, or the distance between the plates.

- In equation form, the general relationship between voltage and electric field is

  $$E = -\frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that E points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Conceptual Questions

1: Discuss how potential difference and electric field strength are related. Give an example.

2: What is the strength of the electric field in a region where the electric potential is constant?

3: Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

Problems & Exercises

1: Show that units of V/m and N/C for electric field strength are indeed equivalent.

2: What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of $1.50 \times 10^4 \text{ V}$?

3: The electric field strength between two parallel conducting plates separated by 4.00 cm is $7.50 \times 10^4 \text{ V/m}$. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

4: How far apart are two conducting plates that have an electric field strength of $4.50 \times 10^3 \text{ V/m}$ between them, if their potential difference is 15.0 kV?

5: (a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air $(3.0 \times 10^6 \text{ V/m})$ if the plates are separated by 2.00 mm and a potential difference of $5.0 \times 10^3 \text{ V}$ is applied? (b) How close together can the plates be with this applied voltage?

6: The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in Chapter 19.5 Capacitors and Dielectrics and Chapter 20.7 Nerve Conduction—Electrocardiograms.) You may assume a uniform electric field.

7: Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in Chapter 20.7 Nerve Conduction—Electrocardiograms.) What is the voltage across an 8.00 nm–thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

8: Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

9: Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0 \times 10^6 \text{ V/m}$.

10: A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

11: An electron is to be accelerated in a uniform electric field having a strength of 2.00×10^6 V/m. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

Glossary

scalar
physical quantity with magnitude but no direction
vector
physical quantity with both magnitude and direction

Solutions

Problems & Exercises

3: (a) 3.00 kV

(b) 750 V

5: (a) No. The electric field strength between the plates is 2.5×10^6 V/m, which is lower than the breakdown strength for air $(3.0 \times 10^6$ V/m$)$.

(b) 1.7 mm

7: 44.0 mV

9: 15 kV

11: (a) 800 KeV

(b) 25.0 km

19.3 Electrical Potential Due to a Point Charge

Summary

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential* V *of a point charge* is

$$V = \frac{kQ}{r} \text{ (Point Charge)},$$

where *k* is a constant equal to $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

Electric Potential *V* of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kQ}{r} \text{ (Point Charge)}.$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas E for a point charge decreases with distance squared:

$$E = \frac{F}{q} = \frac{kQ}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

Example 1: What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a −3.00nC static charge?

Strategy

As we have discussed in Chapter 18 Electric Charge and Electric Field, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

$$\begin{aligned} V &= k\frac{Q}{r} \\ &= (8.99 \times 10^{9}\ \mathrm{N \cdot m^2/C^2})\left(\frac{-3.00\times10^{-9}\ \mathrm{C}}{5.00\times10^{-2}\ \mathrm{m}}\right) \\ &= -539\ \mathrm{V}. \end{aligned}$$

Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example 2: What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See Figure 1.) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

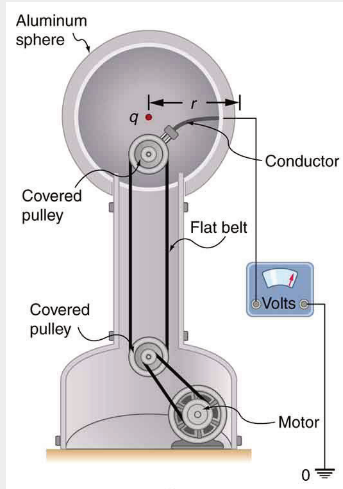


Figure 1. The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{k}{Qr}.$$

Solution

Solving for Q and entering known values gives

$$\begin{aligned} Q &= \frac{rV}{k} \\ &= \frac{(0.125 \text{ m})(100 \times 10^3 \text{ V})}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} \\ &= 1.39 \times 10^{-6} \text{ C} = 1.39 \text{ }\mu\text{C}. \end{aligned}$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in Chapter 19.1 Electric Potential Energy: Potential Difference, this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $\text{PE}_g = mgh$.

Section Summary

- Electric potential of a point charge is $V = kQ/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage

due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

Conceptual Questions

1: In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

2: Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

Problems & Exercises

1: A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

2: What is the potential $0.530 \times 10^{-10}\text{ m}$ from a proton (the average distance between the proton and electron in a hydrogen atom)?

3: (a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

4: How far from a $1.00\,\mu\text{C}$ point charge will the potential be 100 V? At what distance will it be $2.00 \times 10^{2}\text{ V}$?

5: What are the sign and magnitude of a point charge that produces a potential of -2.00 V at a distance of 1.00 mm?

6: If the potential due to a point charge is $5.00 \times 10^{2}\text{ V}$ at a distance of 15.0 m, what are the sign and magnitude of the charge?

7: In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $2.00 \times 10^{-14}\text{ m}$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

8: A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

9: An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object. (a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

10: In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

11: (a) What is the potential between two points situated 10 cm and 20 cm from a $3.0\,\mu\text{C}$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

12: Unreasonable Results

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

Solutions

Problems & Exercises

1: 144 V

3: (a) 1.80 km

(b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

5: $-2.22 \times 10^{-13}\ \text{C}$

7: (a) $3.31 \times 10^{6}\ \text{V}$

(b) 152 MeV

9: (a) $2.78 \times 10^{-7}\ \text{C}$

(b) $2.00 \times 10^{-10}\ \text{C}$

12: (a) $2.96 \times 10^{9}\ \text{m/s}$

(b) This velocity is far too great. It is faster than the speed of light.

(c) The assumption that the speed of the electron is far less than that of light and that the problem does not require a relativistic treatment produces an answer greater than the speed of light.

19.4 Equipotential Lines

Summary

- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider Figure 1, which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called equipotential lines in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by $V = kQ/r$ and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of Figure 1. Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

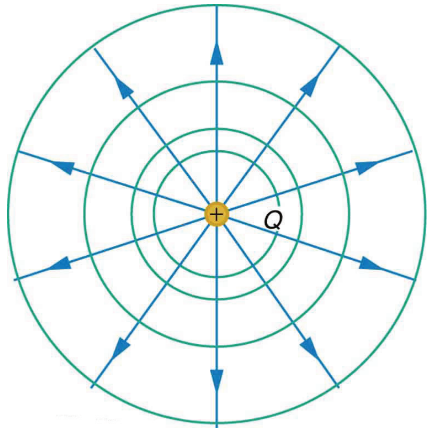


Figure 1. An isolated point charge ***Q*** with its electric field lines in blue and equipotential lines in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is

$$W = -\Delta \text{PE} = -q\Delta V = 0.$$

Work is zero if force is perpendicular to motion. Force is in the same direction as E, so that motion along an equipotential must be perpendicular to E. More precisely, work is related to the electric field by

$$W = Fd\cos\theta = qEd\cos\theta = 0.$$

Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor E nor d is zero, and so $\cos\theta$ must be 0, meaning θ must be 90°. In other words, motion along an equipotential is perpendicular to E.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

Grounding

A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in Figure 1 a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

Figure 2 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in Figure 3(a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in Figure 3(b).

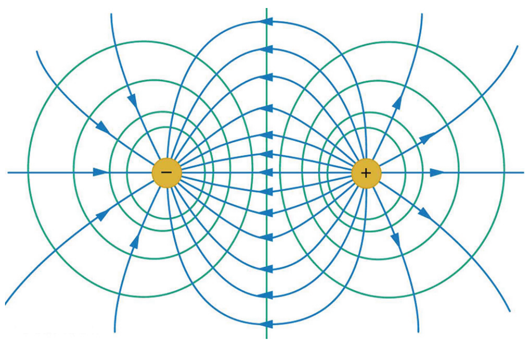


Figure 2. The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.

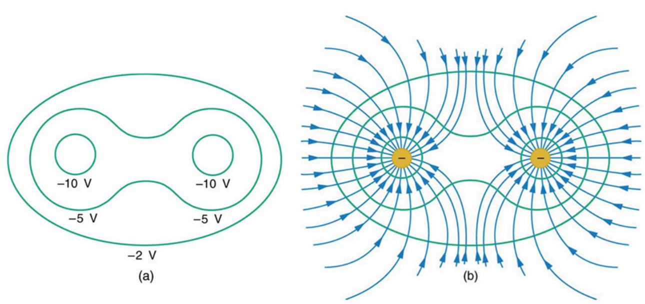


Figure 3. (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges

One of the most important cases is that of the familiar parallel conducting plates shown in Figure 4. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.

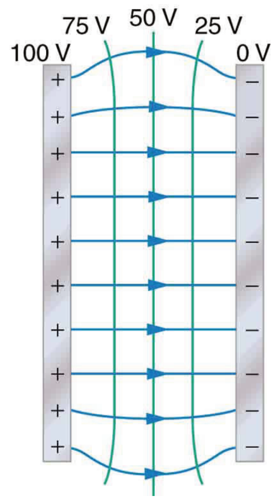


Figure 4. The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in Chapter 19.7 Energy Stored in Capacitors.

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

Figure 5. Charges and Fields

Section Summary

- An equipotential line is a line along which the electric potential is constant.

- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

Conceptual Questions

1: What is an equipotential line? What is an equipotential surface?

2: Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

3: Can different equipotential lines cross? Explain.

Problems & Exercises

1: (a) Sketch the equipotential lines near a point charge $+q$. Indicate the direction of increasing potential. (b) Do the same for a point charge $-3q$.

2: Sketch the equipotential lines for the two equal positive charges shown in Figure 6. Indicate the direction of increasing potential.

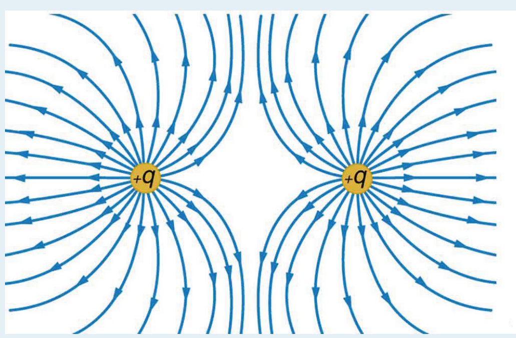


Figure 6. The electric field near two equal positive charges is directed away from each of the charges.

3: Figure 7 shows the electric field lines near two charges q_1 and q_2, the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.

4: Sketch the equipotential lines a long distance from the charges shown in Figure 7. Indicate the direction of increasing potential.

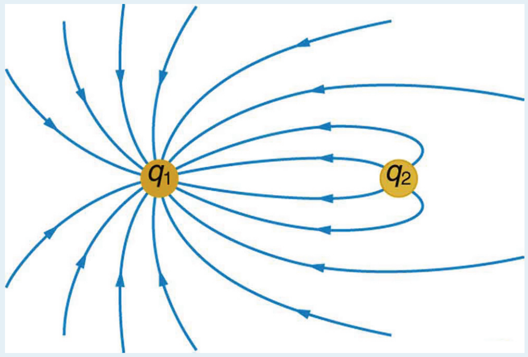


Figure 7. The electric field near two charges.

5: Sketch the equipotential lines in the vicinity of two opposite charges, where the negative charge is three times as great in magnitude as the positive. See Figure 7 for a similar situation. Indicate the direction of increasing potential.

6: Sketch the equipotential lines in the vicinity of the negatively charged conductor in Figure 8. How will these equipotentials look a long distance from the object?

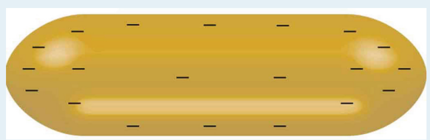


Figure 8. A negatively charged conductor.

7: Sketch the equipotential lines surrounding the two conducting plates shown in Figure 9, given the top plate is positive and the bottom plate has an equal amount of negative charge. Be certain to indicate the distribution of charge on the plates. Is the field strongest where the plates are closest? Why should it be?

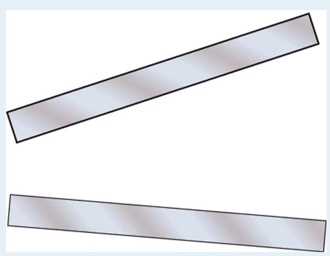


Figure 9.

8: (a) Sketch the electric field lines in the vicinity of the charged insulator in Figure 10. Note its non-uniform charge distribution. (b) Sketch equipotential lines surrounding the insulator. Indicate the direction of increasing potential.

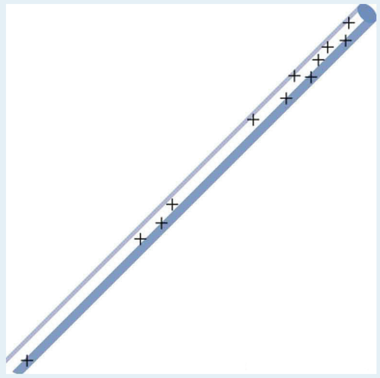


Figure 10. A charged insulating rod such as might be used in a classroom demonstration.

9: The naturally occurring charge on the ground on a fine day out in the open country is $-1.00\ \text{nC/m}^2$. (a) What is the electric field relative to ground at a height of 3.00 m? (b) Calculate the electric potential at this height. (c) Sketch electric field and equipotential lines for this scenario.

10: The lesser electric ray (*Narcine bancroftii*) maintains an incredible charge on its head and a charge equal in magnitude but opposite in sign on its tail (Figure 11). (a) Sketch the equipotential lines surrounding the ray. (b) Sketch the equipotentials when the ray is near a ship with a conducting surface. (c) How could this charge distribution be of use to the ray?

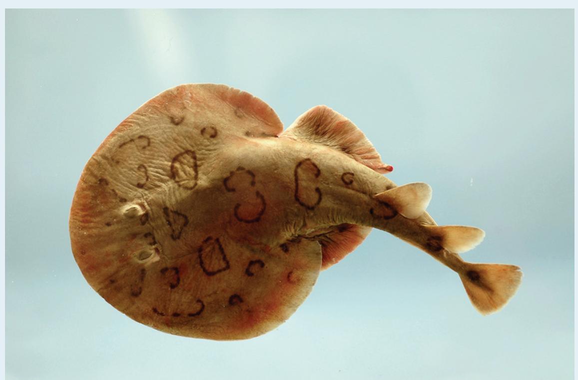


Figure 11. Lesser electric ray (*Narcine bancroftii*) (credit: National Oceanic and Atmospheric Administration, NOAA's Fisheries Collection).

Glossary

equipotential line

a line along which the electric potential is constant

grounding

fixing a conductor at zero volts by connecting it to the earth or ground

19.5 Capacitors and Dielectrics

Summary

- Describe the action of a capacitor and define capacitance.
- Explain parallel plate capacitors and their capacitances.
- Discuss the process of increasing the capacitance of a dielectric.
- Determine capacitance given charge and voltage.

A capacitor is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in Figure 1. (Most of the time an insulator is used between the two plates to provide separation—see the discussion on dielectrics below.) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

Capacitor

A capacitor is a device used to store electric charge.

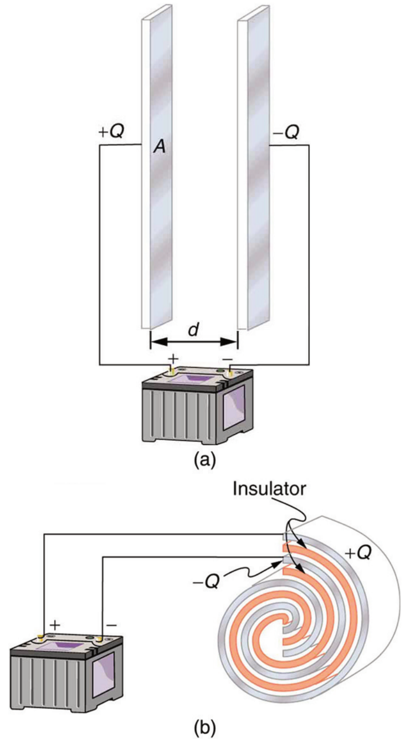


Figure 1. Both capacitors shown here were initially uncharged before being connected to a battery. They now have separated charges of ***+Q*** and ***–Q*** on their two halves. (a) A parallel plate capacitor. (b) A rolled capacitor with an insulating material between its two conducting sheets.

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

The Amount of Charge *Q* a Capacitor Can Store

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

A system composed of two identical, parallel conducting plates separated by a distance, as in Figure 2, is called a parallel plate capacitor. It is easy to see the relationship between the voltage and the stored charge for a parallel plate capacitor, as shown in Figure 2. Each electric field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. (Drawing a single field line per charge is a convenience, only. We can draw many field lines for each charge, but the total number is proportional to the number of charges.) The electric field strength is, thus, directly proportional to Q

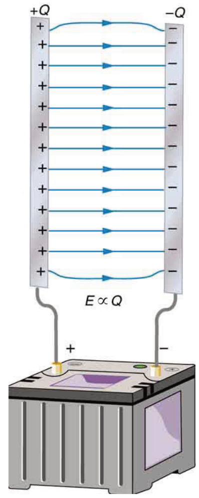


Figure 2. Electric field lines in this parallel plate capacitor, as always, start on positive charges and end on negative charges. Since the electric field strength is proportional to the density of field lines, it is also proportional to the amount of charge on the capacitor.

The field is proportional to the charge:

$$E \propto Q,$$

where the symbol $\propto$ means "proportional to." From the discussion in Chapter 19.2 Electric Potential in a Uniform Electric Field, we know that the voltage across parallel plates is $V = Ed$. Thus,

$$V \propto E.$$

It follows, then, that $V \propto Q$, and conversely,

$$Q \propto V.$$

This is true in general: The greater the voltage applied to any capacitor, the greater the charge stored in it.

Different capacitors will store different amounts of charge for the same applied voltage, depending on their physical characteristics. We define their capacitance C to be such that the charge Q stored in a capacitor is proportional to C. The charge stored in a capacitor is given by

$$Q = CV.$$

This equation expresses the two major factors affecting the amount of charge stored. Those factors are the physical characteristics of the capacitor, C, and the voltage, *VV*. Rearranging the equation, we see that *capacitance* C *is the amount of charge stored per volt,* or

$$C = \frac{Q}{V}.$$

Capacitance

Capacitance C is the amount of charge stored per volt, or

$$C = \frac{Q}{V}.$$

The unit of capacitance is the farad (F), named for Michael Faraday (1791–1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt, or

$$1\text{ F} = \frac{1\text{ C}}{1\text{ V}}.$$

A 1-farad capacitor would be able to store 1 coulomb (a very large amount of charge) with the application of only 1 volt. One farad is, thus, a very large capacitance. Typical capacitors range from fractions of a picofarad $(1\text{ pF} = 10^{-12}\text{ F})$ to millifarads $(1\text{ mF} = 10^{-3}\text{ F})$.

Figure 3 shows some common capacitors. Capacitors are primarily made of ceramic, glass, or plastic, depending upon purpose and size. Insulating materials, called dielectrics, are commonly used in their construction, as discussed below.

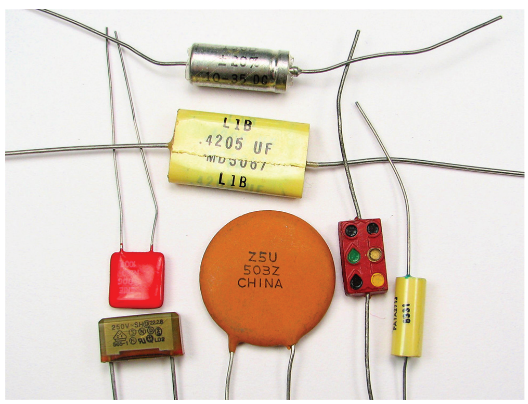


Figure 3. Some typical capacitors. Size and value of capacitance are not necessarily related. (credit: Windell Oskay)

Parallel Plate Capacitor

The parallel plate capacitor shown in Figure 4 has two identical conducting plates, each having a surface area A, separated by a distance d (with no material between the plates). When a voltage V is applied to the capacitor, it stores a charge Q, as shown. We can see how its capacitance depends on A and d by considering the characteristics of the Coulomb force. We know that like charges repel, unlike charges attract, and the force between charges decreases with distance. So it seems quite reasonable that the bigger the plates are, the more charge they can store—because the charges can spread out more. Thus C should be greater for larger A. Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. So C should be greater for smaller d.

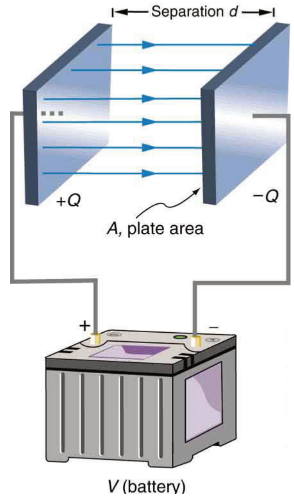


Figure 4. Parallel plate capacitor with plates separated by a distance ***d***. Each plate has an area ***A***.

It can be shown that for a parallel plate capacitor there are only two factors (A and d) that affect its capacitance C. The capacitance of a parallel plate capacitor in equation form is given by

$$C=\varepsilon_0\frac{A}{d}.$$

Capacitance of a Parallel Plate Capacitor

$$C=\varepsilon_0\frac{A}{d}.$$

A is the area of one plate in square meters, and d is the distance between the plates in meters. The constant ε_0 is the permittivity of free space; its numerical value in SI units is $\varepsilon_0=8.85\times10^{-12}\text{ F/m}$. The units of F/m are equivalent to $\text{C}^2/\text{N}\cdot\text{m}^2$. The small numerical value of ε_0 is related to the large size of the farad. A parallel plate capacitor must have a large area to have a capacitance approaching a farad. (Note that the above equation is valid when the parallel plates are separated by air or free space. When another material is placed between the plates, the equation is modified, as discussed below.)

Example 1: Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel plate capacitor with metal plates, each of area 1.00 m^2, separated by 1.00 mm? (b) What charge is stored in this capacitor if a voltage of $3.00\times10^3\text{ V}$ is applied to it?

Strategy

Finding the capacitance C is a straightforward application of the equation $C=\varepsilon_0A/d$. Once C is found, the charge stored can be found using the equation $Q=CV$.

Solution for (a)

Entering the given values into the equation for the capacitance of a parallel plate capacitor yields

$$\begin{aligned} C &= \varepsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\ \frac{\text{F}}{\text{m}}\right)\frac{1.00\ \text{m}^2}{1.00\times10^{-3}\ \text{m}} \\ &= 8.85 \times 10^{-9}\ \text{F} = 8.85\ \text{nF}. \end{aligned}$$

Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very large area thin foils placed close together.

Solution for (b)

The charge stored in any capacitor is given by the equation $Q = CV$. Entering the known values into this equation gives

$$\begin{aligned} Q &= CV = (8.85 \times 10^{-9}\ \text{F})(3.00 \times 10^{3}\ \text{V}) \\ &= 26.6\ \mu\text{C}. \end{aligned}$$

Discussion for (b)

This charge is only slightly greater than those found in typical static electricity. Since air breaks down at about $3.00 \times 10^6\ \text{V/m}$, more charge cannot be stored on this capacitor by increasing the voltage.

Another interesting biological example dealing with electric potential is found in the cell's plasma membrane. The membrane sets a cell off from its surroundings and also allows ions to selectively pass in and out of the cell. There is a potential difference across the membrane of about –70 mV. This is due to the mainly negatively charged ions in the cell and the predominance of positively charged sodium (Na^+) ions outside. Things change when a nerve cell is stimulated. Na^+ ions are allowed to pass through the membrane into the cell, producing a positive membrane potential—the nerve signal. The cell membrane is about 7 to 10 nm thick. An approximate value of the electric field across it is given by

$$E = \frac{V}{d} = \frac{-70\times10^{-3}\ \text{V}}{8\times10^{-9}\ \text{m}} = -9 \times 10^{6}\ \text{V/m}.$$

This electric field is enough to cause a breakdown in air.

Dielectric

The previous example highlights the difficulty of storing a large amount of charge in capacitors. If d is made smaller to produce a larger capacitance, then the maximum voltage must be reduced proportionally to avoid breakdown (since $E = V/d$). An important solution to this difficulty is to put an insulating material, called a dielectric, between the plates of a capacitor and allow d to be as small as possible. Not only does the smaller d make the capacitance greater, but many insulators can withstand greater electric fields than air before breaking down.

There is another benefit to using a dielectric in a capacitor. Depending on the material used, the capacitance is greater than that given by the equation $C = \varepsilon_0 \frac{A}{d}$ by a factor κ, called the *dielectric constant*. A parallel plate capacitor with a dielectric between its plates has a capacitance given by

$$C = \kappa\varepsilon_0 \frac{A}{d}\ \text{(parallel plate capacitor with dielectric)}.$$

Values of the dielectric constant κ for various materials are given in Table 1. Note that κ for vacuum is exactly 1, and so the above equation is valid in that case, too. If a dielectric is used, perhaps by placing Teflon between the plates of the capacitor in Example 1, then the capacitance is greater by the factor κ, which for Teflon is 2.1.

Take-Home Experiment: Building a Capacitor

How large a capacitor can you make using a chewing gum wrapper? The plates will be the aluminum foil, and the separation (dielectric) in between will be the paper.

Material	Dielectric constant κ	Dielectric strength (V/m)
Vacuum	1.00000	—
Air	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Pyrex glass	5.6	14×10^6
Silicon oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Water	80	—

Table 1. Dielectric Constants and Dielectric Strengths for Various Materials at 20ºC

Note also that the dielectric constant for air is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates *except* that the air can become conductive if the electric field strength becomes too great. (Recall that $E = V/d$ for a parallel plate capacitor.) Also shown in Table 1 are maximum electric field strengths in V/m, called dielectric strengths, for several materials. These are the fields above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. For instance, in Example 1, the separation is 1.00 mm, and so the voltage limit for air is

$$\begin{aligned} V &= E \cdot d \\ &= (3 \times 10^6 \text{ V/m})(1.00 \times 10^{-3} \text{ m}) \\ &= 3000 \text{ V} \end{aligned}$$

However, the limit for a 1.00 mm separation filled with Teflon is 60,000 V, since the dielectric strength of Teflon is 60×10^6 V/m. So the same capacitor filled with Teflon has a greater capacitance and can be subjected to a much greater voltage. Using the capacitance we calculated in the above example for the air-filled parallel plate capacitor, we find that the Teflon-filled capacitor can store a maximum charge of

$$\begin{aligned} Q &= CV \\ &= \kappa C_{\text{air}} V \\ &= (2.1)(8.85 \text{ nF})(6.0 \times 10^4 \text{ V}) \\ &= 1.1 \text{ mC}. \end{aligned}$$

This is 42 times the charge of the same air-filled capacitor.

Dielectric Strength

The maximum electric field strength above which an insulating material begins to break down and conduct is called its dielectric strength.

Microscopically, how does a dielectric increase capacitance? Polarization of the insulator is responsible. The more easily it is polarized, the greater its dielectric constant κ. Water, for example, is a polar molecule because one end of the molecule has a slight positive charge and the other end has a slight negative charge. The polarity of water causes it to have a relatively large dielectric constant of 80. The effect of polarization can be best explained in terms of the characteristics of the Coulomb force. Figure 5 shows the separation of charge schematically in the molecules of a dielectric material placed between the charged plates of a capacitor. The Coulomb force between the closest ends of the molecules and the charge on the plates is attractive and very strong, since they are very close together. This attracts more charge onto the plates than if the space were empty and the opposite charges were a distance d away.

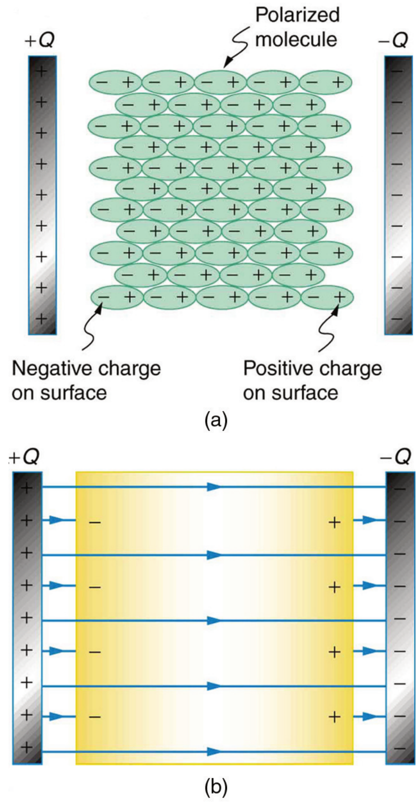


Figure 5. (a) The molecules in the insulating material between the plates of a capacitor are polarized by the charged plates. This produces a layer of opposite charge on the surface of the dielectric that attracts more charge onto the plate, increasing its capacitance. (b) The dielectric reduces the electric field strength inside the capacitor, resulting in a smaller voltage between the plates for the same charge. The capacitor stores the same charge for a smaller voltage, implying that it has a larger capacitance because of the dielectric.

Another way to understand how a dielectric increases capacitance is to consider its effect on the electric field inside the capacitor. Figure 5(b) shows the electric field lines with a dielectric in place. Since the field lines end on charges in the dielectric, there are fewer of them going from one side of the capacitor to the other. So the electric field strength is less than if there were a vacuum between the plates, even though the same charge is on the plates. The voltage between the plates is $V = Ed$, so it too is reduced by the dielectric. Thus there is a smaller voltage V for the same charge Q; since $C = Q/V$, the capacitance C is greater.

The dielectric constant is generally defined to be $\kappa = E_0/E$, or the ratio of the electric field in a vacuum to that in the dielectric material, and is intimately related to the polarizability of the material.

Things Great and Small

The Submicroscopic Origin of Polarization

Polarization is a separation of charge within an atom or molecule. As has been noted, the planetary model of the atom pictures it as having a positive nucleus orbited by negative electrons, analogous to the planets orbiting the Sun. Although this model is not completely accurate, it is very helpful in explaining a vast range of phenomena and will be refined elsewhere, such as in Chapter 30 Atomic Physics. The submicroscopic origin of polarization can be modeled as shown in Figure 6.

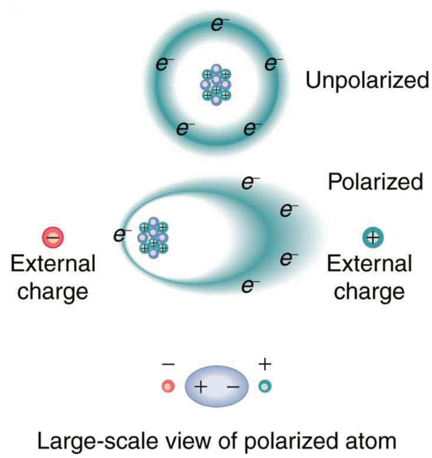


Figure 6. Artist's conception of a polarized atom. The orbits of electrons around the nucleus are shifted slightly by the external charges (shown exaggerated). The resulting separation of charge within the atom means that it is polarized. Note that the unlike charge is now closer to the external charges, causing the polarization.

We will find in Chapter 30 Atomic Physics that the orbits of electrons are more properly viewed as electron clouds with the density of the cloud related to the probability of finding an electron in that location (as opposed to the definite locations and paths of planets in their orbits around the Sun). This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Although the atom remains neutral, it can now be the source of a Coulomb force, since a charge brought near the atom will be closer to one type of charge than the other.

Some molecules, such as those of water, have an inherent separation of charge and are thus called polar molecules. Figure 7 illustrates the separation of charge in a water molecule, which has two hydrogen atoms and one oxygen atom H_2O. The water molecule is not symmetric—the hydrogen atoms are repelled to one side, giving the molecule a boomerang shape. The electrons in a water molecule are more concentrated around the more highly charged oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogen ends slightly positive. The inherent separation of charge in polar molecules makes it easier to align them with external fields and charges. Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants. Those who study chemistry will find that the polar nature of water has many effects. For example, water molecules gather ions much more effectively because they have an electric field and a separation of charge to attract charges of both signs. Also, as brought out in the previous chapter, polar water provides a shield or screening of the electric fields in the highly charged molecules of interest in biological systems.

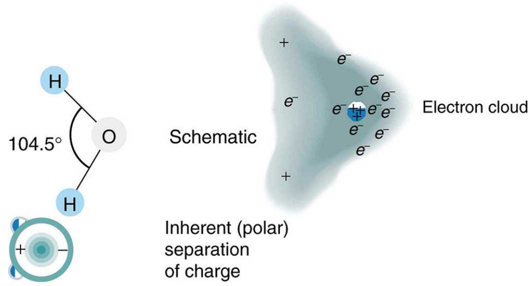


Figure 7. Artist's conception of a water molecule. There is an inherent separation of charge, and so water is a polar molecule. Electrons in the molecule are attracted to the oxygen nucleus and leave an excess of positive charge near the two hydrogen nuclei. (Note that the schematic on the right is a rough illustration of the distribution of electrons in the water molecule. It does not show the actual numbers of protons and electrons involved in the structure.)

PhET Explorations: Capacitor Lab

Explore how a capacitor works! Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electric field in the capacitor. Measure the voltage and the electric field.

PhET Interactive Simulation

Figure 8. Capacitor Lab

Section Summary

- A **capacitor is a device used to store charge.**
- The amount of charge Q a capacitor can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.
- The capacitance C is the amount of charge stored per volt, or $C =$ Formula does not parse, when the plates are separated by air or free space. ε_0 is called the permittivity of free space.
- A parallel plate capacitor with a dielectric between its plates has a capacitance given by

$$C = \kappa\varepsilon_0 \frac{A}{d},$$

where κ is the dielectric constant of the material.

- The maximum electric field strength above which an insulating material begins to break down and conduct is called dielectric strength.

Conceptual Questions

1: Does the capacitance of a device depend on the applied voltage? What about the charge stored in it?

2: Use the characteristics of the Coulomb force to explain why capacitance should be proportional to the plate area of a capacitor. Similarly, explain why capacitance should be inversely proportional to the separation between plates.

3: Give the reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. What is the independent reason that a dielectric material also allows a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V.)

4: How does the polar character of water molecules help to explain water's relatively large dielectric constant? (Figure 7)

5: Sparks will occur between the plates of an air-filled capacitor at lower voltage when the air is humid than when dry. Explain why, considering the polar character of water molecules.

6: Water has a large dielectric constant, but it is rarely used in capacitors. Explain why.

7: Membranes in living cells, including those in humans, are characterized by a separation of charge across the membrane. Effectively, the membranes are thus charged capacitors with important functions related to the potential difference across the membrane. Is energy required to separate these charges in living membranes and, if so, is its source the metabolization of food energy or some other source?

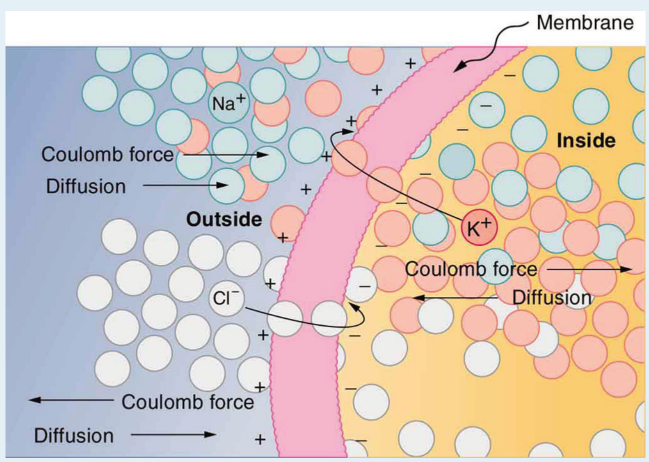


Figure 9. The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+ (sodium ions).

Problems & Exercises

1: What charge is stored in a $180\ \mu F$ capacitor when 120 V is applied to it?

2: Find the charge stored when 5.50 V is applied to an 8.00 pF capacitor.

3: What charge is stored in the capacitor in Example 1?

4: Calculate the voltage applied to a $2.00\ \mu F$ capacitor when it holds $3.10\ \mu C$ of charge.

5: What voltage must be applied to an 8.00 nF capacitor to store 0.160 mC of charge?

6: What capacitance is needed to store $3.00\ \mu C$ of charge at a voltage of 120 V?

7: What is the capacitance of a large Van de Graaff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 MV?

8: Find the capacitance of a parallel plate capacitor having plates of area $5.00\ m^2$ that are separated by 0.100 mm of Teflon.

9: (a)What is the capacitance of a parallel plate capacitor having plates of area $1.50\ m^2$ that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

10: Integrated Concepts

A prankster applies 450 V to an $80.0\ \mu F$ capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. What is the temperature increase of the flesh? Is it reasonable to assume no phase change?

11: Unreasonable Results

(a) A certain parallel plate capacitor has plates of area $4.00\ m^2$, separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

Glossary

capacitor
a device that stores electric charge

capacitance
amount of charge stored per unit volt

dielectric
an insulating material

dielectric strength
the maximum electric field above which an insulating material begins to break down and conduct

parallel plate capacitor
two identical conducting plates separated by a distance

polar molecule
a molecule with inherent separation of charge

Solutions

Problems & Exercises

1: 21.6 mC

3: 80.0 mC

5: 20.0 kV

7: 667 pF

9: (a) $4.4 \text{ } \mu\text{F}$

(b) $4.0 \times 10^{-5} \text{ C}$

11: (a) 14.2 kV

(b) The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon.

(c) The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

19.6 Capacitors in Series and Parallel

Summary

- Derive expressions for total capacitance in series and in parallel.
- Identify series and parallel parts in the combination of connection of capacitors.
- Calculate the effective capacitance in series and parallel given individual capacitances.

Several capacitors may be connected together in a variety of applications. Multiple connections of capacitors act like a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. There are two simple and common types of connections, called *series* and *parallel*, for which we can easily calculate the total capacitance. Certain more complicated connections can also be related to combinations of series and parallel.

Capacitance in Series

Figure 1(a) shows a series connection of three capacitors with a voltage applied. As for any capacitor, the capacitance of the combination is related to charge and voltage by $C=\frac{Q}{V}$.

Note in Figure 1 that opposite charges of magnitude Q flow to either side of the originally uncharged combination of capacitors when the voltage V is applied. Conservation of charge requires that equal-magnitude charges be created on the plates of the individual capacitors, since charge is only being separated in these originally neutral devices. The end result is that the combination resembles a single capacitor with an effective plate separation greater than that of the individual capacitors alone. (See Figure 1(b).) Larger plate separation means smaller capacitance. It is a general feature of series connections of capacitors that the total capacitance is less than any of the individual capacitances.

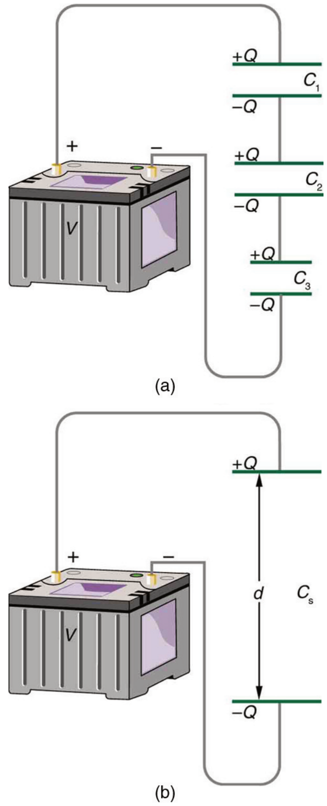


Figure 1. (a) Capacitors connected in series. The magnitude of the charge on each plate is ***Q***. (b) An equivalent capacitor has a larger plate separation ***d***. Series connections produce a total capacitance that is less than that of any of the individual capacitors.

We can find an expression for the total capacitance by considering the voltage across the individual capacitors shown in Figure 1. Solving $C=\frac{Q}{V}$ for V gives $V=\frac{Q}{C}$. The voltages across the individual capacitors are thus $V_1=\frac{Q}{C_1}$, $V_2=\frac{Q}{C_2}$, and $V_3=\frac{Q}{C_3}$. The total voltage is the sum of the individual voltages:

$$V=V_1+V_2+V_3.$$

Now, calling the total capacitance C_S for series capacitance, consider that

$$V=\frac{Q}{C_S}=V_1+V_2+V_3.$$

Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_S}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}.$$

Canceling the Qs, we obtain the equation for the total capacitance in series C_S to be

$$\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots,$$

where "…" indicates that the expression is valid for any number of capacitors connected in series. An expression of this form always results in a total capacitance C_s that is less than any of the individual capacitances C_1, C_2, ..., as the next example illustrates.

Total Capacitance in Series, C_S

Total capacitance in series: $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$

Example 1: What Is the Series Capacitance?

Find the total capacitance for three capacitors connected in series, given their individual capacitances are 1.000, 5.000, and 8.000 μF.

Strategy

With the given information, the total capacitance can be found using the equation for capacitance in series.

Solution

Entering the given capacitances into the expression for $\frac{1}{C_S}$ gives $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$.

$$\frac{1}{C_S} = \frac{1}{1.000\ \mu F} + \frac{1}{5.000\ F} + \frac{1}{8.000\ \mu F} = \frac{1.325}{\mu F}$$

Inverting to find C_S yields $C_S = \frac{\mu F}{1.325} = 0.755\ \mu F$.

Discussion

The total series capacitance C_s is less than the smallest individual capacitance, as promised. In series connections of capacitors, the sum is less than the parts. In fact, it is less than any individual. Note that it is sometimes possible, and more convenient, to solve an equation like the above by finding the least common denominator, which in this case (showing only whole-number calculations) is 40. Thus,

$$\frac{1}{C_S} = \frac{40}{40\ \mu F} + \frac{8}{40\ \mu F} + \frac{5}{40\ \mu F} = \frac{53}{40\ \mu F},$$

so that

$$C_S = \frac{40\ \mu F}{53} = 0.755\ \mu F.$$

Capacitors in Parallel

Figure 2(a) shows a parallel connection of three capacitors with a voltage applied. Here the total capacitance is easier to find than in the series case. To find the equivalent total capacitance C_p, we first note that the voltage across each capacitor is V, the same as that of the source, since they are connected directly to it through a conductor. (Conductors are equipotentials, and so the voltage across the capacitors is the same as that across the voltage source.) Thus the capacitors have the same charges on them as they would have if connected individually to the voltage source. The total charge Q is the sum of the individual charges:

$$Q = Q_1 + Q_2 + Q_3.$$

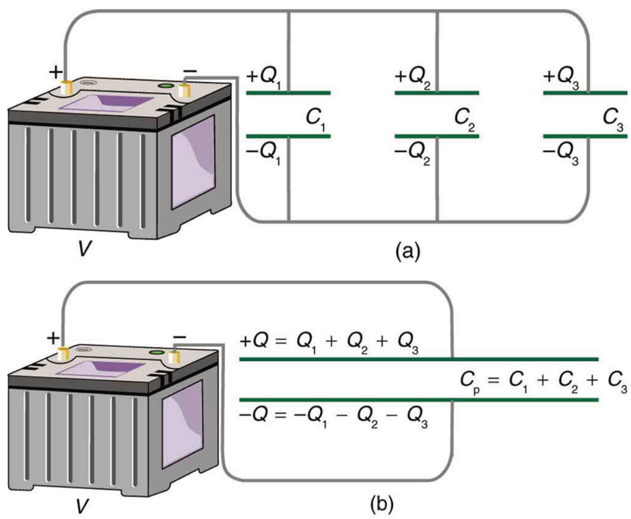


Figure 2. (a) Capacitors in parallel. Each is connected directly to the voltage source just as if it were all alone, and so the total capacitance in parallel is just the sum of the individual capacitances. (b) The equivalent capacitor has a larger plate area and can therefore hold more charge than the individual capacitors.

Using the relationship $Q=CV$, we see that the total charge is $Q=C_pV$, and the individual charges are $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$. Entering these into the previous equation gives

$$C_pV=C_1V+C_2V+C_3V.$$

Canceling V from the equation, we obtain the equation for the total capacitance in parallel C_p:

$$C_p=C_1+C_2+C_3\cdots.$$

Total capacitance in parallel is simply the sum of the individual capacitances. (Again the "…" indicates the expression is valid for any number of capacitors connected in parallel.) So, for example, if the capacitors in the example above were connected in parallel, their capacitance would be

$$C_p=1.000\ \mu\text{F}+5.000\ \mu\text{F}+8.000\ \mu\text{F}=14.000\ \mu\text{F}.$$

The equivalent capacitor for a parallel connection has an effectively larger plate area and, thus, a larger capacitance, as illustrated in Figure 2(b).

Total Capacitance in Parallel, C_p C_p

Total capacitance in parallel $C_p=C_1+C_2+C_3+\cdots$

More complicated connections of capacitors can sometimes be combinations of series and parallel. (See Figure 3.) To find the total capacitance of such combinations, we identify series and parallel parts, compute their capacitances, and then find the total.

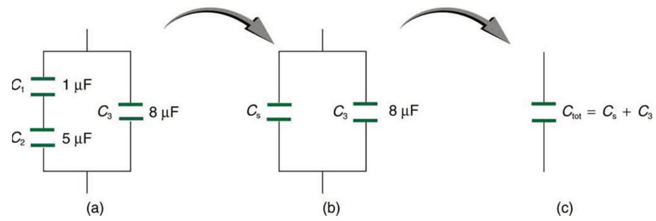


Figure 3. (a) This circuit contains both series and parallel connections of capacitors. See Example 2 for the calculation of the overall capacitance of the circuit. (b) C_1 and C_2 are in series; their equivalent capacitance C_S is less than either of them. (c) Note that C_S is in parallel with C_3. The total capacitance is, thus, the sum of C_S and C_3.

A Mixture of Series and Parallel Capacitance

Find the total capacitance of the combination of capacitors shown in Figure 3. Assume the capacitances in Figure 3 are known to three decimal places ($C_1 = 1.000\ \mu F$, $C_2 = 5.000\ \mu F$, and $C_3 = 8.000\ \mu F$), and round your answer to three decimal places.

Strategy

To find the total capacitance, we first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S in the figure, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their total capacitance is given by $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Entering their values into the equation gives

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu F} + \frac{1}{5.000\ \mu F} = \frac{1.200}{\mu F}.$$

Inverting gives

$$C_S = 0.833\ \mu F.$$

This equivalent series capacitance is in parallel with the third capacitor; thus, the total is the sum

$$\begin{aligned} C_{tot} &= C_S + C_S \\ &= 0.833\ \mu F + 8.000\ \mu F \\ &= 8.833\ \mu F. \end{aligned}$$

Discussion

This technique of analyzing the combinations of capacitors piece by piece until a total is obtained can be applied to larger combinations of capacitors.

Section Summary

- Total capacitance in series $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$
- Total capacitance in parallel $C_p = C_1 + C_2 + C_3 + \cdots$

- If a circuit contains a combination of capacitors in series and parallel, identify series and parallel parts, compute their capacitances, and then find the total.

Conceptual Questions

1: If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

Problems & Exercises

1: Find the total capacitance of the combination of capacitors in Figure 4.

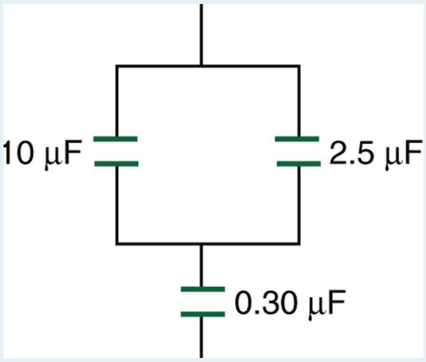


Figure 4. A combination of series and parallel connections of capacitors.

2: Suppose you want a capacitor bank with a total capacitance of 0.750 F and you possess numerous 1.50 mF capacitors. What is the smallest number you could hook together to achieve your goal, and how would you connect them?

3: What total capacitances can you make by connecting a $5.00\ \mu F$ and an $8.00\ \mu F$ capacitor together?

4: Find the total capacitance of the combination of capacitors shown in Figure 5.

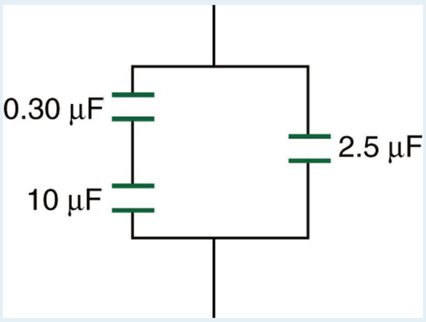


Figure 5. A combination of series and parallel connections of capacitors.

5: Find the total capacitance of the combination of capacitors shown in Figure 6.

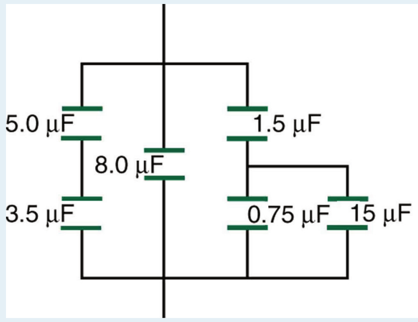


Figure 6. A combination of series and parallel connections of capacitors.

6: Unreasonable Results

(a) An $8.00\ \mu F$ capacitor is connected in parallel to another capacitor, producing a total capacitance of $5.00\ \mu F$. What is the capacitance of the second capacitor? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solutions

Problems & Exercises

1: $0.293\ \mu F$

3: $3.08\ \mu F$ in series combination, $13.0\ \mu F$ in parallel combination

4: $2.79\ \mu F$

6: (a) $-3.00\ \mu F$

(b) You cannot have a negative value of capacitance.

(c) The assumption that the capacitors were hooked up in parallel, rather than in series, was incorrect. A parallel connection always produces a greater capacitance, while here a smaller capacitance was assumed. This could happen only if the capacitors are connected in series.

19.7 Energy Stored in Capacitors

Summary

- List some uses of capacitors.
- Express in equation form the energy stored in a capacitor.
- Explain the function of a defibrillator.

Most of us have seen dramatizations in which medical personnel use a **defibrillator** to pass an electric current through a patient's heart to get it to beat normally. (Review Figure 1.) Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics, such as certain handheld calculators, to supply energy when batteries are charged. (See Figure 1.) Capacitors are also used to supply energy for flash lamps on cameras.

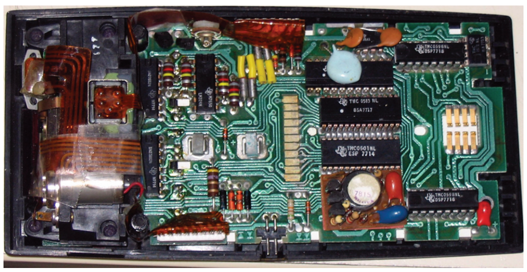


Figure 1. Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged. (credit: Kucharek, Wikimedia Commons)

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge Q and voltage V on the capacitor. We must be careful when applying the equation for electrical potential energy $\Delta\text{PE} = q\Delta V$ to a capacitor. Remember that ΔPE is the potential energy of a charge q going through a voltage ΔV. But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage V on it. The average voltage on the capacitor during the charging process is $V/2$, and so the average voltage experienced by the full charge q is $V/2$. Thus the energy stored in a capacitor, E_{cap}, is

$$E_{\text{cap}} = \frac{QV}{2},$$

where Q is the charge on a capacitor with a voltage V applied. (Note that the energy is not QV, but $QV/2$.) Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$, and so the expression for E_{cap} can be algebraically manipulated into three equivalent expressions:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge and V the voltage on a capacitor C. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

Energy Stored in Capacitors

The energy stored in a capacitor can be expressed in three ways:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

In a defibrillator, the delivery of a large charge in a short burst to a set of paddles across a person's chest can be a lifesaver. The person's heart attack might have arisen from the onset of fast, irregular beating of the heart—cardiac or ventricular fibrillation. The application of a large shock of electrical energy can terminate the arrhythmia and allow the body's pacemaker to resume normal patterns. Today it is common for ambulances to carry a defibrillator, which also uses an electrocardiogram to analyze the patient's heartbeat pattern. Automated external defibrillators (AED) are found in many public places (Figure 2). These are designed to be used by lay persons. The device automatically diagnoses the patient's heart condition and then applies the shock with appropriate energy and waveform. CPR is recommended in many cases before use of an AED.

Figure 2. Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies, Wikimedia Commons)

Example 1: Capacitance in a Heart Defibrillator

A heart defibrillator delivers $4.00 \times 10^2\ \text{J}$ of energy by discharging a capacitor initially at $1.00 \times 10^4\ \text{V}$. What is its capacitance?

Strategy

We are given E_{cap} and V, and we are asked to find the capacitance C. Of the three expressions in the equation for E$_{E_{\text{cap}}}$, the most convenient relationship is

$$E_{\text{cap}} = \frac{CV^2}{2}.$$

Solution

Solving this expression for C and entering the given values yields

$$\begin{aligned} C &= \frac{2E_{\text{cap}}}{V^2} = \frac{2(4.00\times 10^2\ \text{J})}{(1.00\times 10^4\ \text{V})^2} = 8.00 \times 10^{-6}\ \text{F} \\ &= 8.00\ \mu\text{F}. \end{aligned}$$

Discussion

This is a fairly large, but manageable, capacitance at $1.00 \times 10^4\ \text{V}$.

Section Summary

- Capacitors are used in a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps, to supply energy.
- The energy stored in a capacitor can be expressed in three ways:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules when the charge is in coulombs, voltage is in volts, and capacitance is in farads.

Conceptual Questions

1: How does the energy contained in a charged capacitor change when a dielectric is inserted, assuming the capacitor is isolated and its charge is constant? Does this imply that work was done?

2: What happens to the energy stored in a capacitor connected to a battery when a dielectric is inserted? Was work done in the process?

Problems & Exercises

1: (a) What is the energy stored in the $10.0\ \mu F$ capacitor of a heart defibrillator charged to $9.00 \times 10^3\ V$? (b) Find the amount of stored charge.

2: In open heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the $8.00\ \mu F$ capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of stored charge.

3: A $165\ \mu F$ capacitor is used in conjunction with a motor. How much energy is stored in it when 119 V is applied?

4: Suppose you have a 9.00 V battery, a $2.00\ \mu F$ capacitor, and a $7.40\ \mu F$ capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.

5: A nervous physicist worries that the two metal shelves of his wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1.00 \times 10^2\ m^2$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored.

6: Show that for a given dielectric material the maximum energy a parallel plate capacitor can store is directly proportional to the volume of dielectric ($\text{Volume} = A \cdot d$). Note that the applied voltage is limited by the dielectric strength.

7: Construct Your Own Problem

Consider a heart defibrillator similar to that discussed in Example 1. Construct a problem in which you examine the charge stored in the capacitor of a defibrillator as a function of stored energy. Among the things to be considered are the applied voltage and whether it should vary with energy to be delivered, the range of energies involved, and the capacitance of the defibrillator. You may also wish to consider the much smaller energy needed for defibrillation during open-heart surgery as a variation on this problem.

8: Unreasonable Results

(a) On a particular day, it takes $9.60 \times 10^3\ J$ of electric energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Glossary

defibrillator
a machine used to provide an electrical shock to a heart attack victim's heart in order to restore the heart's normal rhythmic pattern

Solutions

Problems & Exercises

1: (a) 405 J

(b) 90.0 mC

2: (a) 3.16 kV

(b) 25.3 mC

4: (a) 1.42×10^{-5} C, 6.38×10^{-5} J
(b) 8.46×10^{-5} C, 3.81×10^{-4} J

5: (a) 4.43×10^{-12} F
(b) 452 V

(c) 4.52×10^{-7} J

8: (a) 133 F

(b) Such a capacitor would be too large to carry with a truck. The size of the capacitor would be enormous.

(c) It is unreasonable to assume that a capacitor can store the amount of energy needed.

PART 13

Chapter 20 Electric Current, Resistance, and Ohm's Law

Figure 1. Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailam power station located along the Krishna River in India, by the movement of charge—that is, by electric current. (credit: Chintohere, Wikimedia Commons)

The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge*. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.

20.1 Current

Summary

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

$$I = \frac{\Delta Q}{\Delta t}$$

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See Figure 1.) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

$$1\text{ A} = 1\text{ C/s}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

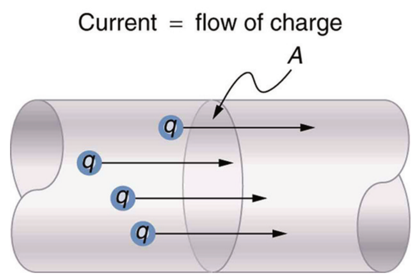


Figure 1. The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example 1: Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q/\Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720\ \text{C}}{4.00\ \text{s}} = 180\ \text{C/s} \\ &= 180\ \text{A}. \end{aligned}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q/\Delta t$ for time Δt, and entering the known values for charge and current gives

$$\begin{aligned} \Delta t &= \frac{\Delta Q}{I} = \frac{1.00\ \text{C}}{0.300 \times 10^{-3}\ \text{C/s}} \\ &= 3.33 \times 10^{3}\ \text{s}. \end{aligned}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It’s because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

Figure 2 shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in Figure 2 (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.

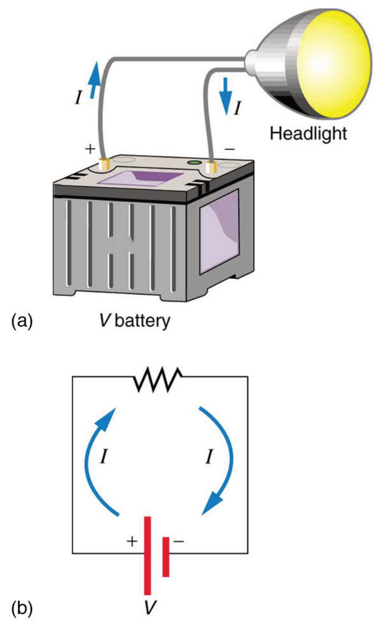


Figure 2. (a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.

Note that the direction of current flow in Figure 2 is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. Figure 3 illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in Figure 3. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.

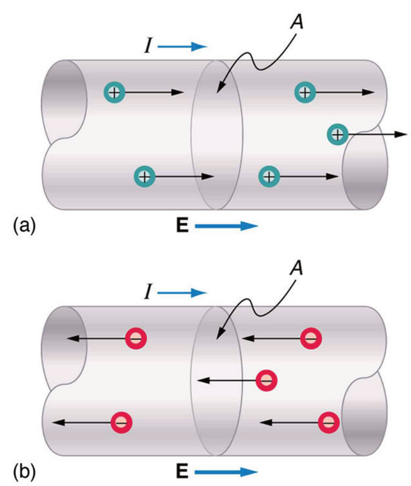


Figure 3. Current ***I*** is the rate at which charge moves through an area ***A***, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example 2: Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the Example 1 example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite,

$I_{\text{electrons}} = -0.300 \times 10^{-3}\ \text{C/s}$.Since each electron ($e^-$) has a charge of $-1.60 \times 10^{-19}\ \text{C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} \$ = \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}}.$$

We divide this by the charge per electron, so that

$$\begin{aligned}\frac{e^-}{\text{s}} &= \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}} \times \frac{1\ e^-}{-1.60\times10^{-19}\ \text{C}}\\ &= 1.88 \times 10^{15}\ \frac{e^-}{\text{s}}\end{aligned}$$

Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of $10^8\ \text{m/s}$, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of $10^{-4}\ \text{m/s}$. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in [link], the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.s

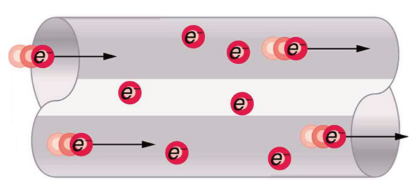


Figure 4. When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. Figure 5 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** v_d is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.

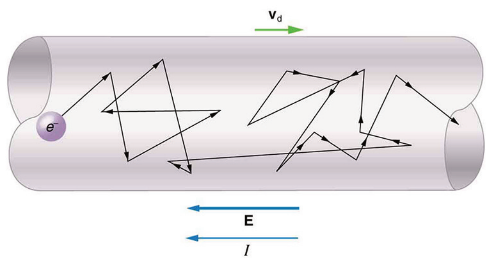


Figure 5. Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, $\mathbf{v_d}$, and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in Figure 6. *The number of free charges per unit volume* is given the symbol n and depends on the material. The shaded segment has a volume Ax, so that the number of free charges in it is nAx. The charge ΔQ in this segment is thus $qnAx$, where q is the amount of charge on each carrier. (Recall that for electrons, q is -1.60×10^{-19} C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt, the current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnAx}{\Delta t}.$$

Note that $x/\Delta t$ is the magnitude of the drift velocity, v_d, since the charges move an average distance x in a time Δt. Rearranging terms gives

$$I = nqAv_d,$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n. The carriers of the current each have charge q and move with a drift velocity of magnitude v_d.

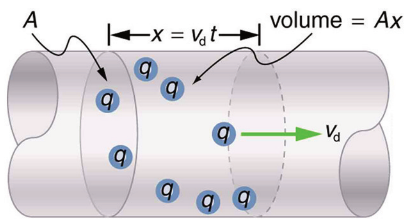


Figure 6. All the charges in the shaded volume of this wire move out in a time ***t***, having a drift velocity of magnitude $\mathbf{v_d = x/t}$. See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a “sea” of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Example 3: Calculating Drift Velocity in a Common Wire

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80 \times 10^3\ \text{kg/m}^3$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_d$. The current $I = 20.0\ \text{A}$ is given, and $q = -1.60 \times 10^{-19}\ \text{C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80 \times 10^3\ \text{kg/m}^3$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02 \times 10^{23}\ \text{atoms/mol}$, to determine n, the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per m^3. We can now find n as follows:

$$\begin{aligned} n &= \frac{1\ e^-}{\text{atom}} \times \frac{6.02 \times 10^{23}\ \text{atoms}}{\text{mol}} \times \frac{1\ \text{mol}}{63.54\ \text{g}} \times \frac{1000\ \text{g}}{\text{kg}} \times \frac{8.80 \times 10^3\ \text{kg}}{1\ \text{m}^3} \\ &= 8.342 \times 10^{28}\ e^-/\text{m}^3 \end{aligned}$$

The cross-sectional area of the wire is

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{2.053 \times 10^{-3}\ \text{m}}{2}\right)^2 \\ &= 3.310 \times 10^{-6}\ \text{m}^2 \end{aligned}$$

Rearranging $I = nqAv_d$ to isolate drift velocity gives

$$v_d = \frac{I}{nqA}$$

$$= \frac{20.0\ \text{A}}{(8.342 \times 10^{28}/\text{m}^3)(-1.60 \times 10^{-19}\ \text{C})(3.310 \times 10^{-6}\ \text{m}^2)}$$

$$= -4.53 \times 10^{-4}\ \text{m/s}$$

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of $10^{-4}\ \text{m/s}$) confirms that the signal moves on the order of 10^{12} times faster (about $10^8\ \text{m/s}$) than the charges that carry it.

Section Summary

- Electric current I is the rate at which charge flows, given by

  $$I = \frac{\Delta Q}{\Delta t},$$

  where ΔQ is the amount of charge passing through an area in time Δt.
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1\ \text{A} = 1\ \text{C/s}$.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_d is the average speed at which these charges move.
- Current I is proportional to drift velocity v_d, as expressed in the relationship $I = nqAv_d$. Here, I is the current through a wire of cross-sectional area A. The wire's material has a free-charge density n, and each carrier has charge q and a drift velocity v_d.
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

Conceptual Questions

1: Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

2: Car batteries are rated in ampere-hours ($A \cdot h$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

3: If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_d = \frac{I}{nqA}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

4: Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

5: In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

6: Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

Problems & Exercises

1: What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

2: A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

3: What is the current when a typical static charge of $0.250\ \mu C$ moves from your finger to a metal doorknob in $1.00\ \mu s$?

4: Find the current when 2.00 nC jumps between your comb and hair over a 0.500 – μs time interval.

5: A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

6: The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

7: (a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2R$.)

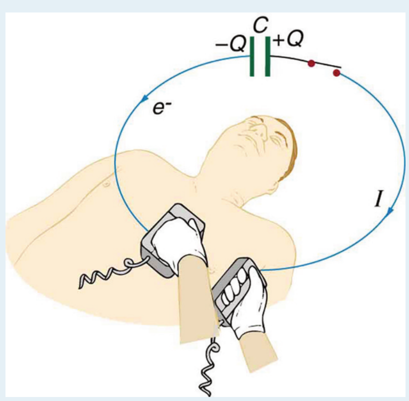


Figure 7. The capacitor in a defibrillation unit drives a current through the heart of a patient

8: During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?

9: (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

10: A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

11: The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

12: Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

13: A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

14: Repeat the above example on Example 3, but for a wire made of silver and given there is one free electron per silver atom.

15: Using the results of the above example on Example 3, find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

16: A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on Example 3 for useful information.)

17: SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See Figure 8.) How many electrons are in the beam?

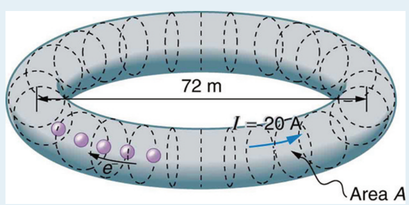


Figure 8. Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

Glossary

electric current

the rate at which charge flows, $I = \Delta Q/\Delta t$

ampere

(amp) the SI unit for current; 1 A = 1 C/s

drift velocity

the average velocity at which free charges flow in response to an electric field

Solutions

Problems & Exercises

1: 0.278 mA

3: 0.250 A

5: 1.50ms

7: (a) $1.67\ \text{k}\Omega$

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation $P = I^2R$), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

9: (a) 0.120 C

(b) 7.50×10^{17} electrons

11: 96.3 s

13: (a)

$7.81 \times 10^{14}\ \text{He}^{++}\ textbfnuclei/s$

(b)

$4.00 \times 10^{3}\ \text{s}$

(c)

$7.71 \times 10^{8}\ \text{s}$

15: $-1.13 \times 10^{-4}\ \text{m/s}$

17: $9.42 \times 10^{13}\ \text{electrons}$

20.2 Ohm's Law: Resistance and Simple Circuits

Summary

- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V.$$

This important relationship is known as Ohm's law. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called resistance RR size 12{R} {}. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}.$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called ohmic. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an ohm and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1\ \frac{V}{A}$$

Figure 1 shows the schematic for a simple circuit. A simple circuit has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.

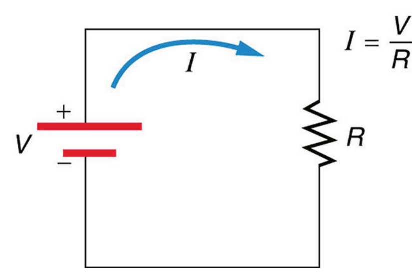


Figure 1. A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Example 1: Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

$R = \frac{V}{I} = \frac{12.0\ \text{V}}{2.50\ \text{A}} = 4.80\ \Omega$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in Chapter 20.3 Resistance and Resistivity, resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\ \Omega$, whereas the resistance of the human heart is about $10^{3}\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in Chapter 20.3 Resistance and Resistivity.

Additional insight is gained by solving $I = V/R$ yielding

$$V = IR.$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I. The phrase IR *drop* is often used for this voltage. For instance, the headlight in Example 1 has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $PE = q\Delta V$, and the same qq size 12{q} {} flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See Figure 2.)

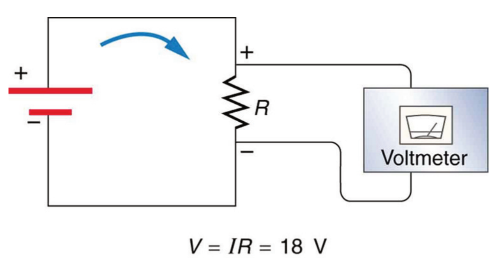


Figure 2. The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

Figure 3. Ohm's Law

Section Summary

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I, voltage V, and resistance R in a simple circuit to be $I=\frac{V}{R}$.
- Resistance has units of ohms (Ω), related to volts and amperes by $1\ \Omega=1\ \text{V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V=IR$.

Conceptual Questions

1: The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

2: How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

Problems & Exercises

1: What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

2: Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

3: What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

4: How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

5: (a) Find the voltage drop in an extension cord having a $0.0600 - \Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

6: A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00 \times 10^{9}\ \Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

Glossary

Ohm's law
an empirical relation stating that the current I is proportional to the potential difference V, ? V; it is often written as I = V/R, where R is the resistance

resistance
the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, R = V/I

ohm
the unit of resistance, given by 1Ω = 1 V/A

ohmic
a type of a material for which Ohm's law is valid

simple circuit
a circuit with a single voltage source and a single resistor

Solutions

Problems & Exercises

1: 0.833 A

3: $7.33 \times 10^{-2}\ \Omega$

5: (a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

20.3 Resistance and Resistivity

Summary

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in Figure 1 is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L, similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A.

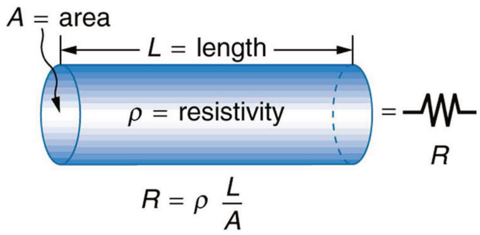


Figure 1. A uniform cylinder of length L and cross-sectional area A. Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow. The longer the cylinder, the greater its resistance. The larger its cross-sectional area A, the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity** ρ of a substance so that the **resistance** R of an object is directly proportional to ρ. Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L, of cross-sectional area A, and made of a material with resistivity ρ, is

$$R = \frac{\rho L}{A}.$$

Table 1 gives representative values of ρ. The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Material	**Resistivity** $\rho(\Omega\cdot\text{m})$
Conductors	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
Semiconductors[1]	
Carbon (pure)	3.5×10^{5}
Carbon	$(3.5-60)\times10^{5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1-600)\times10^{-3}$
Silicon (pure)	2300
Silicon	$0.1-2300$
Insulators	
Amber	5×10^{14}
Glass	$10^{9}-10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11}-10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13}-10^{16}$
Sulfur	10^{15}
Teflon	$>10^{13}$
Wood	$10^{8}-10^{11}$

Table 1. Resistivities ρ of Various materials at 20ºC

Example 1: Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of $0.350\ \Omega$. If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \frac{\rho L}{A}$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \frac{\rho L}{A}$, is

$$A = \frac{\rho L}{R}$$

Substituting the given values, and taking ρ from Table 1, yields

$$\begin{aligned} A &= \frac{(5.6 \times 10^{-8}\ \Omega\cdot\text{m})(4.00 \times 10^{-2}\ \text{m})}{0.350\ \Omega} \\ &= 6.40 \times 10^{-9}\ \text{m}^2 \end{aligned}.$$

The area of a circle is related to its diameter D by

$$A = \frac{\pi D^2}{4}$$

Solving for the diameter D, and substituting the value found for A, gives

$$\begin{aligned} D &= 2\left(\frac{A}{p}\right)^{\frac{1}{2}} = 2\left(\frac{6.40 \times 10^{-9}\ \text{m}^2}{3.14}\right)^{\frac{1}{2}} \\ &= 9.0 \times 10^{-5}\ \text{m} \end{aligned}.$$

Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See Figure 2.) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about 100ºC or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

$$\rho = \rho_0(1 + \alpha\Delta T),$$

where ρ_0 is the original resistivity and α is the **temperature coefficient of resistivity**. (See the values of α in Table 2 below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ. Note that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has α *close to zero (to three digits on the scale in Table 2), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.*

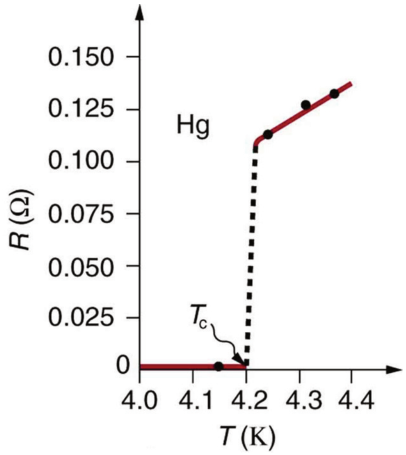


Figure 2. The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Material	**Coefficient α *(1/°C)*** [2]
Conductors	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
Semiconductors	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Table 2: Tempature Coefficients of Resistivity α

Note also that α is negative for the semiconductors listed in Table 2, meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ. For a cylinder we know $R = \rho L/A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ. (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ.) Thus,

$$R = R_0(1 + \alpha\Delta T)$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT. Numerous thermometers are based on the effect of temperature on resistance. (See Figure 3.) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.

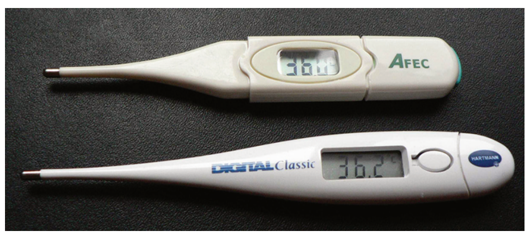


Figure 3. These familiar thermometers are based on the automated measurement of a thermistor's temperature-dependent resistance. (credit: Biol, Wikimedia Commons)

Example 2: Calculating Resistance: Hot-Filament Resistance

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100ºC, for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20ºC) to a typical operating temperature of 2850ºC?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350\ \Omega$, and the temperature change is $\Delta T = 2830°\text{C}$.

Solution

The hot resistance R is obtained by entering known values into the above equation:

$$\begin{aligned} R &= R_0(1 + \alpha\Delta T) \\ &= (0.350\ \Omega)[1 + (4.5 \times 10^{-3}/°C)(2830°C)] \ . \\ &= 4.8\ \Omega \end{aligned}$$

Discussion

This value is consistent with the headlight resistance example in Example 1 Chapter 20.2 Ohm's Law: Resistance and Simple Circuits.

PhET Explorations: Resistance in a Wire

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

Figure 4. Resistance in a Wire

Section Summary

- The resistance R of a cylinder of length L and cross-sectional area A is $R=\frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in Table 1 show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT, resistivity is $\rho=\rho_0(1+\alpha\Delta T)$, where ρ_0 is the original resistivity and $\alpha\alpha$ is the temperature coefficient of resistivity.
- Table 2 gives values for α, the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R=R_0(1+\alpha\Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

Conceptual Questions

1: In which of the three semiconducting materials listed in Table 1 do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

2: Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See Figure 5.)

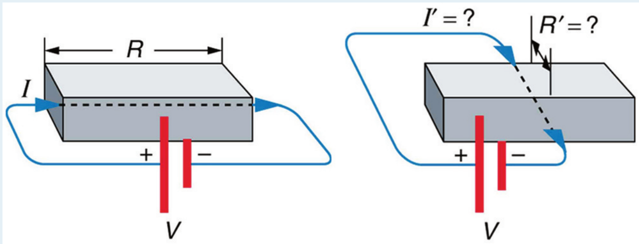


Figure 5. Does current taking two different paths through the same object encounter different resistance?

3: If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

4: Explain why $R = R_0(1 + \alpha\Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0(1 + \alpha\Delta T)$, which gives the temperature variation of resistivity ρ.

Problems & Exercises

1: What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

2: The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

3: If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at 20.0ºC, how long should it be?

4: Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

5: What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00 \times 10^3\ \text{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

6: (a) To what temperature must you raise a copper wire, originally at 20.0ºC, to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

7: A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0ºC. Over what temperature range can it be used?

8: Of what material is a resistor made if its resistance is 40.0% greater at 100ºC than at 20.0ºC?

9: An electronic device designed to operate at any temperature in the range from –10.0ºC to 55.0ºC contains pure carbon resistors. By what factor does their resistance increase over this range?

10: (a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of $77.7\ \Omega$ at 20.0ºC? (b) What is its resistance at 150ºC?

11: Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at 20.0ºC?

12: A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

13: A copper wire has a resistance of $0.500\ \Omega$ at 20.0ºC, and an iron wire has a resistance of $0.525\ \Omega$ at the same temperature. At what temperature are their resistances equal?

14: (a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/°C$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0ºC (normal body temperature)? (b) The negative value for α *may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)*

15: Integrated Concepts

(a) Redo Exercise 2 taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12 \times 10^{-6}/°C$. (b) By what percentage does your answer differ from that in the example?

16: Unreasonable Results

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Footnotes

1. 1 Values depend strongly on amounts and types of impurities
2. 2 Values at 20°C.

Glossary

resistivity
an intrinsic property of a material, independent of its shape or size, directly proportional to the resistance, denoted by ρ
temperature coefficient of resistivity
an empirical quantity, denoted by α, which describes the change in resistance or resistivity of a material with temperature

Solutions

Problems & Exercises

1: $0.104\ \Omega$

3: $2.8 \times 10^{-2}\ \text{m}$

5: $1.10 \times 10^{-3}\ \text{A}$

7: −5ºC to 45ºC

9: 1.03

11: 0.06%

13: −17ºC

15: (a) $4.7\ \Omega$ (total)

(b) 3.0% decrease

20.4 Electric Power and Energy

Summary

- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for electric power? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See Figure 1(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?

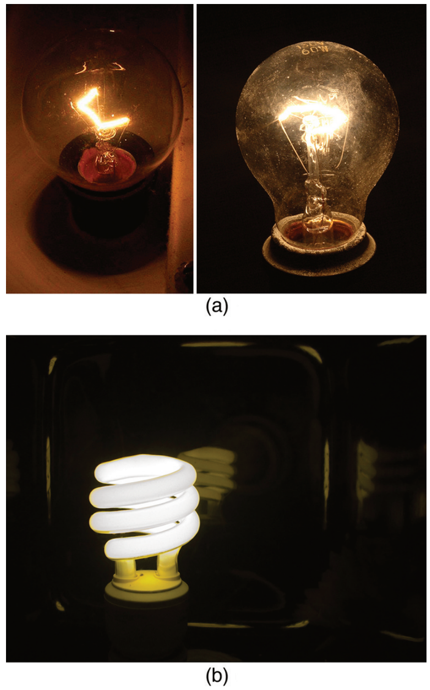


Figure 1. (a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\text{PE} = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{PE}{t} = \frac{qV}{t}.$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = IV.$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\text{ A}\cdot\text{V} = 1\text{ W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20\text{ A})(12\text{ V}) = 240\text{ W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\text{ kA}\cdot\text{V} = 1\text{ kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = IV$$
$$P = \frac{V^2}{R}$$
$$P = I^2R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example 1: Calculating Power Dissipation and Current: Hot and Cold Power

(a) Consider the examples given in Chapter 20.2 Ohm's Law: Resistance and Simple Circuits and Chapter 20.3 Resistance and Resistivity. Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold. (b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2/R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50\ \text{A})(12.0\ \text{V}) = 30.0\ \text{W}.$$

The cold resistance was $0.350\ \Omega$, and so the power it uses when first switched on is

$$P = \frac{V^2}{R} = \frac{(12.0\ \text{V})^2}{0.350\ \Omega} = 411\ \text{W}.$$

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2R$, and enter known values, obtaining

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\ \text{W}}{0.350\ \Omega}} = 34.3\ \text{A}$$

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

$$E = Pt$$

is the energy used by a device using power P for a time interval t. For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1\ \text{kW} \cdot \text{h} = 3.6 \times 10^6\ \text{J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See Figure 1(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example 2: Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E = Pt = (60\text{ W})(1000\text{ h}) = 60,000\text{ W}\cdot\text{ h}.$$

In kilowatt-hours, this is

$$E = 60.0\text{ kW}\cdot\text{ h}.$$

Now the electricity cost is

$$\text{cost} = (60.0\text{ kW}\cdot\text{h})(\$0.12/\text{ kW}\cdot\text{h}) = \$7.20.$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

Section Summary

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are

$$P = IV,$$

$$P = \frac{V^2}{R},$$

  and

$$P = I^2R.$$

- The energy used by a device with a power P over a time t is $E = Pt$.

Conceptual Questions

1: Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

2: The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2R$, which means power increases if resistance increases. Explain why there is no contradiction here.

Probles & Exercises

1: What is the power of a $1.00 \times 10^2 \text{ MV}$ lightning bolt having a current of $2.00 \times 10^4 \text{ A}$?

2: What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

3: A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See Figure 2.)

Figure 2. The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

4: How many watts does a flashlight that has $6.00 \times 10^{2}\ \text{C}$ pass through it in 0.500 h use if its voltage is 3.00 V?

5: Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600 - \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$

6: Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

7: Show that the units $1\ \text{V}^2/\Omega = 1\ \text{W}$, as implied by the equation $P = V^2/R$.

8: Show that the units $1\ \text{A}^2 \cdot \Omega = 1\ \text{W}$, as implied by the equation $P = I^2R$.

9: Verify the energy unit equivalence that $1\ \text{kW} \cdot \text{h} = 3.60 \times 10^{6}\ \text{J}$.

10: Electrons in an X-ray tube are accelerated through $1.00 \times 10^{2}\ \text{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

11: An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\ \text{cents/kW} \cdot \text{h}$? See Figure 3.

Figure 3. On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

12: With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $9.0\ \text{cents/kW} \cdot \text{h}$, how much does this cost?

13: What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs 10 cents/kWh. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

14: Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

15: Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00\text{ A}\cdot\text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

16: A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

17: The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0\text{ cents/kW}\cdot\text{h}$.

18: An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

19: 00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries $1.00 \times 10^2\text{ A}$.

20: Integrated Concepts

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See Figure 4.)

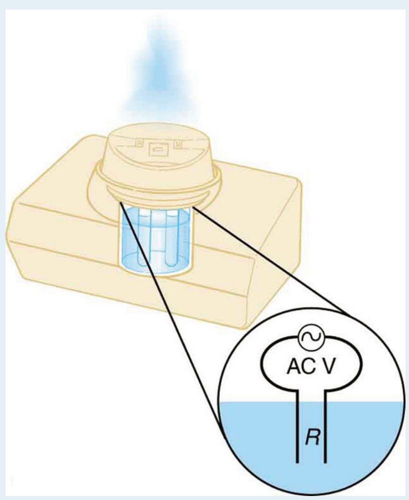


Figure 4. This cold vaporizer passes current directly through water, vaporizing it directly with relatively little temperature increase.

21: Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of $1.00 \times 10^2\text{ MV}$, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0ºC to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

22: Integrated Concepts

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and $3.00 \times 10^2 \text{ g}$ of aluminum from 20.0ºC to 90.0ºC in 5.00 min?

23: Integrated Concepts

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0ºC to 100ºC and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

24: Integrated Concepts

24: Hydroelectric generators (see Figure 5) at Hoover Dam produce a maximum current of $8.00 \times 10^3 \text{ A}$ at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?

Figure 5. Hydroelectric generators at the Hoover dam. (credit: Jon Sullivan)

25: Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a $2.00 \times 10^2 \text{-m}$ -high hill in 2.00 min at a constant 25.0-m/s speed while exerting $5.00 \times 10^2 \text{ N}$ of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a $5.00 \times 10^2 \text{ N}$ force to overcome air resistance and friction? See Figure 6.

Figure 6. This REVAi, an electric car, gets recharged on a street in London. (credit: Frank Hebbert)

26: Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is $5.30 \times 10^4 \text{ kg}$, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

27: Integrated Concepts

(a) An aluminum power transmission line has a resistance of $0.0580 \ \Omega/\text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

28: Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a $1.00 \times 10^2\text{-g}$ aluminum cup containing 350 g of water from 20.0ºC to 95.0ºC in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

29: Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from 10.0ºC to 40.0ºC, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is $9 \text{ cents/kW} \cdot \text{h}$. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

30: Unreasonable Results

(a) What current is needed to transmit $1.00 \times 10^2 \text{ MW}$ of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00 - \Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

31: Unreasonable Results

(a) What current is needed to transmit $1.00 \times 10^2 \text{ MW}$ of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

32: Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

Glossary

electric power

the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

Solutions

Problems & Exercise

1: $2.00 \times 10^{12}\ \text{W}$

5: (a) 1.50 W
(b) 7.50 W

7: $\frac{\text{V}^2}{\Omega} = \frac{\text{V}^2}{\text{V/A}} = \text{AV} = \left(\frac{\text{C}}{\text{s}}\right)\left(\frac{\text{J}}{\text{C}}\right)$

9: $1\ \text{kW}\cdot\text{h} = \left(\frac{1\times10^3}{1\ \text{s}}\right)(1\ \text{h})\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 3.60\times10^6\ \text{J}$

11: $438/y

13: $6.25

15: 1.58 h

17: $3.94 billion/year

19: 25.5 W

21: (a) $2.00 \times 10^9\ \text{J}$

(b) 769 kg

23: 45.0 s

25: (a) 343 A

(b) $2.17 \times 10^3\ \text{A}$

(c) $1.10 \times 10^3\ \text{A}$

27: (a) $1.23 \times 10^3\ \text{kg}$

(b) $2.64 \times 10^3\ \text{kg}$

3o: (a) $2.08 \times 10^5\ \text{A}$

(b) $4.33 \times 10^4\ \text{MW}$

(c) The transmission lines dissipate more power than they are supposed to transmit.

(d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

20.5 Alternating Current versus Direct Current

Summary

- Explain the differences and similarities between AC and DC current.
- Calculate rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

Alternating Current

Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. Direct current (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source. Alternating current (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. Figure 1 shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.

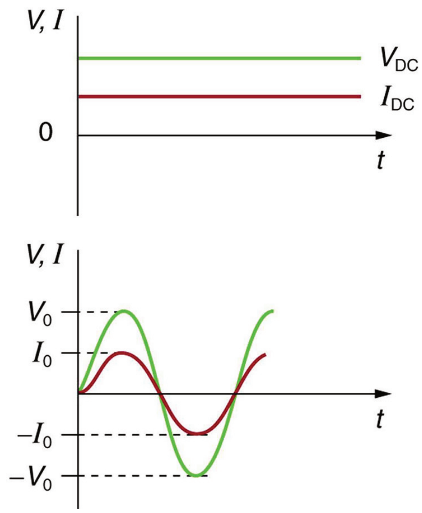


Figure 1. (a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.

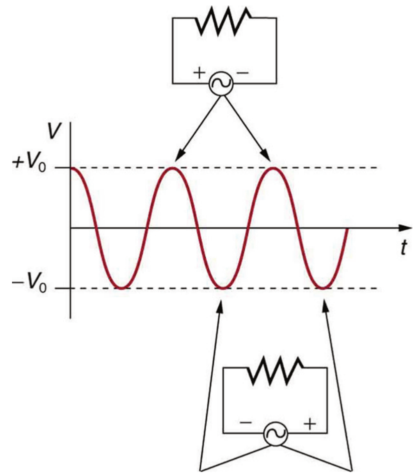


Figure 2. The potential difference ***V*** between the terminals of an AC voltage source fluctuates as shown. The mathematical expression for ***V*** is given by ***V = V₀ sin 2πft***.

Figure 2 shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the AC voltage given by

$$V = V_0 \sin 2\pi ft,$$

where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, $I = V/R$, and so the AC current is

$$I = I_0 \sin 2\pi ft,$$

where I is the current at time t, and $I_0 = V_0/R$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in Figure 1(b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = IV$. Using the expressions for I and V above, we see that the time dependence of power is $P = I_0V_0 \sin^2 2\pi ft$, as shown in Figure 3.

Making Connections: Take-Home Experiment—AC/DC Lights

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car? Explain what you observe. *Warning: Do not look directly at very bright light.*

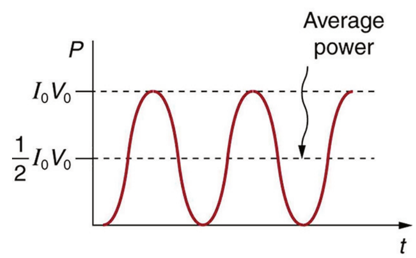


Figure 3. AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and I_0V_0. Average power is $(1/2)I_0V_0$.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in Figure 3, the average power P_{ave} is

$$P_{ave} = \frac{1}{2} I_0V_0.$$

This is evident from the graph, since the areas above and below the $(1/2)I_0V_0$ line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or rms current I_{rms} and average or rms voltage V_{rms} to be, respectively,

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

and

$$V_{rms} = \frac{V_0}{\sqrt{2}}.$$

where rms stands for root mean square, a particular kind of average. In general, to obtain a root mean square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

$$P_{ave} = I_{rms}V_{rms},$$

which gives

$$P_{ave} = \frac{I_0}{2} \cdot \frac{V_0}{2} = \frac{1}{2} I_0V_0,$$

as stated above. It is standard practice to quote I_{rms}, V_{rms}, and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven consumes $P_{ave} = 1.0\ \text{kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

$$I_{rms} = \frac{V\text{rms}}{R}.$$

The various expressions for AC power P_{ave} are

$$P_{ave} = I_{rms}V_{rms},$$

$$P_{ave} = \frac{V^2_{rms}}{R},$$

and

$$P_{ave} = I^2_{rms}R.$$

Example 1: Peak Voltage and Power for AC

(a) What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Strategy

We are told that V_{rms} is 120 V and P_{ave} is 60.0 W. We can use $V_{rms} = \frac{V_0}{\sqrt{2}}$ to find the peak voltage, and we can manipulate the definition of power to find the peak power from the given average power.

Solution for (a)

Solving the equation $V_{rms} = \frac{V_0}{\sqrt{2}}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

$$V_0 = \sqrt{2}V_{rms} = 1.414(120\ \text{V}) = 170\ \text{V}.$$

Discussion for (a)

This means that the AC voltage swings from 170 V to –170 V and back 60 times every second. An equivalent DC voltage is a constant 120 V.

Solution for (b)

Peak power is peak current times peak voltage. Thus,

$$P_0 = I_0V_0 = 2\left(\frac{1}{2}I_0V_0\right) = 2P_{\text{ave}}.$$

We know the average power is 60.0 W, and so

$$P_0 = 2(60.0\text{ W}) = 120\text{ W}.$$

Discussion

So the power swings from zero to 120 W one hundred twenty times per second (twice each cycle), and the power averages 60 W.

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall see. (See Figure 4.) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that it is much easier to increase and decrease AC voltages than DC, so AC is used in most large power distribution systems.

Figure 4. Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers (see Chapter 23.7 Transformers) to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use. (Credit: GeorgHH, Wikimedia Commons)

Example 2: Power Losses Are Less for High-Voltage Transmission

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of $1.00\ \Omega$? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\ \text{MW}$, $V_{\text{rms}} = 200\ \text{kV}$, and the resistance of the lines is $R = 1.00\ \Omega$. Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100 \times 10^6\ \text{W}}{200 \times 10^3\ \text{V}} = 500\ \text{A}.$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I_{\text{rms}}^2 R$. Substituting the known values gives

$$P_{\text{ave}} = I_{\text{rms}}^2 R = (500\ \text{A})^2 (1.00\ \Omega) = 250\ \text{kW}.$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

$$\%\ \text{loss} = \frac{250\ \text{kW}}{100\ \text{MW}} \times 100 = 0.250\%.$$

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

It is widely recognized that high voltages pose greater hazards than low voltages. But, in fact, some high voltages, such as those associated with common static electricity, can be harmless. So it is not voltage alone that determines a hazard. It is not so widely recognized that AC shocks are often more harmful than similar DC shocks. Thomas Edison thought that AC shocks were more harmful and set up a DC power-distribution system in New York City in the late 1800s. There were bitter fights, in particular between Edison and George Westinghouse and Nikola Tesla, who were advocating the use of AC in early power-distribution systems. AC has prevailed largely due to transformers and lower power losses with high-voltage transmission.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

PhET Interactive Simulation

Figure 5. Generator

Section Summary

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi ft$, where V is the voltage at time t, V_0 is the peak voltage, and f is the frequency in hertz.
- In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi ft$, where I is the current at time t, and $I_0 = V_0/R$ is the peak current.
- The average AC power is $P_{ave} = \frac{1}{2}I_0V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{rms} = \frac{I_0}{\sqrt{2}}$ and $V_{rms} = \frac{V_0}{\sqrt{2}}$, where rms stands for root mean square.
- Thus, $P_{ave} = I_{rms}V_{rms}$.
- Ohm's law for AC is $I_{rms} = \frac{V_{rms}}{R}$.
- Expressions for the average power of an AC circuit are $P_{ave} = I_{rms}V_{rms}$, $P_{ave} = \frac{V_{rms}^2}{R}$, and $P_{ave} = I_{rms}^2R$, analogous to the expressions for DC circuits.

Conceptual Questions

1: Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

2: Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

3: You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

Problem Exercises

1: (a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is 2700ºC, what is its resistance at 2600ºC?

2: Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?

3: A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?

4: Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

5: A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

6: In this problem, you will verify statements made at the end of the power losses for Example 2. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00 - \Omega$ transmission line. (c) What percent loss does this represent?

7: A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00\text{ cents/kW}\cdot\text{h}$?

8: What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?

9: What is the peak current through a 500-W room heater that operates on 120-V AC power?

10: Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

11: Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of 5.00 mm^2, is needed if the operating temperature is 500º C? (c) What power will it draw when first switched on?

12: Find the time after $t=0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.

13: (a) At what two times in the first period following $t=0$ does the instantaneous voltage in 60-Hz AC equal V_{rms}? (b) $-V_{\text{rms}}$?

Glossary

direct current

(DC) the flow of electric charge in only one direction

alternating current

(AC) the flow of electric charge that periodically reverses direction

AC voltage

voltage that fluctuates sinusoidally with time, expressed as $V = V_0 \sin 2\pi ft$, *where* V *is the voltage at time* t, V_0 *is the peak voltage, and* f *is the frequency in hertz*

AC current

current that fluctuates sinusoidally with time, expressed as $I = I_0 \sin 2\pi ft$, *where* I *is the current at time* t, I_0 *is the peak current, and* f *is the frequency in hertz*

rms current

the root mean square of the current, $I_{rms} = I_0/\sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage

the root mean square of the voltage, $V_{rms} = V_0/\sqrt{2}$, where V_0 is the peak voltage, in an AC system

Solutions

Problem Exercises

2: 480 V

4: 2.50 ms

6: (a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

8: 2.40 kW

10: (a) 4.0

(b) 0.50

(c) 4.0

12: (a) 1.39 ms

(b) 4.17 ms

(c) 8.33 ms

20.6 Electric Hazards and the Human Body

Summary

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner. Chapter 23.8 Electrical Safety: Systems and Devices will consider systems and devices for preventing electrical hazards.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in Figure 1. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is $0.100\ \Omega$, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

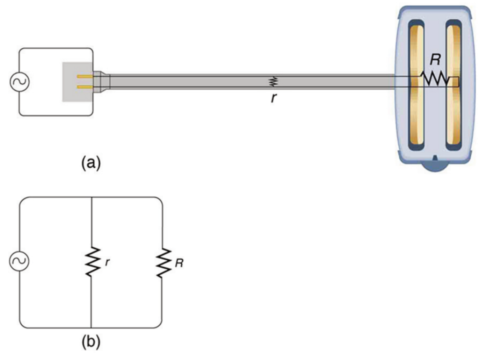


Figure 1. A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance ***r***. Since $\boldsymbol{P = V^2/r}$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2R_w$, where R_w is the resistance of the wires and I the current flowing through them. If either I or R_w is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_w = 2.00\ \Omega$ rather than the $0.100\ \Omega$ it should be. If 10.0 A of current passes through the cord, then $P = I^2R_w = 200\ \text{W}$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100 - \Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000 W$. Fuses and circuit breakers are used to limit excessive currents. (See Figure 2 and Figure 3.) Each device opens the circuit automatically when a sustained current exceeds safe limits.

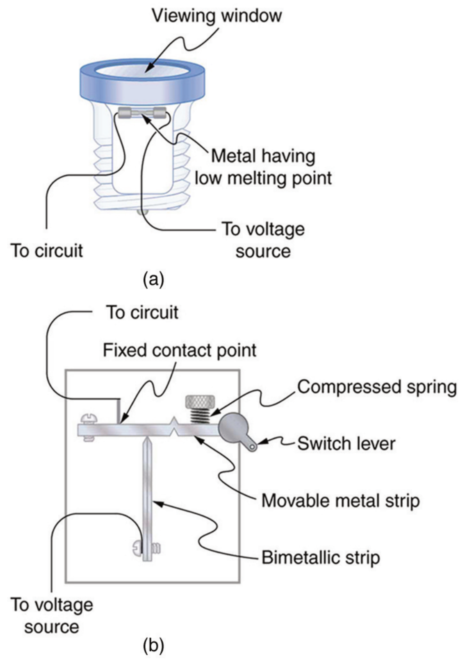


Figure 2. (a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.

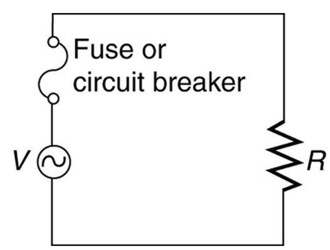


Figure 3. Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

Table 3 gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.

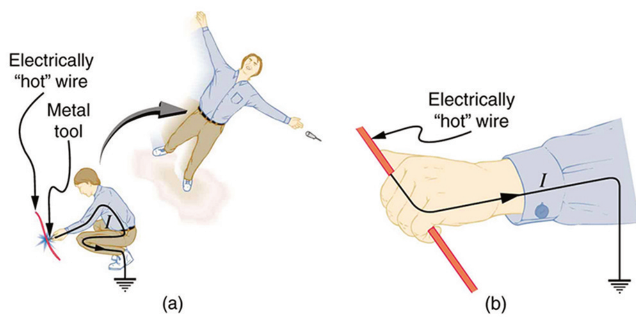


Figure 4. An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Table 3: Effects of Electrical Shock as a Function of Current[1]

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See Figure 4(a).) More frightening, and potentially more dangerous, is the “can’t let go” effect illustrated in Figure 4(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer’s hand may close about the victim’s wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called “ventricular fibrillation.” This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = V/R$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\ \text{k}\Omega$. If he comes into contact with 120-V AC, a current $I = (120\ \text{V})/(200\ \text{k}\Omega) = 0.6\ \text{mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ \text{k}\Omega$ and the same 120 V will produce a current of 12 mA—above the “can’t let go” threshold and potentially dangerous.

Most of the body’s resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in Table 3 produce similar effects. During open-heart surgery, currents as small as $20\ \mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.

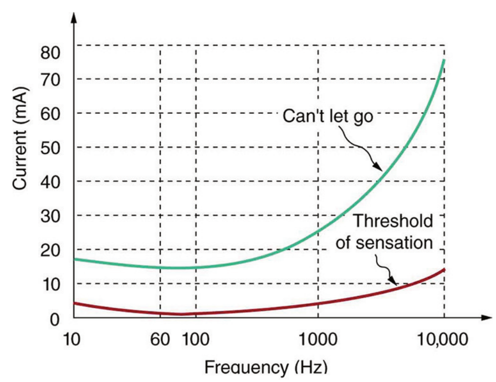


Figure 5. Graph of average values for the threshold of sensation and the “can’t let go” current as a function of frequency. The lower the value, the more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. Figure 5 presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f = 0$), mildly confirming Edison’s claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See Figure 6.) Electrical safety devices and techniques are discussed in detail in Chapter 23.8 Electrical Safety: Systems and Devices.

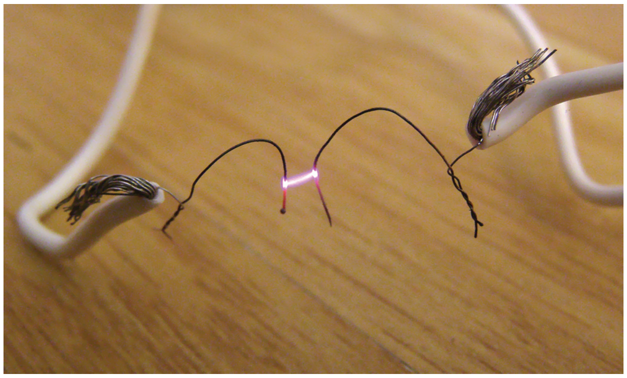


Figure 6. Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

Section Summary

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- Table 3 lists shock hazards as a function of current.
- Figure 5 graphs the threshold current for two hazards as a function of frequency.

Conceptual Questions

1: Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

2: What are the two major hazards of electricity?

3: Why isn't a short circuit a shock hazard?

4: What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

5: An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

6: Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

7: Some devices often used in bathrooms, such as hairdryers, often have safety messages saying "Do not use when the bathtub or basin is full of water." Why is this so?

8: We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

9: Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

10: Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

11: Could a person on intravenous infusion (an IV) be microshock sensitive?

12: In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

Problem Exercises

1: (a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$? (b) What current flows?

2: What voltage is involved in a 1.44-kW short circuit through a $0.100 - \Omega$ resistance?

3: Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\ \mathrm{k\Omega}$; (b) if she is standing barefoot on wet grass and has a resistance of only $4000\ \mathrm{k\Omega}$.

4: While taking a bath, a person touches the metal case of a radio. The path through the person to the drain-pipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

5: Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

6: (a) During surgery, a current as small as $20.0\ \mu\mathrm{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

7: (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

8: A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

9: Integrated Concepts

A short circuit in a 120-V appliance cord has a $0.500 - \Omega$ resistance. Calculate the temperature rise of the 2.00

g of surrounding materials, assuming their specific heat capacity is $0.200 \text{ cal/g} \cdot ^\circ\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

10: Construct Your Own Problem

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

Footnotes

1. 1 For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

Glossary

thermal hazard
a hazard in which electric current causes undesired thermal effects
shock hazard
when electric current passes through a person
short circuit
also known as a "short," a low-resistance path between terminals of a voltage source
microshock sensitive
a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

Solutions

Problem Exercises

1: (a) 230 kW

(b) 960 A

3: (a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

5: $1.20 \times 10^5 \ \Omega$

7: (a) $1.00 \ \Omega$

(b) 14.4 kW

9: Temperature increases 860º C. It is very likely to be damaging.

20.7 Nerve Conduction–Electrocardiograms

Summary

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See Figure 1.) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

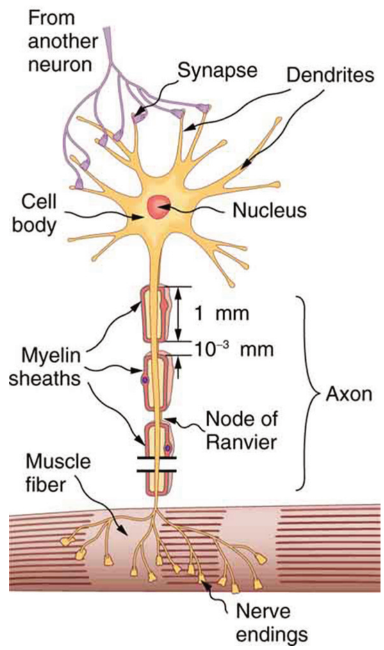


Figure 1. A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor, but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

Figure 2 illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+, K^+, and Cl^- (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). As discussed in Chapter 12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes, free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^-, and impermeable to Na^+. Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.

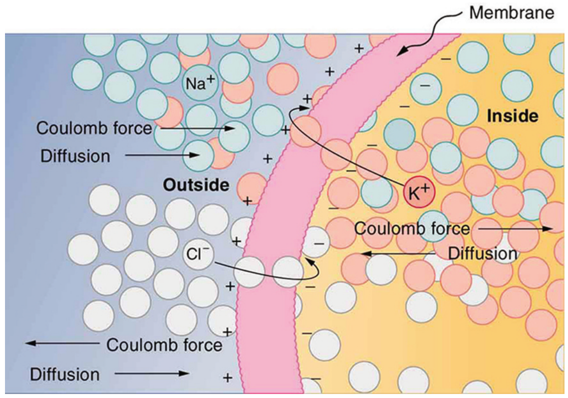


Figure 2. The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the $\mathbf{K^+}$ and $\mathbf{Cl^-}$ ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to $\mathbf{Na^+}$.

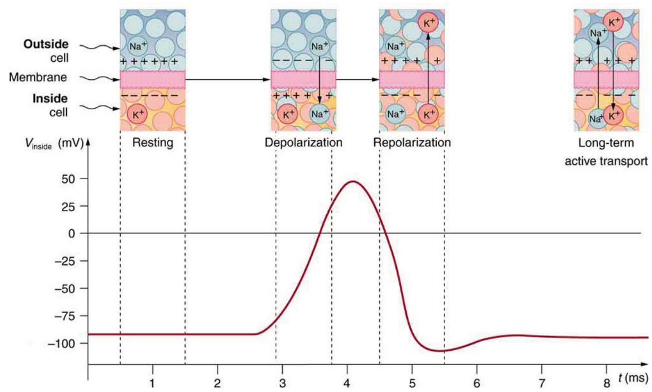


Figure 3. An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to $\mathbf{Na^+}$ ions. Repolarization follows as the membrane again becomes impermeable to $\mathbf{Na^+}$, and $\mathbf{K^+}$ moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field ($E = V/d$) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about –90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+, which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na^+ first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+, and the movement of K^+ quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See Figure 3.) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na^+ and K^+. Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in Figure 4. Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.

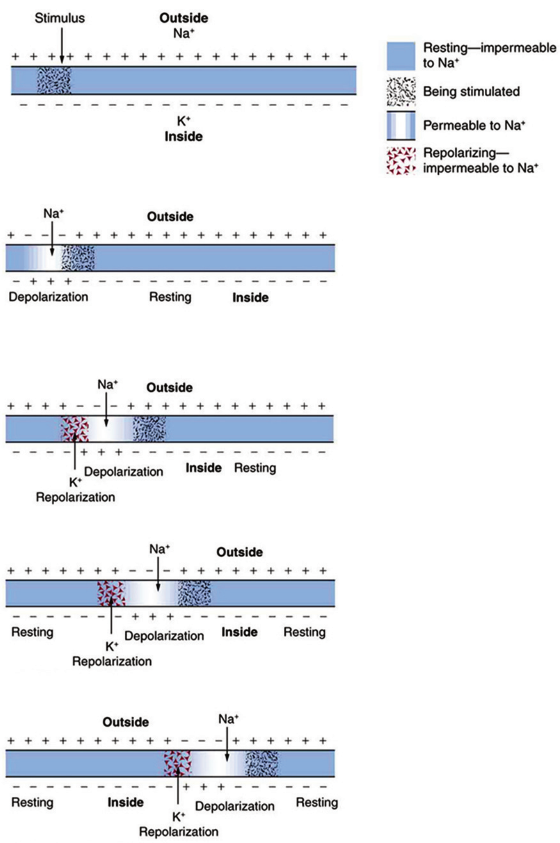


Figure 4. A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to $\mathbf{Na^+}$ and $\mathbf{K^+}$ going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.

Some axons, like that in Figure 1, are sheathed with *myelin*, consisting of fat-containing cells. Figure 5 shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an IR signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see [link]), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.

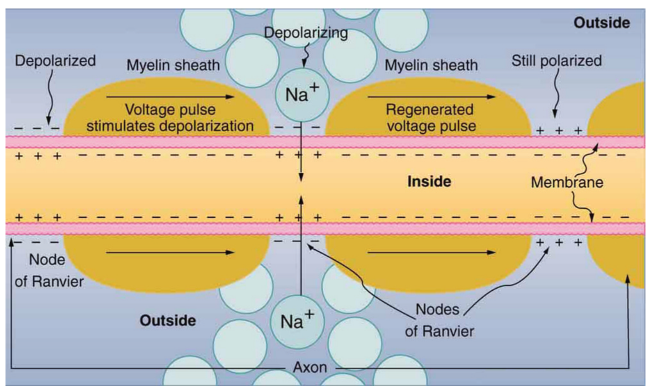


Figure 5. Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.

Figure 6. An electric eel flexes its muscles to create a voltage that stuns prey. (credit: chrisbb, Flickr)

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. Figure 7 is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial (SA) node***,** the heart's natural pacemaker.

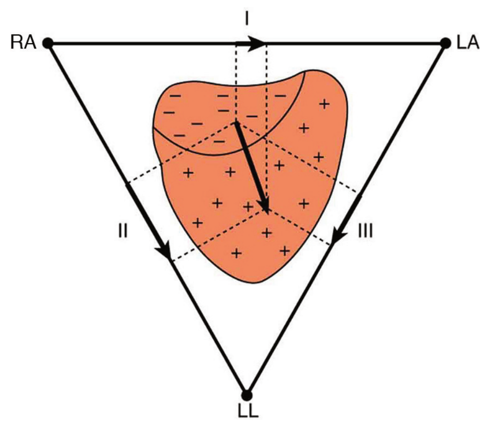


Figure 7. The outer surface of the heart changes from positive to negative during depolarization. This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave. This vector is a voltage (potential difference) vector. Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.

An **electrocardiogram (ECG)** is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in Figure 7 for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

Heart function and its four-chamber action are explored in Chapter 12.4 Viscosity and Laminar Flow; Poiseuille's Law. Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

Figure 8 shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.

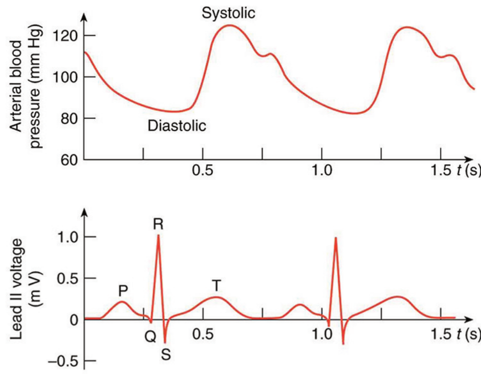


Figure 8. A lead II ECG with corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See Figure 9.

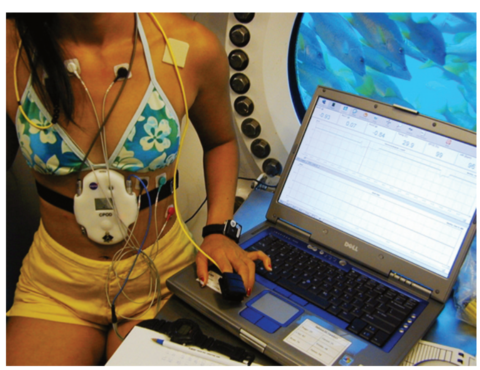


Figure 9. This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs are being recorded by a portable device while living in an underwater habitat. (credit: NASA, Life Sciences Data Archive at Johnson Space Center, Houston, Texas)

PhET Explorations: Neuron

PhET Interactive Simulation

Figure 10. Neuron

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

Section Summary

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

Conceptual Questions

1: Note that in Figure 2, both the concentration gradient and the Coulomb force tend to move Na^+ ions into the cell. What prevents this?

2: Define depolarization, repolarization, and the action potential.

3: Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

Problems & Exercises

1: Integrated Concepts

Use the ECG in Figure 8 to determine the heart rate in beats per minute assuming a constant time between beats.

2: Integrated Concepts

(a) Referring to Figure 8, find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

Glossary

nerve conduction
the transport of electrical signals by nerve cells
bioelectricity
electrical effects in and created by biological systems
semipermeable
property of a membrane that allows only certain types of ions to cross it
electrocardiogram (ECG)
usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

Solutions

Problems & Exercises

1: 80 beats/minute

PART 14

Chapter 21 Circuits and DC Instruments

Figure 1. Electric circuits in a computer allow large amounts of data to be quickly and accurately analyzed.. (credit: Airman 1st Class Mike Meares, United States Air Force)

Electric circuits are commonplace. Some are simple, such as those in flashlights. Others, such as those used in supercomputers, are extremely complex.

This collection of modules takes the topic of electric circuits a step beyond simple circuits. When the circuit is purely resistive, everything in this module applies to both DC and AC. Matters become more complex when capacitance is involved. We do consider what happens when capacitors are connected to DC voltage sources, but the interaction of capacitors and other nonresistive devices with AC is left for a later chapter. Finally, a number of important DC instruments, such as meters that measure voltage and current, are covered in this chapter.

21.1 Resistors in Series and Parallel

Summary

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in Figure 1. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

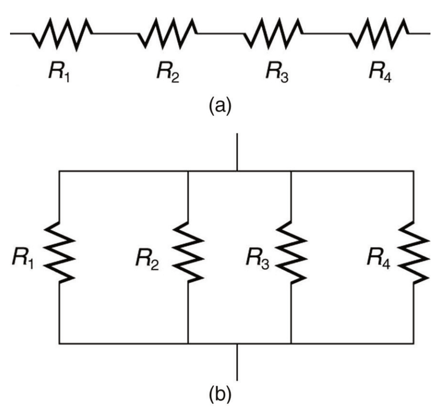


Figure 1. (a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in Figure 1(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

Figure 2 shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)

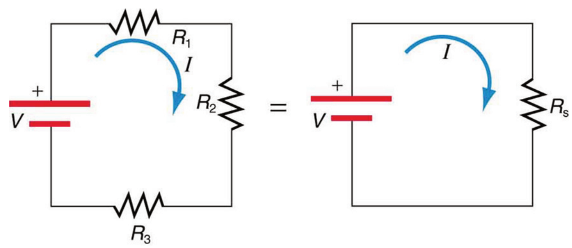


Figure 2. Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in Figure 2.

According to **Ohm's law**, the voltage drop, V, across a resistor when a current flows through it is calculated using the equation $V=IR$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another way to think of this is that V is the voltage necessary to make a current I flow through a resistance R.

So the voltage drop across R_1 is $V_1=IR_1$, that across R_2 is $V_2=IR_2$, and that across R_3 is $V_3=IR_3$. The sum of these voltages equals the voltage output of the source; that is,

$$V=V_1+V_2+V_3.$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $PE=qV$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV, while that dissipated by the resistors is

$$qV_1+qV_2+qV_3.$$

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).$$

Note that for the equivalent single series resistance R_s, we have

$$V = IR_s.$$

This implies that the total or equivalent series resistance R_s of three resistors is

$$R_s = R_1 + R_2 + R_3.$$

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

$ $$R_s = R_1 + R_2 + R_3 + \ldots$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example 1: Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in Figure 2 is 12.0 V, and the resistances are $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\ &= 20.0\ \Omega. \end{aligned}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

$$I = \frac{V}{R_s} = \frac{12.0\text{ V}}{20.0\ \Omega} = 0.600\text{ A}.$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

$$V_1 = IR_1 = (0.600\text{ A})(1.0\ \Omega) = 0.600\text{ V}.$$

Similarly,

$$V_2 = IR_2 = (0.600\text{ A})(6.0\ \Omega) = 3.60\text{ V}$$

and

$$V_3 = IR_3 = (0.600\text{ A})(13.0\ \Omega) = 7.80\text{ V}.$$

Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\text{ V} = 12.0\text{ V}.$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

$$P_1 = I^2R_1 = (0.600\text{ A})^2(1.00\ \Omega) = 0.360\text{ W}.$$

Similarly,

$$P_2 = I^2R_2 = (0.600\text{ A})^2(6.00\ \Omega) = 2.16\text{ W}$$

and

$$P_3 = I^2R_3 = (0.600\text{ A})^2(13.0\ \Omega) = 4.68\text{ W}.$$

Discussion for (d)

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = IV$, where V is the source voltage. This gives

$$P = (0.600\text{ A})(12.0\text{ V}) = 7.20\text{ W}.$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68)\text{ W} = 7.20\text{ W}.$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Major Features of Resistors in Series

1. Series resistances add: $R_s = R_1 + R_2 + R_3 + \dots$.

2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

Resistors in Parallel

Figure 3 shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See Figure 3(b).)

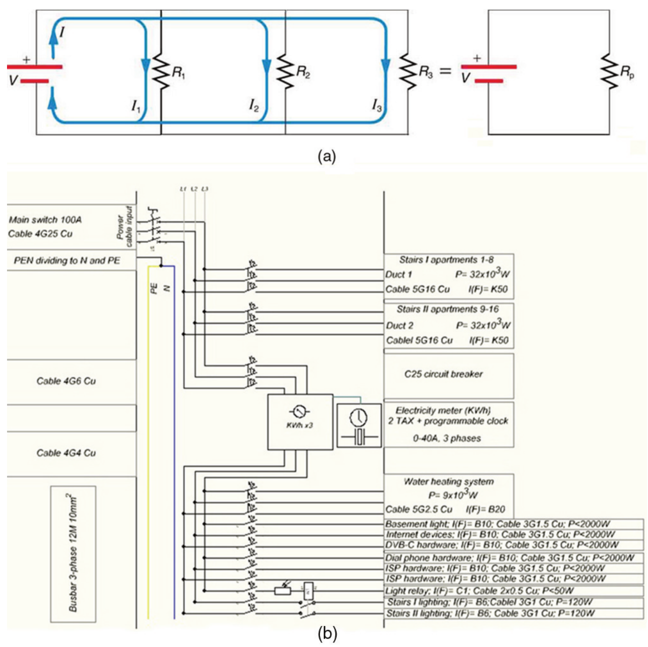


Figure 3. (a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_p, let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

$$I = I_1 + I_2 + I_3.$$

Substituting the expressions for the individual currents gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

Note that Ohm's law for the equivalent single resistance gives

$$I = \frac{V}{R_p} = V\left(\frac{1}{R_p}\right).$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example 2: Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in Figure 3 be the same as the previously considered series connection: $V = 12.0\text{ V}$, $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega}.$$

Thus,

$$\frac{1}{R_p} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p. This yields

$$R_p = \frac{1}{1.2436}\ \Omega = 0.8041\ \Omega.$$

The total resistance with the correct number of significant digits is $R_p = 0.804\ \Omega$

Discussion for (a)

R_p is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_p for the total resistance. This gives

$$I = \frac{V}{R_p} = \frac{12.0\text{ V}}{0.8041\ \Omega} = 14.92\text{ A}$$

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{12.0\ \text{V}}{1.00\ \Omega} = 12.0\ \text{A}\ .$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{12.0\ \text{V}}{6.00\ \Omega} = 2.00\ \text{A}$$

and

$$I_3 = \frac{V}{R_3} = \frac{12.0\ \text{V}}{13.0\ \Omega} = 0.92\ \text{A}\ .$$

Discussion for (c)

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 14.92\ \text{A}.$$

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R^1} = \frac{(12.0\ \text{V})^2}{1.00\ \Omega} = 144\ \text{W}\ .$$

Similarly,

$$P_2 = \frac{V^2}{R^2} = \frac{(12.0\ \text{V})^2}{6.00\ \Omega} = 24.0\ \text{W}$$

and

$$P_3 = \frac{V^2}{R^3} = \frac{(12.0\ \text{V})^2}{13.0\ \Omega} = 11.1\ \text{W}\ .$$

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

$$P = IV = (14.92\ \text{A})(12.0\ \text{V}) = 179\ \text{W}.$$

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

$$P_1 + P_2 + P_3 = 144\ \text{W} + 24.0\ \text{W} + 11.1\ \text{W} = 179\ \text{W}.$$

This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in Figure 4. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

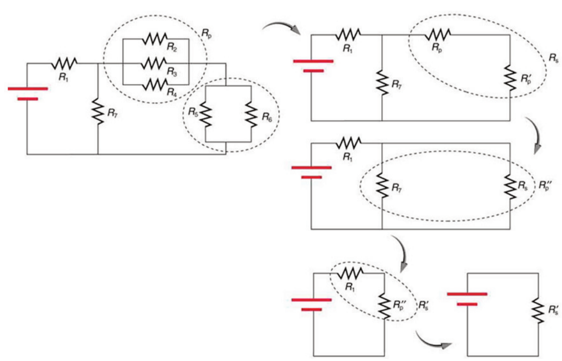


Figure 4. This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

The simplest combination of series and parallel resistance, shown in Figure 5, is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example 3: Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

Figure 5 shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the total resistance. (b) What is the IR drop in R_1? (c) Find the current I_2 through R_2. (d) What power is dissipated by R_2?

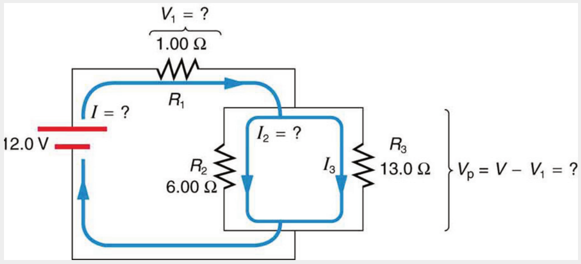


Figure 5. These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1. Thus the total (equivalent) resistance of this combination is

$$R_{tot} = R_1 + R_p.$$

First, we find R_p using the equation for resistors in parallel and entering known values:

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega} = \frac{0.2436}{\Omega}.$$

Inverting gives

$$R_p = \frac{1}{0.2436}\ \Omega = 4.11\ \Omega.$$

So the total resistance is

$$R_{tot} = R_1 + R_p = 1.00\ \Omega + 4.11\ \Omega = 5.11\ \Omega.$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1, we note that the full current I flows through R_1. Thus its IR drop is

$$V_1 = IR_1.$$

We must find I before we can calculate V_1. The total current I is found using Ohm's law for the circuit. That is,

$$I = \frac{V}{R_{tot}} = \frac{12.0\ \text{V}}{5.11\ \Omega} = 2.35\ \text{A}.$$

Entering this into the expression above, we get

$$V_1 = IR_1 = (2.35\ \text{A})(1.00\ \Omega) = 2.35\ \text{V}.$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.

Strategy and Solution for (c)

To find the current through R_2, we must first find the voltage applied to it. We call this voltage V_p, because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1, and so it is

$$V_p = V - V_1 = 12.0\ \text{V} - 2.35\ \text{V} = 9.65\ \text{V}.$$

Now the current I_2 through resistance R_2 is found using Ohm's law:

$$I_2 = \frac{V_p}{R_2} = \frac{9.65\ \text{V}}{6.00\ \Omega} = 1.61\ \text{A}$$

Discussion for (c)

The current is less than the 2.00 A that flowed through R2R2 size 12{R rSub { size 8{2} } } {} when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

$$P_2 = (I_2)^2 R_2 = (1.61\ \text{A})^2 (6.00\ \Omega) = 15.5\ \text{W}.$$

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in Figure 6. The device represented by R_3 has a very low resistance, and so when it is switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

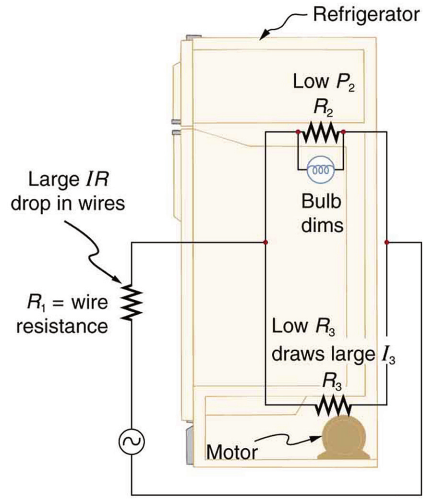


Figure 7. Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant *IR* drop in the wires and reduces the voltage across the light.

Check Your Understanding

1: Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding $latex\boldsymbol{R_p}$, the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance should be greater, whereas total parallel resistance should be

smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

Section Summary

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_s = R_1 + R_2 + R_3 + \cdots$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

Conceptual Questions

1: A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in Figure 7 has on current when open and when closed.

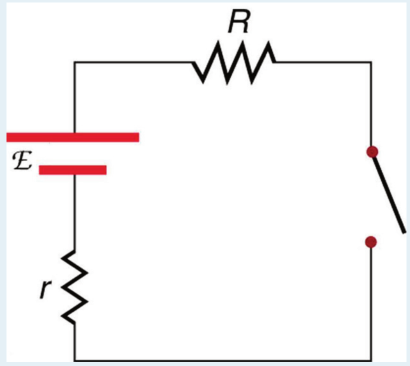


Figure 7. A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

2: What is the voltage across the open switch in Figure 7?

3: There is a voltage across an open switch, such as in Figure 7. Why, then, is the power dissipated by the open switch small?

4: Why is the power dissipated by a closed switch, such as in Figure 7, small?

5: A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in Figure 8. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

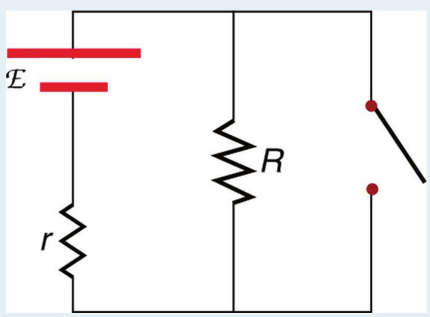


Figure 8. A wiring mistake put this switch in parallel with the device represented by ***R***. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

6: Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

7: Would your headlights dim when you start your car's engine if the wires in your automobile were super-conductors? (Do not neglect the battery's internal resistance.) Explain.

8: Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

9: If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

10: Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

11: Before World War II, some radios got power through a "resistance cord" that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio's tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

12: Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

Problem Exercises

Note: Data taken from figures can be assumed to be accurate to three significant digits.

1: (a) What is the resistance of ten $275 - \Omega$ resistors connected in series? (b) In parallel?

2: (a) What is the resistance of a $1.00 \times 10^3 - \Omega$ a $2.50 - k\Omega$, and a $4.00 - k\Omega$ resistor connected in series? (b) In parallel?

3: What are the largest and smallest resistances you can obtain by connecting a $36.0 - \Omega$, a $50.0 - \Omega$, and a $700 - \Omega$ resistor together?

4: An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

5: Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

6: (a) Given a 48.0-V battery and $24.0 - \Omega$ and $96.0 - \Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

7: Referring to the example combining series and parallel circuits and Figure 5, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

8: Referring to Figure 5: (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

9: Refer to Figure 6 and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

10: A 240-kV power transmission line carrying $5.00 \times 10^2 \text{ A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9 - \Omega$ resistance. Figure 9. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

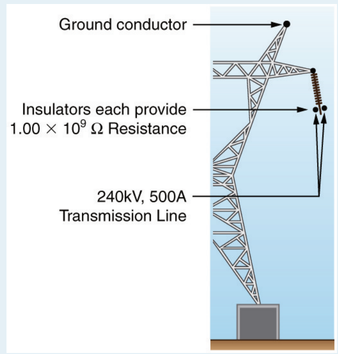


Figure 9. High-voltage (240-kV) transmission line carrying $\mathbf{5.00 \times 10^2 \text{ A}}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $\mathbf{1.00 \times 10^9 \Omega}$ of resistance each.

11: Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 >> R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1. (b) Their parallel resistance is very nearly equal to smaller resistance R_2.

12: Unreasonable Results

Two resistors, one having a resistance of 145Ω, are connected in parallel to produce a total resistance of 150Ω. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

13: Unreasonable Results

Two resistors, one having a resistance of $900 \text{ k}\Omega$, are connected in series to produce a total resistance of $0.500 \text{ M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

series
a sequence of resistors or other components wired into a circuit one after the other
resistor
a component that provides resistance to the current flowing through an electrical circuit
resistance
causing a loss of electrical power in a circuit
Ohm's law
the relationship between current, voltage, and resistance within an electrical circuit: V=IRV=IR size 12{V= ital "IR"} {}

voltage
the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery
voltage drop
the loss of electrical power as a current travels through a resistor, wire or other component
current
the flow of charge through an electric circuit past a given point of measurement
Joule’s law
the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: Pe=IVPe=IV size 12{P rSub { size 8{e} } = ital “IV”} {}
parallel
the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

Solutions

Check Your Understanding

1: No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff’s rules, to be introduced in Chapter 21.3 Kirchhoff’s Rules, will allow you to analyze the circuit.

Problems Exercises

1: (a) $2.75\ \text{k}\Omega$

(b) $27.5\ \Omega$

3: (a) $786\ \Omega$

(b) $20.3\ \Omega$

5: 29.6 W

7: (a) 0.74 A

(b) 0.742 A

9: (a) 60.8 W

(b) 3.18 kW

11: (a) $\begin{array}{c} R_s = R_1 + R_2 \\ \Rightarrow R_s \approx R_1 (R_1 >> R_2) \end{array}$

(b) $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1+R_2}{R_1 R_2}$,

so that

$R_p = \frac{R_1 R_2}{R_1+R_2} \approx \frac{R_1 R_2}{R_1} = R_2\ (R_1 >> R_2)$.

13: (a) $-400\ \text{k}\Omega$

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

21.2 Electromotive Force: Terminal Voltage

Summary

- Compare and contrast the voltage and the electromagnetic force of an electric power source.
- Describe what happens to the terminal voltage, current, and power delivered to a load as internal resistance of the voltage source increases (due to aging of batteries, for example).
- Explain why it is beneficial to use more than one voltage source connected in parallel.

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted.

Furthermore, if you connect an excessive number of 12-V lights in parallel to a car battery, they will be dim even when the battery is fresh and even if the wires to the lights have very low resistance. This implies that the battery's output voltage is reduced by the overload.

The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an **internal resistance**. Let us examine both.

Electromotive Force

You can think of many different types of voltage sources. Batteries themselves come in many varieties. There are many types of mechanical/electrical generators, driven by many different energy sources, ranging from nuclear to wind. Solar cells create voltages directly from light, while thermoelectric devices create voltage from temperature differences.

A few voltage sources are shown in Figure 1. All such devices create a **potential difference** and can supply current if connected to a resistance. On the small scale, the potential difference creates an electric field that exerts force on charges, causing current. We thus use the name **electromotive force**, abbreviated emf.

Emf is not a force at all; it is a special type of potential difference. To be precise, the electromotive force (emf) is the potential difference of a source when no current is flowing. Units of emf are volts.

Figure 1. A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

Internal Resistance

As noted before, a 12-V truck battery is physically larger, contains more charge and energy, and can deliver a larger current than a 12-V motorcycle battery. Both are lead-acid batteries with identical emf, but, because of its size, the truck battery has a smaller internal resistance r. Internal resistance is the inherent resistance to the flow of current within the source itself.

Figure 2 is a schematic representation of the two fundamental parts of any voltage source. The emf (represented by a script E in the figure) and internal resistance r are in series. The smaller the internal resistance for a given emf, the more current and the more power the source can supply.

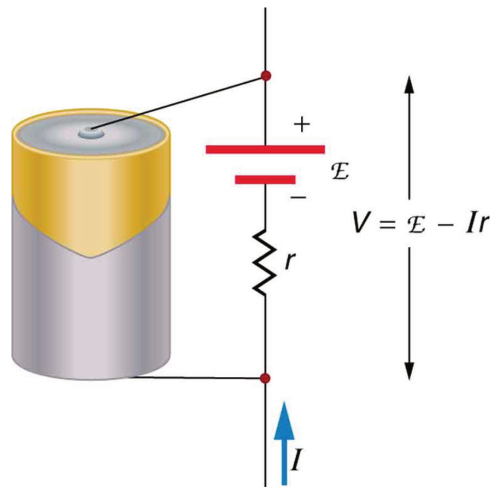


Figure 2. Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of potential difference, and an internal resistance ***r*** related to its construction. (Note that the script E stands for emf.). Also shown are the output terminals across which the terminal voltage ***V*** is measured. Since ***V* = emf − *Ir***, terminal voltage equals emf only if there is no current flowing.

The internal resistance r can behave in complex ways. As noted, r increases as a battery is depleted. But internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted.

Things Great and Small: The Submicroscopic Origin of Battery Potential

Various types of batteries are available, with emfs determined by the combination of chemicals involved. We can view this as a molecular reaction (what much of chemistry is about) that separates charge.

The lead-acid battery used in cars and other vehicles is one of the most common types. A single cell (one of six) of this battery is seen in Figure 3. The cathode (positive) terminal of the cell is connected to a lead oxide plate, while the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

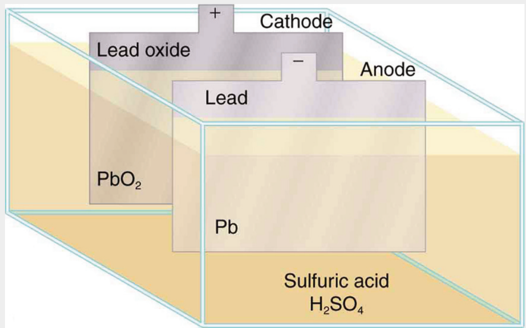


Figure 3. Artist's conception of a lead-acid cell. Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge as well as participating in the chemical reaction.

The details of the chemical reaction are left to the reader to pursue in a chemistry text, but their results at the molecular level help explain the potential created by the battery. Figure 4 shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplied two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction will not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.

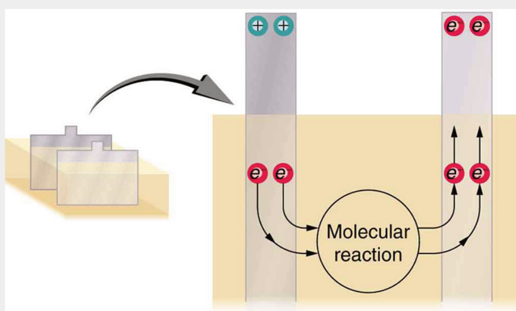


Figure 4. Artist's conception of two electrons being forced onto the anode of a cell and two electrons being removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Why are the chemicals able to produce a unique potential difference? Quantum mechanical descriptions of molecules, which take into account the types of atoms and numbers of electrons in them, are able to predict the energy states they can have and the energies of reactions between them.

In the case of a lead-acid battery, an energy of 2 eV is given to each electron sent to the anode. Voltage is defined as the electrical potential energy divided by charge: $V = \frac{P_E}{q}$. An electron volt is the energy given to a single electron by a voltage of 1 V. So the voltage here is 2 V, since 2 eV is given to each electron. It is the energy produced in each molecular reaction that produces the voltage. A different reaction produces a different energy and, hence, a different voltage.

Terminal Voltage

The voltage output of a device is measured across its terminals and, thus, is called its **terminal voltage** V. Terminal voltage is given by

$$V = \text{emf} - Ir,$$

where r is the internal resistance and I is the current flowing at the time of the measurement.

I is positive if current flows away from the positive terminal, as shown in Figure 2. You can see that the larger the current, the smaller the terminal voltage. And it is likewise true that the larger the internal resistance, the smaller the terminal voltage.

Suppose a load resistance R_{load} is connected to a voltage source, as in Figure 5. Since the resistances are in series, the total resistance in the circuit is $R_{\text{load}}+r$. Thus the current is given by Ohm's law to be

$$I = \frac{\text{emf}}{R_{\text{load}}+r}.$$

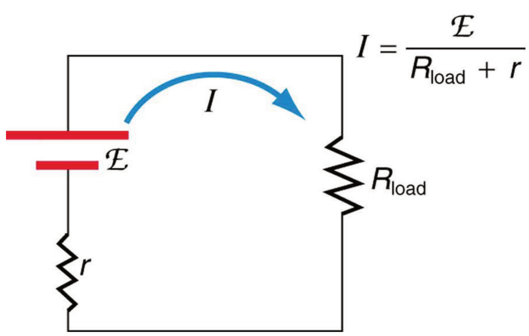


Figure 5. Schematic of a voltage source and its load R_{load}. Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and current delivered to the load. (Note that the script E stands for emf.)

We see from this expression that the smaller the internal resistance r, the greater the current the voltage source supplies to its load R_{load}. As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

Example 1: Calculating Terminal Voltage, Power Dissipation, Current, and Resistance: Terminal Voltage and Load

A certain battery has a 12.0-V emf and an internal resistance of $0.100 \, \Omega$. (a) Calculate its terminal voltage when connected to a $10.0 - \Omega$ load. (b) What is the terminal voltage when connected to a $0.500 - \Omega$ load? (c) What power does the $0.500 - \Omega$ load dissipate? (d) If the internal resistance grows to $0.500 \, \Omega$, find the current, terminal voltage, and power dissipated by a $0.500 - \Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated using the equation $V = \text{emf} - Ir$. Once current is found, the power dissipated by a resistor can also be found.

Solution for (a)

Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0 \text{ V}}{10.1 \ \Omega} = 1.188 \text{ A}.$$

Enter the known values into the equation $V = \text{emf} - Ir$ to get the terminal voltage:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0 \text{ V} - (1.188 \text{ A})(0.100 \ \Omega) \\ &= 11.9 V. \end{aligned}$$

Discussion for (a)

The terminal voltage here is only slightly lower than the emf, implying that $10.0 \, \Omega$ is a light load for this particular battery.

Solution for (b)

Similarly, with $R_{\text{load}} = 0.500 \, \Omega$, the current is

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0 \text{ V}}{0.600 \ \Omega} = 20.0 \text{ A}.$$

The terminal voltage is now

$$\begin{aligned} V &= \text{emf} - Ir = 12.0 \text{ V} - (20.0 \text{ A})(0.100 \ \Omega) \\ &= 10.0 \text{ V}. \end{aligned}$$

Discussion for (b)

This terminal voltage exhibits a more significant reduction compared with emf, implying $0.500 \, \Omega$ is a heavy load for this battery.

Solution for (c)

The power dissipated by the $0.500 - \Omega$ load can be found using the formula \$\boldsymbol{P = I^2R} \$. Entering the known values gives

$$P_{\text{load}} = I^2 R_{\text{load}} = (20.0 \text{ A})^2 (0.500 \ \Omega) = 2.00 \times 10^2 \text{ W}.$$

Discussion for (c)

Note that this power can also be obtained using the expressions $\frac{V^2}{R}$ or IV, where V is the terminal voltage (10.0 V in this case).

Solution for (d)

Here the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

$$I = \frac{\text{emf}}{R_{\text{load}} + r} [latex] = \frac{12.0 \text{ V}}{1.00 \ \Omega} = 12.0 \text{ A}.$$

Now the terminal voltage is

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\ \text{V} - (12.0\ \text{A})(0.500\ \Omega) \\ &= 6.00\ \text{V}, \end{aligned}$$

and the power dissipated by the load is

$$P_{\text{load}} = I^2 R_{\text{load}} = (12.0\ \text{A})^2(0.500\ \Omega) = 72.0\ \text{W}.$$

Discussion for (d)

We see that the increased internal resistance has significantly decreased terminal voltage, current, and power delivered to a load.

Battery testers, such as those in Figure 6, use small load resistors to intentionally draw current to determine whether the terminal voltage drops below an acceptable level. They really test the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.

Figure 6. These two battery testers measure terminal voltage under a load to determine the condition of a battery. The large device is being used by a U.S. Navy electronics technician to test large batteries aboard the aircraft carrier USS *Nimitz* and has a small resistance that can dissipate large amounts of power. (credit: U.S. Navy photo by Photographer's Mate Airman Jason A. Johnston) The small device is used on small batteries and has a digital display to indicate the acceptability of their terminal voltage. (credit: Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to a resistance. This is done routinely in cars and batteries for small electrical appliances and electronic devices, and is represented pictorially in Figure 7. The voltage output of the battery charger must be greater than the emf of the battery to reverse current through it. This will cause the terminal voltage of the battery to be greater than the emf, since $V = \text{emf} - Ir$, and I is now negative.

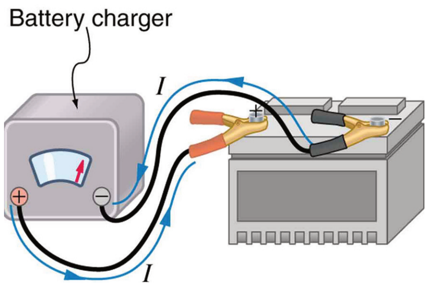


Figure 7. A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

Multiple Voltage Sources

There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. (See Figure 8.) Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

But if the cells oppose one another, such as when one is put into an appliance backward, the total emf is less, since it is the algebraic sum of the individual emfs.

A battery is a multiple connection of voltaic cells, as shown in Figure 9. The disadvantage of series connections of cells is that their internal resistances add. One of the authors once owned a 1957 MGA that had two 6-V batteries in series, rather than a single 12-V battery. This arrangement produced a large internal resistance that caused him many problems in starting the engine.

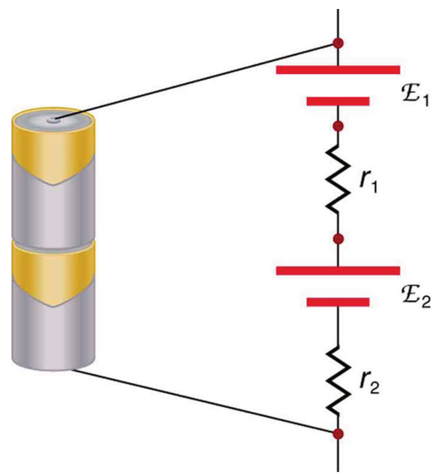


Figure 8. A series connection of two voltage sources. The emfs (each labeled with a script E) and internal resistances add, giving a total emf of $\mathbf{emf_1 + emf_2}$ and a total internal resistance of $\boldsymbol{r_1 + r_2}$.

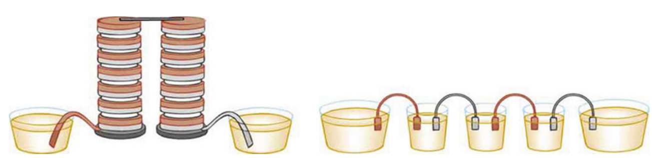


Figure 9. Batteries are multiple connections of individual cells, as shown in this modern rendition of an old print. Single cells, such as AA or C cells, are commonly called batteries, although this is technically incorrect.

If the *series* connection of two voltage sources is made into a complete circuit with the emfs in opposition, then a current of magnitude $I = \frac{\text{emf}_1 - \text{emf}_2}{r_1 + r_2}$ flows. See Figure 10, for example, which shows a circuit exactly analogous to the battery charger discussed above. If two voltage sources in series with emfs in the same sense are connected to a load R_{load}, as in Figure 11, then $I = \frac{\text{emf}_1 + \text{emf}_2}{r_1 + r_2 + R_{\text{load}}}$ flows.

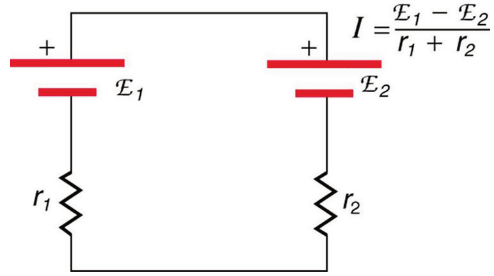


Figure 10. These two voltage sources are connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited to ***I* = (emf$_1$ − emf$_2$)/(*r*$_1$+*r*$_2$)** by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.) A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.

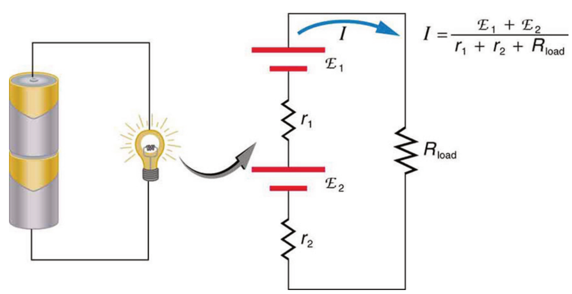


Figure 11. This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series. The current that flows is ***I* = (emf$_1$ + emf$_2$) / (*r*$_1$ + *r*$_2$ + *R*$_{load}$)**. (Note that each emf is represented by script E in the figure.)

Take-Home Experiment: Flashlight Batteries

Find a flashlight that uses several batteries and find new and old batteries. Based on the discussions in this module, predict the brightness of the flashlight when different combinations of batteries are used. Do your predictions match what you observe? Now place new batteries in the flashlight and leave the flashlight switched on for several hours. Is the flashlight still quite bright? Do the same with the old batteries. Is the flashlight as bright when left on for the same length of time with old and new batteries? What does this say for the case when you are limited in the number of available new batteries?

Figure 12 shows two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.

Here, $I = \frac{\text{emf}}{(r_{tot} + R_{load})}$ flows through the load, and r_{tot} is less than those of the individual batteries. For example, some diesel-powered cars use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

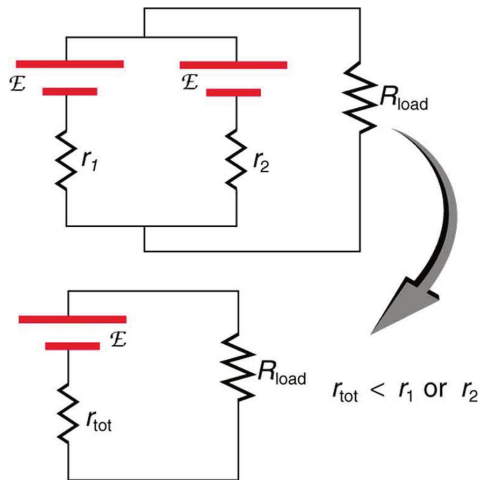


Figure 12. Two voltage sources with identical emfs (each labeled by script E) connected in parallel produce the same emf but have a smaller total internal resistance than the individual sources. Parallel combinations are often used to deliver more current. Here $\boldsymbol{I = (\text{emf})/(r_{\text{tot}}+R_{\text{load}})}$ flows through the load.

Animals as Electrical Detectors

A number of animals both produce and detect electrical signals. Fish, sharks, platypuses, and echidnas (spiny anteaters) all detect electric fields generated by nerve activity in prey. Electric eels produce their own emf through biological cells (electric organs) called electroplaques, which are arranged in both series and parallel as a set of batteries.

Electroplaques are flat, disk-like cells; those of the electric eel have a voltage of 0.15 V across each one. These cells are usually located toward the head or tail of the animal, although in the case of the electric eel, they are found along the entire body. The electroplaques in the South American eel are arranged in 140 rows, with each row stretching horizontally along the body and containing 5,000 electroplaques. This can yield an emf of approximately 600 V, and a current of 1 A—deadly.

The mechanism for detection of external electric fields is similar to that for producing nerve signals in the cell through depolarization and repolarization—the movement of ions across the cell membrane. Within the fish, weak electric fields in the water produce a current in a gel-filled canal that runs from the skin to sensing cells, producing a nerve signal. The Australian platypus, one of the very few mammals that lay eggs, can detect fields of $30\ \frac{\text{mV}}{\text{m}}$, while sharks have been found to be able to sense a field in their snouts as small as $100\ \frac{\text{mV}}{\text{m}}$ (Figure 13). Electric eels use their own electric fields produced by the electroplaques to stun their prey or enemies.

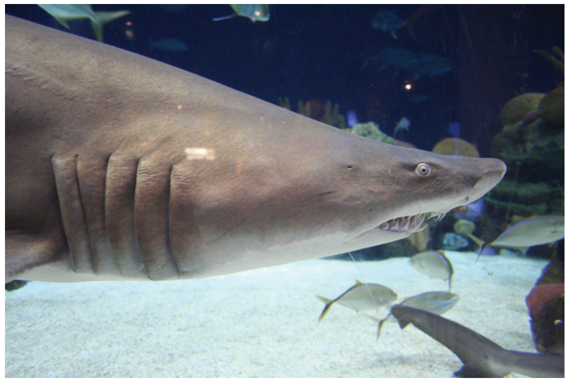


Figure 13. Sand tiger sharks (*Carcharias taurus*), like this one at the Minnesota Zoo, use electroreceptors in their snouts to locate prey. (credit: Jim Winstead, Flickr)

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation (PV), the conversion of sunlight directly into electricity, is based upon the photoelectric effect, in which photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon—either as single-crystal silicon, or as a thin film of silicon deposited upon a glass or metal backing. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight upon the cell (the incident solar radiation—the insolation). Under bright noon sunlight, a current of about 100 mA/cm^2 of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical-energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

The output of the solar cells is direct current. For most uses in a home, AC is required, so a device called an inverter must be used to convert the DC to AC. Any extra output can then be passed on to the outside electrical grid for sale to the utility.

Take-Home Experiment: Virtual Solar Cells

One can assemble a “virtual” solar cell array by using playing cards, or business or index cards, to represent a solar cell. Combinations of these cards in series and/or parallel can model the required array output. Assume each card has an output of 0.5 V and a current (under bright light) of 2 A. Using your cards, how would you arrange them to produce an output of 6 A at 3 V (18 W)?

Suppose you were told that you needed only 18 W (but no required voltage). Would you need more cards to make this arrangement?

Section Summary

- All voltage sources have two fundamental parts—a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r.
- The emf is the potential difference of a source when no current is flowing.
- The numerical value of the emf depends on the source of potential difference.
- The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V and is given by $V = \text{emf} - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source.
- When multiple voltage sources are in series, their internal resistances add and their emfs add algebraically.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

Conceptual Questions

1: Is every emf a potential difference? Is every potential difference an emf? Explain.

2: Explain which battery is doing the charging and which is being charged in Figure 14.

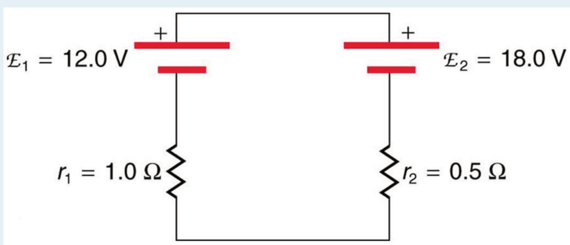


Figure 14.

3: Given a battery, an assortment of resistors, and a variety of voltage and current measuring devices, describe how you would determine the internal resistance of the battery.

4: Two different 12-V automobile batteries on a store shelf are rated at 600 and 850 “cold cranking amps.” Which has the smallest internal resistance?

5: What are the advantages and disadvantages of connecting batteries in series? In parallel?

6: Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck’s other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck’s engine (a very heavy load)?

Problem Exercises

1: Standard automobile batteries have six lead-acid cells in series, creating a total emf of 12.0 V. What is the emf of an individual lead-acid cell?

2: Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.

3: What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is $2.00\ \Omega$?

4: (a) What is the terminal voltage of a large 1.54-V carbon-zinc dry cell used in a physics lab to supply 2.00 A to a circuit, if the cell's internal resistance is $0.100\ \Omega$? (b) How much electrical power does the cell produce? (c) What power goes to its load?

5: What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

6: (a) Find the terminal voltage of a 12.0-V motorcycle battery having a $0.600-\Omega$ internal resistance, if it is being charged by a current of 10.0 A. (b) What is the output voltage of the battery charger?

7: A car battery with a 12-V emf and an internal resistance of $0.050\ \Omega$ is being charged with a current of 60 A. Note that in this process the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 60 A to the starter motor?

8: The hot resistance of a flashlight bulb is $2.30\ \Omega$, and it is run by a 1.58-V alkaline cell having a $0.100-\Omega$ internal resistance. (a) What current flows? (b) Calculate the power supplied to the bulb using I^2R_{bulb}. (c) Is this power the same as calculated using $\frac{V^2}{R_{\text{bulb}}}$?

9: The label on a portable radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf while alkaline cells have a 1.58-V emf. The radio has a $3.20-\Omega$ resistance. (a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio. (b) When using Nicad cells each having an internal resistance of $0.0400\ \Omega$. (c) When using alkaline cells each having an internal resistance of $0.200\ \Omega$. (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

10: An automobile starter motor has an equivalent resistance of $0.0500\ \Omega$ and is supplied by a 12.0-V battery with a $0.0100-\Omega$ internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these calculations for when the battery connections are corroded and add $0.0900\ \Omega$ to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

11: A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.0200\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.100-\Omega$ internal resistance. The load resistance is $10.0\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

12: (a) What is the internal resistance of a voltage source if its terminal voltage drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

13: A person with body resistance between his hands of $10.0\ \text{k}\Omega$ accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is $2000\ \Omega$, what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

14: Electric fish generate current with biological cells called electroplaques, which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of 0.15 V and internal resistance of $0.25\ \Omega$. If the water surrounding the fish has resistance of $800\ \Omega$, how much current can the eel produce in water from near its head to near its tail?

15: Integrated Concepts

A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in ºC/min) will its temperature increase if its mass is 20.0 kg and it has a specific heat of $0.300\ \text{kcal/kg}\cdot^{\circ}\text{C}$, assuming no heat escapes?

16: Unreasonable Results

A 1.58-V alkaline cell with a $0.200-\Omega$ internal resistance is supplying 8.50 A to a load. (a) What is its terminal voltage? (b) What is the value of the load resistance? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable or inconsistent?

17: Unreasonable Results

(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a $15.0-\Omega$ bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

electromotive force (emf)
the potential difference of a source of electricity when no current is flowing; measured in volts
internal resistance
the amount of resistance within the voltage source
potential difference
the difference in electric potential between two points in an electric circuit, measured in volts
terminal voltage
the voltage measured across the terminals of a source of potential difference

Solutions

Problems Exercises

1: 2.00 V

3: 2.9994 V

5: $0.375\ \Omega$

8: (a) 0.658 A

(b) 0.997 W

(c) 0.997 W; yes

10: (a) 200 A

(b) 10.0 V

(c) 2.00 kW

(d) $0.1000\ \Omega$; 80.0 A, 4.0 V, 320 W

12: (a) $0.400\ \Omega$

(b) No, there is only one independent equation, so only r can be found.

16: (a) –0.120 V

(b) $-1.41 \times 10^{-2}\ \Omega$

(c) Negative terminal voltage; negative load resistance.

(d) The assumption that such a cell could provide 8.50 A is inconsistent with its internal resistance.

21.3 Kirchhoff's Rules

Summary

- Analyze a complex circuit using Kirchhoff's rules, using the conventions for determining the correct signs of various terms.

Many complex circuits, such as the one in Figure 1, cannot be analyzed with the series-parallel techniques developed in Chapter 21.1 Resistors in Series and Parallel and Chapter 21.2 Electromotive Force: Terminal Voltage. There are, however, two circuit analysis rules that can be used to analyze any circuit, simple or complex. These rules are special cases of the laws of conservation of charge and conservation of energy. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).

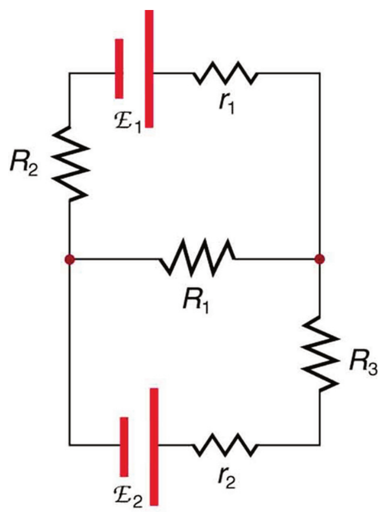


Figure 1. This circuit cannot be reduced to a combination of series and parallel connections. Kirchhoff's rules, special applications of the laws of conservation of charge and energy, can be used to analyze it. (Note: The script E in the figure represents electromotive force, emf.)

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

Explanations of the two rules will now be given, followed by problem-solving hints for applying Kirchhoff's rules, and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) is an application of the conservation of charge to a junction; it is illustrated in Figure 2. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out. Kirchhoff's first rule requires that $I_1 = I_2 + I_3$ (see figure). Equations like this can and will be used to analyze circuits and to solve circuit problems.

Making Connections: Conservation Laws

Kirchhoff's rules for circuit analysis are applications of **conservation laws** to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application.

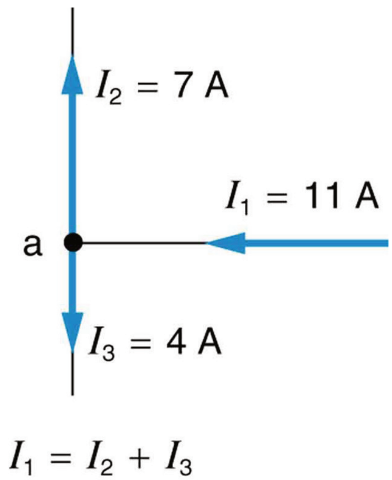


Figure 2. The junction rule. The diagram shows an example of Kirchhoff's first rule where the sum of the currents into a junction equals the sum of the currents out of a junction. In this case, the current going into the junction splits and comes out as two currents, so that $I_1 = I_2 + I_3$. Here I_1 must be 11 A, since I_2 is 7 A and I_3 is 4 A.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) is an application of conservation of energy. The loop rule is stated in terms of potential, V, rather than potential energy, but the two are related since $\mathrm{PE_{elec}} = qV$. Recall that **emf** is the potential difference of a source when no current is flowing. In a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Figure 3 illustrates the changes in potential in a simple series circuit loop.

Kirchhoff's second rule requires $\mathrm{emf} - Ir - IR_1 - IR_2 = 0$. Rearranged, this is $\mathrm{emf} = Ir + IR_1 + IR_2$, which means the emf equals the sum of the IR (voltage) drops in the loop.

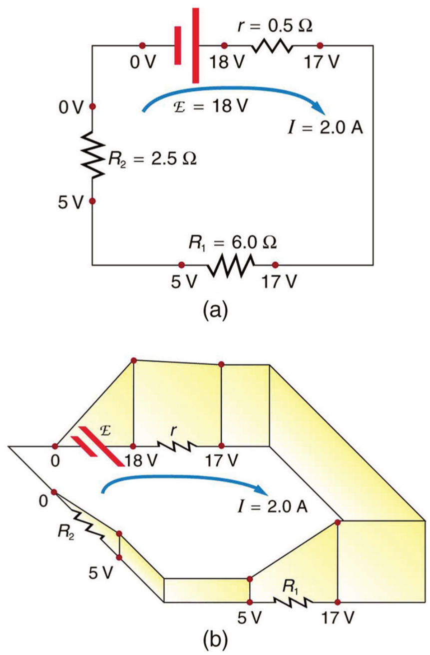


Figure 3. The loop rule. An example of Kirchhoff's second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff's rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

1. When applying Kirchhoff's first rule, the junction rule, you must label the current in each branch and decide in what direction it is going. For example, in Figure 1, Figure 2, and Figure 3, currents are labeled I_1, I_2, I_3, and I, and arrows indicate their directions. There is no risk here, for if you choose the wrong direction, the current will be of the correct magnitude but negative.

2. When applying Kirchhoff's second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise. For example, in Figure 3 the loop was traversed in the same direction as the current (clockwise). Again, there is no risk; going around the circuit in the opposite direction reverses the sign of every term in the equation, which is like multiplying both sides of

the equation by –1.

Figure 4 and the following points will help you get the plus or minus signs right when applying the loop rule. Note that the resistors and emfs are traversed by going from a to b. In many circuits, it will be necessary to construct more than one loop. In traversing each loop, one needs to be consistent for the sign of the change in potential. (See Example 1.)

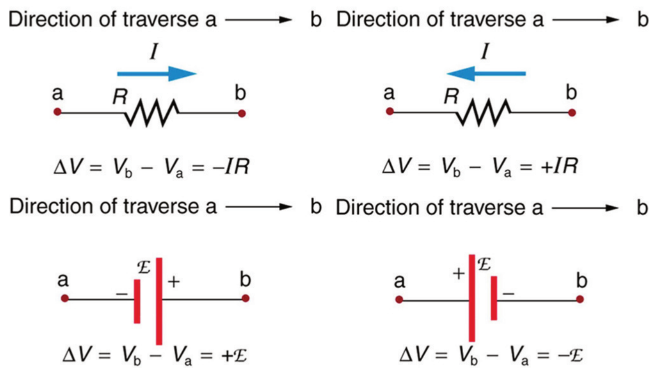


Figure 4. Each of these resistors and voltage sources is traversed from a to b. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

- When a resistor is traversed in the same direction as the current, the change in potential is $-IR$. (See Figure 4.)
- When a resistor is traversed in the direction opposite to the current, the change in potential is $+IR$. (See Figure 4.)
- When an emf is traversed from – to + (the same direction it moves positive charge), the change in potential is +emf. (See Figure 4.)
- When an emf is traversed from + to – (opposite to the direction it moves positive charge), the change in potential is −emf. (See Figure 4.)

Example 1: Calculating Current: Using Kirchhoff's Rules

Find the currents flowing in the circuit in Figure 5.

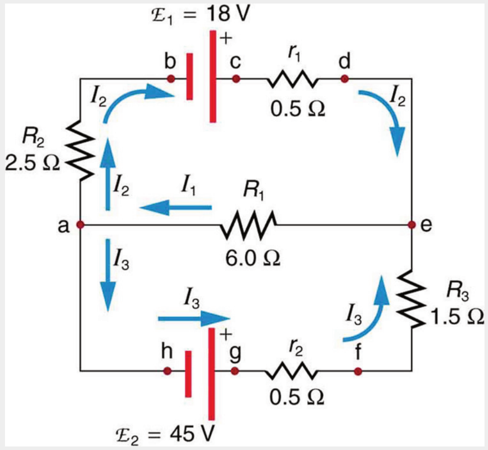


Figure 5. This circuit is similar to that in Figure 1, but the resistances and emfs are specified. (Each emf is denoted by script E.) The currents in each branch are labeled and assumed to move in the directions shown. This example uses Kirchhoff's rules to find the currents.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1, I_2, and I_3 in the figure and assumptions have been made about their directions. Locations on the diagram have been labeled with letters a through h. In the solution we will apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

We begin by applying Kirchhoff's first or junction rule at point a. This gives

$$I_1 = I_2 + I_3,$$

since I_1 flows into the junction, while I_2 and I_3 flow out. Applying the junction rule at e produces exactly the same equation, so that no new information is obtained. This is a single equation with three unknowns—three independent equations are needed, and so the loop rule must be applied.

Now we consider the loop abcdea. Going from a to b, we traverse R_2 in the same (assumed) direction of the current I_2, and so the change in potential is $-I_2R_2$. Then going from b to c, we go from – to +, so that the change in potential is $+\text{emf}_1$. Traversing the internal resistance r_1 from c to d gives $-I_2r_1$. Completing the loop by going from d to a again traverses a resistor in the same direction as its current, giving a change in potential of $-I_1R_1$.

The loop rule states that the changes in potential sum to zero. Thus,

$$-I_2R_2 + \text{emf}_1 - I_2r_1 - I_1R_1 = -I_2(R_2 + r_1) + \text{emf}_1 - I_1R_1 = 0.$$

Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives

$$-3I_2 + 18 - 6I_1 = 0.$$

Now applying the loop rule to aefgha (we could have chosen abcdefgha as well) similarly gives

$$+I_1R_1 + I_3R_3 + I_3r_2 - \text{emf}_2 = +I_1R_1 + I_3(R_3 + r_2) - \text{emf}_2 = 0.$$

Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. With values entered, this becomes

$$+6I_1 + 2I_3 - 45 = 0.$$

These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for I_2:

$$I_2 = 6 - 2I_1.$$

Now solve the third equation for I_3:

$$I_3 = 22.5 - 3I_1.$$

Substituting these two new equations into the first one allows us to find a value for I_1:

$$I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1.$$

Combining terms gives

$6I_1 = 28.5$, and

$$I_1 = 4.75 \text{ A}.$$

Substituting this value for I_1 back into the fourth equation gives

$$I_2 = 6 - 2I_1 = 6 - 9.50$$

$$I_2 = -3.50 \text{ A}.$$

The minus sign means I_2 flows in the direction opposite to that assumed in Figure 5.

Finally, substituting the value for I_1 into the fifth equation gives

$$I_3 = 22.5 - 3I_1 = 22.5 - 14.25$$

$$I_3 = 8.25 \text{ A}.$$

Discussion

Just as a check, we note that indeed $I_1 = I_2 + I_3$. The results could also have been checked by entering all of the values into the equation for the abcdefgha loop.

Problem-Solving Strategies for Kirchhoff's Rules

1. Make certain there is a clear circuit diagram on which you can label all known and unknown resistances, emfs, and currents. If a current is unknown, you must assign it a direction. This is necessary for determining the signs of potential changes. If you assign the direction incorrectly, the current will be found to have a negative value—no harm done.

2. Apply the junction rule to any junction in the circuit. Each time the junction rule is applied, you should get an equation with a current that does not appear in a previous application—if not, then the equation is redundant.

3. Apply the loop rule to as many loops as needed to solve for the unknowns in the problem. (There must be as many independent equations as unknowns.) To apply the loop rule, you must choose a direction to go around the loop. Then carefully and consistently determine the signs of the potential changes for each element using the four bulleted points discussed above in conjunction with Figure 4.

4. Solve the simultaneous equations for the unknowns. This may involve many algebraic steps, requiring careful checking and rechecking.

5. Check to see whether the answers are reasonable and consistent. The numbers should be of the correct order of magnitude, neither exceedingly large nor vanishingly small. The signs should be reasonable—for example, no resistance should be negative. Check to see that the values obtained sat-

isfy the various equations obtained from applying the rules. The currents should satisfy the junction rule, for example.

The material in this section is correct in theory. We should be able to verify it by making measurements of current and voltage. In fact, some of the devices used to make such measurements are straightforward applications of the principles covered so far and are explored in the next modules. As we shall see, a very basic, even profound, fact results—making a measurement alters the quantity being measured.

Check Your Understanding

1: Can Kirchhoff's rules be applied to simple series and parallel circuits or are they restricted for use in more complicated circuits that are not combinations of series and parallel?

Section Summary

- Kirchhoff's rules can be used to analyze any circuit, simple or complex.
- Kirchhoff's first rule—the junction rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule: The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.
- The two rules are based, respectively, on the laws of conservation of charge and energy.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- The simpler series and parallel rules are special cases of Kirchhoff's rules.

Conceptual Questions

1: Can all of the currents going into the junction in Figure 6 be positive? Explain.

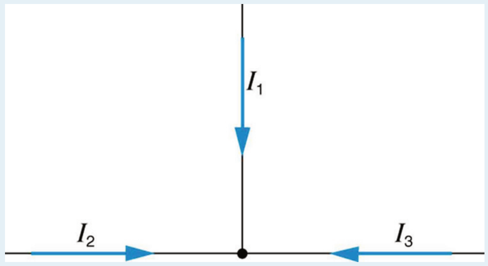


Figure 6.

2: Apply the junction rule to junction b in Figure 7. Is any new information gained by applying the junction rule at e? (In the figure, each emf is represented by script E.)

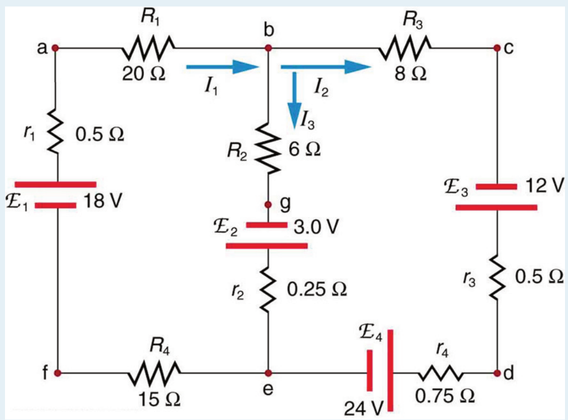


Figure 7.

3: (a) What is the potential difference going from point a to point b in Figure 7? (b) What is the potential difference going from c to b? (c) From e to g? (d) From e to d?

4: Apply the loop rule to loop afedcba in Figure 7.

5: Apply the loop rule to loops abgefa and cbgedc in Figure 7.

Problem Exercises

1: Apply the loop rule to loop abcdefgha in Figure 5.

2: Apply the loop rule to loop aedcba in Figure 5.

3: Verify the second equation in Example 1 by substituting the values found for the currents I_1 and I_2.

4: Verify the third equation in Example 1 by substituting the values found for the currents I_1 and I_3.

5: Apply the junction rule at point a in Figure 8.

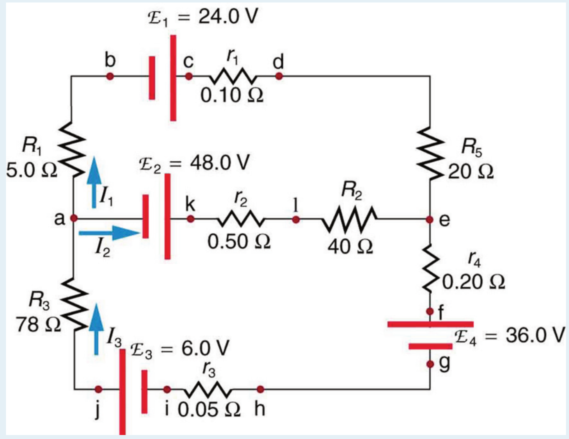


Figure 8.

6: Apply the loop rule to loop abcdefghija in Figure 8.

7: Apply the loop rule to loop akledcba in Figure 8.

8: Find the currents flowing in the circuit in Figure 8. Explicitly show how you follow the steps in the Chapter 21.1 Problem-Solving Strategies for Series and Parallel Resistors.

9: Solve Example 1, but use loop abcdefgha instead of loop akledcba. Explicitly show how you follow the steps in the Chapter 21.1 Problem-Solving Strategies for Series and Parallel Resistors.

10: Find the currents flowing in the circuit in Figure 7.

11: Unreasonable Results

Consider the circuit in Figure 9, and suppose that the emfs are unknown and the currents are given to be $I_1 = 5.00\ \text{A}$, $I_2 = 3.0\ \text{A}$, and $I_3 = -2.00\ \text{A}$. (a) Could you find the emfs? (b) What is wrong with the assumptions?

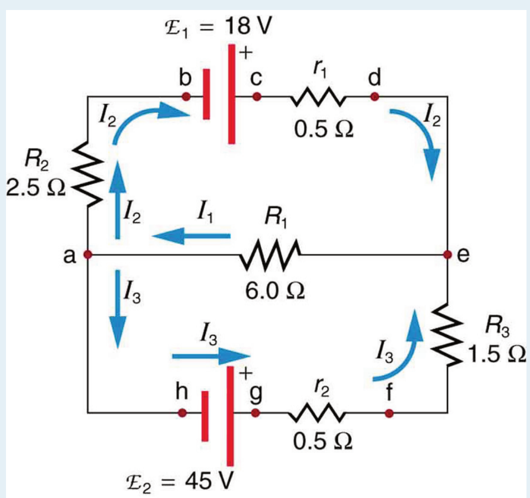


Figure 9.

Glossary

Kirchhoff’s rules

a set of two rules, based on conservation of charge and energy, governing current and changes in potential in an *electric circuit*

junction rule

Kirchhoff's first rule, which applies the conservation of charge to a junction; current is the flow of charge; thus, whatever charge flows into the junction must flow out; the rule can be stated $I_1 = I_2 + I_3$

loop rule

Kirchhoff's second rule, which states that in a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Thus, the emf equals the sum of the IR *(voltage) drops in the loop and can be stated:*

$\text{emf} = Ir + IR_1 + IR_2$

conservation laws

require that energy and charge be conserved in a system

Solutions

Check Your Understanding

1: Kirchhoff's rules can be applied to any circuit since they are applications to circuits of two conservation laws. Conservation laws are the most broadly applicable principles in physics. It is usually mathematically simpler to use the rules for series and parallel in simpler circuits so we emphasize Kirchhoff's rules for use in more complicated situations. But the rules for series and parallel can be derived from Kirchhoff's rules. Moreover, Kirchhoff's rules can be expanded to devices other than resistors and emfs, such as capacitors, and are one of the basic analysis devices in circuit analysis.

Problem Exercises

1: $-I_2R_2 + \text{emf}_1 - I_2r_1 + I_3R_3 + I_3r_2 - \text{emf}_2 = 0$

5: $I_3 = I_1 + I_2$

7: $\text{emf}_2 - I_2r_2 - I_2R_2 + I_1R_5 + I_1r_1 - \text{emf}_1 + I_1R_1 = 0$

9: (a) $I_1 = 4.75\ \text{A}$

(b) $I_2 = -3.5\ \text{A}$

(c) $I_3 = 8.25\ \text{A}$

11: (a) No, you would get inconsistent equations to solve.

(b) $I_1 \neq I_2 + I_3$. The assumed currents violate the junction rule.

21.4 DC Voltmeters and Ammeters

Summary

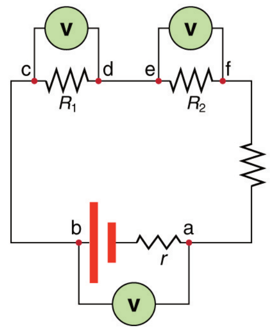
- Explain why a voltmeter must be connected in parallel with the circuit.
- Draw a diagram showing an ammeter correctly connected in a circuit.
- Describe how a galvanometer can be used as either a voltmeter or an ammeter.
- Find the resistance that must be placed in series with a galvanometer to allow it to be used as a voltmeter with a given reading.
- Explain why measuring the voltage or current in a circuit can never be exact.

Voltmeters measure voltage, whereas **ammeters** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are voltmeters or ammeters. (See Figure 1.) The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.

Figure 1. The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of "sender" units, which are hopefully proportional to the amount of gasoline in the tank and the engine temperature. (credit: Christian Giersing)

Voltmeters are connected in parallel with whatever device's voltage is to be measured. A parallel connection is used because objects in parallel experience the same potential difference. (See Figure 2, where the voltmeter is represented by the symbol V.)

Ammeters are connected in series with whatever device's current is to be measured. A series connection is used because objects in series have the same current passing through them. (See Figure 3, where the ammeter is represented by the symbol A.)



(a)

(b)

Figure 2. (a) To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the voltage source or either of the resistors. Note that terminal voltage is measured between points a and b. It is not possible to connect the voltmeter directly across the emf without including its internal resistance, ***r***. (b) A digital voltmeter in use. (credit: Messtechniker, Wikimedia Commons)

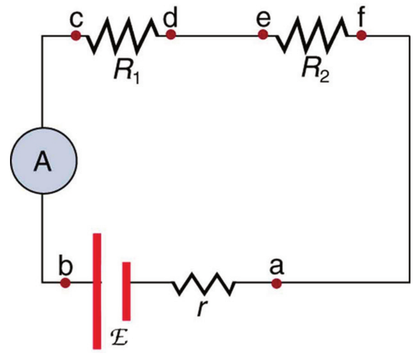


Figure 3. An ammeter (A) is placed in series to measure current. All of the current in this circuit flows through the meter. The ammeter would have the same reading if located between points d and e or between points f and a as it does in the position shown. (Note that the script capital E stands for emf, and ***r*** stands for the internal resistance of the source of potential difference.)

Analog Meters: Galvanometers

Analog meters have a needle that swivels to point at numbers on a scale, as opposed to **digital meters**, which have numerical readouts similar to a hand-held calculator. The heart of most analog meters is a device called a **galvanometer**, denoted by G. Current flow through a galvanometer, I_G, produces a proportional needle deflection. (This deflection is due to the force of a magnetic field upon a current-carrying wire.)

The two crucial characteristics of a given galvanometer are its resistance and current sensitivity. **Current sensitivity** is the current that gives a **full-scale deflection** of the galvanometer's needle, the maximum current that the instrument can measure. For example, a galvanometer with a current sensitivity of $50\ \mu A$ has a maximum deflection of its needle when $50\ \mu A$ flows through it, reads half-scale when $25\ \mu A$ flows through it, and so on.

If such a galvanometer has a $25-\Omega$ resistance, then a voltage of only $V = IR = (50\ \mu A)(25\ \Omega) = 1.25\ mV$ produces a full-scale reading. By connecting resistors to this galvanometer in different ways, you can use it as either a voltmeter or ammeter that can measure a broad range of voltages or currents.

Galvanometer as Voltmeter

Figure 4 shows how a galvanometer can be used as a voltmeter by connecting it in series with a large resistance, R. The value of the resistance R is determined by the maximum voltage to be measured. Suppose you want 10 V to produce a full-scale deflection of a voltmeter containing a $25-\Omega$ galvanometer with a $50-\mu A$ sensitivity. Then 10 V applied to the meter must produce a current of $50\ \mu A$. The total resistance must be

$$R_{tot} = R + r = \frac{V}{I} = \frac{10\ V}{50\ \mu A} = 200\ k\Omega\text{, or}$$

$$R = R_{tot} - r = 200\ k\Omega - 25\ \Omega \approx 200\ k\Omega$$

(R is so large that the galvanometer resistance, r, is nearly negligible.) Note that 5 V applied to this voltmeter produces a half-scale deflection by producing a $25-\mu A$ current through the meter, and so the voltmeter's reading is proportional to voltage as desired.

This voltmeter would not be useful for voltages less than about half a volt, because the meter deflection would be small and difficult to read accurately. For other voltage ranges, other resistances are placed in series with the galvanometer. Many meters have a choice of scales. That choice involves switching an appropriate resistance into series with the galvanometer.

Figure 4. A large resistance ***R*** placed in series with a galvanometer G produces a voltmeter, the full-scale deflection of which depends on the choice of ***R***. The larger the voltage to be measured, the larger ***R*** must be. (Note that ***r*** represents the internal resistance of the galvanometer.)

Galvanometer as Ammeter

The same galvanometer can also be made into an ammeter by placing it in parallel with a small resistance R, often called the **shunt resistance**, as shown in Figure 5. Since the shunt resistance is small, most of the current passes through it, allowing an ammeter to measure currents much greater than those producing a full-scale deflection of the galvanometer.

Suppose, for example, an ammeter is needed that gives a full-scale deflection for 1.0 A, and contains the same $25-\Omega$ galvanometer with its $50-\mu A$ sensitivity. Since R and r are in parallel, the voltage across them is the same.

These IR drops are $IR = I_G r$ so that $IR = \frac{I_G}{I} = \frac{R}{r}$. Solving for R, and noting that I_G is $50\ \mu A$ and I is 0.999950 A, we have

$$R = r\frac{I_G}{I} = (25\ \Omega)\frac{50\ \mu A}{0.999950\ A} = 1.25 \times 10^{-3}\ \Omega\ .$$

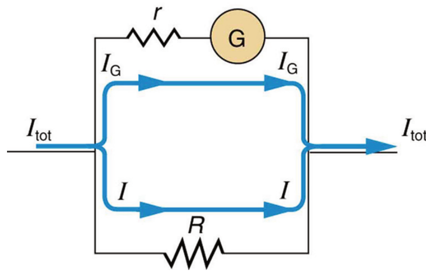


Figure 5. A small shunt resistance ***R*** placed in parallel with a galvanometer G produces an ammeter, the full-scale deflection of which depends on the choice of ***R***. The larger the current to be measured, the smaller ***R*** must be. Most of the current (***I***) flowing through the meter is shunted through ***R*** to protect the galvanometer. (Note that ***r*** represents the internal resistance of the galvanometer.) Ammeters may also have multiple scales for greater flexibility in application. The various scales are achieved by switching various shunt resistances in parallel with the galvanometer—the greater the maximum current to be measured, the smaller the shunt resistance must be.

Taking Measurements Alters the Circuit

When you use a voltmeter or ammeter, you are connecting another resistor to an existing circuit and, thus, altering the circuit. Ideally, voltmeters and ammeters do not appreciably affect the circuit, but it is instructive to examine the circumstances under which they do or do not interfere.

First, consider the voltmeter, which is always placed in parallel with the device being measured. Very little current flows through the voltmeter if its resistance is a few orders of magnitude greater than the device, and so the circuit is not appreciably affected. (See Figure 6(a).) (A large resistance in parallel with a small one has a combined resistance essentially equal to the small one.) If, however, the voltmeter's resistance is comparable to that of the device being measured, then the two in parallel have a smaller resistance, appreciably affecting the circuit. (See Figure 6(b).) The voltage across the device is not the same as when the voltmeter is out of the circuit.

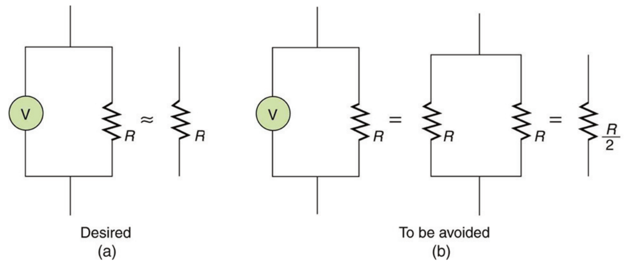


Figure 6. (a) A voltmeter having a resistance much larger than the device (**$R_{Voltmeter}$ >> R**) with which it is in parallel produces a parallel resistance essentially the same as the device and does not appreciably affect the circuit being measured. (b) Here the voltmeter has the same resistance as the device (**$R_{Voltmeter}$? R**), so that the parallel resistance is half of what it is when the voltmeter is not connected. This is an example of a significant alteration of the circuit and is to be avoided.

An ammeter is placed in series in the branch of the circuit being measured, so that its resistance adds to that branch. Normally, the ammeter's resistance is very small compared with the resistances of the devices in the circuit, and so the extra resistance is negligible. (See Figure 7(a).) However, if very small load resistances are involved, or if the ammeter is not as low in resistance as it should be, then the total series resistance is significantly greater, and the current in the branch being measured is reduced. (See Figure 7(b).)

A practical problem can occur if the ammeter is connected incorrectly. If it was put in parallel with the resistor to measure the current in it, you could possibly damage the meter; the low resistance of the ammeter would allow most of the current in the circuit to go through the galvanometer, and this current would be larger since the effective resistance is smaller.

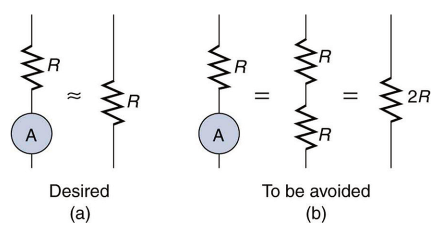


Figure 7. (a) An ammeter normally has such a small resistance that the total series resistance in the branch being measured is not appreciably increased. The circuit is essentially unaltered compared with when the ammeter is absent. (b) Here the ammeter's resistance is the same as that of the branch, so that the total resistance is doubled and the current is half what it is without the ammeter. This significant alteration of the circuit is to be avoided.

One solution to the problem of voltmeters and ammeters interfering with the circuits being measured is to use galvanometers with greater sensitivity. This allows construction of voltmeters with greater resistance and ammeters with smaller resistance than when less sensitive galvanometers are used.

There are practical limits to galvanometer sensitivity, but it is possible to get analog meters that make measurements accurate to a few percent. Note that the inaccuracy comes from altering the circuit, not from a fault in the meter.

Connections: Limits to Knowledge

Making a measurement alters the system being measured in a manner that produces uncertainty in the measurement. For macroscopic systems, such as the circuits discussed in this module, the alteration can usually be made negligibly small, but it cannot be eliminated entirely. For submicroscopic systems, such as atoms, nuclei, and smaller particles, measurement alters the system in a manner that cannot be made arbitrarily small. This actually limits knowledge of the system—even limiting what nature can know about itself. We shall see profound implications of this when the Heisenberg uncertainty principle is discussed in the modules on quantum mechanics.

There is another measurement technique based on drawing no current at all and, hence, not altering the circuit at all. These are called null measurements and are the topic of Chapter 21.5 Null Measurements. Digital meters that employ solid-state electronics and null measurements can attain accuracies of one part in 10^6.

Check Your Understanding

1: Digital meters are able to detect smaller currents than analog meters employing galvanometers. How does this explain their ability to measure voltage and current more accurately than analog meters?

PhET Explorations: Circuit Construction Kit (DC Only), Virtual Lab

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

PhET Interactive Simulation

Figure 8. Circuit Construction Kit (DC Only), Virtual Lab

Section Summary

- Voltmeters measure voltage, and ammeters measure current.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.
- Both can be based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current.

- Standard voltmeters and ammeters alter the circuit being measured and are thus limited in accuracy.

Conceptual Questions

1: Why should you not connect an ammeter directly across a voltage source as shown in Figure 9? (Note that script E in the figure stands for emf.)

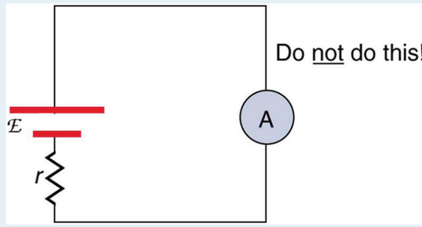


Figure 9.

2: Suppose you are using a multimeter (one designed to measure a range of voltages, currents, and resistances) to measure current in a circuit and you inadvertently leave it in a voltmeter mode. What effect will the meter have on the circuit? What would happen if you were measuring voltage but accidentally put the meter in the ammeter mode?

3: Specify the points to which you could connect a voltmeter to measure the following potential differences in Figure 10: (a) the potential difference of the voltage source; (b) the potential difference across R_1; (c) across R_2; (d) across R_3; (e) across R_2 and R_3. Note that there may be more than one answer to each part.

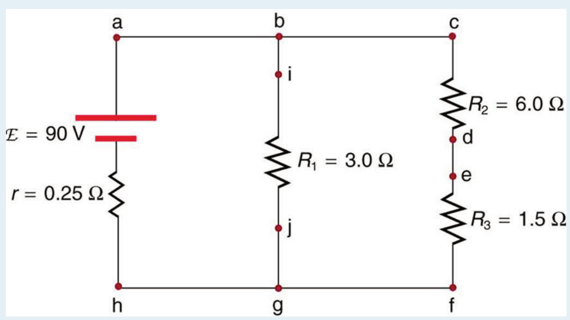


Figure 10.

4: To measure currents in Figure 10, you would replace a wire between two points with an ammeter. Specify the points between which you would place an ammeter to measure the following: (a) the total current; (b) the current flowing through R_1; (c) through R_2; (d) through R_3. Note that there may be more than one answer to each part.

Problem Exercises

1: What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $1.00-\text{M}\Omega$ resistance on its 30.0-V scale?

2: What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $25.0-\text{k}\Omega$ resistance on its 100-V scale?

3: Find the resistance that must be placed in series with a $25.0-\Omega$ galvanometer having a $50.0-\mu A$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 0.100-V full-scale reading.

4: Find the resistance that must be placed in series with a $25.0-\Omega$ galvanometer having a $50.0-\mu A$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit diagram with your solution.

5: Find the resistance that must be placed in parallel with a $25.0-\Omega$ galvanometer having a $50.0-\ A$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 10.0-A full-scale reading. Include a circuit diagram with your solution.

6: Find the resistance that must be placed in parallel with a $25.0-\Omega$ galvanometer having a $50.0-\mu A$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 300-mA full-scale reading.

7: Find the resistance that must be placed in series with a $10.0-\Omega$ galvanometer having a $100-\mu A$ sensitivity to allow it to be used as a voltmeter with: (a) a 300-V full-scale reading, and (b) a 0.300-V full-scale reading.

8: Find the resistance that must be placed in parallel with a $10.0-\Omega$ galvanometer having a $100-\mu A$ sensitivity to allow it to be used as an ammeter with: (a) a 20.0-A full-scale reading, and (b) a 100-mA full-scale reading.

9: Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\ \Omega$ by placing a $1.00-k\Omega$ voltmeter across its terminals. (See Figure 11.) (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

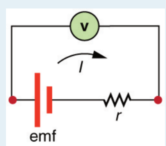


Figure 11.

10: Suppose you measure the terminal voltage of a 3.200-V lithium cell having an internal resistance of $5.00\ \Omega$ by placing a $1.00-k\Omega$ voltmeter across its terminals. (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

11: A certain ammeter has a resistance of $5.00 \times 10^{-5}\ \Omega$ on its 3.00-A scale and contains a $10.0-\Omega$ galvanometer. What is the sensitivity of the galvanometer?

12: A $1.00-M\Omega$ voltmeter is placed in parallel with a $75.0-k\Omega$ resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) What is the resistance of the combination? (c) If the voltage across the combination is kept the same as it was across the $75.0-k\Omega$ resistor alone, what is the percent increase in current? (d) If the current through the combination is kept the same as it was through the $75.0-k\Omega$ resistor alone, what is the percentage decrease in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

13: A $0.0200-\Omega$ ammeter is placed in series with a $10.00-\Omega$ resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) Calculate the resistance of the combination. (c) If the voltage is kept the same across the combination as it was through the $10.00-\Omega$ resistor alone, what is the percent decrease in current? (d) If the current is kept the same through the combination as it was through the $10.00-\Omega$ resistor alone, what is the percent increase in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

14: Unreasonable Results

Suppose you have a $40.0-\Omega$ galvanometer with a $25.0-\mu A$ sensitivity. (a) What resistance would you put in series with it to allow it to be used as a voltmeter that has a full-scale deflection for 0.500 mV? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

15: Unreasonable Results

(a) What resistance would you put in parallel with a $40.0-\Omega$ galvanometer having a $25.0-\mu A$ sensitivity to allow it to be used as an ammeter that has a full-scale deflection for $10.0-\mu A$? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Glossary

voltmeter
an instrument that measures voltage

ammeter
an instrument that measures current

analog meter
a measuring instrument that gives a readout in the form of a needle movement over a marked gauge

digital meter
a measuring instrument that gives a readout in a digital form

galvanometer
an analog measuring device, denoted by G, that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire

current sensitivity
the maximum current that a galvanometer can read

full-scale deflection
the maximum deflection of a galvanometer needle, also known as current sensitivity; a galvanometer with a full-scale deflection of $50\ \mu A$ has a maximum deflection of its needle when $50\ \mu A$ flows through it

shunt resistance
a small resistance R placed in parallel with a galvanometer G to produce an ammeter; the larger the current to be measured, the smaller R must be; most of the current flowing through the meter is shunted through R to protect the galvanometer

Solutions

Check Your Understanding

1: Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult Figure 2 and Figure 3 and their discussion in the text.

Problem Exercises

1: $30\ \mu A$

3: $1.98\ k\Omega$

5: $1.25 \times 10^{-4}\ \Omega$

7: (a) $3.00\ \text{M}\Omega$

(b) $2.99\ \text{k}\Omega$

9: (a) 1.58 mA
(b) 1.5848 V (need four digits to see the difference)

(c) 0.99990 (need five digits to see the difference from unity)

11: $15.0\ \mu\text{A}$

13: (a)

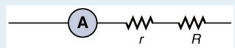


Figure 12.

(b) $10.02\ \Omega$

(c) 0.9980, or a 2.0×10^{-1} percent decrease

(d) 1.002, or a 2.0×10^{-1} percent increase

(e) Not significant.

15: (a) $-66.7\ \Omega$

(b) You can't have negative resistance.

(c) It is unreasonable that I_G is greater than I_{tot} (see Figure 5). You cannot achieve a full-scale deflection using a current less than the sensitivity of the galvanometer.

21.5 Null Measurements

Summary

- Explain why a null measurement device is more accurate than a standard voltmeter or ammeter.
- Demonstrate how a Wheatstone bridge can be used to accurately calculate the resistance in a circuit.

Standard measurements of voltage and current alter the circuit being measured, introducing uncertainties in the measurements. Voltmeters draw some extra current, whereas ammeters reduce current flow. Null measurements balance voltages so that there is no current flowing through the measuring device and, therefore, no alteration of the circuit being measured.

Null measurements are generally more accurate but are also more complex than the use of standard voltmeters and ammeters, and they still have limits to their precision. In this module, we shall consider a few specific types of null measurements, because they are common and interesting, and they further illuminate principles of electric circuits.

The Potentiometer

Suppose you wish to measure the emf of a battery. Consider what happens if you connect the battery directly to a standard voltmeter as shown in Figure 1. (Once we note the problems with this measurement, we will examine a null measurement that improves accuracy.) As discussed before, the actual quantity measured is the terminal voltage V, which is related to the emf of the battery by $V = \text{emf} - Ir$, where I is the current that flows and r is the internal resistance of the battery.

The emf could be accurately calculated if r were very accurately known, but it is usually not. If the current I could be made zero, then $V = \text{emf}$, and so emf could be directly measured. However, standard voltmeters need a current to operate; thus, another technique is needed.

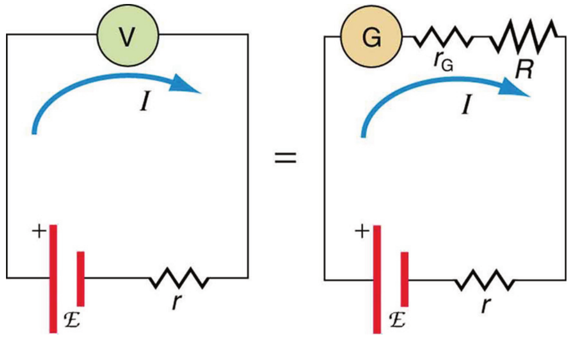


Figure 1. An analog voltmeter attached to a battery draws a small but nonzero current and measures a terminal voltage that differs from the emf of the battery. (Note that the script capital E symbolizes electromotive force, or emf.) Since the internal resistance of the battery is not known precisely, it is not possible to calculate the emf precisely.

A potentiometer is a null measurement device for measuring potentials (voltages). (See Figure 2.) A voltage source is connected to a resistor R, say, a long wire, and passes a constant current through it. There is a steady drop in potential (an IR drop) along the wire, so that a variable potential can be obtained by making contact at varying locations along the wire.

Figure 2(b) shows an unknown emf_x (represented by script E_x in the figure) connected in series with a galvanometer. Note that emf_x opposes the other voltage source. The location of the contact point (see the arrow on the drawing) is adjusted until the galvanometer reads zero. When the galvanometer reads zero, $\text{emf}_x = IR_x$ is the resistance of the section of wire up to the contact point. Since no current flows through the galvanometer, none flows through the unknown emf, and so emf_x is directly sensed.

Now, a very precisely known standard emf_s is substituted for emf_x, and the contact point is adjusted until the galvanometer again reads zero, so that $\text{emf}_s = IR_s$. In both cases, no current passes through the galvanometer, and so the current I through the long wire is the same. Upon taking the ratio $\frac{\text{emf}_x}{\text{emf}_s}$, I cancels, giving

$$\frac{\text{emf}_x}{\text{emf}_s} = \frac{IR_x}{IR_s} = \frac{R_x}{R_s}.$$

Solving for emf_x gives

$$\text{emf}_x = \text{emf}_s \frac{R_x}{R_s}.$$

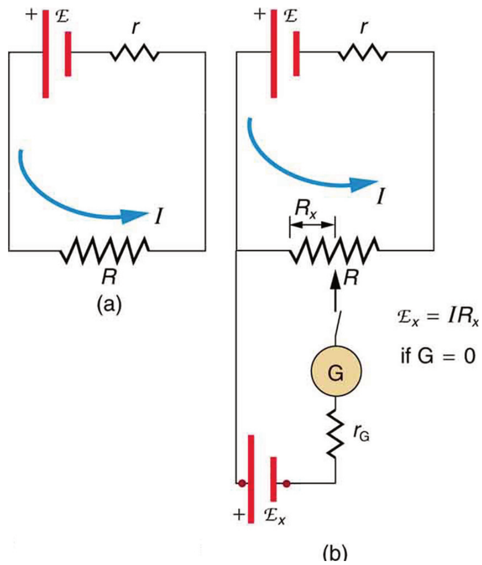


Figure 2. The potentiometer, a null measurement device. (a) A voltage source connected to a long wire resistor passes a constant current ***I*** through it. (b) An unknown emf (labeled script ***E*x** in the figure) is connected as shown, and the point of contact along ***R*** is adjusted until the galvanometer reads zero. The segment of wire has a resistance **Rx** and script ***E*x = *IR*x**, where ***I*** is unaffected by the connection since no current flows through the galvanometer. The unknown emf is thus proportional to the resistance of the wire segment.

Because a long uniform wire is used for R, the ratio of resistances R_x/R_s is the same as the ratio of the lengths of wire that zero the galvanometer for each emf. The three quantities on the right-hand side of the equation are now known or measured, and emf_x can be calculated. The uncertainty in this calculation can be considerably smaller than when using a voltmeter directly, but it is not zero. There is always some uncertainty in the ratio of resistances R_x/R_s and in the standard emf_s. Furthermore, it is not possible to tell when the galvanometer reads exactly zero, which introduces error into both R_x and R_s, and may also affect the current I.

Resistance Measurements and the Wheatstone Bridge

There is a variety of so-called ohmmeters that purport to measure resistance. What the most common ohmmeters actually do is to apply a voltage to a resistance, measure the current, and calculate the resistance using Ohm’s law. Their readout is this calculated resistance. Two configurations for ohmmeters using standard voltmeters and ammeters are shown in Figure 3. Such configurations are limited in accuracy, because the meters alter both the voltage applied to the resistor and the current that flows through it.

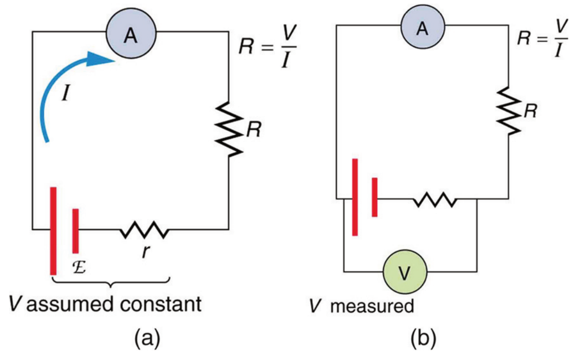


Figure 3. Two methods for measuring resistance with standard meters. (a) Assuming a known voltage for the source, an ammeter measures current, and resistance is calculated as ***R* = *V*/*I***. (b) Since the terminal voltage ***V*** varies with current, it is better to measure it. ***V*** is most accurately known when ***I*** is small, but ***I*** itself is most accurately known when it is large.

The Wheatstone bridge is a null measurement device for calculating resistance by balancing potential drops in a circuit. (See Figure 4.) The device is called a bridge because the galvanometer forms a bridge between two branches. A variety of bridge devices are used to make null measurements in circuits.

Resistors R_1 and R_2 are precisely known, while the arrow through R_3 indicates that it is a variable resistance. The value of R_3 can be precisely read. With the unknown resistance R_x in the circuit, R_3 is adjusted until the galvanometer reads zero. The potential difference between points b and d is then zero, meaning that b and d are at the same potential. With no current running through the galvanometer, it has no effect on the rest of the circuit. So the branches abc and adc are in parallel, and each branch has the full voltage of the source. That is, the IR drops along abc and adc are the same. Since b and d are at the same potential, the IR drop along ad must equal the IR drop along ab. Thus,

$$I_1R_1 = I_2R_3.$$

Again, since b and d are at the same potential, the IR drop along dc must equal the IR drop along bc. Thus,

$$I_1R_2 = I_2R_x.$$

Taking the ratio of these last two expressions gives

$$\frac{I_1R_1}{I_1R_2} = \frac{I_2R_3}{I_2R_x}.$$

Canceling the currents and solving for R_X yields

$$R_x = R_3\frac{R_2}{R_1}.$$

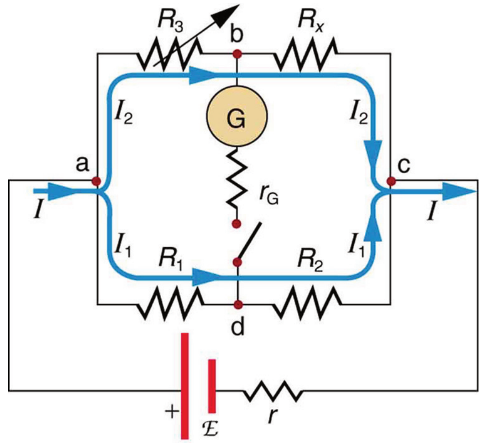


Figure 4. The Wheatstone bridge is used to calculate unknown resistances. The variable resistance $\boldsymbol{R_3}$ is adjusted until the galvanometer reads zero with the switch closed. This simplifies the circuit, allowing $\boldsymbol{R_x}$ to be calculated based on the $\boldsymbol{IR}$ drops as discussed in the text.

This equation is used to calculate the unknown resistance when current through the galvanometer is zero. This method can be very accurate (often to four significant digits), but it is limited by two factors. First, it is not possible to get the current through the galvanometer to be exactly zero. Second, there are always uncertainties in R_1, R_2, and R_3, which contribute to the uncertainty in R_x.

Check Your Understanding

1: Identify other factors that might limit the accuracy of null measurements. Would the use of a digital device that is more sensitive than a galvanometer improve the accuracy of null measurements?

Section Summary

- Null measurement techniques achieve greater accuracy by balancing a circuit so that no current flows through the measuring device.
- One such device, for determining voltage, is a potentiometer.
- Another null measurement device, for determining resistance, is the Wheatstone bridge.
- Other physical quantities can also be measured with null measurement techniques.

Conceptual Questions

1: Why can a null measurement be more accurate than one using standard voltmeters and ammeters? What factors limit the accuracy of null measurements?

2: If a potentiometer is used to measure cell emfs on the order of a few volts, why is it most accurate for the standard emf_s to be the same order of magnitude and the resistances to be in the range of a few ohms?

Problem Exercises

1: What is the emf_x of a cell being measured in a potentiometer, if the standard cell's emf is 12.0 V and the potentiometer balances for $R_x = 5.000\ \Omega$ and $R_s = 2.500\ \Omega$?

2: Calculate the emf_x of a dry cell for which a potentiometer is balanced when $R_x = 1.200\ \Omega$, while an alkaline standard cell with an emf of 1.600 V requires $R_s = 1.247\ \Omega$ to balance the potentiometer.

3: When an unknown resistance R_x is placed in a Wheatstone bridge, it is possible to balance the bridge by adjusting R_3 to be $2500\ \Omega$. What is R_x if $\frac{R_2}{R_1} = 0.625$?

4: To what value must you adjust R_3 to balance a Wheatstone bridge, if the unknown resistance R_x is $100\ \Omega$, R_1 is $50.0\ \Omega$, and R_2 is $175\ \Omega$?

5: (a) What is the unknown emf_x in a potentiometer that balances when R_x is $10.0\ \Omega$, and balances when R_s is $15.0\ \Omega$ for a standard 3.000-V emf? (b) The same emf_x is placed in the same potentiometer, which now balances when R_s is $15.0\ \Omega$ for a standard emf of 3.100 V. At what resistance R_x will the potentiometer balance?

6: Suppose you want to measure resistances in the range from $10.0\ \Omega$ to $10.0\ \text{k}\Omega$ using a Wheatstone bridge that has $\frac{R_2}{R_1} = 2.000$. Over what range should R_3 be adjustable?

Glossary

null measurements
methods of measuring current and voltage more accurately by balancing the circuit so that no current flows through the measurement device

potentiometer
a null measurement device for measuring potentials (voltages)

ohmmeter
an instrument that applies a voltage to a resistance, measures the current, calculates the resistance using Ohm's law, and provides a readout of this calculated resistance

bridge device
a device that forms a bridge between two branches of a circuit; some bridge devices are used to make null measurements in circuits

Wheatstone bridge
a null measurement device for calculating resistance by balancing potential drops in a circuit

Solutions

Check Your Understanding

1: One factor would be resistance in the wires and connections in a null measurement. These are impossible to make zero, and they can change over time. Another factor would be temperature variations in resistance, which can be reduced but not completely eliminated by choice of material. Digital devices sensitive to smaller currents than analog devices do improve the accuracy of null measurements because they allow you to get the current closer to zero.

Problem Exercises

1: 24.0 V

3: $1.56\ \text{k}\Omega$

5: (a) 2.00 V

(b) $9.68\ \Omega$

6: $\text{Range} = 5.00\ \Omega\ \text{to}\ 5.00\ \text{k}\Omega$

21.6 DC Circuits Containing Resistors and Capacitors

Summary

- Explain the importance of the time constant, τ , and calculate the time constant for a given resistance and capacitance.
- Explain why batteries in a flashlight gradually lose power and the light dims over time.
- Describe what happens to a graph of the voltage across a capacitor over time as it charges.
- Explain how a timing circuit works and list some applications.
- Calculate the necessary speed of a strobe flash needed to “stop” the movement of an object over a particular length.

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and a number of other phenomena that involve charging and discharging capacitors are discussed in this module.

RC Circuits

An RC circuit is one containing a **resistor** R and a **capacitor** C. The capacitor is an electrical component that stores electric charge.

Figure 1 shows a simple RC circuit that employs a DC (direct current) voltage source. The capacitor is initially uncharged. As soon as the switch is closed, current flows to and from the initially uncharged capacitor. As charge increases on the capacitor plates, there is increasing opposition to the flow of charge by the repulsion of like charges on each plate.

In terms of voltage, this is because voltage across the capacitor is given by $V_c = Q/C$, where Q is the amount of charge stored on each plate and C is the **capacitance**. This voltage opposes the battery, growing from zero to the maximum emf when fully charged. The current thus decreases from its initial value of $I_0 = \frac{\text{emf}}{R}$ to zero as the voltage on the capacitor reaches the same value as the emf. When there is no current, there is no IR drop, and so the voltage on the capacitor must then equal the emf of the voltage source. This can also be explained with Kirchhoff's second rule (the loop rule), discussed in Chapter 21.3 Kirchhoff's Rules, which says that the algebraic sum of changes in potential around any closed loop must be zero.

The initial current is $I_0 = \frac{\text{emf}}{R}$, because all of the IR drop is in the resistance. Therefore, the smaller the resistance, the faster a given capacitor will be charged. Note that the internal resistance of the voltage source is included in R, as are the resistances of the capacitor and the connecting wires. In the flash camera scenario above, when the batteries powering the camera begin to wear out, their internal resistance rises, reducing the current and lengthening the time it takes to get ready for the next flash.

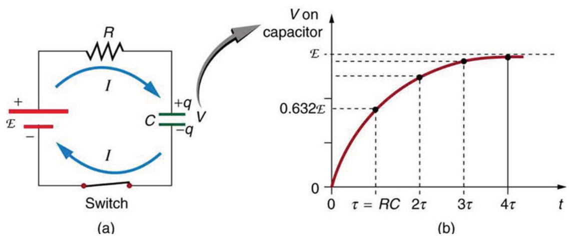


Figure 1. (a) An ***RC*** circuit with an initially uncharged capacitor. Current flows in the direction shown (opposite of electron flow) as soon as the switch is closed. Mutual repulsion of like charges in the capacitor progressively slows the flow as the capacitor is charged, stopping the current when the capacitor is fully charged and ***Q* = *C* ? emf**. (b) A graph of voltage across the capacitor versus time, with the switch closing at time ***t* = 0**. (Note that in the two parts of the figure, the capital script E stands for emf, ***q*** stands for the charge stored on the capacitor, and **τ** is the ***RC*** time constant.)

Voltage on the capacitor is initially zero and rises rapidly at first, since the initial current is a maximum. Figure 1(b) shows a graph of capacitor voltage versus time (t) starting when the switch is closed at $t = 0$. The voltage approaches emf asymptotically, since the closer it gets to emf the less current flows. The equation for voltage versus time when charging a capacitor C through a resistor R, derived using calculus, is

$$V = \text{emf}(1 - e^{-t/RC})\ \text{(charging)},$$

where V is the voltage across the capacitor, emf is equal to the emf of the DC voltage source, and the exponential e = 2.718 … is the base of the natural logarithm. Note that the units of RC are seconds. We define

$$\tau = \text{RC},$$

where τ (the Greek letter tau) is called the time constant for an RC circuit. As noted before, a small resistance R allows the capacitor to charge faster. This is reasonable, since a larger current flows through a smaller resistance. It is also reasonable that the smaller the capacitor C, the less time needed to charge it. Both factors are contained in $\tau = RC$.

More quantitatively, consider what happens when $t = \tau = RC$. Then the voltage on the capacitor is

$$V = \text{emf}\,(1 - e^{-1}) = \text{emf}(1 - 0.368) = 0.632 \cdot \text{emf}$$

This means that in the time $\tau = RC$, the voltage rises to 0.632 of its final value. The voltage will rise 0.632 of the remainder in the next time τ. It is a characteristic of the exponential function that the final value is never reached, but 0.632 of the remainder to that value is achieved in every time, τ. In just a few multiples of the time constant τ, then, the final value is very nearly achieved, as the graph in Figure 1(b) illustrates.

Discharging a Capacitor

Discharging a capacitor through a resistor proceeds in a similar fashion, as Figure 2 illustrates. Initially, the current is $I_0 = \frac{V_0}{R}$, driven by the initial voltage V_0 on the capacitor. As the voltage decreases, the current and hence the rate of discharge decreases, implying another exponential formula for V. Using calculus, the voltage V on a capacitor C being discharged through a resistor R is found to be

$$V = V_0\, e^{-t/RC} \text{ (discharging)}.$$

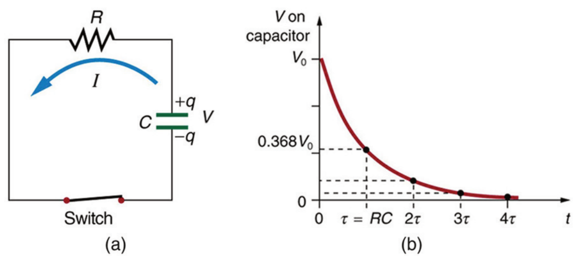


Figure 2. (a) Closing the switch discharges the capacitor ***C*** through the resistor ***R***. Mutual repulsion of like charges on each plate drives the current. (b) A graph of voltage across the capacitor versus time, with $V = V_0$ at $t = 0$. The voltage decreases exponentially, falling a fixed fraction of the way to zero in each subsequent time constant **τ**.

The graph in Figure 2(b) is an example of this exponential decay. Again, the time constant is $\tau = RC$. A small resistance R allows the capacitor to discharge in a small time, since the current is larger. Similarly, a small capacitance requires less time to discharge, since less charge is stored. In the first time interval $\tau = RC$ after the switch is closed, the voltage falls to 0.368 of its initial value, since $V = V_0 \cdot e^{-1} = 0.368V_0$.

During each successive time τ, the voltage falls to 0.368 of its preceding value. In a few multiples of τ, the voltage becomes very close to zero, as indicated by the graph in Figure 2(b).

Now we can explain why the flash camera in our scenario takes so much longer to charge than discharge; the resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower. (You may have noticed this.)

The flash discharge is through a low-resistance ionized gas in the flash tube and proceeds very rapidly. Flash photographs, such as in Figure 3, can capture a brief instant of a rapid motion because the flash can be less than a microsecond in duration. Such flashes can be made extremely intense.

During World War II, nighttime reconnaissance photographs were made from the air with a single flash illuminating more than a square kilometer of enemy territory. The brevity of the flash eliminated blurring due to the surveillance aircraft's motion. Today, an important use of intense flash lamps is to pump energy into a laser. The short intense flash can rapidly energize a laser and allow it to reemit the energy in another form.

Figure 3. This stop-motion photograph of a rufous hummingbird (*Selasphorus rufus*) feeding on a flower was obtained with an extremely brief and intense flash of light powered by the discharge of a capacitor through a gas. (credit: Dean E. Biggins, U.S. Fish and Wildlife Service)

Integrated Concept Problem: Calculating Capacitor Size—Strobe Lights

High-speed flash photography was pioneered by Doc Edgerton in the 1930s, while he was a professor of electrical engineering at MIT. You might have seen examples of his work in the amazing shots of hummingbirds in motion, a drop of milk splattering on a table, or a bullet penetrating an apple (see Figure 3). To stop the motion and capture these pictures, one needs a high-intensity, very short pulsed flash, as mentioned earlier in this module.

Suppose one wished to capture the picture of a bullet (moving at $5.0 \times 10^2\ \text{m/s}$) that was passing through an apple. The duration of the flash is related to the RC time constant, τ. What size capacitor would one need in the RC circuit to succeed, if the resistance of the flash tube was $10.0\ \Omega$? Assume the apple is a sphere with a diameter of $8.0 \times 10^{-2}\ \text{m}$.

Strategy

We begin by identifying the physical principles involved. This example deals with the strobe light, as discussed above. Figure 2 shows the circuit for this probe. The characteristic time τ of the strobe is given as $\tau = RC$.

Solution

We wish to find C, but we don't know τ. We want the flash to be on only while the bullet traverses the apple. So we need to use the kinematic equations that describe the relationship between distance x, velocity v, and time t:

$$x = vt \text{ or } t = \frac{x}{v}.$$

The bullet's velocity is given as $5.0 \times 10^2\ \text{m/s}$, and the distance x is $8.0 \times 10^{-2}\ \text{m}$. The traverse time, then, is

$$t = \frac{x}{v} = \frac{8.0 \times 10^{-2}\ \text{m}}{5.0 \times 10^2\ \text{m/s}} = 1.6 \times 10^{-4}\ \text{s}.$$

We set this value for the crossing time t equal to τ. Therefore,

$$C = \frac{t}{R} = \frac{1.6 \times 10^{-4}\ \text{s}}{10.0\ \Omega} = 16\ \mu\text{F}.$$

(Note: Capacitance C is typically measured in farads, F, defined as Coulombs per volt. From the equation, we see that C can also be stated in units of seconds per ohm.)

Discussion

The flash interval of $160\ \mu s$ (the traverse time of the bullet) is relatively easy to obtain today. Strobe lights have opened up new worlds from science to entertainment. The information from the picture of the apple and bullet was used in the Warren Commission Report on the assassination of President John F. Kennedy in 1963 to confirm that only one bullet was fired.

RC Circuits for Timing

RC circuits are commonly used for timing purposes. A mundane example of this is found in the ubiquitous intermittent wiper systems of modern cars. The time between wipes is varied by adjusting the resistance in an RC circuit. Another example of an RC circuit is found in novelty jewelry, Halloween costumes, and various toys that have battery-powered flashing lights. (See Figure 4 for a timing circuit.)

A more crucial use of RC circuits for timing purposes is in the artificial pacemaker, used to control heart rate. The heart rate is normally controlled by electrical signals generated by the sino-atrial (SA) node, which is on the wall of the right atrium chamber. This causes the muscles to contract and pump blood. Sometimes the heart rhythm is abnormal and the heartbeat is too high or too low.

The artificial pacemaker is inserted near the heart to provide electrical signals to the heart when needed with the appropriate time constant. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during exercise to meet the body's increased needs for blood and oxygen.

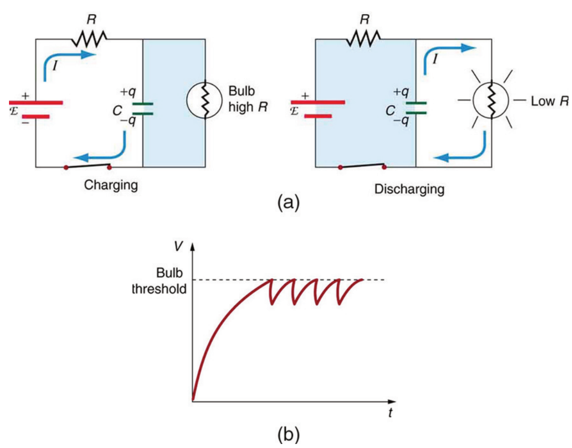


Figure 4. (a) The lamp in this ***RC*** circuit ordinarily has a very high resistance, so that the battery charges the capacitor as if the lamp were not there. When the voltage reaches a threshold value, a current flows through the lamp that dramatically reduces its resistance, and the capacitor discharges through the lamp as if the battery and charging resistor were not there. Once discharged, the process starts again, with the flash period determined by the ***RC*** constant **τ**. (b) A graph of voltage versus time for this circuit.

Calculating Time: *RC* Circuit in a Heart Defibrillator

A heart defibrillator is used to resuscitate an accident victim by discharging a capacitor through the trunk of her body. A simplified version of the circuit is seen in Figure 2. (a) What is the time constant if an $8.00-\mu F$ capacitor is used and the path resistance through her body is $1.00\times10^3\ \Omega$? (b) If the initial voltage is 10.0 kV, how long does it take to decline to $5.00\times10^2\ V$?

Strategy

Since the resistance and capacitance are given, it is straightforward to multiply them to give the time constant asked for in part (a). To find the time for the voltage to decline to $5.00\times10^2\ V$, we repeatedly multiply the initial voltage by 0.368 until a voltage less than or equal to $5.00\times10^2\ V$ is obtained. Each multiplication corresponds to a time of τ seconds.

Solution for (a)

The time constant τ is given by the equation $\tau = RC$. Entering the given values for resistance and capacitance (and remembering that units for a farad can be expressed as s/Ω) gives

$$\tau = RC = (1.00\times10^3\ \Omega)(8.00\ \mu F) = 8.00\ ms.$$

Solution for (b)

In the first 8.00 ms, the voltage (10.0 kV) declines to 0.368 of its initial value. That is:

$$V = 0.368V_0 = 3.680\times10^3\ V \text{ at } t = 8.00\ ms.$$

(Notice that we carry an extra digit for each intermediate calculation.) After another 8.00 ms, we multiply by 0.368 again, and the voltage is

$$\begin{aligned} V' &= 0.368\ V \\ &= (0.368)\ (3.680\times10^3\ V) \\ &= 1.354\times10^3\ V \text{ at } t = 16.0\ ms. \end{aligned}$$

Similarly, after another 8.00 ms, the voltage is

$$\begin{aligned} V'' &= 0.368V' = (0.368)(1.354\times10^3\ V) \\ &= 498\ V \text{ at } t = 24.0\ ms. \end{aligned}$$

Discussion

So after only 24.0 ms, the voltage is down to 498 V, or 4.98% of its original value. Such brief times are useful in heart defibrillation, because the brief but intense current causes a brief but effective contraction of the heart. The actual circuit in a heart defibrillator is slightly more complex than the one in Figure 2, to compensate for magnetic and AC effects that will be covered in Chapter 22 Magnetism.

Check Your Understanding

1: When is the potential difference across a capacitor an emf?

PhET Explorations: Circuit Construction Kit (DC only)

An electronics kit in your computer! Build circuits with resistors, light bulbs, batteries, and switches. Take measurements with the realistic ammeter and voltmeter. View the circuit as a schematic diagram, or switch to a life-like view.

PhET Interactive Simulation

Figure 5. Circuit Construction Kit (DC only)

Section Summary

- An RC circuit is one that has both a resistor and a capacitor.
- The time constant τ for an RC circuit is $\tau = RC$.
- When an initially uncharged ($V_0 = 0$ at $t = 0$) capacitor in series with a resistor is charged by a DC voltage source, the voltage rises, asymptotically approaching the emf of the voltage source; as a function of time,

$$V = \text{emf}(1 - e^{-t/RC})(\text{charging}).$$

- Within the span of each time constant τ, the voltage rises by 0.632 of the remaining value, approaching the final voltage asymptotically.
- If a capacitor with an initial voltage V_0 is discharged through a resistor starting at $t = 0$, then its voltage decreases exponentially as given by

$$V = V_0 e^{-t/RC}(\text{discharging}).$$

- In each time constant τ, the voltage falls by 0.368 of its remaining initial value, approaching zero asymptotically.

Conceptual Questions

1: Regarding the units involved in the relationship $\tau = RC$, verify that the units of resistance times capacitance are time, that is, $\Omega \cdot \text{F} = \text{s}$.

2: The RC time constant in heart defibrillation is crucial to limiting the time the current flows. If the capacitance in the defibrillation unit is fixed, how would you manipulate resistance in the circuit to adjust the RC constant τ? Would an adjustment of the applied voltage also be needed to ensure that the current delivered has an appropriate value?

3: When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the RC constant of the circuit—it is not possible to measure time variations shorter than RC. How would you manipulate R and C in the circuit to allow the necessary measurements?

4: Draw two graphs of charge versus time on a capacitor. Draw one for charging an initially uncharged capacitor in series with a resistor, as in the circuit in Figure 1, starting from $t = 0$. Draw the other for discharging a capacitor through a resistor, as in the circuit in Figure 2, starting at $t = 0$, with an initial charge Q_0. Show at least two intervals of τ.

5: When charging a capacitor, as discussed in conjunction with Figure 1, how long does it take for the voltage on the capacitor to reach emf? Is this a problem?

6: When discharging a capacitor, as discussed in conjunction with Figure 2, how long does it take for the voltage on the capacitor to reach zero? Is this a problem?

7: Referring to Figure 1, draw a graph of potential difference across the resistor versus time, showing at least two intervals of τ. Also draw a graph of current versus time for this situation.

8: A long, inexpensive extension cord is connected from inside the house to a refrigerator outside. The refrigerator doesn't run as it should. What might be the problem?

9: In Figure 4, does the graph indicate the time constant is shorter for discharging than for charging? Would you expect ionized gas to have low resistance? How would you adjust R to get a longer time between flashes? Would adjusting R affect the discharge time?

10: An electronic apparatus may have large capacitors at high voltage in the power supply section, presenting a shock hazard even when the apparatus is switched off. A "bleeder resistor" is therefore placed across such a capacitor, as shown schematically in Figure 6, to bleed the charge from it after the apparatus is off. Why must the bleeder resistance be much greater than the effective resistance of the rest of the circuit? How does this affect the time constant for discharging the capacitor?

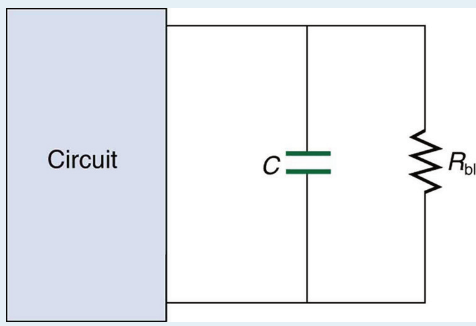


Figure 6. A bleeder resistor R_{bl} discharges the capacitor in this electronic device once it is switched off.

Problem Exercises

1: The timing device in an automobile's intermittent wiper system is based on an RC time constant and utilizes a $0.500-\mu F$ capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

2: A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

3: The duration of a photographic flash is related to an RC time constant, which is $0.100\ \mu s$ for a certain camera. (a) If the resistance of the flash lamp is $0.0400\ \Omega$ during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is $800\ k\Omega$?

4: A 2.00- and a $7.50-\mu F$ capacitor can be connected in series or parallel, as can a 25.0- and a $100-k\Omega$ resistor. Calculate the four RC time constants possible from connecting the resulting capacitance and resistance in series.

5: After two time constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C, charged through a resistance R?

6: A $500-\Omega$ resistor, an uncharged $1.50-\mu F$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

7: A heart defibrillator being used on a patient has an RC time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an $8.00-\mu F$ capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is 12.0 kV, how long does it take to decline to $6.00 \times 10^2 \text{ V}$?

8: An ECG monitor must have an RC time constant less than $1.00 \times 10^2 \ \mu s$ to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patient's chest) is $1.00 \text{ k}\Omega$, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

9: Figure 7 shows how a bleeder resistor is used to discharge a capacitor after an electronic device is shut off, allowing a person to work on the electronics with less risk of shock. (a) What is the time constant? (b) How long will it take to reduce the voltage on the capacitor to 0.250% (5% of 5%) of its full value once discharge begins? (c) If the capacitor is charged to a voltage V_0 through a $100-\Omega$ resistance, calculate the time it takes to rise to 0.865 V (This is about two time constants.)

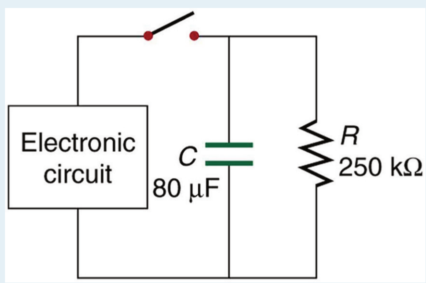


Figure 7.

10: Using the exact exponential treatment, find how much time is required to discharge a $250-\mu F$ capacitor through a $500-\Omega$ resistor down to 1.00% of its original voltage.

11: Using the exact exponential treatment, find how much time is required to charge an initially uncharged 100-pF capacitor through a $75.0-\text{M}\Omega$ resistor to 90.0% of its final voltage.

12: Integrated Concepts

If you wish to take a picture of a bullet traveling at 500 m/s, then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a $600-\mu F$ capacitor, what is the resistance in the flash tube?

13: Integrated Concepts

A flashing lamp in a Christmas earring is based on an RCRC size 12{ ital "RC"} {} discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s, during which it produces an average 0.500 W from an average 3.00 V. (a) What energy does it dissipate? (b) How much charge moves through the lamp? (c) Find the capacitance. (d) What is the resistance of the lamp?

14: Integrated Concepts

A $160-\mu F$ capacitor charged to 450 V is discharged through a $31.2-\text{k}\Omega$ resistor. (a) Find the time constant. (b)

Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is $1.67\ \frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}}$, noting that most of the thermal energy is retained in the short time of the discharge. (c) Calculate the new resistance, assuming it is pure carbon. (d) Does this change in resistance seem significant?

16: Unreasonable Results

(a) Calculate the capacitance needed to get an RC time constant of $1.00 \times 10^3\ \text{s}$ with a $0.100 - \Omega$ resistor. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

17: Construct Your Own Problem

Consider a camera's flash unit. Construct a problem in which you calculate the size of the capacitor that stores energy for the flash lamp. Among the things to be considered are the voltage applied to the capacitor, the energy needed in the flash and the associated charge needed on the capacitor, the resistance of the flash lamp during discharge, and the desired RC time constant.

18: Construct Your Own Problem

Consider a rechargeable lithium cell that is to be used to power a camcorder. Construct a problem in which you calculate the internal resistance of the cell during normal operation. Also, calculate the minimum voltage output of a battery charger to be used to recharge your lithium cell. Among the things to be considered are the emf and useful terminal voltage of a lithium cell and the current it should be able to supply to a camcorder.

Glossary

RC circuit
a circuit that contains both a resistor and a capacitor
capacitor
an electrical component used to store energy by separating electric charge on two opposing plates
capacitance
the maximum amount of electric potential energy that can be stored (or separated) for a given electric potential

Solutions

Check Your Understanding

1: Only when the current being drawn from or put into the capacitor is zero. Capacitors, like batteries, have internal resistance, so their output voltage is not an emf unless current is zero. This is difficult to measure in practice so we refer to a capacitor's voltage rather than its emf. But the source of potential difference in a capacitor is fundamental and it is an emf.

Problem Exercises

1: $\text{range}\,4.00\ \text{to}\,30.0\ \text{M}\Omega$

3: (a) $2.50\ \mu\text{F}$

(b) 2.00 s

5: 86.5%

7: (a) $1.25\ \text{k}\Omega$

(b) 30.0 ms

9: (a) 20.0 s

(b) 120 s

(c) 16.0 ms

11: $1.73 \times 10^{-2}\ \text{s}$

12: $3.33 \times 10^{-3}\ \Omega$

15: (a) 4.99 s

(b) $3.87\ ^\circ\text{C}$

(c) $31.1\ \text{k}\Omega$

(d) No

PART 15

Chapter 22 Magnetism

Introduction to Magnetism

class=”introduction”
class=”section-summary” title=”Section Summary”class=”conceptual-questions” title=”Conceptual Questions”class=”problems-exercises” title=”Problems & Exercises”

The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth’s magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, via Flickr)

One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of “north” and “south” being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists' understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn't have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.

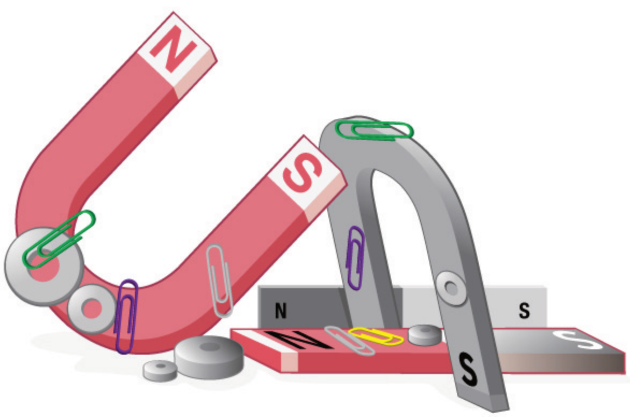
Engineering of technology like iPods would not be possible without a deep understanding magnetism. (credit: Jesse! S?, Flickr)

22.1 Magnets

Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.

Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).



All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the north magnetic pole and the south magnetic pole (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

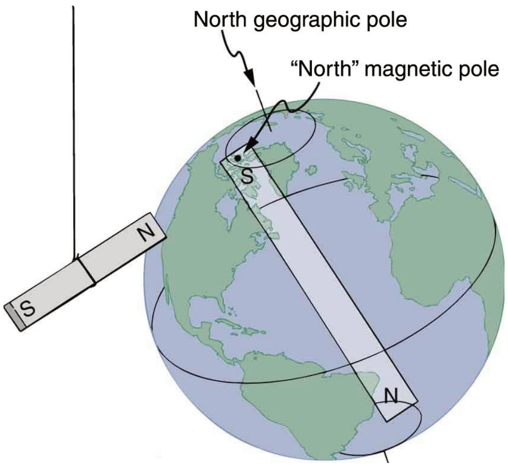
Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and − charges can be separated.

One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled

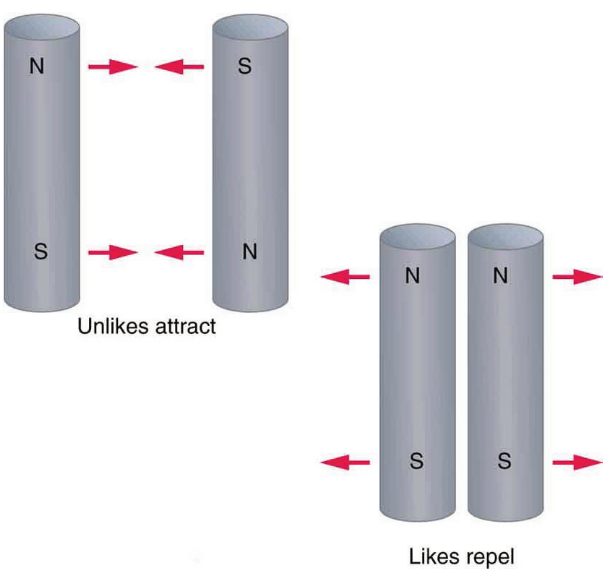
N and S for north-seeking and south-seeking poles, respectively.



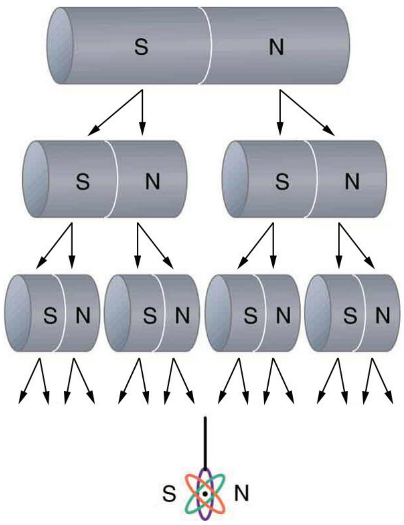
Misconception Alert: Earth’s Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term “North Pole” has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, “North magnetic pole” is actually a misnomer—it should be called the South magnetic pole.

Unlike poles attract, whereas like poles repel.



North and south poles always occur in pairs. Attempts to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.



The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

Section Summary

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth's geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

Conceptual Questions

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

Glossary

north magnetic pole
the end or the side of a magnet that is attracted toward Earth's geographic north pole
south magnetic pole
the end or the side of a magnet that is attracted toward Earth's geographic south pole

22.2 Ferromagnets and Electromagnets

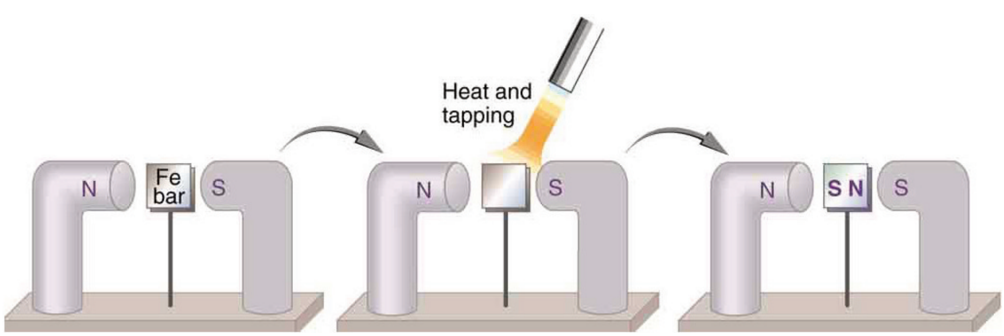
Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

Ferromagnets

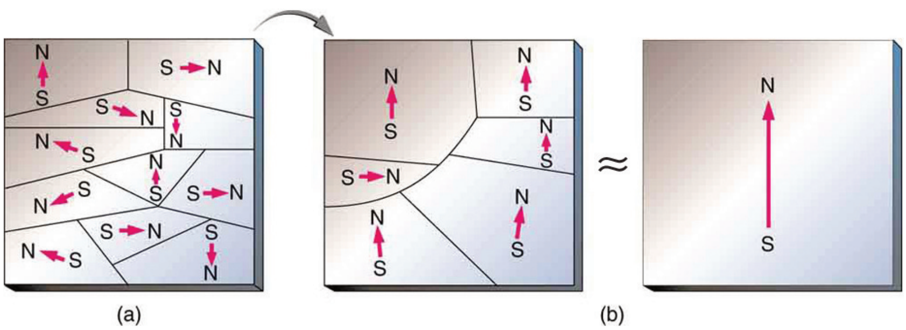
Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called ferromagnetic, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be magnetized themselves—that is, they can be induced to be magnetic or made into permanent magnets.

An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.



When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [link]. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [link]. The regions within the material called domains act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [link](b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.

(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.



Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of the domains. There is a well-defined temperature for ferromagnetic materials, which is called the Curie temperature, above which they cannot be magnetized. The Curie temperature for iron is 1043 K (770ºC)(770ºC) size 12{ (“770”°C) } {}, which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

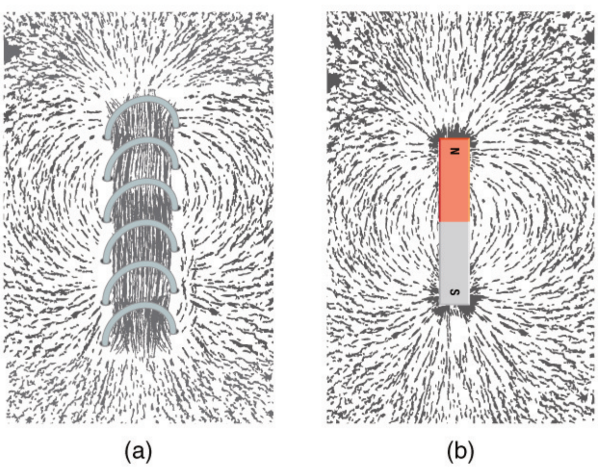
Electromagnets

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. Electromagnetism is the use of electric current to make magnets. These temporarily induced magnets are called electromagnets. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [link]).

Instrument for magnetic resonance imaging (MRI). The device uses a superconducting cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney. (credit: Bill McChesney, Flickr)

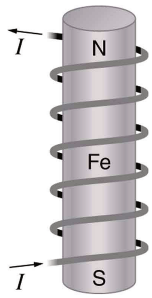
[link] shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.

Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.



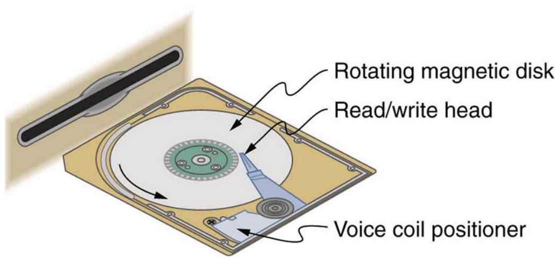
Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [link].) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

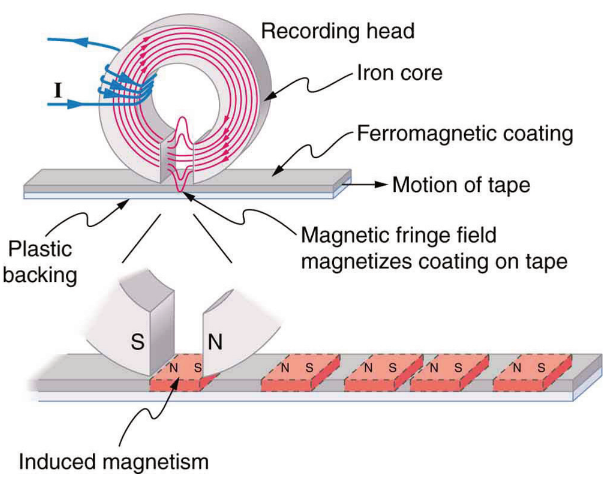
An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet.



[link] shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.

An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog (with a varying strength), such as on audiotapes.





Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets?

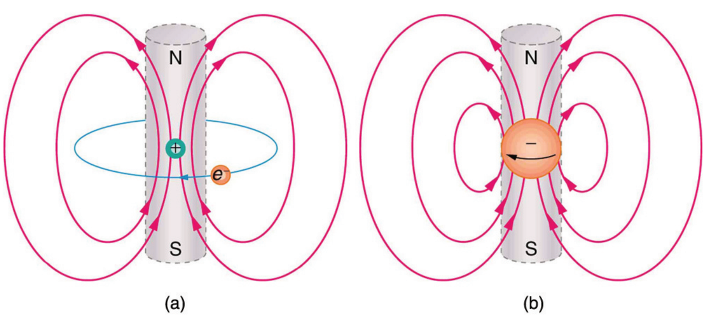
[link] shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called magnetic monopoles, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do *not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Electric Currents and Magnetism

Electric current is the source of all magnetism.

(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.



PhET Explorations: Magnets and Electromagnets

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

Magnets and Electromagnets

PhET Interactive Simulation

Section Summary

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

Glossary

ferromagnetic
materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects
magnetized
to be turned into a magnet; to be induced to be magnetic
domains
regions within a material that behave like small bar magnets
Curie temperature
the temperature above which a ferromagnetic material cannot be magnetized
electromagnetism
the use of electrical currents to induce magnetism
electromagnet
an object that is temporarily magnetic when an electrical current is passed through it
magnetic monopoles
an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

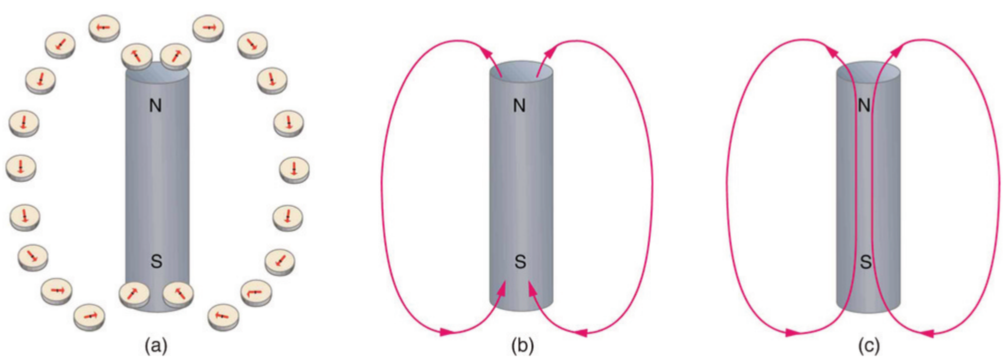
22.3 Magnetic Fields and Magnetic Field Lines

Magnetic Fields and Magnetic Field Lines

- Define magnetic field and describe the magnetic field lines of various magnetic fields.

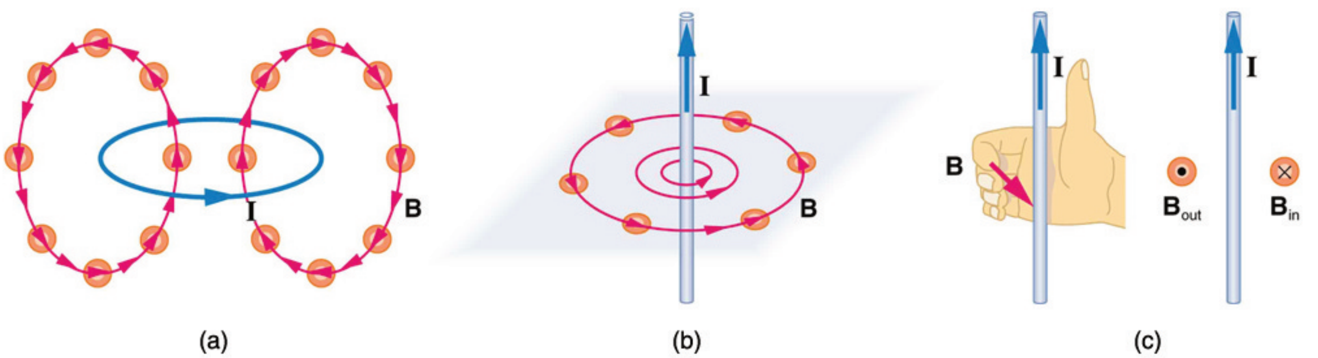
Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a magnetic field to represent magnetic forces. The pictorial representation of magnetic field lines is very useful in visualizing the strength and direction of the magnetic field. As shown in [link], the direction of magnetic field lines is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the *B*-field.

Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.



Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) [link] shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper.

Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).



Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

Section Summary

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:
  1. The field is tangent to the magnetic field line.
  2. Field strength is proportional to the line density.
  3. Field lines cannot cross.
  4. Field lines are continuous loops.

Conceptual Questions

Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

Is the Earth’s magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

Glossary

magnetic field
the representation of magnetic forces
***B*-field**
another term for magnetic field
magnetic field lines
the pictorial representation of the strength and the direction of a magnetic field
direction of magnetic field lines
the direction that the north end of a compass needle points

22.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Right Hand Rule 1

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the magnetic force FF size 12{F} {} on a charge qq size 12{q} {} moving at a speed vv size 12{v} {} in a magnetic field of strength BB size 12{B} {} is given by

F=qvBsinθ,F=qvBsinθ, size 12{F= ital “qvB””sin”θ} {}

where θθ size 12{θ} {} is the angle between the directions of vv and B.B. size 12{B} {} This force is often called the Lorentz force. In fact, this is how we define the magnetic field strength BB size 12{B} {}—in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength BB size 12{B} {} is called the tesla (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve F=qvBsinθF=qvBsinθ size 12{F= ital “qvB””sin”θ} {} for BB size 12{B} {}.

B=FqvsinθB=Fqvsinθ size 12{B= { {F} over { ital “qv””sin”θ} } } {}

Because
sinθsinθ size 12{θ} {}
is unitless, the tesla is

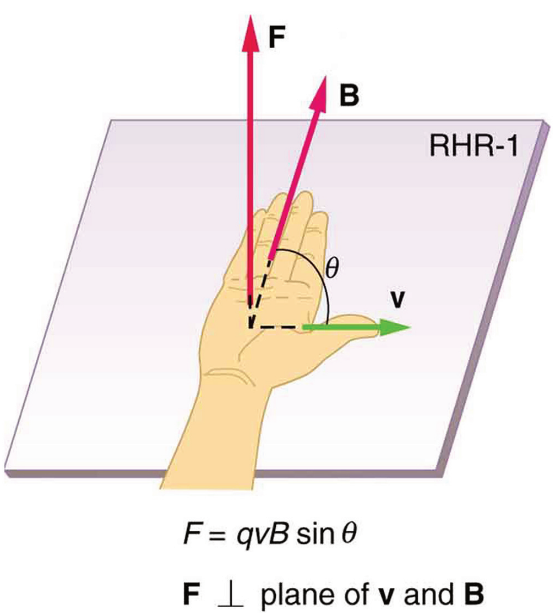
1 T=1 NC?m/s=1 NA?m1 T=1 NC?m/s=1 NA?m size 12{“1 T”= { {“1 N”} over {C cdot “m/s”} } = { {1″ N”} over {A cdot m} } } {}

(note that C/s = A).

Another smaller unit, called the gauss (G), where 1 G=10−4T1 G=10−4T size 12{1`G=”10″ rSup { size 8{ – 4} } `T} {}, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth’s magnetic field on its surface is only about 5×10−5T5×10−5T size 12{5 times “10” rSup { size 8{ – 5} } `T} {}, or 0.5 G.

The *direction* of the magnetic force FF size 12{F} {} is perpendicular to the plane formed by vv size 12{v} {} and BB, as determined by the right hand rule 1 (or RHR-1), which is illustrated in [link]. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of vv, the fingers in the direction of BB, and a perpendicular to the palm points in the direction of FF. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.

Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by vv and BB size 12{B} {} and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to qq size 12{q} {}, vv size 12{v} {}, BB size 12{B} {}, and the sine of the angle between vv size 12{v} {} and BB size 12{B} {}.



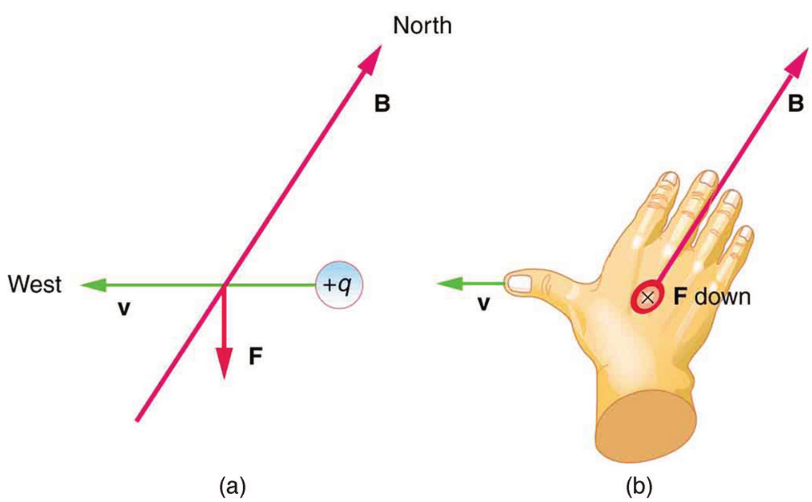
Making Connections: Charges and Magnets

There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Calculating Magnetic Force: Earth's Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth's small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth's magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in [link].)

A positively charged object moving due west in a region where the Earth's magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up.



Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation F=qvBsinθF=qvBsinθ size 12{F= ital "qvB""sin"θ} {} to find the force.

Solution

The magnetic force is

F=qvbsinθ.F=qvbsinθ. size 12{F= ital "qvb""sin"θ} {}

We see that sinθ=1sinθ=1 size 12{"sin"θ=1} {}, since the angle between the velocity and the direction of the field is 90º90º size 12{"90" rSup { size 8{ circ } } } {}. Entering the other given quantities yields

F=20×10–9C10 m/s5×10–5T=1×10–11C?m/sNC?m/s=1×10–11N.F=20×10–9C10 m/s5×10–5T=1×10–11C?m/

sNC?m/s=1×10–11N.alignl { stack {
size 12{F= left (“20” times “10” rSup { size 8{ – 9 } } `C right ) left (“10″`”m/s” right) left (5 times “10” rSup { size 8{ – 5} } `T right)} {} #
” “=1 times “10” rSup { size 8{ – “11”} } ` left (C cdot “m/s” right ) left ( { {N} over {C cdot “m/s”} } right )=1 times “10” rSup { size 8{ – “11”} } `N “.” {}
} } {}

Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth’s field varies with location and is given to only one digit.) The Earth’s magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in Force on a Moving Charge in a Magnetic Field: Examples and Applications.

Section Summary

- Magnetic fields exert a force on a moving charge *q*, the magnitude of which is
  F=qvBsinθ,F=qvBsinθ, size 12{F= ital “qvB””sin”θ} {}

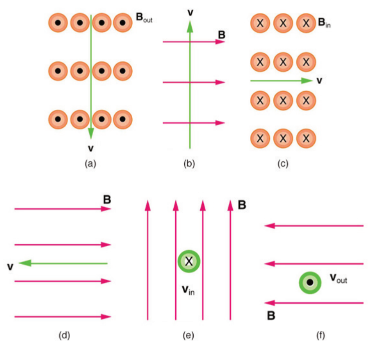
  where θθ size 12{θ} {} is the angle between the directions of vv size 12{v} {} and BB size 12{B} {}.
- The SI unit for magnetic field strength BB size 12{B} {} is the tesla (T), which is related to other units by 1 T=1 NC?m/s=1 NA?m.1 T=1 NC?m/s=1 NA?m.
- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of vv size 12{v} {}, the fingers in the direction of BB size 12{B} {}, and a perpendicular to the palm points in the direction of FF size 12{F} {}.
- The force is perpendicular to the plane formed by vv and BB size 12{B} {}. Since the force is zero if vv size 12{v} {} is parallel to BB size 12{B} {}, charged particles often follow magnetic field lines rather than cross them.

Conceptual Questions

If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

Problems & Exercises

What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in [link]?



(a) Left (West)

(b) Into the page

(c) Up (North)

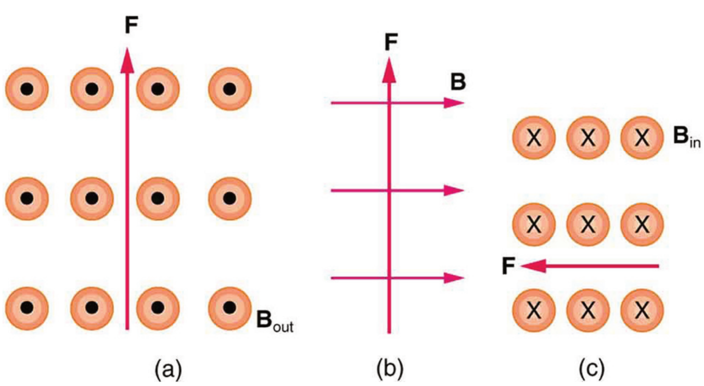
(d) No force

(e) Right (East)

(f) Down (South)

Repeat [link] for a negative charge.

What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in [link], assuming it moves perpendicular to B?B? size 12{B?} {}



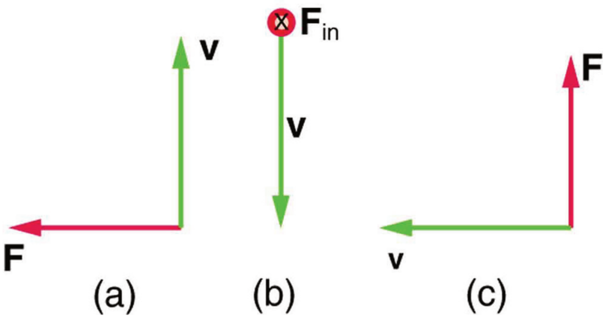
(a) East (right)

(b) Into page

(c) South (down)

Repeat [link] for a positive charge.

What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming BB size 12{B} {} is perpendicular to vv size 12{v} {}?



(a) Into page

(b) West (left)

(c) Out of page

Repeat [link] for a negative charge.

What is the maximum force on an aluminum rod with a 0.100-μC0.100-μC size 12{0 "." "100""-μC"} {} charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?

7.50×10−7 N7.50×10−7 N size 12{7 "." "50" times "10" rSup { size 8{ – 7} } " N"} {} perpendicular to both the magnetic field lines and the velocity

(a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a 0.500-μC0.500-μC size 12{0 "." "500""-μC"} {} charge and flies due west at a speed of 660 m/s over the Earth's south magnetic pole, where the 8.00×10−5-T8.00×10−5-T size 12{8 "." "00" times "10" rSup { size 8{ – 5} } "-T"} {} magnetic field points straight up. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

(a) A cosmic ray proton moving toward the Earth at 5.00×107m/s5.00×107m/s size 12{5 "." "00" times "10" rSup { size 8{7} } `"m/s"} {} experiences a magnetic force of 1.70×10−16N1.70×10−16N size 12{1 "." "70" times "10" rSup { size 8{ – "16"} } `N} {}. What is the strength of the magnetic field if there is a 45º45º size 12{"45" rSup { size 8{ circ } } } {} angle between it and the proton's velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth's magnetic field on its surface? Discuss.

(a) 3.01×10−5 T3.01×10−5 T size 12{3 "." "01" times "10" rSup { size 8{ – 5} } " T"} {}

(b) This is slightly less then the magnetic field strength of 5×10−5T5×10−5T size 12{5 times "10" rSup { size 8{ – 5} } `T} {} at the surface of the Earth, so it is consistent.

An electron moving at 4.00×103m/s4.00×103m/s size 12{4 "." "00" times "10" rSup { size 8{3} } `"m/s"} {}

in a 1.25-T magnetic field experiences a magnetic force of 1.40×10−16N1.40×10−16N size 12{1 "." "40" times "10" rSup { size 8{ – "16"} } `N} {}. What angle does the velocity of the electron make with the magnetic field? There are two answers.

(a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than 1.00×10−12N1.00×10−12N size 12{1 "." "00" times "10" rSup { size 8{ – "12"} } `N} {}. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth's field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

(a) 6.67×10−10 C6.67×10−10 C (taking the Earth's field to be 5.00×10−5 T5.00×10−5 T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } " T"} {})

(b) Less than typical static, therefore difficult

Glossary

right hand rule 1 (RHR-1)
the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity vv size 12{v} {} and the fingers point in the direction of the magnetic field BB size 12{B} {}, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm
Lorentz force
the force on a charge moving in a magnetic field
tesla
T, the SI unit of the magnetic field strength; 1 T=1 NA?m1 T=1 NA?m
magnetic force
the force on a charge produced by its motion through a magnetic field; the Lorentz force
gauss
G, the unit of the magnetic field strength; 1 G=10–4T1 G=10–4T size 12{"1 G"="10" rSup { size 8{ – 4} } `T} {}

22.5 Force on a Moving Charge in a Magnetic Field: Examples and Applications

Force on a Moving Charge in a Magnetic Field: Examples and Applications

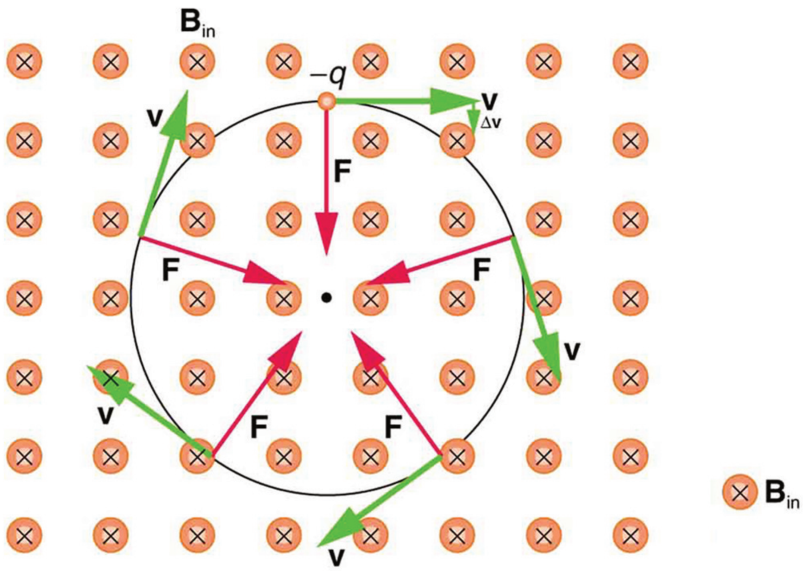
- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in [link] shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.

Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be used to find the mass, charge, and energy of the particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform BB size 12{B} {}-field, such as shown in [link]. (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force Fc=mv2/rFc=mv2/r size 12{F rSub { size 8{c} } = ital "mv" rSup { size 8{2} } /r} {}. Noting that sinθ=1sinθ=1 size 12{"sin"θ=1} {}, we see that F=qvBF=qvB size 12{F= ital "qvB"} {}.

A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.



Because the magnetic force FF size 12{F} {} *supplies the centripetal force FcFc size 12{F rSub { size 8{c} } } {}, we have*

qvB=mv2r.qvB=mv2r. size 12{ ital "qvB"= { { ital "mv" rSup { size 8{2} } } over {r} } "." } {}

Solving for rr size 12{r} {} yields

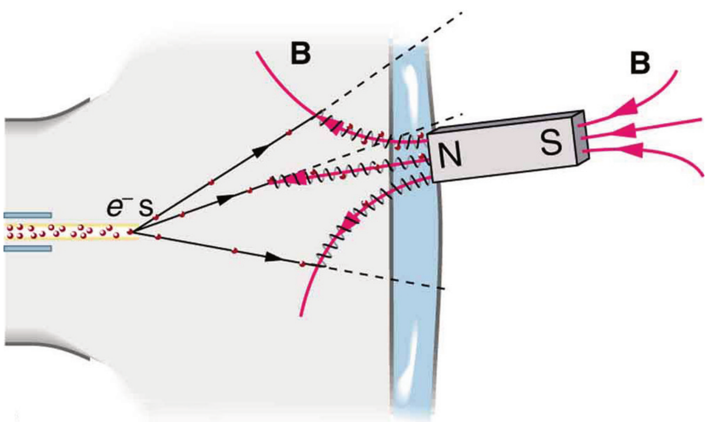
r=mvqB.r=mvqB. size 12{r= { { ital "mv"} over { ital "qB"} } "." } {}

Here, rr size 12{r} {} is the radius of curvature of the path of a charged particle with mass mm size 12{m} {} and charge qq size 12{q} {}, moving at a speed vv size 12{v} {} perpendicular to a magnetic field of strength BB size 12{B} {}. If the velocity is not perpendicular to the magnetic field, then vv size 12{v} {} is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in [link] (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. ***(Don't try this at home, as it will permanently magnetize and ruin the TV.)*** To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of 6.00×107m/s6.00×107m/s size 12{6 "." "00" times "10" rSup { size 8{7} } `"m/s"} {} (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength B=0.500 TB=0.500 T size 12{B=0 "." "500" T} {} (obtainable with permanent magnets).

Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.



Strategy

We can find the radius of curvature
rr directly from the equation
r=mvqBr=mvqB, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of vv size 12{v} {} and BB size 12{B} {} gives us

r=mvqB=9.11×10−31kg6.00×107m/s1.60×10−19C0.500T=6.83×10−4mr=mvqB=9.11×10−31kg6.00×107m/s1.60×10−19C0.500T=6.83×10−4malignl { stack {
size 12{r= { { ital "mv"} over { ital "qB"} } = { { left (9 "." "11" times "10" rSup { size 8{ – "31"} } `"kg" right ) left (6 "." "00" times "10" rSup { size 8{7} } `"m/s" right)} over { left (1 "." "60" times "10" rSup { size 8{ – "19"} } `C right ) left (0 "." "500"`T right)} } } {} #
=6 "." "83" times "10" rSup { size 8{ – 4} } `m {}
} } {}

or

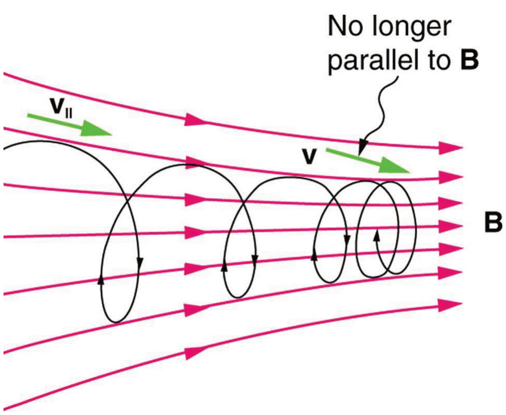
r=0.683 mm.r=0.683 mm. size 12{r=0 "." "683"" mm"} {}

Discussion

The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

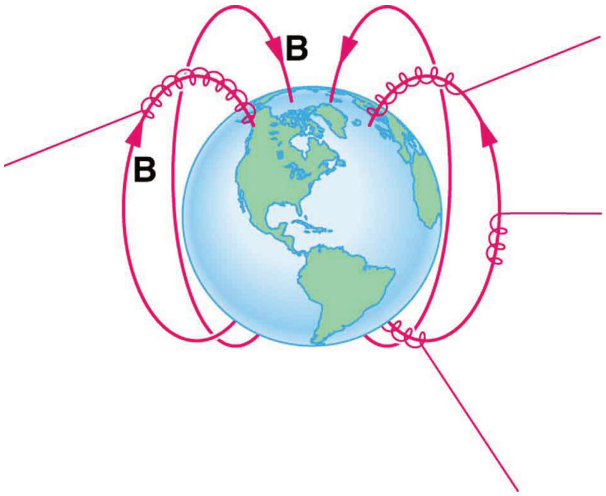
[link] shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.

When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a "magnetic mirror."



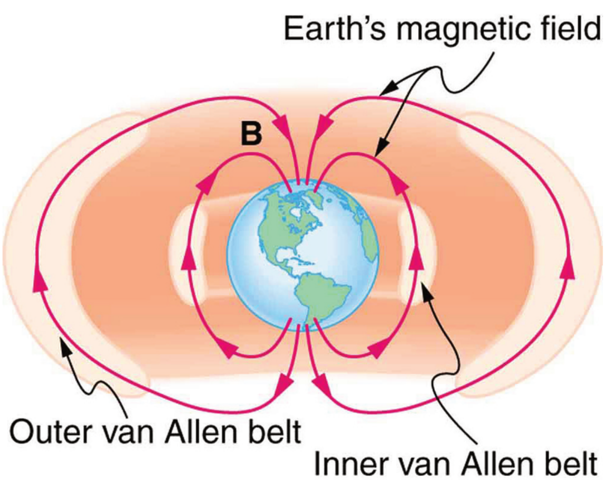
The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in [link]. Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.

Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)



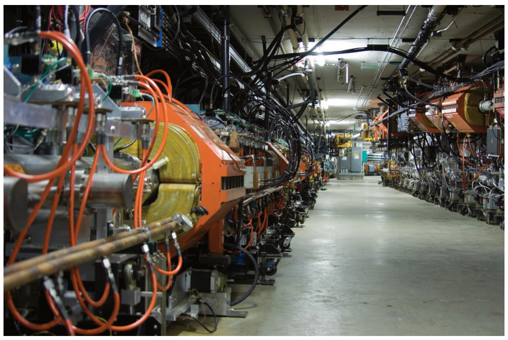
Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See [link].) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.

The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface, the other about 16,000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space.



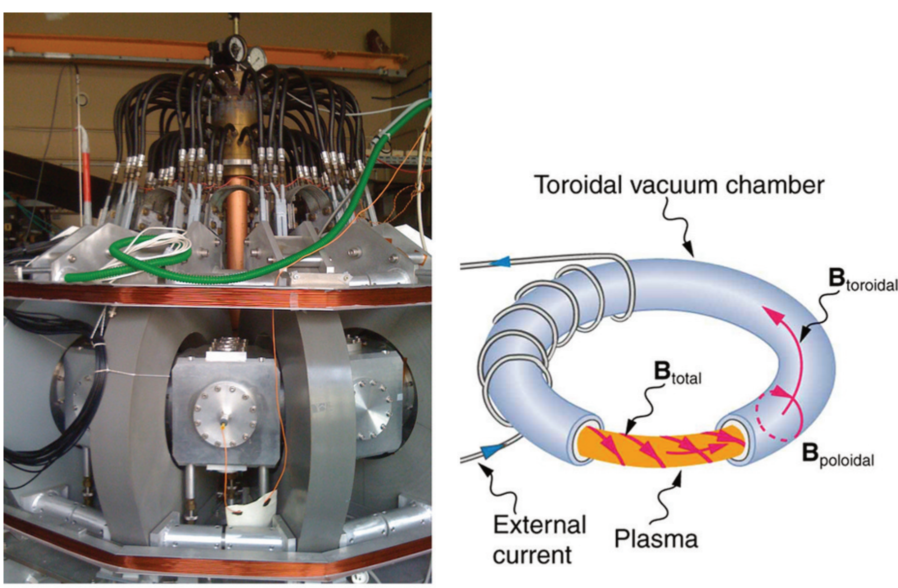
Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See [link].) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcrim, Flickr)



Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [link].) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.

Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)



(a) (b)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See More Applications of Magnetism.) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

Section Summary

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius

r=mvqB,r=mvqB, size 12{r= { { ital "mv"} over { ital "qB"} } ,} {}

where vv size 12{v} {} is the component of the velocity perpendicular to BB size 12{B} {} for a charged particle with mass *mm size 12{m} {}* and charge *qq size 12{q} {}*.

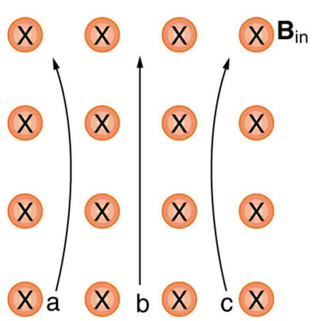
Conceptual Questions

How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

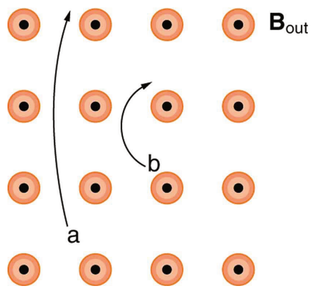
High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

What are the signs of the charges on the particles in [link]?



Which of the particles in [link] has the greatest velocity, assuming they have identical charges and masses?



Which of the particles in [link] has the greatest mass, assuming all have identical charges and velocities?

While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

Problems & Exercises

If you need additional support for these problems, see More Applications of Magnetism.

A cosmic ray electron moves at 7.50×106m/s7.50×106m/s size 12{7 “.” “50” times “10” rSup { size 8{6} } `”m/s”} {} perpendicular to the Earth's magnetic field at an altitude where field strength is 1.00×10−5T1.00×10−5T size 12{1 “.” “00” times “10” rSup { size 8{ – 5} } `T} {}. What is the radius of the circular path the electron follows?

4.27 m

A proton moves at 7.50×107m/s7.50×107m/s size 12{7 “.” “50” times “10” rSup { size 8{7} } `”m/s”} {} perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

(a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at 5.00×107m/s5.00×107m/s size 12{5 “.” “00” times “10” rSup { size 8{7} } `”m/s”} {} in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

(a) 0.261 T

(b) This strength is definitely obtainable with today's technology. Magnetic field strengths of 0.500 T are obtainable with permanent magnets.

(a) An oxygen-16 ion with a mass of 2.66×10−26kg2.66×10−26kg size 12{2 “.” “66” times “10” rSup { size 8{ – “26”} } `”kg”} {} travels at 5.00×106m/s5.00×106m/s size 12{5 “.” “00” times “10” rSup { size 8{6} } `”m/s”} {} perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in [link]?

4.36×10−4 m4.36×10−4 m size 12{4 "." "36" times "10" rSup { size 8{ – 4} } " m"} {}

A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of 4.00×106m/s4.00×106m/s size 12{4 "." "00" times "10" rSup { size 8{6} } `"m/s"} {}? (b) What is the voltage between the plates if they are separated by 1.00 cm?

An electron in a TV CRT moves with a speed of 6.00×107m/s6.00×107m/s size 12{6 "." "00" times "10" rSup { size 8{7} } `"m/s"} {}, in a direction perpendicular to the Earth's field, which has a strength of 5.00×10−5T5.00×10−5T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } `T} {}. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

(a) 3.00 kV/m

(b) 30.0 V

(a) At what speed will a proton move in a circular path of the same radius as the electron in [link]? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is 2.66×10−26kg,2.66×10−26kg, size 12{2 "." "66" times "10" rSup { size 8{ – "26"} } `"kg"} {} and they are singly charged and travel at 5.00×106m/s5.00×106m/s size 12{5 "." "00" times "10" rSup { size 8{6} } `"m/s"} {} in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

0.173 m

(a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are 3.90×10−25kg3.90×10−25kg size 12{3 "." "90" times "10" rSup { size 8{ – "25"} } `"kg"} {} and 3.95×10−25kg3.95×10−25kg size 12{3 "." "95" times "10" rSup { size 8{ – "25"} } `"kg"} {}, respectively, and they travel at 3.00×105m/s3.00×105m/s size 12{3 "." "00" times "10" rSup { size 8{5} } `"m/s"} {} in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

22.6 The Hall Effect

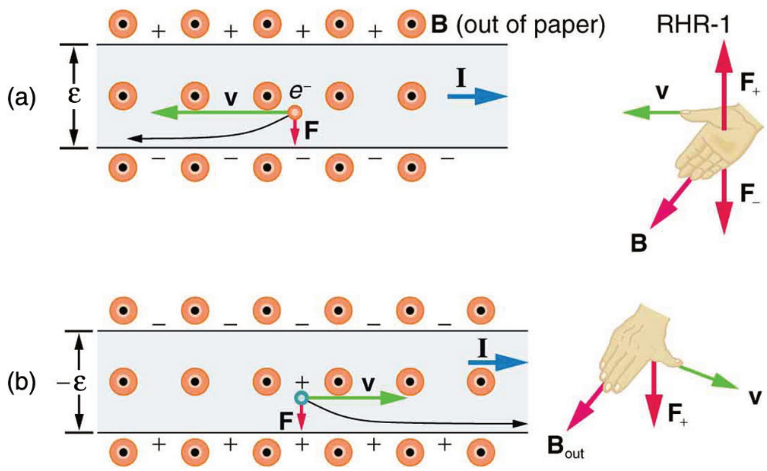
The Hall Effect

- Describe the Hall effect.
- Calculate the Hall emf across a current-carrying conductor.

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

[link] shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage εε size 12{ε} {}*, known as the Hall emf, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the Hall effect, after Edwin Hall, the American physicist who discovered it in 1879.

The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage εε, the Hall emf, across the conductor. (b) Positive charges moving to the right (conventional current also to the right) are moved to the side, producing a Hall emf of the opposite sign, –ε–ε. Thus, if the direction of the field and current are known, the sign of the charge carriers can be determined from the Hall effect.



One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in [link](b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, εε size 12{ε} {}, across a conductor. Consider the balance of forces on a moving charge in a situation where BB size 12{B} {}, vv size 12{v} {}, and ll size 12{l} {} are mutually perpendicular, such as shown in [link]. Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, F=qvBF=qvB size 12{F= ital "qvB"} {}, and the electric force, Fe=qEFe=qE size 12{F rSub { size 8{e} } = ital "qE"} {}, eventually grows to equal it. That is,

qE=qvBqE=qvB size 12{ ital "qE"= ital "qvB"} {}

or

E=vB.E=vB. size 12{E= ital "vB"} {}

Note that the electric field EE size 12{E} {} is uniform across the conductor because the magnetic field BB size 12{B} {} is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is E=ε/lE=ε/l size 12{E=ε/l} {}, where ll size 12{l} {} is the width of the conductor and εε size 12{ε} {} is the Hall emf. Entering this into the last expression gives

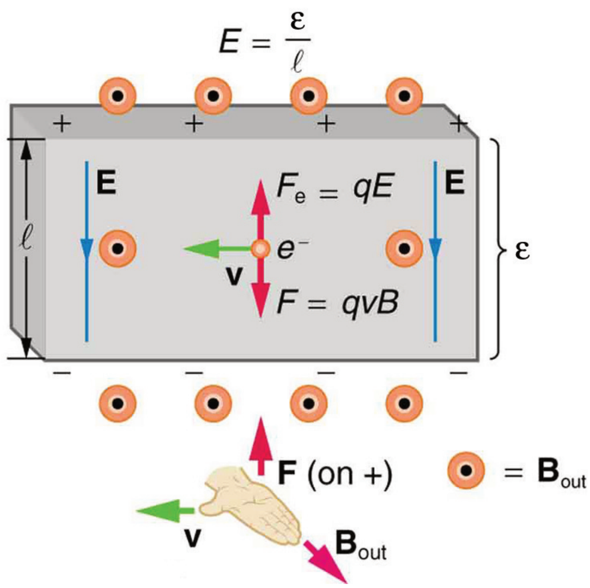
εl=vB.εl=vB. size 12{ { {ε} over {l} } = ital "vB" "." } {}

Solving this for the Hall emf yields

ε=Blv(B,v,andl,mutually perpendicular),ε=Blv(B,v,andl,mutually perpendicular), size 12{ε= ital "Blv""` (
B,`v,`"and"`l,`"mutually perpendicular") ,} {}

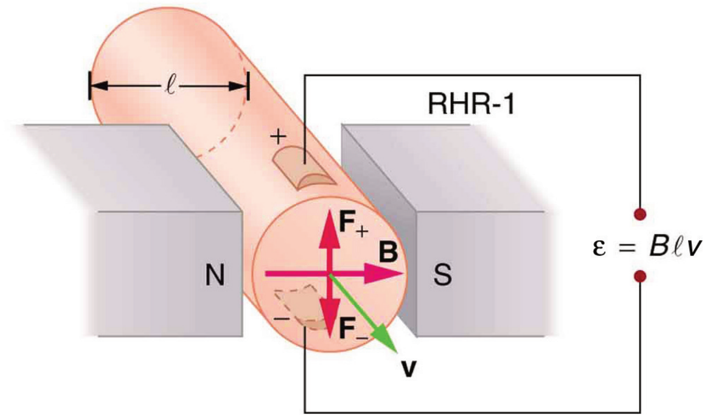
where εε size 12{ε} {} is the Hall effect voltage across a conductor of width ll size 12{l} {} through which charges move at a speed vv size 12{v} {}.

The Hall emf εε size 12{ε} {} produces an electric force that balances the magnetic force on the moving charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached.



One of the most common uses of the Hall effect is in the measurement of magnetic field strength BB size 12{B} {}. Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See [link].) A magnetic field applied perpendicular to the flow direction produces a Hall emf εε size 12{ε} {} as shown. Note that the sign of εε size 12{ε} {} depends not on the sign of the charges, but only on the directions of BB size 12{B} {} and vv size 12{v} {}. The magnitude of the Hall emf is ε=Blvε=Blv size 12{ε= ital "Blv"} {}, where ll size 12{l} {} is the pipe diameter, so that the average velocity vv size 12{v} {} can be determined from εε size 12{ε} {} providing the other factors are known.

The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf εε size 12{ε} {} is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity vv size 12{v} {}.



Calculating the Hall emf: Hall Effect for Blood Flow

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in [link]. What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because BB size 12{B} {}, vv size 12{v} {}, and ll size 12{l} {} are mutually perpendicular, the equation ε=Blvε=Blv size 12{ε= ital "Blv"} {} can be used to find εε size 12{ε} {}.

Solution

Entering the given values for BB size 12{B} {}, vv size 12{v} {}, and *ll size 12{l} {}* gives

ε=Blv=0.100 T4.00×10−3m0.200 m/s=80.0 μVε=Blv=0.100 T4.00×10−3m0.200 m/s=80.0 μValignl { stack {
size 12{ε= ital "Blv"= left (0 "." "100"`T right ) left (4 "." "00" times "10" rSup { size 8{ – 3} } `m right) left (0 "." "200"`"m/s" right)} {} #
="80" "." 0`"μV" {}
} } {}

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. εε size 12{ε} {} is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

Section Summary

- The Hall effect is the creation of voltage εε size 12{ε} {}, known as the Hall emf, across a current-carrying conductor by a magnetic field.

- The Hall emf is given by
  ε=Blv(B,v,andl,mutually perpendicular)ε=Blv(B,v,andl,mutually perpendicular) size 12{ε= ital “Blv”“` ( B,`v,`”and”`l,`”mutually perpendicular”) } {}

  for a conductor of width *ll size 12{l} {}* through which charges move at a speed vv size 12{v} {}.

Conceptual Questions

Discuss how the Hall effect could be used to obtain information on free charge density in a conductor. (Hint: Consider how drift velocity and current are related.)

Problems & Exercises

A large water main is 2.50 m in diameter and the average water velocity is 6.00 m/s. Find the Hall voltage produced if the pipe runs perpendicular to the Earth’s 5.00×10−5-T5.00×10−5-T size 12{5 “.” “00” times “10” rSup { size 8{ – 5} } “-T”} {} field.

7.50×10−4V7.50×10−4V

What Hall voltage is produced by a 0.200-T field applied across a 2.60-cm-diameter aorta when blood velocity is 60.0 cm/s?

(a) What is the speed of a supersonic aircraft with a 17.0-m wingspan, if it experiences a 1.60-V Hall voltage between its wing tips when in level flight over the north magnetic pole, where the Earth’s field strength is 8.00×10−5T?8.00×10−5T? size 12{8 “.” “00” times “10” rSup { size 8{ – 5} } `”T?”} {} (b) Explain why very little current flows as a result of this Hall voltage.

(a) 1.18×10^{3} m/s

(b) Once established, the Hall emf pushes charges one direction and the magnetic force acts in the opposite direction resulting in no net force on the charges. Therefore, no current flows in the direction of the Hall emf. This is the same as in a current-carrying conductor—current does not flow in the direction of the Hall emf.

A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?

Calculate the Hall voltage induced on a patient’s heart while being scanned by an MRI unit. Approximate the conducting path on the heart wall by a wire 7.50 cm long that moves at 10.0 cm/s perpendicular to a 1.50-T magnetic field.

11.3 mV

A Hall probe calibrated to read 1.00 μV1.00 μV size 12{1 "." "00"`"μV"} {} when placed in a 2.00-T field is placed in a 0.150-T field. What is its output voltage?

Using information in [link], what would the Hall voltage be if a 2.00-T field is applied across a 10-gauge copper wire (2.588 mm in diameter) carrying a 20.0-A current?

1.16 μV1.16 μV size 12{1 "." "47"`"μV"} {}

Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)

A patient with a pacemaker is mistakenly being scanned for an MRI image. A 10.0-cm-long section of pacemaker wire moves at a speed of 10.0 cm/s perpendicular to the MRI unit's magnetic field and a 20.0-mV Hall voltage is induced. What is the magnetic field strength?

2.00 T

Glossary

Hall effect
the creation of voltage across a current-carrying conductor by a magnetic field
Hall emf
the electromotive force created by a current-carrying conductor by a magnetic field, ε=Blvε=Blv size 12{ε= ital "Blv"} {}

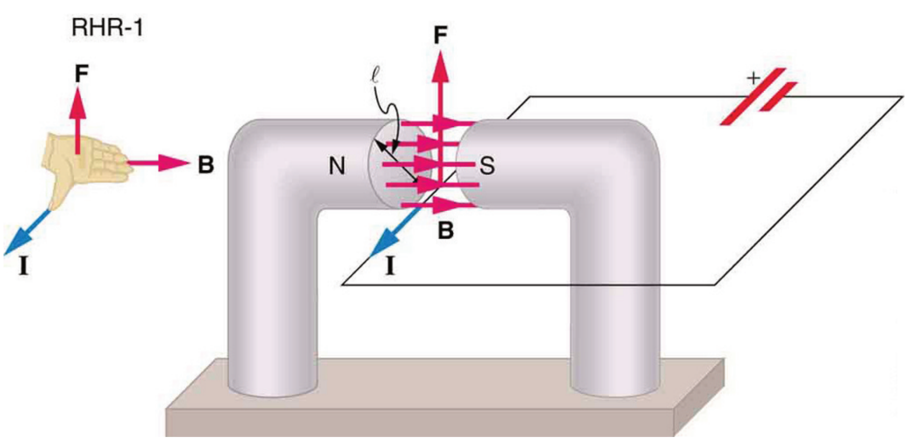
22.7 Magnetic Force on a Current-Carrying Conductor

Magnetic Force on a Current-Carrying Conductor

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.

The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.



We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity vdvd is given by F=qvdBsinθF=qvdBsinθ. Taking BB size 12{B} {} to be uniform over a length of wire ll and zero elsewhere, the total magnetic force on the wire is then F=(qvdBsinθ)(N)F=(qvdBsinθ)(N) size 12{F= (ital "qv" rSub { size 8{d} } B"sin"θ) (N) } {}, where NN size 12{N} {} *is the number of charge carriers in the section of wire of length ll size 12{l} {}. Now, N=nVN=nV size 12{N= ital "nV"} {}, where nn size 12{n} {} is the number of charge carriers per unit volume and VV size 12{V} {} is the volume of wire in the field.*

Noting that V=AlV=Al size 12{V= ital "Al"} {}, where AA size 12{A} {} is the cross-sectional area of the wire, then the force on the wire is F=(qvdBsinθ)(nAl)F=(qvdBsinθ)(nAl). Gathering terms,

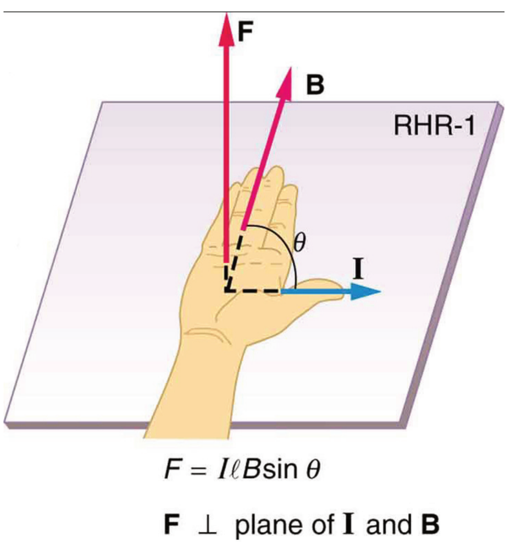
F=(nqAvd)lBsinθ.F=(nqAvd)lBsinθ. size 12{F= (ital "nqAv" rSub { size 8{d} }) ital "lB""sin"θ} {}

Because nqAvd=InqAvd=I size 12{ ital "nqAv" rSub { size 8{d} } =I} {} (see Current),

F=IlBsinθF=IlBsinθ size 12{F= ital "IlB""sin"θ} {}

is the equation for *magnetic force on a length ll of wire carrying a current II in a uniform magnetic field BB*, as shown in [link]. If we divide both sides of this expression by ll, we find that the magnetic force per unit length of wire in a uniform field is Fl=IBsinθFl=IBsinθ size 12{ { {F} over {l} } = ital "IB""sin"θ} {}. The direction of this force is given by RHR-1, with the thumb in the direction of the current II size 12{I} {}. Then, with the fingers in the direction of BB size 12{B} {}, a perpendicular to the palm points in the direction of FF size 12{F} {}, as in [link].

The force on a current-carrying wire in a magnetic field is F=IlBsinθF=IlBsinθ size 12{F= ital "IlB""sin"θ} {}. Its direction is given by RHR-1.



Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in [link], given B=1.50 TB=1.50 T size 12{B=1 "." "50"" T"} {}, l=5.00 cml=5.00 cm size 12{l=5 "." "00"" cm"} {}, and I=20.0AI=20.0A size 12{I="20" "." 0 A} {}.

Strategy

The force can be found with the given information by using F=IlBsinθF=IlBsinθ size 12{F= ital "IlB""sin"θ} {} and noting that the angle θθ size 12{θ} {} between II size 12{I} {} and BB size 12{B} {} is 90º90º, so that sinθ=1sinθ=1.

Solution

Entering the given values into F=IlBsinθF=IlBsinθ size 12{F= ital "IlB""sin"θ} {} yields

F=IlBsinθ=20.0 A0.0500 m1.50 T1 .F=IlBsinθ=20.0 A0.0500 m1.50 T1 . size 12{F= ital "IlB""sin"θ= left ("20" "." 0" A" right) left (0 "." "0500"" m" right) left (1 "." "50"" T" right) left (1 right)} {}

The units for tesla are 1 T=NA?m1 T=NA?m size 12{"1 T"= { {N} over {A cdot m} } } {}; thus,

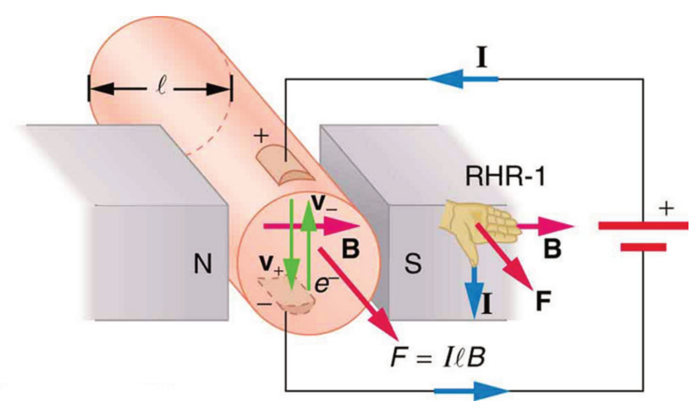
F=1.50 N.F=1.50 N. size 12{F=1 "." "50"" N"} {}

Discussion

This large magnetic field creates a significant force on a small length of wire.

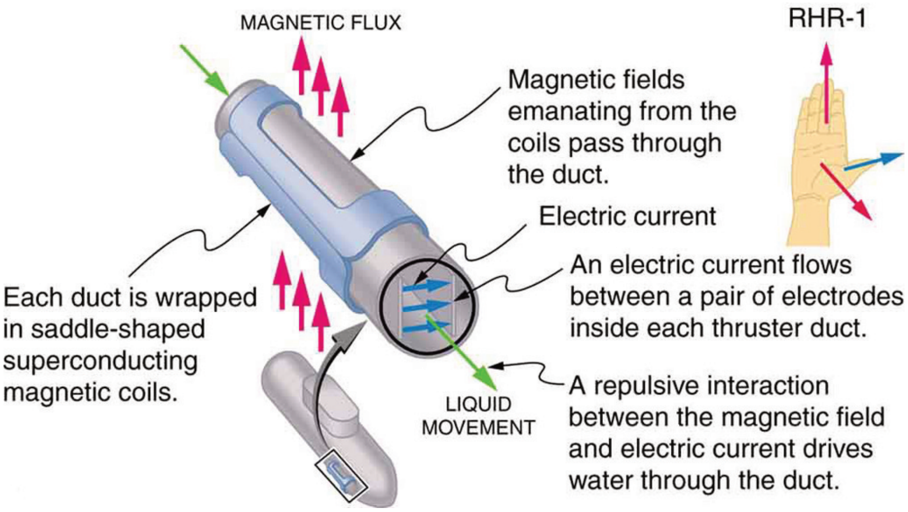
Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See [link].)

Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.



A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See [link].) Existing MHD drives are heavy and inefficient—much development work is needed.

An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.



Section Summary

- The magnetic force on current-carrying conductors is given by
  F=IlBsinθ,F=IlBsinθ, size 12{F= ital "IlB""sin"θ} {}

  where *II size 12{I} {}* is the current, ll size 12{l} {} is the length of a straight conductor in a uniform magnetic field *BB size 12{B} {}*, and *θθ size 12{θ} {}* is the angle between *II size 12{I} {}* and *BB size 12{B} {}*. The force follows RHR-1 with the thumb in the direction of *II size 12{I} {}*.

Conceptual Questions

Draw a sketch of the situation in [link] showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

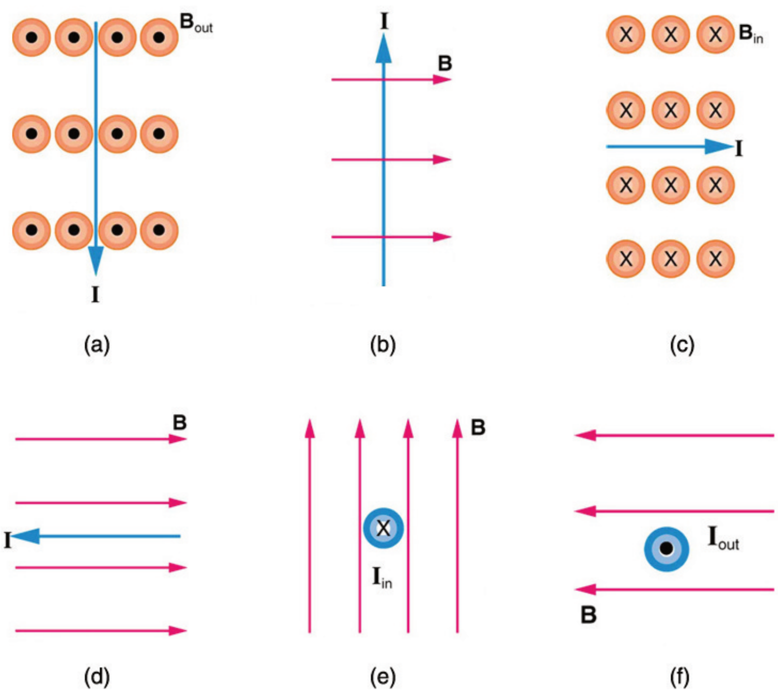
Verify that the direction of the force in an MHD drive, such as that in [link], does not depend on the sign of the charges carrying the current across the fluid.

Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

Problems & Exercises

What is the direction of the magnetic force on the current in each of the six cases in [link]?



(a) west (left)

(b) into page

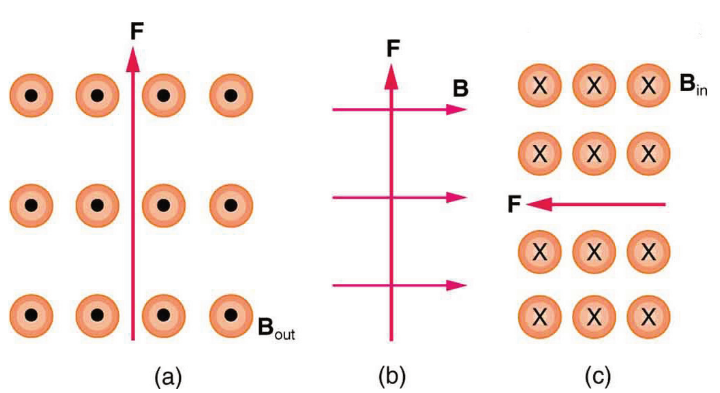
(c) north (up)

(d) no force

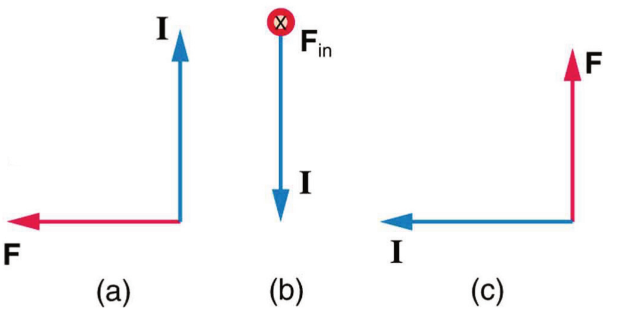
(e) east (right)

(f) south (down)

What is the direction of a current that experiences the magnetic force shown in each of the three cases in [link], assuming the current runs perpendicular to BB size 12{B} {}?



What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in [link], assuming BB size 12{B} {} is perpendicular to II size 12{I} {}?



(a) into page

(b) west (left)

(c) out of page

(a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's 3.00×10−5-T3.00×10−5-T size 12{3 "." "00" times "10" rSup { size 8{ – 5} } "-T"} {} field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

(a) A DC power line for a light-rail system carries 1000 A at an angle of 30.0º30.0º to the Earth's 5.00×10−5-T5.00×10−5-T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } "-T"} {} field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

(a) 2.50 N

(b) This is about half a pound of force per 100 m of wire, which is much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

1.80 T

(a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60º60º with the Earth's 5.50×10−5 T5.50×10−5 T field. What is the current when the wire experiences a force of 7.00×10−3N7.00×10−3N? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

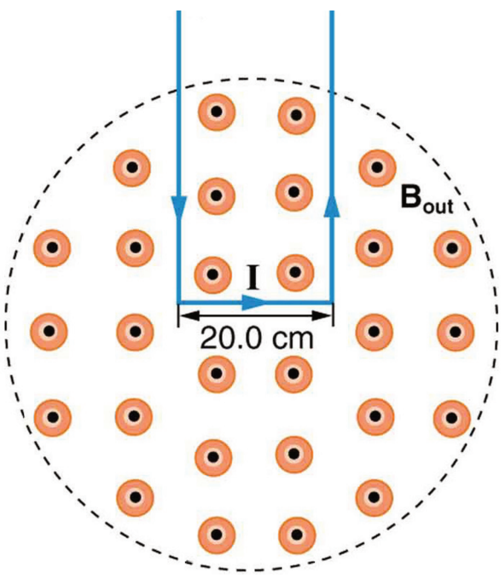
(a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90º90º size 12{“90″°} {} with the field?

(a) 30º30º size 12{“30″°} {}

(b) 4.80 N

The force on the rectangular loop of wire in the magnetic field in [link] can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?

A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region.



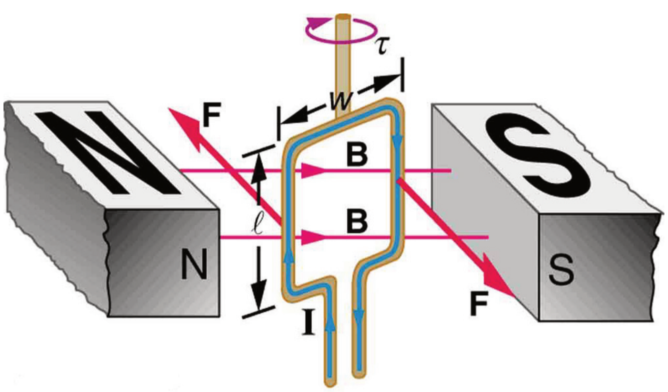
22.8 Torque on a Current Loop: Motors and Meters

Torque on a Current Loop: Motors and Meters

- Describe how motors and meters work in terms of torque on a current loop.
- Calculate the torque on a current-carrying loop in a magnetic field.

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See [link].)

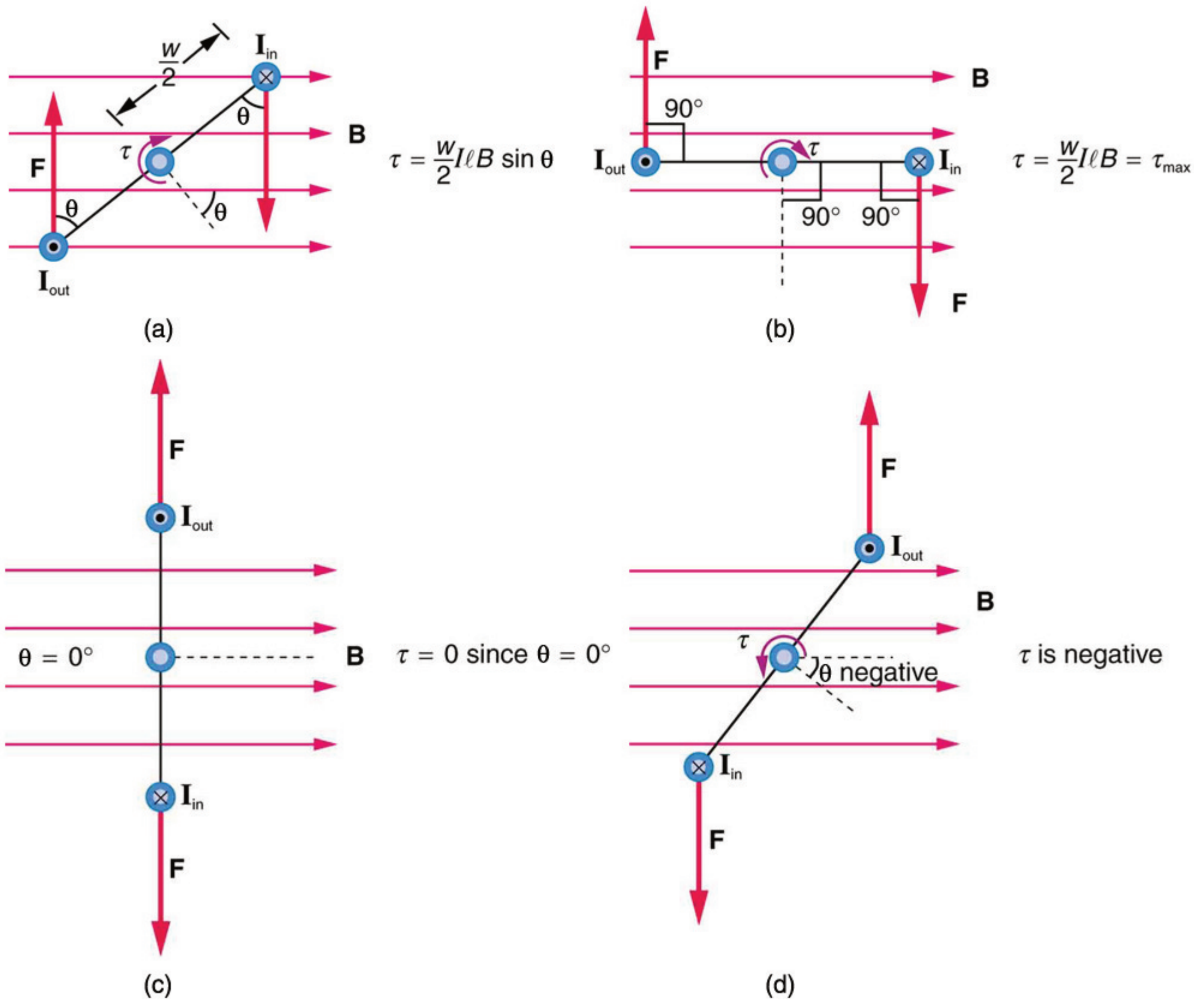
Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above.



Let us examine the force on each segment of the loop in [link] to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width ww and height ll. First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. [link] shows views of the loop from above. Torque is defined as τ=rFsinθτ=rFsinθ size 12{τ= ital “rF””sin”θ} {}, where FF size 12{F} {} is the force, rr is the distance from the pivot that the force is applied, and θθ is the angle between rr and FF. As seen in [link](a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since r=w/2r=w/2, the torque on each vertical segment is (w/2)Fsinθ(w/2)Fsinθ, and the two add to give a total torque.

τ=w2Fsinθ+w2Fsinθ=wFsinθτ=w2Fsinθ+w2Fsinθ=wFsinθ size 12{τ= { {w} over {2} } F”sin”θ+ { {w} over {2} } F”sin”θ= ital “wF””sin”θ} {}

Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle θθ size 12{θ} {} with the field that is the same as the angle between w/2w/2 size 12{w/2} {} and FF size 12{F} {}. (b) The maximum torque occurs when θθ size 12{θ} {} is a right angle and sinθ=1sinθ=1 size 12{“sin”θ=1} {}. (c) Zero (minimum) torque occurs when θθ size 12{θ} {} is zero and sinθ=0sinθ=0. (d) The torque reverses once the loop rotates past θ=0θ=0.



Now, each vertical segment has a length ll size 12{l} {} that is perpendicular to BB size 12{B} {}, so that the force on each is F=IlBF=IlB size 12{F= ital “IlB”} {}. Entering FF size 12{F} {} into the expression for torque yields

τ=wIlBsinθ.τ=wIlBsinθ. size 12{τ= ital “wIlB””sin”θ} {}

If we have a multiple loop of NN size 12{N} {} turns, we get NN size 12{N} {} times the torque of one loop. Finally, note that the area of the loop is A=wlA=wl size 12{A= ital “wl”} {}; the expression for the torque becomes τ=NIABsinθ.τ=NIABsinθ. size 12{τ= ital “NIAB””sin”θ} {}

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current II size 12{I} {}, has NN size 12{N} {} turns, each of area AA size 12{A} {}, and the perpendicular to the loop makes an angle θθ size 12{θ} {} with the field BB size 12{B} {}. The net force on the loop is zero.

Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Strategy

Torque on the loop can be found using τ=NIABsinθτ=NIABsinθ size 12{τ= ital “NIAB””sin”θ} {}. Maximum torque occurs when θ=90ºθ=90º and sinθ=1sinθ=1 size 12{“sin”θ=1} {}.

Solution

For sinθ=1sinθ=1 size 12{“sin”θ=1} {}, the maximum torque is

τmax=NIAB.τmax=NIAB. size 12{τ rSub { size 8{“max”} } = ital “NIAB”} {}

Entering known values yields

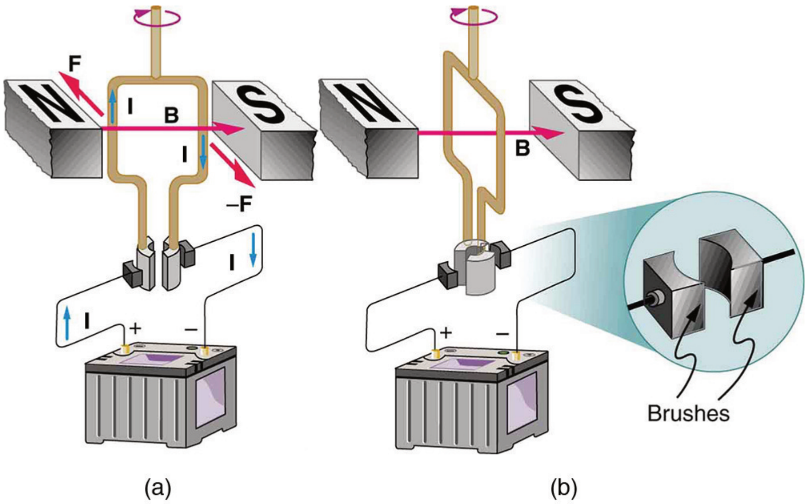
τmax=10015.0 A0.100 m22.00 T=30.0 N?m.τmax=10015.0 A0.100 m22.00 T=30.0 N?m.alignl { stack {
size 12{τ rSub { size 8{“max”} } = left (“100” right) left (“15” “.” 0″ A” right) left (0 “.” “100”” m” rSup {
size 8{2} } right) left (2 “.” “00”” T” right)} {} #
” “=”30” “.” “0 N” cdot m “.” {}
} } {}

Discussion

This torque is large enough to be useful in a motor.

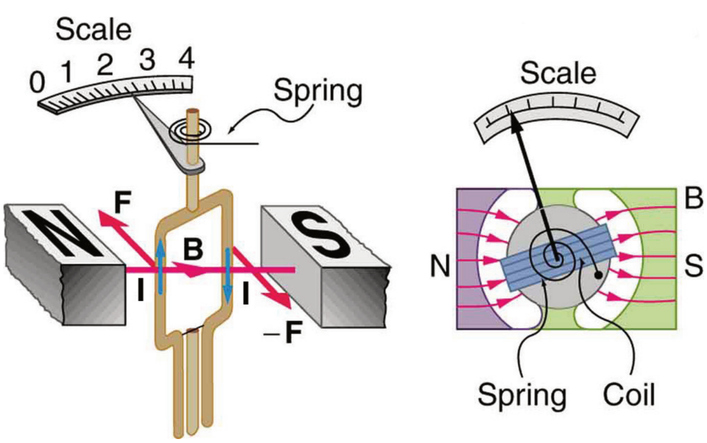
The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at θ=0θ=0 size 12{θ=0} {}. The torque then *reverses* its direction once the coil rotates past θ=0θ=0 size 12{θ=0} {}. (See [link](d).) This means that, unless we do something, the coil will oscillate back and forth about equilibrium at θ=0θ=0 size 12{θ=0} {}. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through θ=0θ=0 size 12{θ=0} {} with automatic switches called *brushes*. (See [link].)

(a) As the angular momentum of the coil carries it through θ=0θ=0 size 12{θ=0} {}, the brushes reverse the current to keep the torque clockwise. (b) The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque.



Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. [link] shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θθ size 12{θ} {} by making BB size 12{B} {} perpendicular to the loop over a large angular range. Thus the torque is proportional to II size 12{I} {} and not θθ size 12{θ} {}. A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to II size 12{I} {}. If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area AA size 12{A} {}, high magnetic field BB size 12{B} {}, and low-resistance coils.

Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of BB size 12{B} {} perpendicular to the loop constant, so that the torque does not depend on θθ size 12{θ} {} and the deflection against the return spring is proportional only to the current II size 12{I} {}.



Section Summary

- The torque ττ size 12{τ} {} on a current-carrying loop of any shape in a uniform magnetic field. is τ=NIABsinθ,τ=NIABsinθ, size 12{τ= ital “NIAB””sin”θ} {}

  where NN size 12{N} {} is the number of turns, II size 12{I} {} is the current, AA size 12{A} {} is the area of the loop, BB size 12{B} {} is the magnetic field strength, and θθ size 12{θ} {} is the angle between the perpendicular to the loop and the magnetic field.

Conceptual Questions

Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor’s current loop in [link] are vertical and produce no torque about the axis of rotation.

Problems & Exercises

(a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

(a) τ τ size 12{” τ”} {} decreases by 5.00% if B decreases by 5.00%

(b) 5.26% increase

(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θθ size 12{θ} {} is 10.9º?10.9º? size 12{“10” “.” 9°?} {}

Find the current through a loop needed to create a maximum torque of 9.00 N?m.9.00 N?m. size 12{9 “.” “00”`N cdot m “.” } {} The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

10.0 A

Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of 300 N?m300 N?m size 12{3″00″`N cdot m} {} if the loop is carrying 25.0 A.

Since the equation for torque on a current-carrying loop is τ=NIABsinθτ=NIABsinθ size 12{τ= ital “NIAB””sin”θ} {}, the units of N?mN?m size 12{N cdot m} {} must equal units of A?m2TA?m2T size 12{A cdot m rSup { size 8{2} } `T} {}. Verify this.

A?m2?T=A?m2NA?m=N?mA?m2?T=A?m2NA?m=N?m size 12{A cdot m rSup { size 8{2} } cdot T=A cdot m rSup { size 8{2} } left ({ {N} over {A cdot m} } right)=N cdot m} {}.

(a) At what angle θθ size 12{θ} {} is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop 0.650×10−15m0.650×10−15m size 12{0 "." "650" times "10" rSup { size 8{ – "15"} } `m} {} in radius with a current of 1.05×104A1.05×104A size 12{1 "." "05" times "10" rSup { size 8{4} } `A} {} (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

3.48×10−26N?m3.48×10−26N?m size 12{3 "." "48" times "10" rSup { size 8{ – "26"} } `N cdot m} {}

(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of 3.00×10−5T3.00×10−5T size 12{3 "." "00" times "10" rSup { size 8{ – 5} } `T} {}. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?

Repeat [link], but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle 45.0º45.0º size 12{"45" "." 0°} {} below the horizontal and with a strength of 6.00×10−5T6.00×10−5T size 12{6 "." "00" times "10" rSup { size 8{ – 5} } `T} {}.

(a) 0.666 N?m0.666 N?m size 12{0 "." "666"`N cdot m} {} west

(b) This is not a very significant torque, so practical use would be limited. Also, the current would need to be alternated to make the loop rotate (otherwise it would oscillate).

Glossary

motor
loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process
meter
common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to II and not θθ, so the needle deflection is proportional to the current

22.9 Magnetic Fields Produced by Currents: Ampere’s Law

Magnetic Fields Produced by Currents: Ampere’s Law

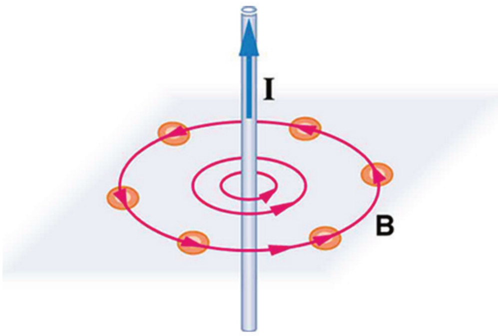
- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth’s field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

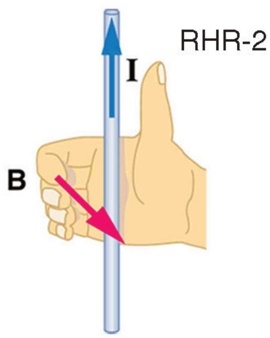
Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in [link]. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The right hand rule 2 (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.

(a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.



(a)



(b)

The magnetic field strength (magnitude) produced by a long straight current-carrying wire is found by experiment to be

B=μ0I2πr(long straight wire),B=μ0I2πr(long straight wire), size 12{B= { {μ rSub { size 8{0} } I} over {2πr} } “ (“long straight wire”) ,} {}

where II size 12{I} {} is the current, rr size 12{r} {} is the shortest distance to the wire, and the constant μ0=4π×10−7T?m/Aμ0=4π×10−7T?m/A is the permeability of free space.
(μ0(μ0 size 12{ (μ rSub { size 8{0} } } {} is one of the basic constants in nature. We will see later that μ0μ0 size 12{μ rSub { size 8{0} } } {} is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire rr size 12{r} {}, not on position along the wire.

Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth’s at a distance of 5.0 cm from the wire.

Strategy

The Earth’s field is about 5.0×10−5T5.0×10−5T, and so here BB size 12{B} {} due to the wire is taken to be 1.0×10−4T1.0×10−4T. The equation B=μ0I2πrB=μ0I2πr can be used to find
II, since all other quantities are known.

Solution

Solving for II size 12{I} {} and entering known values gives

I=2πrBμ0=2π5.0×10−2m1.0×10−4T4π×10−7T?m/A=25 A.I=2πrBμ0=2π5.0×10−2m1.0×10−4T4π×10−7T?m/A=25 A.

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the Biot-Savart law. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called Ampere's law, which relates magnetic field and current in a general way. Ampere's law in turn is a part of Maxwell's equations, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in Magnetic Fields and Magnetic Field Lines, while concentrating on the fields created in certain important situations.

Making Connections: Relativity

Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

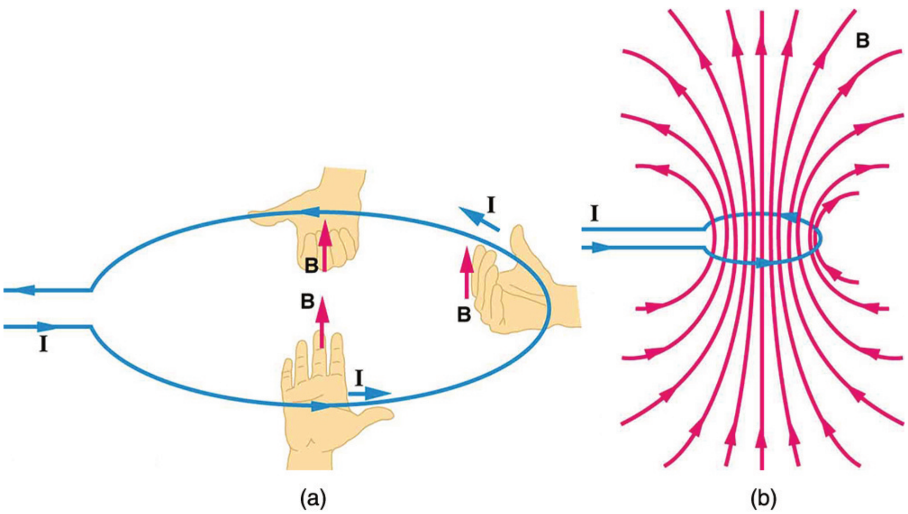
Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in [link]. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in Magnetic Fields and Magnetic Field Lines are needed for more detail. There is a simple formula for the magnetic field strength at the center of a circular loop. It is

B=μ0I2R(at center of loop),B=μ0I2R(at center of loop), size 12{B= { {μ rSub { size 8{0} } I} over {2R} } ` ("at center of loop") ,} {}

where RR size 12{R} {} is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have NN size 12{N} {} *loops; then, the field is B=Nμ0I/(2R)B=Nμ0I/(2R). Note that the larger the loop, the smaller the field at its center, because the current is farther away.*

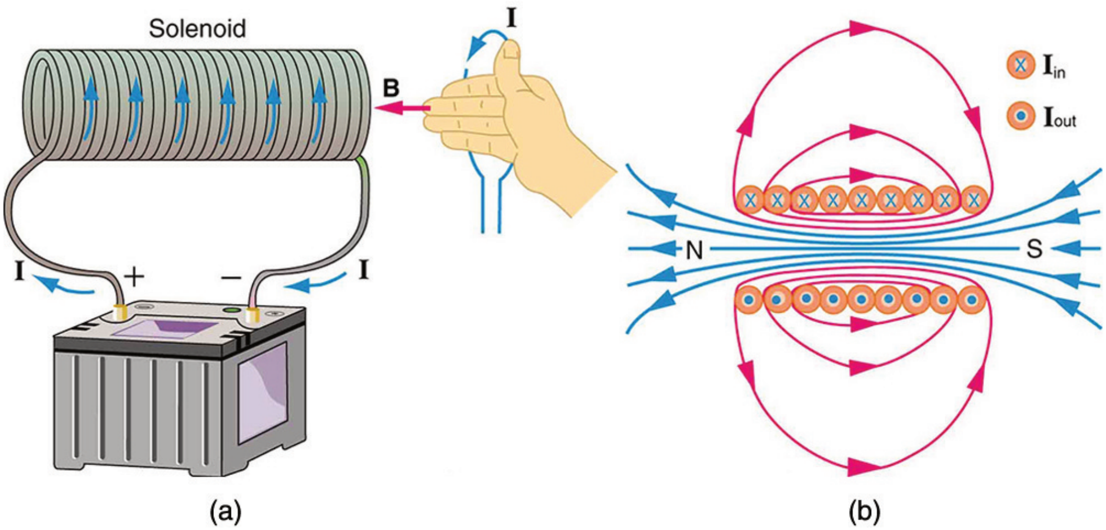
(a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a Hall probe completes the picture. The field is similar to that of a bar magnet.



Magnetic Field Produced by a Current-Carrying Solenoid

A solenoid is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. [link] shows how the field looks and how its direction is given by RHR-2.

(a) Because of its shape, the field inside a solenoid of length ll size 12{l} {} is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.



The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops and bar magnets, but the magnetic field strength inside a solenoid is simply

B=μ0nI(inside a solenoid),B=μ0nI(inside a solenoid), size 12{B=μ rSub { size 8{0} } ital "nI"` ("inside a solenoid") ,} {}

where nn size 12{n} {} is the number of loops per unit length of the solenoid (n=N/l(n=N/l size 12{ (n=N/l} {}, with NN size 12{N} {} being the number of loops and ll size 12{l} {} the length). Note that BB size 12{B} {} is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as [link] implies.

Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use B=μ0nIB=μ0nI size 12{B=μ rSub { size 8{0} } ital "nI"} {}. First, we note the number of loops per unit length is

n=Nl=20002.00 m=1000 m−1=10 cm−1.n=Nl=20002.00 m=1000 m−1=10 cm−1. size 12{n rSup { size 8{ – 1} } = { {N} over {l} } = { {"2000"} over {2 "." "00" m} } ="1000″" m" rSup { size 8{ – 1} } ="10″" cm" rSup { size 8{ – 1} } "." } {}

Solution

Substituting known values gives

B=μ0nI=4π×10−7T?m/A1000m−11600 A=2.01 T.B=μ0nI=4π×10−7T?m/A1000m−11600 A=2.01 T.

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

Generator

PhET Interactive Simulation

Section Summary

- The strength of the magnetic field created by current in a long straight wire is given by
  B=μ0I2πr(long straight wire),B=μ0I2πr(long straight wire),

  where II size 12{I} {} is the current, rr size 12{r} {} is the shortest distance to the wire, and the constant μ0=4π×10−7T?m/Aμ0=4π×10−7T?m/A size 12{μ rSub { size 8{0} } =4π times "10" rSup { size 8{ – 7} } `T cdot "m/A"} {} is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.

- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by
  B=μ0I2R(at center of loop),B=μ0I2R(at center of loop), size 12{B= { {μ rSub { size 8{0} } I} over {2R} } " " ("at center of loop") ,} {}

  where RR size 12{R} {} is the radius of the loop. This equation becomes B=μ0nI/(2R)B=μ0nI/(2R) size 12{B=μ rSub { size 8{0} } ital "nI"/ (2R) } {} for a flat coil of NN size 12{N} {} loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.
- The magnetic field strength inside a solenoid is
  B=μ0nI(inside a solenoid),B=μ0nI(inside a solenoid), size 12{B=μ rSub { size 8{0} } ital "nI"" " ("inside a solenoid") ,} {}

  where nn size 12{n} {} is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

Conceptual Questions

Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in [link]). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

Glossary

right hand rule 2 (RHR-2)
a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops
magnetic field strength (magnitude) produced by a long straight current-carrying wire
defined as B=μ0I2πrB=μ0I2πr, where II is the current, rr is the shortest distance to the wire, and μ0μ0 is the permeability of free space
permeability of free space
the measure of the ability of a material, in this case free space, to support a magnetic field; the constant μ0=4π×10−7T?m/Aμ0=4π×10−7T?m/A
magnetic field strength at the center of a circular loop
defined as B=μ0I2RB=μ0I2R where RR is the radius of the loop
solenoid
a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it
magnetic field strength inside a solenoid
defined as B=μ0nIB=μ0nI where nn is the number of loops per unit length of the solenoid (n=N/l(n=N/l, with NN being the number of loops and ll the length)

Biot-Savart law

a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Ampere's law

the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

Maxwell's equations

a set of four equations that describe electromagnetic phenomena

22.10 Magnetic Force between Two Parallel Conductors

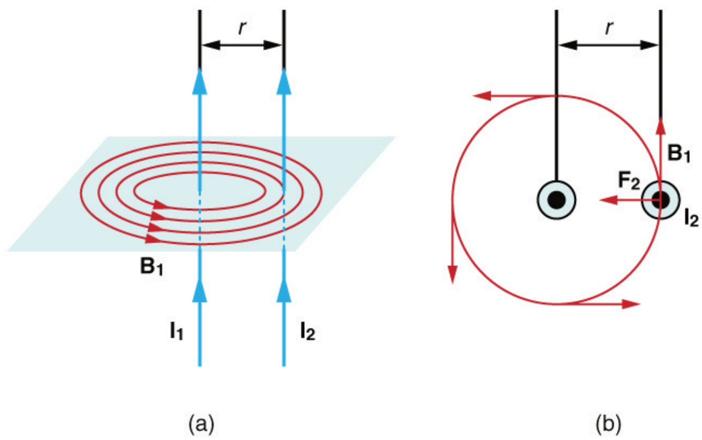
Magnetic Force between Two Parallel Conductors

- Describe the effects of the magnetic force between two conductors.
- Calculate the force between two parallel conductors.

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance rr size 12{r} {} can be found by applying what we have developed in preceding sections. [link] shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F2F2 size 12{F rSub { size 8{2} } } {}). The field due to I1I1 size 12{I rSub { size 8{1} } } {} at a distance rr size 12{r} {} is given to be

B1=μ0I12πr.B1=μ0I12πr. size 12{B rSub { size 8{1} } = { {μ rSub { size 8{0} } I rSub { size 8{1} } } over {2πr} } "." } {}
(a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.



This field is uniform along wire 2 and perpendicular to it, and so the force F2F2 size 12{F rSub { size 8{2} } } {} it exerts on wire 2 is given by F=IlBsinθF=IlBsinθ size 12{F= ital "IlB""sin"θ} {} with sinθ=1sinθ=1 size 12{"sin"θ=1} {}:

F2=I2lB1.F2=I2lB1. size 12{F rSub { size 8{2} } =I rSub { size 8{2} } ital "lB" rSub { size 8{1} } } {}

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write FF size 12{F} {} for the magnitude of F2F2 size 12{F rSub { size 8{2} } } {}. (Note that F1=−F2F1=−F2 size 12{F rSub { size 8{1} } = – F rSub { size 8{2} } } {}.) Since the wires are very long, it is convenient to think in terms of F/lF/l size 12{F/ l} {}, the force per unit length. Substituting the expression for B1B1 size 12{B rSub { size 8{1} } } {} into the last equation and rearranging terms gives

Fl=μ0I1I22πr.Fl=μ0I1I22πr. size 12{ { {F} over {l} } = { {μ rSub { size 8{0} } I rSub { size 8{1} } I rSub { size 8{2} } } over {2πr} } "." } {}

F/lF/l size 12{F/l} {} is the force per unit length between two parallel currents I1I1 size 12{I rSub { size 8{1} } } {} and I2I2 size 12{I rSub { size 8{2} } } {} separated by a distance rr size 12{r} {}. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

Fl=4π×10−7T?m/A1 A22π1 m=2×10−7 N/m.Fl=4π×10−7T?m/A1 A22π1 m=2×10−7 N/m. size 12{ { {F} over {l} } = { { left (4π times "10" rSup { size 8{ – 7} } `T cdot "m/A" right ) left (1`A right) rSup { size 8{2} } } over { left (2π right) left (1" m" right)} } =2 times "10" rSup { size 8{ – 7} } " N/m" "." } {}

Since μ0μ0 size 12{μ rSub { size 8{0} } } {} is exactly 4π×10−7T?m/A4π×10−7T?m/A by definition, and because
1 T=1 N/A?m1 T=1 N/A?m size 12{1″ T”=” 1N/” left (A cdot m right)} {}, the force per meter is exactly 2×10−7N/m2×10−7N/m size 12{2 times “10” rSup { size 8{ – 7} } `”N/m”} {}. This is the basis of the operational definition of the ampere.

The Ampere

The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly 2×10−7 N/m2×10−7 N/m size 12{2 times “10” rSup { size 8{ – 7} } ” N/m”} {} on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, 1 C=1 A?s1 C=1 A?s size 12{1`C=1`A cdot s} {}. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

Section Summary

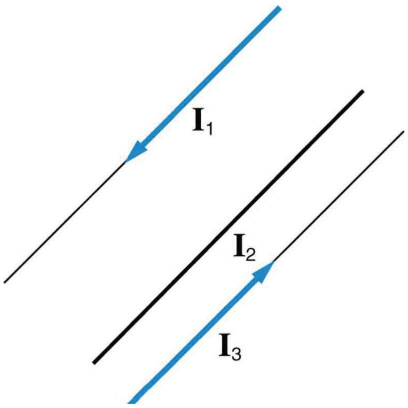
- The force between two parallel currents I1I1 size 12{I rSub { size 8{1} } } {} and I2I2 size 12{I rSub { size 8{2} } } {}, separated by a distance rr size 12{r} {}, has a magnitude per unit length given by Fl=μ0I1I22πr.Fl=μ0I1I22πr. size 12{ { {F} over {l} } = { {μ rSub { size 8{0} } I rSub { size 8{1} } I rSub { size 8{2} } } over {2πr} } } {}
- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

Conceptual Questions

Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

If you have three parallel wires in the same plane, as in [link], with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.

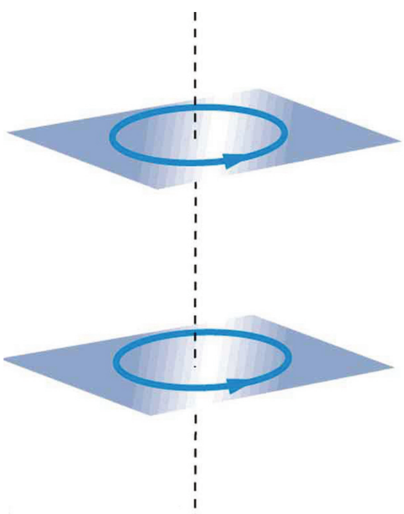
Three parallel coplanar wires with currents in the outer two in opposite directions.



Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.

Use the right hand rules to show that the force between the two loops in [link] is attractive if the currents are in the same direction and repulsive if they are in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.

Two loops of wire carrying currents can exert forces and torques on one another.



If one of the loops in [link] is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?

Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

Problems & Exercises

(a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires? (b) Discuss the practical consequences of this force, if any.

(a) 8.53 N, repulsive

(b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?

A 2.50-m segment of wire supplying current to the motor of a submerged submarine carries 1000 A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

400 A in the opposite direction

The wire carrying 400 A to the motor of a commuter train feels an attractive force of $4.00\times10^{-3}\text{N/m}$4.00×10−3N/m size 12{4 “.” “00” times “10” rSup { size 8{ – 3} } `”N/m”} {} due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?

An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?

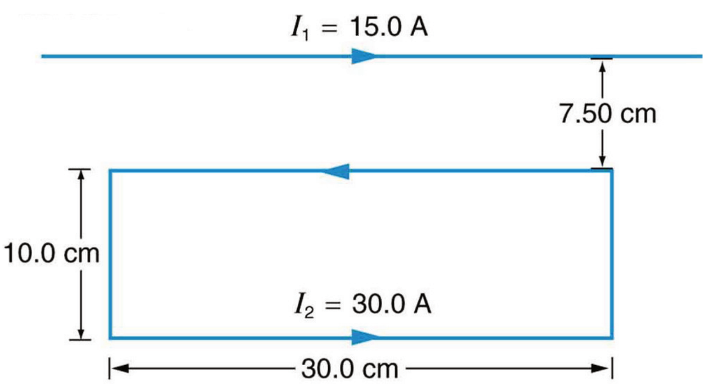
(a) 1.67×10^{-3} N/m1.67×10−3N/m size 12{1 "." "67" times "10" rSup { size 8{ – 3} } `"N/m"} {}

(b) 3.33×10^{-3} N/m3.33×10−3 N/m size 12{3 "." "33" times "10" rSup { size 8{ – 3} } " N/m"} {}

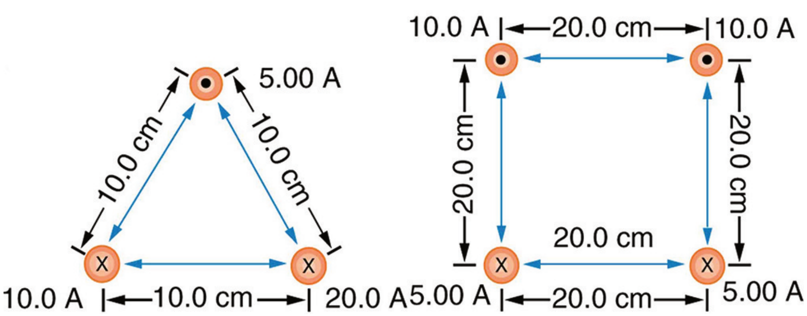
(c) Repulsive

(d) No, these are very small forces

[link] shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?



Find the direction and magnitude of the force that each wire experiences in [link](a) by, using vector addition.



(a) Top wire: 2.65×10^{-4} N/m2.65×10−4N/m s, 10.9º10.9º size 12{"10" "." 9°} {} to left of up

(b) Lower left wire: 3.61×10^{-4} N/m3.61×10−4N/m size 12{3 "." "61" times "10" rSup { size 8{ – 4} } `"N/m"} {}, 13.9º13.9º size 12{"13" "." 9°} {} down from right

(c) Lower right wire: 3.46×10^{-4} N/m3.46×10−4N/m size 12{3 "." "46" times "10" rSup { size 8{ – 4} } `"N/m"} {}, 30.0º30.0º size 12{"30" "." 0°} {} down from left

Find the direction and magnitude of the force that each wire experiences in [link](b), using vector addition.

22.11 More Applications of Magnetism

More Applications of Magnetism

- Describe some applications of magnetism.

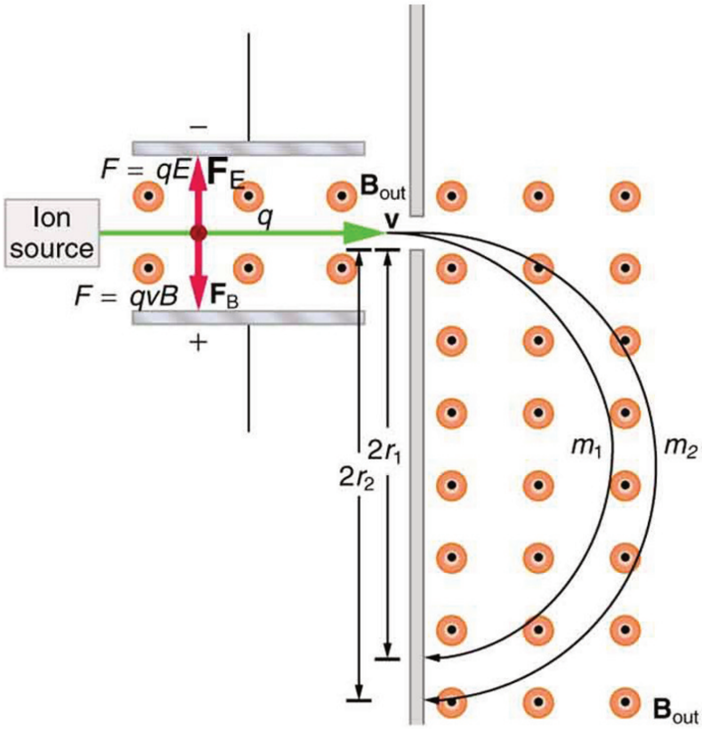
Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius rr size 12{r} {}.

r=mvqBr=mvqB size 12{r= { { ital “mv”} over { ital “qB”} } } {}

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if vv size 12{v} {}, qq size 12{q} {}, and BB size 12{B} {} can be fixed, then the radius of the path rr size 12{r} {} is simply proportional to the mass mm size 12{m} {} of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See [link].) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity vv size 12{v} {}, and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of vv size 12{v} {} to get through.

This mass spectrometer uses a velocity selector to fix vv size 12{v} {} so that the radius of the path is proportional to mass.



The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force F=qEF=qE size 12{F= ital “qE”} {} equals the magnetic force F=qvBF=qvB size 12{F= ital “qvB”} {}, so that qE=qvBqE=qvB size 12{ ital “qE”= ital “qvB”} {}. Noting that qq size 12{q} {} cancels, we see that

v=EBv=EB size 12{v= { {E} over {B} } } {}

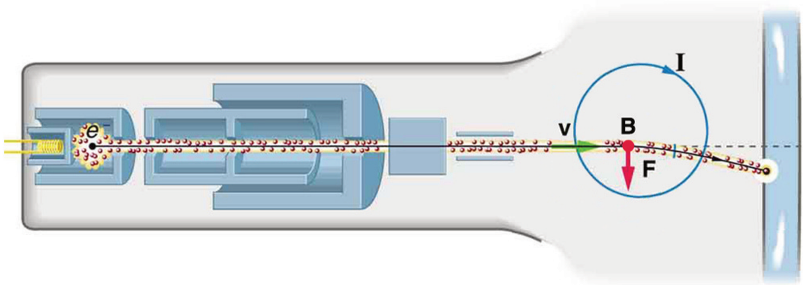
is the velocity particles must have to make it through the velocity selector, and further, that vv size 12{v} {} can be selected by varying EE size 12{E} {} and BB size 12{B} {}. In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge qq size 12{q} {}, but since qq size 12{q} {} is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. [link] shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.

The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.



Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called nuclear magnetic resonance (NMR) in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to “flip” the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in Oscillatory Motion and Waves) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR).*

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word “nuclear” and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These “slices” or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body. Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New “open-MRI” machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called “functional MRI” (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

Other Medical Uses of Magnetic Fields

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about 10−610−6 size 12{“10” rSup { size 8{ – 6} } } {} to 10−810−8 size 12{“10” rSup { size 8{ – 8} } } {}*less* than the Earth’s magnetic field. Recording of the heart’s magnetic field as it beats is called a magnetocardiogram (MCG), while measurements of the brain's magnetic field is called a magnetoencephalogram (MEG). Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how things change both inside and outside. Use the field meter to measure how the magnetic field changes.

Magnet and Compass

PhET Interactive Simulation

Section Summary

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude
  v=EB.v=EB. size 12{v= { {E} over {B} } "." } {}

Conceptual Questions

Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

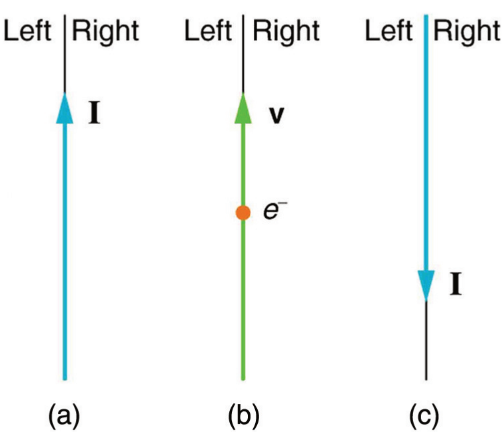
You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

Problems & Exercises

Indicate whether the magnetic field created in each of the three situations shown in [link] is into or out of the page on the left and right of the current.

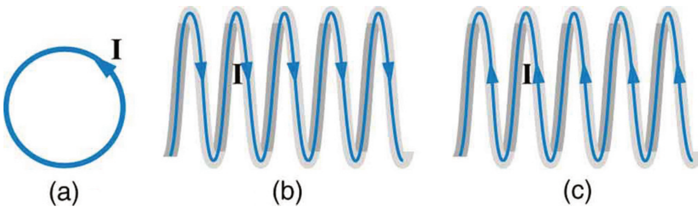


(a) right-into page, left-out of page

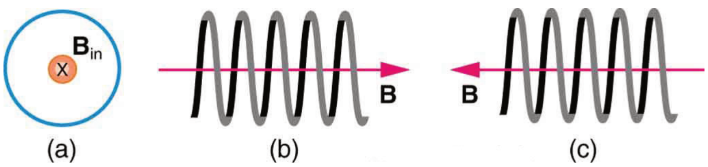
(b) right-out of page, left-into page

(c) right-out of page, left-into page

What are the directions of the fields in the center of the loop and coils shown in [link]?



What are the directions of the currents in the loop and coils shown in [link]?



(a) clockwise

(b) clockwise as seen from the left

(c) clockwise as seen from the right

To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop 0.650×10−15m0.650×10−15m in radius carrying 1.05×104A1.05×104A size 12{1 "." "05" times "10" rSup { size 8{4} } `A} {}. What is the field at the center of such a loop?

1.01×1013 T1.01×1013 T size 12{1 "." "01" times "10" rSup { size 8{"13″} } " T"} {}

Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

(a) 4.80×10−4T4.80×10−4T size 12{4 "." "80" times "10" rSup { size 8{ – 4} } `T} {}

(b) Zero

(c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?

What current is needed in the solenoid described in [link] to produce a magnetic field 104104 size 12{“10” rSup { size 8{4} } } {} times the Earth’s magnetic field of 5.00×10−5T5.00×10−5T size 12{5 “.” “00” times “10” rSup { size 8{ – 5} } `T} {}?

39.8 A

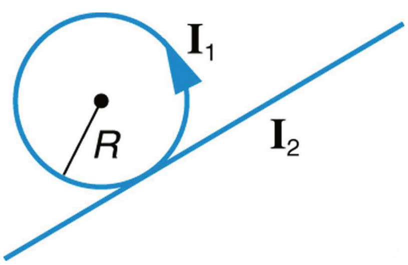
How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth’s (5.00×10−5T)?(5.00×10−5T)? size 12{ (5 “.” “00” times “10” rSup { size 8{ – 5} } `T) ?} {} Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

Measurements affect the system being measured, such as the current loop in [link]. (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

(a) 3.14×10−5T3.14×10−5T size 12{3 “.” “14” times “10” rSup { size 8{ – 5} } `T} {}

(b) 0.314 T

[link] shows a long straight wire just touching a loop carrying a current I1I1 size 12{I rSub { size 8{1} } } {}. Both lie in the same plane. (a) What direction must the current I2I2 size 12{I rSub { size 8{2} } } {} in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of I1/I2I1/I2 size 12{I rSub { size 8{1} } /I rSub { size 8{2} } } {} that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?



Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [link](a), using the rules of vector addition to sum the contributions from each wire.

7.55×10−5T7.55×10−5T, 23.4º23.4º

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [link](b), using the rules of vector addition to sum the contributions from each wire.

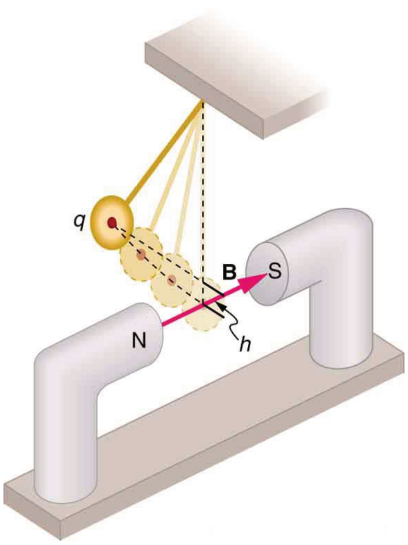
What current is needed in the top wire in [link](a) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

10.0 A

Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

Integrated Concepts

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in [link]. What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive 0.250 μC0.250 μC size 12{0 "." "250"`"μC"} {} charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.



(a) 9.09×10−7N9.09×10−7N size 12{9 "." "09" times "10" rSup { size 8{ – 7} } `N} {} upward

(b) 3.03×10−5m/s23.03×10−5m/s2 size 12{3 "." "03" times "10" rSup { size 8{ – 5} } `"m/s" rSup { size 8{2} } } {}

Integrated Concepts

(a) What voltage will accelerate electrons to a speed of 6.00×10−7m/s6.00×10−7m/s size 12{6 "." "00" times "10" rSup { size 8{ – 7} } `"m/s"} {}? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

Integrated Concepts

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

60.2 cm

Integrated Concepts

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

Integrated Concepts

(a) Using the values given for an MHD drive in [link], and assuming the force is uniformly applied to the fluid, calculate the pressure created in N/m2.N/m2. size 12{"N/m" rSup { size 8{2} } "." } {} (b) Is this a significant fraction of an atmosphere?

(a) 1.02×103N/m21.02×103N/m2 size 12{1 "." "02" times "10" rSup { size 8{3} } `"N/m" rSup { size 8{2} } } {}

(b) Not a significant fraction of an atmosphere

Integrated Concepts

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying 50 μA50 μA size 12{"50"`"μA"} {} in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads 50 μA50 μA size 12{"50"`"μA"} {} full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the 60º60º size 12{"60"°} {} arc moved?

Integrated Concepts

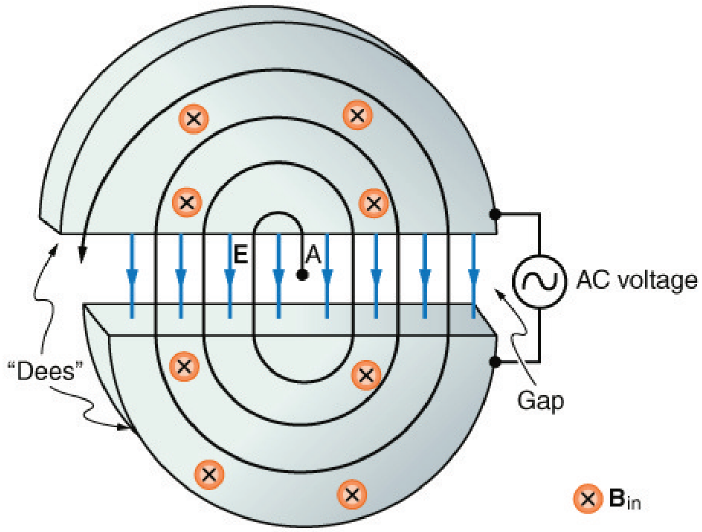
A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

17.0×10−4%/ºC17.0×10−4%/ºC size 12{"17" "." 0 times "10" rSup { size 8{ – 4} } "%/"°C} {}

Integrated Concepts

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is T=2πm/(qB)T=2πm/(qB) size 12{T=2πm/ (ital "qB") } {}. (b) What is the frequency *ff size 12{f} {}*? (c) What is the angular velocity *ωω size 12{ω} {}*? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. ([link].)

Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.



Integrated Concepts

A cyclotron accelerates charged particles as shown in [link]. Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

18.3 MHz

Integrated Concepts

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal 5.00×10−5T5.00×10−5T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } `T} {} field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

Integrated Concepts

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is 3.00×10−5T3.00×10−5T size 12{3 "." "00" times "10" rSup { size 8{ – 5} } `T} {}. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

(a) Straight up

(b) 6.00×10−4N/m6.00×10−4N/m size 12{6 "." "00" times "10" rSup { size 8{ – 4} } `"N/m"} {}

(c) 94.1 μm94.1 μm size 12{"94" "." 1`"μm"} {}

(d)2.47 Ω/m, 49.4 V/m

Integrated Concepts

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's 3.00×10−5T3.00×10−5T size 12{3 "." "00" times "10" rSup { size 8{ – 5} } `T} {} field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

Unreasonable Results

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's 5.00×10−5T5.00×10−5T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } `T} {} field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

(a) 571 C

(b) Impossible to have such a large separated charge on such a small object.

(c) The 1.00-N force is much too great to be realistic in the Earth's field.

Unreasonable Results

A charged particle having mass 6.64×10−27kg6.64×10−27kg size 12{6 "." "64" times "10" rSup { size 8{ – "27"} } `"kg"} {} (that of a helium atom) moving at 8.70×105m/s8.70×105m/s size 12{8 "." "70" times "10" rSup { size 8{5} } `"m/s"} {} perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Unreasonable Results

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's 5.00×10−5T5.00×10−5T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } `T} {} field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

(a) 2.40×106 m/s2.40×106 m/s size 12{2 "." "40" times "10" rSup { size 8{6} } " m" rSup { size 8{3} } "/s"} {}

(b) The speed is too high to be practical ≤≤ 1% speed of light

(c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

Unreasonable Results

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

Unreasonable Results

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a $5.00\times10^{-5}\,T$5.00×10−5T size 12{5 “.” “00” times “10” rSup { size 8{ – 5} } `T} {} field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

(a) 25.0 kA

(b) This current is unreasonably high. It implies a total power delivery in the line of 50.0×10^9 W, which is much too high for standard transmission lines.

(c)100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation

from
opposing
currents.
The
surveyor's
concerns
are
not
a
problem
for
his
magnetic
field
measurements.

Construct Your Own Problem

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

Construct Your Own Problem

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

Glossary

magnetic resonance imaging (MRI)
a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs
nuclear magnetic resonance (NMR)
a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms
magnetocardiogram (MCG)
a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)
a measurement of the brain's magnetic field

PART 16

Chapter 23 Electromagnetic Induction, AC Circuits, and Electrical Technologies

Introduction to Electromagnetic Induction, AC Circuits and Electrical Technologies

class=”introduction” class=”section-summary” title=”Section Summary”class=”conceptual-questions” title=”Conceptual Questions”class=”problems-exercises” title=”Problems & Exercises”

These wind turbines in the Thames Estuary in the UK are an example of induction at work. Wind pushes the blades of the turbine, spinning a shaft attached to magnets. The magnets spin around a conductive coil, inducing an electric current in the coil, and eventually feeding the electrical grid. (credit: modification of work by Petr Kratochvil)

Nature’s displays of symmetry are beautiful and alluring. A butterfly’s wings exhibit an appealing symmetry in a complex system. (See [link].) The laws of physics display symmetries at the most basic level—these symmetries are a source of wonder and imply deeper meaning. Since we place a high value on symmetry, we look for it when we explore nature. The remarkable thing is that we find it.

Physics, like this butterfly, has inherent symmetries. (credit: Thomas Bresson)

The hint of symmetry between electricity and magnetism found in the preceding chapter will be elaborated upon in this chapter. Specifically, we know that a current creates a magnetic field. If nature is symmetric here, then perhaps a magnetic field can create a current. The Hall effect is a voltage caused by a magnetic force. That voltage could drive a current. Historically, it was very shortly after Oersted discovered currents cause magnetic fields that other scientists asked the following question: Can magnetic fields cause currents? The answer was soon found by experiment to be yes. In 1831, some 12 years after Oersted's discovery, the English scientist Michael Faraday (1791–1862) and the American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating emfs (electromotive force) and, hence, currents with magnetic fields is known as induction; this process is also called magnetic induction to distinguish it from charging by induction, which utilizes the Coulomb force.

Today, currents induced by magnetic fields are essential to our technological society. The ubiquitous generator—found in automobiles, on bicycles, in nuclear power plants, and so on—uses magnetism to generate current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances. Not so familiar perhaps, but important nevertheless, is that the behavior of AC circuits depends strongly on the effect of magnetic fields on currents.

Glossary

induction
(magnetic induction) the creation of emfs and hence currents by magnetic fields

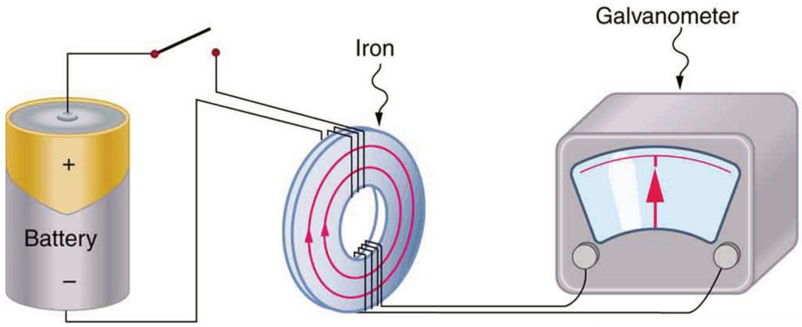
23.1 Induced Emf and Magnetic Flux

Induced Emf and Magnetic Flux

- Calculate the flux of a uniform magnetic field through a loop of arbitrary orientation.
- Describe methods to produce an electromotive force (emf) with a magnetic field or magnet and a loop of wire.

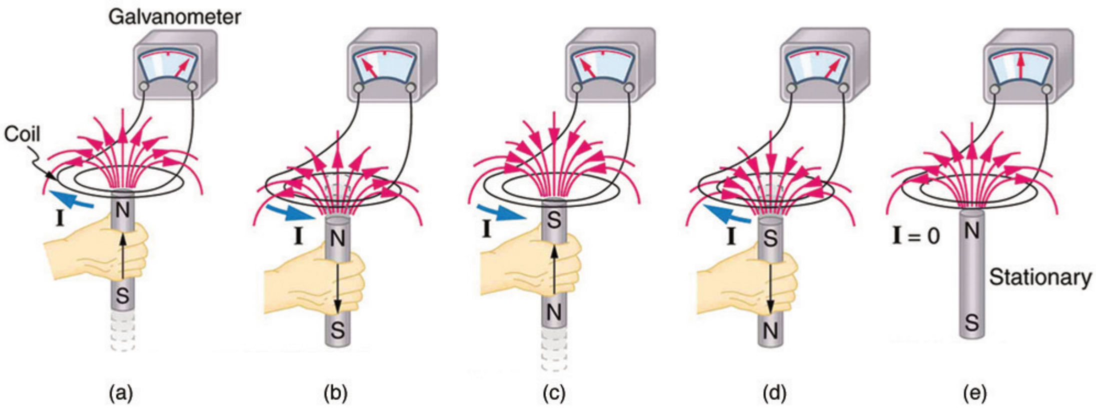
The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in [link]. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. (You can also observe this in a physics lab.) Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current. More basic than the current that flows is the emf*that causes it. The current is a result of an emf induced by a changing magnetic field, whether or not there is a path for current to flow.*

Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an emf and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows through the galvanometer when the switch remains closed or open.



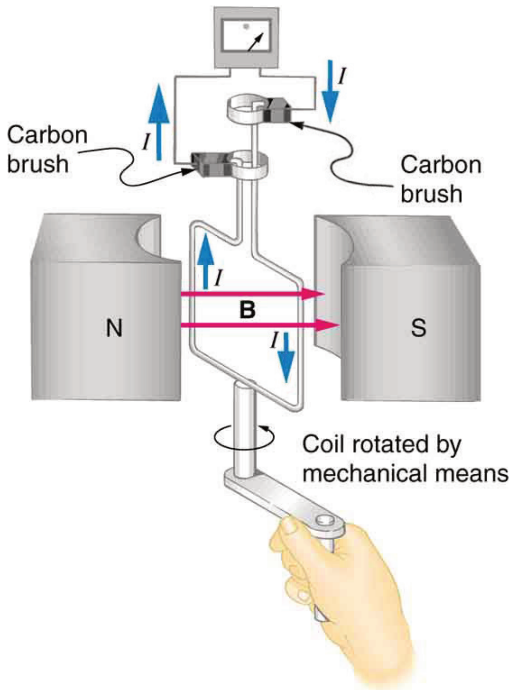
An experiment easily performed and often done in physics labs is illustrated in [link]. An emf is induced in the coil when a bar magnet is pushed in and out of it. Emfs of opposite signs are produced by motion in opposite directions, and the emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.

Movement of a magnet relative to a coil produces emfs as shown. The same emfs are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion.



The method of inducing an emf used in most electric generators is shown in [link]. A coil is rotated in a magnetic field, producing an alternating current emf, which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor (another symmetry).

Rotation of a coil in a magnetic field produces an emf. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

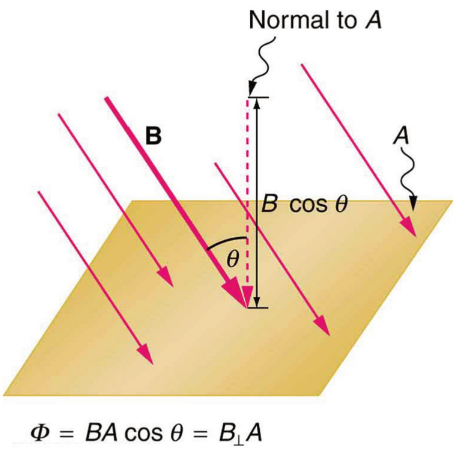


So we see that changing the magnitude or direction of a magnetic field produces an emf. Experiments revealed that there is a crucial quantity called the magnetic flux, ΦΦ size 12{Φ} {} , given by

Φ=BAcosθ,Φ=BAcosθ, size 12{Φ= ital “BA””cos”θ} {}

where *BB size 12{B} {}* is the magnetic field strength over an area *AA size 12{A} {}*, at an angle θθ with the perpendicular to the area as shown in [link]. **Any change in magnetic flux ΦΦ size 12{Φ} {} induces an emf.** This process is defined to be electromagnetic induction. Units of magnetic flux *ΦΦ size 12{Φ} {}* are T?m2T?m2 size 12{T cdot m rSup { size 8{2} } } {}. As seen in [link], Bcosθ=BBcosθ=B size 12{B”cos”θ=B rSub { size 8{ ortho } } } {}?, which is the component of *BB size 12{B} {}* perpendicular to the area *AA size 12{A} {}*. Thus magnetic flux is Φ=B?AΦ=B?A size 12{Φ=B rSub { size 8{ ortho } } A} {}, the product of the area and the component of the magnetic field perpendicular to it.

Magnetic flux ΦΦ size 12{Φ} {} is related to the magnetic field and the area over which it exists. The flux Φ=BAcosθΦ=BAcosθ size 12{Φ= ital “BA””cos”θ} {} is related to induction; any change in ΦΦ size 12{Φ} {} induces an emf.



All induction, including the examples given so far, arises from some change in magnetic flux *ΦΦ size 12{Φ} {}*. For example, Faraday changed *BB size 12{B} {}* and hence *ΦΦ size 12{Φ} {}* when opening and closing the switch in his apparatus (shown in [link]). This is also true for the bar magnet and coil shown in [link]. When rotating the coil of a generator, the angle θθ size 12{θ} {} and, hence, *ΦΦ size 12{Φ} {}* is changed. Just how great an emf and what direction it takes depend on the change in *ΦΦ size 12{Φ} {}* and how rapidly the change is made, as examined in the next section.

Section Summary

- The crucial quantity in induction is magnetic flux *ΦΦ size 12{Φ} {}*, defined to be Φ=BAcosθΦ=BAcosθ size 12{Φ= ital "BA""cos"θ} {}, where *BB size 12{B} {}* is the magnetic field strength over an area *AA size 12{A} {}* at an angle θθ size 12{θ} {} with the perpendicular to the area.
- Units of magnetic flux *ΦΦ size 12{Φ} {}* are T?m2T?m2 size 12{T cdot m rSup { size 8{2} } } {}.
- Any change in magnetic flux *ΦΦ size 12{Φ} {}* induces an emf—the process is defined to be electromagnetic induction.

Conceptual Questions

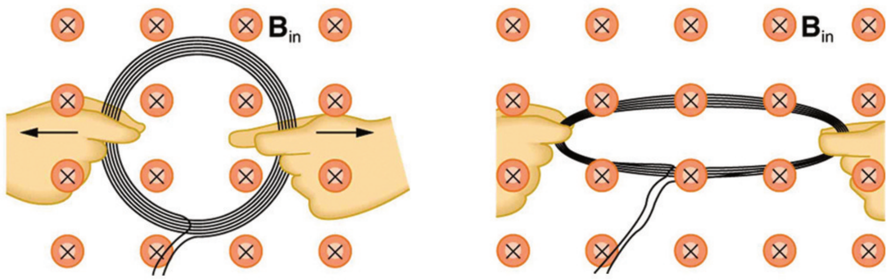
How do the multiple-loop coils and iron ring in the version of Faraday's apparatus shown in [link] enhance the observation of induced emf?

When a magnet is thrust into a coil as in [link](a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

Explain how magnetic flux can be zero when the magnetic field is not zero.

Is an emf induced in the coil in [link] when it is stretched? If so, state why and give the direction of the induced current.

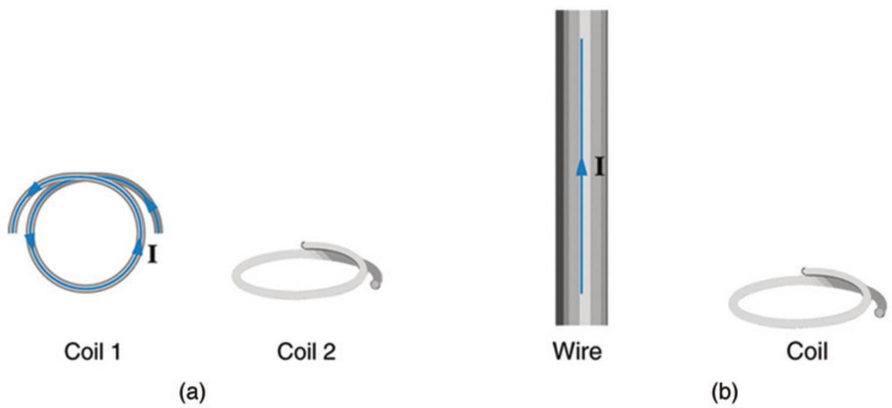
A circular coil of wire is stretched in a magnetic field.



Problems & Exercises

What is the value of the magnetic flux at coil 2 in [link] due to coil 1?

(a) The planes of the two coils are perpendicular. (b) The wire is perpendicular to the plane of the coil.



Zero

What is the value of the magnetic flux through the coil in [link](b) due to the wire?

Glossary

magnetic flux
the amount of magnetic field going through a particular area, calculated with Φ=BAcosθΦ=BAcosθ size 12{Φ= ital "BA""cos"θ} {} where BB size 12{B} {} is the magnetic field strength over an area AA size 12{A} {} at an angle θθ size 12{θ} {} with the perpendicular to the area
electromagnetic induction
the process of inducing an emf (voltage) with a change in magnetic flux

23.2 Faraday's Law of Induction: Lenz's Law

Faraday's Law of Induction: Lenz's Law

- Calculate emf, current, and magnetic fields using Faraday's Law.
- Explain the physical results of Lenz's Law

Faraday's and Lenz's Law

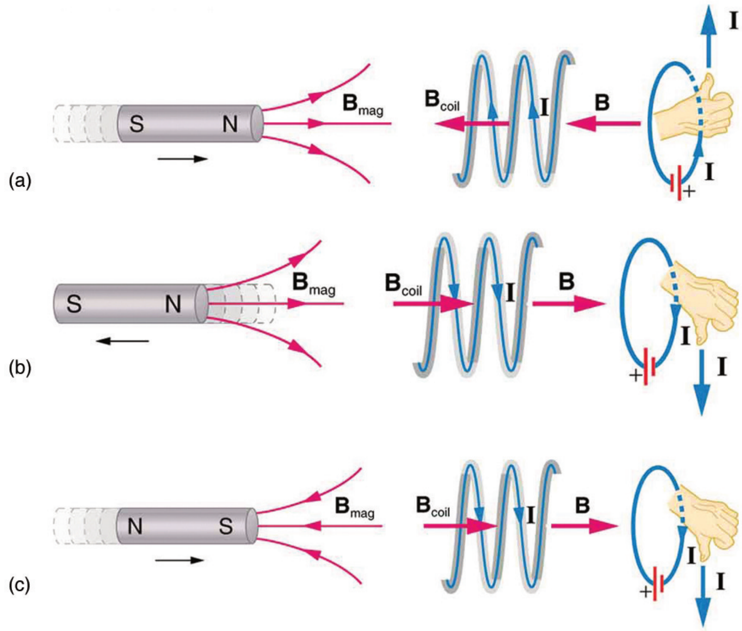
Faraday's experiments showed that the emf *induced by a change in magnetic flux depends on only a few factors. First, emf is directly proportional to the change in flux ΔΦΔΦ size 12{ΔΦ} {}. Second, emf is greatest when the change in time ΔtΔt size 12{Δt} {} is smallest—that is, emf is inversely proportional to ΔtΔt size 12{Δt} {}. Finally, if a coil has NN turns, an emf will be produced that is NN size 12{N} {} times greater than for a single coil, so that emf is directly proportional to NN size 12{N} {}. The equation for the emf induced by a change in magnetic flux is*

emf=−NΔΦΔt.emf=−NΔΦΔt. size 12{"emf"= – N { {ΔΦ} over {Δt} } } {}

This relationship is known as Faraday's law of induction. The units for emf are volts, as is usual.

The minus sign in Faraday's law of induction is very important. The minus means that *the emf creates a current I and magnetic field B that oppose the change in flux ΔΦΔΦ size 12{ΔΦ} {}—this is known as Lenz's law*. The direction (given by the minus sign) of the emf*is so important that it is called Lenz's law after the Russian Heinrich Lenz (1804–1865), who, like Faraday and Henry,****independently investigated aspects of induction. Faraday was aware of the direction, but Lenz stated it so clearly that he is credited for its discovery. (See [link].)***

(a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of *Lenz's law—induction opposes any change in flux*. (b) and (c) are two other situations. Verify for yourself that the direction of the induced BcoilBcoil size 12{B rSub { size 8{"coil"} } } {} shown indeed opposes the change in flux and that the current direction shown is consistent with RHR-2.



Problem-Solving Strategy for Lenz's Law

To use Lenz's law to determine the directions of the induced magnetic fields, currents, and emfs:

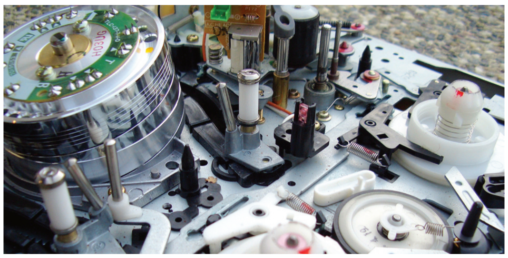
1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the magnetic field B.
3. Determine whether the flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field B. It opposes the *change* in flux by adding or subtracting from the original field.
5. Use RHR-2 to determine the direction of the induced current I that is responsible for the induced magnetic field B.
6. The direction (or polarity) of the induced emf will now drive a current in this direction and can be represented as current emerging from the positive terminal of the emf and returning to its negative terminal.

For practice, apply these steps to the situations shown in [link] and to others that are part of the following text material.

Applications of Electromagnetic Induction

There are many applications of Faraday's Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape, coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet ([link]). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces an emf in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.

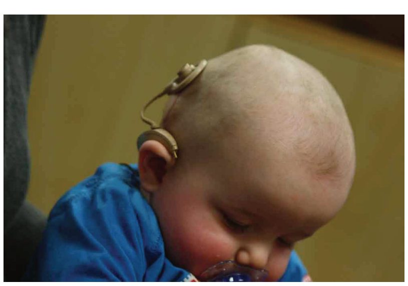
Recording and playback heads used with audio and video magnetic tapes. (credit: Steve Jurvetson)



Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0's or 1's are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.

Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)



Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic simulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea (“the cessation of breath”) affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant’s chest has an alternating current running through it. The expansion and contraction of the infant’s chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Making Connections: Conservation of Energy

Lenz’s law is a manifestation of the conservation of energy. The induced emf produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz’s law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced emf were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

Calculating Emf: How Great Is the Induced Emf?

Calculate the magnitude of the induced emf when the magnet in [link](a) is thrust into the coil, given the following information: the single loop coil has a radius of 6.00 cm and the average value of $B\cos\theta$$B\cos\theta$ size 12{B”cos”θ} {} (this is given, since the bar magnet’s field is complex) increases from 0.0500 T to 0.250 T in 0.100 s.

Strategy

To find the *magnitude* of emf, we use Faraday’s law of induction as stated by $\text{emf}=-N\frac{\Delta\Phi}{\Delta t}$$\text{emf}=-N\frac{\Delta\Phi}{\Delta t}$, but without the minus sign that indicates direction:

$\text{emf}=N\frac{\Delta\Phi}{\Delta t}.$$\text{emf}=N\frac{\Delta\Phi}{\Delta t}.$

Solution

We are given that $N=1$$N=1$ size 12{N=1} {} and $\Delta t=0.100\text{ s}$$\Delta t=0.100\text{ s}$, but we must determine the change in flux $\Delta\Phi$$\Delta\Phi$ size 12{ΔΦ} {} before we can find emf. Since the area of the loop is fixed, we see that

$\Delta\Phi=\Delta(BA\cos\theta)=A\Delta(B\cos\theta).$$\Delta\Phi=\Delta(BA\cos\theta)=A\Delta(B\cos\theta).$ size 12{ΔΦ=Δ (BA”cos”θ) =AΔ (B”cos”θ) } {}

Now $\Delta(B\cos\theta)=0.200\text{ T}$$\Delta(B\cos\theta)=0.200\text{ T}$ size 12{Δ (B”cos”θ) =0 “.” “200”`T} {}, since it was given that $B\cos\theta$$B\cos\theta$ size 12{B”cos”θ} {} changes from 0.0500 to 0.250 T. The area of the loop is $A=\pi r^2=(3.14\ldots)(0.060\text{ m})^2=1.13\times10^{-2}\text{ m}^2$$A=\pi r^2=(3.14\ldots)(0.060\text{ m})^2=1.13\times10^{-2}\text{ m}^2$ size 12{A=πr rSup { size 8{2} } = (3 “.” “14” “.”

"." ".") (0 "." "060"`m ) rSup { size 8{2} } =1 "." "13" times "10" rSup { size 8{ – 2} } `m rSup { size 8{2} } } {}. Thus,

$\Delta\Phi=(1.13\times10^{-2}\text{ m}^2)(0.200\text{ T}).\Delta\Phi=(1.13\times10^{-2}\text{ m}^2)(0.200\text{ T})$. size 12{ΔΦ= (1 "." "13" times "10" rSup { size 8{ – 2} } " m" rSup { size 8{2} }) (0 "." "200"" T") } {}

Entering the determined values into the expression for emf gives

$\text{Emf}=N\frac{\Delta\Phi}{\Delta t}=\frac{(1.13\times10^{-2}\text{ m}^2)(0.200\text{T})}{0.100\text{s}}=22.6\text{mV}.\text{Emf}=N\frac{\Delta\Phi}{\Delta t}=\frac{(1.13\times10^{-2}\text{ m}^2)(0.200\text{T})}{0.100\text{s}}=22.6\text{mV}.$ size 12{E=N { {ΔΦ} over {Δt} } = { { (1 "." "13" times "10" rSup { size 8{ – 2} } " m" rSup { size 8{2} }) (0 "." "200"" T") } over {0 "." "100"" s"} } ="22″ "." 6″ mV"} {}

Discussion

While this is an easily measured voltage, it is certainly not large enough for most practical applications. More loops in the coil, a stronger magnet, and faster movement make induction the practical source of voltages that it is.

PhET Explorations: Faraday's Electromagnetic Lab

Play with a bar magnet and coils to learn about Faraday's law. Move a bar magnet near one or two coils to make a light bulb glow. View the magnetic field lines. A meter shows the direction and magnitude of the current. View the magnetic field lines or use a meter to show the direction and magnitude of the current. You can also play with electromagnets, generators and transformers!

Faraday's Electromagnetic Lab

PhET Interactive Simulation

Section Summary

- Faraday's law of induction states that the emf*induced by a change in magnetic flux is* $\text{emf}=-N\frac{\Delta\Phi}{\Delta t}\text{emf}=-N\frac{\Delta\Phi}{\Delta t}$ *size 12{"emf"= – N { {ΔΦ} over {Δt} } } {}when flux changes by* $\Delta\Phi\Delta\Phi$ *size 12{ΔΦ} {} in a time* $\Delta t\Delta t$ *size 12{Δt} {}.*
- If emf is induced in a coil, NN is its number of turns.
- The minus sign means that the emf *creates a current* II *size 12{I} {} and magnetic field* BB *size 12{B} {}*

that oppose the change in flux ΔΦΔΦ size 12{ΔΦ} {} —this opposition is known as Lenz's law.

Conceptual Questions

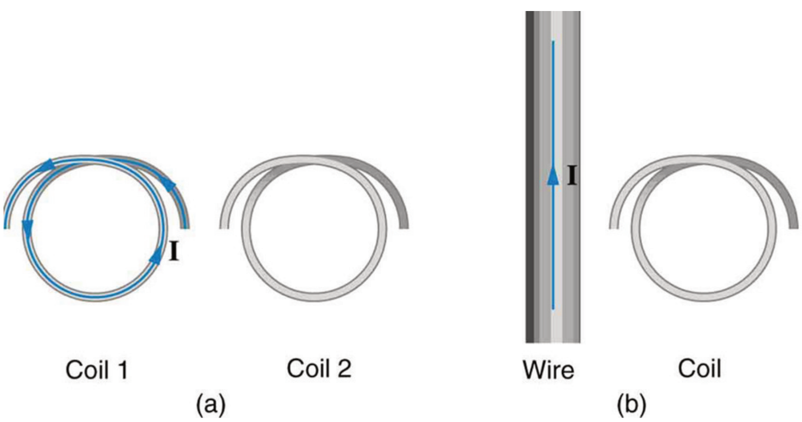
A person who works with large magnets sometimes places her head inside a strong field. She reports feeling dizzy as she quickly turns her head. How might this be associated with induction?

A particle accelerator sends high-velocity charged particles down an evacuated pipe. Explain how a coil of wire wrapped around the pipe could detect the passage of individual particles. Sketch a graph of the voltage output of the coil as a single particle passes through it.

Problems & Exercises

Referring to [link](a), what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant? Explicitly show how you follow the steps in the Problem-Solving Strategy for Lenz's Law.

(a) The coils lie in the same plane. (b) The wire is in the plane of the coil



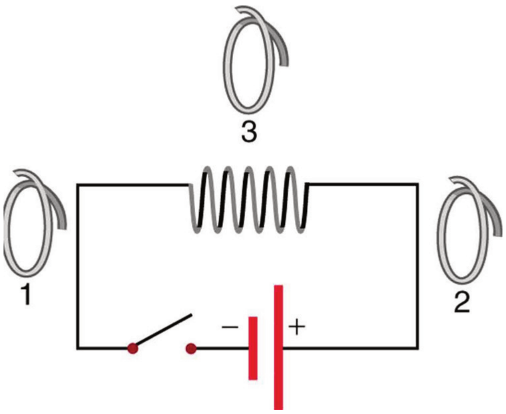
(a) CCW

(b) CW

(c) No current induced

Referring to [link](b), what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction? Explicitly show how you follow the steps in the Problem-Solving Strategy for Lenz's Law.

Referring to [link], what are the directions of the currents in coils 1, 2, and 3 (assume that the coils are lying in the plane of the circuit): (a) When the switch is first closed? (b) When the switch has been closed for a long time? (c) Just after the switch is opened?



(a) 1 CCW, 2 CCW, 3 CW

(b) 1, 2, and 3 no current induced

(c) 1 CW, 2 CW, 3 CCW

Repeat the previous problem with the battery reversed.

Verify that the units of $\Delta\Phi\Delta\Phi$ size 12{ΔΦ} {}/ $\Delta t\Delta t$ size 12{Δt} {} are volts. That is, show that 1T?m2/s=1 V1T?m2/s=1 V size 12{1`T cdot m rSup { size 8{2} } /s=1`V} {}.

Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of
0.250m20.250m2.
It is stretched to have no area in 0.100 s. What is the direction and magnitude of the induced emf if the uniform magnetic field has a strength of 1.50 T?

(a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) Discuss whether this current would significantly change the temperature of the ring.

(a) 3.04 mV

(b) As a lower limit on the ring, estimate R = 1.00 mΩ. The heat transferred will be 2.31 mJ. This is not a significant amount of heat.

Integrated Concepts

Referring to the situation in the previous problem: (a) What current is induced in the ring if its resistance is 0.0100 ΩΩ size 12{ %OMEGA } {}? (b) What average power is dissipated? (c) What magnetic field is induced at the center of the ring? (d) What is the direction of the induced magnetic field relative to the MRI's field?

An emf is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's 5.00×10−5T5.00×10−5T size 12{5

"." "00" times "10" rSup { size 8{ – 5} } `T} {} magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

0.157 V

A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average emf of 10,000 V.

Integrated Concepts

Approximately how does the emf induced in the loop in [link](b) depend on the distance of the center of the loop from the wire?

proportional to 1 r1 r size 12{ { {1} over { ital " r"} } } {}

Integrated Concepts

(a) A lightning bolt produces a rapidly varying magnetic field. If the bolt strikes the earth vertically and acts like a current in a long straight wire, it will induce a voltage in a loop aligned like that in [link](b). What voltage is induced in a 1.00 m diameter loop 50.0 m from a 2.00×106A2.00×106A size 12{2 "." "00" times "10" rSup { size 8{6} } `A} {} lightning strike, if the current falls to zero in
25.0 μs25.0 μs?
(b) Discuss circumstances under which such a voltage would produce noticeable consequences.

Glossary

Faraday's law of induction
the means of calculating the emf in a coil due to changing magnetic flux, given by emf=−NΔΦΔtemf=−NΔΦΔt size 12{"emf"= – N { {ΔΦ} over {Δt} } } {}
Lenz's law
the minus sign in Faraday's law, signifying that the emf induced in a coil opposes the change in magnetic flux

23.3 Motional Emf

Motional Emf

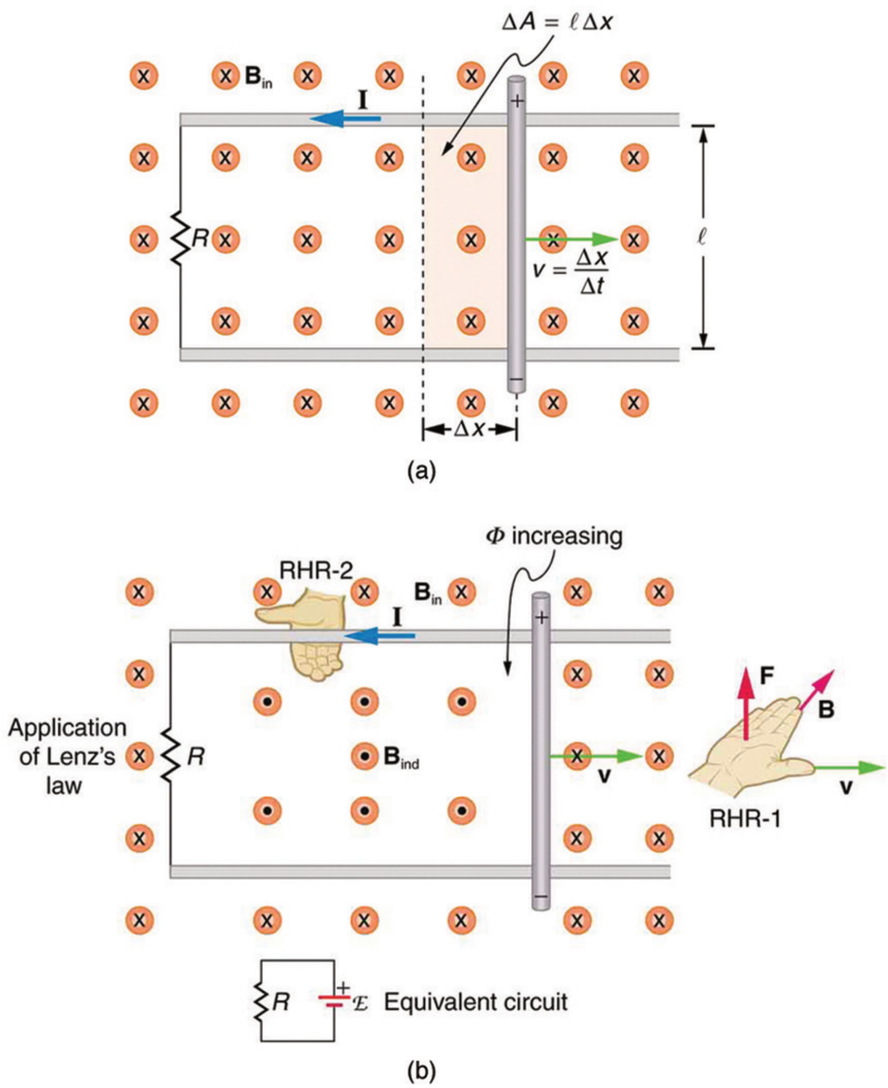
- Calculate emf, force, magnetic field, and work due to the motion of an object in a magnetic field.

As we have seen, any change in magnetic flux induces an emf opposing that change—a process known as induction. Motion is one of the major causes of induction. For example, a magnet moved toward a coil induces an emf, and a coil moved toward a magnet produces a similar emf. In this section, we concentrate on motion in a magnetic field that is stationary relative to the Earth, producing what is loosely called *motional emf*.

One situation where motional emf occurs is known as the Hall effect and has already been examined. Charges moving in a magnetic field experience the magnetic force F=qvBsinθF=qvBsinθ size 12{F= ital "qvB""sin"θ} {}, which moves opposite charges in opposite directions and produces an emf=Bℓvemf=Bℓv size 12{"emf"=Bℓv} {}. We saw that the Hall effect has applications, including measurements of BB size 12{B} {} and vv size 12{v} {}. We will now see that the Hall effect is one aspect of the broader phenomenon of induction, and we will find that motional emf can be used as a power source.

Consider the situation shown in [link]. A rod is moved at a speed
vv
along a pair of conducting rails separated by a distance
ℓℓ
in a uniform magnetic field BB size 12{B} {}. The rails are stationary relative to BB size 12{B} {} and are connected to a stationary resistor RR size 12{R} {}. The resistor could be anything from a light bulb to a voltmeter. Consider the area enclosed by the moving rod, rails, and resistor. BB size 12{B} {} is perpendicular to this area, and the area is increasing as the rod moves. Thus the magnetic flux enclosed by the rails, rod, and resistor is increasing. When flux changes, an emf *is induced according to Faraday's law of induction.*

(a) A motional emf=Bℓvemf=Bℓv size 12{"emf"=Bℓv} {} is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field BB size 12{B} {} is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them. (b) Lenz's law gives the directions of the induced field and current, and the polarity of the induced emf. Since the flux is increasing, the induced field is in the opposite direction, or out of the page. RHR-2 gives the current direction shown, and the polarity of the rod will drive such a current. RHR-1 also indicates the same polarity for the rod. (Note that the script E symbol used in the equivalent circuit at the bottom of part (b) represents emf.)



To find the magnitude of emf induced along the moving rod, we use Faraday's law of induction without the sign:

emf=NΔΦΔt.emf=NΔΦΔt. size 12{"emf"=N { {ΔΦ} over {Δt} } } {}

Here and below, "emf" implies the magnitude of the emf. In this equation, N=1N=1 size 12{N=1} {} and the flux Φ=BAcosθΦ=BAcosθ size 12{Φ= ital "BA""cos"θ} {}. We have θ=0ºθ=0º
and cosθ=1cosθ=1, since BB is perpendicular to AA.
Now ΔΦ=Δ(BA)=BΔAΔΦ=Δ(BA)=BΔA size 12{ΔΦ=Δ (ital "BA") =BΔA} {}, since BB size 12{B} {} is uniform. Note that the area swept out by the rod is ΔA=ℓΔxΔA=ℓΔx size 12{ΔA=ℓΔx} {}. Entering these quantities into the expression for *emf yields*

emf=BΔAΔt=BℓΔxΔt.emf=BΔAΔt=BℓΔxΔt. size 12{"emf"= { {BΔA} over {Δt} } =B { {ℓΔx} over {Δt} } } {}

Finally, note that Δx/Δt=vΔx/Δt=v size 12{Δx/Δt=v} {}, *the velocity of the rod. Entering this into the last expression shows that* emf=Bℓv(B,ℓ, andv perpendicular)emf=Bℓv(B,ℓ, andv perpendicular) size 12{“emf”=Bℓv} {}

is the motional emf. This is the same expression given for the Hall effect previously.

Making Connections: Unification of Forces

There are many connections between the electric force and the magnetic force. The fact that a moving electric field produces a magnetic field and, conversely, a moving magnetic field produces an electric field is part of why electric and magnetic forces are now considered to be different manifestations of the same force. This classic unification of electric and magnetic forces into what is called the electromagnetic force is the inspiration for contemporary efforts to unify other basic forces.

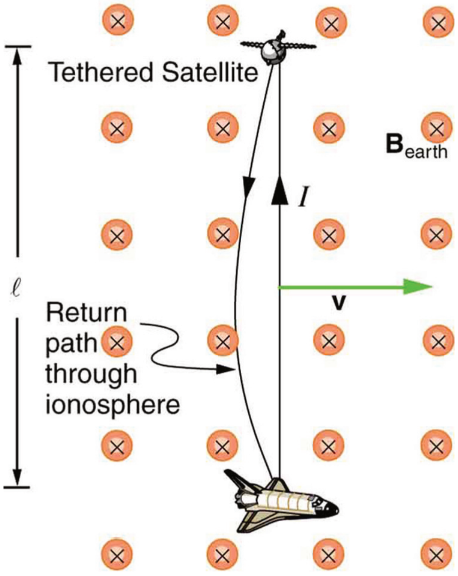
To find the direction of the induced field, the direction of the current, and the polarity of the induced emf, we apply Lenz’s law as explained in Faraday’s Law of Induction: Lenz’s Law. (See [link](b).) Flux is increasing, since the area enclosed is increasing. Thus the induced field must oppose the existing one and be out of the page. And so the RHR-2 requires that *I* be counterclockwise, which in turn means the top of the rod is positive as shown.

Motional emf also occurs if the magnetic field moves and the rod (or other object) is stationary relative to the Earth (or some observer). We have seen an example of this in the situation where a moving magnet induces an emf in a stationary coil. It is the relative motion that is important. What is emerging in these observations is a connection between magnetic and electric fields. A moving magnetic field produces an electric field through its induced emf. We already have seen that a moving electric field produces a magnetic field—moving charge implies moving electric field and moving charge produces a magnetic field.

Motional emfs in the Earth’s weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1 m rod moving at 3.0 m/s perpendicular to the Earth’s field gives emf=Bℓv=(5.0×10−5T)(1.0 m)(3.0 m/s)=150 μVemf=Bℓv=(5.0×10−5T)(1.0 m)(3.0 m/s)=150 μV size 12{“emf”=Bℓv= (5 “.” 0 times “10” rSup { size 8{ – 5} } T) (1 “.” 0`m ) ( 3 “.” 0`”m/s”) =”150″`”μV”} {}. This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The Tethered Satellite was to be let out on a 20 km length of wire as shown in [link], to create a 5 kV emf by moving at orbital speed through the Earth’s field. This emf could be used to convert some of the shuttle’s kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path for the current to flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in [link], without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force F=IℓBsinθF=IℓBsinθ size 12{F=IℓB”sin”θ} {} does the work that reduces the shuttle’s kinetic and potential energy and allows it to be converted to electrical energy. The tests were both unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. [link] indicates feasibility in principle.

Calculating the Large Motional Emf of an Object in Orbit

Motional emf as electrical power conversion for the space shuttle is the motivation for the Tethered Satellite experiment. A 5 kV emf was predicted to be induced in the 20 km long tether while moving at orbital speed in the Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.



Calculate the motional emf induced along a 20.0 km long conductor moving at an orbital speed of 7.80 km/s perpendicular to the Earth's 5.00×10^{-5} T magnetic field.

Strategy

This is a straightforward application of the expression for motional emf— $\text{emf}=B\ell v$.

Solution

Entering the given values into $\text{emf}=B\ell v$ gives

$$\begin{aligned}\text{emf} &= B\ell v\\ &= (5.00\times10^{-5}\ \text{T})(2.0\times10^{4}\ \text{m})(7.80\times10^{3}\ \text{m/s})\\ &= 7.80\times10^{3}\ \text{V}.\end{aligned}$$

Discussion

The value obtained is greater than the 5 kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to the Earth's field. The 7.80 kV value is the maximum emf obtained when $\theta=90^\circ$ and $\sin\theta=1$.

Section Summary

- An emf induced by motion relative to a magnetic field
  BB
  is called a *motional emf* and is given by
  emf=Bℓv(B, ℓ, andv perpendicular),emf=Bℓv(B, ℓ, andv perpendicular), size 12{“emf”=Bℓv} {}

  where ℓℓ size 12{ℓ} {} is the length of the object moving at speed vv size 12{v} {} relative to the field.

Conceptual Questions

Why must part of the circuit be moving relative to other parts, to have usable motional emf? Consider, for example, that the rails in [link] are stationary relative to the magnetic field, while the rod moves.

A powerful induction cannon can be made by placing a metal cylinder inside a solenoid coil. The cylinder is forcefully expelled when solenoid current is turned on rapidly. Use Faraday's and Lenz's laws to explain how this works. Why might the cylinder get live/hot when the cannon is fired?

An induction stove heats a pot with a coil carrying an alternating current located beneath the pot (and without a hot surface). Can the stove surface be a conductor? Why won't a coil carrying a direct current work?

Explain how you could thaw out a frozen water pipe by wrapping a coil carrying an alternating current around it. Does it matter whether or not the pipe is a conductor? Explain.

Problems & Exercises

Use Faraday's law, Lenz's law, and RHR-1 to show that the magnetic force on the current in the moving rod in [link] is in the opposite direction of its velocity.

If a current flows in the Satellite Tether shown in [link], use Faraday's law, Lenz's law, and RHR-1 to show that there is a magnetic force on the tether in the direction opposite to its velocity.

(a) A jet airplane with a 75.0 m wingspan is flying at 280 m/s. What emf is induced between wing tips if the vertical component of the Earth's field is 3.00×10−5T3.00×10−5T size 12{3 "." "00" times "10" rSup { size 8{ – 5} } `T} {}? (b) Is an emf of this magnitude likely to have any consequences? Explain.

(a) 0.630 V

(b) No, this is a very small emf.

(a) A nonferrous screwdriver is being used in a 2.00 T magnetic field. What maximum emf can be induced along its 12.0 cm length when it moves at 6.00 m/s? (b) Is it likely that this emf will have any consequences or even be noticed?

At what speed must the sliding rod in [link] move to produce an emf of 1.00 V in a 1.50 T field, given the rod's length is 30.0 cm?

2.22 m/s

The 12.0 cm long rod in [link] moves at 4.00 m/s. What is the strength of the magnetic field if a 95.0 V emf is induced?

Prove that when BB size 12{B} {}, ℓℓ size 12{ℓ} {}, and vv size 12{v} {} are not mutually perpendicular, motional emf is given by *emf=Bℓvsinθemf=Bℓvsinθ size 12{"emf"=Bℓv"sin"θ} {}. If vv size 12{v} {} is perpendicular to BB size 12{B} {}, then θθ size 12{θ} {} is the angle between ℓℓ size 12{ℓ} {} and BB size 12{B} {}. If ℓℓ size 12{ℓ} {} is perpendicular to BB size 12{B} {}, then θθ size 12{θ} {} is the angle between vv size 12{v} {} and BB size 12{B} {}.*

In the August 1992 space shuttle flight, only 250 m of the conducting tether considered in [link] could be let out. A 40.0 V motional emf was generated in the Earth's 5.00×10−5T5.00×10−5T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } `T} {} field, while moving at 7.80×103m/s7.80×103m/s size 12{7 "." "80" times "10" rSup { size 8{3} } `"m/s"} {}. What was the angle between the shuttle's velocity and the Earth's field, assuming the conductor was perpendicular to the field?

Integrated Concepts

Derive an expression for the current in a system like that in [link], under the following conditions. The resistance between the rails is
RR, the rails and the moving rod are identical in cross section
AA and have the same resistivity
ρρ. The distance between the rails is l, and the rod moves at constant speed
vv perpendicular to the uniform field
BB. At time zero, the moving rod is next to the resistance RR.

Integrated Concepts

The Tethered Satellite in [link] has a mass of 525 kg and is at the end of a 20.0 km long, 2.50 mm diameter cable with the tensile strength of steel. (a) How much does the cable stretch if a 100 N force is exerted to pull the satellite in? (Assume the satellite and shuttle are at the same altitude above the Earth.) (b) What is the effective force constant of the cable? (c) How much energy is stored in it when stretched by the 100 N force?

Integrated Concepts

The Tethered Satellite discussed in this module is producing 5.00 kV, and a current of 10.0 A flows. (a) What magnetic drag force does this produce if the system is moving at 7.80 km/s? (b) How much kinetic energy is removed from the system in 1.00 h, neglecting any change in altitude or velocity during that time? (c) What is the change in velocity if the mass of the system is 100,000 kg? (d) Discuss the long term consequences (say, a week-long mission) on the space shuttle's orbit, noting what effect a decrease in velocity has and assessing the magnitude of the effect.

(a) 10.0 N

(b) 2.81×108J2.81×108J

(c) 0.36 m/s

(d) For a week-long mission (168 hours), the change in velocity will be 60 m/s, or approximately 1%. In general, a decrease in velocity would cause the orbit to start spiraling inward because the velocity would no longer be sufficient to keep the circular orbit. The long-term consequences are that the shuttle would require a little more fuel to maintain the desired speed, otherwise the orbit would spiral slightly inward.

23.4 Eddy Currents and Magnetic Damping

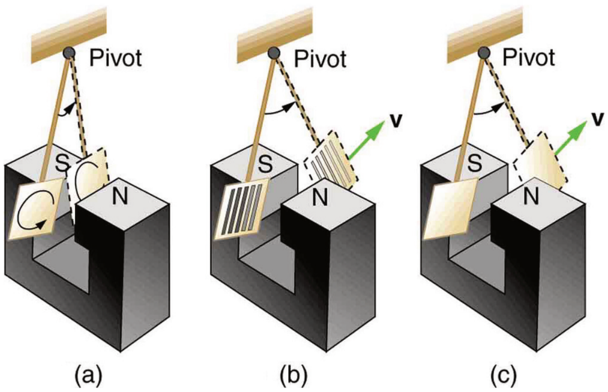
Eddy Currents and Magnetic Damping

- Explain the magnitude and direction of an induced eddy current, and the effect this will have on the object it is induced in.
- Describe several applications of magnetic damping.

Eddy Currents and Magnetic Damping

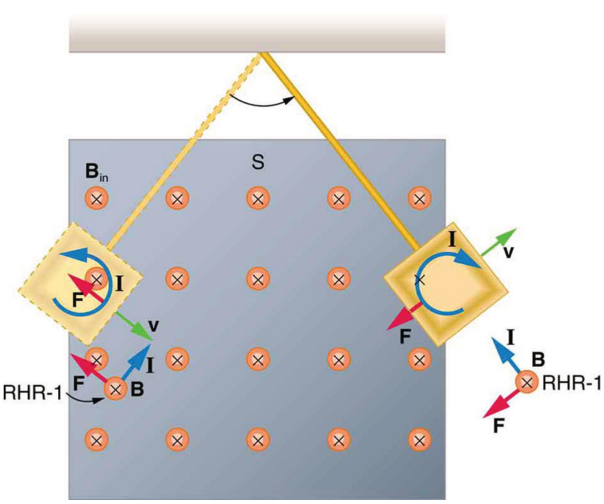
As discussed in Motional Emf, motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current loop in the conductor, we refer to that current as an eddy current. Eddy currents can produce significant drag, called magnetic damping, on the motion involved. Consider the apparatus shown in [link], which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics lab activity.) If the bob is metal, there is significant drag on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in [link](b), there is a much smaller effect due to the magnet. There is no discernible effect on a bob made of an insulator. Why is there drag in both directions, and are there any uses for magnetic drag?

A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents. (b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.



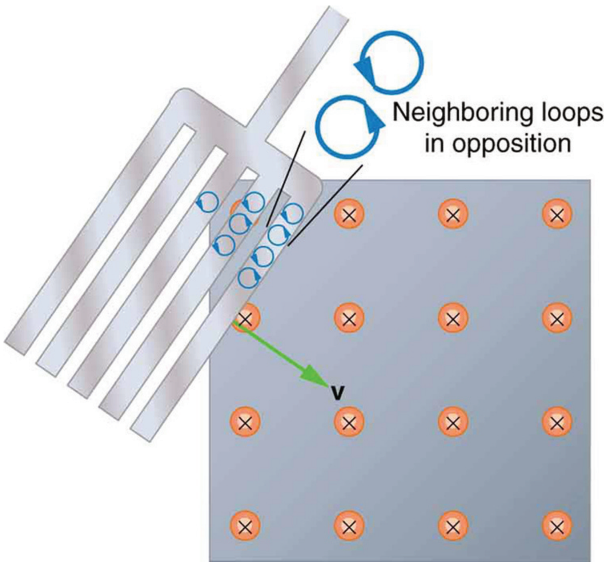
[link] shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, and so an eddy current is set up (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.

A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.



When a slotted metal plate enters the field, as shown in [link], an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, and so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they can be slotted or constructed of thin layers of conducting material separated by insulating sheets.

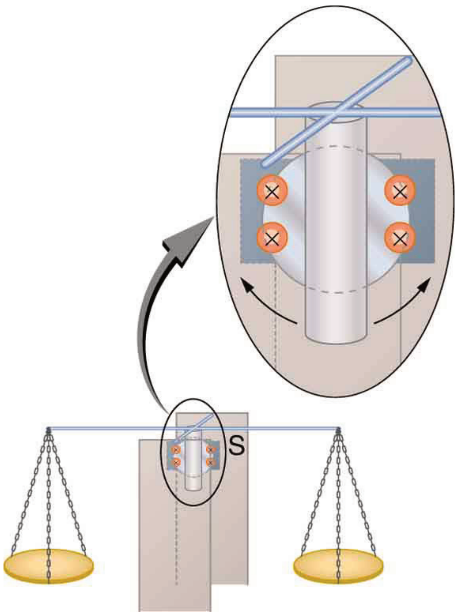
Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to cancel, thereby making magnetic drag almost zero.



Applications of Magnetic Damping

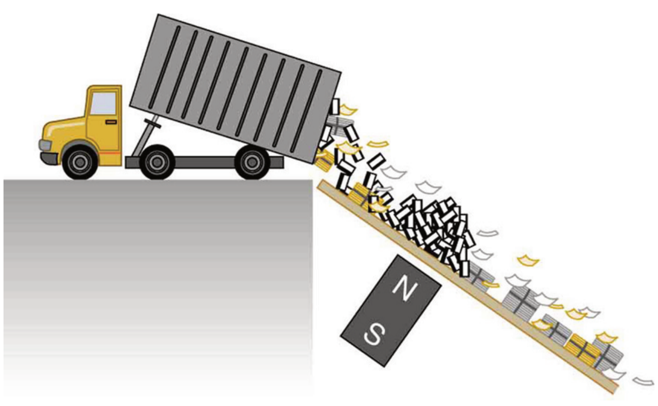
One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive. (See [link].) In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.

Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.



Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals. (See [link].) This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.

Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.



Other major applications of eddy currents are in metal detectors and braking systems in trains and roller coasters. Portable metal detectors ([link]) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current will be induced in a piece of metal close to the detector which will cause a change in the induced current within the secondary coil, leading to some sort of signal like a shrill noise. Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the force produced decreases with speed. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare earth magnets such as neodymium magnets are used in roller coasters. [link] shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) which pass through the magnetic field slowing the vehicle down in much the same way as with the pendulum bob shown in [link].

A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)

The rows of rare earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer, Wikimedia Commons)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times but the base of the pot needs to be ferromagnetic, iron or steel for induction to work.

Section Summary

- Current loops induced in moving conductors are called eddy currents.
- They can create significant drag, called magnetic damping.

Conceptual Questions

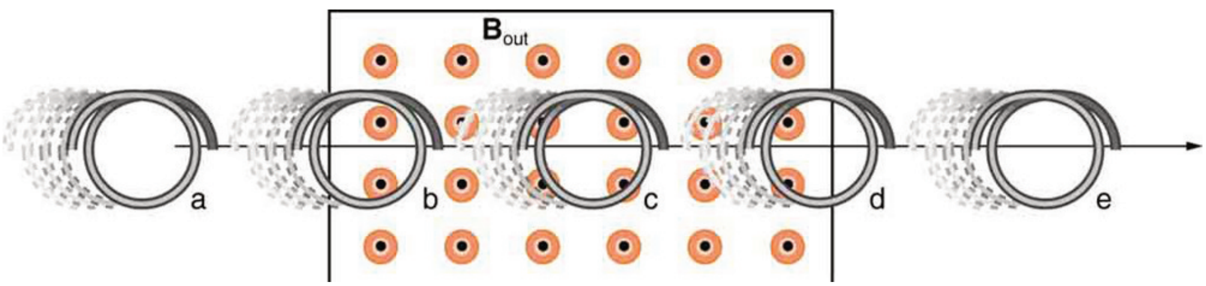
Explain why magnetic damping might not be effective on an object made of several thin conducting layers separated by insulation.

Explain how electromagnetic induction can be used to detect metals? This technique is particularly important in detecting buried landmines for disposal, geophysical prospecting and at airports.

Problems & Exercises

Make a drawing similar to [link], but with the pendulum moving in the opposite direction. Then use Faraday's law, Lenz's law, and RHR-1 to show that magnetic force opposes motion.

A coil is moved into and out of a region of uniform magnetic field.



A coil is moved through a magnetic field as shown in [link]. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?

Glossary

eddy current
a current loop in a conductor caused by motional emf
magnetic damping
the drag produced by eddy currents

23.5 Electric Generators

Electric Generators

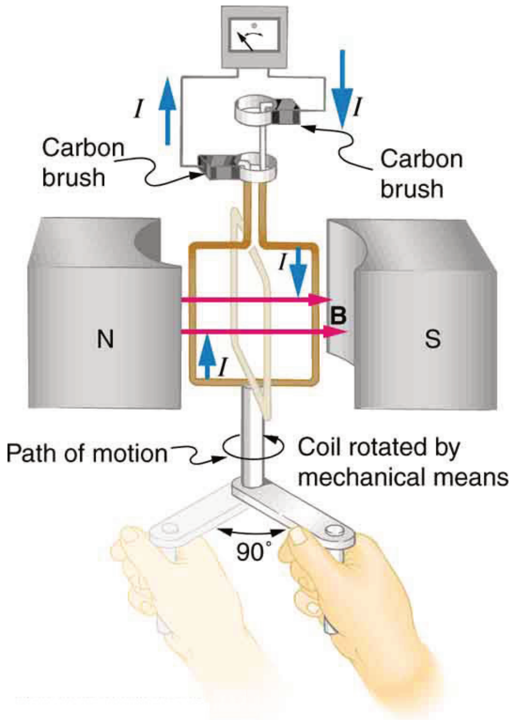
- Calculate the emf induced in a generator.
- Calculate the peak emf which can be induced in a particular generator system.

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in Induced Emf and Magnetic Flux. We will now explore generators in more detail. Consider the following example.

Calculating the Emf Induced in a Generator Coil

The generator coil shown in [link] is rotated through one-fourth of a revolution (from $\theta=0^\circ$ to $\theta=90^\circ$) in 15.0 ms. The 200-turn circular coil has a 5.00 cm radius and is in a uniform 1.25 T magnetic field. What is the average emf induced?

When this generator coil is rotated through one-fourth of a revolution, the magnetic flux Φ changes from its maximum to zero, inducing an emf.



Strategy

We use Faraday’s law of induction to find the average emf induced over a time ΔtΔt size 12{Δt} {}:

emf=−NΔΦΔt.emf=−NΔΦΔt. size 12{“emf”= – N { {ΔΦ} over {Δt} } } {}

We know that N=200N=200 size 12{N=”200″} {} and Δt=15.0msΔt=15.0ms size 12{Δt=”15″ “.” 0`”ms”} {}, and so we must determine the change in flux ΔΦΔΦ size 12{ΔΦ} {} to find emf.

Solution

Since the area of the loop and the magnetic field strength are constant, we see that

ΔΦ=Δ(BAcosθ)=ABΔ(cosθ).ΔΦ=Δ(BAcosθ)=ABΔ(cosθ). size 12{ΔΦ=Δ (ital “BA””cos”θ) = ital “AB”Δ (“cos”θ) } {}

Now, Δ(cosθ)=−1.0Δ(cosθ)=−1.0 size 12{Δ (“cos”θ) = – 1 “.” 0} {}, since it was given that θθ goes from 0º0º to 90º90º . Thus ΔΦ=−ABΔΦ=−AB size 12{ΔΦ= – ital “AB”} {}, and

emf=NABΔt.emf=NABΔt. size 12{“emf”=N { { ital “AB”} over {Δt} } } {}

The area of the loop is A=πr2=(3.14…)(0.0500m)2=7.85×10−3m2A=πr2=(3.14…)(0.0500m)2=7.85×10−3m2 size 12{A=πr rSup { size 8{2} } = (3 “.” “14” “.” “.” “.”) (0 “.” “0500”`m ) rSup { size 8{2} } =7 “.” “85” times “10” rSup { size 8{ – 3} } `m rSup { size 8{2} } } {}. Entering this value gives

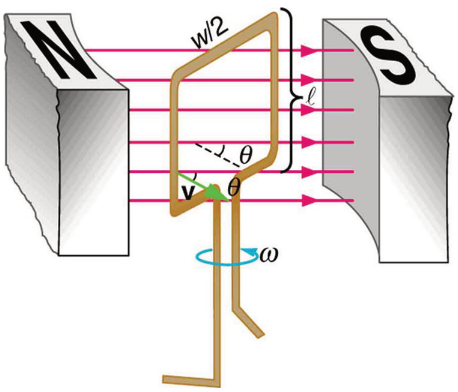
emf=200(7.85×10−3m2)(1.25 T)15.0×10−3 s=131V.emf=200(7.85×10−3m2)(1.25 T)15.0×10−3 s=131V. size 12{“emf”=”200” { { (7 “.” “85” times “10” rSup { size 8{ – 3} } ” m” rSup { size 8{2} }) (1 “.” “25”” T”) } over {“15” “.” 0 times “10” rSup { size 8{ – 3} } ” s”} } =”131″” V”} {}

Discussion

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in [link] is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf *as a function of time by considering the motional emf on a rotating rectangular coil of width ww size 12{w} {} and height ℓℓ size 12{l} {} in a uniform magnetic field, as illustrated in [link].*

A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.



Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be *emf=Bℓvemf=Bℓv size 12{“emf”=Bℓv} {}, where the velocity v is perpendicular to the magnetic field BB size 12{B} {}. Here the velocity is at an angle θθ size 12{θ} {} with BB size 12{B} {}, so that its component perpendicular to BB size 12{B} {} is vsinθvsinθ size 12{v”sin”θ} {} (see [link]). Thus in this case the emf induced on each side is emf=Bℓvsinθemf=Bℓvsinθ size 12{“emf”=Bℓv”sin”θ} {}, and they are in the same direction. The total emf around the loop is then*

emf=2Bℓvsinθ.emf=2Bℓvsinθ. size 12{“emf”=2Bℓv”sin”θ} {}

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity ωω size 12{ω} {}. The angle θθ size 12{θ} {} is related to angular velocity by *θ=ωtθ=ωt size 12{θ=ωt} {}*, so that

emf=2Bℓvsinωt.emf=2Bℓvsinωt. size 12{“emf”=Bℓv”sin”ωt} {}

Now, linear velocity
vv

is related to angular velocity
ωω by v=rωv=rω size 12{v=rω} {}. Here r=w/2r=w/2 size 12{r=w/2} {}, so that *v=(w/2)ωv=(w/2)ω size 12{v= (w/2) ω} {}*, and

emf=2Bℓw2ωsinωt=(ℓw)Bωsinωt.emf=2Bℓw2ωsinωt=(ℓw)Bωsinωt. size 12{“emf”=2Bℓ { {w} over {2} } ω”sin”ωt= (ℓw) Bω”sin”ωt} {}

Noting that the area of the loop is A=ℓwA=ℓw size 12{A=ℓw} {}, and allowing for NN size 12{N} {} loops, we find that

emf=NABωsinωtemf=NABωsinωt size 12{“emf”= ital “NAB”ω”sin”ωt} {}

is the emf induced in a generator coil of NN size 12{N} {} turns and area AA size 12{A} {} rotating at a constant angular velocity
ωω
in a uniform magnetic field BB size 12{B} {}. This can also be expressed as

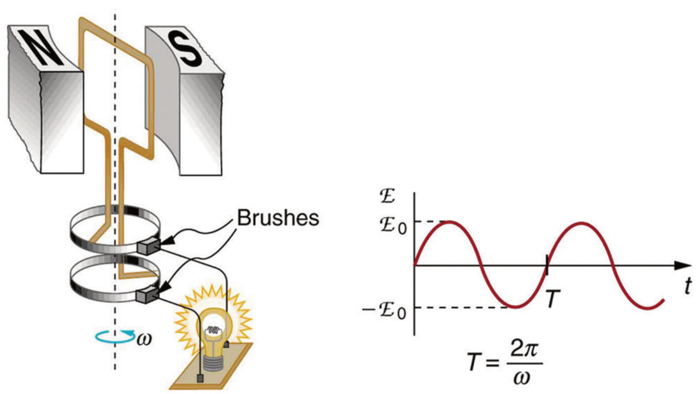
emf=emf0sinωt,emf=emf0sinωt, size 12{“emf”=”emf” rSub { size 8{0} } “sin”ωt} {}

where

emf0=NABωemf0=NABω size 12{“emf” rSub { size 8{0} } = ital “NAB”ω} {}

is the maximum (peak) emf. Note that the frequency of the oscillation is *f=ω/2πf=ω/2π size 12{f=ω/2π} {}*, and the period is T=1/f=2π/ωT=1/f=2π/ω size 12{T=1/f=2π/ω} {}. [link] shows a graph of emf as a function of time, and it now seems reasonable that AC voltage is sinusoidal.

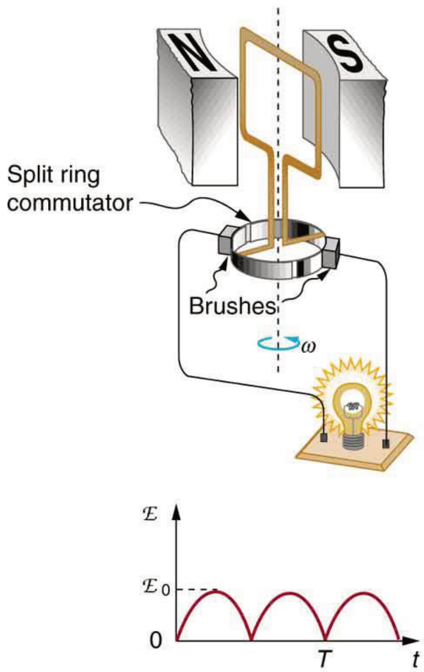
The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time. emf0emf0 size 12{“emf” rSub { size 8{0} } } {} is the peak emf. The period is T=1/f=2π/ωT=1/f=2π/ω size 12{T=1/f=2π/ω} {}, where ff size 12{f} {} is the frequency. Note that the script E stands for emf.



The fact that the peak emf, emf0=NABωemf0=NABω size 12{“emf” rSub { size 8{0} } = ital “NAB”ω} {}, makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater ωω size 12{ω} {}), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties. One of the authors as a juvenile found it amusing to ride his bicycle fast enough to burn out his lights, until he had to ride home lightless one dark night.

[link] shows a scheme by which a generator can be made to produce pulsed DC. More elaborate arrangements of multiple coils and split rings can produce smoother DC, although electronic rather than mechanical means are usually used to make ripple-free DC.

Split rings, called commutators, produce a pulsed DC emf output in this configuration.



Calculating the Maximum Emf of a Generator

Calculate the maximum emf, emf0emf0 size 12{“emf” rSub { size 8{0} } } {}, of the generator that was the subject of [link].

Strategy

Once ωω size 12{ω} {}, the angular velocity, is determined, emf0=NABωemf0=NABω size 12{“emf” rSub { size 8{0} } = ital “NAB”ω} {} can be used to find emf0emf0 size 12{“emf” rSub { size 8{0} } } {}. All other quantities are known.

Solution

Angular velocity is defined to be the change in angle per unit time:

ω=ΔθΔt.ω=ΔθΔt. size 12{ω= { {Δθ} over {Δt} } } {}

One-fourth of a revolution is π/2π/2 size 12{l} {} radians, and the time is 0.0150 s; thus, ω=π/2 rad0.0150 s=104.7 rad/s.ω=π/2 rad0.0150 s=104.7 rad/s.

104.7 rad/s is exactly 1000 rpm. We substitute this value for ωω size 12{ω} {} and the information from the previous example into emf0=NABωemf0=NABω size 12{“emf” rSub { size 8{0} } = ital “NAB”ω} {}, yielding

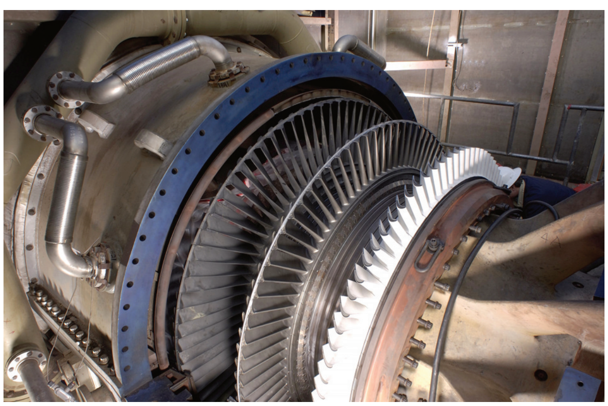
emf0=NABω=200(7.85×10−3m2)(1.25T)(104.7rad/
s)=206V.emf0=NABω=200(7.85×10−3m2)(1.25T)(104.7rad/s)=206V.alignl { stack {
size 12{“emf” rSub { size 8{0} } = ital “NAB”ω} {} #
” “=”200” (7 “.” “85” times “10” rSup { size 8{ – 3} } ” m” rSup { size 8{2} }) (1 “.” “25”” T”) (“104” “.”
7″ rad/s”) {} #
” “=”206″” V” {}
} } {}

Discussion

The maximum emf is greater than the average emf of 131 V found in the previous example, as it should be.

In real life, electric generators look a lot different than the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the burning of fossil fuels, or the kinetic energy of wind. [link] shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator.

Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft which is connected to the generator. (credit: Nabonaco, Wikimedia Commons)



Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In Back Emf, we shall further explore the action of a motor as a generator.

Section Summary

- An electric generator rotates a coil in a magnetic field, inducing an emf*given as a function of time by emf=NABωsinωt,emf=NABωsinωt, size 12{“emf”= ital “NAB”ω”sin”ωt} {}*

where AA size 12{A} {} is the area of an NN size 12{N} {}-turn coil rotated at a constant angular velocity ωω size 12{ω} {} in a uniform magnetic field BB size 12{B} {}.

- The peak emf emf0emf0 size 12{“emf” rSub { size 8{0} } } {} of a generator is emf0=NABω.emf0=NABω. size 12{“emf” rSub { size 8{0} } = ital “NAB”ω} {}

Conceptual Questions

Using RHR-1, show that the emfs in the sides of the generator loop in [link] are in the same sense and thus add.

The source of a generator’s electrical energy output is the work done to turn its coils. How is the work needed to turn the generator related to Lenz’s law?

Problems & Exercises

Calculate the peak voltage of a generator that rotates its 200-turn, 0.100 m diameter coil at 3600 rpm in a 0.800 T field.

474 V

At what angular velocity in rpm will the peak voltage of a generator be 480 V, if its 500-turn, 8.00 cm diameter coil rotates in a 0.250 T field?

What is the peak emf generated by rotating a 1000-turn, 20.0 cm diameter coil in the Earth’s 5.00×10−5T5.00×10−5T size 12{5 “.” “00” times “10” rSup { size 8{ – 5} } `T} {} magnetic field, given the plane of the coil is originally perpendicular to the Earth’s field and is rotated to be parallel to the field in 10.0 ms?

0.247 V

What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)

(a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) Is this number of turns of wire practical for a 1.00 by 3.00 cm coil?

(a) 50

(b) yes

Integrated Concepts

This problem refers to the bicycle generator considered in the previous problem. It is driven by a 1.60 cm diameter wheel that rolls on the outside rim of the bicycle tire. (a) What is the velocity of the bicycle if the generator’s angular velocity is 1875 rad/s? (b) What is the maximum emf of the generator when the bicycle moves at 10.0

m/s, noting that it was 18.0 V under the original conditions? (c) If the sophisticated generator can vary its own magnetic field, what field strength will it need at 5.00 m/s to produce a 9.00 V maximum emf?

(a) A car generator turns at 400 rpm when the engine is idling. Its 300-turn, 5.00 by 8.00 cm rectangular coil rotates in an adjustable magnetic field so that it can produce sufficient voltage even at low rpms. What is the field strength needed to produce a 24.0 V peak emf? (b) Discuss how this required field strength compares to those available in permanent and electromagnets.

(a) 0.477 T

(b) This field strength is small enough that it can be obtained using either a permanent magnet or an electromagnet.

Show that if a coil rotates at an angular velocity ωω, the period of its AC output is
2π/ω2π/ω.

A 75-turn, 10.0 cm diameter coil rotates at an angular velocity of 8.00 rad/s in a 1.25 T field, starting with the plane of the coil parallel to the field. (a) What is the peak emf? (b) At what time is the peak emf first reached? (c) At what time is the emf first at its most negative? (d) What is the period of the AC voltage output?

(a) 5.89 V

(b) At t=0

(c) 0.393 s

(d) 0.785 s

(a) If the emf of a coil rotating in a magnetic field is zero at t=0t=0 size 12{t=0} {}, and increases to its first peak at t=0.100mst=0.100ms size 12{t=0 "." "100"`"ms"} {}, what is the angular velocity of the coil? (b) At what time will its next maximum occur? (c) What is the period of the output? (d) When is the output first one-fourth of its maximum? (e) When is it next one-fourth of its maximum?

Unreasonable Results

A 500-turn coil with a 0.250m20.250m2 size 12{0 "." "250"`m rSup { size 8{2} } } {} area is spun in the Earth's 5.00×10−5T5.00×10−5T size 12{5 "." "00" times "10" rSup { size 8{ – 5} } `T} {} field, producing a 12.0 kV maximum emf. (a) At what angular velocity must the coil be spun? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

(a) 1.92×106rad/s1.92×106rad/s size 12{1 "." "92" times "10" rSup { size 8{6} } `"rad/s"} {}

(b) This angular velocity is unreasonably high, higher than can be obtained for any mechanical system.

(c) The assumption that a voltage as great as 12.0 kV could be obtained is unreasonable.

Glossary

electric generator
a device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field
emf induced in a generator coil
emf=NABωsinωtemf=NABωsinωt size 12{"emf"= ital "NAB"ω"sin"ωt} {}, where AA is the area of an NN-turn coil rotated at a constant angular velocity
ωω in a uniform magnetic field BB, over a period of time tt
peak emf
emf0=NABωemf0=NABω size 12{"emf" rSub { size 8{0} } = ital "NAB"ω} {}

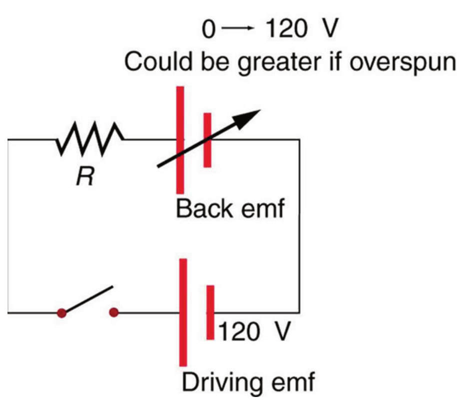
23.6 Back Emf

Back Emf

- Explain what back emf is and how it is induced.

It has been noted that motors and generators are very similar. Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Furthermore, motors and generators have the same construction. When the coil of a motor is turned, magnetic flux changes, and an emf (consistent with Faraday's law of induction) is induced. The motor thus acts as a generator whenever its coil rotates. This will happen whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor will be opposed by the motor's self-generated emf, called the back emf of the motor. (See [link].)

The coil of a DC motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the one driving the motor. Back emf is zero when the motor is not turning, and it increases proportionally to the motor's angular velocity.



Back emf is the generator output of a motor, and so it is proportional to the motor's angular velocity ωω size 12{ω} {}. It is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster and faster, the back emf grows, always opposing the driving emf, and reduces the voltage across the coil and the amount of current it draws. This effect is noticeable in a number of situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the IRIR drop produced in feeder lines by the large current drawn by the motor. When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, P=I2RP=I2R size 12{P = I rSup { size 8{2} } R} {}), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it will increase its angular velocity ωω size 12{ω} {} until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Consider, for example, the motor coils represented in [link]. The coils have a 0.400Ω0.400Ω size 12{ %OMEGA } {} equivalent resistance and are driven by a 48.0 V emf. Shortly after being turned on, they draw a current I=V/R=(48.0V)/(0.400Ω)=120AI=V/R=(48.0V)/(0.400Ω)=120A size 12{I = ital "V/R"= ("48" "." 0`V ) ( 0 "." "400"` %OMEGA) ="120"`A} {} and, thus, dissipate P=I2R=5.76kWP=I2R=5.76kW size 12{P = I rSup { size 8{2} } R=5 "." "76"`"kW"} {} of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is 40.0 V. Then at operating speed, the total voltage across the coils is 8.0 V (48.0 V minus the 40.0 V back emf), and the current drawn is I=V/R=(8.0V)/(0.400Ω)=20AI=V/R=(8.0V)/(0.400Ω)=20A size 12{I = ital "V/R"= (8 "." 0`V ) ( 0 "." "400"` %OMEGA) ="20"`A} {}. Under normal load, then, the power dissipated is P=IV=(20A)/(8.0V)=160WP=IV=(20A)/(8.0V)=160W size 12{P = ital "IV"= ( "20"`A) (8 "." 0`V ) ="160"`W} {}. The latter will not cause a problem for this motor, whereas the former 5.76 kW would burn out the coils if sustained.

Section Summary

- Any rotating coil will have an induced emf—in motors, this is called back emf, since it opposes the emf input to the motor.

Conceptual Questions

Suppose you find that the belt drive connecting a powerful motor to an air conditioning unit is broken and the motor is running freely. Should you be worried that the motor is consuming a great deal of energy for no useful purpose? Explain why or why not.

Problems & Exercises

Suppose a motor connected to a 120 V source draws 10.0 A when it first starts. (a) What is its resistance? (b) What current does it draw at its normal operating speed when it develops a 100 V back emf?

(a) 12.00 Ω12.00 Ω

(b) 1.67 A

A motor operating on 240 V electricity has a 180 V back emf at operating speed and draws a 12.0 A current. (a) What is its resistance? (b) What current does it draw when it is first started?

What is the back emf of a 120 V motor that draws 8.00 A at its normal speed and 20.0 A when first starting?

72.0 V

The motor in a toy car operates on 6.00 V, developing a 4.50 V back emf at normal speed. If it draws 3.00 A at normal speed, what current does it draw when starting?

Integrated Concepts

The motor in a toy car is powered by four batteries in series, which produce a total emf of 6.00 V. The motor draws 3.00 A and develops a 4.50 V back emf at normal speed. Each battery has a
$0.100\ \Omega$0.100 Ω
internal resistance. What is the resistance of the motor?

$0.100\ \Omega$0.100 Ω

Glossary

back emf
the emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor

23.7 Transformers

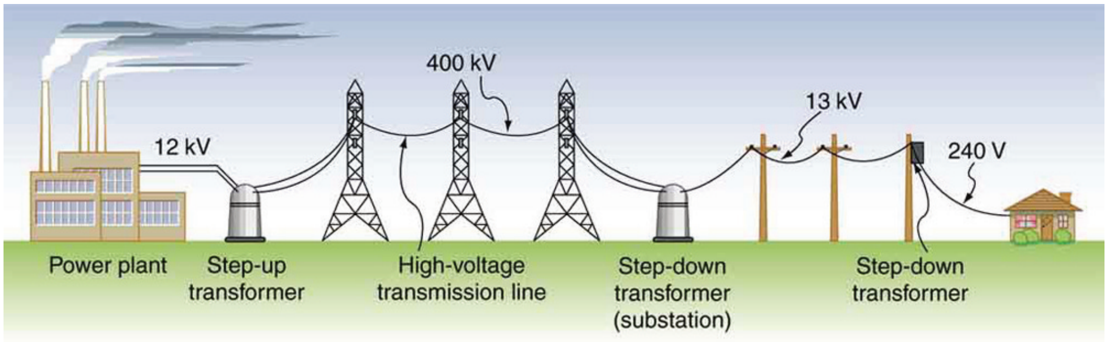
Transformers

- Explain how a transformer works.
- Calculate voltage, current, and/or number of turns given the other quantities.

Transformers do what their name implies—they transform voltages from one value to another (The term voltage is used rather than emf, because transformers have internal resistance). For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in [link]) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in [link]. Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.

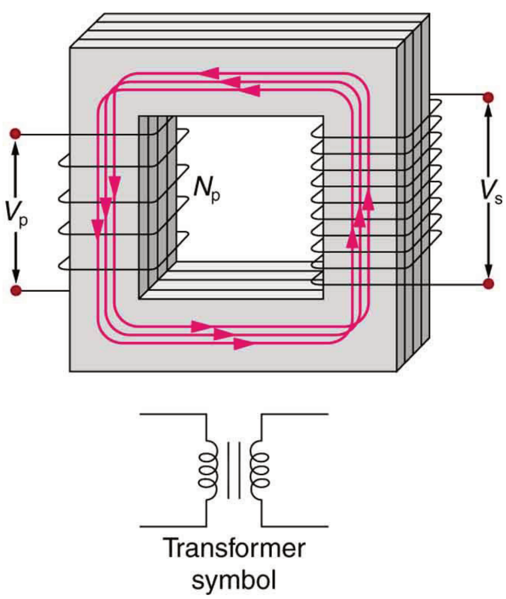
The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V AC. Most are in the 3 to 12 V range. (credit: Shop Xtreme)

Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.



The type of transformer considered in this text—see [link]—is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.

A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.



For the simple transformer shown in [link], the output voltage VsVs size 12{V rSub { size 8{s} } } {} depends almost entirely on the input voltage VpVp size 12{V rSub { size 8{p} } } {} and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage VsVs size 12{V rSub { size 8{s} } } {} to be

Vs=−NsΔΦΔt,Vs=−NsΔΦΔt, size 12{V rSub { size 8{s} } = – N rSub { size 8{s} } { {ΔΦ} over {Δt} } } {}

where NsNs size 12{N rSub { size 8{s} } } {} is the number of loops in the secondary coil and ΔΦΔΦ size 12{ΔΦ} {}/ ΔtΔt size 12{Δt} {} is the rate of change of magnetic flux. Note that the output voltage equals the induced emf (Vs=emfsVs=emfs size 12{V rSub { size 8{s} } =”emf” rSub { size 8{s} } } {}), provided coil resistance is small (a reasonable assumption for transformers). The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so

ΔΦ/ΔtΔΦ/Δt size 12{ΔΦ} {}

is the same on either side. The input primary voltage VpVp size 12{V rSub { size 8{p} } } {} is also related to changing flux by

Vp=−NpΔΦΔt.Vp=−NpΔΦΔt. size 12{V rSub { size 8{p} } = – N rSub { size 8{p} } { {ΔΦ} over {Δt} } } {}

The reason for this is a little more subtle. Lenz’s law tells us that the primary coil opposes the change in flux caused by the input voltage VpVp size 12{V rSub { size 8{p} } } {}, hence the minus sign (This is an example of *self-inductance*, a topic to be explored in some detail in later sections). Assuming negligible coil resistance, Kirchhoff’s loop rule tells us that the induced emf exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

VsVp=NsNp.VsVp=NsNp. size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } {}

This is known as the transformer equation, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A step-up transformer is one that increases voltage, whereas a step-down transformer decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%. Equating the power input and output,

Pp=IpVp=IsVs=Ps.Pp=IpVp=IsVs=Ps. size 12{P rSub { size 8{p} } =I rSub { size 8{p} } V rSub { size 8{p} } =I rSub { size 8{s} } V rSub { size 8{s} } =P rSub { size 8{s} } } {}

Rearranging terms gives

VsVp=IpIs.VsVp=IpIs. size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {I rSub { size 8{p} } } over {I rSub { size 8{s} } } } } {}

Combining this with VsVp=NsNpVsVp=NsNp size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } {}, we find that

IsIp=NpNsIsIp=NpNs size 12{ { {I rSub { size 8{s} } } over {I rSub { size 8{p} } } } = { {N rSub { size 8{p} } } over {N rSub { size 8{s} } } } } {}

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve VsVp=NsNpVsVp=NsNp size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } {} for NsNs size 12{N rSub { size 8{s} } } {}, the number of loops in the secondary, and enter the known values. This gives

Ns=NpVsVp=(50)100,000 V120 V=4.17×104.Ns=NpVsVp=(50)100,000 V120 V=4.17×104.alignl { stack {
size 12{N rSub { size 8{s} } =N rSub { size 8{p} } { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } }
{} #
" "= ("50") { {"100,000 V"} over {"120″" V"} } =4 "." "17" times "10" rSup { size 8{4} } {}
} } {}

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving IsIp=NpNsIsIp=NpNs size 12{ { {I rSub { size 8{s} } } over {I rSub { size 8{p} } } } = { {N rSub { size 8{p} } } over {N rSub { size 8{s} } } } } {} for IsIs size 12{I rSub { size 8{s} } } {} and entering known values. This gives

Is=IpNpNs=(10.00 A)504.17×104=12.0 mA.Is=IpNpNs=(10.00 A)504.17×104=12.0 mA.alignl { stack {
size 12{I rSub { size 8{s} } =I rSub { size 8{p} } { {N rSub { size 8{p} } } over {N rSub { size 8{s} } } } } {}
#
" "= ("10" "." "00 A") { {"50"} over {4 "." "17" times "10" rSup { size 8{4} } } } ="12″ "." 0″ mA" {}
} } {}

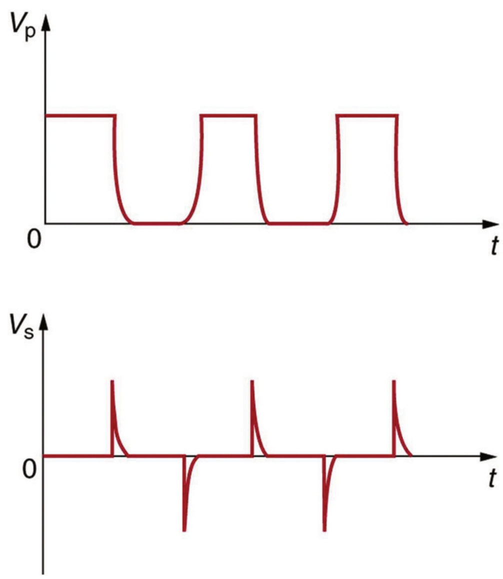
Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is Pp=IpVp=(10.00A)(120V)=1.20kWPp=IpVp=(10.00A)(120V)=1.20kW size 12{P rSub { size 8{p} } =I rSub { size 8{p} } V rSub { size 8{p} } = ("10" "." "00"`A ) ( "120"`V) =1 "." "20"`"kW"} {}. This equals the power output Pp=IsVs=(12.0mA)(100kV)=1.20kWPp=IsVs=(12.0mA)(100kV)=1.20kW size 12{P rSub { size 8{p} }

=I rSub { size 8{s} } V rSub { size 8{s} } = (“12” “.” 0`”mA” ) ( “100”`”kV”) =1 “.” “20”`”kW”} {}, as we assumed in the derivation of the equations used.

The fact that transformers are based on Faraday’s law of induction makes it clear why we cannot use transformers to change DC voltages. If there is no change in primary voltage, there is no voltage induced in the secondary. One possibility is to connect DC to the primary coil through a switch. As the switch is opened and closed, the secondary produces a voltage like that in [link]. This is not really a practical alternative, and AC is in common use wherever it is necessary to increase or decrease voltages.

Transformers do not work for pure DC voltage input, but if it is switched on and off as on the top graph, the output will look something like that on the bottom graph. This is not the sinusoidal AC most AC appliances need.



Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries (total emf of 12.5 V DC) needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving VsVp=NsNpVsVp=NsNp size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } {} for NsNs size 12{N rSub { size 8{s} } } {} and entering known values gives

Ns=NpVsVp=(200)15.0 V120 V=25.Ns=NpVsVp=(200)15.0 V120 V=25.alignl { stack {
size 12{N rSub { size 8{s} } =N rSub { size 8{p} } { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } } {} #
” “= (“200”) { {“15” “.” “0 V”} over {“120 V”} } =”25″ {}

} } {}

Strategy and Solution for (b)

The current input can be obtained by solving IsIp=NpNsIsIp=NpNs size 12{ { {I rSub { size 8{s} } } over {I rSub { size 8{p} } } } = { {N rSub { size 8{p} } } over {N rSub { size 8{s} } } } } {} for IpIp size 12{I rSub { size 8{p} } } {} and entering known values. This gives

Ip=IsNsNp=(16.0 A)25200=2.00 A.Ip=IsNsNp=(16.0 A)25200=2.00 A.alignl { stack {
size 12{I rSub { size 8{p} } =I rSub { size 8{s} } { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } } {}
#
” “= (“16” “.” “0 A”) { {“25”} over {“200”} } =2 “.” “00”” A” {}
} } {}

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in (Pp=PsPp=Ps size 12{P rSub { size 8{p} } =P rSub { size 8{s} } } {})—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Transformers have many applications in electrical safety systems, which are discussed in Electrical Safety: Systems and Devices.

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

Generator

PhET Interactive Simulation

Section Summary

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by VsVp=NsNp,VsVp=NsNp, size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } {}

  where VpVp size 12{V rSub { size 8{p} } } {} and VsVs size 12{V rSub { size 8{s} } } {} are the voltages across primary and secondary coils having NpNp size 12{N rSub { size 8{p} } } {} and NsNs size 12{N rSub { size 8{s} } } {} turns.
- The currents IpIp size 12{I rSub { size 8{p} } } {} and IsIs size 12{I rSub { size 8{s} } } {} in the primary and secondary coils are related by IsIp=NpNsIsIp=NpNs size 12{ { {I rSub { size 8{s} } } over {I rSub { size 8{p} } } } = { {N rSub { size 8{p} } } over {N rSub { size 8{s} } } } } {}.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

Conceptual Questions

Explain what causes physical vibrations in transformers at twice the frequency of the AC power involved.

Problems & Exercises

A plug-in transformer, like that in [link], supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

(a) 30.0

(b) 9.75×10−2A9.75×10−2A size 12{9 "." "75" times "10" rSup { size 8{ – 2} } `A} {}

An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

(a) 20.0 mA

(b) 2.40 W

(c) Yes, this amount of power is quite reasonable for a small appliance.

(a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

(a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

(a) 0.063 A

(b) Greater input current needed.

A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines. (a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

(a) 2.2

(b) 0.45

(c) 0.20, or 20.0%

If the power output in the previous problem is 1000 MW and line resistance is 2.00 Ω2.00 Ω, what were the old and new line losses?

Unreasonable Results

The 335 kV AC electricity from a power transmission line is fed into the primary coil of a transformer. The ratio of the number of turns in the secondary to the number in the primary is Ns/Np=1000Ns/Np=1000 size 12{N rSub { size 8{s} } /N rSub { size 8{p} } =”1000″} {}. (a) What voltage is induced in the secondary? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

(a) 335 MV

(b) way too high, well beyond the breakdown voltage of air over reasonable distances

(c) input voltage is too high

Construct Your Own Problem

Consider a double transformer to be used to create very large voltages. The device consists of two stages. The first is a transformer that produces a much larger output voltage than its input. The output of the first transformer is used as input to a second transformer that further increases the voltage. Construct a problem in which you calculate the output voltage of the final stage based on the input voltage of the first stage and the number of turns or loops in both parts of both transformers (four coils in all). Also calculate the maximum output current of the final stage based on the input current. Discuss the possibility of power losses in the devices and the effect on the output current and power.

Glossary

transformer
a device that transforms voltages from one value to another using induction
transformer equation
the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; VsVp=NsNpVsVp=NsNp size 12{ { {V rSub { size 8{s} } } over {V rSub { size 8{p} } } } = { {N rSub { size 8{s} } } over {N rSub { size 8{p} } } } } {}
step-up transformer
a transformer that increases voltage
step-down transformer
a transformer that decreases voltage

23.8 Electrical Safety: Systems and Devices

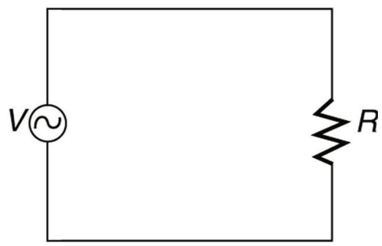
Electrical Safety: Systems and Devices

- Explain how various modern safety features in electric circuits work, with an emphasis on how induction is employed.

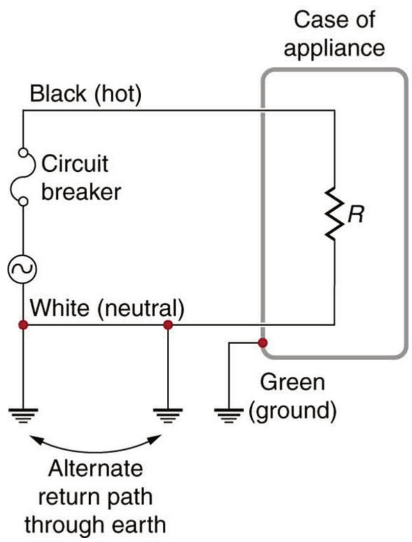
Electricity has two hazards. A thermal hazard occurs when there is electrical overheating. A shock hazard occurs when electric current passes through a person. Both hazards have already been discussed. Here we will concentrate on systems and devices that prevent electrical hazards.

[link] shows the schematic for a simple AC circuit with no safety features. This is not how power is distributed in practice. Modern household and industrial wiring requires the three-wire system, shown schematically in [link], which has several safety features. First is the familiar *circuit breaker* (or *fuse*) to prevent thermal overload. Second, there is a protective *case* around the appliance, such as a toaster or refrigerator. The case's safety feature is that it prevents a person from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.

Schematic of a simple AC circuit with a voltage source and a single appliance represented by the resistance RR size 12{R} {}. There are no safety features in this circuit.

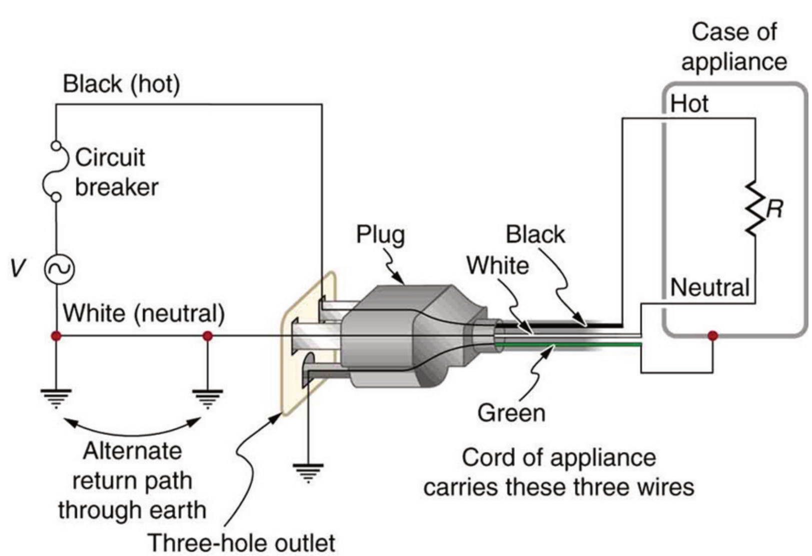


The three-wire system connects the neutral wire to the earth at the voltage source and user location, forcing it to be at zero volts and supplying an alternative return path for the current through the earth. Also grounded to zero volts is the case of the appliance. A circuit breaker or fuse protects against thermal overload and is in series on the active (live/hot) wire. Note that wire insulation colors vary with region and it is essential to check locally to determine which color codes are in use (and even if they were followed in the particular installation).



There are *three connections to earth or ground (hereafter referred to as "earth/ground") shown in [link]. Recall that an earth/ground connection is a low-resistance path directly to the earth. The two earth/ground connections on the neutral wire force it to be at zero volts relative to the earth, giving the wire its name. This wire is therefore safe to touch even if its insulation, usually white, is missing. The neutral wire is the return path for the current to follow to complete the circuit. Furthermore, the two earth/ground connections supply an alternative path through the earth, a good conductor, to complete the circuit. The earth/ground connection closest to the power source could be at the generating plant, while the other is at the user's location. The third earth/ground is to the case of the appliance, through the green earth/ground wire, forcing the case, too, to be at zero volts. The live or hot wire (hereafter referred to as "live/hot") supplies voltage and current to operate the appliance. [link] shows a more pictorial version of how the three-wire system is connected through a three-prong plug to an appliance.*

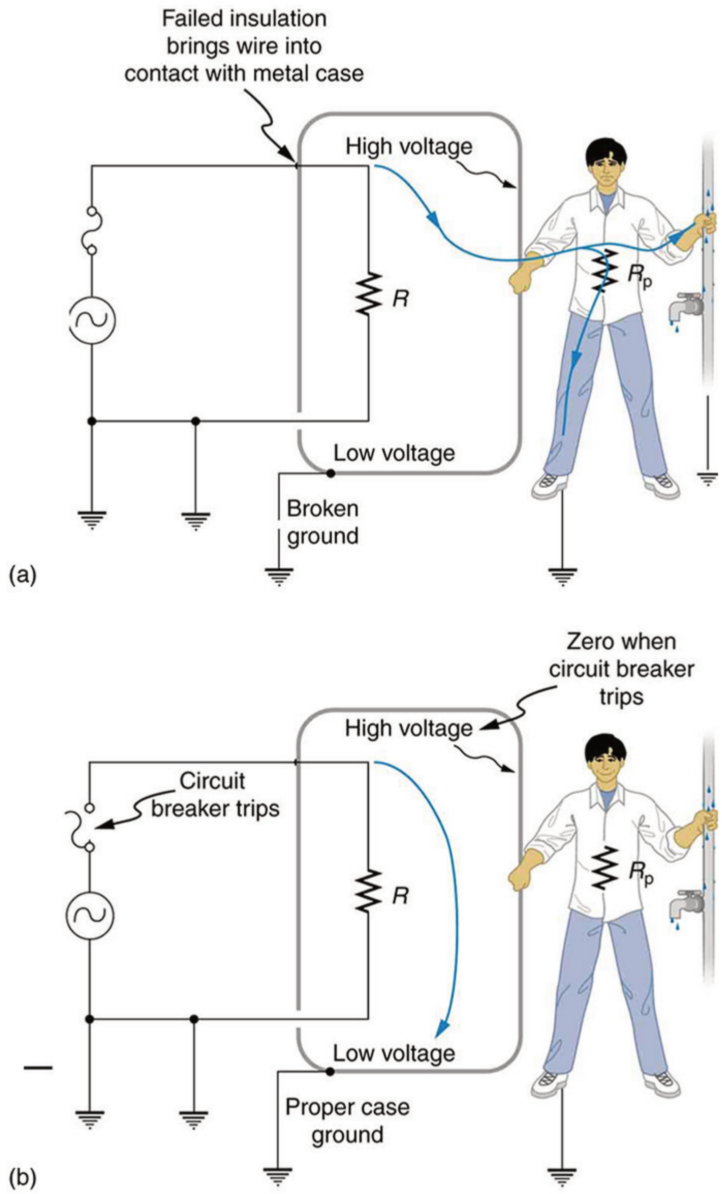
The standard three-prong plug can only be inserted in one way, to assure proper function of the three-wire system.



A note on insulation color-coding: Insulating plastic is color-coded to identify live/hot, neutral and ground wires but these codes vary around the world. Live/hot wires may be brown, red, black, blue or grey. Neutral wire may be blue, black or white. Since the same color may be used for live/hot or neutral in different parts of the world, it is essential to determine the color code in your region. The only exception is the earth/ground wire which is often green but may be yellow or just bare wire. Striped coatings are sometimes used for the benefit of those who are colorblind.

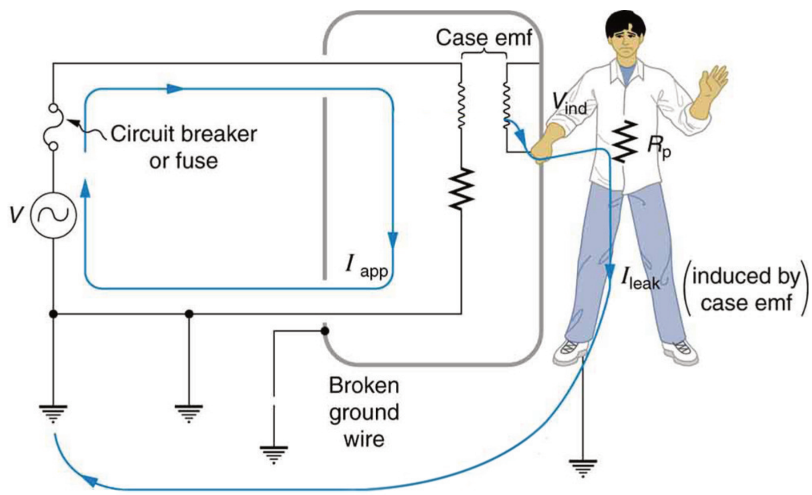
The three-wire system replaced the older two-wire system, which lacks an earth/ground wire. Under ordinary circumstances, insulation on the live/hot and neutral wires prevents the case from being directly in the circuit, so that the earth/ground wire may seem like double protection. Grounding the case solves more than one problem, however. The simplest problem is worn insulation on the live/hot wire that allows it to contact the case, as shown in [link]. Lacking an earth/ground connection (some people cut the third prong off the plug because they only have outdated two hole receptacles), a severe shock is possible. This is particularly dangerous in the kitchen, where a good connection to earth/ground is available through water on the floor or a water faucet. With the earth/ground connection intact, the circuit breaker will trip, forcing repair of the appliance. Why are some appliances still sold with two-prong plugs? These have nonconducting cases, such as power tools with impact resistant plastic cases, and are called *doubly insulated*. Modern two-prong plugs can be inserted into the asymmetric standard outlet in only one way, to ensure proper connection of live/hot and neutral wires.

Worn insulation allows the live/hot wire to come into direct contact with the metal case of this appliance. (a) The earth/ground connection being broken, the person is severely shocked. The appliance may operate normally in this situation. (b) With a proper earth/ground, the circuit breaker trips, forcing repair of the appliance.



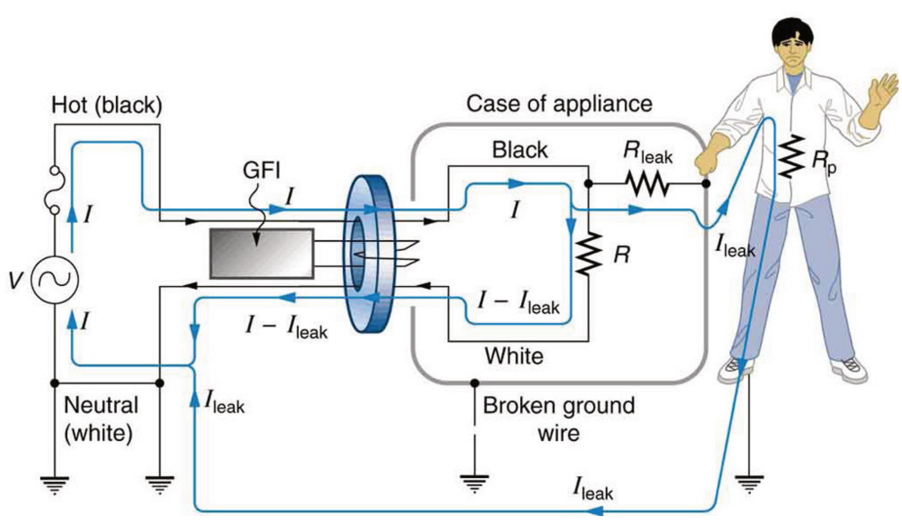
Electromagnetic induction causes a more subtle problem that is solved by grounding the case. The AC current in appliances can induce an emf on the case. If grounded, the case voltage is kept near zero, but if the case is not grounded, a shock can occur as pictured in [link]. Current driven by the induced case emf is called a *leakage current*, although current does not necessarily pass from the resistor to the case.

AC currents can induce an emf on the case of an appliance. The voltage can be large enough to cause a shock. If the case is grounded, the induced emf is kept near zero.



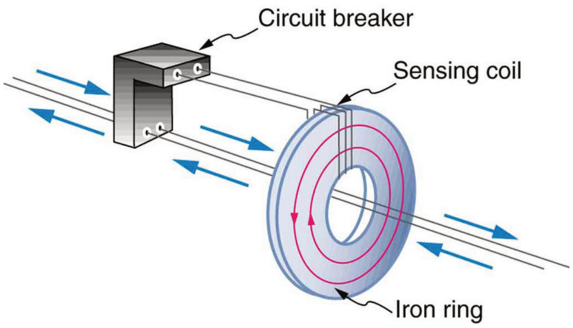
A *ground fault interrupter* (GFI) is a safety device found in updated kitchen and bathroom wiring that works based on electromagnetic induction. GFIs compare the currents in the live/hot and neutral wires. When live/hot and neutral currents are not equal, it is almost always because current in the neutral is less than in the live/hot wire. Then some of the current, again called a leakage current, is returning to the voltage source by a path other than through the neutral wire. It is assumed that this path presents a hazard, such as shown in [link]. GFIs are usually set to interrupt the circuit if the leakage current is greater than 5 mA, the accepted maximum harmless shock. Even if the leakage current goes safely to earth/ground through an intact earth/ground wire, the GFI will trip, forcing repair of the leakage.

A ground fault interrupter (GFI) compares the currents in the live/hot and neutral wires and will trip if their difference exceeds a safe value. The leakage current here follows a hazardous path that could have been prevented by an intact earth/ground wire.



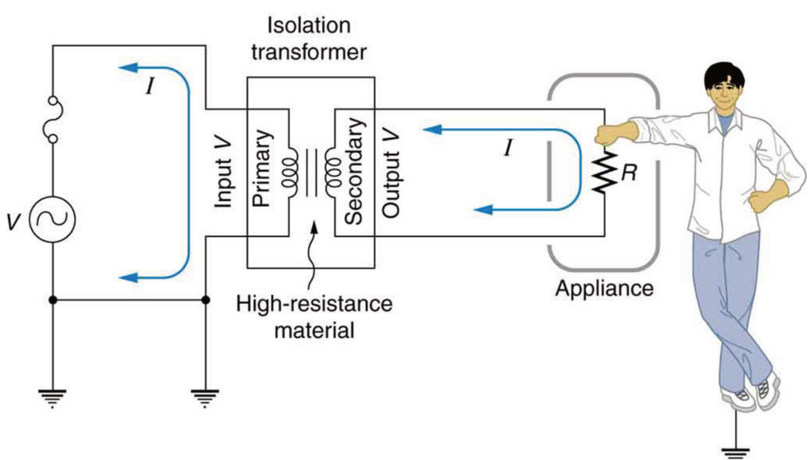
[link] shows how a GFI works. If the currents in the live/hot and neutral wires are equal, then they induce equal and opposite emfs in the coil. If not, then the circuit breaker will trip.

A GFI compares currents by using both to induce an emf in the same coil. If the currents are equal, they will induce equal but opposite emfs.



Another induction-based safety device is the *isolation transformer*, shown in [link]. Most isolation transformers have equal input and output voltages. Their function is to put a large resistance between the original voltage source and the device being operated. This prevents a complete circuit between them, even in the circumstance shown. There is a complete circuit through the appliance. But there is not a complete circuit for current to flow through the person in the figure, who is touching only one of the transformer's output wires, and neither output wire is grounded. The appliance is isolated from the original voltage source by the high resistance of the material between the transformer coils, hence the name isolation transformer. For current to flow through the person, it must pass through the high-resistance material between the coils, through the wire, the person, and back through the earth—a path with such a large resistance that the current is negligible.

An isolation transformer puts a large resistance between the original voltage source and the device, preventing a complete circuit between them.



The basics of electrical safety presented here help prevent many electrical hazards. Electrical safety can be pursued to greater depths. There are, for example, problems related to different earth/ground connections for appliances in close proximity. Many other examples are found in hospitals. Microshock-sensitive patients, for instance, require special protection. For these people, currents as low as 0.1 mA may cause ventricular fibrillation. The interested reader can use the material presented here as a basis for further study.

Section Summary

- Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and earth/ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault interrupter (GFI) prevents shock by detecting the loss of current to unintentional paths.
- An isolation transformer insulates the device being powered from the original source, also to prevent shock.
- Many of these devices use induction to perform their basic function.

Conceptual Questions

Does plastic insulation on live/hot wires prevent shock hazards, thermal hazards, or both?

Why are ordinary circuit breakers and fuses ineffective in preventing shocks?

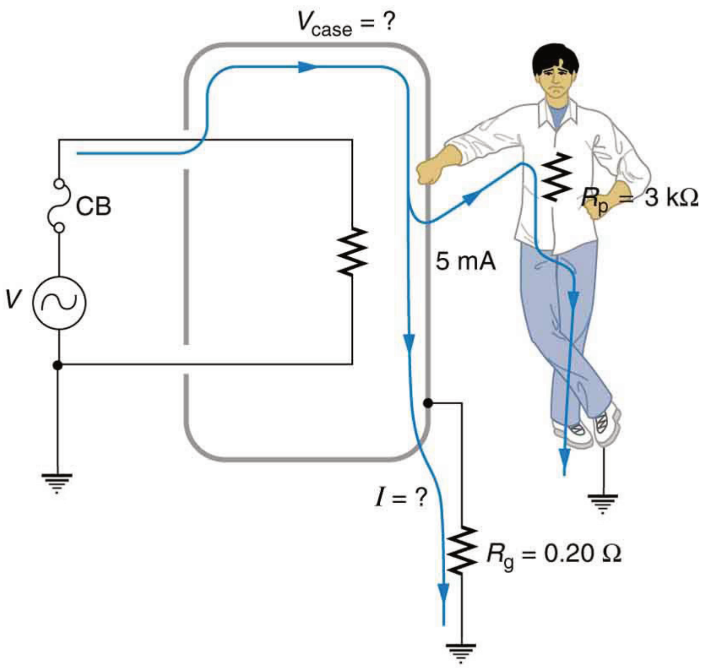
A GFI may trip just because the live/hot and neutral wires connected to it are significantly different in length. Explain why.

Problems & Exercises

Integrated Concepts

A short circuit to the grounded metal case of an appliance occurs as shown in [link]. The person touching the case is wet and only has a
$3.00\ \text{k}\Omega$ resistance to earth/ground. (a) What is the voltage on the case if 5.00 mA flows through the person? (b) What is the current in the short circuit if the resistance of the earth/ground wire is $0.200\ \Omega$? (c) Will this trigger the 20.0 A circuit breaker supplying the appliance?

A person can be shocked even when the case of an appliance is grounded. The large short circuit current produces a voltage on the case of the appliance, since the resistance of the earth/ground wire is not zero.



(a) 15.0 V

(b) 75.0 A

(c) yes

Glossary

thermal hazard
the term for electrical hazards due to overheating
shock hazard
the term for electrical hazards due to current passing through a human
three-wire system
the wiring system used at present for safety reasons, with live, neutral, and ground wires

23.9 Inductance

Inductance

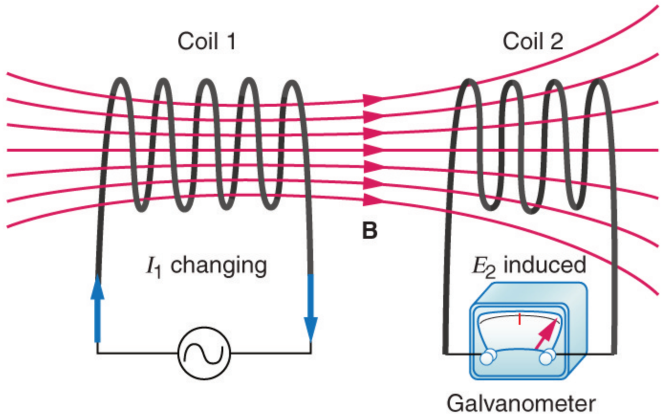
- Calculate the inductance of an inductor.
- Calculate the energy stored in an inductor.
- Calculate the emf generated in an inductor.

Inductors

Induction is the process in which an emf is induced by changing magnetic flux. Many examples have been discussed so far, some more effective than others. Transformers, for example, are designed to be particularly effective at inducing a desired voltage and current with very little loss of energy to other forms. Is there a useful physical quantity related to how "effective" a given device is? The answer is yes, and that physical quantity is called inductance.

Mutual inductance is the effect of Faraday's law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer. See [link], where simple coils induce emfs in one another.

These coils can induce emfs in one another like an inefficient transformer. Their mutual inductance M indicates the effectiveness of the coupling between them. Here a change in current in coil 1 is seen to induce an emf in coil 2. (Note that "E2E2 size 12{E rSub { size 8{2} } } {}
induced" represents the induced emf in coil 2.)



In the many cases where the geometry of the devices is fixed, flux is changed by varying current. We therefore concentrate on the rate of change of current, ΔI/ΔtΔI/Δt size 12{ΔI} {}, as the cause of induction. A change in the current I1I1 size 12{I rSub { size 8{1} } } {} in one device, coil 1 in the figure, induces an emf2emf2 size 12{"emf" rSub { size 8{2} } } {} in the other. We express this in equation form as

emf2=−MΔI1Δt,emf2=−MΔI1Δt, size 12{"emf" rSub { size 8{2} } = – M { {ΔI rSub { size 8{1} } } over {Δt} } } {}

where MM size 12{M} {} is defined to be the mutual inductance between the two devices. The minus sign is an expression of Lenz's law. The larger the mutual inductance MM size 12{M} {}, the more effective the coupling. For example, the coils in [link] have a small MM size 12{M} {} compared with the transformer coils in [link]. Units for MM size 12{M} {} are (V?s)/A=Ω?s(V?s)/A=Ω?s size 12{ (V cdot s) "/A"= %OMEGA cdot s} {}, which is named a henry (H), after Joseph Henry. That is, 1 H=1Ω?s1 H=1Ω?s size 12{1`H=1` %OMEGA cdot s} {}.

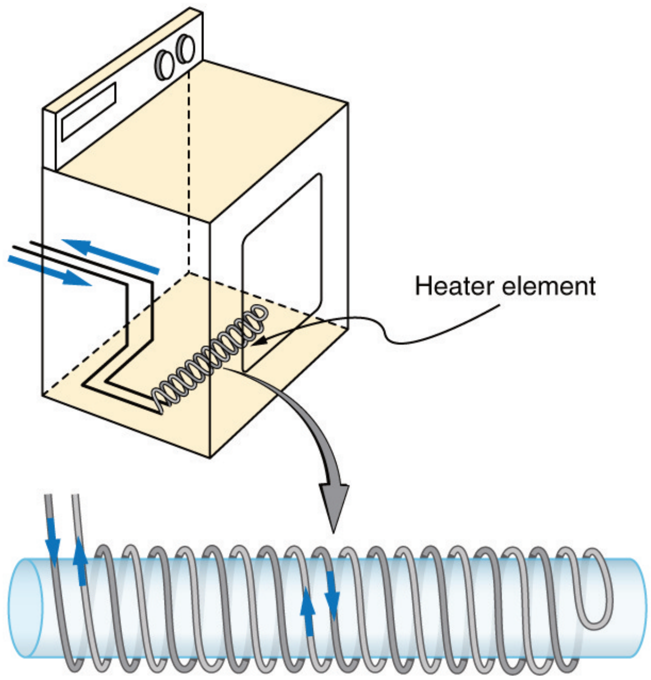
Nature is symmetric here. If we change the current I2I2 size 12{I rSub { size 8{2} } } {} in coil 2, we induce an emf1emf1 size 12{"emf" rSub { size 8{1} } } {} in coil 1, which is given by

emf1=−MΔI2Δt,emf1=−MΔI2Δt, size 12{"emf" rSub { size 8{1} } = – M { {ΔI rSub { size 8{2} } } over {Δt} } } {}

where MM size 12{M} {} is the same as for the reverse process. Transformers run backward with the same effectiveness, or mutual inductance MM size 12{M} {}.

A large mutual inductance MM size 12{M} {} may or may not be desirable. We want a transformer to have a large mutual inductance. But an appliance, such as an electric clothes dryer, can induce a dangerous emf on its case if the mutual inductance between its coils and the case is large. One way to reduce mutual inductance MM size 12{M} {} *is to counterwind coils to cancel the magnetic field produced. (See [link].)*

The heating coils of an electric clothes dryer can be counter-wound so that their magnetic fields cancel one another, greatly reducing the mutual inductance with the case of the dryer.



Self-inductance, the effect of Faraday's law of induction of a device on itself, also exists. When, for example, current through a coil is increased, the magnetic field and flux also increase, inducing a counter emf, as required by Lenz's law. Conversely, if the current is decreased, an emf is induced that opposes the decrease. Most devices have a fixed geometry, and so the change in flux is due entirely to the change in current ΔIΔI size 12{ΔI} {} through the device. The induced emf is related to the physical geometry of the device and the rate of change of current. It is given by

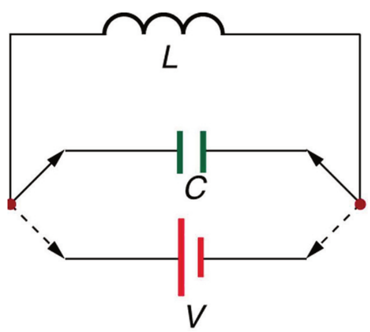
emf=−LΔIΔt,emf=−LΔIΔt, size 12{"emf"= – L { {ΔI} over {Δt} } } {}

where LL size 12{L} {} *is the self-inductance of the device. A device that exhibits significant self-inductance is called an inductor, and given the symbol in [link].**The minus sign is an expression of Lenz's law, indicating that emf opposes the change in current. Units of self-inductance are henries (H) just as for mutual inductance. The larger the self-inductance LL size 12{L} {} of a device, the greater its opposition to any change in current through it. For example, a large coil with many turns and an iron core has a large LL size 12{L} {} and will not allow current to change quickly. To avoid this effect, a small LL size 12{L} {} must be achieved, such as by counterwinding coils as in [link].*

A 1 H inductor is a large inductor. To illustrate this, consider a device with L=1.0 HL=1.0 H size 12{L=1 "." 0`H} {} that has a 10 A current flowing through it. What happens if we try to shut off the current rapidly, perhaps in only 1.0 ms? An emf, given by emf=−L(ΔI/Δt)emf=−L(ΔI/Δt) size 12{"emf"= – L (ΔI/Δt) } {}, will oppose the change. Thus an emf will be induced given by *emf=−L(ΔI/Δt)=(1.0 H)[(10 A)/(1.0 ms)]=10,000 Vemf=−L(ΔI/Δt)=(1.0 H)[(10 A)/(1.0 ms)]=10,000 V. The positive sign means this large voltage is in the same direction as the current, opposing its decrease. Such large emfs can cause arcs, damaging switching equipment, and so it may be necessary to change current more slowly.*

There are uses for such a large induced voltage. Camera flashes use a battery, two inductors that function as a transformer, and a switching system or oscillator to induce large voltages. (Remember that we need a changing magnetic field, brought about by a changing current, to induce a voltage in another coil.) The oscillator system will do this many times as the battery voltage is boosted to over one thousand volts. (You may hear the high pitched whine from the transformer as the capacitor is being charged.) A capacitor stores the high voltage for later use in powering the flash. (See [link].)

Through rapid switching of an inductor, 1.5 V batteries can be used to induce emfs of several thousand volts. This voltage can be used to store charge in a capacitor for later use, such as in a camera flash attachment.



It is possible to calculate LL size 12{L} {} for an inductor given its geometry (size and shape) and knowing the magnetic field that it produces. This is difficult in most cases, because of the complexity of the field created. So in this text the inductance LL size 12{L} {} is usually a given quantity. One exception is the solenoid, because it has a very uniform field inside, a nearly zero field outside, and a simple shape. It is instructive to derive an equation for its inductance. We start by noting that the induced emf is given by Faraday's law of induction as *emf=−N(ΔΦ/Δt)emf=−N(ΔΦ/Δt) size 12{"emf"= – N (ΔΦ/Δt) } {} and, by the definition of self-inductance, as emf=−L(ΔI/Δt)emf=−L(ΔI/Δt) size 12{"emf"= – L (ΔI/Δt) } {}. Equating these yields*

emf=−NΔΦΔt=−LΔIΔt.emf=−NΔΦΔt=−LΔIΔt. size 12{"emf"= – N { {ΔΦ} over {Δt} } = – L { {ΔI} over {Δt} } } {}

Solving for LL size 12{L} {} gives

L=NΔΦΔI.L=NΔΦΔI. size 12{L=N { {ΔΦ} over {ΔI} } } {}

This equation for the self-inductance LL size 12{L} {} of a device is always valid. It means that self-inductance LL size 12{L} {} depends on how effective the current is in creating flux; the more effective, the greater ΔΦΔΦ size 12{ΔΦ} {}/ ΔIΔI size 12{ΔI} {} is.

Let us use this last equation to find an expression for the inductance of a solenoid. Since the area AA of a solenoid is fixed, the change in flux is

ΔΦ=Δ(BA)=AΔBΔΦ=Δ(BA)=AΔB.

To find
ΔBΔB, we note that the magnetic field of a solenoid is given by B=μ0nI=μ0NIℓB=μ0nI=μ0NIℓ size 12{B=μ rSub

{ size 8{0} } ital “nI”=μ rSub { size 8{0} } { { ital “NI”} over {ℓ} } } {}. (Here n=N/ℓn=N/ℓ size 12{n=N/ℓ} {}, where
NN is the number of coils and
ℓℓ is the solenoid’s length.) Only the current changes, so that ΔΦ=AΔB=μ0NAΔIℓΔΦ=AΔB=μ0NAΔIℓ size 12{ΔΦ=AΔB=μ rSub { size 8{0} } ital “NA” { {ΔI} over {ℓ} } } {}. Substituting
ΔΦΔΦ into L=NΔΦΔIL=NΔΦΔI size 12{L=N { {ΔΦ} over {ΔI} } } {} gives

L=NΔΦΔI=Nμ0NAΔIℓΔI.L=NΔΦΔI=Nμ0NAΔIℓΔI. size 12{L=N { {ΔΦ} over {ΔI} } =N { {μ rSub { size 8{0} } ital “NA” { {ΔI} over {ℓ} } } over {ΔI} } } {}

This simplifies to

L=μ0N2Aℓ(solenoid).L=μ0N2Aℓ(solenoid). size 12{L= { {μ rSub { size 8{0} } N rSup { size 8{2} } A} over {ℓ} } } {}

This is the self-inductance of a solenoid of cross-sectional area AA and length
ℓℓ. Note that the inductance depends only on the physical characteristics of the solenoid, consistent with its definition.

Calculating the Self-inductance of a Moderate Size Solenoid

Calculate the self-inductance of a 10.0 cm long, 4.00 cm diameter solenoid that has 200 coils.

Strategy

This is a straightforward application of L=μ0N2AℓL=μ0N2Aℓ size 12{L= { {μ rSub { size 8{0} } N rSup { size 8{2} } A} over {ℓ} } } {}, since all quantities in the equation except LL size 12{L} {} are known.

Solution

Use the following expression for the self-inductance of a solenoid:

L=μ0N2Aℓ.L=μ0N2Aℓ. size 12{L= { {μ rSub { size 8{0} } N rSup { size 8{2} } A} over {ℓ} } } {}

The cross-sectional area in this example is A=πr2=(3.14…)(0.0200 m)2=1.26×10−3m2A=πr2=(3.14…)(0.0200 m)2=1.26×10−3m2 size 12{A=πr rSup { size 8{2} } = (3 “.” “14” “.” “.” “.”) (0 “.” “0200”`m ) rSup { size 8{2} } =1 “.” “26” times “10” rSup { size 8{ – 3} } `m rSup { size 8{2} } } {}, NN is given to be 200, and the length ℓℓ is 0.100 m. We know the permeability of free space is μ0=4π×10−7T?m/Aμ0=4π×10−7T?m/A. Substituting these into the expression for
LL gives

L=(4π×10−7 T?m/A)(200)2(1.26×10−3 m2)0.100 m=0.632 mH.L=(4π×10−7 T?m/A)(200)2(1.26×10−3 m2)0.100 m=0.632 mH.

Discussion

This solenoid is moderate in size. Its inductance of nearly a millihenry is also considered moderate.

One common application of inductance is used in traffic lights that can tell when vehicles are waiting at the intersection. An electrical circuit with an inductor is placed in the road under the place a waiting car will stop over. The body of the car increases the inductance and the circuit changes sending a signal to the traffic lights to change colors. Similarly, metal detectors used for airport security employ the same technique. A coil or inductor in the metal detector frame acts as both a transmitter and a receiver. The pulsed signal in the transmitter coil induces a signal in the receiver. The self-inductance of the circuit is affected by any metal object in the path. Such detectors can be adjusted for sensitivity and also can indicate the approximate location of metal found on a person. (But they will not be able to detect any plastic explosive such as that found on the “underwear bomber.”) See [link].

The familiar security gate at an airport can not only detect metals but also indicate their approximate height above the floor. (credit: Alexbuirds, Wikimedia Commons)

Energy Stored in an Inductor

We know from Lenz’s law that inductances oppose changes in current. There is an alternative way to look at this opposition that is based on energy. Energy is stored in a magnetic field. It takes time to build up energy, and it also takes time to deplete energy; hence, there is an opposition to rapid change. In an inductor, the magnetic field is directly proportional to current and to the inductance of the device. It can be shown that the energy stored in an inductor EindEind size 12{E rSub { size 8{“ind”} } } {} is given by

Eind=12LI2.Eind=12LI2. size 12{E rSub { size 8{“ind”} } = { {1} over {2} } ital “LI” rSup { size 8{2} } } {}

This expression is similar to that for the energy stored in a capacitor.

Calculating the Energy Stored in the Field of a Solenoid

How much energy is stored in the 0.632 mH inductor of the preceding example when a 30.0 A current flows through it?

Strategy

The energy is given by the equation Eind=12LI2Eind=12LI2 size 12{E rSub { size 8{“ind”} } = { {1} over {2} } ital “LI” rSup { size 8{2} } } {}, and all quantities except EindEind size 12{E rSub { size 8{“ind”} } } {} are known.

Solution

Substituting the value for LL size 12{L} {} found in the previous example and the given current into Eind=12LI2Eind=12LI2 size 12{E rSub { size 8{“ind”} } = { {1} over {2} } ital “LI” rSup { size 8{2} } } {} gives

Eind=12LI2=0.5(0.632×10−3 H)(30.0 A)2=0.284 J.Eind=12LI2=0.5(0.632×10−3 H)(30.0 A)2=0.284 J.

Discussion

This amount of energy is certainly enough to cause a spark if the current is suddenly switched off. It cannot be built up instantaneously unless the power input is infinite.

Section Summary

- Inductance is the property of a device that tells how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices in inducing emfs in each other.
- A change in current ΔI1/ΔtΔI1/Δt size 12{ΔI rSub { size 8{1} } /Δt} {} in one induces an emf emf2emf2 size 12{“emf” rSub { size 8{2} } } {} in the second:
  emf2=−MΔI1Δt,emf2=−MΔI1Δt, size 12{“emf” rSub { size 8{2} } = – M { {ΔI rSub { size 8{1} } } over {Δt} } } {}

  where
  MM is defined to be the mutual inductance between the two devices, and the minus sign is due to Lenz’s law.
- Symmetrically, a change in current ΔI2/ΔtΔI2/Δt size 12{ΔI rSub { size 8{2} } /Δt} {} through the second device induces an emf emf1emf1 size 12{“emf” rSub { size 8{1} } } {} in the first:
  emf1=−MΔI2Δt,emf1=−MΔI2Δt, size 12{“emf” rSub { size 8{1} } = – M { {ΔI rSub { size 8{2} } } over {Δt} } } {}

  where
  MM is the same mutual inductance as in the reverse process.
- Current changes in a device induce an emf in the device itself.
- Self-inductance is the effect of the device inducing emf in itself.
- The device is called an inductor, and the emf *induced in it by a change in current through it is*emf=−LΔIΔt,emf=−LΔIΔt, size 12{“emf”= – L { {ΔI} over {Δt} } } {}where LL size 12{L} {} is the self-inductance of the inductor, and ΔI/ΔtΔI/Δt size 12{ΔI/Δt} {} is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz’s law.
- The unit of self- and mutual inductance is the henry (H), where 1 H=1 Ω?s1 H=1 Ω?s size 12{1`H=1` %OMEGA cdot s} {}.
- The self-inductance LL size 12{L} {} of an inductor is proportional to how much flux changes with current. For an NN size 12{N} {}-turn inductor,

L=NΔΦΔI.L=NΔΦΔI. size 12{L=N { {ΔΦ} over {ΔI} } } {}

- The self-inductance of a solenoid is
  L=μ0N2Aℓ(solenoid),L=μ0N2Aℓ(solenoid), size 12{L= { {μ rSub { size 8{0} } N rSup { size 8{2} } A} over {ℓ} } } {}

  where NN size 12{N} {} is its number of turns in the solenoid, AA size 12{A} {} is its cross-sectional area, ℓℓ size 12{ℓ} {} is its length, and μ0=4π×10−7T?m/Aμ0=4π×10−7T?m/A size 12{μ rSub { size 8{0} } =4π times "10" rSup { size 8{"-7"} } `T cdot "m/A"} {} is the permeability of free space.

- The energy stored in an inductor EindEind size 12{E rSub { size 8{"ind"} } } {} is
  Eind=12LI2.Eind=12LI2. size 12{E rSub { size 8{"ind"} } = { {1} over {2} } ital "LI" rSup { size 8{2} } } {}

Conceptual Questions

How would you place two identical flat coils in contact so that they had the greatest mutual inductance? The least?

How would you shape a given length of wire to give it the greatest self-inductance? The least?

Verify, as was concluded without proof in [link], that units of T?m2/A=Ω?s=HT?m2/A=Ω?s=H size 12{T cdot m rSup { size 8{2} } /A= %OMEGA cdot s=H} {}.

Problems & Exercises

Two coils are placed close together in a physics lab to demonstrate Faraday's law of induction. A current of 5.00 A in one is switched off in 1.00 ms, inducing a 9.00 V emf in the other. What is their mutual inductance?

1.80 mH

If two coils placed next to one another have a mutual inductance of 5.00 mH, what voltage is induced in one when the 2.00 A current in the other is switched off in 30.0 ms?

The 4.00 A current through a 7.50 mH inductor is switched off in 8.33 ms. What is the emf induced opposing this?

3.60 V

A device is turned on and 3.00 A flows through it 0.100 ms later. What is the self-inductance of the device if an induced 150 V emf opposes this?

Starting with emf2=−MΔI1Δtemf2=−MΔI1Δt size 12{"emf" rSub { size 8{2} } = – M { {ΔI rSub { size 8{1} } } over {Δt} } } {}, show that the units of inductance are (V?s)/A=Ω?s(V?s)/A=Ω?s size 12{ (V cdot s) "/A"= %OMEGA cdot s} {}.

Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

A large research solenoid has a self-inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) In view of the answer to the last part, is it surprising that shutting it down this quickly is difficult?

(a) 31.3 kV

(b) 125 kJ

(c) 1.56 MW

(d) No, it is not surprising since this power is very high.

(a) Calculate the self-inductance of a 50.0 cm long, 10.0 cm diameter solenoid having 1000 loops. (b) How much energy is stored in this inductor when 20.0 A of current flows through it? (c) How fast can it be turned off if the induced emf cannot exceed 3.00 V?

A precision laboratory resistor is made of a coil of wire 1.50 cm in diameter and 4.00 cm long, and it has 500 turns. (a) What is its self-inductance? (b) What average emf is induced if the 12.0 A current through it is turned on in 5.00 ms (one-fourth of a cycle for 50 Hz AC)? (c) What is its inductance if it is shortened to half its length and counter-wound (two layers of 250 turns in opposite directions)?

(a) 1.39 mH

(b) 3.33 V

(c) Zero

The heating coils in a hair dryer are 0.800 cm in diameter, have a combined length of 1.00 m, and a total of 400 turns. (a) What is their total self-inductance assuming they act like a single solenoid? (b) How much energy is stored in them when 6.00 A flows? (c) What average emf opposes shutting them off if this is done in 5.00 ms (one-fourth of a cycle for 50 Hz AC)?

When the 20.0 A current through an inductor is turned off in 1.50 ms, an 800 V emf is induced, opposing the change. What is the value of the self-inductance?

60.0 mH

How fast can the 150 A current through a 0.250 H inductor be shut off if the induced emf cannot exceed 75.0 V?

Integrated Concepts

A very large, superconducting solenoid such as one used in MRI scans, stores 1.00 MJ of energy in its magnetic field when 100 A flows. (a) Find its self-inductance. (b) If the coils “go normal,” they gain resistance and start

to dissipate thermal energy. What temperature increase is produced if all the stored energy goes into heating the 1000 kg magnet, given its average specific heat is
200 J/kg·ºC200 J/kg·ºC?

(a) 200 H

(b) 5.00ºC5.00ºC

Unreasonable Results

A 25.0 H inductor has 100 A of current turned off in 1.00 ms. (a) What voltage is induced to oppose this? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Glossary

inductance
a property of a device describing how efficient it is at inducing emf in another device
mutual inductance
how effective a pair of devices are at inducing emfs in each other
henry
the unit of inductance; 1H=1Ω?s1H=1Ω?s size 12{1`H=1` %OMEGA cdot s} {}
self-inductance
how effective a device is at inducing emf in itself
inductor
a device that exhibits significant self-inductance
energy stored in an inductor
self-explanatory; calculated by Eind=12LI2Eind=12LI2 size 12{E rSub { size 8{“ind”} } = { {1} over {2} } ital “LI” rSup { size 8{2} } } {}

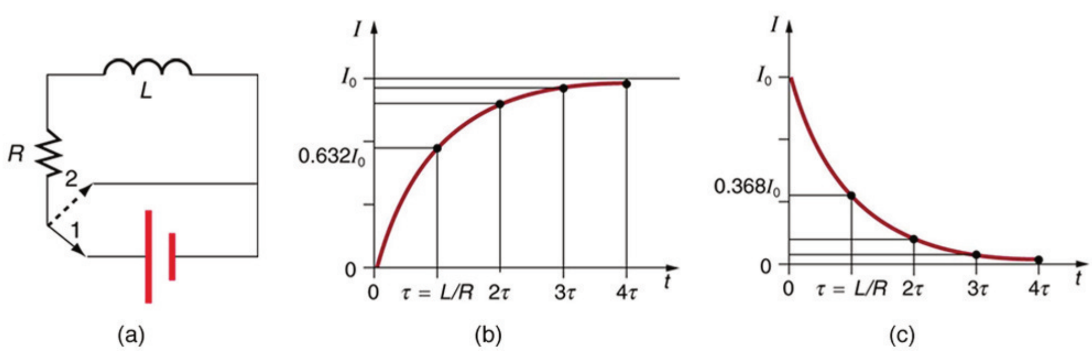
23.10 RL Circuits

RL Circuits

- Calculate the current in an RL circuit after a specified number of characteristic time steps.
- Calculate the characteristic time of an RL circuit.
- Sketch the current in an RL circuit over time.

We know that the current through an inductor LL size 12{L} {} cannot be turned on or off instantaneously. The change in current changes flux, inducing an emf opposing the change (Lenz's law). How long does the opposition last? Current *will* flow and *can* be turned off, but how long does it take? [link] shows a switching circuit that can be used to examine current through an inductor as a function of time.

(a) An *RL* circuit with a switch to turn current on and off. When in position 1, the battery, resistor, and inductor are in series and a current is established. In position 2, the battery is removed and the current eventually stops because of energy loss in the resistor. (b) A graph of current growth versus time when the switch is moved to position 1. (c) A graph of current decay when the switch is moved to position 2.



When the switch is first moved to position 1 (at *t=0t=0 size 12{t=0} {}*), the current is zero and it eventually rises to I0=V/RI0=V/R size 12{I rSub { size 8{0} } = ital "V/R"} {}, where RR is the total resistance of the circuit. The opposition of the inductor LL size 12{L} {} is greatest at the beginning, because the amount of change is greatest. The opposition it poses is in the form of an induced emf, which decreases to zero as the current approaches its final value. The opposing emf is proportional to the amount of change left. This is the hallmark of an exponential behavior, and it can be shown with calculus that

I=I0(1−e−t/τ) (turning on),I=I0(1−e−t/τ) (turning on), size 12{I=I rSub { size 8{0} } (1 – e rSup { size 8{ – t/τ} }) } {}

is the current in an *RL* circuit when switched on (Note the similarity to the exponential behavior of the voltage on a charging capacitor). The initial current is zero and approaches I0=V/RI0=V/R size 12{I rSub { size 8{0} } = ital "V/R"} {} with a characteristic time constant
ττ
for an *RL* circuit, given by

τ=LR,τ=LR, size 12{τ= { {L} over {R} } } {}

where ττ size 12{τ} {} has units of seconds, since
1H=1Ω·s1H=1Ω·s.
In the first period of time ττ size 12{τ} {}, the current rises from zero to 0.632I00.632I0 size 12{0 "." "632"I rSub { size 8{0} } } {}, since I=I0(1−e−1)=I0(1−0.368)=0.632I0I=I0(1−e−1)=I0(1−0.368)=0.632I0 size 12{I=I rSub { size 8{0} } (1 – e rSup { size 8{ – 1} }) =I rSub { size 8{0} } (1 – 0 "." "368") =0 "." "632"I rSub { size 8{0} } } {}. The current will go 0.632 of the remainder in the next time ττ size 12{τ} {}. A well-known property of the exponential is that the final value is never exactly reached, but 0.632 of the remainder to that value is achieved in every characteristic time ττ size 12{τ} {}. In just a few multiples of the time ττ size 12{τ} {}, the final value is very nearly achieved, as the graph in [link](b) illustrates.

The characteristic time ττ size 12{τ} {} depends on only two factors, the inductance LL size 12{L} {} and the resistance RR size 12{R} {}. The greater the inductance LL size 12{L} {}, the greater ττ size 12{τ} {} is, which makes sense since a large inductance is very effective in opposing change. The smaller the resistance RR size 12{R} {}, the greater ττ size 12{τ} {} is. Again this makes sense, since a small resistance means a large final current and a greater change to get there. In both cases—large LL size 12{L} {} and small RR size 12{R} {} —more energy is stored in the inductor and more time is required to get it in and out.

When the switch in [link](a) is moved to position 2 and cuts the battery out of the circuit, the current drops because of energy dissipation by the resistor. But this is also not instantaneous, since the inductor opposes the decrease in current by inducing an emf in the same direction as the battery that drove the current. Furthermore, there is a certain amount of energy, (1/2)LI02(1/2)LI02 size 12{ ("1/2") ital "LI" rSub { size 8{0} } rSup { size 8{2} } } {}, stored in the inductor, and it is dissipated at a finite rate. As the current approaches zero, the rate of decrease slows, since the energy dissipation rate is I2RI2R size 12{ I rSup { size 8{2} } R} {}. Once again the behavior is exponential, and
II
is found to be

I=I0e−t/τ (turning off).I=I0e−t/τ (turning off). size 12{I=I rSub { size 8{0} } e rSup { size 8{ – t/τ} } } {}

(See [link](c).) In the first period of time τ=L/Rτ=L/R size 12{τ=L/R} {} after the switch is closed, the current falls to 0.368 of its initial value, since I=I0e−1=0.368I0I=I0e−1=0.368I0 size 12{I=I rSub { size 8{0} } e rSup { size 8{ – 1} } =0 "." "368"I rSub { size 8{0} } } {}. In each successive time ττ size 12{τ} {}, the current falls to 0.368 of the preceding value, and in a few multiples of ττ size 12{τ} {}, the current becomes very close to zero, as seen in the graph in [link](c).

Calculating Characteristic Time and Current in an *RL* Circuit

(a) What is the characteristic time constant for a 7.50 mH inductor in series with a 3.00 Ω3.00 Ω resistor? (b) Find the current 5.00 ms after the switch is moved to position 2 to disconnect the battery, if it is initially 10.0 A.

Strategy for (a)

The time constant for an *RL* circuit is defined by τ=L/Rτ=L/R size 12{τ=L/R} {}.

Solution for (a)

Entering known values into the expression for ττ size 12{τ} {} given in τ=L/Rτ=L/R size 12{τ=L/R} {} yields

τ=LR=7.50 mH3.00Ω=2.50 ms.τ=LR=7.50 mH3.00Ω=2.50 ms. size 12{τ= { {L} over {R} } = { {7 “.” “50”” mH”} over {3 “.” “00 ” %OMEGA } } =2 “.” “50”” ms”} {}

Discussion for (a)

This is a small but definitely finite time. The coil will be very close to its full current in about ten time constants, or about 25 ms.

Strategy for (b)

We can find the current by using I=I0e−t/τI=I0e−t/τ size 12{I=I rSub { size 8{0} } e rSup { size 8{ – t/τ} } } {}, or by considering the decline in steps. Since the time is twice the characteristic time, we consider the process in steps.

Solution for (b)

In the first 2.50 ms, the current declines to 0.368 of its initial value, which is

I=0.368I0=(0.368)(10.0 A)=3.68 A at t=2.50 ms.I=0.368I0=(0.368)(10.0 A)=3.68 A at t=2.50 ms.

After another 2.50 ms, or a total of 5.00 ms, the current declines to 0.368 of the value just found. That is,

I′=0.368I=(0.368)(3.68 A)=1.35 A at t=5.00 ms.I′=0.368I=(0.368)(3.68 A)=1.35 A at t=5.00 ms.alignl { stack {
size 12{ { {I}} sup { ‘ }=0 “.” “368”I= (0 “.” “368”) (3 “.” “68”” A”) } {} #
size 12{” “=1 “.” “35”” A at “t=5 “.” “00”” ms”} {}
} } {}

Discussion for (b)

After another 5.00 ms has passed, the current will be 0.183 A (see [link]); so, although it does die out, the current certainly does not go to zero instantaneously.

In summary, when the voltage applied to an inductor is changed, the current also changes, *but the change in current lags the change in voltage in an RL circuit*. In Reactance, Inductive and Capacitive, we explore how an *RL* circuit behaves when a sinusoidal AC voltage is applied.

Section Summary

- When a series connection of a resistor and an inductor—an *RL* circuit—is connected to a voltage source, the time variation of the current is
  I=I0(1−e−t/τ)   (turning on).I=I0(1−e−t/τ)   (turning on). size 12{I=I rSub { size 8{0} } (1 – e rSup { size 8{ – t/τ} }) } {}

  where I0=V/RI0=V/R size 12{I rSub { size 8{0} } =V/R} {} is the final current.

- The characteristic time constant ττ size 12{τ} {} is τ=LRτ=LR size 12{τ= { {L} over {R} } } {} , where LL is the inductance and RR is the resistance.
- In the first time constant ττ size 12{τ} {}, the current rises from zero to 0.632I00.632I0 size 12{0 "." "632"I rSub { size 8{0} } } {}, and 0.632 of the remainder in every subsequent time interval ττ size 12{τ} {}.
- When the inductor is shorted through a resistor, current decreases as
  I=I0e−t/τ   (turning off).I=I0e−t/τ   (turning off). size 12{I=I rSub { size 8{0} } e rSup { size 8{ – t/τ} } } {}

  Here I0I0 size 12{I rSub { size 8{0} } } {} is the initial current.

- Current falls to 0.368I00.368I0 size 12{0 "." "368"I rSub { size 8{0} } } {} in the first time interval ττ size 12{τ} {}, and 0.368 of the remainder toward zero in each subsequent time ττ size 12{τ} {}.

Problem Exercises

If you want a characteristic *RL* time constant of 1.00 s, and you have a 500 Ω500 Ω resistor, what value of self-inductance is needed?

500 H

Your *RL* circuit has a characteristic time constant of 20.0 ns, and a resistance of 5.00 MΩ5.00 MΩ. (a) What is the inductance of the circuit? (b) What resistance would give you a 1.00 ns time constant, perhaps needed for quick response in an oscilloscope?

A large superconducting magnet, used for magnetic resonance imaging, has a 50.0 H inductance. If you want current through it to be adjustable with a 1.00 s characteristic time constant, what is the minimum resistance of system?

50.0 Ω50.0 Ω

Verify that after a time of 10.0 ms, the current for the situation considered in [link] will be 0.183 A as stated.

Suppose you have a supply of inductors ranging from 1.00 nH to 10.0 H, and resistors ranging from
0.100 Ω0.100 Ω to

1.00 MΩ1.00 MΩ. What is the range of characteristic *RL* time constants you can produce by connecting a single resistor to a single inductor?

1.00×10–18 s1.00×10–18 s size 12{1 "." "00" times "10" rSup { size 8{"-15″} } " s"} {} to 0.100 s

(a) What is the characteristic time constant of a 25.0 mH inductor that has a resistance of
4.00 Ω4.00 Ω? (b) If it is connected to a 12.0 V battery, what is the current after 12.5 ms?

What percentage of the final current
I0I0
flows through an inductor LL size 12{L} {} in series with a resistor RR size 12{R} {}, three time constants after the circuit is completed?

95.0%

The 5.00 A current through a 1.50 H inductor is dissipated by a 2.00 Ω2.00 Ω resistor in a circuit like that in [link] with the switch in position 2. (a) What is the initial energy in the inductor? (b) How long will it take the current to decline to 5.00% of its initial value? (c) Calculate the average power dissipated, and compare it with the initial power dissipated by the resistor.

(a) Use the exact exponential treatment to find how much time is required to bring the current through an 80.0 mH inductor in series with a
15.0 Ω15.0 Ω
resistor to 99.0% of its final value, starting from zero. (b) Compare your answer to the approximate treatment using integral numbers of ττ size 12{τ} {}. (c) Discuss how significant the difference is.

(a) 24.6 ms

(b) 26.7 ms

(c) 9% difference, which is greater than the inherent uncertainty in the given parameters.

(a) Using the exact exponential treatment, find the time required for the current through a 2.00 H inductor in series with a
0.500 Ω0.500 Ω
resistor to be reduced to 0.100% of its original value. (b) Compare your answer to the approximate treatment using integral numbers of ττ size 12{τ} {}. (c) Discuss how significant the difference is.

Glossary

characteristic time constant
denoted by ττ size 12{τ} {}, of a particular series RL circuit is calculated by τ=LRτ=LR size 12{τ= { {L} over {R} } } {}, where LL size 12{L} {} is the inductance and RR is the resistance

23.11 Reactance, Inductive and Capacitive

Reactance, Inductive and Capacitive

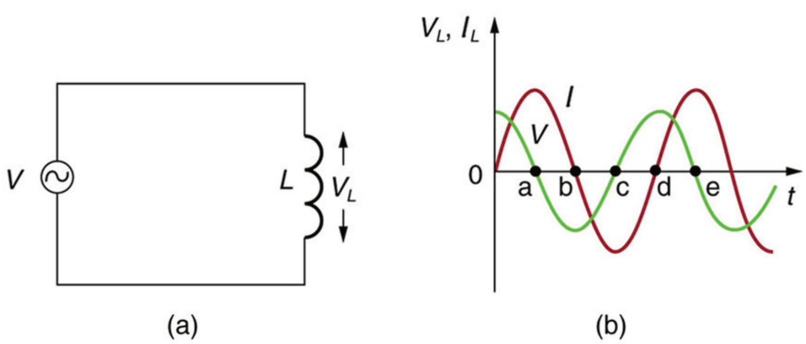
- Sketch voltage and current versus time in simple inductive, capacitive, and resistive circuits.
- Calculate inductive and capacitive reactance.
- Calculate current and/or voltage in simple inductive, capacitive, and resistive circuits.

Many circuits also contain capacitors and inductors, in addition to resistors and an AC voltage source. We have seen how capacitors and inductors respond to DC voltage when it is switched on and off. We will now explore how inductors and capacitors react to sinusoidal AC voltage.

Inductors and Inductive Reactance

Suppose an inductor is connected directly to an AC voltage source, as shown in [link]. It is reasonable to assume negligible resistance, since in practice we can make the resistance of an inductor so small that it has a negligible effect on the circuit. Also shown is a graph of voltage and current as functions of time.

(a) An AC voltage source in series with an inductor having negligible resistance. (b) Graph of current and voltage across the inductor as functions of time.



The graph in [link](b) starts with voltage at a maximum. Note that the current starts at zero and rises to its peak *after* the voltage that drives it, just as was the case when DC voltage was switched on in the preceding section. When the voltage becomes negative at point a, the current begins to decrease; it becomes zero at point b, where voltage is its most negative. The current then becomes negative, again following the voltage. The voltage becomes positive at point c and begins to make the current less negative. At point d, the current goes through zero just as the voltage reaches its positive peak to start another cycle. This behavior is summarized as follows:

AC Voltage in an Inductor

When a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90º90º phase angle.

Current lags behind voltage, since inductors oppose change in current. Changing current induces a back emf V=−L(ΔI/Δt)V=−L(ΔI/Δt) size 12{V= – L (ΔI/Δt) } {}. This is considered to be an effective resistance of the inductor to AC. The rms current II size 12{I} {} through an inductor LL size 12{L} {} is given by a version of Ohm’s law:

I=VXL,I=VXL, size 12{I= { {V} over {X rSub { size 8{L} } } } } {}

where
VV is the rms voltage across the inductor and XLXL size 12{X rSub { size 8{L} } } {} is defined to be

XL=2πfL,XL=2πfL, size 12{X rSub { size 8{L} } =2π ital “fL”} {}

with ff size 12{f} {} the frequency of the AC voltage source in hertz (An analysis of the circuit using Kirchhoff’s loop rule and calculus actually produces this expression). XLXL size 12{X rSub { size 8{L} } } {} is called the inductive reactance, because the inductor reacts to impede the current. XLXL size 12{X rSub { size 8{L} } } {} has units of ohms (1 H=1 Ω?s1 H=1 Ω?s, so that frequency times inductance has units of (cycles/s)(Ω?s)=Ω(cycles/s)(Ω?s)=Ω size 12{ (“cycles/s”) (` %OMEGA cdot s) = %OMEGA } {}), consistent with its role as an effective resistance. It makes sense that XLXL size 12{X rSub { size 8{L} } } {} is proportional to LL size 12{L} {}, since the greater the induction the greater its resistance to change. It is also reasonable that XLXL size 12{X rSub { size 8{L} } } {} is proportional to frequency ff size 12{f} {}, since greater frequency means greater change in current. That is, ΔI/ΔtΔI/Δt size 12{ΔI} {} is large for large frequencies (large ff size 12{f} {}, small ΔtΔt size 12{Δt} {}). The greater the change, the greater the opposition of an inductor.

Calculating Inductive Reactance and then Current

(a) Calculate the inductive reactance of a 3.00 mH inductor when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?

Strategy

The inductive reactance is found directly from the expression XL=2πfLXL=2πfL size 12{X rSub { size 8{L} } =2π ital “fL”} {}. Once XLXL size 12{X rSub { size 8{L} } } {} has been found at each frequency, Ohm’s law as stated in the Equation I=V/XLI=V/XL size 12{I=V/X rSub { size 8{L} } } {} can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and inductance into Equation $X_L=2\pi fL$XL=2πfL size 12{X rSub { size 8{L} } =2π ital “fL”} {} gives

$X_L=2\pi fL=6.28(60.0/\text{ s})(3.00\text{ mH})=1.13\ \Omega$ at 60 Hz.XL=2πfL=6.28(60.0/ s)(3.00 mH)=1.13 Ω at 60 Hz.

Similarly, at 10 kHz,

$X_L=2\pi fL=6.28(1.00\times10^4/\text{s})(3.00\text{ mH})=188\ \Omega$ at 10 kHz.XL=2πfL=6.28(1.00×104/s)(3.00 mH)=188 Ω at 10 kHz. size 12{X rSub { size 8{L} } =2π ital “fL”=6 “.” “28” (3 “.” “00”” mH”) =”188″ %OMEGA } {}

Solution for (b)

The rms current is now found using the version of Ohm’s law in Equation $I=V/X_L$I=V/XL size 12{I=V/X rSub { size 8{L} } } {}, given the applied rms voltage is 120 V. For the first frequency, this yields

$I=\frac{V}{X_L}=\frac{120\text{ V}}{1.13\ \Omega}=106\text{ A}$ at 60 Hz.I=VXL=120 V1.13 Ω=106 A at 60 Hz.

Similarly, at 10 kHz,

$I=\frac{V}{X_L}=\frac{120\text{ V}}{188\ \Omega}=0.637\text{ A}$ at 10 kHz.I=VXL=120 V188 Ω=0.637 A at 10 kHz. size 12{I= { {V} over {X rSub { size 8{L} } } } = { {“120″” V”} over {“188 ” %OMEGA } } =0 “.” “637”” A”} {}

Discussion

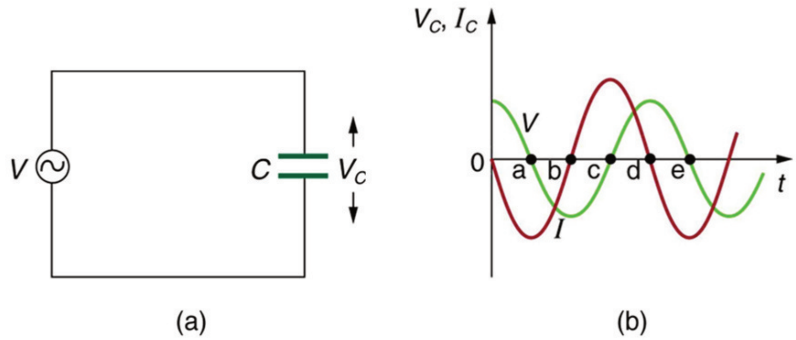
The inductor reacts very differently at the two different frequencies. At the higher frequency, its reactance is large and the current is small, consistent with how an inductor impedes rapid change. Thus high frequencies are impeded the most. Inductors can be used to filter out high frequencies; for example, a large inductor can be put in series with a sound reproduction system or in series with your home computer to reduce high-frequency sound output from your speakers or high-frequency power spikes into your computer.

Note that although the resistance in the circuit considered is negligible, the AC current is not extremely large because inductive reactance impedes its flow. With AC, there is no time for the current to become extremely large.

Capacitors and Capacitive Reactance

Consider the capacitor connected directly to an AC voltage source as shown in [link]. The resistance of a circuit like this can be made so small that it has a negligible effect compared with the capacitor, and so we can assume negligible resistance. Voltage across the capacitor and current are graphed as functions of time in the figure.

(a) An AC voltage source in series with a capacitor *C* having negligible resistance. (b) Graph of current and voltage across the capacitor as functions of time.



The graph in [link] starts with voltage across the capacitor at a maximum. The current is zero at this point, because the capacitor is fully charged and halts the flow. Then voltage drops and the current becomes negative as the capacitor discharges. At point a, the capacitor has fully discharged (*Q=0Q=0 size 12{Q=0} {}* on it) and the voltage across it is zero. The current remains negative between points a and b, causing the voltage on the capacitor to reverse. This is complete at point b, where the current is zero and the voltage has its most negative value. The current becomes positive after point b, neutralizing the charge on the capacitor and bringing the voltage to zero at point c, which allows the current to reach its maximum. Between points c and d, the current drops to zero as the voltage rises to its peak, and the process starts to repeat. Throughout the cycle, the voltage follows what the current is doing by one-fourth of a cycle:

AC Voltage in a Capacitor

When a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by
a
90º90º
phase angle.

The capacitor is affecting the current, having the ability to stop it altogether when fully charged. Since an AC voltage is applied, there is an rms current, but it is limited by the capacitor. This is considered to be an effective resistance of the capacitor to AC, and so the rms current II size 12{I} {} in the circuit containing only a capacitor CC size 12{C} {} is given by another version of Ohm's law to be

I=VXC,I=VXC, size 12{I= { {V} over {X rSub { size 8{C} } } } } {}

where VV size 12{V} {} is the rms voltage and XCXC size 12{X rSub { size 8{C} } } {} is defined (As with XLXL size 12{X rSub { size 8{L} } } {}, this expression for XCXC size 12{X rSub { size 8{C} } } {} results from an analysis of the circuit using Kirchhoff's rules and calculus) to be

XC=12πfC,XC=12πfC, size 12{X rSub { size 8{C} } = { {1} over {2π ital "fC"} } } {}

where XCXC size 12{X rSub { size 8{C} } } {} is called the capacitive reactance, because the capacitor reacts to impede the current. XCXC size 12{X rSub { size 8{C} } } {} has units of ohms (verification left as an exercise for the reader). XCXC size 12{X rSub { size 8{C} } } {} is inversely proportional to the capacitance CC size 12{C} {}; the larger the capacitor, the greater the charge it can store and the greater the current that can flow. It is also inversely proportional to the frequency ff size 12{f} {}; the greater the frequency, the less time there is to fully charge the capacitor, and so it impedes current less.

Calculating Capacitive Reactance and then Current

(a) Calculate the capacitive reactance of a 5.00 mF capacitor when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current if the applied rms voltage is 120 V?

Strategy

The capacitive reactance is found directly from the expression in XC=12πfCXC=12πfC size 12{X rSub { size 8{C} } = { {1} over {2π ital "fC"} } } {}. Once
XCXC

has been found at each frequency, Ohm's law stated as I=V/XCI=V/XC size 12{I=V/X rSub { size 8{C} } } {} can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and capacitance into XC=12πfCXC=12πfC size 12{X rSub { size 8{C} } = { {1} over {2π ital "fC"} } } {} gives

XC=12πfC=16.28(60.0/s)(5.00 μF)=531 Ω at 60 Hz.XC=12πfC=16.28(60.0/s)(5.00 μF)=531 Ω at 60 Hz.alignl { stack {
size 12{X rSub { size 8{C} } = { {1} over {2π ital "fC"} } } {} #
" "= { {1} over {6 "." "28" ("60" "." 0/s) (5 "." "00" μF) } } ="531 " %OMEGA " at 60 Hz" {}
} } {}

Similarly, at 10 kHz,

XC=12πfC=16.28(1.00×104/s)(5.00 μF)=3.18 Ω at 10 kHz.XC=12πfC=16.28(1.00×104/s)(5.00 μF)=3.18 Ω at 10 kHz.alignl { stack {
size 12{X rSub { size 8{C} } = { {1} over {2π ital "fC"} } = { {1} over {6 "." "28" (1 "." "00" times "10" rSup { size 8{4} } /s) (5 "." "00" μF) } } } {} #
" "=3 "." "18" %OMEGA " at 10 kHz" {}
} } {}

Solution for (b)

The rms current is now found using the version of Ohm's law in I=V/XCI=V/XC size 12{I=V/X rSub { size 8{C} } } {}, given the applied rms voltage is 120 V. For the first frequency, this yields

I=VXC=120 V531 Ω=0.226 A at 60 Hz.I=VXC=120 V531 Ω=0.226 A at 60 Hz. size 12{I= { {V} over {X rSub { size 8{C} } } } = { {"120"" V"} over {"531 " %OMEGA } } =0 "." "226"" A"} {}

Similarly, at 10 kHz,

I=VXC=120 V3.18 Ω=37.7 A at 10 kHz.I=VXC=120 V3.18 Ω=37.7 A at 10 kHz. size 12{I= { {V} over {X rSub { size 8{C} } } } = { {“120″” V”} over {3 “.” “18 ” %OMEGA } } =”37″ “.” 7″ A”} {}

Discussion

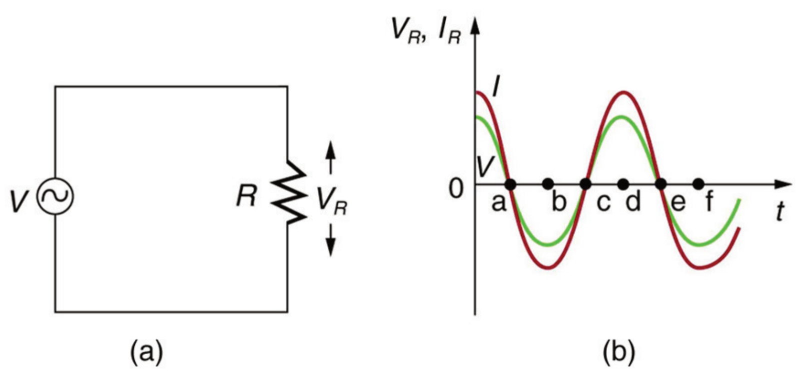
The capacitor reacts very differently at the two different frequencies, and in exactly the opposite way an inductor reacts. At the higher frequency, its reactance is small and the current is large. Capacitors favor change, whereas inductors oppose change. Capacitors impede low frequencies the most, since low frequency allows them time to become charged and stop the current. Capacitors can be used to filter out low frequencies. For example, a capacitor in series with a sound reproduction system rids it of the 60 Hz hum.

Although a capacitor is basically an open circuit, there is an rms current in a circuit with an AC voltage applied to a capacitor. This is because the voltage is continually reversing, charging and discharging the capacitor. If the frequency goes to zero (DC), XCXC size 12{X rSub { size 8{C} } } {} tends to infinity, and the current is zero once the capacitor is charged. At very high frequencies, the capacitor’s reactance tends to zero—it has a negligible reactance and does not impede the current (it acts like a simple wire). *Capacitors have the opposite effect on AC circuits that inductors have*.

Resistors in an AC Circuit

Just as a reminder, consider [link], which shows an AC voltage applied to a resistor and a graph of voltage and current versus time. The voltage and current are exactly *in phase* in a resistor. There is no frequency dependence to the behavior of plain resistance in a circuit:

(a) An AC voltage source in series with a resistor. (b) Graph of current and voltage across the resistor as functions of time, showing them to be exactly in phase.



AC Voltage in a Resistor

When a sinusoidal voltage is applied to a resistor, the voltage is exactly in phase with the current—they have a 0º0º phase angle.

Section Summary

- For inductors in AC circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90º90º phase angle.
- The opposition of an inductor to a change in current is expressed as a type of AC resistance.
- Ohm's law for an inductor is
  I=VXL,I=VXL, size 12{I= { {V} over {X rSub { size 8{L} } } } } {}

  where VV size 12{V} {} is the rms voltage across the inductor.
- XLXL size 12{X rSub { size 8{L} } } {} is defined to be the inductive reactance, given by
  XL=2πfL,XL=2πfL, size 12{X rSub { size 8{L} } =2π ital "fL"} {}

  with ff size 12{f} {} the frequency of the AC voltage source in hertz.
- Inductive reactance XLXL size 12{X rSub { size 8{L} } } {} has units of ohms and is greatest at high frequencies.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by a 90º90º phase angle.
- Since a capacitor can stop current when fully charged, it limits current and offers another form of AC resistance; Ohm's law for a capacitor is
  I=VXC,I=VXC, size 12{I= { {V} over {X rSub { size 8{C} } } } } {}

  where VV size 12{V} {} is the rms voltage across the capacitor.
- XCXC size 12{X rSub { size 8{C} } } {} is defined to be the capacitive reactance, given by
  XC=12πfC.XC=12πfC. size 12{X rSub { size 8{C} } = { {1} over {2π ital "fC"} } } {}
- XCXC size 12{X rSub { size 8{C} } } {} has units of ohms and is greatest at low frequencies.

Conceptual Questions

Presbycusis is a hearing loss due to age that progressively affects higher frequencies. A hearing aid amplifier is designed to amplify all frequencies equally. To adjust its output for presbycusis, would you put a capacitor in series or parallel with the hearing aid's speaker? Explain.

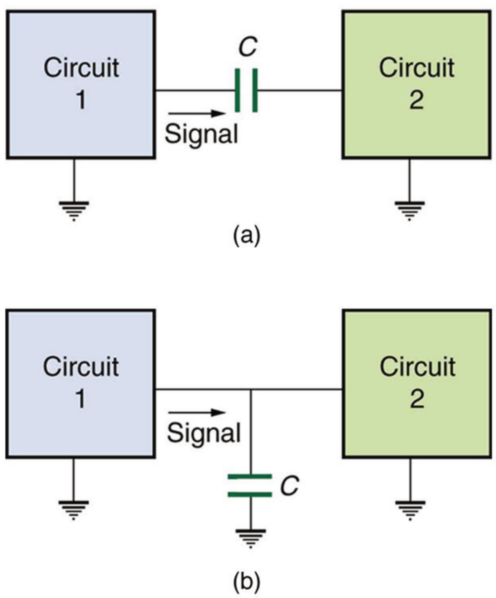
Would you use a large inductance or a large capacitance in series with a system to filter out low frequencies, such as the 100 Hz hum in a sound system? Explain.

High-frequency noise in AC power can damage computers. Does the plug-in unit designed to prevent this damage use a large inductance or a large capacitance (in series with the computer) to filter out such high frequencies? Explain.

Does inductance depend on current, frequency, or both? What about inductive reactance?

Explain why the capacitor in [link](a) acts as a low-frequency filter between the two circuits, whereas that in [link](b) acts as a high-frequency filter.

Capacitors and inductors. Capacitor with high frequency and low frequency.



If the capacitors in [link] are replaced by inductors, which acts as a low-frequency filter and which as a high-frequency filter?

Problems & Exercises

At what frequency will a 30.0 mH inductor have a reactance of
100 Ω100 Ω?

531 Hz

What value of inductance should be used if a
20.0 kΩ20.0 kΩ
reactance is needed at a frequency of 500 Hz?

What capacitance should be used to produce a
2.00 MΩ2.00 MΩ reactance at 60.0 Hz?

1.33 nF

At what frequency will an 80.0 mF capacitor have a reactance of 0.250 Ω0.250 Ω?

(a) Find the current through a 0.500 H inductor connected to a 60.0 Hz, 480 V AC source. (b) What would the current be at 100 kHz?

(a) 2.55 A

(b) 1.53 mA

(a) What current flows when a 60.0 Hz, 480 V AC source is connected to a
0.250 μF0.250 μF
capacitor? (b) What would the current be at 25.0 kHz?

A 20.0 kHz, 16.0 V source connected to an inductor produces a 2.00 A current. What is the inductance?

63.7 μH63.7 μH

A 20.0 Hz, 16.0 V source produces a 2.00 mA current when connected to a capacitor. What is the capacitance?

(a) An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a
2.00 kΩ2.00 kΩ reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?

(a) 21.2 mH

(b) 8.00 Ω8.00 Ω

The capacitor in [link](a) is designed to filter low-frequency signals, impeding their transmission between circuits. (a) What capacitance is needed to produce a
100 kΩ100 kΩ reactance at a frequency of 120 Hz? (b) What would its reactance be at 1.00 MHz? (c) Discuss the implications of your answers to (a) and (b).

The capacitor in [link](b) will filter high-frequency signals by shorting them to earth/ground. (a) What capacitance is needed to produce a reactance of
10.0 mΩ10.0 mΩ for a 5.00 kHz signal? (b) What would its reactance be at 3.00 Hz? (c) Discuss the implications of your answers to (a) and (b).

(a) 3.18 mF

(b) 16.7 Ω16.7 Ω

Unreasonable Results

In a recording of voltages due to brain activity (an EEG), a 10.0 mV signal with a 0.500 Hz frequency is applied to a capacitor, producing a current of 100 mA. Resistance is negligible. (a) What is the capacitance? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Construct Your Own Problem

Consider the use of an inductor in series with a computer operating on 60 Hz electricity. Construct a problem in which you calculate the relative reduction in voltage of incoming high frequency noise compared to 60 Hz voltage. Among the things to consider are the acceptable series reactance of the inductor for 60 Hz power and the likely frequencies of noise coming through the power lines.

Glossary

inductive reactance

the opposition of an inductor to a change in current; calculated by XL=2πfLXL=2πfL size 12{X rSub { size 8{L} } =2π ital “fL”} {}

capacitive reactance

the opposition of a capacitor to a change in current; calculated by XC=12πfCXC=12πfC size 12{X rSub { size 8{C} } = { {1} over {2π ital “fC”} } } {}

23.12 RLC Series AC Circuits

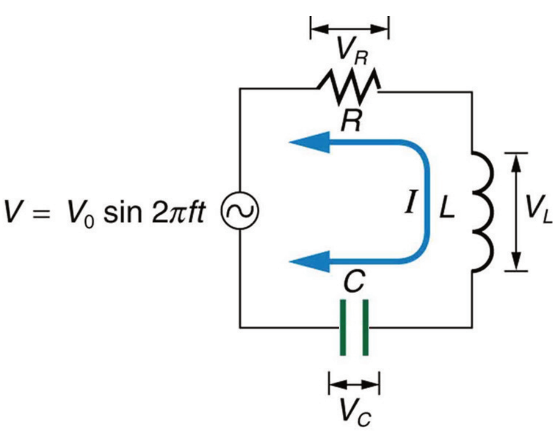
RLC Series AC Circuits

- Calculate the impedance, phase angle, resonant frequency, power, power factor, voltage, and/or current in a RLC series circuit.
- Draw the circuit diagram for an RLC series circuit.
- Explain the significance of the resonant frequency.

Impedance

When alone in an AC circuit, inductors, capacitors, and resistors all impede current. How do they behave when all three occur together? Interestingly, their individual resistances in ohms do not simply add. Because inductors and capacitors behave in opposite ways, they partially to totally cancel each other's effect. [link] shows an *RLC* series circuit with an AC voltage source, the behavior of which is the subject of this section. The crux of the analysis of an *RLC* circuit is the frequency dependence of XLXL size 12{X rSub { size 8{L} } } {} and XCXC size 12{X rSub { size 8{C} } } {}, and the effect they have on the phase of voltage versus current (established in the preceding section). These give rise to the frequency dependence of the circuit, with important "resonance" features that are the basis of many applications, such as radio tuners.

An *RLC* series circuit with an AC voltage source.



The combined effect of resistance RR size 12{R} {}, inductive reactance XLXL size 12{X rSub { size 8{L} } } {}, and capacitive reactance XCXC size 12{X rSub { size 8{C} } } {} is defined to be impedance, an AC analogue to resistance in a DC circuit. Current, voltage, and impedance in an *RLC* circuit are related by an AC version of Ohm's law:

I0=V0Z or Irms=VrmsZ.I0=V0Z or Irms=VrmsZ. size 12{I rSub { size 8{0} } = { {V rSub { size 8{0} } } over {Z} } " or "I rSub { size 8{ ital "rms"} } = { {V rSub { size 8{ ital "rms"} } } over {Z} } "." } {}

Here I0I0 size 12{I rSub { size 8{0} } } {} is the peak current, V0V0 size 12{V rSub { size 8{0} } } {} the peak source voltage, and ZZ is the impedance of the circuit. The units of impedance are ohms, and its effect on the circuit is as you might expect: the greater the impedance, the smaller the current. To get an expression for ZZ size 12{Z} {} in terms of RR, XLXL size 12{X rSub { size 8{L} } } {}, and XCXC size 12{X rSub { size 8{C} } } {}, we will now examine how the voltages across the various components are related to the source voltage. Those voltages are labeled VRVR size 12{V rSub { size 8{R} } } {}, VLVL size 12{V rSub { size 8{L} } } {}, and VCVC size 12{V rSub { size 8{C} } } {} in [link].

Conservation of charge requires current to be the same in each part of the circuit at all times, so that we can say the currents in RR size 12{R} {}, LL size 12{L} {}, and CC size 12{C} {} are equal and in phase. But we know from the preceding section that the voltage across the inductor VLVL size 12{V rSub { size 8{L} } } {} leads the current by one-fourth of a cycle, the voltage across the capacitor VCVC size 12{V rSub { size 8{C} } } {} follows the current by one-fourth of a cycle, and the voltage across the resistor VRVR size 12{V rSub { size 8{R} } } {} is exactly in phase with the current. [link] shows these relationships in one graph, as well as showing the total voltage around the circuit V=VR+VL+VCV=VR+VL+VC size 12{V=V rSub { size 8{R} } +V rSub { size 8{L} } +V rSub { size 8{C} } } {}, where all four voltages are the instantaneous values. According to Kirchhoff's loop rule, the total voltage around the circuit VV is also the voltage of the source.

You can see from [link] that while VRVR size 12{V rSub { size 8{R} } } {} is in phase with the current, VLVL size 12{V rSub { size 8{L} } } {} leads by 90º90º, and VCVC size 12{V rSub { size 8{C} } } {} follows by 90º90º. Thus VLVL size 12{V rSub { size 8{L} } } {} and VCVC size 12{V rSub { size 8{C} } } {} are 180º180º out of phase (crest to trough) and tend to cancel, although not completely unless they have the same magnitude. Since the peak voltages are not aligned (not in phase), the peak voltage V0V0 size 12{V rSub { size 8{0} } } {} of the source does *not* equal the sum of the peak voltages across RR size 12{R} {}, LL size 12{L} {}, and CC size 12{C} {}. The actual relationship is

V0=V0R2+(V0L−V0C)2,V0=V0R2+(V0L−V0C)2, size 12{V rSub { size 8{0} } = sqrt {V rSub { size 8{0R} } "" lSup { size 8{2} } + (V rSub { size 8{0L} } – V rSub { size 8{0C} }) rSup { size 8{2} } } ,} {}

where V0RV0R size 12{V rSub { size 8{0R} } } {}, V0LV0L size 12{V rSub { size 8{0L} } } {}, and V0CV0C size 12{V rSub { size 8{0C} } } {} are the peak voltages across RR size 12{R} {}, LL size 12{L} {}, and CC size 12{C} {}, respectively. Now, using Ohm's law and definitions from Reactance, Inductive and Capacitive, we substitute V0=I0ZV0=I0Z size 12{V rSub { size 8{0} } =I rSub { size 8{0} } Z} {} into the above, as well as V0R=I0RV0R=I0R size 12{V rSub { size 8{0R} } =I rSub { size 8{0} } R} {}, V0L=I0XLV0L=I0XL size 12{V rSub { size 8{0L} } =I rSub { size 8{0} } X rSub { size 8{L} } } {}, and V0C=I0XCV0C=I0XC size 12{V rSub { size 8{0C} } =I rSub { size 8{0} } X rSub { size 8{C} } } {}, yielding

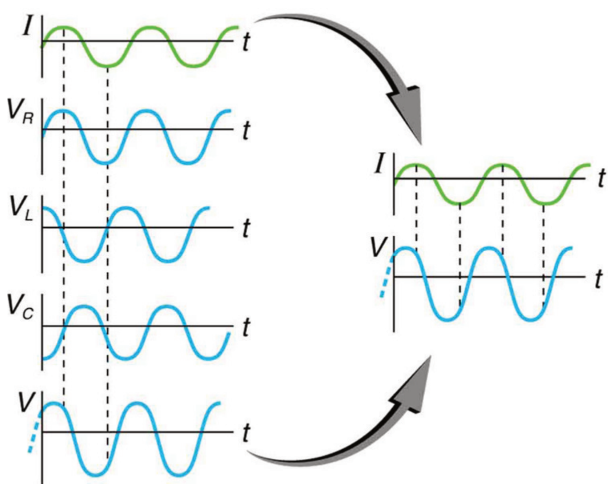
I0Z=I02R2+(I0XL−I0XC)2=I0R2+(XL−XC)2.I0Z=I02R2+(I0XL−I0XC)2=I0R2+(XL−XC)2. size 12{I rSub { size 8{0} } Z= sqrt {I rSub { size 8{0} rSup { size 8{2} } } R rSup { size 8{2} } + (I rSub { size 8{0} } X rSub { size 8{L} } – I rSub { size 8{0} } X rSub { size 8{C} }) rSup { size 8{2} } } =I rSub { size 8{0} } sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {}

I0I0 size 12{I rSub { size 8{0} } } {} cancels to yield an expression for ZZ:

Z=R2+(XL−XC)2,Z=R2+(XL−XC)2, size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {}

which is the impedance of an *RLC* series AC circuit. For circuits without a resistor, take R=0R=0; for those without an inductor, take XL=0XL=0 size 12{X rSub { size 8{L} } =0} {}; and for those without a capacitor, take XC=0XC=0 size 12{X rSub { size 8{C} } =0} {}.

This graph shows the relationships of the voltages in an *RLC* circuit to the current. The voltages across the circuit elements add to equal the voltage of the source, which is seen to be out of phase with the current.



Calculating Impedance and Current

An *RLC* series circuit has a 40.0 Ω40.0 Ω resistor, a 3.00 mH inductor, and a
5.00 μF5.00 μF capacitor. (a) Find the circuit’s impedance at 60.0 Hz and 10.0 kHz, noting that these frequencies and the values for
LL and
CC
are the same as in [link] and [link]. (b) If the voltage source has Vrms=120VVrms=120V size 12{V rSub { size 8{“rms”} } =”120″`V} {}, what is IrmsIrms size 12{I rSub { size 8{“rms”} } } {} at each frequency?

Strategy

For each frequency, we use Z=R2+(XL−XC)2Z=R2+(XL−XC)2 size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {} to find the impedance and then Ohm’s law to find current. We can take advantage of the results of the previous two examples rather than calculate the reactances again.

Solution for (a)

At 60.0 Hz, the values of the reactances were found in [link] to be XL=1.13ΩXL=1.13Ω size 12{X rSub { size 8{L} } =1 "." "13" %OMEGA } {} and in [link] to be XC=531 ΩXC=531 Ω size 12{X rSub { size 8{C} } ="531 " %OMEGA } {}. Entering these and the given 40.0 Ω40.0 Ω for resistance into Z=R2+(XL−XC)2Z=R2+(XL−XC)2 size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {} yields

Z=R2+(XL−XC)2=(40.0Ω)2+(1.13Ω−531Ω)2=531Ω at 60.0
Hz.Z=R2+(XL−XC)2=(40.0Ω)2+(1.13Ω−531Ω)2=531Ω at 60.0 Hz.alignl { stack {
size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } }
{} #
" "= sqrt { ("40" "." 0` %OMEGA) rSup { size 8{2} } + (1 "." "13" %OMEGA – "531" %OMEGA) rSup {
size 8{2} } } {} #
" "="531″ %OMEGA " at 60″ "." "0 Hz" {}
} } {}

Similarly, at 10.0 kHz, XL=188ΩXL=188Ω size 12{X rSub { size 8{L} } ="188″ %OMEGA } {} and XC=3.18ΩXC=3.18Ω size 12{X rSub { size 8{C} } =3 "." "18" %OMEGA } {}, so that

Z=(40.0Ω)2+(188Ω−3.18Ω)2=190Ω at 10.0 kHz.Z=(40.0Ω)2+(188Ω−3.18Ω)2=190Ω at 10.0 kHz.alignl { stack
{
size 12{Z= sqrt { ("40" "." 0` %OMEGA) rSup { size 8{2} } + ("188" %OMEGA – 3 "." "18" %OMEGA)
rSup { size 8{2} } } } {} #
" "="190″ %OMEGA " at 10″ "." "0 kHz" {}
} } {}

Discussion for (a)

In both cases, the result is nearly the same as the largest value, and the impedance is definitely not the sum of the individual values. It is clear that XLXL size 12{X rSub { size 8{L} } } {} dominates at high frequency and XCXC size 12{X rSub { size 8{C} } } {} dominates at low frequency.

Solution for (b)

The current IrmsIrms size 12{I rSub { size 8{"rms"} } } {} can be found using the AC version of Ohm's law in Equation Irms=Vrms/ZIrms=Vrms/Z size 12{I rSub { size 8{"rms"} } =V rSub { size 8{"rms"} } /Z} {}:

Irms=VrmsZ=120 V531 Ω=0.226 AIrms=VrmsZ=120 V531 Ω=0.226 A size 12{I rSub { size 8{"rms"} } = { {V rSub { size 8{"rms"} } } over {Z} } = { {"120"" V"} over {"531 " %OMEGA } } =0 "." "226"" A"} {} at 60.0 Hz

Finally, at 10.0 kHz, we find

Irms=VrmsZ=120 V190 Ω=0.633 AIrms=VrmsZ=120 V190 Ω=0.633 A size 12{I rSub { size 8{“rms”} } = { {V rSub { size 8{“rms”} } } over {Z} } = { {“120″” V”} over {“190 ” %OMEGA } } =0 “.” “633”” A”} {} at 10.0 kHz

Discussion for (a)

The current at 60.0 Hz is the same (to three digits) as found for the capacitor alone in [link]. The capacitor dominates at low frequency. The current at 10.0 kHz is only slightly different from that found for the inductor alone in [link]. The inductor dominates at high frequency.

Resonance in *RLC* Series AC Circuits

How does an *RLC* circuit behave as a function of the frequency of the driving voltage source? Combining Ohm’s law, Irms=Vrms/ZIrms=Vrms/Z size 12{I rSub { size 8{“rms”} } =V rSub { size 8{“rms”} } /Z} {}, and the expression for impedance ZZ from Z=R2+(XL−XC)2Z=R2+(XL−XC)2 size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {} gives

Irms=VrmsR2+(XL−XC)2.Irms=VrmsR2+(XL−XC)2. size 12{I rSub { size 8{“rms”} } = { {V rSub { size 8{“rms”} } } over { sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } } } {}

The reactances vary with frequency, with XLXL size 12{X rSub { size 8{L} } } {} large at high frequencies and XCXC size 12{X rSub { size 8{C} } } {} large at low frequencies, as we have seen in three previous examples. At some intermediate frequency f0f0 size 12{f rSub { size 8{0} } } {}, the reactances will be equal and cancel, giving *Z=RZ=R size 12{Z=R} {}* —this is a minimum value for impedance, and a maximum value for IrmsIrms size 12{I rSub { size 8{“rms”} } } {} results. We can get an expression for f0f0 size 12{f rSub { size 8{0} } } {} by taking

XL=XC.XL=XC. size 12{X rSub { size 8{L} } =X rSub { size 8{C} } } {}

Substituting the definitions of XLXL size 12{X rSub { size 8{L} } } {} and XCXC size 12{X rSub { size 8{C} } } {},

2πf0L=12πf0C.2πf0L=12πf0C. size 12{2πf rSub { size 8{0} } L= { {1} over {2πf rSub { size 8{0} } C} } } {}

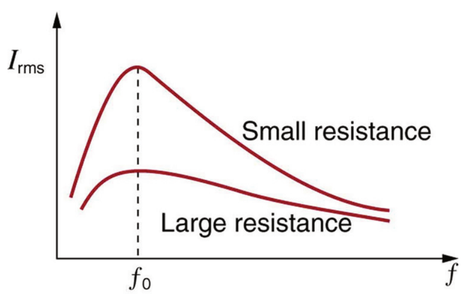
Solving this expression for f0f0 size 12{f rSub { size 8{0} } } {} yields

f0=12πLC,f0=12πLC, size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital “LC”} } } } {}

where f0f0 size 12{f rSub { size 8{0} } } {} is the resonant frequency of an *RLC* series circuit. This is also the *natural frequency* at which the circuit would oscillate if not driven by the voltage source. At f0f0 size 12{f rSub { size 8{0} } } {}, the effects of the inductor and capacitor cancel, so that *Z=RZ=R size 12{Z=R} {}*, and IrmsIrms size 12{I rSub { size 8{“rms”} } } {} is a maximum.

Resonance in AC circuits is analogous to mechanical resonance, where resonance is defined to be a forced oscillation—in this case, forced by the voltage source—at the natural frequency of the system. The receiver in a radio is an *RLC* circuit that oscillates best at its f0f0 size 12{f rSub { size 8{0} } } {}. A variable capacitor is often used to adjust f0f0 size 12{f rSub { size 8{0} } } {} to receive a desired frequency and to reject others. [link] is a graph of current as a function of frequency, illustrating a resonant peak in IrmsIrms size 12{I rSub { size 8{"rms"} } } {} at f0f0 size 12{f rSub { size 8{0} } } {}. The two curves are for two different circuits, which differ only in the amount of resistance in them. The peak is lower and broader for the higher-resistance circuit. Thus the higher-resistance circuit does not resonate as strongly and would not be as selective in a radio receiver, for example.

A graph of current versus frequency for two *RLC* series circuits differing only in the amount of resistance. Both have a resonance at f0f0 size 12{f rSub { size 8{0} } } {}, but that for the higher resistance is lower and broader. The driving AC voltage source has a fixed amplitude V0V0 size 12{V rSub { size 8{0} } } {}.



Calculating Resonant Frequency and Current

For the same *RLC* series circuit having a 40.0 Ω40.0 Ω resistor, a 3.00 mH inductor, and a 5.00 μF5.00 μF capacitor: (a) Find the resonant frequency. (b) Calculate IrmsIrms size 12{I rSub { size 8{"rms"} } } {} at resonance if VrmsVrms size 12{V rSub { size 8{"rms"} } } {} is 120 V.

Strategy

The resonant frequency is found by using the expression in f0=12πLCf0=12πLC size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital "LC"} } } } {}. The current at that frequency is the same as if the resistor alone were in the circuit.

Solution for (a)

Entering the given values for LL and CC into the expression given for f0f0 size 12{f rSub { size 8{0} } } {} in f0=12πLCf0=12πLC size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital "LC"} } } } {} yields

f0=12πLC=12π(3.00×10−3 H)(5.00×10−6 F)=1.30 kHz.f0=12πLC=12π(3.00×10−3 H)(5.00×10−6 F)=1.30 kHz.alignl { stack {
size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital "LC"} } } } {} #
" "= { {1} over {2π sqrt { (3 "." "00" times "10" rSup { size 8{ – 3} } " H") (5 "." "00" times "10" rSup { size 8{ – 6} } " F") } } } =1 "." "30"" kHz" {}
} } {}

Discussion for (a)

We see that the resonant frequency is between 60.0 Hz and 10.0 kHz, the two frequencies chosen in earlier examples. This was to be expected, since the capacitor dominated at the low frequency and the inductor dominated at the high frequency. Their effects are the same at this intermediate frequency.

Solution for (b)

The current is given by Ohm's law. At resonance, the two reactances are equal and cancel, so that the impedance equals the resistance alone. Thus,

Irms=VrmsZ=120 V40.0 Ω=3.00 A.Irms=VrmsZ=120 V40.0 Ω=3.00 A. size 12{I rSub { size 8{"rms"} } = { {V rSub { size 8{"rms"} } } over {Z} } = { {"120″" V"} over {"40" "." "0 " %OMEGA } } =3 "." "00"" A"} {}

Discussion for (b)

At resonance, the current is greater than at the higher and lower frequencies considered for the same circuit in the preceding example.

Power in *RLC* Series AC Circuits

If current varies with frequency in an *RLC* circuit, then the power delivered to it also varies with frequency. But the average power is not simply current times voltage, as it is in purely resistive circuits. As was seen in [link], voltage and current are out of phase in an *RLC* circuit. There is a phase angle ϕϕ size 12{ϕ} {} between the source voltage VV size 12{V} {} and the current II size 12{I} {}, which can be found from

cosϕ=RZ.cosϕ=RZ. size 12{"cos"ϕ= { {R} over {Z} } } {}

For example, at the resonant frequency or in a purely resistive circuit *Z=RZ=R size 12{Z=R} {}*, so that cosϕ=1cosϕ=1 size 12{"cos"ϕ=1} {}. This implies that ϕ=0ºϕ=0º size 12{ϕ=0 rSup { size 8{ circ } } } {} and that voltage and current are in phase, as expected for resistors. At other frequencies, average power is less than at resonance. This is both because voltage and current are out of phase and because IrmsIrms size 12{I rSub { size 8{"rms"} } } {} is lower. The fact that source voltage and current are out of phase affects the power delivered to the circuit. It can be shown that the *average power* is

Pave=IrmsVrmscosϕ,Pave=IrmsVrmscosϕ, size 12{P rSub { size 8{"ave"} } =I rSub { size 8{"rms"} } V rSub { size 8{"rms"} } "cos"ϕ} {}

Thus cosϕcosϕ size 12{"cos"ϕ} {} is called the power factor, which can range from 0 to 1. Power factors near 1 are desirable when designing an efficient motor, for example. At the resonant frequency, cosϕ=1cosϕ=1 size 12{"cos"ϕ=1} {}.

Calculating the Power Factor and Power

For the same *RLC* series circuit having a 40.0 Ω40.0 Ω resistor, a 3.00 mH inductor, a 5.00 μF5.00 μF capacitor, and a voltage source with a VrmsVrms of 120 V: (a) Calculate the power factor and phase angle for f=60.0Hzf=60.0Hz size 12{f="60″ "." 0`"Hz"} {}. (b) What is the average power at 50.0 Hz? (c) Find the average power at the circuit's resonant frequency.

Strategy and Solution for (a)

The power factor at 60.0 Hz is found from

cosϕ=RZ.cosϕ=RZ. size 12{"cos"ϕ= { {R} over {Z} } } {}

We know Z= 531 ΩZ= 531 Ω from [link], so that

cosϕ=40.0Ω531 Ω=0.0753 at 60.0 Hz.cosϕ=40.0Ω531 Ω=0.0753 at 60.0 Hz. size 12{"cos"Ø= { {"40" "." 0 %OMEGA } over {5″31 " %OMEGA } } =0 "." "0753"} {}

This small value indicates the voltage and current are significantly out of phase. In fact, the phase angle is

ϕ=cos−10.0753=85.7º at 60.0 Hz.ϕ=cos−10.0753=85.7º at 60.0 Hz. size 12{ϕ="cos" rSup { size 8{ – 1} } 0 "." "0753"="85″ "." 7 rSup { size 8{ circ } } } {}

Discussion for (a)

The phase angle is close to
90º90º, consistent with the fact that the capacitor dominates the circuit at this low frequency (a pure *RC* circuit has its voltage and current 90º90º out of phase).

Strategy and Solution for (b)

The average power at 60.0 Hz is

Pave=IrmsVrmscosϕ.Pave=IrmsVrmscosϕ. size 12{P rSub { size 8{"ave"} } =I rSub { size 8{"rms"} } V rSub { size 8{"rms"} } "cos"ϕ} {}

IrmsIrms size 12{I rSub { size 8{"rms"} } } {} was found to be 0.226 A in [link]. Entering the known values gives

Pave=(0.226 A)(120 V)(0.0753)=2.04 W at 60.0 Hz.Pave=(0.226 A)(120 V)(0.0753)=2.04 W at 60.0 Hz. size 12{P rSub { size 8{"ave"} } = (0 "." "226"" A") ("120"" V") (0 "." "0753") =2 "." "04"" W"} {}

Strategy and Solution for (c)

At the resonant frequency, we know cosϕ=1cosϕ=1 size 12{"cos"ϕ=1} {}, and IrmsIrms size 12{I rSub { size 8{"rms"} } } {} was found to be 6.00 A in [link]. Thus,

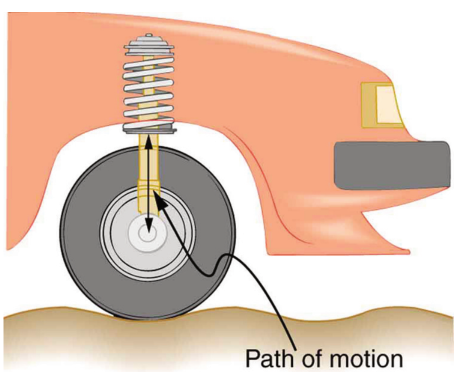
Pave=(3.00 A)(120 V)(1)=360 WPave=(3.00 A)(120 V)(1)=360 W size 12{P rSub { size 8{"ave"} } = (3 "." "00"" A") ("120"" V") (1) ="350″" W"} {} at resonance (1.30 kHz)

Discussion

Both the current and the power factor are greater at resonance, producing significantly greater power than at higher and lower frequencies.

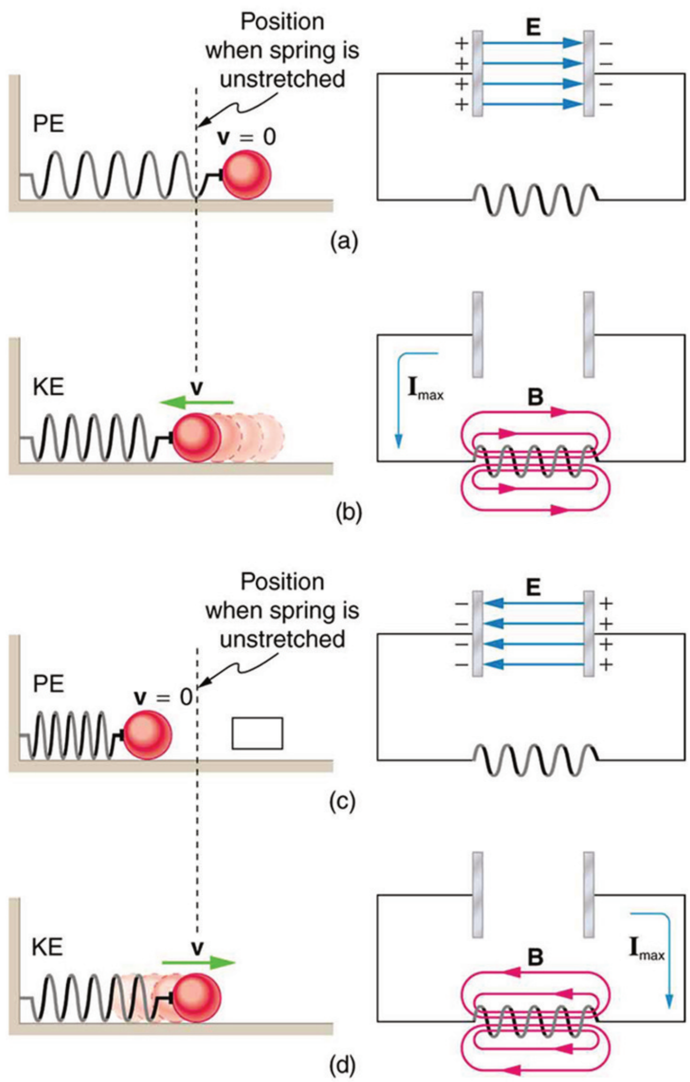
Power delivered to an *RLC* series AC circuit is dissipated by the resistance alone. The inductor and capacitor have energy input and output but do not dissipate it out of the circuit. Rather they transfer energy back and forth to one another, with the resistor dissipating exactly what the voltage source puts into the circuit. This assumes no significant electromagnetic radiation from the inductor and capacitor, such as radio waves. Such radiation can happen and may even be desired, as we will see in the next chapter on electromagnetic radiation, but it can also be suppressed as is the case in this chapter. The circuit is analogous to the wheel of a car driven over a corrugated road as shown in [link]. The regularly spaced bumps in the road are analogous to the voltage source, driving the wheel up and down. The shock absorber is analogous to the resistance damping and limiting the amplitude of the oscillation. Energy within the system goes back and forth between kinetic (analogous to maximum current, and energy stored in an inductor) and potential energy stored in the car spring (analogous to no current, and energy stored in the electric field of a capacitor). The amplitude of the wheels' motion is a maximum if the bumps in the road are hit at the resonant frequency.

The forced but damped motion of the wheel on the car spring is analogous to an *RLC* series AC circuit. The shock absorber damps the motion and dissipates energy, analogous to the resistance in an *RLC* circuit. The mass and spring determine the resonant frequency.



A pure *LC* circuit with negligible resistance oscillates at f0f0 size 12{f rSub { size 8{0} } } {}, the same resonant frequency as an *RLC* circuit. It can serve as a frequency standard or clock circuit—for example, in a digital wrist-watch. With a very small resistance, only a very small energy input is necessary to maintain the oscillations. The circuit is analogous to a car with no shock absorbers. Once it starts oscillating, it continues at its natural frequency for some time. [link] shows the analogy between an *LC* circuit and a mass on a spring.

An *LC* circuit is analogous to a mass oscillating on a spring with no friction and no driving force. Energy moves back and forth between the inductor and capacitor, just as it moves from kinetic to potential in the mass-spring system.



PhET Explorations: Circuit Construction Kit (AC+DC), Virtual Lab

Build circuits with capacitors, inductors, resistors and AC or DC voltage sources, and inspect them using lab instruments such as voltmeters and ammeters.

Circuit Construction Kit (AC+DC), Virtual Lab

PhET Interactive Simulation

Section Summary

- The AC analogy to resistance is impedance ZZ, the combined effect of resistors, inductors, and capacitors, defined by the AC version of Ohm's law:
  I0=V0Z or Irms=VrmsZ,I0=V0Z or Irms=VrmsZ, size 12{I rSub { size 8{0} } = { {V rSub { size 8{0} } } over {Z} } " or "I rSub { size 8{ ital "rms"} } = { {V rSub { size 8{ ital "rms"} } } over {Z} } ,} {}

  where I0I0 size 12{I rSub { size 8{0} } } {} is the peak current and V0V0 size 12{V rSub { size 8{0} } } {} is the peak source voltage.
- Impedance has units of ohms and is given by Z=R2+(XL−XC)2Z=R2+(XL−XC)2 size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {}.
- The resonant frequency f0f0 size 12{f rSub { size 8{0} } } {}, at which XL=XCXL=XC size 12{X rSub { size 8{L} } =X rSub { size 8{C} } } {}, is
  f0=12πLC.f0=12πLC. size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital "LC"} } } } {}
- In an AC circuit, there is a phase angle *ϕϕ size 12{ϕ} {}* between source voltage VV size 12{V} {} and the current II size 12{I} {}, which can be found from
  cosϕ=RZ,cosϕ=RZ, size 12{"cos"ϕ= { {R} over {Z} } } {}
- ϕ=0ºϕ=0º size 12{ϕ=0 rSup { size 8{ circ } } } {} for a purely resistive circuit or an *RLC* circuit at resonance.
- The average power delivered to an *RLC* circuit is affected by the phase angle and is given by
  Pave=IrmsVrmscosϕ,Pave=IrmsVrmscosϕ, size 12{P rSub { size 8{"ave"} } =I rSub { size 8{"rms"} } V rSub { size 8{"rms"} } "cos"ϕ} {}

  cosϕcosϕ size 12{"cos"ϕ} {} is called the power factor, which ranges from 0 to 1.

Conceptual Questions

Does the resonant frequency of an AC circuit depend on the peak voltage of the AC source? Explain why or why not.

Suppose you have a motor with a power factor significantly less than 1. Explain why it would be better to improve the power factor as a method of improving the motor's output, rather than to increase the voltage input.

Problems & Exercises

An *RL* circuit consists of a

40.0 Ω40.0 Ω resistor and a
3.00 mH inductor. (a) Find its impedance ZZ at 60.0 Hz and 10.0 kHz. (b) Compare these values of ZZ with those found in [link] in which there was also a capacitor.

(a) 40.02 Ω40.02 Ω at 60.0 Hz, 193 Ω193 Ω at 10.0 kHz

(b) At 60 Hz, with a capacitor, Z=531 ΩZ=531 Ω, over 13 times as high as without the capacitor. The capacitor makes a large difference at low frequencies. At 10 kHz, with a capacitor Z=190 ΩZ=190 Ω, about the same as without the capacitor. The capacitor has a smaller effect at high frequencies.

An *RC* circuit consists of a
40.0 Ω40.0 Ω resistor and a
5.00 μF5.00 μF capacitor. (a) Find its impedance at 60.0 Hz and 10.0 kHz. (b) Compare these values of ZZ with those found in [link], in which there was also an inductor.

An *LC* circuit consists of a 3.00mH3.00mH size 12{3 "." "00" μH} {} inductor and a 5.00μF5.00μF size 12{5 "." "00" μF} {} capacitor. (a) Find its impedance at 60.0 Hz and 10.0 kHz. (b) Compare these values of ZZ size 12{Z} {} with those found in [link] in which there was also a resistor.

(a) 529 Ω529 Ω at 60.0 Hz, 185 Ω185 Ω at 10.0 kHz

(b) These values are close to those obtained in [link] because at low frequency the capacitor dominates and at high frequency the inductor dominates. So in both cases the resistor makes little contribution to the total impedance.

What is the resonant frequency of a 0.500 mH inductor connected to a
40.0 μF40.0 μF capacitor?

To receive AM radio, you want an *RLC* circuit that can be made to resonate at any frequency between 500 and 1650 kHz. This is accomplished with a fixed 1.00 μH1.00 μH inductor connected to a variable capacitor. What range of capacitance is needed?

9.30 nF to 101 nF

Suppose you have a supply of inductors ranging from 1.00 nH to 10.0 H, and capacitors ranging from 1.00 pF to 0.100 F. What is the range of resonant frequencies that can be achieved from combinations of a single inductor and a single capacitor?

What capacitance do you need to produce a resonant frequency of 1.00 GHz, when using an 8.00 nH inductor?

3.17 pF

What inductance do you need to produce a resonant frequency of 60.0 Hz, when using a 2.00 μF2.00 μF capacitor?

The lowest frequency in the FM radio band is 88.0 MHz. (a) What inductance is needed to produce this resonant frequency if it is connected to a 2.50 pF capacitor? (b) The capacitor is variable, to allow the resonant frequency to be adjusted to as high as 108 MHz. What must the capacitance be at this frequency?

(a) 1.31 μH1.31 μH

(b) 1.66 pF

An *RLC* series circuit has a
2.50 Ω2.50 Ω resistor, a
100 μH100 μH inductor, and an
80.0 μF80.0 μF capacitor.(a) Find the circuit’s impedance at 120 Hz. (b) Find the circuit’s impedance at 5.00 kHz. (c) If the voltage source has Vrms=5.60VVrms=5.60V size 12{V rSub { size 8{“rms”} } =5 “.” “60”`V} {}, what is IrmsIrms size 12{I rSub { size 8{“rms”} } } {} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is IrmsIrms size 12{I rSub { size 8{“rms”} } } {} at resonance?

An *RLC* series circuit has a 1.00 kΩ1.00 kΩ resistor, a 150 μH150 μH inductor, and a 25.0 nF capacitor. (a) Find the circuit’s impedance at 500 Hz. (b) Find the circuit’s impedance at 7.50 kHz. (c) If the voltage source has Vrms=408VVrms=408V size 12{V rSub { size 8{“rms”} } =”408″`V} {}, what is IrmsIrms size 12{I rSub { size 8{“rms”} } } {} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is IrmsIrms size 12{I rSub { size 8{“rms”} } } {} at resonance?

(a) 12.8 kΩ12.8 kΩ

(b) 1.31 kΩ1.31 kΩ

(c) 31.9 mA at 500 Hz, 312 mA at 7.50 kHz

(d) 82.2 kHz

(e) 0.408 A

An *RLC* series circuit has a
2.50 Ω2.50 Ω resistor, a
100 μH100 μH inductor, and an
80.0 μF80.0 μF capacitor. (a) Find the power factor at f=120 Hzf=120 Hz. (b) What is the phase angle at 120 Hz? (c) What is the average power at 120 Hz? (d) Find the average power at the circuit’s resonant frequency.

An *RLC* series circuit has a
1.00 kΩ1.00 kΩ resistor, a
150 μH150 μH inductor, and a 25.0 nF capacitor. (a) Find the power factor at f=7.50 Hzf=7.50 Hz. (b) What is the phase angle at this frequency? (c) What is the average power at this frequency? (d) Find the average power at the circuit’s resonant frequency.

(a) 0.159

(b) 80.9º80.9º

(c) 26.4 W

(d) 166 W

An *RLC* series circuit has a
200 Ω200 Ω
resistor and a 25.0 mH inductor. At 8000 Hz, the phase angle is 45.0º45.0º. (a) What is the impedance? (b) Find the circuit’s capacitance. (c) If Vrms=408VVrms=408V size 12{V rSub { size 8{“rms”} } =”408″`V} {} is applied, what is the average power supplied?

Referring to [link], find the average power at 10.0 kHz.

16.0 W

Glossary

impedance
the AC analogue to resistance in a DC circuit; it is the combined effect of resistance, inductive reactance, and capacitive reactance in the form Z=R2+(XL−XC)2Z=R2+(XL−XC)2 size 12{Z= sqrt {R rSup { size 8{2} } + (X rSub { size 8{L} } – X rSub { size 8{C} }) rSup { size 8{2} } } } {}
resonant frequency
the frequency at which the impedance in a circuit is at a minimum, and also the frequency at which the circuit would oscillate if not driven by a voltage source; calculated by f0=12πLCf0=12πLC size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital “LC”} } } } {}
phase angle
denoted by ϕϕ size 12{ϕ} {}, the amount by which the voltage and current are out of phase with each other in a circuit
power factor
the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase; calculated by cosϕcosϕ size 12{“cos”ϕ} {}

PART 17

Chapter 24 Electromagnetic Waves

Introduction to Electromagnetic Waves

class=”introduction”
class=”section-summary” title=”Section Summary”class=”conceptual-questions” title=”Conceptual Questions”class=”problems-exercises” title=”Problems & Exercises”

Human eyes detect these orange “sea goldie” fish swimming over a coral reef in the blue waters of the Gulf of Eilat (Red Sea) using visible light. (credit: Daviddarom, Wikimedia Commons)

A photo showing many orange and pale blue colored fish, swimming over a coral reef in the blue waters of the Gulf of Eilat.

The beauty of a coral reef, the warm radiance of sunshine, the sting of sunburn, the X-ray revealing a broken bone, even microwave popcorn—all are brought to us by electromagnetic waves. The list of the various types of electromagnetic waves, ranging from radio transmission waves to nuclear gamma-ray (γγ size 12{g} {}-ray) emissions, is interesting in itself.

Even more intriguing is that all of these widely varied phenomena are different manifestations of the same thing—electromagnetic waves. (See [link].) What are electromagnetic waves? How are they created, and how do they travel? How can we understand and organize their widely varying properties? What is their relationship to electric and magnetic effects? These and other questions will be explored.

Misconception Alert: Sound Waves vs. Radio Waves

Many people confuse sound waves with radio waves, one type of electromagnetic (EM) wave. However, sound and radio waves are completely different phenomena. Sound creates pressure variations (waves) in matter, such as air or water, or your eardrum. Conversely, radio waves are *electromagnetic waves, like visible light, infrared, ultraviolet, X-rays, and gamma rays. EM waves don’t need a medium in which to propagate; they can travel through a vacuum, such as outer space.*

A radio works because sound waves played by the D.J. at the radio station are converted into electromagnetic waves, then encoded and transmitted in the radio-frequency range. The radio in your car receives the radio waves, decodes the information, and uses a speaker to change it back into a sound wave, bringing sweet music to your ears.

Discovering a New Phenomenon

It is worth noting at the outset that the general phenomenon of electromagnetic waves was predicted by theory before it was realized that light is a form of electromagnetic wave. The prediction was made by James Clerk Maxwell in the mid-19th century when he formulated a single theory combining all the electric and magnetic effects known by scientists at that time. “Electromagnetic waves” was the name he gave to the phenomena his theory predicted.

Such a theoretical prediction followed by experimental verification is an indication of the power of science in general, and physics in particular. The underlying connections and unity of physics allow certain great minds to solve puzzles without having all the pieces. The prediction of electromagnetic waves is one of the most spectacular examples of this power. Certain others, such as the prediction of antimatter, will be discussed in later modules.

The electromagnetic waves sent and received by this 50-foot radar dish antenna at Kennedy Space Center in Florida are not visible, but help track expendable launch vehicles with high-definition imagery. The first use of this C-band radar dish was for the launch of the Atlas V rocket sending the New Horizons probe toward Pluto. (credit: NASA)

The large, round dish antenna looking like a giant white saucer is shown. It rests on a pillar shaped structure with a moveable tracking system that allows it to point towards a target object, send out electromagnetic waves, and collect any signals that bounce back from the target object.

24.1 Maxwell's Equations: Electromagnetic Waves Predicted and Observed

Maxwell's Equations: Electromagnetic Waves Predicted and Observed

- Restate Maxwell's equations.

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See [link].) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by Maxwell's equations, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn's rings.

James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic waves. (credit: G. J. Stodart)

This black and white engraving shows physicist James Clerk Maxwell as a Victorian era gentleman dressed in bowtie, vest, and jacket, and sporting a full, graying beard and moustache.

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. Maxwell's equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Maxwell's Equations

1. Electric field lines originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge, and the strength of the force is related to the electric

constant ε0ε0 size 12{ε rSub { size 8{0} } } {}, also known as the permittivity of free space. From Maxwell's first equation we obtain a special form of Coulomb's law known as Gauss's law for electricity.

2. Magnetic field lines are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the magnetic constant μ0μ0 size 12{μ rSub { size 8{0} } } {}, also known as the permeability of free space. This second of Maxwell's equations is known as Gauss's law for magnetism.

3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. This third of Maxwell's equations is Faraday's law of induction, and includes Lenz's law.

4. Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell's equations encompasses Ampere's law and adds another source of magnetism—changing electric fields.

Maxwell's equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday's law of induction and had been suspected for some time, but fits beautifully into Maxwell's equations.

Symmetry is apparent in nature in a wide range of situations. In contemporary research, symmetry plays a major part in the search for sub-atomic particles using massive multinational particle accelerators such as the new Large Hadron Collider at CERN.

Making Connections: Unification of Forces

Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at a speed given by the equation

c =1μ0ε0.c =1μ0ε0. size 12{"c "= { {1} over { sqrt {μ rSub { size 8{0} } ε rSub { size 8{0} } } } } } {}

When the values for μ0μ0 size 12{μ rSub { size 8{0} } } {} and ε0ε0 size 12{ε rSub { size 8{0} } } {} are entered into the equation for cc, we find that

c =1(8.85×10−12C2N?m2)(4π×10−7T?mA)= 3.00× 108 m/s,c =1(8.85×10−12C2N?m2)(4π×10−7T?mA)= 3.00× 108 m/s, size 12{"c "= { {1} over { sqrt { (8 "." "85" times "10" rSup { size 8{-"12″} } { {C rSup { size

8{2} } } over {N cdot m rSup { size 8{2} } } }) (4π´”10″ rSup { size 8{-7} } { {T cdot m} over {A} }) } } }
=” 3″ “.” “00”´” 10″ rSup { size 8{8} } ” m/s”} {}

which is the speed of light. In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell’s theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell’s death.

Hertz’s Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC RLCRLC size 12{ ital “RLC”} {} (resistor-inductor-capacitor) circuit that resonates at a known frequency f0=12πLCf0=12πLC size 12{f rSub { size 8{0} } = { {1} over {2π sqrt { ital “LC”} } } } {} and connected it to a loop of wire as shown in [link]. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another RLCRLC size 12{ ital “RLC”} {} circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.

The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An RLCRLC size 12{ ital “RLC”} {} circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

The circuit diagram shows a simple circuit containing an alternating voltage source, a resistor R, capacitor C and a transformer, which provides the impedance. The transformer is shown to consist of two coils separated by a core. In parallel with the transformer is connected a wire loop labeled as Loop one Transmitter with a small gap that creates sparks across the gap. The sparks create electromagnetic waves, which are transmitted through the air to a similar loop next to it labeled as Loop two Receiver. These waves induce sparks in Loop two, and are detected by the tuner shown as a rectangular box connected to it.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation υ=fλυ=fλ size 12{υ=fλ} {} (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz (1 Hz= 1 cycle/sec1 Hz= 1 cycle/sec size 12{1″ Hz"=1″ cycle/sec"} {}), is named in his honor.

Section Summary

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light cc. They were predicted by Maxwell, who also showed that
  c =1μ0ε0,c =1μ0ε0, size 12{"c "= { {1} over { sqrt {μ rSub { size 8{0} } ε rSub { size 8{0} } } } } } {}

  where μ0μ0 size 12{μ rSub { size 8{0} } } {} is the permeability of free space and ε0ε0 size 12{ε rSub { size 8{0} } } {} is the permittivity of free space.
- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

Problems & Exercises

Verify that the correct value for the speed of light cc is obtained when numerical values for the permeability and permittivity of free space (μ0μ0 size 12{μ rSub { size 8{0} } } {} and ε0ε0 size 12{ε rSub { size 8{0} } } {}) are entered into the equation c =1μ0ε0c =1μ0ε0 size 12{"c "= { {1} over { sqrt {μ rSub { size 8{0} } ε rSub { size 8{0} } } } } } {}.

Show that, when SI units for μ0μ0 size 12{μ rSub { size 8{0} } } {} and ε0ε0 size 12{ε rSub { size 8{0} } } {} are entered, the units given by the right-hand side of the equation in the problem above are m/s.

Glossary

electromagnetic waves
radiation in the form of waves of electric and magnetic energy
Maxwell's equations
a set of four equations that comprise a complete, overarching theory of electromagnetism
***RLC* circuit**
an electric circuit that includes a resistor, capacitor and inductor
hertz
an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

speed of light

in a vacuum, such as space, the speed of light is a constant 3 x 10^8 m/s

electromotive force (emf)

energy produced per unit charge, drawn from a source that produces an electrical current

electric field lines

a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

magnetic field lines

a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

24.2 Production of Electromagnetic Waves

Production of Electromagnetic Waves

- Describe the electric and magnetic waves as they move out from a source, such as an AC generator.
- Explain the mathematical relationship between the magnetic field strength and the electrical field strength.
- Calculate the maximum strength of the magnetic field in an electromagnetic wave, given the maximum electric field strength.

We can get a good understanding of electromagnetic waves (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in [link].

This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field (EE size 12{E} {}) propagates away from the antenna at the speed of light, forming part of an electromagnetic wave.

A long straight gray wire with an A C generator at its center, functioning as a broadcast antenna for electromagnetic waves, is shown. The wave distributions at four different times are shown in four different parts. Part a of the diagram shows a long straight gray wire with an A C generator at its center. The time is marked t equals zero. The bottom part of the antenna is positive and the upper end of the antenna is negative. An electric field E acting upward is shown by an upward arrow. Part b of the diagram shows a long straight gray wire with an A C generator at its center. The time is marked t equals capital T divided by four. The antenna has no polarity marked and a wave is shown to emerge from the A C source. An electric field E acting upward as shown by an upward arrow. The electric field E propagates away from the antenna at the speed of light, forming part of the electromagnetic wave from the A C source. A quarter portion of the wave is shown above the horizontal axis. Part c of the diagram shows a long straight gray wire with an A C generator at its center. The time is marked t equals capital T divided by two. The bottom part of the antenna is negative and the upper end of the antenna is positive and a wave is shown to emerge from the A C source. The electric field E propagates away from the antenna at the speed of light, forming part of the electromagnetic wave from the A C source. A quarter portion of the wave is shown below the horizontal axis and a quarter portion of the wave is above the horizontal axis. Part d of the diagram shows a long straight gray wire with an AC generator at its center. The time is marked t equals capital T. The bottom part of the antenna is positive and the upper end of the antenna is negative. A wave is shown to emerge from the A C source. The electric field E propagates away from the antenna at the speed of light, forming part of the electromagnetic wave from the A C source. A quarter portion of the wave is shown above the horizontal axis followed by a half wave below the horizontal axis and then again a quarter of a wave above the horizontal axis.

The electric field (EE size 12{E} {}) shown surrounding the wire is produced by the charge distribution on the wire. Both the EE size 12{E} {} and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

There is an associated magnetic field (BB size 12{B} {}) which propagates outward as well (see [link]). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

Closer examination of the one complete cycle shown in [link] reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time t=0t=0 size 12{t=0} {}, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or EE size 12{E} {}-field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum EE size 12{E} {}-field has moved away at speed cc size 12{c} {}.

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an amplitude proportional to the maximum separation of charge. Its wavelengthλλ size 12{ left (λ right)} {} is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and frequencyff size 12{ left (f right)} {} are inversely proportional.)

Electric and Magnetic Waves: Moving Together

Following Ampere's law, current in the antenna produces a magnetic field, as shown in [link]. The relationship between EE size 12{E} {} and BB size 12{B} {} is shown at one instant in [link] (a). As the current varies, the magnetic field varies in magnitude and direction.

(a) The current in the antenna produces the circular magnetic field lines. The current (II size 12{I} {}) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields (EE size 12{E} {} and BB size 12{B} {}) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

Part a of the diagram shows a long straight gray wire with an A C generator at its center, functioning as a broadcast antenna. The antenna has a current I flowing vertically upward. The bottom end of the antenna is negative and the upper end of the antenna is positive. An electric field is shown to act vertically downward. The magnetic field lines B produced in the antenna are circular in direction around the wire. Part b of the diagram shows a long straight gray wire with an A C generator at its center, functioning as a broadcast antenna. The electric field E and magnetic field B near the wire are shown perpendicular to each other. Part c of the diagram shows a long straight gray wire with an A C generator at its center, functioning as a broadcast antenna. The current is shown to flow in the antenna. The magnetic field varies with the current and propagates away from the antenna as a sine wave in the horizontal plane. The vibrations in the wave are marked as small arrows along the wave.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in [link] (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in [link]. The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a transverse wave.

A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields (EE size 12{E} {} and BB size 12{B} {}) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

A part of the electromagnetic wave sent out from the antenna at one instant in time is shown. The wave is shown with the variation of two components, E and B, moving with velocity c. E is a sine wave in one plane with small arrows showing the vibrations of particles in the plane. B is a sine wave in a plane perpendicular to the E wave. The B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because E and B are perpendicular to each other. E and B are in phase, and they are perpendicular to one another and to the direction of propagation.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in [link] to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a standing wave, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a resonant phenomenon and when we tune radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Relating EE size 12{E} {}-Field and BB size 12{B} {}-Field Strengths

There is a relationship between the EE size 12{E} {}- and BB size 12{B} {}-field strengths in an electromagnetic wave. This can be understood by again considering the antenna just described. The stronger the EE size 12{E} {}-field created by a separation of charge, the greater the current and, hence, the greater the BB size 12{B} {}-field created.

Since current is directly proportional to voltage (Ohm's law) and voltage is directly proportional to EE size 12{E} {}-field strength, the two should be directly proportional. It can be shown that the magnitudes of the fields do have a constant ratio, equal to the speed of light. That is,

EB=cEB=c size 12{ { {E} over {B} } =c} {}

is the ratio of EE size 12{E} {}-field strength to BB size 12{B} {}-field strength in any electromagnetic wave. This is true at all times and at all locations in space. A simple and elegant result.

Calculating BB size 12{B} {}-Field Strength in an Electromagnetic Wave

What is the maximum strength of the BB size 12{B} {}-field in an electromagnetic wave that has a maximum EE size 12{E} {}-field strength of 1000 V/m1000 V/m size 12{“1000” {V} slash {m} } {}?

Strategy

To find the BB size 12{B} {}-field strength, we rearrange the above equation to solve for BB size 12{B} {}, yielding

B=Ec.B=Ec. size 12{B= { {E} over {c} } } {}

Solution

We are given EE size 12{E} {}, and cc size 12{c} {} is the speed of light. Entering these into the expression for BB size 12{B} {} yields

B=1000 V/m3.00 × 108 m/s= 3.33×10-6 T,B=1000 V/m3.00 × 108 m/s= 3.33×10-6 T, size 12{B = { {“1000 V/ m”} over {3 “.” “00 ” times ” 10″ rSup { size 8{8} } ” m/s”} } =” 3″ “.” “33” times “10” rSup { size 8{ +- 6} } ” T”} {}

Where T stands for Tesla, a measure of magnetic field strength.

Discussion

The BB size 12{B} {}-field strength is less than a tenth of the Earth’s admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field. Note that as this wave spreads out, say with distance from an antenna, its field strengths become progressively weaker.

The result of this example is consistent with the statement made in the module Maxwell’s Equations: Electromagnetic Waves Predicted and Observed that changing electric fields create relatively weak magnetic fields. They can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

Take-Home Experiment: Antennas

For your TV or radio at home, identify the antenna, and sketch its shape. If you don’t have cable, you might have an outdoor or indoor TV antenna. Estimate its size. If the TV signal is between 60 and 216 MHz for basic channels, then what is the wavelength of those EM waves?

Try tuning the radio and note the small range of frequencies at which a reasonable signal for that station is received. (This is easier with digital readout.) If you have a car with a radio and extendable antenna, note the quality of reception as the length of the antenna is changed.

PhET Explorations: Radio Waves and Electromagnetic Fields

Broadcast radio waves from KPhET. Wiggle the transmitter electron manually or have it oscillate automatically.

Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.

Radio Waves and Electromagnetic Fields

PhET Interactive Simulation

Section Summary

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.
- The strengths of the electric and magnetic parts of the wave are related by
  EB= c,EB= c, size 12{ { {E} over {B} } = ital " c"} {}

  which implies that the magnetic field BB size 12{B} {} is very weak relative to the electric field EE size 12{E} {}.

Conceptual Questions

The direction of the electric field shown in each part of [link] is that produced by the charge distribution in the wire. Justify the direction shown in each part, using the Coulomb force law and the definition of E=F/qE=F/q size 12{E= {F} slash {q} } {}, where qq size 12{q} {} is a positive test charge.

Is the direction of the magnetic field shown in [link] (a) consistent with the right-hand rule for current (RHR-2) in the direction shown in the figure?

Why is the direction of the current shown in each part of [link] opposite to the electric field produced by the wire's charge separation?

In which situation shown in [link] will the electromagnetic wave be more successful in inducing a current in the wire? Explain.

Electromagnetic waves approaching long straight wires.

Part a of the diagram shows an electromagnetic wave approaching a long straight vertical wire. The wave is shown with the variation of two components E and B. E is a sine wave in vertical plane with small arrows showing the vibrations of particles in the plane. B is a sine wave in a horizontal plane perpendicular to the E wave. The B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because E and B are perpendicular to each other. The direction of propagation of wave is shown perpendicular to E and B waves. Part b of the diagram shows an electromagnetic wave approaching a long straight vertical wire. The wave is shown with the variation of two components E and B. E is a sine wave in horizontal plane with small arrows showing the vibrations of particles in the plane. B is a sine wave in a vertical plane perpendicular to the E wave. The B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because E and B are perpendicular to each other. The direction of propagation of wave is shown perpendicular to E and B waves.

In which situation shown in [link] will the electromagnetic wave be more successful in inducing a current in the loop? Explain.

Electromagnetic waves approaching a wire loop.

Part a of the diagram shows an electromagnetic wave approaching a receiver loop connected to a tuner. The wave is shown with the variation of two components E and B. E is a sine wave in vertical plane with small arrows showing the vibrations of particles in the plane. B is a sine wave in a horizontal plane perpendicular to the E wave. The B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because E and B are perpendicular to each other. The direction of propagation of wave is shown perpendicular to E and B waves. Part b of the diagram shows an electromagnetic wave approaching a receiver loop connected to a tuner. The wave is shown with the variation of two components E and B. E is a sine wave in horizontal plane with small arrows showing the vibrations of particles in the plane. B is a sine wave in a vertical plane perpendicular to the E wave. The B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because E and B are perpendicular to each other. The direction of propagation of wave is shown perpendicular to E and B waves.

Should the straight wire antenna of a radio be vertical or horizontal to best receive radio waves broadcast by a vertical transmitter antenna? How should a loop antenna be aligned to best receive the signals? (Note that the direction of the loop that produces the best reception can be used to determine the location of the source. It is used for that purpose in tracking tagged animals in nature studies, for example.)

Under what conditions might wires in a DC circuit emit electromagnetic waves?

Give an example of interference of electromagnetic waves.

[link] shows the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.

An overhead view of two radio broadcast antennas sending the same signal, and the interference pattern they produce.

The picture shows an overhead view of a radio broadcast antenna sending signals in the form of waves. Two waves are shown in the diagram with concentric circular wave fonts. The crest and trough are marked as bold and dashed circles respectively. The points where the bold circles of the two different waves meet are marked as points of constructive interference. Arrows point outward from the antenna, joining these points. These arrows show the directions of constructive interference.

Can an antenna be any length? Explain your answer.

Problems & Exercises

What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of 5.00×10^{-4} T5.00×10−4 T size 12{5 "." "00"´"10″ rSup { size 8{-4} } " T"} {} (about 10 times the Earth's)?

150 kV/m

The maximum magnetic field strength of an electromagnetic field is 5×10^{-6}T5×10−6T size 12{5 times "10" rSup { size 8{ – 6} } T} {}. Calculate the maximum electric field strength if the wave is traveling in a medium in which the speed of the wave is
$0.75c$0.75c size 12{c} {}.

Verify the units obtained for magnetic field strength BB in [link] (using the equation $B=\frac{E}{c}$B=Ec) are in fact teslas (T).

Glossary

electric field
*a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge*
electric field strength
the magnitude of the electric field, denoted E-field

magnetic field

*a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle*

magnetic field strength

the magnitude of the magnetic field, denoted B-field

transverse wave

a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

standing wave

a wave that oscillates in place, with nodes where no motion happens

wavelength

the distance from one peak to the next in a wave

amplitude

the height, or magnitude, of an electromagnetic wave

frequency

the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

resonant

a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

oscillate

to fluctuate back and forth in a steady beat

24.3 The Electromagnetic Spectrum

The Electromagnetic Spectrum

- List three “rules of thumb” that apply to the different frequencies along the electromagnetic spectrum.
- Explain why the higher the frequency, the shorter the wavelength of an electromagnetic wave.
- Draw a simplified electromagnetic spectrum, indicating the relative positions, frequencies, and spacing of the different types of radiation bands.
- List and explain the different methods by which electromagnetic waves are produced across the spectrum.

In this module we examine how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on, so that we can understand some of their similarities as well as some of their differences. We will also find that there are many connections with previously discussed topics, such as wavelength and resonance. A brief overview of the production and utilization of electromagnetic waves is found in [link].

Electromagnetic Waves

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Radio & TV	Accelerating charges	Communications Remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges & thermal agitation	Communications Ovens Radar	Deep heating	Cell phone use
Infrared	Thermal agitations & electronic transitions	Thermal imaging Heating	Absorbed by atmosphere	Greenhouse effect
Visible light	Thermal agitations & electronic transitions	All pervasive	Photosynthesis Human vision	
Ultraviolet	Thermal agitations & electronic transitions	Sterilization Cancer control	Vitamin D production	Ozone depletion Cancer causing
X-rays	Inner electronic transitions and fast collisions	Medical Security	Medical diagnosis Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicineSecurity	Medical diagnosis Cancer therapy	Cancer causing Radiation damage

Connections: Waves

There are many types of waves, such as water waves and even earthquakes. Among the many shared attributes of waves are propagation speed, frequency, and wavelength. These are always related by the expression vW=fλvW=fλ size 12{v rSub { size 8{W} } =fλ} {}. This module concentrates on EM waves, but other modules contain examples of all of these characteristics for sound waves and submicroscopic particles.

As noted before, an electromagnetic wave has a frequency and a wavelength associated with it and travels at the speed of light, or cc size 12{c} {}. The relationship among these wave characteristics can be described by vW=fλvW=fλ size 12{v rSub { size 8{W} } =fλ} {}, where vWvW size 12{v rSub { size 8{W} } } {} is the propagation speed of the wave, ff size 12{f} {} is the frequency, and λλ size 12{λ} {} is the wavelength. Here vW=cvW=c size 12{v rSub { size 8{W} } =c} {}, so that for all electromagnetic waves,

c=fλ.c=fλ. size 12{c = fλ} {}

Thus, for all electromagnetic waves, the greater the frequency, the smaller the wavelength.

[link] shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum. Many of the characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths, as we shall see.

The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

An electromagnetic spectrum is shown. Different wave category regions are indicated using double sided arrows based on the values of their wavelength, energy, and frequency; the visual strip is also shown. The radio wave region is further segmented into AM radio, FM radio, and microwaves bands.

Electromagnetic Spectrum: Rules of Thumb

Three rules that apply to electromagnetic waves in general are as follows:

- High-frequency electromagnetic waves are more energetic and are more able to penetrate than low-frequency waves.
- High-frequency electromagnetic waves can carry more information per unit time than low-frequency waves.
- The shorter the wavelength of any electromagnetic wave probing a material, the smaller the detail it is possible to resolve.

Note that there are exceptions to these rules of thumb.

Transmission, Reflection, and Absorption

What happens when an electromagnetic wave impinges on a material? If the material is transparent to the particular frequency, then the wave can largely be transmitted. If the material is opaque to the frequency, then the wave can be totally reflected. The wave can also be absorbed by the material, indicating that there is some interaction between the wave and the material, such as the thermal agitation of molecules.

Of course it is possible to have partial transmission, reflection, and absorption. We normally associate these properties with visible light, but they do apply to all electromagnetic waves. What is not obvious is that something that is transparent to light may be opaque at other frequencies. For example, ordinary glass is transparent to visible light but largely opaque to ultraviolet radiation. Human skin is opaque to visible light—we cannot see through people—but transparent to X-rays.

Radio and TV Waves

The broad category of radio waves is defined to contain any electromagnetic wave produced by currents in wires and circuits. Its name derives from their most common use as a carrier of audio information (i.e., radio). The name is applied to electromagnetic waves of similar frequencies regardless of source. Radio waves from outer space, for example, do not come from alien radio stations. They are created by many astronomical phenomena, and their study has revealed much about nature on the largest scales.

There are many uses for radio waves, and so the category is divided into many subcategories, including microwaves and those electromagnetic waves used for AM and FM radio, cellular telephones, and TV.

The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. (See [link].) These extremely long wavelength electromagnetic waves (about 6000 km!) are one means of energy loss in long-distance power transmission.

This high-voltage traction power line running to Eutingen Railway Substation in Germany radiates electromagnetic waves with very long wavelengths. (credit: Zonk43, Wikimedia Commons)

A high-voltage traction power line is shown to the side of a roadway. The power line in the photo has two transmission poles supporting the cables.

There is an ongoing controversy regarding potential health hazards associated with exposure to these electromagnetic fields (EE size 12{E} {}-fields). Some people suspect that living near such transmission lines may cause a variety of illnesses, including cancer. But demographic data are either inconclusive or simply do not support the hazard theory. Recent reports that have looked at many European and American epidemiological studies have found no increase in risk for cancer due to exposure to EE size 12{E} {}-fields.

Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it, and very long wavelengths are needed to reach a submarine under the surface. (See [link].)

Very long wavelength radio waves are needed to reach this submarine, requiring extremely low frequency signals (ELF). Shorter wavelengths do not penetrate to any significant depth.

The picture of a submarine under water is shown. The submarine is shown to receive extremely low frequency signals shown as a curvy line from the ocean surface to the submarine in the ocean depth.

AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for amplitude modulation, which is the method for placing information on these waves. (See [link].) A carrier wave having the basic frequency of the radio station, say 1530 kHz, is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.

A radio receiver tuned to have the same resonant frequency as the carrier wave can pick up the signal, while rejecting the many other frequencies impinging on its antenna. The receiver's circuitry is designed to respond to variations in amplitude of the carrier wave to replicate the original audio signal. That audio signal is amplified to drive a speaker or perhaps to be recorded.

Amplitude modulation for AM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

Part a of the diagram shows a carrier wave along the horizontal axis. The wave is shown to have a high frequency as the vibrations are closely spaced. The wave has constant amplitude represented by uniform height of crest and trough. Part b of the diagram shows an audio wave with a lower frequency. The wave is on the upper side of horizontal axis. The amplitude of the wave is not uniform. It has an initial small rise and fall followed by a steep rise and a gradual fall in the wave. Part c of the diagram shows the amplitude modulated wave. It is the resultant wave obtained by mixing of the waves in part a and part b. The amplitude of the resultant wave is non uniform, similar to the audio wave. The frequency of the amplitude modulated wave is equal to the frequency of the carrier wave. The wave spreads on both sides of the horizontal axis.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for frequency modulation, another method of carrying information. (See [link].) Here a carrier wave having the basic frequency of the radio station, perhaps 105.1 MHz, is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.

Frequency modulation for FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Part a of the diagram shows a carrier wave along the horizontal axis. The wave is shown to have a high frequency as the vibrations are closely spaced. The wave has constant amplitude represented by uniform height of crest and trough. Part b of the diagram shows an audio wave with a lower frequency as shown by widely spaced vibrations. The wave has constant amplitude, represented by uniform length of crest and trough. Part c shows the frequency modulated wave obtained from waves in part a and part b. The amplitude of the resultant wave is similar to the source waves but the frequency varies. Frequency maxima are shown as closely spaced vibrations and frequency minima are shown as widely spaced vibrations. These maxima and minima are shown to alternate.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. Thus the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio. The reason is that amplitudes of waves add. So an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be made to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. It is thus easier to reject noise from FM, since noise produces a variation in amplitude.

Television is also broadcast on electromagnetic waves. Since the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz. (The entire FM radio band lies between channels 88 MHz and 174 MHz.) These TV channels are called VHF (for very high frequency). Other channels called UHF (for ultra high frequency) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies and is rapidly evolving with the use of the high-definition or HD format.

Calculating Wavelengths of Radio Waves

Calculate the wavelengths of a 1530-kHz AM radio signal, a 105.1-MHz FM radio signal, and a 1.90-GHz cell phone signal.

Strategy

The relationship between wavelength and frequency is c=fλc=fλ size 12{c = fλ} {}, where c=3.00×108m/sc=3.00×108m/s size 12{c=3 “.” “00” times “10” rSup { size 8{8} } {m} slash {s} } {} is the speed of light (the speed of light is only very slightly smaller in air than it is in a vacuum). We can rearrange this equation to find the wavelength for all three frequencies.

Solution

Rearranging gives

λ=cf.λ=cf. size 12{λ= { {c} over {f} } } {}

(a) For the f=1530 kHzf=1530 kHz size 12{f=”1530″” kHz”} {} AM radio signal, then,

λ=3.00×108 m/s1530×103 cycles/s=196 m.λ=3.00×108 m/s1530×103 cycles/s=196 m. size 12{ matrix {
λ {} # = { {3 “.” “00” times “10” rSup { size 8{8} } ” m/s”} over {“1530” times “10” rSup { size 8{3} } ” cycles/s”} } {} ##
{} # =”196 m” “.” {}
} } {}

(b) For the f=105.1 MHzf=105.1 MHz size 12{f=”105″ “.” 1`”MHz”} {} FM radio signal,

λ=3.00×108 m/s105.1×106 cycles/s=2.85 m.λ=3.00×108 m/s105.1×106 cycles/s=2.85 m. size 12{ matrix {
λ {} # = { {3 "." "00" times "10" rSup { size 8{8} } " m/s"} over {"1530" times "10" rSup { size 8{3} } " cycles/s"} } {} ##
{} # ="196 m" "." {}
} } {}

(c) And for the f=1.90 GHzf=1.90 GHz size 12{f=1 "." "90"`"GHz"} {} cell phone,

λ=3.00×108 m/s1.90×109 cycles/s=0.158 m.λ=3.00×108 m/s1.90×109 cycles/s=0.158 m. size 12{ matrix {
λ {} # = { {3 "." "00" times "10" rSup { size 8{8} } " m/s"} over {"1530" times "10" rSup { size 8{3} } " cycles/s"} } {} ##
{} # ="196 m" "." {}
} } {}

Discussion

These wavelengths are consistent with the spectrum in [link]. The wavelengths are also related to other properties of these electromagnetic waves, as we shall see.

The wavelengths found in the preceding example are representative of AM, FM, and cell phones, and account for some of the differences in how they are broadcast and how well they travel. The most efficient length for a linear antenna, such as discussed in Production of Electromagnetic Waves, is λ/2λ/2 size 12{ {λ} slash {2} } {}, half the wavelength of the electromagnetic wave. Thus a very large antenna is needed to efficiently broadcast typical AM radio with its carrier wavelengths on the order of hundreds of meters.

One benefit to these long AM wavelengths is that they can go over and around rather large obstacles (like buildings and hills), just as ocean waves can go around large rocks. FM and TV are best received when there is a line of sight between the broadcast antenna and receiver, and they are often sent from very tall structures. FM, TV, and mobile phone antennas themselves are much smaller than those used for AM, but they are elevated to achieve an unobstructed line of sight. (See [link].)

(a) A large tower is used to broadcast TV signals. The actual antennas are small structures on top of the tower—they are placed at great heights to have a clear line of sight over a large broadcast area. (credit: Ozizo, Wikimedia Commons) (b) The NTT Dokomo mobile phone tower at Tokorozawa City, Japan. (credit: tokoroten, Wikimedia Commons)

The first photograph shows a large tower used to broadcast TV signals. The tower is alternately painted red and white along the length. The antennas are shown as small structures on top of the tower. The second photograph shows a photo of a mobile phone tower. The tower has two ring shaped structures at its top most point.

Radio Wave Interference

Astronomers and astrophysicists collect signals from outer space using electromagnetic waves. A common problem for astrophysicists is the “pollution” from electromagnetic radiation pervading our surroundings from communication systems in general. Even everyday gadgets like our car keys having the facility to lock car doors remotely and being able to turn TVs on and off using remotes involve radio-wave frequencies. In order to prevent interference between all these electromagnetic signals, strict regulations are drawn up for different organizations to utilize different radio frequency bands.

One reason why we are sometimes asked to switch off our mobile phones (operating in the range of 1.9 GHz) on airplanes and in hospitals is that important communications or medical equipment often uses similar radio frequencies and their operation can be affected by frequencies used in the communication devices.

For example, radio waves used in magnetic resonance imaging (MRI) have frequencies on the order of 100 MHz, although this varies significantly depending on the strength of the magnetic field used and the nuclear type being scanned. MRI is an important medical imaging and research tool, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei (usually protons or hydrogen nuclei).

The wavelength of 100-MHz radio waves is 3 m, yet using the sensitivity of the resonant frequency to the magnetic field strength, details smaller than a millimeter can be imaged. This is a good example of an exception to a rule of thumb (in this case, the rubric that details much smaller than the probe’s wavelength cannot be detected). The intensity of the radio waves used in MRI presents little or no hazard to human health.

Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about 109Hz109Hz size 12{“10″ rSup { size 8{9} } `”Hz”} {} to the highest practical LCLC size 12{ ital “LC”} {} resonance at nearly 1012Hz1012Hz size 12{“10” rSup { size 8{“12″} } `”Hz”} {}. Since they have high frequencies, their wavelengths are short compared with those of other radio waves—hence the name “microwave.”

Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by thermal agitation. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

Radar is a common application of microwaves that was first developed in World War II. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can be used to determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems are used to map the Earth and other planets, with a resolution limited by wavelength. (See [link].) The shorter the wavelength of any probe, the smaller the detail it is possible to observe.

An image of Sif Mons with lava flows on Venus, based on Magellan synthetic aperture radar data combined with radar altimetry to produce a three-dimensional map of the surface. The Venusian atmosphere is opaque to visible light, but not to the microwaves that were used to create this image. (credit: NSSDC, NASA/JPL)

A photograph of the surface of planet Venus is shown. The lava flows on Venus are shown as orange red color of the surface.

Heating with Microwaves

How does the ubiquitous microwave oven produce microwaves electronically, and why does food absorb them preferentially? Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves are then used to induce an alternating electric field in the oven.

Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar molecules, in trying to keep orienting themselves with the electric field, absorb these energies and increase their temperatures—called dielectric heating.

The energy thereby absorbed results in thermal agitation heating food and not the plate, which does not contain water. Hot spots in the food are related to constructive and destructive interference patterns. Rotating antennas and food turntables help spread out the hot spots.

Another use of microwaves for heating is within the human body. Microwaves will penetrate more than shorter wavelengths into tissue and so can accomplish “deep heating” (called microwave diathermy). This is used for treating muscular pains, spasms, tendonitis, and rheumatoid arthritis.

Making Connections: Take-Home Experiment—Microwave Ovens

1. Look at the door of a microwave oven. Describe the structure of the door. Why is there a metal grid on the door? How does the size of the holes in the grid compare with the wavelengths of microwaves used in microwave ovens? What is this wavelength?
2. Place a glass of water (about 250 ml) in the microwave and heat it for 30 seconds. Measure the temperature gain (the ΔTΔT size 12{DT} {}). Assuming that the power output of the oven is 1000 W, calculate the efficiency of the heat-transfer process.
3. Remove the rotating turntable or moving plate and place a cup of water in several places along a line parallel with the opening. Heat for 30 seconds and measure the ΔTΔT size 12{DT} {} for each position. Do you see cases of destructive interference?

Microwaves generated by atoms and molecules far away in time and space can be received and detected by electronic circuits. Deep space acts like a blackbody with a 2.7 K temperature, radiating most of its energy in the microwave frequency range. In 1964, Penzias and Wilson detected this radiation and eventually recognized that it was the radiation of the Big Bang's cooled remnants.

Infrared Radiation

The microwave and infrared regions of the electromagnetic spectrum overlap (see [link]). Infrared radiation is generally produced by thermal motion and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means "below red." Frequencies at its upper limit are too high to be produced by accelerating electrons in circuits, but small systems, such as atoms and molecules, can vibrate fast enough to produce these waves.

Water molecules rotate and vibrate particularly well at infrared frequencies, emitting and absorbing them so efficiently that the emissivity for skin is e=0.97e=0.97 size 12{e=0 "." "97"} {} in the infrared. Night-vision scopes can detect the infrared emitted by various warm objects, including humans, and convert it to visible light.

We can examine radiant heat transfer from a house by using a camera capable of detecting infrared radiation. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, some of which are called quartz heaters, to preferentially warm us because we absorb infrared better than our surroundings.

The Sun radiates like a nearly perfect blackbody (that is, it has e=1e=1 size 12{e=1} {}), with a 6000 K surface temperature. About half of the solar energy arriving at the Earth is in the infrared region, with most of the rest in the visible part of the spectrum, and a relatively small amount in the ultraviolet. On average, 50 percent of the incident solar energy is absorbed by the Earth.

The relatively constant temperature of the Earth is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by CO2CO2 size 12{"CO" rSub { size 8{2} } } {} and H2OH2O size 12{H rSub { size 8{2} } O} {} in the atmosphere and then radiated back to Earth or into outer space. This radiation back to Earth is known as the greenhouse effect, and it maintains the surface temperature of the Earth about 40ºC40ºC size 12{"40″°C} {} higher than it would be if there is no absorption. Some scientists think that the increased concentration of CO2CO2 size 12{"CO" rSub { size 8{2} } } {} and other greenhouse gases in the atmosphere, resulting from increases in fossil fuel burning, has increased global average temperatures.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.

[link] shows this part of the spectrum, together with the colors associated with particular pure wavelengths. We usually refer to visible light as having wavelengths of between 400 nm and 750 nm. (The retina of the eye actually responds to the lowest ultraviolet frequencies, but these do not normally reach the retina because they are absorbed by the cornea and lens of the eye.)

Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.

A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

The visible strip of the electromagnetic spectrum is highlighted and shown in the picture. The wave length range is from eight hundred nanometers on the left to three hundred nanometers on the right. The divisions between infrared, visible, and ultraviolet are not perfectly distinct. The colors in the visible strip are also not perfectly distinct; they are marked as bands labeled from red on the left to violet on the right.

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Visible light is the most predominant and we enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis makes use of parts of the visible spectrum to make sugars.

Integrated Concept Problem: Correcting Vision with Lasers

During laser vision correction, a brief burst of 193-nm ultraviolet light is projected onto the cornea of a patient. It makes a spot 0.80 mm in diameter and evaporates a layer of cornea 0.30μm0.30μm size 12{0 “.” “30” mm} {} thick. Calculate the energy absorbed, assuming the corneal tissue has the same properties as water; it is initially at 34ºC34ºC size 12{“34″°C} {}. Assume the evaporated tissue leaves at a temperature of 100ºC100ºC size 12{“100″°C} {}.

Strategy

The energy from the laser light goes toward raising the temperature of the tissue and also toward evaporating it. Thus we have two amounts of heat to add together. Also, we need to find the mass of corneal tissue involved.

Solution

To figure out the heat required to raise the temperature of the tissue to 100ºC100ºC size 12{“100″°C} {}, we can apply concepts of thermal energy. We know that

$Q = mc\Delta T$,$Q = mc\Delta T$, size 12{ ital “Q “= ital ” mc”ΔT} {}

where Q is the heat required to raise the temperature, ΔTΔT size 12{DT} {} is the desired change in temperature, mm size 12{m} {} is the mass of tissue to be heated, and cc size 12{c} {} is the specific heat of water equal to 4186 J/kg/K.

Without knowing the mass mm at this point, we have

Q=m(4186 J/kg/K)(100ºC–34ºC)=m(276,276 J/kg)=m(276 kJ/kg).Q=m(4186 J/kg/K)(100ºC–34ºC)=m(276,276 J/kg)=m(276 kJ/kg). size 12{Q=m ("4186 J/kg/K") ("100"°C – "34"°C) =m ("276,276 J/kg") =m ("276 kJ") } {}

The latent heat of vaporization of water is 2256 kJ/kg, so that the energy needed to evaporate mass mm size 12{m} {} is

Qv=mLv=m(2256 kJ/kg).Qv=mLv=m(2256 kJ/kg). size 12{Q rSub { size 8{v} } = mL rSub { size 8{v} } = m ("2256") " kJ"} {}

To find the mass mm size 12{m} {}, we use the equation ρ=m/Vρ=m/V size 12{ρ= {m} slash {V} } {}, where ρρ size 12{ρ} {} is the density of the tissue and VV size 12{V} {} is its volume. For this case,

m=ρV = (1000 kg/m3)(area×thickness(m3))=(1000 kg/m3)(π(0.80×10–3 m)2/4)(0.30×10–6 m)=0.151×10–9 kg.m=ρV = (1000 kg/m3)(area×thickness(m3))=(1000 kg/m3)(π(0.80×10–3 m)2/4)(0.30×10–6 m)=0.151×10–9 kg.alignl { stack {
size 12{m = ρ"V "=" 1000 kg/m" rSup { size 8{3} } ("area"´"thickness") } {} #
="1000" (p (0 "." "80"´"10" rSup { size 8{ +- 3} } " m") rSup { size 8{2} } /4) (0 "." "30"´"10" rSup { size 8{-6} } " m") {} #
=0 "." "151"´"10" rSup { size 8{ +- 9} } " kg" "." {}
} } {}

Therefore, the total energy absorbed by the tissue in the eye is the sum of QQ size 12{Q} {} and QvQv size 12{Q rSub { size 8{v} } } {}:

Qtot=m(cΔT + Lv)=(0.151×10−9kg)(276 kJ/kg+2256 kJ/kg)=382×10−9 kJ.Qtot=m(cΔT + Lv)=(0.151×10−9kg)(276 kJ/kg+2256 kJ/kg)=382×10−9 kJ. size 12{Q rSub { size 8{"tot"} } = m (cD"T "+" L" rSub { size 8{v} }) =0 "." "151"´"10" rSup { size 8{-9} } ("276"+"2256") ="382"´"10" rSup { size 8{-9} } " kJ"} {}

Discussion

The lasers used for this eye surgery are excimer lasers, whose light is well absorbed by biological tissue. They evaporate rather than burn the tissue, and can be used for precision work. Most lasers used for this type of eye surgery have an average power rating of about one watt. For our example, if we assume that each laser burst from this pulsed laser lasts for 10 ns, and there are 400 bursts per second, then the average power is Qtot×400=150 mWQtot×400=150 mW size 12{Q rSub { size 8{"tot"} } ´"400"="150"" mW"} {}.

Optics is the study of the behavior of visible light and other forms of electromagnetic waves. Optics falls into two distinct categories. When electromagnetic radiation, such as visible light, interacts with objects that are large compared with its wavelength, its motion can be represented by straight lines like rays. Ray optics is the study of such situations and includes lenses and mirrors.

When electromagnetic radiation interacts with objects about the same size as the wavelength or smaller, its wave nature becomes apparent. For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. Physical or wave optics is the study of such situations and includes all wave characteristics.

Take-Home Experiment: Colors That Match

When you light a match you see largely orange light; when you light a gas stove you see blue light. Why are the colors different? What other colors are present in these?

Ultraviolet Radiation

Ultraviolet means “above violet.” The electromagnetic frequencies of ultraviolet radiation (UV) extend upward from violet, the highest-frequency visible light. Ultraviolet is also produced by atomic and molecular motions and electronic transitions. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies, which overlap with the lowest X-ray frequencies. It was recognized as early as 1801 by Johann Ritter that the solar spectrum had an invisible component beyond the violet range.

Solar UV radiation is broadly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone (O3O3 size 12{O rSub { size 8{3} } } {}) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth’s surface is UV-A.

Human Exposure to UV Radiation

It is largely exposure to UV-B that causes skin cancer. It is estimated that as many as 20% of adults will develop skin cancer over the course of their lifetime. Again, treatment is often successful if caught early. Despite very little UV-B reaching the Earth’s surface, there are substantial increases in skin-cancer rates in countries such as Australia, indicating how important it is that UV-B and UV-C continue to be absorbed by the upper atmosphere.

All UV radiation can damage collagen fibers, resulting in an acceleration of the aging process of skin and the formation of wrinkles. Because there is so little UV-B and UV-C reaching the Earth’s surface, sunburn is caused by large exposures, and skin cancer from repeated exposure. Some studies indicate a link between overexposure to the Sun when young and melanoma later in life.

The tanning response is a defense mechanism in which the body produces pigments to absorb future exposures in inert skin layers above living cells. Basically UV-B radiation excites DNA molecules, distorting the DNA helix, leading to mutations and the possible formation of cancerous cells.

Repeated exposure to UV-B may also lead to the formation of cataracts in the eyes—a cause of blindness among people living in the equatorial belt where medical treatment is limited. Cataracts, clouding in the eye’s lens and a loss of vision, are age related; 60% of those between the ages of 65 and 74 will develop cataracts. However, treatment is easy and successful, as one replaces the lens of the eye with a plastic lens. Prevention is important. Eye protection from UV is more effective with plastic sunglasses than those made of glass.

A major acute effect of extreme UV exposure is the suppression of the immune system, both locally and throughout the body.

Low-intensity ultraviolet is used to sterilize haircutting implements, implying that the energy associated with ultraviolet is deposited in a manner different from lower-frequency electromagnetic waves. (Actually this is true for all electromagnetic waves with frequencies greater than visible light.)

Flash photography is generally not allowed of precious artworks and colored prints because the UV radiation from the flash can cause photo-degradation in the artworks. Often artworks will have an extra-thick layer of glass in front of them, which is especially designed to absorb UV radiation.

UV Light and the Ozone Layer

If all of the Sun's ultraviolet radiation reached the Earth's surface, there would be extremely grave effects on the biosphere from the severe cell damage it causes. However, the layer of ozone (O3O3 size 12{O rSub { size 8{3} } } {}) in our upper atmosphere (10 to 50 km above the Earth) protects life by absorbing most of the dangerous UV radiation.

Unfortunately, today we are observing a depletion in ozone concentrations in the upper atmosphere. This depletion has led to the formation of an "ozone hole" in the upper atmosphere. The hole is more centered over the southern hemisphere, and changes with the seasons, being largest in the spring. This depletion is attributed to the breakdown of ozone molecules by refrigerant gases called chlorofluorocarbons (CFCs).

The UV radiation helps dissociate the CFC's, releasing highly reactive chlorine (Cl) atoms, which catalyze the destruction of the ozone layer. For example, the reaction of CFCl3CFCl3 size 12{"CFCl" rSub { size 8{3} } } {} with a photon of light hvhv size 12{ left (ital "hv" right)} {} can be written as:

CFCl3+ hv→ CFCl2+ Cl.CFCl3+ hv→ CFCl2+ Cl. size 12{"CFCl" rSub { size 8{3} } +" h"v" " rightarrow " CFCl" rSub { size 8{2} } +" Cl"} {}

The Cl atom then catalyzes the breakdown of ozone as follows:

Cl + O3→ ClO + O2 and ClO + O3→ Cl + 2O2.Cl + O3→ ClO + O2 and ClO + O3→ Cl + 2O2. size 12{"Cl "+" O" rSub { size 8{3} } " " rightarrow " ClO "+" O" rSub { size 8{2} } " and ClO "+" O" rSub { size 8{3} } " " rightarrow " Cl "+" 2O" rSub { size 8{2} } } {}

A single chlorine atom could destroy ozone molecules for up to two years before being transported down to the surface. The CFCs are relatively stable and will contribute to ozone depletion for years to come. CFCs are found in refrigerants, air conditioning systems, foams, and aerosols.

International concern over this problem led to the establishment of the "Montreal Protocol" agreement (1987) to phase out CFC production in most countries. However, developing-country participation is needed if worldwide production and elimination of CFCs is to be achieved. Probably the largest contributor to CFC emissions today is India. But the protocol seems to be working, as there are signs of an ozone recovery. (See [link].)

This map of ozone concentration over Antarctica in October 2011 shows severe depletion suspected to be caused by CFCs. Less dramatic but more general depletion has been observed over northern latitudes, suggesting the effect is global. With less ozone, more ultraviolet radiation from the Sun reaches the surface, causing more damage. (credit: NASA Ozone Watch)

The map shows the variation in concentration of ozone over Antarctica. The scale for the total ozone level is depicted below the graph in Dobson units. The values are marked in colors of spectrum with the lowest value is marked in violet and the maximum value in red. The Antarctica region is marked in violet showing lesser ozone concentration and more ultraviolet rays. The region around Antarctica is in green, showing slightly higher concentration of ozone.

Benefits of UV Light

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin (epidermis) results from exposure to UVB radiation, generally from sunlight. A number of studies indicate lack of vitamin D can result in the development of a range of cancers (prostate, breast, colon), so a certain amount of UV exposure is helpful. Lack of vitamin D is also linked to osteoporosis. Exposures (with no sunscreen) of 10 minutes a day to arms, face, and legs might be sufficient to provide the accepted dietary level. However, in the winter time north of about 37º37º size 12{“37″°} {} latitude, most UVB gets blocked by the atmosphere.

UV radiation is used in the treatment of infantile jaundice and in some skin conditions. It is also used in sterilizing workspaces and tools, and killing germs in a wide range of applications. It is also used as an analytical tool to identify substances.

When exposed to ultraviolet, some substances, such as minerals, glow in characteristic visible wavelengths, a process called fluorescence. So-called black lights emit ultraviolet to cause posters and clothing to fluoresce in the visible. Ultraviolet is also used in special microscopes to detect details smaller than those observable with longer-wavelength visible-light microscopes.

Things Great and Small: A Submicroscopic View of X-Ray Production

X-rays can be created in a high-voltage discharge. They are emitted in the material struck by electrons in the discharge current. There are two mechanisms by which the electrons create X-rays.

The first method is illustrated in [link]. An electron is accelerated in an evacuated tube by a high positive voltage. The electron strikes a metal plate (e.g., copper) and produces X-rays. Since this is a high-voltage discharge, the electron gains sufficient energy to ionize the atom.

Artist’s conception of an electron ionizing an atom followed by the recapture of an electron and emission of an X-ray. An energetic electron strikes an atom and knocks an electron out of one of the orbits closest to the nucleus. Later, the atom captures another electron, and the energy released by its fall into a low orbit generates a high-energy EM wave called an X-ray.

An atom is shown. The nucleus is in the center as a cluster of small spheres packed together. Four electron orbits are shown around the nucleus. The one close to the nucleus is circular. All the other orbits are elliptical in nature and inclined at various angles. An electron, represented as a tiny sphere, is shown to strike the atom. An electron is shown knocked out from the closest orbit. A second image of the same atom illustrates another electron striking innermost orbit; a wavy red arrow representing an x ray is shooting away from the innermost orbit.

In the case shown, an inner-shell electron (one in an orbit relatively close to and tightly bound to the nucleus) is ejected. A short time later, another electron is captured and falls into the orbit in a single great plunge. The energy released by this fall is given to an EM wave known as an X-ray. Since the orbits of the atom are unique to the type of atom, the energy of the X-ray is characteristic of the atom, hence the name characteristic X-ray.

The second method by which an energetic electron creates an X-ray when it strikes a material is illustrated in [link]. The electron interacts with charges in the material as it penetrates. These collisions transfer kinetic energy from the electron to the electrons and atoms in the material.

Artist’s conception of an electron being slowed by collisions in a material and emitting X-ray radiation. This energetic electron makes numerous collisions with electrons and atoms in a material it penetrates. An accelerated charge radiates EM waves, a second method by which X-rays are created.

A picture showing an electron represented as a tiny sphere shown to strike the atoms in the material represented as spheres slightly larger in size than the electron. A ray of X ray is shown to come out from the material shown by a wavy arrow.

A loss of kinetic energy implies an acceleration, in this case decreasing the electron's velocity. Whenever a charge is accelerated, it radiates EM waves. Given the high energy of the electron, these EM waves can have high energy. We call them X-rays. Since the process is random, a broad spectrum of X-ray energy is emitted that is more characteristic of the electron energy than the type of material the electron encounters. Such EM radiation is called "bremsstrahlung" (German for "braking radiation").

X-Rays

In the 1850s, scientists (such as Faraday) began experimenting with high-voltage electrical discharges in tubes filled with rarefied gases. It was later found that these discharges created an invisible, penetrating form of very high frequency electromagnetic radiation. This radiation was called an X-ray, because its identity and nature were unknown.

As described in Things Great and Small, there are two methods by which X-rays are created—both are submicroscopic processes and can be caused by high-voltage discharges. While the low-frequency end of the X-ray range overlaps with the ultraviolet, X-rays extend to much higher frequencies (and energies).

X-rays have adverse effects on living cells similar to those of ultraviolet radiation, and they have the additional liability of being more penetrating, affecting more than the surface layers of cells. Cancer and genetic defects can be induced by exposure to X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained. However, questions have risen in recent years as to accidental overexposure of some people during CT scans—a mistake at least in part due to poor monitoring of radiation dose.

The ability of X-rays to penetrate matter depends on density, and so an X-ray image can reveal very detailed density information. [link] shows an example of the simplest type of X-ray image, an X-ray shadow on film. The amount of information in a simple X-ray image is impressive, but more sophisticated techniques, such as CT scans, can reveal three-dimensional information with details smaller than a millimeter.

This shadow X-ray image shows many interesting features, such as artificial heart valves, a pacemaker, and the wires used to close the sternum. (credit: P. P. Urone)

An X ray image of the chest is shown. It shows the section of the heart with artificial heart valves, a pacemaker, and the wires used to close the sternum.

The use of X-ray technology in medicine is called radiology—an established and relatively cheap tool in comparison to more sophisticated technologies. Consequently, X-rays are widely available and used extensively in medical diagnostics. During World War I, mobile X-ray units, advocated by Madame Marie Curie, were used to diagnose soldiers.

Because they can have wavelengths less than 0.01 nm, X-rays can be scattered (a process called X-ray diffraction) to detect the shape of molecules and the structure of crystals. X-ray diffraction was crucial to Crick, Watson, and Wilkins in the determination of the shape of the double-helix DNA molecule.

X-rays are also used as a precise tool for trace-metal analysis in X-ray induced fluorescence, in which the energy of the X-ray emissions are related to the specific types of elements and amounts of materials present.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted. The most penetrating nuclear radiation was called a gamma ray **(γγ size 12{g} {} ray)** (again a name given because its identity and character were unknown), and it was later found to be an extremely high frequency electromagnetic wave.

In fact, γγ size 12{g} {} rays are any electromagnetic radiation emitted by a nucleus. This can be from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ-rayγ-ray size 12{γ”-ray”} {} frequency range overlaps the upper end of the X-ray range, but γγ size 12{g} {} rays can have the highest frequency of any electromagnetic radiation.

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γγ size 12{g} {} rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

[link] shows a medical image based on γγ size 12{g} {} rays. Food spoilage can be greatly inhibited by exposing it to large doses of γγ size 12{γ} {} radiation, thereby obliterating responsible microorganisms. Damage to food cells through irradiation occurs as well, and the long-term hazards of consuming radiation-preserved food are unknown and controversial for some groups. Both X-ray and γ-rayγ-ray size 12{g”-ray”} {} technologies are also used in scanning luggage at airports.

This is an image of the γγ size 12{g} {} rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Bone cancer is evidenced by nonuniform concentration in similar structures. For example, some ribs are darker than others. (credit: P. P. Urone)

A skeletal image of a human body is shown. The image represents gamma rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Some parts of the image are darker than the others. The ribs are shown darker than the leg and hand bones.

Detecting Electromagnetic Waves from Space

A final note on star gazing. The entire electromagnetic spectrum is used by researchers for investigating stars, space, and time. As noted earlier, Penzias and Wilson detected microwaves to identify the background radiation originating from the Big Bang. Radio telescopes such as the Arecibo Radio Telescope in Puerto Rico and Parkes Observatory in Australia were designed to detect radio waves.

Infrared telescopes need to have their detectors cooled by liquid nitrogen to be able to gather useful signals. Since infrared radiation is predominantly from thermal agitation, if the detectors were not cooled, the vibrations of the molecules in the antenna would be stronger than the signal being collected.

The most famous of these infrared sensitive telescopes is the James Clerk Maxwell Telescope in Hawaii. The earliest telescopes, developed in the seventeenth century, were optical telescopes, collecting visible light. Telescopes in the ultraviolet, X-ray, and γγ size 12{g} {}-ray regions are placed outside the atmosphere on satellites orbiting the Earth.

The Hubble Space Telescope (launched in 1990) gathers ultraviolet radiation as well as visible light. In the X-ray region, there is the Chandra X-ray Observatory (launched in 1999), and in the γγ size 12{g} {}-ray region, there is the new Fermi Gamma-ray Space Telescope (launched in 2008—taking the place of the Compton Gamma Ray Observatory, 1991–2000.).

PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

Color Vision

PhET Interactive Simulation

Section Summary

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by vW=fλvW=fλ size 12{v rSub { size 8{W} } =fλ} {}, so that for electromagnetic waves,
  c=fλ,c=fλ, size 12{c = fλ} {}

  where ff size 12{f} {} is the frequency, λλ size 12{λ} {} is the wavelength, and cc size 12{c} {} is the speed of light.
- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.
- Any electromagnetic wave produced by currents in wires is classified as a radio wave, the lowest frequency electromagnetic waves. Radio waves are divided into many types, depending on their applications, ranging up to microwaves at their highest frequencies.
- Infrared radiation lies below visible light in frequency and is produced by thermal motion and the vibration and rotation of atoms and molecules. Infrared’s lower frequencies overlap with the highest-frequency microwaves.
- Visible light is largely produced by electronic transitions in atoms and molecules, and is defined as being detectable by the human eye. Its colors vary with frequency, from red at the lowest to violet at the highest.
- Ultraviolet radiation starts with frequencies just above violet in the visible range and is produced primarily by electronic transitions in atoms and molecules.
- X-rays are created in high-voltage discharges and by electron bombardment of metal targets. Their lowest frequencies overlap the ultraviolet range but extend to much higher values, overlapping at the high end with gamma rays.
- Gamma rays are nuclear in origin and are defined to include the highest-frequency electromagnetic radiation of any type.

Conceptual Questions

If you live in a region that has a particular TV station, you can sometimes pick up some of its audio portion on your FM radio receiver. Explain how this is possible. Does it imply that TV audio is broadcast as FM?

Explain why people who have the lens of their eye removed because of cataracts are able to see low-frequency ultraviolet.

How do fluorescent soap residues make clothing look “brighter and whiter” in outdoor light? Would this be effective in candlelight?

Give an example of resonance in the reception of electromagnetic waves.

Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light or infrared and X-rays).

Why don’t buildings block radio waves as completely as they do visible light?

Make a list of some everyday objects and decide whether they are transparent or opaque to each of the types of electromagnetic waves.

Your friend says that more patterns and colors can be seen on the wings of birds if viewed in ultraviolet light. Would you agree with your friend? Explain your answer.

The rate at which information can be transmitted on an electromagnetic wave is proportional to the frequency of the wave. Is this consistent with the fact that laser telephone transmission at visible frequencies carries far more conversations per optical fiber than conventional electronic transmission in a wire? What is the implication for ELF radio communication with submarines?

Give an example of energy carried by an electromagnetic wave.

In an MRI scan, a higher magnetic field requires higher frequency radio waves to resonate with the nuclear type whose density and location is being imaged. What effect does going to a larger magnetic field have on the most efficient antenna to broadcast those radio waves? Does it favor a smaller or larger antenna?

Laser vision correction often uses an excimer laser that produces 193-nm electromagnetic radiation. This wavelength is extremely strongly absorbed by the cornea and ablates it in a manner that reshapes the cornea to correct vision defects. Explain how the strong absorption helps concentrate the energy in a thin layer and thus give greater accuracy in shaping the cornea. Also explain how this strong absorption limits damage to the lens and retina of the eye.

Problems & Exercises

(a) Two microwave frequencies are authorized for use in microwave ovens: 900 and 2560 MHz. Calculate the wavelength of each. (b) Which frequency would produce smaller hot spots in foods due to interference effects?

(a) 33.3 cm (900 MHz) 11.7 cm (2560 MHz)

(b) The microwave oven with the smaller wavelength would produce smaller hot spots in foods, corresponding to the one with the frequency 2560 MHz.

(a) Calculate the range of wavelengths for AM radio given its frequency range is 540 to 1600 kHz. (b) Do the same for the FM frequency range of 88.0 to 108 MHz.

A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?

26.96 MHz

Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.

Combing your hair leads to excess electrons on the comb. How fast would you have to move the comb up and down to produce red light?

5.0×10145.0×1014 Hz

Electromagnetic radiation having a 15.0−μm15.0−μm size 12{“15” “.” 0-mm} {} wavelength is classified as infrared radiation. What is its frequency?

Approximately what is the smallest detail observable with a microscope that uses ultraviolet light of frequency 1.20×1015 Hz1.20×1015 Hz size 12{1 “.” “20”´”10″ rSup { size 8{“15″} } ” Hz”} {}?

λ=cf=3.00×108 m/s1.20×1015 Hz=2.50×10–7 mλ=cf=3.00×108 m/s1.20×1015 Hz=2.50×10–7 m size 12{λ= { {c} over {f} } = { {3 “.” “00”´”10″ rSup { size 8{8} } ” m/s”} over {1 “.” “20”´”10″ rSup { size 8{“15″} } ” Hz”} } = {underline {2 “.” “50”´”10″ rSup { size 8{ +- 7} } ” m”}} } {}

A radar used to detect the presence of aircraft receives a pulse that has reflected off an object 6×10−5 s6×10−5 s size 12{6´”10″ rSup { size 8{-5} } ” s”} {} after it was transmitted. What is the distance from the radar station to the reflecting object?

Some radar systems detect the size and shape of objects such as aircraft and geological terrain. Approximately what is the smallest observable detail utilizing 500-MHz radar?

0.600 m

Determine the amount of time it takes for X-rays of frequency 3×1018 Hz3×1018 Hz size 12{3´”10″ rSup { size 8{“18″} } ” Hz”} {} to travel (a) 1 mm and (b) 1 cm.

If you wish to detect details of the size of atoms (about 1×10−10 m1×10−10 m size 12{1´”10″ rSup { size 8{- “10″} } ” m”} {}) with electromagnetic radiation, it must have a wavelength of about this size. (a) What is its frequency? (b) What type of electromagnetic radiation might this be?

(a) f=cλ=3.00×108 m/s1×10-10 m=3×1018 Hzf=cλ=3.00×108 m/s1×10-10 m=3×1018 Hz size 12{f= { {c} over {λ} } = { {3 “.” “00”´”10″ rSup { size 8{8} } ” m/s”} over {1´”10″ rSup { size 8{ +- “10”} } ” m”} } = {underline {3´”10″ rSup { size 8{“18″} } ” Hz”}} } {}

(b) X-rays

If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is 1.50×1011 m1.50×1011 m size 12{1 “.” “50”´”10″ rSup { size 8{“11″} } ” m”} {} away?

Distances in space are often quoted in units of light years, the distance light travels in one year. (a) How many meters is a light year? (b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.00×1062.00×106 size 12{2 “.” “00”´”10″ rSup { size 8{6} } } {} light years away? (c) The most distant galaxy yet discovered is 12.0×10912.0×109 size 12{“12” “.” 0´”10″ rSup { size 8{9} } } {} light years away. How far is this in meters?

A certain 50.0-Hz AC power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m. (a) What is the wavelength of this very low frequency electromagnetic wave? (b) What is its maximum magnetic field strength?

(a) 6.00×106 m6.00×106 m size 12{5 “.” “00” times “10” rSup { size 8{6} } ” m”} {}

(b) 4.33×10−5 T4.33×10−5 T size 12{4 “.” “33” times “10” rSup { size 8{ – 5} } ” T”} {}

During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person’s chest, creating a 1.00-Hz electromagnetic wave. (a) What is the maximum electric field strength created? (b) What is the corresponding maximum magnetic field strength in the electromagnetic wave? (c) What is the wavelength of the electromagnetic wave?

(a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength (λ/4λ/4 size 12{λ/4} {}) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band? (b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

(a) 1.50×10^{6} Hz, AM band

(b) The resonance of currents on an antenna that is 1/4 their wavelength is analogous to the fundamental resonant mode of an air column closed at one end, since the tube also has a length equal to 1/4 the wavelength of the fundamental oscillation.

(a) What is the wavelength of 100-MHz radio waves used in an MRI unit? (b) If the frequencies are swept over a ±1.00±1.00 size 12{ +- 1 “.” “00”%} {} range centered on 100 MHz, what is the range of wavelengths broadcast?

(a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?

(a) 1.55×1015 Hz1.55×1015 Hz size 12{1 "." "55" times "10" rSup { size 8{"15″} } " Hz"} {}

(b) The shortest wavelength of visible light is 380 nm, so that

λvisibleλUV=380 nm193 nm=1.97.λvisibleλUV=380 nm193 nm=1.97.alignl { stack {
size 12{ { {λ rSub { size 8{"visible"} } } over {λ rSub { size 8{"UV"} } } } } {} #
= { {"380 nm"} over {"193 nm"} } {} #
=1 "." "97" "." {}
} } {}

In other words, the UV radiation is 97% more accurate than the shortest wavelength of visible light, or almost twice as accurate!

TV-reception antennas for VHF are constructed with cross wires supported at their centers, as shown in [link]. The ideal length for the cross wires is one-half the wavelength to be received, with the more expensive antennas having one for each channel. Suppose you measure the lengths of the wires for particular channels and find them to be 1.94 and 0.753 m long, respectively. What are the frequencies for these channels?

A television reception antenna has cross wires of various lengths to most efficiently receive different wavelengths.

The picture of a television reception antenna mounted on the roof of a house. An enlarged image of the antenna is also shown. The antenna has a long horizontal rod having smaller cross wires of decreasing length from left to right. The cross wires are numbered from two to thirteen.

Conversations with astronauts on lunar walks had an echo that was used to estimate the distance to the Moon. The sound spoken by the person on Earth was transformed into a radio signal sent to the Moon, and transformed back into sound on a speaker inside the astronaut's space suit. This sound was picked up by the microphone in the space suit (intended for the astronaut's voice) and sent back to Earth as a radio echo of sorts. If the round-trip time was 2.60 s, what was the approximate distance to the Moon, neglecting any delays in the electronics?

3.90×108 m3.90×108 m size 12{3 "." "90" times "10" rSup { size 8{8} } " m"} {}

Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. (a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to 0.100 ns? (b) What percent accuracy is this, given the average distance to the Moon is 3.84×108 m3.84×108 m size 12{3 “.” “84”´”10″ rSup { size 8{8} } ” m”} {}?

Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object. (a) How far away is the planet Venus if the echo time is 1000 s? (b) What is the echo time for a car 75.0 m from a Highway Police radar unit? (c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

(a) 1.50×1011 m1.50×1011 m size 12{1 “.” “50” times “10” rSup { size 8{“11″} } ” m”} {}

(b) 0.500 μs0.500 μs size 12{0 “.” “500 “ms} {}

(c) 66.7 ns

Integrated Concepts

(a) Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (b) Compare this with the ratio of highest to lowest frequencies the ear can hear.

Integrated Concepts

(a) Calculate the rate in watts at which heat transfer through radiation occurs (almost entirely in the infrared) from 1.0 m21.0 m2 size 12{1 “.” 0″ m” rSup { size 8{2} } } {} of the Earth’s surface at night. Assume the emissivity is 0.90, the temperature of the Earth is 15ºC15ºC size 12{“15″°C} {}, and that of outer space is 2.7 K. (b) Compare the intensity of this radiation with that coming to the Earth from the Sun during the day, which averages about 800 W/m2800 W/m2 size 12{“800″” W/m” rSup { size 8{2} } } {}, only half of which is absorbed. (c) What is the maximum magnetic field strength in the outgoing radiation, assuming it is a continuous wave?

(a) −3.5×102 W/m2−3.5×102 W/m2 size 12{-3 “.” 5´”10″ rSup { size 8{2} } ” W/m” rSup { size 8{2} } } {}

(b) 88%

(c) 1.7μT1.7μT size 12{1 “.” 7 mT} {}

Glossary

electromagnetic spectrum
the full range of wavelengths or frequencies of electromagnetic radiation
radio waves
electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena
microwaves
electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

thermal agitation

the thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

radar

a common application of microwaves. Radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm

infrared radiation (IR)

a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from 0.74μm0.74μm size 12{0 “.” “74” μm} {} to 300μm300μm size 12{“300” μm} {}

ultraviolet radiation (UV)

electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

visible light

the narrow segment of the electromagnetic spectrum to which the normal human eye responds

amplitude modulation (AM)

a method for placing information on electromagnetic waves by modulating the amplitude of a carrier wave with an audio signal, resulting in a wave with constant frequency but varying amplitude

extremely low frequency (ELF)

electromagnetic radiation with wavelengths usually in the range of 0 to 300 Hz, but also about 1kHz

carrier wave

an electromagnetic wave that carries a signal by modulation of its amplitude or frequency

frequency modulation (FM)

a method of placing information on electromagnetic waves by modulating the frequency of a carrier wave with an audio signal, producing a wave of constant amplitude but varying frequency

TV

video and audio signals broadcast on electromagnetic waves

very high frequency (VHF)

TV channels utilizing frequencies in the two ranges of 54 to 88 MHz and 174 to 222 MHz

ultra-high frequency (UHF)

TV channels in an even higher frequency range than VHF, of 470 to 1000 MHz

X-ray

invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γγ size 12{g} {}-ray range

gamma ray

(γγ size 12{g} {} ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γγ-ray frequency range overlaps the upper end of the X-ray range, but γγ size 12{g} {} rays can have the highest frequency of any electromagnetic radiation

24.4 Energy in Electromagnetic Waves

Energy in Electromagnetic Waves

- Explain how the energy and amplitude of an electromagnetic wave are related.
- Given its power output and the heating area, calculate the intensity of a microwave oven's electromagnetic field, as well as its peak electric and magnetic field strengths

Anyone who has used a microwave oven knows there is energy in electromagnetic waves. Sometimes this energy is obvious, such as in the warmth of the summer sun. Other times it is subtle, such as the unfelt energy of gamma rays, which can destroy living cells.

Electromagnetic waves can bring energy into a system by virtue of their electric and magnetic fields. These fields can exert forces and move charges in the system and, thus, do work on them. If the frequency of the electromagnetic wave is the same as the natural frequencies of the system (such as microwaves at the resonant frequency of water molecules), the transfer of energy is much more efficient.

Connections: Waves and Particles

The behavior of electromagnetic radiation clearly exhibits wave characteristics. But we shall find in later modules that at high frequencies, electromagnetic radiation also exhibits particle characteristics. These particle characteristics will be used to explain more of the properties of the electromagnetic spectrum and to introduce the formal study of modern physics.

Another startling discovery of modern physics is that particles, such as electrons and protons, exhibit wave characteristics. This simultaneous sharing of wave and particle properties for all submicroscopic entities is one of the great symmetries in nature.

Energy carried by a wave is proportional to its amplitude squared. With electromagnetic waves, larger EE size 12{E} {}-fields and BB size 12{B} {}-fields exert larger forces and can do more work.

The propagation of two electromagnetic waves is shown in three dimensional planes. The first wave shows with the variation of two components E and B. E is a sine wave in one plane with small arrows showing the vibrations of particles in the plane. B is a sine wave in a plane perpendicular to the E wave. The B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because E and B are perpendicular to each other. The direction of propagation of wave is shown perpendicular to E and B waves. The energy carried is given as E sub u. The second wave shows with the variation of the components two E and two B, that is, E and B waves with double the amplitude of the first case. Two E is a sine wave in one plane with small arrows showing the vibrations of particles in the plane. Two B is a sine wave in a plane perpendicular to the two E wave. The two B wave has arrows to show the vibrations of particles in the plane. The waves are shown intersecting each other at the junction of the planes because two E and two B waves are perpendicular to each other. The direction of propagation of wave is shown perpendicular to two E and two B waves. The energy carried is given as four E sub u.

But there is energy in an electromagnetic wave, whether it is absorbed or not. Once created, the fields carry energy away from a source. If absorbed, the field strengths are diminished and anything left travels on. Clearly, the larger the strength of the electric and magnetic fields, the more work they can do and the greater the energy the electromagnetic wave carries.

A wave's energy is proportional to its amplitude squared (E2E2 size 12{E rSup { size 8{2} } } {} or B2B2 size 12{B rSup { size 8{2} } } {}). This is true for waves on guitar strings, for water waves, and for sound waves, where amplitude is proportional to pressure. In electromagnetic waves, the amplitude is the maximum field strength of the electric and magnetic fields. (See [link].)

Thus the energy carried and the intensity II size 12{I} {} of an electromagnetic wave is proportional to E2E2 size 12{E rSup { size 8{2} } } {} and B2B2 size 12{B rSup { size 8{2} } } {}. In fact, for a continuous sinusoidal electromagnetic wave, the average intensity IaveIave size 12{I rSub { size 8{"ave"} } } {} is given by

Iave=cε0E022,Iave=cε0E022, size 12{I rSub { size 8{"ave"} } = { {ce rSub { size 8{0} } E rSub { size 8{0} } rSup { size 8{2} } } over {2} } } {}

where cc size 12{c} {} is the speed of light, ε0ε0 size 12{ε rSub { size 8{0} } } {} is the permittivity of free space, and E0E0 size 12{E rSub { size 8{0} } } {} is the maximum electric field strength; intensity, as always, is power per unit area (here in W/m2W/m2 size 12{“W/m” rSup { size 8{2} } } {}).

The average intensity of an electromagnetic wave IaveIave size 12{I rSub { size 8{“ave”} } } {} can also be expressed in terms of the magnetic field strength by using the relationship B=E/cB=E/c size 12{B= {E} slash {c} } {}, and the fact that ε0=1/μ0c2ε0=1/μ0c2 size 12{ε rSub { size 8{0} } = {1} slash {μ rSub { size 8{0} } } c rSup { size 8{2} } } {}, where μ0μ0 size 12{μ rSub { size 8{0} } } {} is the permeability of free space. Algebraic manipulation produces the relationship

Iave=cB022μ0,Iave=cB022μ0, size 12{I rSub { size 8{“ave”} } = { { ital “cB” rSub { size 8{0} } rSup { size 8{2} } } over {2μ rSub { size 8{0} } } } } {}

where B0B0 size 12{B rSub { size 8{0} } } {} is the maximum magnetic field strength.

One more expression for IaveIave size 12{I rSub { size 8{“ave”} } } {} in terms of both electric and magnetic field strengths is useful. Substituting the fact that c?B0=E0c?B0=E0 size 12{c cdot B rSub { size 8{0} } =E rSub { size 8{0} } } {}, the previous expression becomes

Iave=E0B02μ0.Iave=E0B02μ0. size 12{I rSub { size 8{“ave”} } = { {E rSub { size 8{0} } B rSub { size 8{0} } } over {2μ rSub { size 8{0} } } } } {}

Whichever of the three preceding equations is most convenient can be used, since they are really just different versions of the same principle: Energy in a wave is related to amplitude squared. Furthermore, since these equations are based on the assumption that the electromagnetic waves are sinusoidal, peak intensity is twice the average; that is, I0=2IaveI0=2Iave size 12{I rSub { size 8{0} } =2I rSub { size 8{“ave”} } } {}.

Calculate Microwave Intensities and Fields

On its highest power setting, a certain microwave oven projects 1.00 kW of microwaves onto a 30.0 by 40.0 cm area. (a) What is the intensity in W/m2W/m2 size 12{“W/m” rSup { size 8{2} } } {}? (b) Calculate the peak electric field strength E0E0 size 12{E rSub { size 8{0} } } {} in these waves. (c) What is the peak magnetic field strength B0B0 size 12{B rSub { size 8{0} } } {}?

Strategy

In part (a), we can find intensity from its definition as power per unit area. Once the intensity is known, we can use the equations below to find the field strengths asked for in parts (b) and (c).

Solution for (a)

Entering the given power into the definition of intensity, and noting the area is 0.300 by 0.400 m, yields

I=PA=1.00 kW0.300 m×0.400 m.I=PA=1.00 kW0.300 m×0.400 m. size 12{I= { {P} over {A} } = { {1 “.” “00”” kW”} over {0 “.” “300 m”×0 “.” “400 m”} } } {}

Here I=IaveI=Iave size 12{I=I rSub { size 8{“ave”} } } {}, so that

Iave=1000 W0.120 m2=8.33×103 W/m2.Iave=1000 W0.120 m2=8.33×103 W/m2. size 12{I rSub { size 8{“ave”} } = { {“1000″” W”} over {0 “.” “120”” m” rSup { size 8{2} } } } =8 “.” “33”×”10″ rSup { size 8{3} } ” W/m” rSup { size 8{2} } } {}

Note that the peak intensity is twice the average:

I0=2Iave=1.67×104W/m2.I0=2Iave=1.67×104W/m2. size 12{I rSub { size 8{0} } =2I rSub { size 8{“ave”} } =1 “.” “67” times “10” rSup { size 8{4} } {W} slash {m rSup { size 8{2} } } } {}

Solution for (b)

To find E0E0 size 12{E rSub { size 8{0} } } {}, we can rearrange the first equation given above for IaveIave size 12{I rSub { size 8{“ave”} } } {} to give

E0=2Iavecε01/2.E0=2Iavecε01/2. size 12{E rSub { size 8{0} } = left ({ {2I rSub { size 8{“ave”} } } over {ce rSub { size 8{0} } } } right) rSup { size 8{ {1}wideslash {2} } } } {}

Entering known values gives

E0=2(8.33×103 W/m2)(3.00×108 m/s)(8.85×10–12C2/N?m2)=2.51×103 V/m.E0=2(8.33×103 W/m2)(3.00×108 m/s)(8.85×10–12C2/N?m2)=2.51×103 V/m.alignl { stack {
size 12{E rSub { size 8{0} } = sqrt { { {2 (8 “.” “33”´”10″ rSup { size 8{3} } ” W/m” rSup { size 8{2} }) } over { (3 “.” “00”´”10″ rSup { size 8{8} } ” m/s”) (8 “.” “85”´”10″ rSup { size 8{ +- 2} } C rSup { size 8{2} } /N cdot m rSup { size 8{2} }) } } } } {} #
=2 “.” “51”´”10″ rSup { size 8{3} } ” V/m” “.” {}
} } {}

Solution for (c)

Perhaps the easiest way to find magnetic field strength, now that the electric field strength is known, is to use the relationship given by

B0=E0c.B0=E0c. size 12{B rSub { size 8{0} } = { {E rSub { size 8{0} } } over {c} } } {}

Entering known values gives

B0=2.51×103 V/m3.0×108 m/s=8.35×10−6 T.B0=2.51×103 V/m3.0×108 m/s=8.35×10−6 T.alignl { stack {
size 12{B rSub { size 8{0} } = { {2 “.” “51”´”10″ rSup { size 8{3} } ” V/m”} over {3 “.” 0´”10″ rSup { size 8{8} } ” m/s”} } } {} #
=8 “.” “35”´”10″ rSup { size 8{-6} } ” T” “.” {}
} } {}

Discussion

As before, a relatively strong electric field is accompanied by a relatively weak magnetic field in an electromagnetic wave, since B=E/cB=E/c size 12{B= {E} slash {c} } {}, and cc size 12{c} {} is a large number.

Section Summary

- The energy carried by any wave is proportional to its amplitude squared. For electromagnetic waves, this means intensity can be expressed as
  Iave=cε0E022,Iave=cε0E022, size 12{I rSub { size 8{“ave”} } = { {ce rSub { size 8{0} } E rSub { size 8{0} } rSup { size 8{2} } } over {2} } } {}

  where IaveIave size 12{I rSub { size 8{“ave”} } } {} is the average intensity in W/m2W/m2 size 12{“W/m” rSup { size 8{2} } } {}, and E0E0 size 12{E rSub { size 8{0} } } {} is the maximum electric field strength of a continuous sinusoidal wave.

- This can also be expressed in terms of the maximum magnetic field strength B0B0 size 12{B rSub { size 8{0} } } {} as
  Iave=cB022μ0Iave=cB022μ0 size 12{I rSub { size 8{“ave”} } = { { ital “cB” rSub { size 8{0} } rSup { size 8{2} } } over {2m rSub { size 8{0} } } } } {}

  and in terms of both electric and magnetic fields as

  Iave=E0B02μ0.Iave=E0B02μ0. size 12{I rSub { size 8{“ave”} } = { {E rSub { size 8{0} } B rSub { size 8{0} } } over {2m rSub { size 8{0} } } } } {}

- The three expressions for IaveIave size 12{I rSub { size 8{“ave”} } } {} are all equivalent.

Problems & Exercises

What is the intensity of an electromagnetic wave with a peak electric field strength of 125 V/m?

I=cε0E022=3.00×108 m/s8.85×10–12 C2/N?m2125 V/m22=20.7 W/m2I=cε0E022=3.00×108 m/s8.85×10–12 C2/N?m2125 V/m22=20.7 W/m2

Find the intensity of an electromagnetic wave having a peak magnetic field strength of 4.00×10−9 T4.00×10−9 T size 12{4 “.” “00”´”10″ rSup { size 8{-9} } ” T”} {}.

Assume the helium-neon lasers commonly used in student physics laboratories have power outputs of 0.250 mW. (a) If such a laser beam is projected onto a circular spot 1.00 mm in diameter, what is its intensity? (b) Find the peak magnetic field strength. (c) Find the peak electric field strength.

(a) I=PA=Pπr2=0.250×10−3 Wπ0.500×10−3 m2=318 W/m2I=PA=Pπr2=0.250×10−3 Wπ0.500×10−3 m2=318 W/m2 size 12{I= { {P} over {A} } = { {P} over {p r rSup { size 8{2} } } } = { {0 “.” “250”´”10″ rSup { size 8{-3} } ” W”} over {∂ left (0 “.” “500”´”10″ rSup { size 8{-3} } ” m” right) rSup { size 8{2} } } } =”318 W/m” rSup { size 8{2} } } {}

(b) Iave=cB022μ0?B0=2μ0Ic1/2=24π×10−7 T?m/A318.3 W/m23.00×108 m/s1/2=1.63×10−6 TIave=cB022μ0?B0=2μ0Ic1/2=24π×10−7 T?m/A318.3 W/m23.00×108 m/s1/2=1.63×10−6 Talignl { stack {
size 12{I rSub { size 8{“ave”} } = { { ital “cB” rSub { size 8{0} rSup { size 8{2} } } } over {2m rSub { size 8{0} } } } drarrow B rSub { size 8{0} } = left ({ {2m rSub { size 8{0} } I} over {c} } right) rSup { size 8{1/2} } } {} #
= left [{ {2 left (4¶´”10″ rSup { size 8{-7} } ” T” cdot “m/A” right) left (“318” “.” “3 W/m” rSup { size 8{2} } right)} over {3 “.” “00”´”10″ rSup { size 8{8} } ” m/s”} } right] rSup { size 8{ {1} slash {2} } } {} #
= {underline {1 “.” “63”´”10″ rSup { size 8{-6} } ” T”}} {}
} } {}

(c) E0=cB0=3.00×108 m/s1.633×10−6 T=4.90×102 V/mE0=cB0=3.00×108 m/s1.633×10−6 T=4.90×102 V/malignl { stack {
size 12{E rSub { size 8{0} } = ital “cB” rSub { size 8{0} } = left (3 “.” “00”´”10″ rSup { size 8{8} } ” m/s” right) left (1 “.” “633”´”10″ rSup { size 8{-6} } ” T” right)} {} #
= {underline {4 “.” “90”´”10″ rSup { size 8{2} } ” V/m”}} {}
} } {}

An AM radio transmitter broadcasts 50.0 kW of power uniformly in all directions. (a) Assuming all of the radio waves that strike the ground are completely absorbed, and that there is no absorption by the atmosphere or other objects, what is the intensity 30.0 km away? (Hint: Half the power will be spread over the area of a hemisphere.) (b) What is the maximum electric field strength at this distance?

Suppose the maximum safe intensity of microwaves for human exposure is taken to be 1.00 W/m21.00 W/m2 size 12{1 “.” “00””W/m” rSup { size 8{2} } } {}. (a) If a radar unit leaks 10.0 W of microwaves (other than those sent by its antenna) uniformly in all directions, how far away must you be to be exposed to an intensity considered to be safe? Assume that the power spreads uniformly over the area of a sphere with no complications from absorption or reflection. (b) What is the maximum electric field strength at the safe intensity? (Note that early radar units leaked more than modern ones do. This caused identifiable health problems, such as cataracts, for people who worked near them.)

(a) 89.2 cm

(b) 27.4 V/m

A 2.50-m-diameter university communications satellite dish receives TV signals that have a maximum electric field strength (for one channel) of 7.50μV/m7.50μV/m size 12{7 “.” “50” mV/m} {}. (See [link].) (a) What is the intensity of this wave? (b) What is the power received by the antenna? (c) If the orbiting satellite broadcasts uniformly over an area of 1.50×1013 m21.50×1013 m2 size 12{1 “.” “50”´”10″ rSup { size 8{“13″} } ” m” rSup { size 8{2} } } {} (a large fraction of North America), how much power does it radiate?

Satellite dishes receive TV signals sent from orbit. Although the signals are quite weak, the receiver can detect them by being tuned to resonate at their frequency.

A large, round dish antenna looking like a giant white saucer is shown. It rests on a pillar like structure based on the ground. It is shown to receive TV signals in the form of electromagnetic waves shown as wavy arrows.

Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to ignite nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of 1.00×1011 V/m1.00×1011 V/m size 12{1 "." "00"´"10″ rSup { size 8{"11″} } " V"/m} {} for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on a 1.00-mm21.00-mm2 size 12{1 "." "00""-mm" rSup { size 8{2} } } {} area?

(a) 333 T

(b) 1.33×1019W/m21.33×1019W/m2 size 12{1 "." "33"´"10″ rSup { size 8{"19"} } "W/m" rSup { size 8{2} } } {}

(c) 13.3 kJ

Show that for a continuous sinusoidal electromagnetic wave, the peak intensity is twice the average intensity (I0=2IaveI0=2Iave size 12{I rSub { size 8{0} } =2I rSub { size 8{"ave"} } } {}), using either the fact that E0=2ErmsE0=2Erms size 12{E rSub { size 8{0} } = sqrt {2} E rSub { size 8{"rms"} } } {}, or B0=2BrmsB0=2Brms size 12{B rSub { size 8{0} } = sqrt {2} B rSub { size 8{"rms"} } } {}, where rms means average (actually root mean square, a type of average).

Suppose a source of electromagnetic waves radiates uniformly in all directions in empty space where there are no absorption or interference effects. (a) Show that the intensity is inversely proportional to r2r2 size 12{r rSup { size 8{2} } } {}, the distance from the source squared. (b) Show that the magnitudes of the electric and magnetic fields are inversely proportional to rr size 12{r} {}.

(a) I=PA=P4πr2?1r2I=PA=P4πr2?1r2 size 12{I= { {P} over {A} } = { {P} over {4π r rSup { size 8{2} } } } prop { {1} over {r rSup { size 8{2} } } } } {}

(b) I?E02, B02?E02, B02?1r2?E0,B0?1rI?E02, B02?E02, B02?1r2?E0,B0?1r

Integrated Concepts

An LCLC size 12{ ital "LC"} {} circuit with a 5.00-pF capacitor oscillates in such a manner as to radiate at a wavelength of 3.30 m. (a) What is the resonant frequency? (b) What inductance is in series with the capacitor?

Integrated Concepts

What capacitance is needed in series with an 800−μH800−μH size 12{“800”-mH} {} inductor to form a circuit that radiates a wavelength of 196 m?

13.5 pF

Integrated Concepts

Police radar determines the speed of motor vehicles using the same Doppler-shift technique employed for ultra-sound in medical diagnostics. Beats are produced by mixing the double Doppler-shifted echo with the original frequency. If 1.50×109-Hz1.50×109-Hz size 12{1 “.” “50”´”10″ rSup { size 8{9} } “-Hz”} {} microwaves are used and a beat frequency of 150 Hz is produced, what is the speed of the vehicle? (Assume the same Doppler-shift formulas are valid with the speed of sound replaced by the speed of light.)

Integrated Concepts

Assume the mostly infrared radiation from a heat lamp acts like a continuous wave with wavelength 1.50μm1.50μm. (a) If the lamp’s 200-W output is focused on a person’s shoulder, over a circular area 25.0 cm in diameter, what is the intensity in W/m2W/m2 size 12{“W/m” rSup { size 8{2} } } {}? (b) What is the peak electric field strength? (c) Find the peak magnetic field strength. (d) How long will it take to increase the temperature of the 4.00-kg shoulder by 2.00º C2.00º C, assuming no other heat transfer and given that its specific heat is 3.47×103 J/kg?ºC3.47×103 J/kg?ºC?

(a) 4.07 kW/m24.07 kW/m2 size 12{4 “.” “07””kW/m” rSup { size 8{2} } } {}

(b) 1.75 kV/m

(c) 5.84μT5.84μT size 12{5 “.” “84” mT} {}

(d) 2 min 19 s

Integrated Concepts

On its highest power setting, a microwave oven increases the temperature of 0.400 kg of spaghetti by 45.0ºC45.0ºC size 12{“45” “.” 0°C} {} in 120 s. (a) What was the rate of power absorption by the spaghetti, given that its specific heat is 3.76×103 J/kg?ºC3.76×103 J/kg?ºC? (b) Find the average intensity of the microwaves, given that they are absorbed over a circular area 20.0 cm in diameter. (c) What is the peak electric field strength of the microwave? (d) What is its peak magnetic field strength?

Integrated Concepts

Electromagnetic radiation from a 5.00-mW laser is concentrated on a 1.00-mm21.00-mm2 size 12{1 “.” “00””-mm” rSup { size 8{2} } } {} area. (a) What is the intensity in W/m2W/m2 size 12{“W/m” rSup { size 8{2} } } {}? (b) Suppose a 2.00-nC static charge is in the beam. What is the maximum electric force it experiences? (c) If the static charge moves at 400 m/s, what maximum magnetic force can it feel?

(a) 5.00×103 W/m25.00×103 W/m2 size 12{5 “.” “00”´”10″ rSup { size 8{3} } ” W/m” rSup { size 8{2} } } {}

(b) 3.88×10−6 N3.88×10−6 N size 12{3 “.” “88” times “10” rSup { size 8{ – 6} } ” N”} {}

(c) 5.18×10−12 N5.18×10−12 N size 12{ {underline {5 “.” “18” times “10” rSup { size 8{ – “12”} } ” N”}} } {}

Integrated Concepts

A 200-turn flat coil of wire 30.0 cm in diameter acts as an antenna for FM radio at a frequency of 100 MHz. The magnetic field of the incoming electromagnetic wave is perpendicular to the coil and has a maximum strength of 1.00×10−12 T1.00×10−12 T size 12{1 “.” “00”×”10″ rSup { size 8{-“12″} } ” T”} {}. (a) What power is incident on the coil? (b) What average emf is induced in the coil over one-fourth of a cycle? (c) If the radio receiver has an inductance of 2.50μH2.50μH size 12{2 “.” “50” mH} {}, what capacitance must it have to resonate at 100 MHz?

Integrated Concepts

If electric and magnetic field strengths vary sinusoidally in time, being zero at t=0t=0 size 12{t=0} {}, then E=E0sin2πftE=E0sin2πft size 12{E=E rSub { size 8{0} } “sin”2π ital “ft”} {} and B=B0sin2πftB=B0sin2πft size 12{B=B rSub { size 8{0} } “sin”2p ital “ft”} {}. Let f=1.00 GHzf=1.00 GHz size 12{f=1 “.” “00”” GHz”} {} here. (a) When are the field strengths first zero? (b) When do they reach their most negative value? (c) How much time is needed for them to complete one cycle?

(a) t=0t=0 size 12{t=0} {}

(b) 7.50×10−10 s7.50×10−10 s size 12{7 “.” “50” times “10” rSup { size 8{ – “10”} } ” s”} {}

(c) 1.00×10−9 s1.00×10−9 s size 12{1 “.” “00” times “10” rSup { size 8{ – 9} } ” s”} {}

Unreasonable Results

A researcher measures the wavelength of a 1.20-GHz electromagnetic wave to be 0.500 m. (a) Calculate the speed at which this wave propagates. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Unreasonable Results

The peak magnetic field strength in a residential microwave oven is 9.20×10−5 T9.20×10−5 T size 12{9 “.” “20”´”10″ rSup { size 8{-5} } ” T”} {}. (a) What is the intensity of the microwave? (b) What is unreasonable about this result? (c) What is wrong about the premise?

(a) 1.01×106 W/m21.01×106 W/m2 size 12{1 “.” “01” times “10” rSup { size 8{6} } ” W/m” rSup { size 8{2} } } {}

(b) Much too great for an oven.

(c) The assumed magnetic field is unreasonably large.

Unreasonable Results

An LCLC size 12{ ital “LC”} {} circuit containing a 2.00-H inductor oscillates at such a frequency that it radiates at a 1.00-m wavelength. (a) What is the capacitance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Unreasonable Results

An LCLC size 12{ ital “LC”} {} circuit containing a 1.00-pF capacitor oscillates at such a frequency that it radiates at a 300-nm wavelength. (a) What is the inductance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

(a) 2.53×10−20H2.53×10−20H size 12{2 “.” “53” times “10” rSup { size 8{ – “20”} } H} {}

(b) L is much too small.

(c) The wavelength is unreasonably small.

Create Your Own Problem

Consider electromagnetic fields produced by high voltage power lines. Construct a problem in which you calculate the intensity of this electromagnetic radiation in W/m2W/m2 size 12{“W/m” rSup { size 8{2} } } {} based on the measured magnetic field strength of the radiation in a home near the power lines. Assume these magnetic field strengths are known to average less than a μTμT size 12{μT} {}. The intensity is small enough that it is difficult to imagine mechanisms for biological damage due to it. Discuss how much energy may be radiating from a section of power line several hundred meters long and compare this to the power likely to be carried by the lines. An idea of how much power this is can be obtained by calculating the approximate current responsible for μTμT size 12{μT} {} fields at distances of tens of meters.

Create Your Own Problem

Consider the most recent generation of residential satellite dishes that are a little less than half a meter in diameter. Construct a problem in which you calculate the power received by the dish and the maximum electric field strength of the microwave signals for a single channel received by the dish. Among the things to be considered are the power broadcast by the satellite and the area over which the power is spread, as well as the area of the receiving dish.

Glossary

maximum field strength
the maximum amplitude an electromagnetic wave can reach, representing the maximum amount of electric force and/or magnetic flux that the wave can exert
intensity
the power of an electric or magnetic field per unit area, for example, Watts per square meter

Appendix A Atomic Masses

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^{1}H	1.007 825	99.985%	
	Deuterium	2	^{2}H or D	2.014 102	0.015%	
	Tritium	3	^{3}H or T	3.016 050	β^-	12.33 y
2	Helium	3	^{3}He	3.016 030	$1.38 \times 10^{-4}\%$	
		4	^{4}He	4.002 603	$\approx 100\%$	
3	Lithium	6	^{6}Li	6.015 121	7.5%	
		7	^{7}Li	7.016 003	92.5%	
4	Beryllium	7	^{7}Be	7.016 928	EC	53.29 d
		9	^{9}Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		31	$^{31}\mathrm{Si}$	30.975 362	β^-	
15	Phosphorus	31	$^{31}\mathrm{P}$	30.973 762	100%	
		32	$^{32}\mathrm{P}$	31.973 907	β^-	14.28 d
16	Sulfur	32	$^{32}\mathrm{S}$	31.972 070	95.02%	
		35	$^{35}\mathrm{S}$	34.969 031	β^-	87.4 d
17	Chlorine	35	$^{35}\mathrm{Cl}$	34.968 852	75.77%	
		37	$^{37}\mathrm{Cl}$	36.965 903	24.23%	
18	Argon	40	$^{40}\mathrm{Ar}$	39.962 384	99.60%	
19	Potassium	39	$^{39}\mathrm{K}$	38.963 707	93.26%	
		40	$^{40}\mathrm{K}$	39.963 999	0.0117%, EC, β^-	1.28×10^9 y
20	Calcium	40	$^{40}\mathrm{Ca}$	39.962 591	96.94%	
21	Scandium	45	$^{45}\mathrm{Sc}$	44.955 910	100%	
22	Titanium	48	$^{48}\mathrm{Ti}$	47.947 947	73.8%	
23	Vanadium	51	$^{51}\mathrm{V}$	50.943 962	99.75%	
24	Chromium	52	$^{52}\mathrm{Cr}$	51.940 509	83.79%	
25	Manganese	55	$^{55}\mathrm{Mn}$	54.938 047	100%	
26	Iron	56	$^{56}\mathrm{Fe}$	55.934 939	91.72%	
27	Cobalt	59	$^{59}\mathrm{Co}$	58.933 198	100%	
		60	$^{60}\mathrm{Co}$	59.933 819	β^-	5.271 y
28	Nickel	58	$^{58}\mathrm{Ni}$	57.935 346	68.27%	
		60	$^{60}\mathrm{Ni}$	59.930 788	26.10%	
29	Copper	63	$^{63}\mathrm{Cu}$	62.939 598	69.17%	
		65	$^{65}\mathrm{Cu}$	64.927 793	30.83%	
30	Zinc	64	$^{64}\mathrm{Zn}$	63.929 145	48.6%	
		66	$^{66}\mathrm{Zn}$	65.926 034	27.9%	
31	Gallium	69	$^{69}\mathrm{Ga}$	68.925 580	60.1%	
32	Germanium	72	$^{72}\mathrm{Ge}$	71.922 079	27.4%	
		74	$^{74}\mathrm{Ge}$	73.921 177	36.5%	
33	Arsenic	75	$^{75}\mathrm{As}$	74.921 594	100%	
34	Selenium	80	$^{80}\mathrm{Se}$	79.916 520	49.7%	
35	Bromine	79	$^{79}\mathrm{Br}$	78.918 336	50.69%	
36	Krypton	84	$^{84}\mathrm{Kr}$	83.911 507	57.0%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	
38	Strontium	86	^{86}Sr	85.909 267	9.86%	
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	4.2×10^6y
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	4.4×10^{14}y
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	2.5×10^{21}y
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Er	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	4.6×10^{10}y
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	
		198	^{198}Au	197.968 217	β^-	2.696 d
80	Mercury	199	^{199}Hg	198.968 253	16.87%	
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	1.60×10^3y
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	1.41×10^{10}y
91	Protactinium	231	^{231}Pa	231.035 880	α	3.28×10^4y
92	Uranium	233	^{233}U	233.039 628	α	1.59×10^3y
		235	^{235}U	235.043 924	0.720%, α	7.04×10^8y
		236	^{236}U	236.045 562	α	2.34×10^7y
		238	^{238}U	238.050 784	99.2745%, α	4.47×10^9y
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	2.41×10^4y
95	Americium	243	^{243}Am	243.061 375	α, fission	7.37×10^3y
96	Curium	245	^{245}Cm	245.065 483	α	8.50×10^3y
97	Berkelium	247	^{247}Bk	247.070 300	α	1.38×10^3y
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α, fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α, fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$

Table 1: Atomic Masses

Appendix B Selected Radioactive Isotopes

Decay modes are α, β^-, β^+, electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^\pm$ decays are the maxima; average energies are roughly one-half the maxima.

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ-Ray Energy(MeV)	Percent
^{3}H	12.33 y	β^-	0.0186	100%			
^{14}C	5730 y	β^-	0.156	100%			
^{13}N	9.96 min	β^+	1.20	100%			
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27	100%
^{32}P	14.28 d	β^-	1.71	100%			
^{35}S	87.4 d	β^-	0.167	100%			
^{36}Cl	3.00×10^{5} y	β^-	0.710	100%			
^{40}K	1.28×10^{9} y	β^-	1.31	89%			
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373	87%
						0.618	87%
^{45}Ca	165 d	β^-	0.257	100%			
^{51}Cr	27.70 d	EC			γ	0.320	10%
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33	28%
						1.43	28%
^{52}Fe	8.27 h	β^+	1.80	43%		0.169	43%
						0.378	43%
^{59}Fe	44.6 d	β^-	0.273	45%	γs	1.10	57%
			0.466	55%		1.29	43%
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17	100%
						1.33	100%
^{65}Zn	244.1 d	EC			γ	1.12	51%
^{67}Ga	78.3 h	EC			γs	0.0933	70%
						0.185	35%
						0.300	19%
						others	
^{75}Se	118.5 d	EC			γs	0.121	20%
						0.136	65%
						0.265	68%
						0.280	20%
						others	
^{86}Rb	18.8 d	β^-	0.69	9%	γ	1.08	9%
			1.77	91%			
^{85}Sr	64.8 d	EC			γ	0.514	100%

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ-Ray Energy(MeV)	Percent
^{90}Sr	28.8 y	β^-	0.546	100%			
^{90}Y	64.1 h	β^-	2.28	100%			
^{99m}Tc	6.02 h	IT			γ	0.142	100%
^{113m}In	99.5 min	IT			γ	0.392	100%
^{123}I	13.0 h	EC			γ	0.159	≈ 100%
^{131}I	8.040 d	β^-	0.248	7%	γ s	0.364	85%
			0.607	93%		others	
			others				
^{129}Cs	32.3 h	EC			γ s	0.0400	35%
						0.372	32%
						0.411	25%
						others	
^{137}Cs	30.17 y	β^- s	0.511	95%	γ	0.662	95%
			1.17	5%			
^{140}Ba	12.79 d	β^-	1.035	≈ 100&	γ s	0.030	25%
						0.044	65%
						0.537	24%
						others	
^{198}Au	2.696 d	β^-	1.161	≈ 100%	γ	0.412	≈ 100%
^{197}Hg	64.1 h	EC			γ	0.0733	100%
^{210}Po	138.38 d	α	5.41	100%			
^{226}Ra	1.60×10^3y	α s	4.68	5%	γ	0.186	100%
			4.87	95%			
^{235}U	7.038×10^8y	α	4.68	≈ 100%	γ s	numerous	<0.400%
^{238}U	4.468×10^9y	α s	4.22	23%	γ	0.050	23%
			4.27	77%			
^{237}Np	2.14×10^6y	α s	numerous		γ s	numerous	<0.250%
			4.96 (max.)				
^{239}Pu	2.41×10^4y	α s	5.19	11%	γ s	7.5×10^{-5}	73%
			5.23	15%		0.013	15%
			5.24	73%		0.052	10%
						others	
^{243}Am	7.37×10^3y	α s	Max. 5.44		γ s	0.075	

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent	γ-Ray Energy(MeV)	Percent
			5.37	88%	others	
			5.32	11%		
			others			

Table 2. Selected Radioactive Isotopes

Appendix C Useful Information

This appendix is broken into several tables.

- Table 3, Important Constants
- Table 4, Submicroscopic Masses
- Table 5, Solar System Data
- Table 6, Metric Prefixes for Powers of Ten and Their Symbols
- Table 7, The Greek Alphabet
- Table 8, SI units
- Table 9, Selected British Units
- Table 10, Other Units
- Table 11, Useful Formulae

Table 3. Important Constants[1]

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^8 \mathrm{m/s}$	$3.00 \times 10^8 \mathrm{m/s}$
G	Gravitational constant	$6.67408(31) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$	$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
N_A	Avogadro’s number	$6.02214129(27) \times 10^{23}$	6.02×10^{23}
k	Boltzmann’s constant	$1.3806488(13) \times 10^{-23}\ \mathrm{J/K}$	$1.38 \times 10^{-23}\ \mathrm{J/K}$
R	Gas constant	$8.3144621(75)\ \mathrm{J/mol \cdot K}$	$8.31\ \mathrm{J/mol \cdot K} = 1.99 \mathrm{cal/mol \cdot K} = 0.0821 \mathrm{atm \cdot L/mol \cdot K}$
σ	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8}\ \mathrm{W/m^2 \cdot K}$	$5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K}$
k	Coulomb force constant	$8.987551788 \cdots \times 10^9\ \mathrm{N \cdot m^2/C^2}$	$8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$
q_e	Charge on electron	$-1.602176565(35) \times 10^{-19}\ \mathrm{C}$	$-1.60 \times 10^{-19}\ \mathrm{C}$
ε_0	Permittivity of free space	$8.854187817 \cdots \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$	$8.85 \cdots \times 10^{-12}\ \mathrm{C^2/Nm^2}$
μ_0	Permeability of free space	$4\pi \times 10^{-7}\ \mathrm{T \cdot\ m/A}$	$1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}$
h	Planck’s constant	$6.62606957(29) \times 10^{-34}\ \mathrm{J \cdot s}$	$6.63 \times 10^{-34}\ \mathrm{J \cdot s}$

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40) \times 10^{-31} \text{kg}$	$9.11 \times 10^{-31} \text{kg}$
m_p	Proton mass	$1.672621777(74) \times 10^{-27} \text{kg}$	$1.6726 \times 10^{-27} \text{kg}$
m_n	Neutron mass	$1.674927351(74) \times 10^{-27} \text{kg}$	$1.6749 \times 10^{-27} \text{kg}$
u	Atomic mass unit	$1.660538921(73) \times 10^{-27} \text{kg}$	$1.6605 \times 10^{-27} \text{kg}$

Table 4. Submicroscopic Masses[2]

Sun	mass	$1.99 \times 10^{30} \text{kg}$
	average radius	$6.96 \times 10^{8} \text{m}$
	Earth-sun distance (average)	$1.496 \times 10^{11} \text{m}$
Earth	mass	$5.9736 \times 10^{24} \text{kg}$
	average radius	$6.376 \times 10^{6} \text{m}$
	orbital period	$3.16 \times 10^{7} \text{s}$
Moon	mass	$7.35 \times 10^{22} \text{kg}$
	average radius	$1.74 \times 10^{6} \text{m}$
	orbital period (average)	$2.36 \times 10^{6} \text{s}$
	Earth-moon distance (average)	$3.84 \times 10^{8} \text{m}$

Table 5. Solar System Data

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	μ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}
—	—	$10^{0}(=1)$	femto	f	10^{-15}

Table 6. Metric Prefixes for Powers of Ten and Their Symbols

Alpha	Α	α	Eta	Η	η	Nu	Ν	ν	Tau	Τ	τ
Beta	Β	β	Theta	Θ	θ	Xi	Ξ	ξ	Upsilon	Υ	υ
Gamma	Γ	γ	Iota	Ι	ι	Omicron	Ο	o	Phi	Φ	ϕ
Delta	Δ	δ	Kappa	Κ	κ	Pi	Π	π	Chi	Χ	χ
Epsilon	Ε	ϵ	Lambda	Λ	λ	Rho	Ρ	ρ	Psi	Ψ	ψ
Zeta	Ζ	ζ	Mu	Μ	μ	Sigma	Σ	σ	Omega	Ω	ω

Table 7. The Greek Alphabet

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$\mathrm{N = kg \cdot m/s^2}$	newton
	Energy	$\mathrm{J = kg \cdot m^2/s^2}$	joule
	Power	$\mathrm{W = J/s}$	watt
	Pressure	$\mathrm{Pa = N/m^2}$	pascal
	Frequency	$\mathrm{Hz = 1/s}$	hertz
	Electronic potential	$\mathrm{V = J/C}$	volt
	Capacitance	$\mathrm{F = C/V}$	farad
	Charge	$\mathrm{C = s \cdot A}$	coulomb
	Resistance	$\mathrm{\Omega = V/A}$	ohm
	Magnetic field	$\mathrm{T = N/(A \cdot m)}$	tesla
	Nuclear decay rate	$\mathrm{Bq = 1/s}$	becquerel

Table 8. SI Units

Length	1 inch (in.) = 2.54 cm (exactly)
	1 foot (ft) = 0.3048 m
	1 mile (mi) = 1.609 km
Force	1 pound (lb) = 4.448 N
Energy	1 British thermal unit (Btu) = 1.055×10^3 J
Power	1 horsepower (hp) = 746 W
Pressure	1 lb/in^2 = 6.895×10^3 Pa

Table 9. Selected British Units

Length	1 light year (ly) $= 9.46 \times 10^{15}$ m
	1 astronomical unit (au) $= 1.50 \times 10^{11}$ m
	1 nautical mile $= 1.852$ km
	1 angstrom $(\mathring{A}) = 10^{-10}$m
Area	1 acre (ac) $= 4.05 \times 10^{3}\text{m}^2$
	1 square foot $(\text{ft}^2) = 9.29 \times 10^{-2}\ \text{m}^2$
	1 barn $(b) = 10^{-28}\ \text{m}^2$
Volume	1 liter $(L) = 10^{-3}\ \text{m}^3$
	1 U.S. gallon (gal) $= 3.785 \times 10^{-3}\ \text{m}^3$
Mass	1 solar mass $= 1.99 \times 10^{30}$ kg
	1 metric ton $= 10^{3}$ kg
	1 atomic mass unit $(u) = 1.6605 \times 10^{-27}$ kg
Time	1 year (y) $= 3.16 \times 10^{7}$ s
	1 day (d) $= 86,400$ s
Speed	1 mile per hour (mph) $= 1.609$ km/h
	1 nautical mile per hour (naut) $= 1.852$ km/h
Angle	1 degree $(^\circ) = 1.745 \times 10^{-2}$ rad
	1 minute of arc (ʻ) = 1/60 degree
	1 second of arc (“) = 1/60 minute of arc
	1 grad $= 1.571 \times 10^{-2}$ rad
Energy	1 kiloton TNT (kT) $= 4.2 \times 10^{12}$ J
	1 kilowatt hour $(\text{kW} \cdot h) = 3.60 \times 10^{6}$ J
	1 food calorie (kcal) $= 4186$ J
	1 calorie (cal) $= 4.186$ J
	1 electron volt (eV) $= 1.60 \times 10^{-19}$ J
Pressure	1 atmosphere (atm) $= 1.013 \times 10^{5}$ Pa
	1 millimeter of mercury (mm Hg) $= 133.3$ Pa
	1 torricelli (torr) $= 1$ mm Hg $= 133.3$ Pa
Nuclear decay rate	1 curie (Ci) $= 3.70 \times 10^{10}$ Bq

Table 9. Other Units

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter$d$	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius r	$A = 4\pi r^2$
Volume of a sphere with radius r	$V = (4/3)(\pi r^3)$

Table 10. Useful Formulae

Footnotes

1. 1 Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.
2. 2 Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Appendix D Glossary of Key Symbols and Notation

In this glossary, key symbols and notation are briefly defined.

Symbol	Definition
$\overline{\text{any symbol}}$	average (indicated by a bar over a symbol—e.g., $\bar{v}$ is average velocity)
$^\circ\text{C}$	Celsius degree
$^\circ\text{F}$	Fahrenheit degree
$//$	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus
$_0$	zero as a subscript denotes an initial value
α	alpha rays
α	angular acceleration
α	temperature coefficient(s) of resistivity
β	beta rays
β	sound level
β	volume coefficient of expansion
β^-	electron emitted in nuclear beta decay
β^+	positron decay
γ	gamma rays
γ	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
ΔE	change in energy between the initial and final orbits of an electron in an atom
ΔE	uncertainty in energy
Δm	difference in mass between initial and final products
ΔN	number of decays that occur
Δp	change in momentum
Δp	uncertainty in momentum
ΔPE_g	change in gravitational potential energy
$\Delta \theta$	rotation angle
Δs	distance traveled along a circular path
Δt	uncertainty in time
Δt_0	proper time as measured by an observer at rest relative to the process
ΔV	potential difference

Symbol	Definition
Δx	uncertainty in position
ϵ_0	permittivity of free space
η	viscosity
θ	angle between the force vector and the displacement vector
θ	angle between two lines
θ	contact angle
θ	direction of the resultant
θ_b	Brewster's angle
θ_c	critical angle
κ	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
λ_n	wavelength in a medium
μ_0	permeability of free space
μ_k	coefficient of kinetic friction
μ_s	coefficient of static friction
v_e	electron neutrino
π^+	positive pion
π^-	negative pion
π^0	neutral pion
ρ	density
ρ_c	critical density, the density needed to just halt universal expansion
ρ_{fl}	fluid density
$\bar{\rho}_{obj}$	average density of an object
ρ/ρ_w	specific gravity
τ	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
τ	characteristic time for a resistor and capacitor (RC) circuit
τ	torque
Υ	upsilon meson
Φ	magnetic flux
ϕ	phase angle
Ω	ohm (unit)
ω	angular velocity

Symbol	Definition
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
a_{B}	Bohr radius
a_{c}	centripetal acceleration
a_{t}	tangential acceleration
AC	alternating current
AM	amplitude modulation
atm	atmosphere
B	baryon number
B	blue quark color
$\overline{B}$	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
B_{int}	electron’s intrinsic magnetic field
B_{orb}	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons
BE/A	binding energy per nucleon
Bq	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
C_{p}	total capacitance in parallel
C_{s}	total capacitance in series
CG	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat
c	speed of light
Cal	kilocalorie

Symbol	Definition
cal	calorie
COP_{hp}	heat pump’s coefficient of performance
COP_{ref}	coefficient of performance for refrigerators and air conditioners
$\cos\theta$	cosine
$\cot\theta$	cotangent
$\csc\theta$	cosecant
D	diffusion constant
d	displacement
d	quark flavor down
dB	decibel
d_i	distance of an image from the center of a lens
d_o	distance of an object from the center of a lens
DC	direct current
E	electric field strength
ϵ	emf (voltage) or Hall electromotive force
emf	electromotive force
E	energy of a single photon
E	nuclear reaction energy
E	relativistic total energy
E	total energy
E_0	ground state energy for hydrogen
E_0	rest energy
EC	electron capture
E_{cap}	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
Eff_C	Carnot efficiency
E_{in}	energy consumed (food digested in humans)
E_{ind}	energy stored in an inductor
E_{out}	energy output
e	emissivity of an object
e^+	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)

Symbol	Definition
F	focal point of a lens
$\mathbf{F}$	force
F	magnitude of a force
F	restoring force
F_{B}	buoyant force
F_{c}	centripetal force
F_{i}	force input
$\mathbf{F}_{\mathrm{net}}$	net force
F_{o}	force output
FM	frequency modulation
f	focal length
f	frequency
f_0	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
f_0	threshold frequency for a particular material (photoelectric effect)
f_1	fundamental
f_2	first overtone
f_3	second overtone
f_{B}	beat frequency
f_{k}	magnitude of kinetic friction
f_{s}	magnitude of static friction
G	gravitational constant
G	green quark color
$\overline{G}$	antigreen (magenta) antiquark color
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck’s constant
hf	photon energy
h_{i}	height of the image
h_{o}	height of the object
I	electric current

Symbol	Definition
I	intensity
I	intensity of a transmitted wave
I	moment of inertia (also called rotational inertia)
I_0	intensity of a polarized wave before passing through a filter
I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave
I_{rms}	average current
J	joule
J/ψ	Joules/psi meson
K	kelvin
k	Boltzmann constant
k	force constant of a spring
K_α	x rays created when an electron falls into an $n=1$ shell vacancy from the $n=3$ shell
K_β	x rays created when an electron falls into an $n=2$ shell vacancy from the $n=3$ shell
kcal	kilocalorie
KE	translational kinetic energy
$\text{KE}+\text{PE}$	mechanical energy
KE_e	kinetic energy of an ejected electron
KE_{rel}	relativistic kinetic energy
KE_{rot}	rotational kinetic energy
$\overline{\text{KE}}$	thermal energy
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
ℓ	angular momentum quantum number
L_α	x rays created when an electron falls into an $n=2$ shell from the $n=3$ shell
L_e	electron total family number
L_μ	muon family total number
L_τ	tau family total number
L_f	heat of fusion
L_f and L_v	latent heat coefficients
L_{orb}	orbital angular momentum

Symbol	Definition
L_s	heat of sublimation
L_v	heat of vaporization
L_z	z – component of the angular momentum
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m(^AX)$	atomic mass of a nuclide
MA	mechanical advantage
m_e	magnification of the eyepiece
m_e	mass of the electron
m_ℓ	angular momentum projection quantum number
m_n	mass of a neutron
m_o	magnification of the objective lens
mol	mole
m_p	mass of a proton
m_s	spin projection quantum number
N	magnitude of the normal force
N	newton
$\mathbf{N}$	normal force
N	number of neutrons
n	index of refraction
n	number of free charges per unit volume
N_A	Avogadro’s number
N_r	Reynolds number
$\mathrm{N \cdot m}$	newton-meter (work-energy unit)
$\mathrm{N \cdot m}$	newtons times meters (SI unit of torque)
OE	other energy

Symbol	Definition
P	power
P	power of a lens
P	pressure
$\mathbf{p}$	momentum
p	momentum magnitude
p	relativistic momentum
$\mathbf{p}_{\mathrm{tot}}$	total momentum
$\mathbf{p}'_{\mathrm{tot}}$	total momentum some time later
p_{abs}	absolute pressure
p_{atm}	atmospheric pressure
p_{atm}	standard atmospheric pressure
PE	potential energy
$\mathrm{PE}_{\mathrm{el}}$	elastic potential energy
$\mathrm{PE}_{\mathrm{elec}}$	electric potential energy
PE_s	potential energy of a spring
P_g	gauge pressure
P_{in}	power consumption or input
P_{out}	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge
q	electron charge
q_p	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror
R	red quark color
$\overline{R}$	antired (cyan) quark color
R	resistance
R	resultant or total displacement

Symbol	Definition
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
r_n	radius of the *n*th H-atom orbit
R_p	total resistance of a parallel connection
R_s	total resistance of a series connection
R_s	Schwarzschild radius
S	entropy
$\mathbf{S}$	intrinsic spin (intrinsic angular momentum)
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
s	quark flavor strange
s	second (fundamental SI unit of time)
s	spin quantum number
$\mathbf{s}$	total displacement
$\sec\theta$	secant
$\sin\theta$	sine
s_z	*z*-component of spin angular momentum
T	period—time to complete one oscillation
T	temperature
T_c	critical temperature—temperature below which a material becomes a superconductor

Symbol	Definition
T	tension
T	tesla (magnetic field strength *B*)
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan\theta$	tangent
U	internal energy
u	quark flavor up
u	unified atomic mass unit
$\mathbf{u}$	velocity of an object relative to an observer
$\mathbf{u}'$	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
$\mathbf{v}$	relative velocity between two observers
v	speed of light in a material
$\mathbf{v}$	velocity
$\bar{\mathbf{v}}$	average fluid velocity
$V_B - V_A$	change in potential
$\mathbf{v}_d$	drift velocity
V_p	transformer input voltage
V_{rms}	rms voltage
V_s	transformer output voltage
$\mathbf{v}_{\mathbf{tot}}$	total velocity
$v_{\mathbf{w}}$	propagation speed of sound or other wave
$\mathbf{v}_{\mathbf{w}}$	wave velocity
W	work
W	net work done by a system
W	watt
w	weight
w_{fl}	weight of the fluid displaced by an object
W_c	total work done by all conservative forces

Symbol	Definition
W_{nc}	total work done by all nonconservative forces
W_{out}	useful work output
X	amplitude
X	symbol for an element
$^{Z}_{A}X_{N}$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position
x	horizontal axis
X_C	capacitive reactance
X_L	inductive reactance
x_{rms}	root mean square diffusion distance
y	vertical axis
Y	elastic modulus or Young’s modulus
Z	atomic number (number of protons in a nucleus)
Z	impedance