Strength of Materials Supplement for Power Engineering

Strength of Materials Supplement for Power Engineering

BCIT | School of Energy | Power Engineering

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Contents

About the Book

Strength of Materials is a topic taught in all Mechanical Engineering related programs and as such is part of the Power and Process Engineering curriculum in British Columbia Institute of Technology (BCIT). The course continues developing the engineering foundation set in Principles of Statics and Applied Mechanics, taught to second year Power and Process students in Level 1. Successful completion of the course helps Power Engineering (PE) graduates fast track their careers as the topic is part of their 2^{nd} and 1^{st} class examinations curriculum.

The delivery of Strength of Materials course to Power Engineering students is centered around the open textbook written by Dr. Barry Dupen, Associate Professor in the Mechanical Engineering Technology Department of Indiana University – Purdue University Fort Wayne. The text book is licensed under Creative Commons Attribution [1] and is available for download here:

Applied Strength of Materials for Engineering Technology

It was Dr. Dupen's intention to not include end of the chapter questions as he provides new sets with every course delivery. This gave us the opportunity and challenge to create our own problem sets. The problems were developed (or selected) in such way that they are relevant to Power Engineers. At the same time, when we felt that further summaries or procedures would enhance the learning process and help to our students, we added them. The focus of the "Supplement" is mostly on the "Applied" attribute of Strength of Materials discipline while still summarizing some theoretical aspects.

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Credits

- The cover picture, my own image, shows the “HÜTTE” Engineering Handbook and the slide rule used by my father in his civil engineering career.
- Selected images used in the Supplement were taken from Public Domain and credited appropriately.
- The rest of the images are my own work.

Preface

A fact well know to students and acknowledged by educators is that college and university textbooks are expensive and not having access to one may affect students' success. Typically, these books establish the knowledge foundation of lucrative careers and are also part of the technical library of many working professionals.

In BCIT we have delivered this course following the "Applied Strength of Materials" textbook written by Robert L. Mott. In my years of teaching the subject, alongside with the students, we have used the 4th and 5th edition of the textbook. I often noticed that students could not afford the bookstore textbook or they had "international" paperback editions. When the cost of such a textbook was about 15% of the original one, I found it hard to reason with them. For a few years the textbook was discontinued by the publisher and our institute managed to secure internal publishing rights on a cost-recovery basis. Things changed again when the textbook was upgraded to the 6th Edition, undoubtedly improved but also more expensive. And this brings us to the current textbook…

I was pleasantly surprised to find a first-rate Strength of Materials textbook that covers all the topics that our students need (and then some)… and free. The textbook that we follow in this course is written by Dr. Barry Dupen, Associate Professor in the Mechanical Engineering Technology Department of Indiana University – Purdue University Fort Wayne.

Acknowledgements

The adoption of this textbook and the further development of study materials for our students was made possible with support from the Open Education Working Group at BCIT. Funding for this project was provided through an OER grant from the B.C. Open Textbook Project.

I would like to take this opportunity to thank Dr. Barry Dupen for sharing his engineering knowledge and work with the rest of us and for making this book freely available to all students.

I personally feel I'm indebted to the Pearson Prentice Hall publishers and the author of Applied Strength of Materials textbook, Robert L. Mott, Professor Emeritus. While lecturing with this textbook, not only that I witnessed students improve their critical thinking and problem solving skills, but also helped me transfer my university acquired knowledge to tangible engineering solutions.

Special appreciation goes to my colleagues:

- Mr. Serhat Beyenir – for encouragement, support and help when I needed it
- Dr. Sanja Boskovic – for inspiration, motivation, and for reviewing this work

To all students who joined me in this journey, thank you!

Editors

The following students brought suggestions and contributions to this book:

2017 – 2018 Power and Process, 2nd year students				
Kevin Abian	Erfan Atrchi	Bir Bhatia	Andriy Chervatyuk	Robert Dean
Eric Miska	Akheem White	Mitchell Hudson	Aleksey Bykov	Neil Cahoon
Gundeep Gill	Michael Huang	Colton Jansen	Troy Moltz	Ryley Partington

Main Body

Introduction and Units

Units

Learning Objectives

At the end of this introductory chapter you should be able to

- Demonstrate familiarity with the study procedure, performance expectations and components of the course
- Perform units conversions problems within SI and US Customary systems

Study procedure

Delivery of this course is based on the Applied Strength of Materials for Engineering Technology, by Dr. Barry Dupen. This resource will be referred to as the "textbook".

To complement the textbook students have access to the current resource, further identified as "supplement". This consists of summaries of main concepts developed in the textbook and assigned problems.

For best results students should adhere to the following sequence:

1. Before class, study the theory and review the sample problems in the textbook. Some topics were already covered in Applied Physics but you will benefit from a brief review.
2. To reinforce the concepts, review the key notes in the supplement. Take notes of the concepts you found challenging and ask for clarifications in class.
3. Classroom lectures:
    - Instructor will review the theoretical concepts and answer questions
    - Instructor will demonstrate solving selected problems. When needed, instructor's notes

will be published on line.

  - Students will solve assigned problems in small groups, with guidance from instructor

4. Individual work

  - Students will solve assigned problems on their own, for self-evaluation.
  - Instructor will provide guidance and feedback during posted office hours or Tutorial Sessions

Course evaluation:

- Each chapter will be assessed through quizzes or assignments.
- Midterm and Final examinations. Combined passing score is 60%.
- Attendance will be monitored but is not mandatory.

Recommendations

Strength of Materials is a “methodical” discipline. This means that it deals in general with standard/classical questions that usually have an established method of solving them. When solving problems students often follow steps and procedures that were previously demonstrated in class or in the textbook. These approaches are logical and never students would be expected to memorize them. However, it is important for students to practice solving questions on their own since this will help them see patterns in questions, provide them with problems solving experience and help them complete the exercise in the allotted time.

For best results, students are encouraged to work after classes between 2 and 3 hours for each hour of lecture. This effort will be different for each student. To manage your time more efficiently consider attending the weekly scheduled tutorials.

Units and conversions

Like in many other engineering disciplines calculations may be performed in both systems of units, US Customary and SI. While Canada has officially adopted the SI (metric) system in 1970, economic cooperation with US companies requires engineering graduates to be fluent in both systems. Some computational software that you will use may be available only in US Customary units, being developed in US, and mostly for American users. It is therefore imperative to be able to complete calculations in both systems of units and to be able to convert between systems.

Please note:

When solving problems, if the data is given in SI units, complete the solution in SI units. Similarly for US Customary units; there is not need to switch the system of units.

In the metric system prefixes are added to base and derived units to form names and symbols that are multiples of SI units. The following table shows the commonly used SI prefixes.

Prefix	**Symbol**	**Multiplying Factor**
Giga X	GX	10^9 = 1000 000 000
Mega X	MX	10^6 = 1000 000
Kilo X	kX	10^3 = 1000
Hecto X	hX	10^2 = 100
Deca X	daX	10^1 = 10
Base SI Unit “*X*”	“X” can be m, g, W, J, etc.	10^0 = 1
Deci	dX	10^{-1} = 0.1
Centi	cX	10^{-2} = 0.01
Milli	mX	10^{-3} = 0.001
Micro	μX	10^{-6} = 0.000 001

There are different ways to perform units conversions but in the end, they all lead to the same answer. The following are simple examples to demonstrate the procedure.

Examples – SI system:

1. Convert 0.2 km to cm

$$0.2\ \cancel{km} \times \frac{1000\ \cancel{m}}{1\ \cancel{km}} \times \frac{100\ cm}{1\ \cancel{m}} = 20\ 000\ cm$$

- When performing SI conversions it is easy to see if your answer is reasonable or not. For instance if you move from a large unit (kilo) to a smaller one (centi), the resulting value should be greater.
- Looking at Fig. 1, you may also consider moving the decimal point to the right, three steps from Kilo to base and two more steps from base to your final answer. This is an alternative approach to performing SI conversions.

2. Convert 50 000 cW to kW

$$50\,000\ \cancel{cW} \times \frac{1\ \cancel{W}}{100\ \cancel{cW}} \times \frac{1\ kW}{1000\ \cancel{W}} = 0.5\ kW$$

- Note that some units may be presented with a less commonly used prefixes. For instance, while "centimeter" is frequently used, "centiwatts" not so much. However, you should be able to identify the prefix and the unit it applies to.

3. Convert 300 000 cm^3 to dam^3

$$300\,000\ cm^3 \times \frac{1\ m^3}{(100\ cm)^3} \times \frac{1\ dam^3}{(10\ m)^3} = 0.0003\ dam^3$$

- You may look at this conversion as follows:

$$300\,000\ cm^3 \times \frac{1m \times 1m \times 1m}{100cm \times 100cm \times 100cm} \times \frac{1dam \times 1dam \times 1dam}{10m \times 10m \times 10m} = 0.0003\ dam^3$$

- Pay extra attention when using powers, as in volume or area conversions.

For the purpose of this course most of the US Customary conversions will deal with linear dimensions. The conversion factors we use are presented in Appendix A. It is desirable to remember the most used factors such as 1 ft = 12 in or 1 yd = 3 ft.

Examples – US Customary system

4. Convert 1.2 yards to inches

$$1.2\ yd \times \frac{3\ ft}{1\ yd} \times \frac{12\ in}{1\ ft} = 43.2\ in$$

5. Convert 2 square feet to square inches

$$2\ ft^2 \times \frac{12{\times}12\ in^2}{1\ ft^2} = 288\ in^2$$

Assigned Problems

Problem 1: The hoop stress in a pressure vessel is calculated with the formula $\sigma_{hoop} = \frac{p \times d_i}{2 \times t}$ where p is the design pressure, d_i is the inside diameter and t is the wall thickness.

1. If p = 4450 kPa, d_i = 1.8 m and t = 20 mm, determine the hoop stress in the wall, in MPa.
2. If p = 645 psi, d_i = 6 feet and t = ¾ in, determine the hoop stress in the wall, in ksi.

Problem 2: To determine the dead load on a foundation you are required to estimate the weigh of a spherical tank ($V=4/3\ \pi r^3$), full with a liquid of given density. Tank mass is negligible compared to the mass of the product. Determine its weight based on the following:

1. Diameter = 200 cm, density = 1.12 g/cm^3. Answer in N.
2. Diameter = 80 in., density = 70 lb/ft^3. Answer in lb.

Problem 3: Suggest one improvement to this chapter.

The improvements have to be specific and clear, for example:

- correct this typo
- replace this phrase with this
- add this explanation to this section
- add this problem to the chapter problems
- etc

You may use screen captures to identify the section that you would like improved or expanded.

Stress and Strain

Stress-Strain

Learning Objectives

After completing this chapter you should be able to:

- Define normal and shear stress and strain and discuss the relationship between design stress, yield stress and ultimate stress
- Design members under tension, compression and shear loads
- Determine members deformation under tension and compression

Mechanical stress

This section discusses the effects of mechanical loads (forces) acting on members. Next chapter will cover the effects of thermal loads (thermal expansion).

Normal, tensile and compressive stresses

Tension or compression in a member generate normal stresses; they are called "normal" because the cross-section that resists the load is perpendicular (normal) to the direction of the applied forces. Both tensile and compressive stresses are calculated with:

$$Normal\ Stress, \sigma = \frac{Normal\ Force, P}{Area, A}$$

If a member has a variable cross-section, the area that must be used in calculations is the minimum cross-sectional area; this will give you the maximum stress in the member, which ultimately will govern the design.

Shear stresses

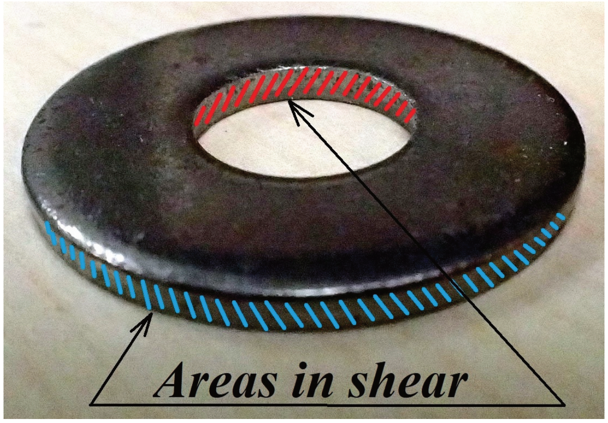
In shear the cross-section area that resists the load is parallel with the direction of applied forces. In addition to that, when estimating the shear area you must factor in how many cross-sections contribute to the overall strength of the assembly.

For instance, if you consider the pin of a door hinge as subjected to a shear load, you have to count how many cross-sections resist the load.

The formula for calculating the shear stress is the same:

$$Shear\ Stress, \tau = \frac{Shear\ Force, P}{Total\ Shear\ Area, A}$$

In a punching operation the area that resists the shear is in the shape of a cylinder for a round hole (think of a cookie cutter). Therefore the area in shear will be found from multiplying the circumference of the shape by the thickness of the plate.



Please note:

When looking at textbook figures you will observe that two forces are indicated. This does not mean that the force you use in the formula is (2 × Force P), but simply indicates that one is the Action force and the second one is the Reaction.

Strain and modulus of elasticity

Normal strain

A member in tension or compression will elastically deform proportional with, among other parameters, the original length. Strain, also called unit deformation, is a non-dimensional parameter expressed as:

$$Strain, \varepsilon = \frac{Change\ in\ Length}{Original\ Length} = \frac{\Delta L}{L}$$

If you choose to use a negative value for compression strain (reduction in length) then you must also express the equivalent compression stress as a negative value.

Modulus of elasticity

The stress – strain curve is generated from the tensile test. Over the elastic region of the graph the deformation is direct proportional with the load. Dividing the load by the cross-section area (constant) and the deformation by the original length (constant) leads to a graphical representation of Strain vs. Stress. The constant ratio of stress and strain is Young's Modulus or Elastic Modulus, a property of each material.

$$Elastic\ Modulus, E = \frac{Stress, \sigma}{Strain, \varepsilon}$$

Elastic deformation

Combining the above two relations for strain and Modulus of Elasticity leads to a unified formula for elastic deformation in tension or compression.

$$\Delta L = \delta = \frac{\sigma \times L}{E} = \frac{F \times L}{A \times E}$$

This relation is applicable to members with uniform cross-sections, homogeneous material, subject to tensile or compressive loads that results in stresses below the proportional limit (straight line in the σ-ε curve).

Design stress and safety factors

These topics were covered in 1^{st} year Strength of Materials and are presented here as a brief review.

Members subjected to an excessive stress may fail by breaking, when actual working stress is greater than the ultimate stress, or due to excessive deformation that renders then inoperable. Consider a heavy condensate line that sags beyond an acceptable limit and while it doesn't break, the flange connections at the end of the lines will develop leaks due to angular movement.

Design stress, σ_d, is the maximum level of actual/working stress that is considered acceptable from a safety point of view. The design stress is determined by:

- Material properties, Ultimate Tensile Strength or Yield Strength, depending if breakage must be avoided or deformation must be limited
- Safety factor (or design factor) N, ratio of maximum strength to the intended load.

The safety factor is chosen by the designer based on experience, judgment AND guidelines/rules from relevant codes and standards, based on several criteria such as risk of injuries, design data accuracy, probability, industry standards, and last but not least, cost. Safety factors standards were set by structural engineers, based on rigorous estimates and backed by years of experience. Standards are continuously evolving reflecting new and improved design philosophies. Example:

- published by ANSI / AISC , such as Specification for Structural Steel Buildings

Design cases

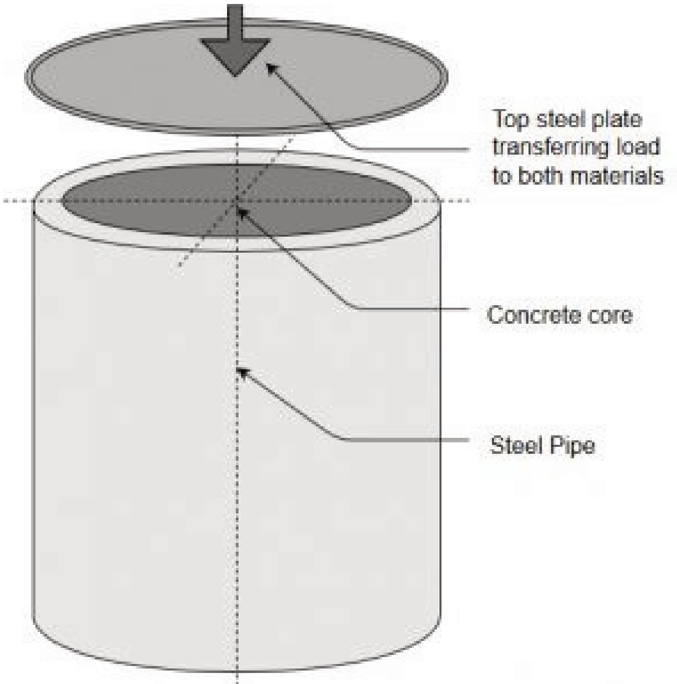
When solving problems students may encounter different scenarios. While the theoretical concepts are the same, the paths to final answers may be different, as required by each approach.

1. Estimating if a design/construction is safe or not
    a. Given: loads magnitude and distribution, material properties, member shape and dimensions
    b. Find: actual stress and compare to the design stress; alternatively find the safety factor and decide if it is acceptable based on applicable standards
2. Selecting a suitable material
    a. Given: loads magnitude and distribution, member shape and dimensions
    b. Find: what material type or grade will provide a strength (yield or ultimate) greater than required, while considering the selected or specified safety factor
3. Determining the shape and dimensions of member's cross-section
    a. Given: loads magnitude and distribution, material properties
    b. Find: the shape and dimensions of the member so that actual cross-sectional area is greater than minimum required.
4. Evaluating maximum allowable load on a component
    a. Given: load type and distribution, material properties, member shape and dimensions
    b. Find: maximum load magnitude that leads to an acceptable stress

Members made from two different materials

There are cases when a member under normal stresses is made out of two (or more) materials. One of the objective of such problems is to find the stress in each component.

For example, you may have a short column made from a steel pipe filled with concrete, as in the figure. Given the total load, materials properties and geometrical dimensions, we must find the individual stress in each component.



Both, the steel pipe and the concrete core work together in supporting the load therefore we must find additional relations that combine the two problems into one . Typically, we look for:

- a relation that describes the force distribution between the two materials
- a relation that correlates the deformations of each material

For this particular problem we may say that:

Equation 1: Total load P = load supported by steel P_{steel} + load supported by concrete $P_{concrete}$

therefore $P = \text{Stress}_{steel} \times \text{Area}_{steel} + \text{Stress}_{concrete} \times \text{Area}_{concrete}$

Equation 2: The deformations of both materials are the same

therefore $\text{Strain}_{steel} = \text{Strain}_{concrete}$

Considering that Elastic Modulus = Stress / Strain, equation (2) yields a relation between the stress and elasticity of both materials

$$\sigma_{steel} = \sigma_{concrete} \times \frac{E_{steel}}{E_{concrete}}$$

Substituting this last relation into equation (1) and solving for $\text{Stress}_{concrete}$ leads to a relation as follows

$$\sigma_{concrete} = \frac{F \times E_{concrete}}{A_{steel} \times E_{steel} + A_{concrete} \times E_{concrete}}$$

Further, Stress $_{steel}$ can be found.

Note that depending on the problem, the original two relations may be different therefore a full step-by-step derivation may be required each time.

Reasonable answers

When solving normal stress – strain problems, especially in the SI system, you should be able to judge if your answers are reasonable or not.

Example: A 1 m long, 20 mm diameter, A 36 Carbon Steel bar (Materials Properties in Appendix B, Table B2) suspends a 6 tons load. Evaluate the stress and the strain in the bar.

$$\sigma = \frac{P}{A} = \frac{6000\ kg \times 9.81\ {}^{m}/_{s^2}}{\frac{\pi}{4} \times 0.020^2 m^2} = \frac{58.86\ kN}{0.0003\ m^2} = 187.5\ MPa$$

$$\varepsilon = \frac{\sigma}{E} = \frac{187.5\ MPa}{207\ GPa} = 0.0009$$

Note that typically loads are in kN, cross-section areas in 10^{-3} m^2 and resulting stresses in MPa.

Also, since Elastic Moduli are in GPa, the strain (non-dimensional) will be in range of 10^{-3}. This bar will stretch 0.9 mm under the given load.

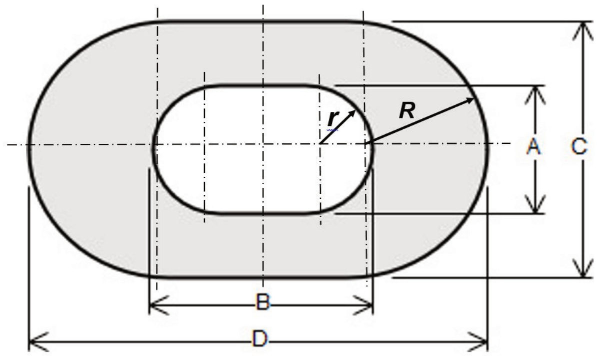
Assigned Problems

When solving these questions you are required to use the textbook Appendices. They are valuable references for material properties, geometrical dimensions, etc.

Problem 1: A condensate line 152 mm nominal size made of schedule 40 carbon steel pipe is supported by threaded rod hangers spaced at 2.5 m center-to-center. The hangers are carbon steel, 50 cm long, with a root diameter of 12 mm. Calculate the stress and the strain in the hangers. Use E=200 GPa for the hangers material.

Problem 2: A clevis fastener with a 1/2 inch pin is used in a shop lifting machine. If the pin is made of A36 steel determine the maximum safe load, using a safety factor of 2.5 based on the yield strength.

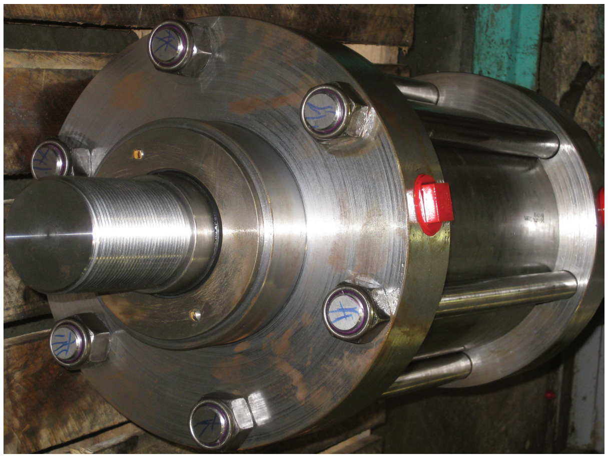
Problem 3: A boiler is supported on several short columns as indicated in the figure, made out of Class 35 gray cast iron. Each column supports a load of 50 tonnes. The required safety factor for this construction is 3. Are the columns safe?



Use the following dimensions: A = 30 mm, B = 80 mm, C = 50 mm, D = 140 mm

Problem 4: A tension member in a roof truss is subject to a load of 25 kips. The construction requires using L2x2x1/4 angle, with a cross-section of 0.944 in^2. For building-like structures American Institute of Steel Construction recommends using a design stress of $0.60 \times S_y$. Using Appendix B table B2 specify a suitable steel material.

Problem 5: A tie rod hydraulic cylinder as in the figure is made from a 6 inch Schedule 40 stainless steel pipe, 15 inches long. The six tie rods are 1/2-13 UNC threaded rods with a root diameter of 0.4822 inch and a thread pitch of 13 TPI. When assembling the cylinder a clamping force equivalent to one full nut turn from hand-tight position is required.



Determine the stress in the cylinder and in the tie rods. Also calculate the strain in each component using an elastic modulus of $E_{ss} = 28 \times 10^6$ psi and $E_{rod} = 30 \times 10^6$ psi.

Problem 6: Suggest one improvement to this chapter.

Thermal Expansion Stress

Thermal Expansion

Learning Objectives

At the end of this section you should be able to calculate problems involving

- Unrestricted thermal expansion
- Restricted thermal expansion

All materials subject to a temperature change will expand or contract proportional with their length and temperature difference. Some materials will expand or contract more than others; the qualitative property that indicates how much will they expand is known as the Linear Thermal Expansion Coefficient (***α***), measured in m/(m ºC) or (in/in ºF). Also units like 1/ºC or 1/ºF can be used.

The change in length due to thermal expansion is calculated with:

$$\Delta L \; (or \; \delta) = \alpha \times L \times \Delta T$$

where δ is the change in length, L is the original length (makes sure both are in the same units) and ΔT is the temperature difference.

For example if steel has a thermal expansion coefficient of 11.7×10^{-6} 1/ºC it means that a 1 m long bar subject to a temperature increase of 1ºC will expand 11.7×10^{-6} m, or 0.0117 mm. This may seem like a negligible amount but if you consider a steam pipe of 50 m long installed at 12ºC and operating at 212ºC (2000 kPa saturation pressure), the thermal expansion would be equivalent to 11.7 cm, or an equivalent strain of 0.002. This is very important for the piping designers because they have to allow for this expansion or factor it in the stress calculations.

Volumetric thermal expansion of solids (isotropic materials) is calculated in a similar way using (3×α) as expansion coefficient. When calculating liquids volumetric expansion, the volumetric expansion coefficient is β, with typical values as listed in The Engineering Toolbox.

Pipelines expansion

Typically pipelines are relatively long and may see a significant temperature increase between installation and operating temperatures. As result, high magnitude thermal expansion stresses may develop if the supports are not adequately designed. In addition to that, the expansion of the pipe increases the load on machinery and vessels nozzles.

Pipe cold springing

There is an abundance of articles and discussions on this topic in piping design groups, easily accessible through an internet search using key word strings "pipe cold springing" or "pipe cold pull"; it is also addressed in ASME B31.3.

"Pipe cold springing is defined as the process of intentional deformation (usually accomplished by cutting short or long the pipe runs between two anchors) of piping during assembly to produce a desired initial displacement and stress. It is also defined as the intentional stressing and elastic deformation of the piping system during the erection cycle to permit the system to attain more favorable reactions and stresses in the operating condition." [1]

Operating engineers are advised to be familiar with this practice since it may be used in steam pipes. There have been circumstances when hired contractors when disassembling steam lines complained about lines not being fitted properly; the pipes would sprung back when unbolted. Avoid costly repairs and unnecessary alterations by being familiar with this procedure and by knowing your plant.

Thermal stresses in composite bars

"The composite tube consists of two different alloys metallurgically bonded together to achieve good thermal transfer properties. One alloy is used to withstand corrosion, while the other is often an approved pressure vessel material.

Typical applications for composite tubes are steam boilers with corrosive conditions, such as:

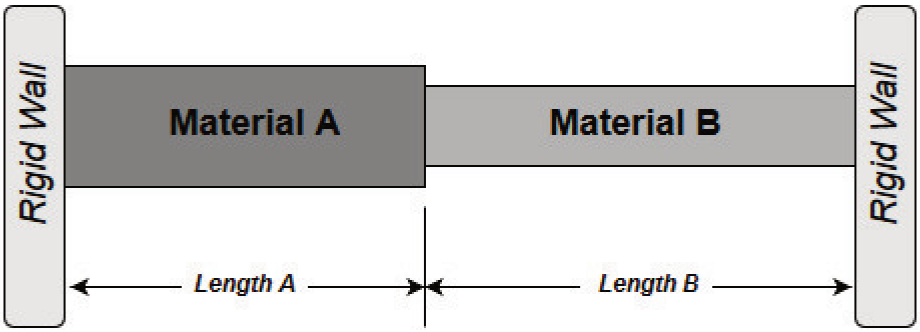
- Black liquor recovery boilers (BLRB)
- Syngas coolers
- Waste heat boilers
- Waste-to-energy boilers

Composite tube (compound tube) are suitable for applications where the conditions outside and inside the tube require material properties that cannot be met by one material only." [2]

While Power Engineering students may not see a direct application of these principles, the following types of problems are part of their 2nd and 1st class curriculum.

Case A

The following diagram represent a typical restricted thermal expansion scenario, with compound bars:

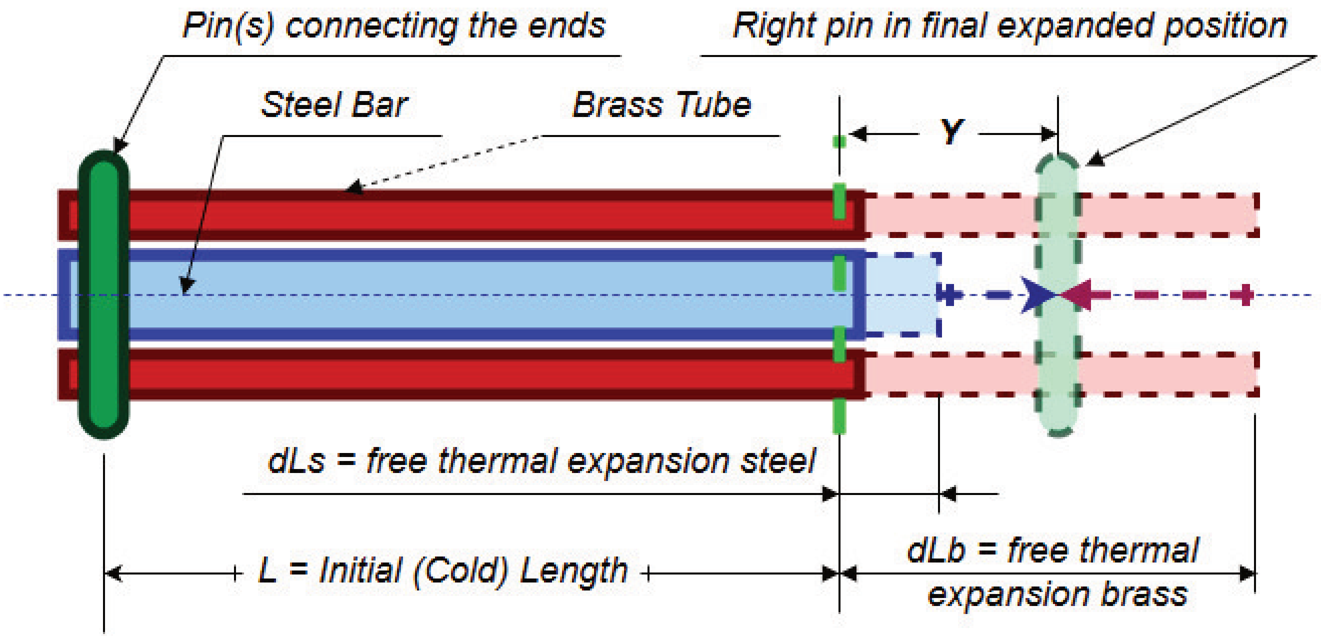


Given all the materials properties and dimensions, the objective is to calculate the stress in each section when the temperature is increased by a given ΔT.

When the bars are heated, each will attempt tend to expand equivalent to their unrestricted ΔL. Given that the expansion is restricted, each bar will be subject to compression which in turn generates compression stresses. The sum of the two corresponding (yet imaginary) compression deformations will be equal to the sum easily quantifiable unrestricted thermal expansions. Furthermore, considering that the forces applied by each bar are the same (static/balanced system), this compression force can easily be calculated. Factoring in the cross-sectional areas of each bar leads to finding the stress developed in each material.

Case B

In the second scenario, a bar is pinned at both ends inside a tube of a different material. When heated up, one material would expand freely more than the other. The lower expansion material will be pulled outwards in tension by the second one that attempts to expand more. In turn, the material that would freely expand more it is pulled in (compression) by the material that expands less. See the following figure for clarification.



Objective of this exercise is to find the stresses generated in each material. The approach at solving this problem is as follows.

- When heated up, the brass tube is pulled by the steel rod, generating a compression stress in the tube. The brass tube restricted deformation will be ($dL_b - Y$), where Y is the actual combined deformation of the composite bar.
- Similarly, the steel bar is pulled out by the brass tube, generating a tensile stress. The actual (restricted) deformation of the steel bar is ($Y\text{-}dL_s$).
    - Note than in the above dL is the free thermal expansion of each material.
- From the diagram, $(dL_b - Y) + (Y - dL_s) = dL_b - dL_s$
- Substitute in the above $dL=\alpha \times L \times \Delta T$ for each material, divide equation by L = initial length and find:
    - $\varepsilon_{brass} + \varepsilon_{steel} = (\alpha_{brass} - \alpha_{steel}) \times \Delta T$
- Substitute E = σ / ε in the above for each material and the resulting equation represents a relation between the stress in each material, function only of the known elastic/thermal expansion properties and the temperature difference.
    - $(\sigma_{brass} / E_{brass}) + (\sigma_{steel} / E_{steel}) = (\alpha_{brass} - \alpha_{steel}) \times \Delta T$ ***(eqn. B1)***
- The second equation comes from the observation that outward pull force of the brass is equal to the inward pull of the steel. This can be expressed as:
    - $\sigma_{brass} \times A_{brass} = \sigma_{steel} \times A_{steel}$ ***(eqn. B2)***
- Solve equation B2 for σ_{brass} and substitute into equation B1. Solve equation B1 for α_{steel} and your final result is dependent only on materials properties, cross-sections and temperature difference.
- Once σ_{steel} answer is found, go back to equation B2 and find σ_{brass}.
- This may appear as a math/algebra demanding problem, and it is; however, it is a classical problem

with a standard solution which means that every question will be solve following the same approach.

Assigned Problems

When solving the following problems find the required data in the textbook appendices, provided external resources or other reputable sources; always quote the source.

Problem 1: A firetube boiler is powered using bunker fuel oil. The storage tank is of an open top construction, 2 m diameter and 3 m height. Oil is added when the ambient temperature is 10ºC. During start-up the temperature suddenly rises to 35ºC. How many centimeters below the tank top can you fill in the tank, so that you maximize the oil volume while avoiding any spillage? Coefficient of linear expansion of tank material is 12×10^{-6} /ºC and volumetric expansion coefficient for oil is 9×10^{-4} /ºC.

Problem 2: During installation, a turbine casing bolt is heated to 250ºC and the nut is tightened so that no stress is produced (hand-tight). When it cools down to the operation temperature of 50ºC the bolt adequately secures the assembly. Determine the tensile stress and strain in the bolt and the force carried by the bolt. The bolt effective length is 300 mm, diameter 50 mm and E_{bolt} = 200 GPa.

Problem 3: A new above ground pipeline will transport crude oil from Northern Alberta, south. To compensate for thermal expansion each straight section of the pipeline will be equipped with corrugated expansion joints that allow 23 mm axial expansion and 18 mm axial compression (figure). The pipeline will be installed early summer when ambient temperatures may conservatively assume to be 23ºC. The pipe is DN 600 Sch 40 and Carbon Steel material.

Determine the maximum straight pipe length between two anchor points (in m) for extreme Alberta temperatures while assuming that due to bush-fires the pipe metal temperature can reach as high as 100ºC. For your specified pipe length what would be the maximum stress developed in the material if the thermal expansion is restricted?

Problem 4: An 8″ Schedule 40 straight length of steam pipe is fitted between two fixed anchor supports with no allowance for expansion. If the compressive stress in the pipe must be limited to 50.7 ksi when in operation at 430ºF, determine the initial tensile stress that must be applied during installation at 60ºF. What equivalent tensile force is required by this cold-springing installation?

Problem 5: A single-pass double pipe heat exchanger is constructed using 1″ nominal thickness ASTM B88 Type K Copper tubing for the internal tube and 2″ nominal size, medium wall thickness, steel tubing for the external shell. The length of the heat exchanger is 24″. The heat exchanger is assembled stress free at 20ºC but in operation the tubes wall temperatures reach 120ºC. Determine the stresses generated by thermal expansion in both, steel and copper tubing. Use:

- $\alpha_{steel} = 6.5\times10^{-6}$ in/inºF; $E_{steel} = 30\times10^{6}$ psi; dimensional data from The Engineering Toolbox Steel

Tubes

- α_{copper} = 9.4×10^{-6} in/in^{o}F; E_{copper} = 17×10^{6} psi; dimensional data from Appendix D5 or The Engineering Toolbox Copper Tubes

Problem 6: Recommend one improvement to this chapter.

Pressure Vessels

Vessels

Learning Objectives

At the end of this chapter you should be able to

- Identify thin wall or thick wall pressure vessels
- Discuss the difference between longitudinal and circumferential stress
- Demonstrate the derivation of the stress formulas in a thin wall pressure vessel
- Perform thin wall pressure vessel design calculations

Thin-walled and thick-walled pressure vessels

The distinction between thin vs. thick wall pressure vessels is determined by the ratio between the mean radius of the vessel and the thickness of the wall. If this ratio is greater than 10, the vessel is considered a thin wall pressure vessel. If the ratio is less than 10, the vessel is considered a thick wall pressure vessel.

$$\frac{R_i\ (Radius_{inside})}{t\ (wall\ thickness)} \geq 10\ means\ Thin\ Wall\ P.V.$$

In operation, in a thin wall pressure vessel, stresses developed in the (thin) wall can conservatively be assumed to be uniform. These are the stresses students are familiar calculating using ASME Section I PG-27 or Section VIII Div. I UG-27. In fact, most of the pressure vessels power engineers will work with are of a thin-wall type.

In contrast, a thick wall pressure vessel develops a greater (circumferential) stress on the inside surface of the vessel and it reduces towards the outside diameter. The design calculations for this type of vessels are only covered in the ASME Section VIII (Pressure Vessels) code, Mandatory Appendix 1 (Supplementary Design Formulas).

Development of stress formula in a pressure vessel

Circumferential stresses (longitudinal joints)

The circumferential stress (or hoop stress) acting on a longitudinal cross-section is derived in the textbook as:

$$\sigma_{hoop} = \frac{p \times d_i}{2 \times t} = \frac{p \times r_i}{t}$$

Design problems most typically deal with finding the minimum required wall thickness, therefore the above formula is more useful expressed as:

$$t = \frac{p \times d_i}{2 \times \sigma_{hoop}} = \frac{p \times r_i}{\sigma_{hoop}}$$

Hoop stress formula from ASME Section VIII Div. 1 UG-27 is:

$$t = \frac{P \times R}{S \times E - 0.6P}$$

Efficiency "E" is a factor that accounts for loss of material strength due to welds or ligaments. Also note that applying "-0.6P" to the denominator leads to a thicker shell compared to the theoretical formula, and therefore more conservative (or safer). Before using the formula check if the relation is applicable (thin wall).

ASME Section I (Power Boilers) calculates the shell thickness only based on circumferential stress, as follows:

$$t = \frac{P \times R}{S \times E - (1 - y)P} + C$$

The formulas are quite similar; in the above "y" is a temperature coefficient and C is an added allowance for corrosion or structural stability. Again, the code formula leads to a thicker shell than simply based on derivations.

Longitudinal stress (circumferential joints)

Longitudinal stress demonstrated and derived in the textbook is derived as:

$$\sigma_{long} = \frac{p \times r_i}{2 \times t} \; or \; t = \frac{p \times r_i}{2 \times \sigma_{long}}$$

Note that longitudinal stresses are 50% of the hoop stresses and therefore they rarely govern the design. This is the reason ASME Section I does not even require evaluating this stress.

ASME Section VIII Div. 1 requires estimating the vessel thickness based on both stresses, and choosing the largest of the two values. Formula is:

$$t = \frac{P \times R}{2S \times E + 0.4P}$$

Spherical pressure vessels

Spherical pressure vessel stress is calculated in the same way as the longitudinal stress. You may conclude that a spherical pressure vessel will require a thinner shell, theoretically one half, than a cylindrical pressure vessel operating at the same pressure and temperature, and therefore it would be a preferred shape. Reality is that while most of that is true, it is difficult to manufacture a spherical shell.

Follow the links for examples of pressure vessels:

- A pressure vessel constructed of a horizontal steel cylinder.
- Spherical gas container.
- LNG carrier ship.

Assigned Problems

- The Pressure Vessel problems must be solved using the theoretical formulas developed in the textbook and NOT the ASME code formulas.
- Always check first if you are dealing with a thin-walled or a thick-walled pressure vessel.
    - Thick wall formulas will be provided if necessary.
- Cylindrical vessels require both calculations (longitudinal and circumferential joints); you specify the final answer (minimum wall thickens or MAWP maximum allowable working pressure)

Problem 1: Derive in detail the formulas for longitudinal and circumferential stresses acting on a cylindrical pressure vessel. Briefly discuss the results.

Problem 2: A seamless pipe of 508 mm outside diameter is used as a header in a large power plant carrying steam at 2 MPa pressure. The standard lengths of pipe are butt-welded together to build a continuous pipe. The pipe material, SA-106 Grade C, has minimum Tensile Strength of 485 MPa and a safety factor of 3.5 based is specified. The allowable stress for the butt-welds is 110 MPa. Specify the minimum pipe wall thickness.

Problem 3: A cylindrical tank 36″ diameter and 12 feet long, is used as a compressed air accumulator. The tank is made of ASTM SA-36 rolled steel plate with a wall thickness of 3/4″. Find the maximum allowable working pressure in the tank using a safety factor of 3.5 based on the Ultimate Strength.

Problem 4: A large spherical storage tank for compressed nitrogen is 8.6 m diameter and is constructed using AISI 1040 cold rolled steel plate of 12 mm thickness. The maximum pressure in the tank is 0.66 MPa. If a design factor of 4 based on the Yield Strength is required, does the tank meet the specifications?

Problem 5: Recommend one improvement to this chapter.

Properties of Areas

Inertia Moment

Learning Objectives

Upon completion of this chapter you should be able to

- Determine the centroid location of a given cross-section
- Calculate the moment of inertia for a given cross-section, with both SI and US Customary units

Finding the location of the centroid is needed when calculating the moment of inertia (or second moment of areas) of beams subjected to bending. For convenience, you may used the table provided in Appendix 1.

The geometric properties of areas for common shapes are given in textbook Appendix C. Common industrial shapes like W-beams and pipes are listed in Appendix D.

Centroids of composite areas

Determining the location of the centroid of a composite area uses the concept of moment of an area; this is why textbooks may refer to this as "first moments of areas". Mathematically this principle is expressed as:

$$Y = \frac{\sum(A_i \times y_i)}{\sum A_i}$$

where:

- Y is the distance to the centroid from some reference axis. Commonly, the reference axis is the base of the figure.
- A_i is the area of one part of the composite area. Typically, the composite areas are split into common shapes of known geometric properties, summarized in the textbook Appendix C.
- y_i is the distance from the reference axis (commonly the base of the figure) to the centroid of each part of the composite figure.
- $\Sigma(A_i)$ is the entire area of the composite area.

When determining the location of a centroid please observe the following rules:

- If the cross-section has one axis of symmetry then the centroid will be located on this axis.
- if the cross-section has two axes of symmetry then the centroid will be located at the intersection of the two axes.
- If the cross-section is not symmetric about any axis then two calculations are required:
  - one for determining the centroid location Y
  - one for determining the centroid location X, commonly measured from the extreme left end. For this second calculation imagine that you rotate the figure 90º counter-clockwise and repeat the first calculation
- If the composite area has a part that is removed from the figure (a void), this missing part can be treated as a negative area.

Moments of inertia of composite areas

In rotational kinetics we learned that the "rotational" moment of inertia of a flywheel (function of its mass, size and shape) represents a resistance to change in its motion. This moment of inertia multiplied by the angular acceleration α, gives an inertia-moment reaction that attempts to balance the accelerating moment action (accelerating torque). In general, a moment of inertia is a resistance to change.

Beams are subject to bending and as a result they tend to deform (deflect). The moment of inertia of a beam cross-section can be related to the stiffness of the beam. The deflection of the beam is inverse proportional to the moment of inertia.

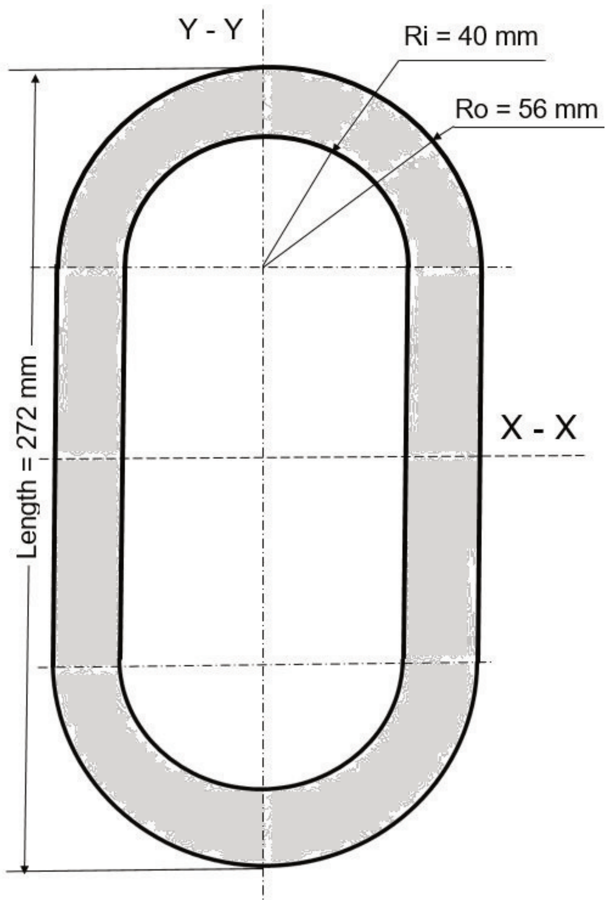
Formulas for moments of inertia of simple shapes are given in Textbook Appendix C. They will also be provided in the exam.

When dealing with a composite area, divide the shape into basic parts for which the moment of inertia can be easily calculated. The combined moment of inertia of the entire shape is the sum of moments of inertia of constituent parts plus their corresponding transfer term. The transfer term is calculated as the area of the part multiplied by the squared distance between the centroid of the part and the common centroid of the entire area. This transfer term represents the additional stiffness of each part due to its relative distance from the common centroid. The table given in Supplement Appendix 1 can be used for calculations; it is useful when the shape is more complex.

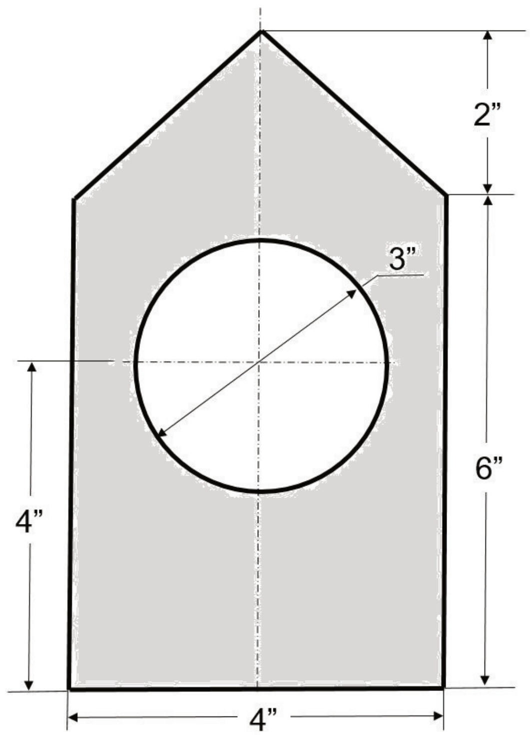
Assigned Problems

When completing these exercises please make sure that you clearly identify and number the parts of your composite area.

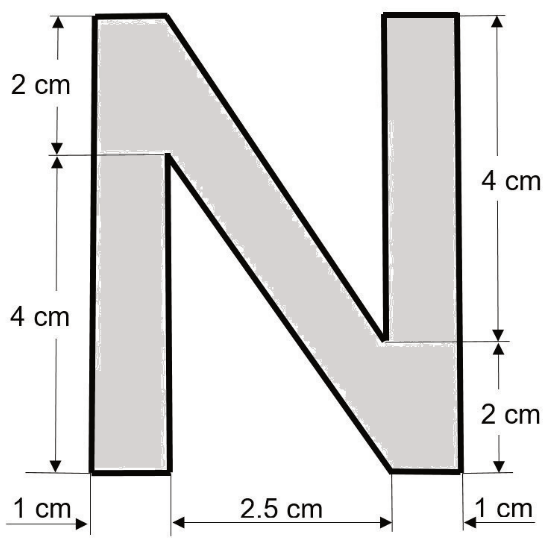
Problem 1: Determine the moment of inertia about the vertical and horizontal centroidal axes for the following figure.



Problem 2: For the following cross-section determine the location (elevation) of the centroid and the moments of inertia with respect to the horizontal and vertical centroidal axes.



Problem 3: For the following figure determine Y, the vertical location of the centroid, and calculate the moment of inertia with respect to the horizontal centroidal axis..



Problem 4: Suggest one improvement to this chapter; this may include an original cross-section.

Beam Reactions and Diagrams

Diagrams

Learning Objectives

At the end of this chapter you should be able to:

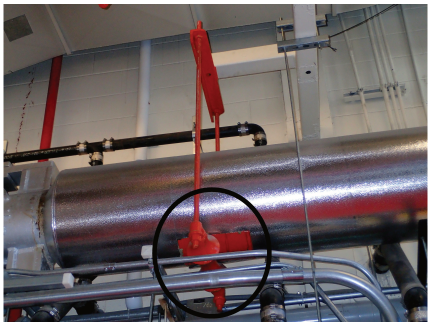
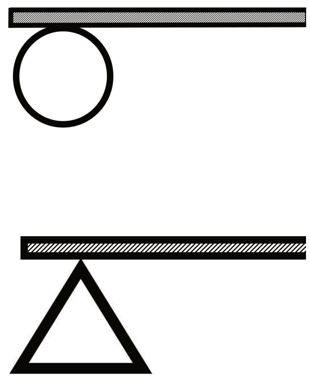
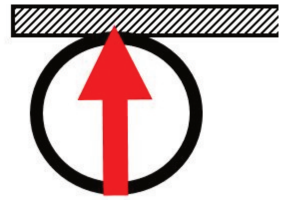
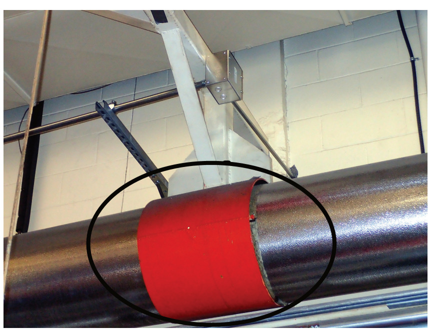
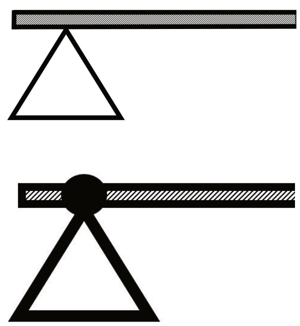
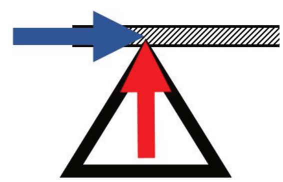
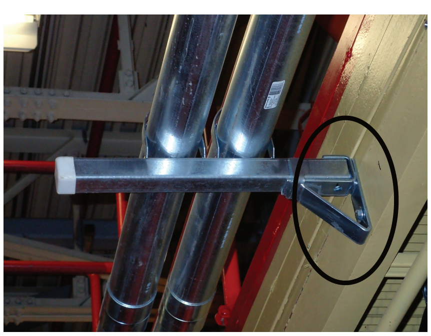
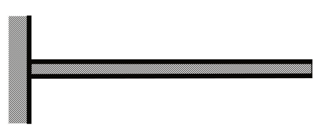
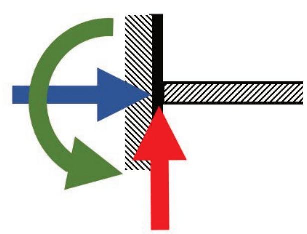
- Determine the reactions of simply supported, overhanging and cantilever beams
- Calculate and draw the shearing force and bending moment diagrams of beams subject to concentrated loads, uniform distributed loads and combinations of the two.

Beams review

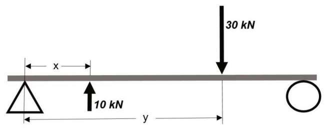
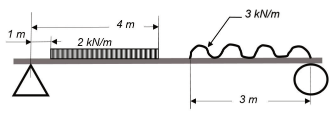
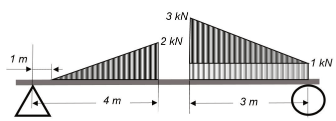
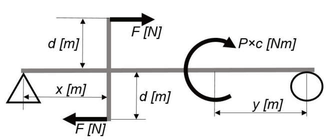
Beams are structural elements with various engineering applications like roofs, bridges, mechanical assemblies, etc. In general, a beam is slender, straight, rigid, built from isotropic materials, and most important, subjected to loads perpendicular to their longitudinal axis. If instead of perpendicular loads the same structural member would be subjected to longitudinal loads it would be called column or post. If the same member would be subjected to a torque, it would be called and treated as a shaft. Therefore, when identifying mechanical or structural components, consideration of the manner of loading is very important.

Note that when it comes to orientation, beams can be horizontal, vertical or any inclination in between (like submerged plates analyzed in fluid mechanics)… provided the loading is perpendicular to their major axis.

Beam supports:

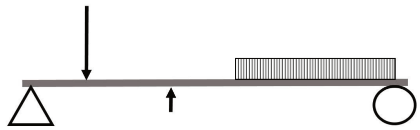
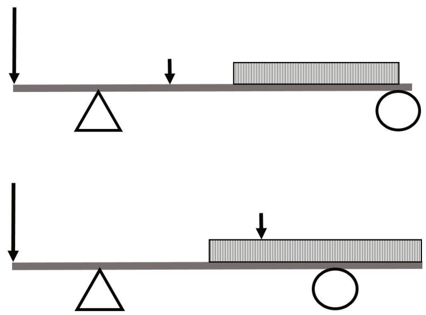
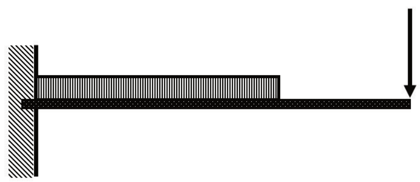
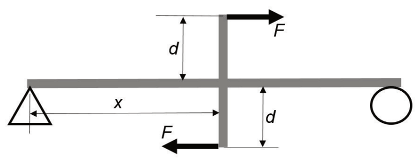
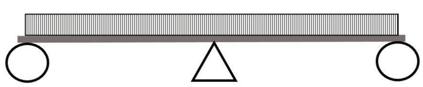
Support Type	***Looks like***	***Symbol***	***Reactions***
Roller, also called • simple • movable • sliding			 • Vertical reaction only • Allows horizontal movement • Allows rotation
Pinned, also called • hinged			 • Vertical reaction • Horizontal reaction • Allows rotation
Fixed			 • Vertical reaction • Horizontal reaction • Moment reaction

Beam Loads[1]:

Loads	***Symbol***	***Examples***	***Covered***
Point, also called • concentrated		• vehicle wheels • columns • person on diving board	Yes
Uniform Distributed		• beam weight • snow load on roof truss	Yes
Variable Distributed		• hydrostatic load on submerged surface • aggregate pile • beam of variable cross-section	Yes
Concentrated Moments		• machine components	No

Beam types:

1. Click on the diagrams to expand

Types	***Diagram***	***Examples***	***Covered***
Simple beams, or simply supported		• short span bridge	Yes
Overhanging beams		• patio support beams • diving board	Yes
Cantilever beams		• building entrance covers • overhanging street signs	Yes
Compound beams		• machine components	No
Continuous beams		• long span bridge • pipe supports	No

Solving for beam reactions

When solving for reactions, the following steps are recommended:

1. Draw the beam free body diagram
2. Replace the uniform distributed load (if any) with the equivalent point load
3. Solve $\Sigma M_A = 0$ (sum of moments about support A). This will give you R_B (reaction at support B).
4. Solve $\Sigma M_B = 0$. This will give you R_A.
5. Using R_A and R_B found at steps 3 and 4 check if $\Sigma V = 0$ (sum of all vertical forces) is satisfied.
    a. Note that steps 4 and 5 can be reversed.

b. For a cantilever beam use $\Sigma V = 0$ to find the vertical reaction at the wall and $\Sigma M_{wall} = 0$ to find the moment reaction at the wall. There is no other equation to validate your results.

Shear forces and bending moments diagrams

Please note:

"Shearing forces are internal forces developed in the material of a beam to balance externally applied forces in order to secure equilibrium of all parts of the beam.

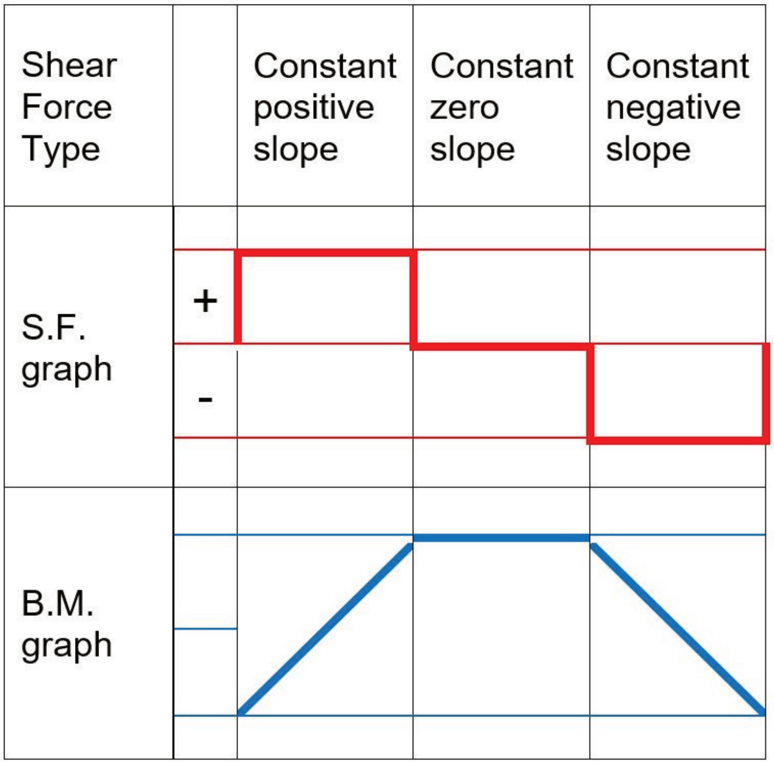
Bending moments are internal moments developed in the material of a beam to balance the tendency for external forces to cause rotation of any part of the beam." [3]

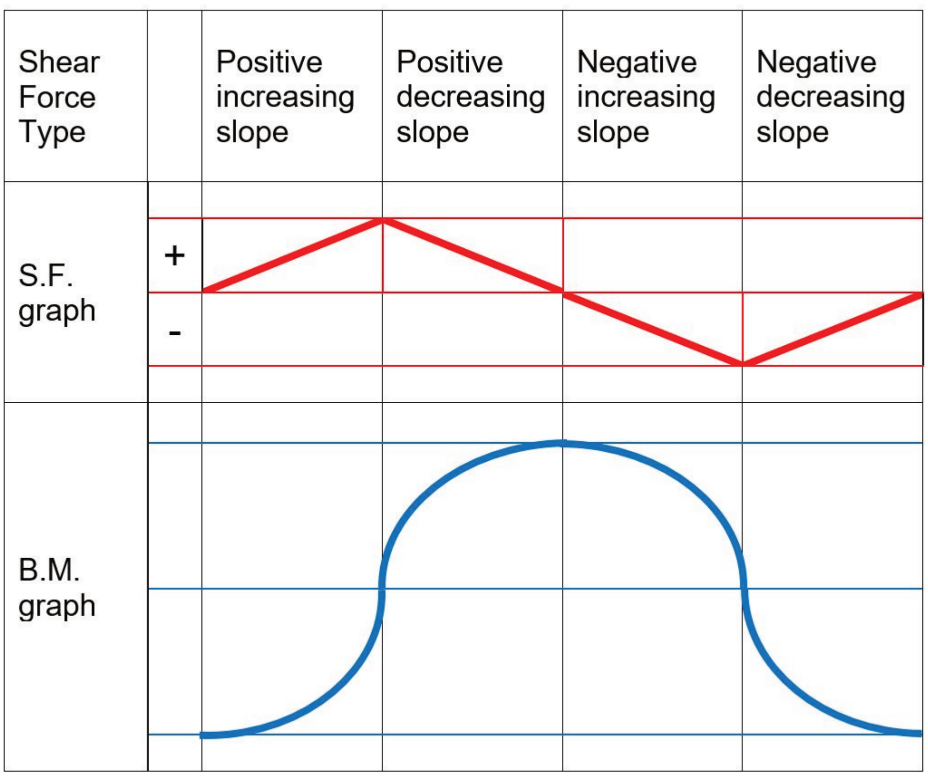
The shear force at any section of a beam may be found by summing all the vertical forces to the left **or** to the right of the section under consideration.

Similarly, the bending moment at any section of a beam may be found by adding the moments from the left **or** from the right of the section considered. The moment's pivot point is the location under consideration.

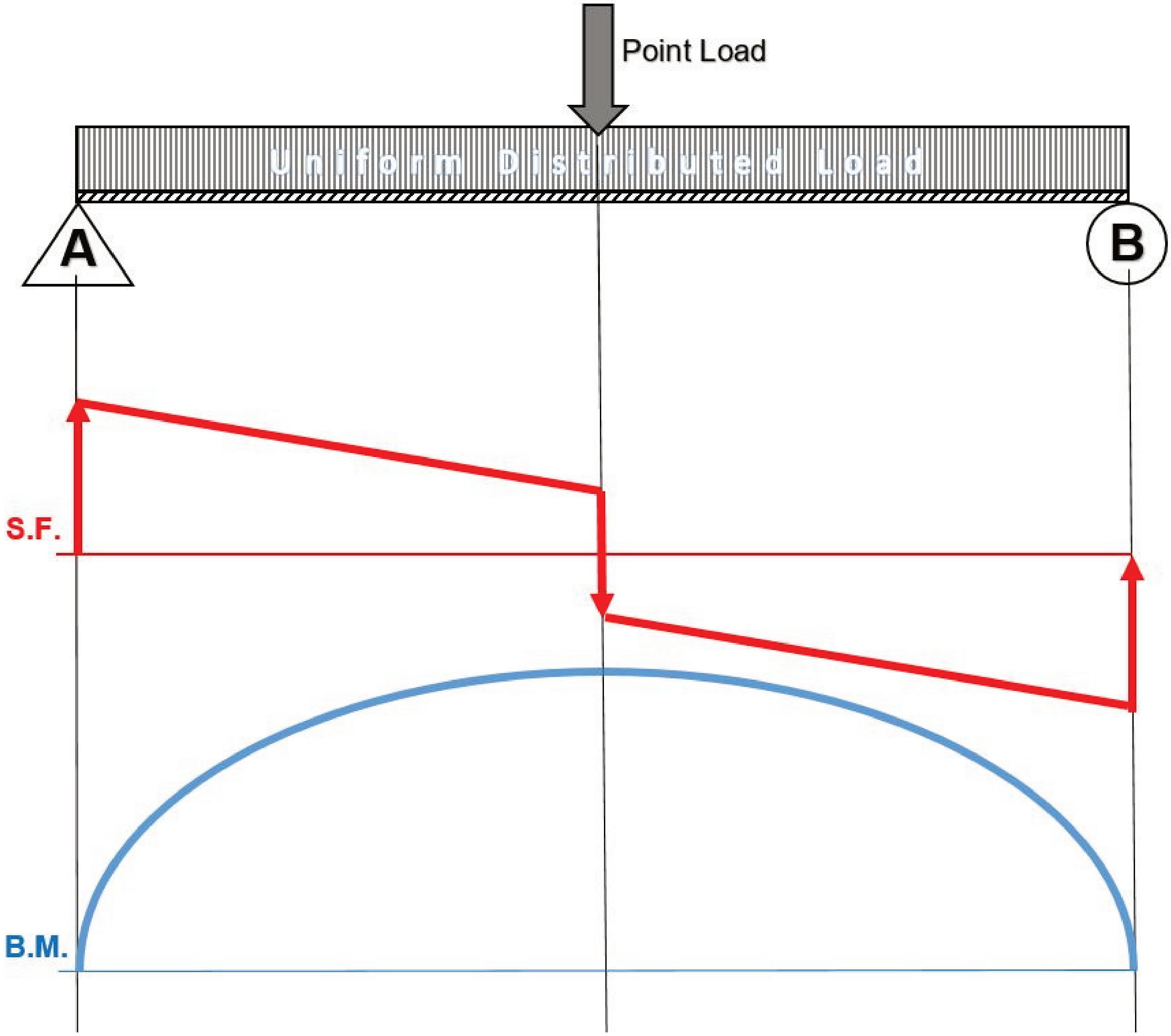
By convention, internal shearing forces acting downward are considered positive. They counteract upward external forces. Therefore, when representing the shear forces you can draw them in the direction of external forces. This is visually easier than following the sign convention.

Clockwise moments, conventionally, are considered negative while counter-clockwise moments are considered positive. When representing the bending moment variation, consult the following table showing qualitative bending moment curves dependent on the shape of the shear force graphs.

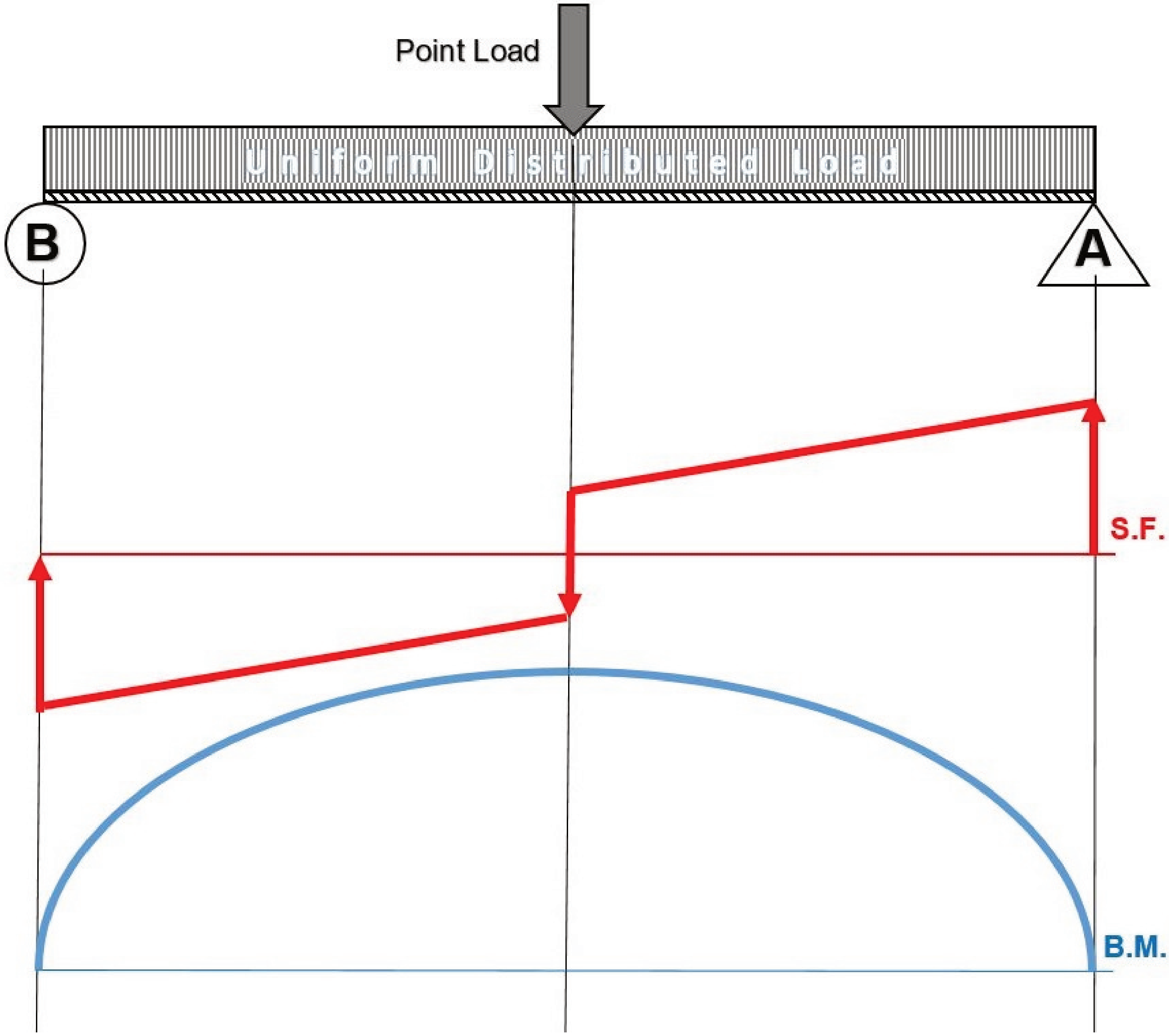




When drawing the shear force and bending moment diagrams, while the sign convention is important, consistency is crucial. For instance, consider a simple beam loaded with a point load applied on a UD load. Starting the diagrams at support A, looking towards the page, will generate the following:



Now, flip the beam horizontally 180º (or change the observation point, looking at the beam from the opposite side) and draw the diagrams, starting from the same point A. The diagrams will appear as follows:



Note that, while the shear force diagrams appeared to be mirrored images (flipped horizontally), the bending moment diagram is not affected. Additionally, the most important result of this analysis, illustrates that maximum shear force and bending moment magnitudes will always be the same.

Beam diagrams check points

When drawing the beam diagrams please observe the following:

Shear Forces Diagrams:

- At the ends of a simply supported beam the shear force is zero.
- At the wall of a cantilever beam the shear force equals the vertical reaction at the wall. At the beam's free end the shear force is zero.

- On any beam segment where no loads are applied, the shear force remains constant (horizontal line).
- A point load or reaction on a shear force diagram generates an abrupt change in the graph, in the direction of the applied load.
- A uniform distributed load acting on a beam is represented by a straight line shear force with a negative or positive slope, equal to the load per unit length.

Bending Moments Diagram:

- At the ends of a simply supported beam the bending moments are zero.
- At the wall of a cantilever beam, the bending moment equals the moment reaction. At the free end, the bending moment is zero.
- At the location where the shear force crosses the zero axis the corresponding bending moment has a maximum value.
- The shape of the bending moment curve between two points on the beam is as shown in the above two tables.
- ***The change in bending moment between two points on the beam equals the area under the shear force diagram between the same two points.***

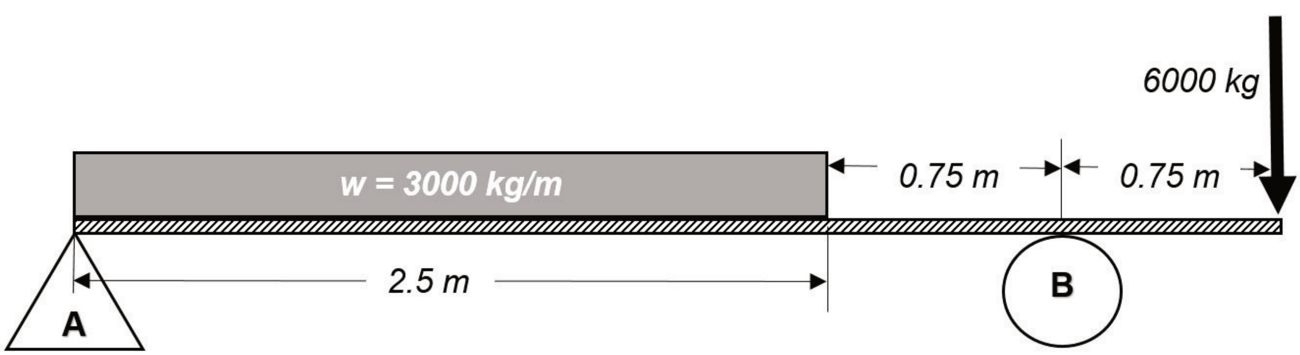
The above guidelines will assist you in generating the beam diagrams; they also serve as a check.

Assigned Problems

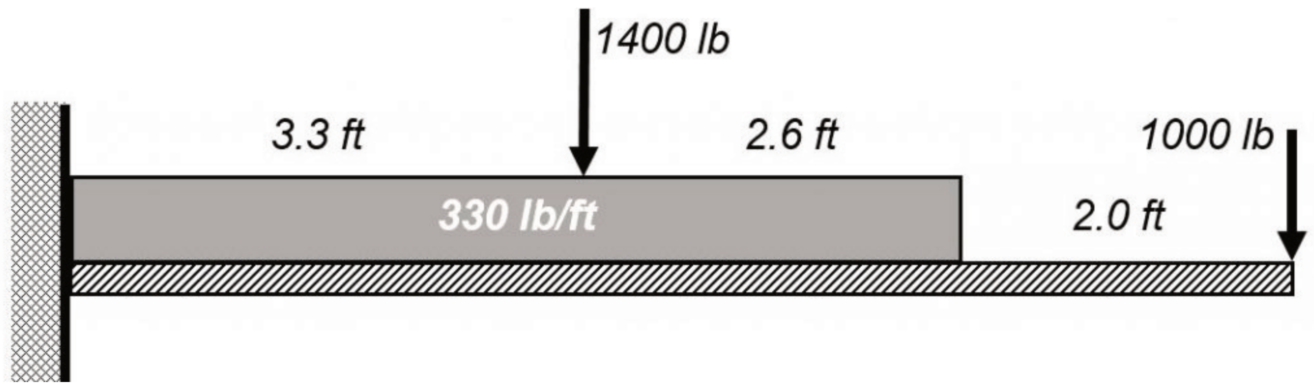
Calculate the beam reactions and draw the shear force and bending moment diagrams for the following beams.

When solving beam diagrams in class and at home you may check your answers by using this free online beam calculator: SkyCiv Cloud Engineering Software

Problem 1: State the maximum shear force and bending moment values.



Problem 2: State the maximum shear force and bending moment values.



Problem 3: A 24 meters long beam is simply supported at 3 meters from each end. The beam carries a point load of 18 kN at the left end and 22 kN at the right end of the beam. The beam weighs 400 kg/m. Sketch the beam diagrams and determine the location on the beam where the bending moment is zero.

Problem 4: A simple overhanging beam 112 ft long overhangs the left support by 14 ft. The beam carries a concentrated load of 90 kips 12 ft from the right end and a uniform distributed load of 12 kips/ft over a 40 ft section from the left end. Sketch the beam diagrams and determine the shear force and the bending moment at a section 50 ft from the left end.

Problem 5: Suggest an improvement to this chapter.

Beam Stress due to Bending Moments

Bending Stress

Learning Objectives

After completing this chapter you should be able to:

- Use the flexure formula to calculate maximum bending stress
- Design beams carrying loads safely
- Determine the required Section Modulus of a beam
- Select standard structural shapes to be used in a given beam problem

Consider a simply supported beam subjected to external downward loads. The beam will deform (deflect) in such a way that the top surface of the beam cross-section will be under compression while the bottom surface will be in tension. At some location along the vertical axis of the beam, the stress will be zero; this location is the centroid of the cross-section, also called the neutral axis.

Flexure formula

To determine the maximum stress due to bending the **flexure formula** is used:

$$\sigma_{max} = \frac{M \times c}{I_x} = \frac{M}{Z_x}$$

where:

- σ_{max} is the maximum stress at the farthest surface from the neutral axis (it can be top or bottom)
- M is the bending moment along the length of the beam where the stress is calculated
  - if the maximum bending stress is required then M is the maximum bending moment acting on the beam
- I_x is the moment of inertia about x (horizontal) centroidal axis
- c is the maximum distance from the centroidal axis to the extreme fiber (again, this can be to the top or bottom of the shape)
- Z_x is called **section modulus** and is a term that combines the moment of inertia and the distance to the extreme fiber ($\boldsymbol{Z_x = I_x / c}$)

The flexure formula is valid if the following criteria are met:

- the beam is straight, relatively long and narrow and of uniform cross-section
- all the loads act perpendicular to the longitudinal axis of the beam
- the resulting stress is below the limit of proportionality of the material
- the beam material is homogeneous and has equal strength in tension and compression
  - if the material has different strengths in tension and compression (example cast iron or other anisotropic materials) then separate calculations are required for both tension and compression surfaces
- no twisting, buckling or crippling occurs

Design cases

Design problems may follow different scenarios:

- calculate the beam cross-sectional dimensions (find the minimum section modulus Z and choose a standard shape of greater stiffness), given the beam geometry, loading and material.
- select the beam material (find maximum working stress and choose a material of greater strength), given the beam dimensions, loading and dimensions/shape.

- determine if a beam is safe (find actual working stress and compare to design stress), given the beam dimensions, loading and material.

Assigned Problems

Note: if not specified, use $\sigma_{design} = 0.6\times\sigma_{YS,}$ where σ_{YS} is the Yield Strength, from textbook Appendix B.

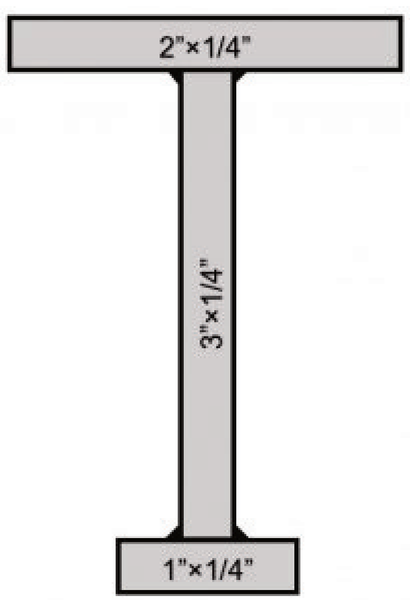
Problem 1: A simply supported beam, 9.9 meters long, is loaded with concentrated loads as follows:

- 40 kN a@ 1.2 m from left end
- 10 kN @ 3.7 m from left end
- 10 kN @ 6.2 m from left end
- 10 kN @ 8.7 m from left end

The beam is constructed using W200×100 I-beam profile from AISI-1020 cold rolled material. **AISC** recommends that the maximum bending stress for building-like structures under static loads be kept below $0.66\times S_y$. Does this construction meet the design requirements?

Problem 2: A pipeline is simply supported above ground on horizontal beams, 4.5 m long. Each beam carries the weight of 20 m Sch 40 DN-600 pipe (see PanGlobal Academic Extract), filled with oil of 0.9 **SG**. Assuming that the load acts at the center of the beam, calculate the required section modulus of the beam to limit the bending stress to 140 MPa; then select the lightest SI W-beam that satisfies the criteria.

Problem 3: The figure shows the cross-section of a beam built from aluminum 6061-T6. The beam is used as a 45 in. long cantilever. Compute the the maximum allowable uniformly distributed load it could carry while limiting the stress due to bending to one-fifth of the ultimate strength.



Problem 4: Design a walkway to span a newly installed pipeline in your plant. Rigid supports are available on each side of the pipeline, 14 ft apart. The walkway has to be 3.5 ft wide and be able to support a uniformly distributed load of 60 lb/ft^2 over its entire surface. Design only the deck boards and the side beams. Use any timber sizes and material grades from textbook Appendix E or others of your own design.

Problem 5: Suggest one beam design problem that you would consider relevant and useful for Power Engineers.

Beam Deflection

Deflection

Learning Objectives

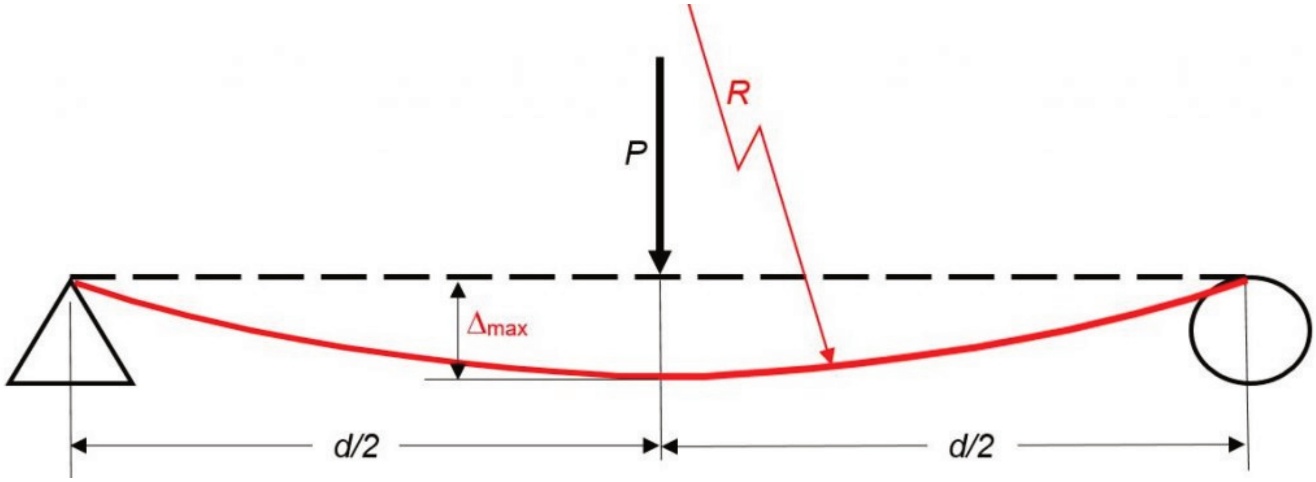
Upon completion of this chapter you should be able to calculate:

- The radius of curvature of a deflected beam using theoretical relations
- The maximum deflection of a simply supported beam
- The maximum deflection of various beams using Formula Method and textbook Appendices

Elastic properties of materials are quantified through their Modulus of Elasticity. All materials are elastic to some extent, for example $E_{steel} \approx 210$ GPa, $E_{cast\ iron} \approx 160$ GPa, $E_{aluminum} \approx 70$ GPa, $E_{concrete} \approx 40$ GPa. In real situations beams subjected to external loads will deflect proportionally to the bending moment and inversely to their stiffness. The overall stiffness of a beam can be expressed as $\boldsymbol{E \times I_c}$ where E can be regarded as the material stiffness and I_c as the cross-sectional, or geometrical stiffness.

Radius of curvature

Review the derivation of the beam deflection covered in detail in Textbook Chapter 10. In practical situations, beam deformation is very small when compared to its length, and as a result the radius of curvature is relatively large.



This radius of curvature can be calculated with

$$R = \frac{E \times I_c}{M}$$

where:

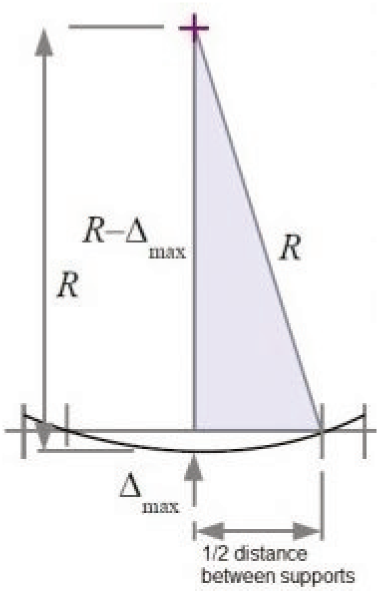
- E is the modulus of elasticity (resistance due to material properties)
- I_c is the moment of inertia about the centroidal axis (resistance due to section geometry)
- M is the bending moment at the section of interest

If the beam is loaded in such a way that the bending moment is constant over a section of the beam (horizontal line in the BM diagram) then the deflection is a circular arc and the radius of curvature is constant.

Take a moment and analyze the above formula… increasing the beam stiffness ($E \times I_c$) will reduce the deflection (large R), while a greater bending moment leads to a smaller radius of curvature (greater deflection/sagging).

Beam deflection

Consider a simply supported beam as in the above diagram. Once the radius of curvature is found, the maximum deflection (at mid span) can easily be geometrically calculated as follows:



$$R - \Delta_{max} = \sqrt{R^2 - (0.5d)^2}$$

$$\Delta_{max} = R - (R - \Delta_{max})$$

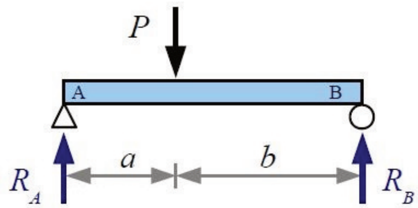
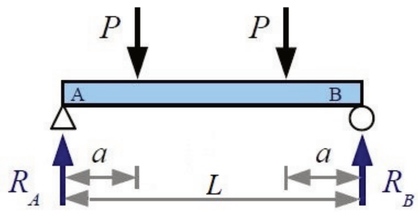
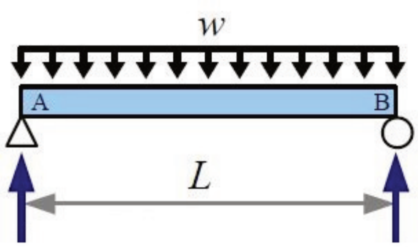
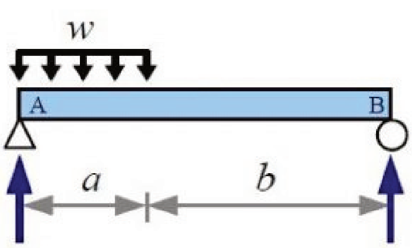
Formula method for simple cases

The Radius of Curvature formula is valid solely for cases where the bending moment is constant. For other cases, geometrical or integration based techniques are involved in determining the beam deflection. Results of these calculations presented in algebraic form are given in engineering handbook of formulas. Most common cases are summarized in textbook Appendix F.

When using “off-the-shelf” formulas, you must first match the beam geometry and loading to one of the given cases. If you are dealing with a more complex loading, such as point loads over-imposed on a distributed load, you can analyze the two loads separately and for the total deflection simply add the constituents.

Assigned Problems

For each problem determine the maximum deflection using the beam equations and compare with the value found using the radius of curvature.

#	Case:	Loading & Dimensions	Shape & Material
Problem 1		• P = 50 kN • a = 2 m; b = 3.5 m	• W 200×59 • AISI 1040, cold rolled
Problem 2		• P = 5000 lb. • a = 2 ft. • L = 10 ft.	• Pipe 6″ Sch. 40 • SS 304, cold rolled
Problem 3		• w = 250 lbs/ft • L = 35 ft.	• W 12×30 • Aluminum 6061-T6
Problem 4		• w = 4400 N/m • a = 4 m; b = 8 m	• Pipe DN 102, Sch 80 • AISI 1020, cold rolled

Problem 5: Recommend one improvement to this chapter.

Torsion in Round Shafts

Torsion

Learning Objectives

At the end of this chapter you should be able to complete torsion calculations using:

- General torsion equation
- Polar moment of inertia
- Modulus of elasticity in shear

Shafts are mechanical components, usually of circular cross-section, used to transmit power/torque through their rotational motion. In operation they are subjected to:

- torsional shear stresses within the cross-section of the shaft, with a maximum at the outer surface of the shaft
- bending stresses (for example a transmission gear shaft supported in bearings)
- vibrations due to critical speeds

This chapter will focus exclusively on evaluating shear stresses in a shaft.

General torsion equation

All torsion problems that you are expected to answer can be solved using the following formula:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G \times \theta}{L}$$

where:

- T = torque or twisting moment, [N×m, lb×in]
- J = polar moment of inertia or polar second moment of area about shaft axis, [m^4, in^4]
- τ = shear stress at outer fibre, [Pa, psi]
- r = radius of the shaft, [m, in]
- G = modulus of rigidity (PanGlobal and Reed's) or shear modulus (everybody else), [Pa, psi]
- θ = angle of twist, [rad]
- L = length of the shaft, [m, in]

The nomenclature above follows the same convention as PanGlobal Power Engineering Training System.

Most common torsion problems will indicate the transmitted power (kW) at a certain rotational speed (rad/s or RPM). The equivalent torque can be found with:

$$T\ [Nm] = \frac{P[W]}{n[rad\ /\ s]}$$

where $n[rad/s] = N[rev/min] \times 2\pi/60$.

Polar moment of inertia

Similar to the moments of inertia that you learned before in rotational kinetics and bending of beams, the polar moment of inertia represents a resistance to twisting deformation in the shaft. General formulas for polar moment of inertia are given in Textbook Appendix C.

Note the difference between bending moments of inertia I_c and polar moments of inertia J, and use them appropriately. For instance, if you are dealing with a circular bar:

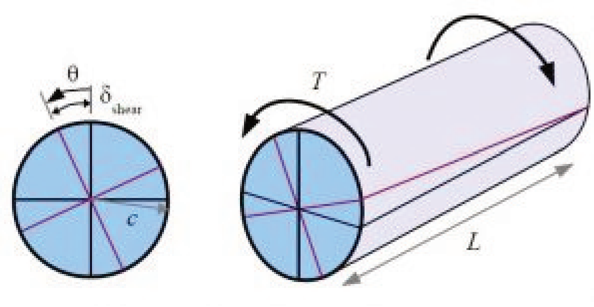
- $I_c = \pi\, d^4 / 64$, if the bar is used as a beam
- $J = \pi\, d^4 / 32$, if the bar is used as a shaft

Shear modulus

Called Modulus of Rigidity in PanGlobal and Reed's, the shear modulus is defined (similarly as E) as ratio of shear stress to the shear strain. It is expressed in GPa or psi and typical values are given in Textbook Appendix B. Typical values are lower than Young's Modulus E, for instance ASTM A36 steel has $E_{A36} = 207\ GPa$ and $G_{A36} = 83\ GPa$.

Angle of twist

The torque deformation of a shaft due is measured by the twist angle at the end of the shaft. This angle of twist depends on the length of the shaft, as shown in the following figure:



[1]

by Barry Dupen

The angle of twist, [radians] is used in the general torsion equation and in estimating the shear strain, γ (gamma), non-dimensional.

$$\gamma = \frac{r \times \theta}{L}$$

Assigned Problems [2]

Solve the following problems using the General Torsion Equation.

Problem 1: To improve an engine transmission, a solid shaft will be replaced with a hollow shaft of better quality steel resulting in an increase in the allowable stress of 24%. In order to keep the existing bearings, the new shaft will have the same outside diameter as the existing, solid shaft. Determine:

(a) the bore diameter of the hollow shaft in terms of outside diameter

(b) the weight savings in percentage, assuming that the steel densities of both shafts are identical

Problem 2: A turbine – generator transmission is rated for 3500 kW at 160 RPM. The shafts, 180 mm diameter and 2 m long, are connected through a flanged coupling with 6 coupling bolts of 40 mm diameter arranged on a pitch circle of 340 mm. If the shaft shear modulus is 85 GPa determine:

1. Textbook figure, page 58

(a) the maximum shear stress in the shaft

(b) the shear stress in the bolts

Problem 3: Two identical hollow shafts are connected by a flanged coupling. The outside diameter of the shafts is 240 mm and the coupling has 6 bolts of 72 mm each on a bolt circle of 480 mm. Determine the inside diameter of the hollow shafts, which results in the same shear stress in both, shafts and bolts.

Problem 4: A brass liner, 24 mm thick, is shrunk over a solid shaft of 220 mm diameter. Taking G_{steel} = 85 GPa and G_{brass} = 37 GPa, determine the maximum shear stress in the shaft and liner if the transmitted torque is 240 kN×m. Also determine the angle of twist if the shaft length is 3.4 m.

Problem 5: Suggest one improvement to this chapter.

Bolted and Welded Joints

Joints

Learning Objectives

At the end of this section you will be able to

- calculate the allowable load of bolted lap joints
- calculate the allowable load of welded joints

Bolted joints

Two end plates bolted together and subject to symmetrical tensile loads react to the applied forces through the shear resistance of the bolts and the friction force developed between the plates. The friction force is difficult to evaluate since it depends on the relative roughness of the contact surface and may also be affected by environmental changes (bolts thermal expansion reduces the friction force). As a result, conservative load calculations rely only on the shear resistance of the bolts (or rivets); the extra joint capacity due to friction increases the safety factor.

There are various scenarios that may lead to bolted joints failure, all described in the textbook. When completing these calculations please note the following:

- nomenclature is listed in the textbook on page 6.
- material properties are taken from Textbook Appendices B3 and B4

The following is a summary of the required calculations. The lowest value represents the maximum allowable load.

Shear Failure of the bolts $\boldsymbol{P_s = n \times A_B \times \tau_{all} \times N}$

- τ_{all}, allowable bolt shear stress depends on the the shear location; this can be along the threaded section or the smooth section of the bolt.

Bearing Failure of the Plates $\boldsymbol{P_P = d \times t \times \sigma_{-all} \times N}$

- σ_{-all}, allowable bearing stress is 1.5 times the ultimate tensile strength of the plate material.

Gross Tensile Failure of the Plates $\boldsymbol{P_G = b \times t \times \sigma_{G\text{-}all}}$

- $\sigma_{G\text{-}all}$, allowable gross tensile stress of the plate is 60% of the yield strength of the material

Net Tensile Failure of the Plates $\boldsymbol{P_N = (b \times t - N_F \times d_H \times t) \times \sigma_{N\text{-}all}}$

- $\sigma_{N\text{-}all}$, allowable net tensile stress is half of the ultimate tensile strength of the plate material

Welded joints

Welded joints are often preferred to bolted joints because they are simpler, easier to complete, relatively stronger and can provide a sealed assembly. However, they cannot be dismantled for maintenance or replacing parts.

When completing weld calculations please note the following:

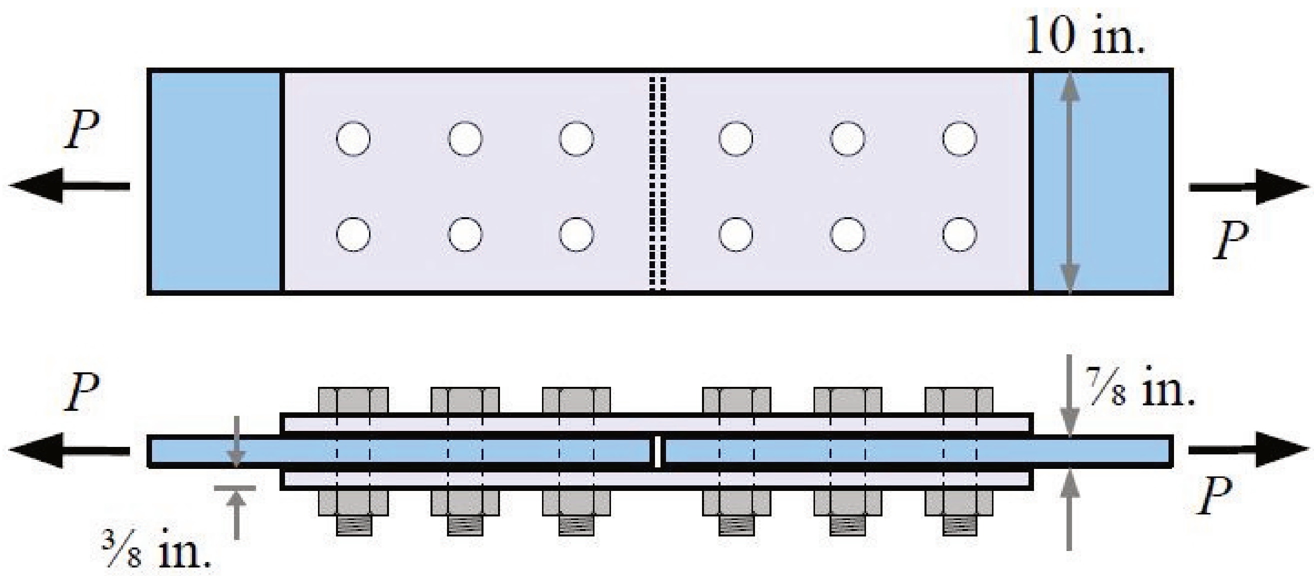
- nomenclature is listed in the textbook on page 6.
- use Appendix B6 for common weld and plate size; use Appendix B5 for weld strength of common electrodes

Weld Strength $\boldsymbol{P_{weld} = L \times f_{weld}}$

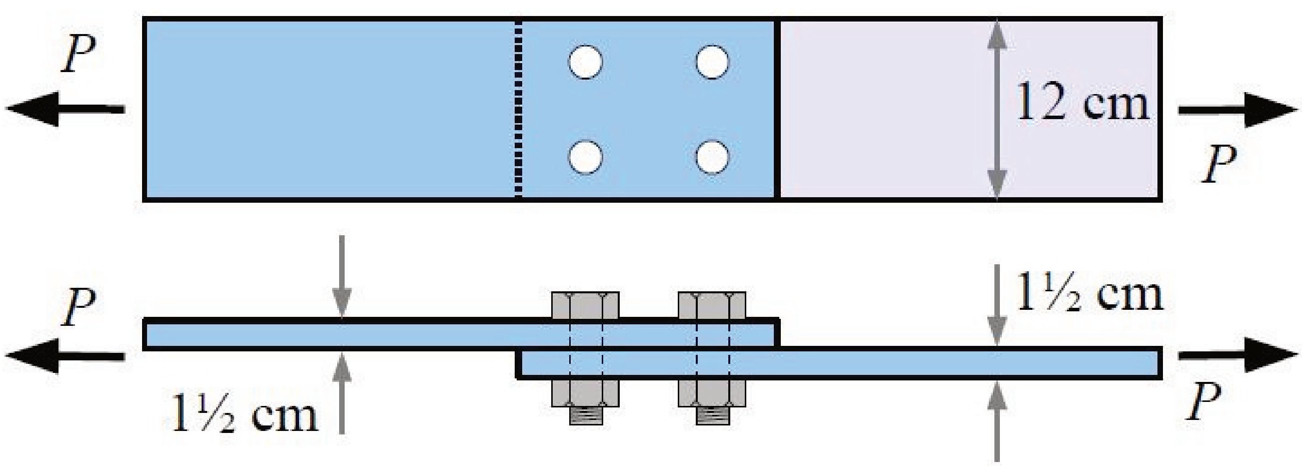
Gross Tensile Strength of the Plates – same as for bolted joints

Assigned Problems [1]

Problem 1: Two A992 steel plates are joined with two A992 steel splice plates and twelve 1 in. diameter A490 steel bolts with threads in the shear planes. Calculate the maximum load that the joint can support, in kips.

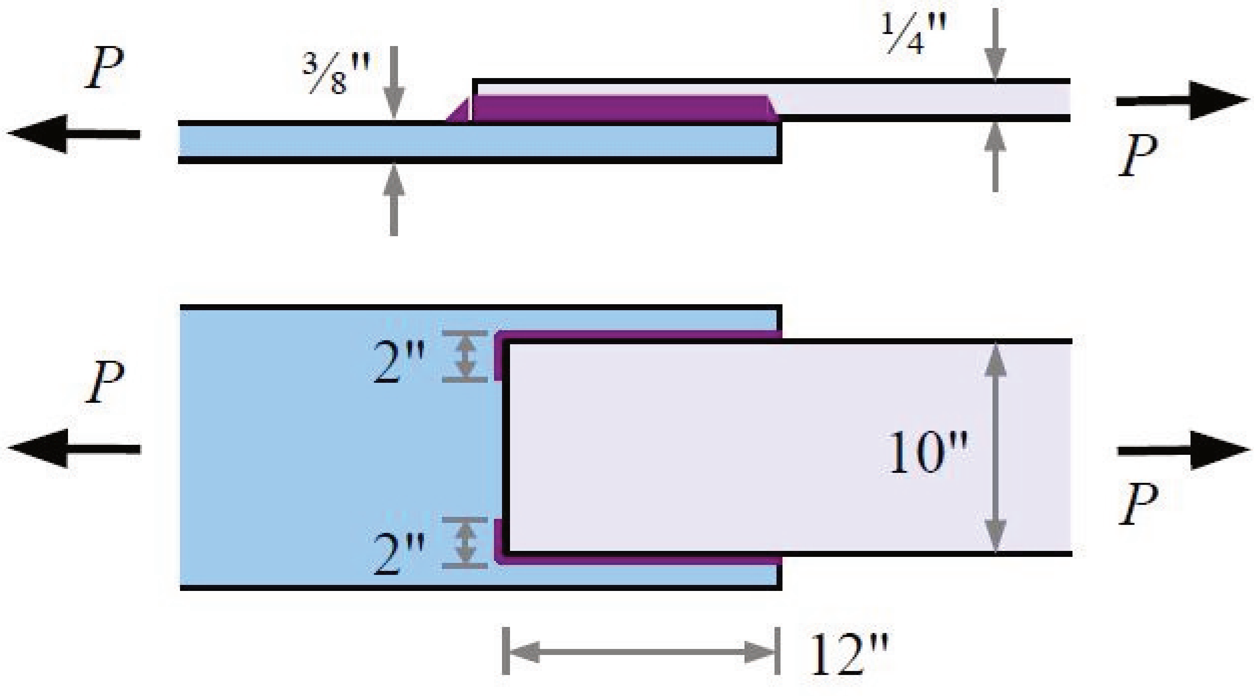


Problem 2: Two A36 steel plates form a lap joint with four 20 mm diameter A307 steel bolts. Calculate the maximum load that the joint can support, in kips.

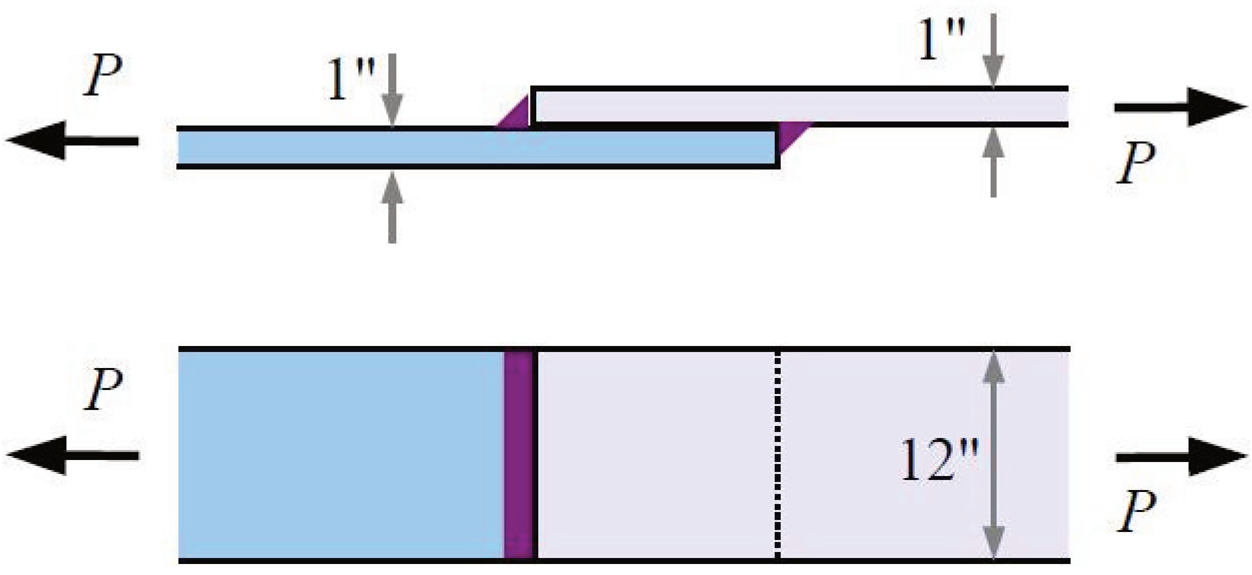


Problem 3: Two A36 steel plates are welded with an E70 electrode.

- What is the minimum recommended weld size for this joint? [in.]
- What is the joint strength? [kips]



Problem 4: Two A36 steel plates are welded as shown with a 3/8 in. fillet weld using an E60 electrode. What is the joint strength?



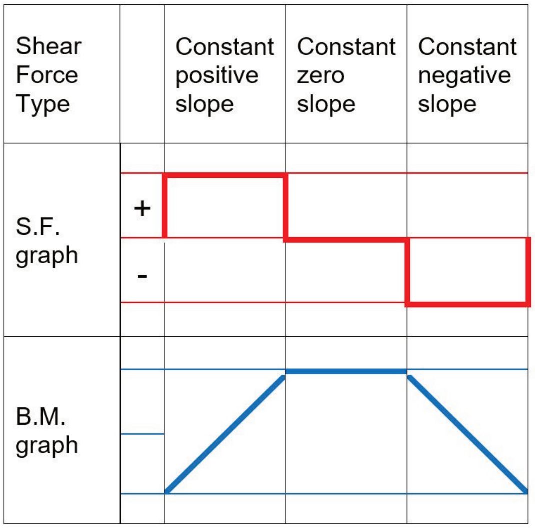
Problem 5: Suggest on improvement to this chapter.

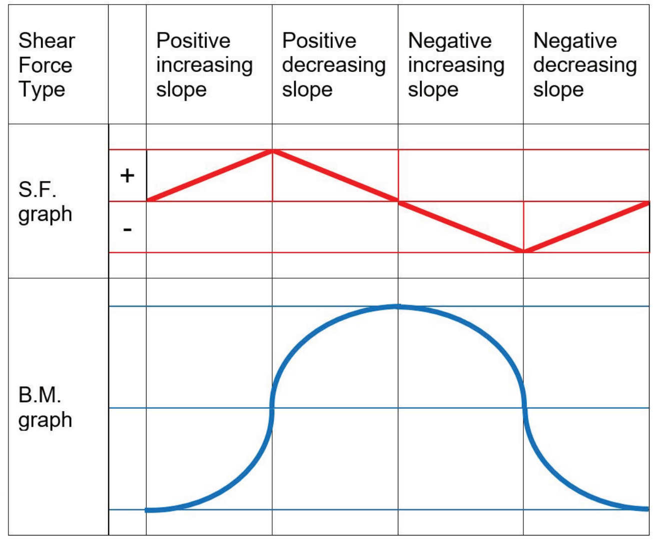
Appendices

Appendix 1: Table for calculating centroid location and moments of inertia

Part	A_i	y_i	$A_i \times y_i$	I_i	d_i	$A_i \times d_i^2$	$I_i + A_i \times d_i^2$
1							
2							
3							
4							
5							
6							
7							
	Draw a simple figure and identify the parts numbers here.						
ΣA_i:		$\Sigma A_i \times y_i$:					
INDICATE THE REFERENCE AXIS USED IN CALCULATION HERE: ______________________							
				Moment of Inertia of Areas I_x =			
Dist. to Centroid from Ref. Axis Y =							

Appendix 2: Beam Diagrams





Appendix 3: Online engineering calculators[1]

- SkyCiv Cloud Engineering Software
    - Beam calculator
    - Moment of Inertia Calculator
    - Steel Beam Sizes
- Advanced Mechanical Engineering Solutions
- Calculator Edge

1. We DO NOT endorse these websites, use them to check your calculations

Answers to Chapter Questions

Units

Problem 1: σ_a = 200 MPa; σ_b = 31 ksi

Problem 2: $W_a = 46\times10^3$ N; $W_b = 10.9\times10^3$ lb

Stress and Strain

Problem 1: Stress = 10.1 MPa; Strain = 0.05×10^{-3} *(hint, pipe full)*

Problem 2: Load = 5655 lb.

Problem 3: It is safe *(hint, σ = 115 MPa)*

Problem 4: S_y = 44 ksi, any material with S_y > 44 ksi will be adequate

Problem 5: σ_{rod} = 127 ksi, $\sigma_{cylinder}$ = 25 ksi, $\varepsilon_{rod} = 4.2\times10^{-3}$, $\varepsilon_{cylinder} = 8.9\times10^{-4}$

Thermal Expansion

Problem 1: 6.34 cm below tank top

Problem 2: 468 MPa; 0.00234; 919 kN

Problem 3: 20 m; 186.3 MPa, using -54ºC in the winter, $\alpha = 11.7\times10^{-6}$ /ºC

Problem 4: 21.45 ksi; 180.2 kips, assuming Carbon Steel $E = 30\times10^6$ psi, $\alpha = 6.5\times10^{-6}$ /ºC

Problem 5: σ_{steel} = 1727 psi; σ_{copper} = 7895 psi

Pressure Vessels

Problem 2: DN500 Schedule 10 pipe

Problem 3: MAWP = 690 psi

Problem 4: Does not meet specifications

Properties of Areas

Problem 1: 7132.9 cm^4; 1761.9 cm^4

Problem 2: 112.85 in^4; 30.7 in^4

Problem 3: 44.32 cm^4

Beam Reactions and Diagrams

Problem 1: SF_{max} = 58.8 kN; BM_{max} = 44.14 kN×m

Problem 2: SF_{max} = 4347 lbs; BM_{max} = 18263 lbs×ft

Problem 3: SF_{max} = 35.98 kN; BM_{max} = 83.6 kN×m

Problem 4: SF_{max} = 293.6 kips ; BM_{max} = 2420.6 kips×ft

Beam Stress due to Bending Moments

Problem 1: 72.3 MPa, safe

Problem 2: $S_x = 766.9\times10^3$ mm^3, W410×46.1

Problem 3: 128.6 lb/ft

Problem 4: Various solutions, example: Side beams Hemlock for S_{min} = 126 in^3 ? choose 4″×16″; deck boards 2″×12″ σ_{req} = 497.6 psi ? choose Eastern White Pine

Beam Deflection

Problem 1: 1.9 cm

Problem 2: 0.26″

Problem 3: 3.54″

Problem 4: depends on location, 37 cm @ 5.27 m from left end

Torsion in Round Shafts

Problem 1: 0.66×OD, 66% weight savings

Problem 2: 182.4 MPa, 162.97 MPa

Problem 3: 224.6 mm

Problem 4: 75.38 MPa, 39.95 MPa

Bolted and Welded Joints

Problem 1: TBD

Problem 2: TBD

Problem 3: TBD

Problem 4: TBD

Authors

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Dr. Dupen is an Associate Professor of Mechanical Engineering Technology at Indiana University – Purdue University Fort Wayne (IPFW). He has nine years' experience as a metallurgist, materials engineer, and materials laboratory manager in the automotive industry. His primary interests lie in materials engineering, mechanics, and engineering technology education. He is also an experienced contra dance caller.

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Alex Podut is a Power Engineering Instructor at the British Columbia Institute of Technology (BCIT), School of Energy. He teaches physics, applied mechanics, and strength of materials; he also collaborates on computer technology courses. As one of the members of the Distance Education group, he provides guidance and support to adult learners enrolled in Power Engineering Department's correspondence courses.

Prior to joining BCIT in 2005, Alex worked as a mechanical engineer for companies such as NORAM Engineering and Constructors and Wellons Canada. He specialized in mechanical equipment, heat exchanger design, equipment specifications, ASME and API vessels calculations and project management.

He holds a Master of Applied Science Degree awarded in 2002 by the University of British Columbia, Mechanical Engineering Department and an European Master of Science Degree in Energy Management and Mechanical Engineering awarded in 1994 by a consortium of European Universities and UNESCO's Division of Engineering and Technology.

Alex enjoys all that Pacific Coast offers, from skiing and camping to boating and motorcycling.

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3. Mott, Robert L., *Applied Strength of Materials Fifth Edition (*Pearson Education Inc., 2008), 258.

Revisions History

Revisions

Revision #	Date	Completed
1	March 2018	• First draft complete • Partially delivered the course
2	June 2018	• Completed course delivery • Editorial corrections done

Outstanding items:

- Edit equations using LaTex editor
- Create more image files to replace the copyrighted images
- Inspect missing footnotes